UNIT 8

Chords and Arcs of a Circle

Students' Learning Outcomes

After completing this unit, the students will be able to:

- Solve problems by using the properties of a circle:
  - One and only one circle can pass through three non- collinear points.
  - A straight line, drawn from the centre of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.
  - Perpendicular from the centre of a circle on a chord bisects it.
  - If two chords of a circle are congruent, then they will be equidistant from the centre.
  - Two chords of a circle which are equidistant from the centre are congruent.
  - If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
  - If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent.
  - Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres).
  - If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, then the chords are equal.
- Apply concepts of chords and arcs of a circle to real life world problems (such as decorative features, rainbow, bridges, roller coaster track).

INTRODUCTION

This unit explores the fascinating properties of circles and their practical applications in real-world contexts. Students will learn how to solve problems by understanding key geometric facts, such as the uniqueness of a circle passing through three non-collinear points and the relationships between chords, arcs and the centre of a circle. They will investigate how perpendiculars from the centre affect chords and how congruence in chords and arcs leads to equal distances and angles. These concepts are not only fundamental in geometry but also useful in analyzing and designing real-life structures and patterns like rainbows, bridges, roller coaster tracks and decorative elements. Through engaging activities and problem-solving tasks, students will deepen their understanding of circular geometry and its relevance in everyday life.

Circle and its Components

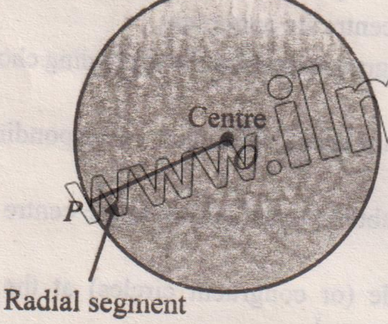
Circle A circle is the set of all points that are at the same distance from a fixed point.

We also define circle as:

A circle is the locus of a point in a plane which moves in such a way that its distance from a fixed point is always constant.

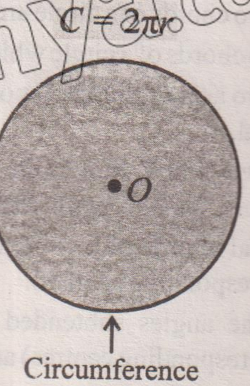
Radial segment

The line segment joining any point of the circle to its centre is called **radial segment** and its measure is called **radius** of the circle.



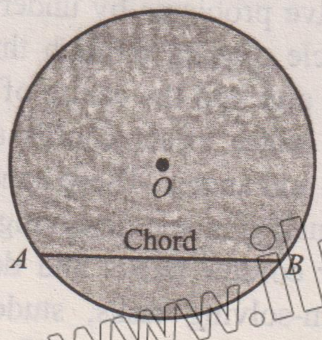
Circumference

The length of the boundary of a circle is called circumference.



Chord

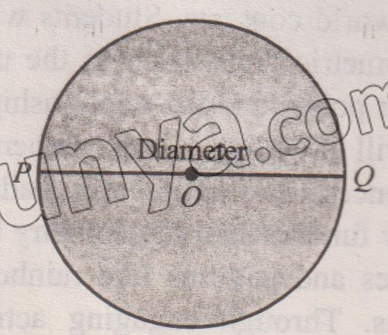
A line segment joining any two distinct points of a circle. In the below figure, $\overline{AB}$ is a chord of the circle.



Diameter

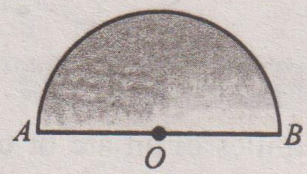
A chord passing through the centre of circle is called diameter.

$\overline{PQ}$ is diameter of the circle.



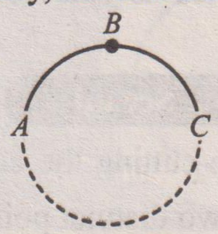
Semi-Circle

A semi-circle is half of the circle, that is a figure bounded by a diameter and circumference cut by the diameter.



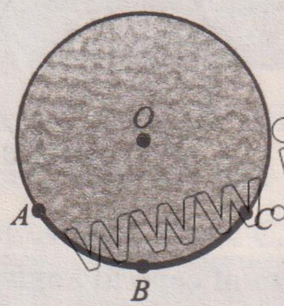
Arc

Any part of the circumference of a circle is called an arc. ABC is an arc of the circle. Similarly, AB and BC are also arcs.



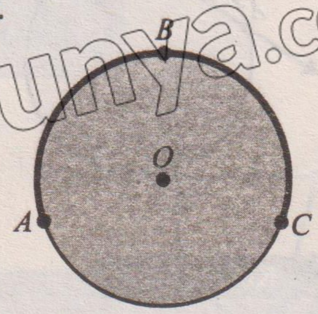
Minor arc

A minor arc is an arc which is smaller than the arc of a semi-circle. ABC is a minor arc.



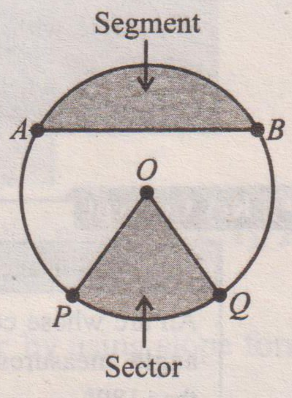
Major arc

A major arc is an arc which is greater than the arc of a semi-circle. ABC is a major arc.



Segment

The area (region) enclosed between a chord and the arc which it cuts off is called a segment of the circle.

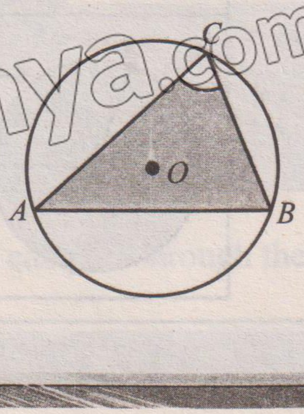


Sector

The area (region) enclosed between two radial segments (Radii) and an arc of a circle is called sector of the circle.

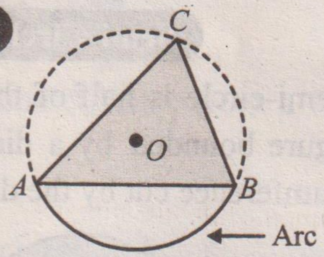
Angle in the Segment

If a point on the arc of a segment is joined with the end points of a chord, the angle formed by the line segments so drawn is called angle in the segment. $\angle ACB$ is an angle in the segment ACB. This is also called **circum angle.**



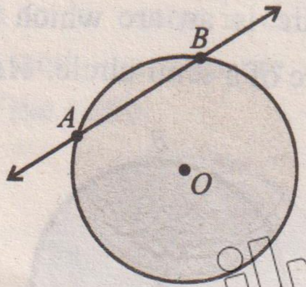
Angle Standing upon an Arc

If a point on the circumference of a circle is joined to the extremities of an arc, the angle so formed is said to angle stand upon the arc.



Secant

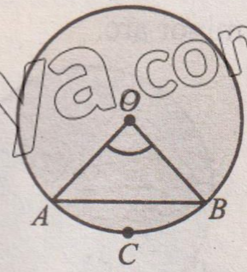
A straight line cutting the circumference of a circle at two distinct points is called a secant. $\overleftrightarrow{AB}$ is a secant.



Central Angle

If the extremities of an arc or of a chord are joined with the centre of a circle, angle so formed is called a **central angle.**

$\angle AOB$ is a central angle of chord AB or of arc ACB.

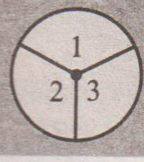


Key Concept

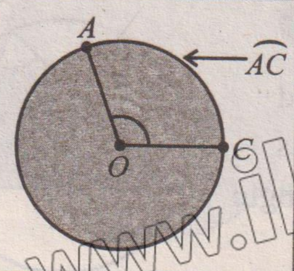
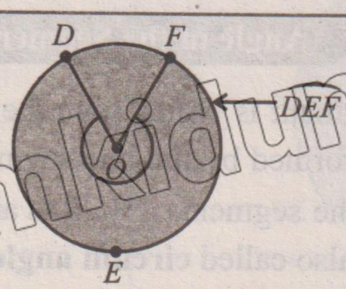
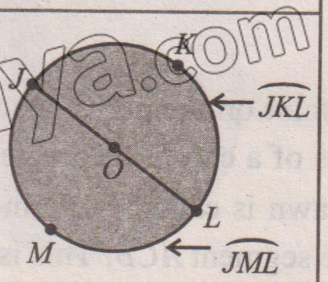
Sum of central angles

The sum of the measures of the central angles of a circle with no interior points in common is 360°.

i.e. $m\angle 1 + m\angle 2 + m\angle 3 = 360°$



Key Concept

Minor arc	Major arc	Semi-circle
An arc whose central angle measures less than 180°.	An arc whose central angle measures greater than 180°.	An arc whose central angle measures 180°.
		

Challenge!

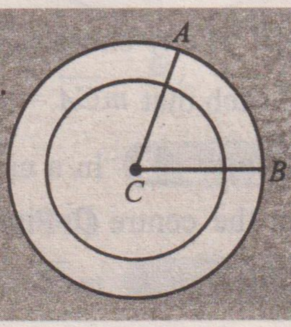
What is the difference between chord and secant?

Do you know?

(i) Circles with same centre are called concentric circles.

(ii) Any two circles are similar.

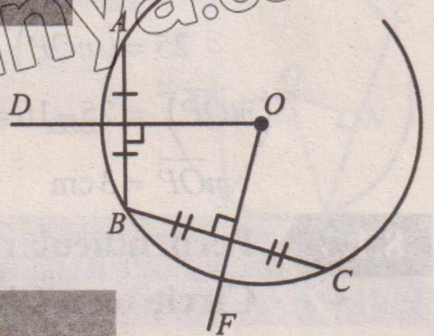
(iii) A cricle whose radius is zero is called point circle.



8.1 Theorems (Properties) of a Circle

8.1.1 One and Only One Circle Can Pass Through Any Three Non-Collinear Points.

If *A, B* and *C* are three points not in the same straight line, then one and only one circle can pass through *A, B* and *C*.



Note

- A circle cannot be drawn through more than two points in a straight line.
- If two circles have three points common, then they coincide.
- Two circles cannot intersect in more than two points.

Example 1 Prove that a unique circle can pass through the points $A(1, 1)$, $B(4, 2)$ and $C(3, 5)$.

Solution We will check the points are non-collinear by using slope formula.

$$\text{Slope of } \overline{AB} = \frac{2-1}{4-1} = \frac{1}{3}$$

$$\text{Slope of } \overline{BC} = \frac{5-2}{3-4} = \frac{3}{-1} = -3$$

$$\text{Slope of } \overline{AB} \neq \text{Slope of } \overline{BC}$$

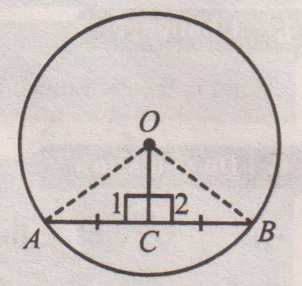
Remember!

If $A(x_1, y_1)$ and $B(x_2, y_2)$,

then Slope of $\overline{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{y_1 - y_2}{x_1 - x_2}$

Hence, the given points are non-collinear and unique circle can pass through them.

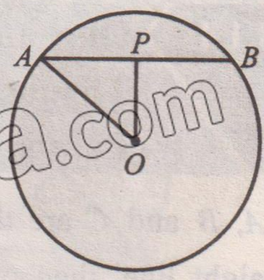
8.1.2 A Straight Line, Drawn from the Centre of a Circle to Bisect a Chord (which is not a diameter) is Perpendicular to the Chord.



If $\overline{AB}$ is any chord (not a diameter) and $\overline{OC}$ meets $\overline{AB}$ at C such that $m\overline{CA} = m\overline{CB}$, then $\overline{OC} \perp \overline{AB}$

Example 2 In a circle, $m\overline{AB} = 8$ cm is bisected at point P by a line segment from the centre O. Find $m\overline{OP}$ if radius is 5 cm.

Solution $m\overline{AB} = 8$ cm, $m\overline{OA} = 5$cm, $m\overline{AP} = 4$ cm



Since triangle APO is right angled triangle.
So, by Pythagoras theorem

$$\left(m\overline{OA}\right)^2 = \left(m\overline{OP}\right)^2 + \left(m\overline{AP}\right)^2$$

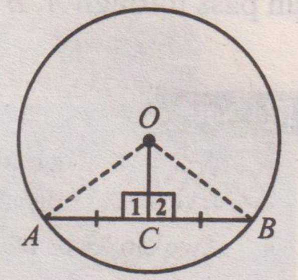
$$5^2 = \left(m\overline{OP}\right)^2 + 4^2$$

$$25 = \left(m\overline{OP}\right)^2 + 16$$

$$\left(m\overline{OP}\right)^2 = 25 - 16 = 9$$

$$m\overline{OP} = 3\text{ cm}$$

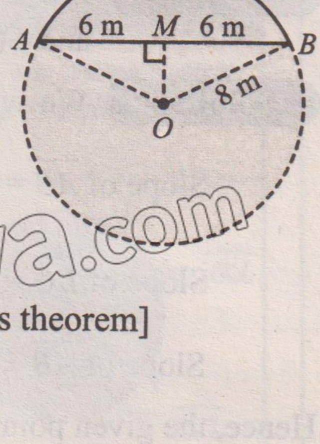
8.1.3 Perpendicular from the Centre of a Circle on a Chord Bisects it.



If $\overline{OC} \perp \overline{AB}$, then $\overline{OC}$ bisects $\overline{AB}$

i.e. $m\overline{CA} = m\overline{CB}$

Example 3 In a circular fountain of radius 8 m, a pipe AB spans across the fountain. If $m\overline{AB} = 12$ m and O is centre of the circle. Find the distance from O to the chord AB.



Solution Given that $m\overline{AB} = 12$ m

Distance from O to chord AB is perpendicular so by theorem, M is the mid-point.

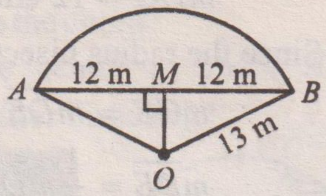
$$m\overline{OM} = \sqrt{(m\overline{OB})^2 - (m\overline{BM})^2} \quad \text{[By Pythagoras theorem]}$$

$$= \sqrt{(8)^2 - (6)^2}$$

$$= \sqrt{64 - 36} = \sqrt{28}$$

$$= 5.29\text{ m}$$

Example 4 Chord AB of 24 m is across a circular arch of radius 13 m. Find the height of the chord from the centre.



Solution $m\overline{AB} = 24$ m across a circular arch. O is centre of the circle of the arch that $m\overline{OB} = 13$ m.

$\overline{OM} \perp \overline{AB}$, therefore, $m\overline{MA} = m\overline{MB} = \dfrac{24}{2} = 12$ m

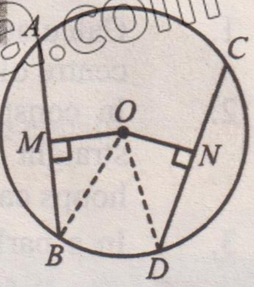
Now, $\left(m\overline{OM}\right)^2 = \left(m\overline{OB}\right)^2 - \left(m\overline{MB}\right)^2$ [By Pythagoras theorem]

$$\left(m\overline{OM}\right)^2 = (13)^2 - (12)^2$$

$$m\overline{OM} = \sqrt{(13)^2 - (12)^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = 5 \text{ m}$$

8.1.4 If Two Chords of a Circle Are Congruent, then they will be Equidistant from the Centre.

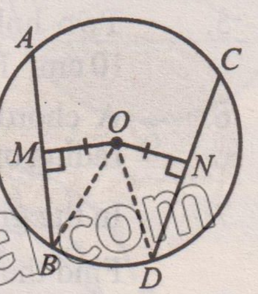


If $\overline{AB}$ and $\overline{CD}$ are congruent (equal in measures) chords of a circle with centre O. i.e.. $m\overline{AB} = m\overline{CD}$, $\overline{OM} \perp \overline{AB}$ and $\overline{ON} \perp \overline{CD}$, then $m\overline{OM} = m\overline{ON}$

Example 5 Two equal-length ropes (chords) are tied in a circular playground. One is 8 m from the centre. Where should the second be placed?

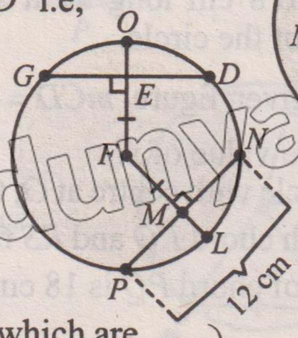
Solution Since chords are congruent, they must be equidistant from the centre. So, the second chord must be 8 m from the centre.

8.1.5 Two Chords of a Circle which are Equidistant from the Centre of a Circle, are Congruent.



If $\overline{AB}$ and $\overline{CD}$ are chords of a circle with centre O, $\overline{OM} \perp \overline{AB}$, $\overline{ON} \perp \overline{CD}$ and $m\overline{OM} = m\overline{ON}$, then $\overline{AB} \cong \overline{CD}$ i.e; $m\overline{AB} = m\overline{CD}$

Example 6 In a circle with centre at F, $m\overline{NP} = 12$ cm and $m\overline{EF} = m\overline{FM}$. Find the length of DE.



Solution As $m\overline{EF} = m\overline{FM}$

then $m\overline{DG} = m\overline{NP}$ (Two chord of a circle which are equidistant from the centre are congruent.)

$m\overline{DG} = 12 \text{ cm}$ $\quad\because\quad m\overline{NP} = 12 \text{ cm}$

Since the radius bisects $\overline{DG}$, so

$m\overline{GE} = m\overline{DE}$

$\Rightarrow \quad m\overline{DE} = \frac{1}{2} m\overline{DG} \quad \Rightarrow \quad \frac{1}{2}(12 \text{ cm}) = 6 \text{ cm}$

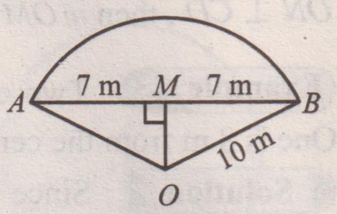
Example 7 In a round table design, two decorations are placed symmetrically 4 cm from the centre. One chord is 14 cm. What is the length of the other?

Solution Since both decorations are equidistant from the centre, therefore chords are congruent.

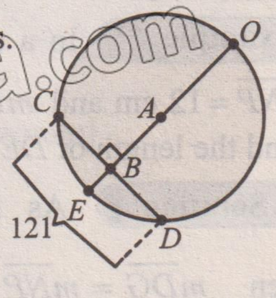
So, the length of second chord is 14 cm.

EXERCISE 8.1

1. Calculate the length of a chord which stands at a distance of 5 cm from the centre of a circle whose radius is 13 cm.
2. In construction, three steel rods are fixed at points A, B, and C (not in a straight line). A circular hoop needs to pass through all three. How many hoops can be used?
3. In a park, lamp posts are 14 m apart on the edge of a circular part of radius 10 m as shown in figure. Find the distance of the chord from the centre of the park.



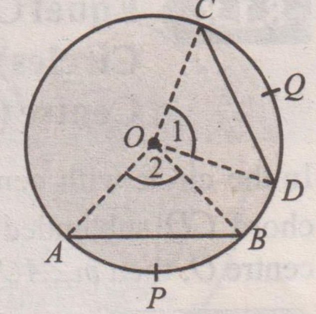
4. In a circle, chords AB and CD both have length 10 cm. If the distance from the centre to $\overline{AB}$ is 6 cm, what is the distance from centre to $\overline{CD}$?
5. Two holes A and B are drilled 12 cm apart on a circular tabletop of radius 10 cm. Find the perpendicular distance from the centre to AB.
6. A chord 8 cm long is at a distance of 3 cm from the centre. Calculate the radius of the circle.
7. In the given figure, $m\overline{CD} = 121$ units and $m\overline{BC} = 3x$ units. Find the value of x.
8. In a circle with centre at O, the perpendicular distance of the each chord PQ and RS from the centre is 6 cm. If the length of chord PQ is 18 cm, find the length of the other chord.



9. A line from the centre of a circle cuts a 10 cm chord at right angle where radius of the circle is 6cm. What is the length from the centre to the chord?

10. In a circle, a perpendicular is drawn from the centre to chord AB. If $m\overline{AB} = 12\text{cm}$, what is the length of each segment after bisecting?

8.1.6 If Two Arcs of a Circle (or of Congruent Circles) are Congruent, then the Corresponding Chords are Equal.



In the circle with centre O, $\widehat{APB} \cong \widehat{CQD}$ and $\overline{AB}$, $\overline{CD}$ are their corresponding chords, then $\overline{AB} \cong \overline{CD}$

Example 8 In a circular garden, two pair of decorative lights are placed such that the arcs between each pair (A and B, C and D) are equal in the length. If the straight-line distance between points A and B is 8 metres, what is the distance points C and D?

Solution Given that

$$m\overline{AB} = 8 \text{ metres}$$

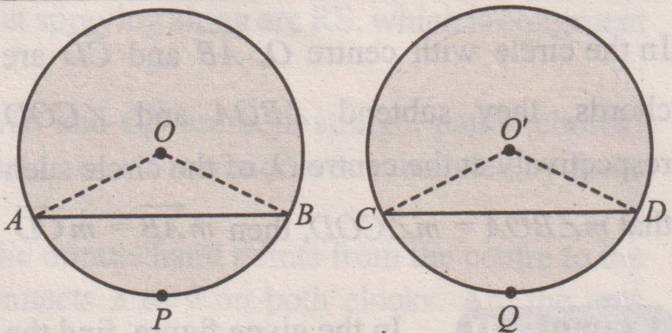
Since the arcs AB and CD are congruent, so their chords must be equal.

$$m\overline{AB} = m\overline{CD}$$

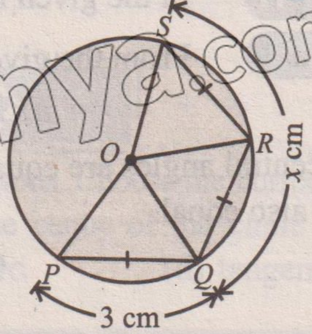
Hence, $m\overline{CD} = 8 \text{ m}$

8.1.7 If Two Chords of a Circle (or of Congruent Circles) are Congruent (Equal in Measurement) then their Corresponding Arcs (Minor, Major) are Congruent.

If circles with centre O and O' are congruent and Chord $AB \cong$ Chord CD, then $\widehat{APB} \cong \widehat{CQD}$



Example 9 Find the value of x in the given figure.



Solution Given that:

Chord $PQ \cong$ Chord $QR \cong$ Chord RS

As, we know that equal chords have equal arcs.

$\therefore$ Arc $PQ \cong$ Arc $QR \cong$ Arc RS

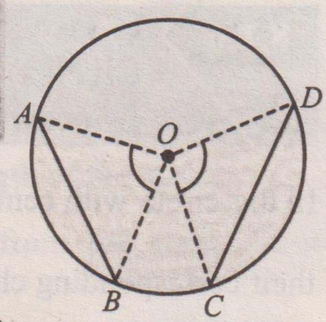
But Arc $PQ = 3$ cm

$\therefore$ Arc $QR = 3$ cm and Arc $RS = 3$ cm

From figure, we have

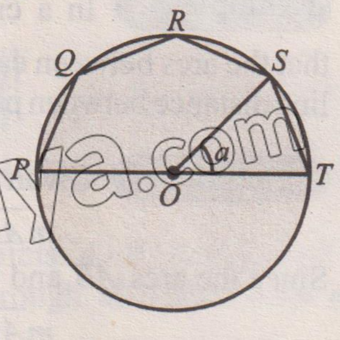
$$\text{Arc } QR + \text{Arc } RS = x$$
$$3\text{ cm} + 3\text{ cm} = x$$
$$x = 6\text{ cm}$$

8.1.8 Equal Chords of a Circle (or of Congruent Circles) Subtend Equal Angles at the Centre (at the corresponding Centres).



In the circle with centre O, $m\overline{AB} = m\overline{CD}$, chord AB and chord CD subtended $\angle AOB$ and $\angle COD$ respectively at the centre O, then $m\angle AOB = m\angle COD$

Example 10 In the given figure, $\overline{PT}$ is a diameter of circle. If $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$, then find a.



Solution $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$ (given)

$\therefore$ $m\angle POQ = m\angle QOR = m\angle ROS = m\angle SOT = a$

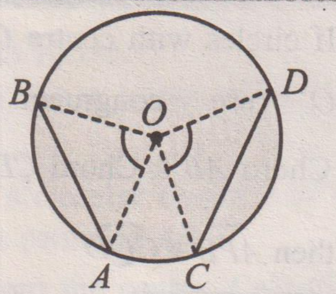
(equal chords have equal angles at the centre)

But

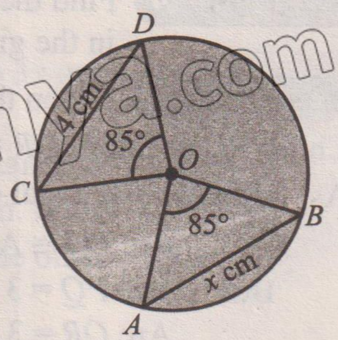
$$m\angle POQ + m\angle QOR + m\angle ROS + m\angle SOT = 180°$$
$$4a = 180°$$
$$a = 45°$$

8.1.9 If the Angles Subtended by Two Chords of a Circle (or Congruent Circles) at the Centre (Corresponding Centres) are Equal, then the Chords are Equal.

In the circle with centre O, $\overline{AB}$ and $\overline{CD}$ are its chords, they subtend $\angle BOA$ and $\angle COD$ respectively at the centre O, of the circle such that $m\angle BOA = m\angle COD$, then $m\overline{AB} = m\overline{CD}$



Example 11 In the given figure, find the value of x.



Solution From the given figure

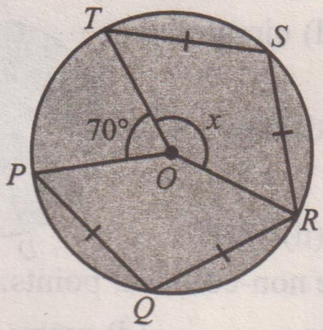
$$m\angle AOB = m\angle COD = 85°$$

$\because$ If central angles are equal, then corresponding chords are also equal.

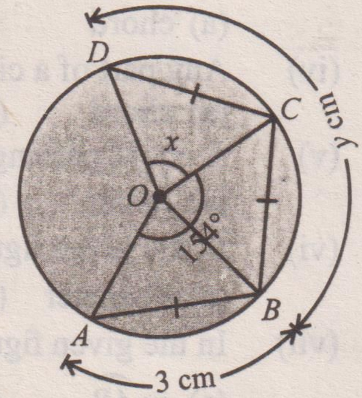
$$m\overline{AB} = m\overline{CD}$$
$$x = 4\text{ cm}$$

EXERCISE 8.2

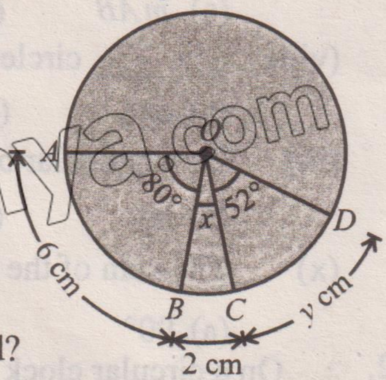
1. In the given figure, if $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$ and $m\angle POT = 70°$, then find the value of x.



2. In the given figure, find the values of x and y.



3. Find the values of x and y in the given figure, such that $m\widehat{AB} = m\widehat{BD}$



4. Two congruent arcs in a circular track subtend angles of 60° each at the centre. If the length of one chord of the circle is 10 metres, what is the length of other chord?
5. In a circular fountain, two water jets are installed such that they spray water along arcs of equal length. If one jet sprays between points P and Q and the straight-line distance (chord PQ) is 12 metres, what is the straight-line distance (chord RS) covered by the second jet spraying along arc RS, which is congruent to arc PQ?
6. In a circular park, two walkways $\overline{AB}$ and $\overline{CD}$ are both straight paths (chords) of length 14 metres. What can be said about the minor arcs subtended by these walkways?
7. In two congruent circular clocks, the minute hand points from the centre to the 3 on both. A decorative string connects 3 to 9 on both clocks. Are the arcs from 3 to 9 on both clocks congruent?

REVIEW EXERCISE 8

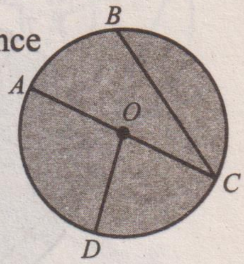
1. Four possible answers are given for the following questions. Choose the correct answer.
   (i) Distance of a point on the circumference to the centre of the circle is called:
   (a) radius (b) arc (c) chord (d) tangent

(ii) Radii of same circles are:
(a) all unequal (b) all equal
(c) half of each chord (d) double of the diameter

(iii) The boundary of the circle is called:
(a) chord (b) segment (c) circumference (d) diameter

(iv) Any part of a circumference is called:
(a) chord (b) diameter (c) radius (d) arc

(v) A chord passing through the centre of the circle is called:
(a) radius (b) diameter (c) secant (d) circumference



(vi) In the given figure, what is AB?
(a) diameter (b) tangent (c) chord (d) arc

(vii) In the given figure, major arc is:
(a) $m\widehat{AB}$ (b) $m\widehat{BC}$ (c) $m\widehat{BDC}$ (d) $m\widehat{AD}$

(viii) ________ circle(s) can pass through the three non-collinear points.
(a) one (b) two (c) three (d) many

(ix) Perpendicular bisector of a chord always passes through the _____ of circle.
(a) arc (b) radius (c) centre (d) circumference

(x) The sum of the measures of central angles of a circle is:
(a) 90° (b) 180° (c) 270° (d) 360°

2. On a circular clock with a radius of 6cm, the points from 2 to 10 form a chord that is 10cm long. Find the perpendicular distance from the centre of clock to the chord.

3. A chord 6 cm long is at a distance of 4 cm from the centre. Calculate the radius of the circle.

4. In a circular park, two benches are placed so that the chords formed by their positions are both 10 m long. What can you say about the angles subtended at the centre by each bench?

5. Jannat is designing a triangular garden with corners at points $A(2, 1)$, $B(4, 4)$ and $C(1, 5)$. Can she install a circular fountain that touches all three corners?

6. Two chords, PQ and RS, each measure 18cm in length. If the distance of PQ from the centre of circle is 7cm. Find the distance of RS from the centre.

7. A tree branch lies across a circular pond, forming a 20 m chord. A measuring rod having length 10 m is perpendicular to chord from the centre. What is the radius of the pond?

8. A steel bar 6 m long lies inside a circular structure with ends on the circle. It is bisected by a perpendicular rod from the centre. Find the radius of the circular structure if the perpendicular distance from the centre to the bar is 4 m.