UNIT 7

Trigonometry

Students' Learning Outcomes

After completing this unit, the students will be able to:

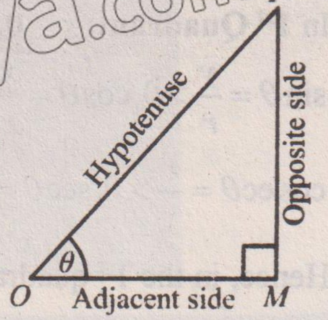
- Extend sine and cosine functions to angles between 90° and 180°.
- Solve problems using the laws of sine, cosine and the area formulas for any triangle.
- Solve simple trigonometric problems in three dimensions.
- Apply concepts of trigonometry to real life world problems (such as video games, flight engineering, navigation and sound waves).
- Interpret and use three figure bearings.
- Solve real life problems involving bearing.
- Apply the concepts of bearing to real world problems.

INTRODUCTION

This unit explores the extended applications of trigonometry beyond right-angled triangles by developing an understanding of sine and cosine functions for angles between 90° and 180°. Students will learn to apply the laws of sine and cosine, as well as area formulas, to solve problems involving any type of triangle. The unit further introduces the use of trigonometric problems in three-dimensional contexts and emphasizes concept of trigonometry in real-world scenarios such as navigation, flight engineering, video game design and sound wave analysis. Students will also gain proficiency in interpreting and using three-figure bearings and applying this knowledge to solve practical bearing-related problems, enhancing spatial reasoning and problem-solving skills.

7.1 Trigonometric Ratios or Functions

Consider an angle θ of right angled triangle POM, with right angle at M, where $\overline{OM}$ is called adjacent side, $\overline{PM}$ is called opposite side and $\overline{PO}$ is called hypotenuse. Then,



(i) sine θ ($\sin\theta$) = $\dfrac{\text{opp.}}{\text{hyp.}}$ (ii) cosine θ ($\cos\theta$) = $\dfrac{\text{adj.}}{\text{hyp.}}$

(iii) cosecant θ ($\text{cosec}\,\theta$) = $\dfrac{1}{\sin\theta} = \dfrac{\text{hyp.}}{\text{opp.}}$ (iv) secant θ ($\sec\theta$) = $\dfrac{1}{\cos\theta} = \dfrac{\text{hyp.}}{\text{adj.}}$

(v) tangent θ ($\tan\theta$) = $\dfrac{\text{opp.}}{\text{adj.}}$ (vi) cotangent θ ($\cot\theta$) = $\dfrac{1}{\tan\theta} = \dfrac{\text{adj.}}{\text{opp.}}$

The above six trigonometric ratios are also called trigonometric functions.

Remember!

$\sin\theta$ does not mean the product of sin and θ.
The $\sin\theta$ is correctly read as sin of angle θ.

Note

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

7.1.1 Trigonometric Values for Special Angles

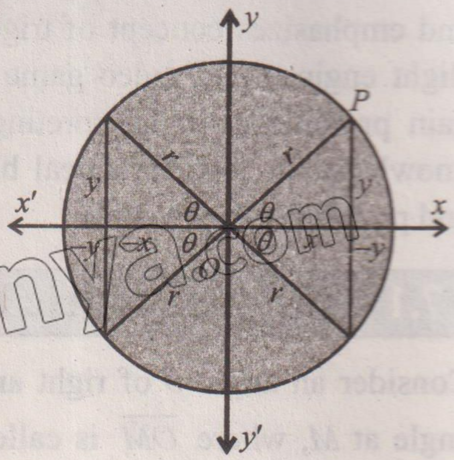
Trigonometric values for 0°, 30°, 45°, 60°, 90° and 180° are given in the following table:

Angle θ / Ratios	0°	30°	45°	60°	90°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined	0

7.1.2 Signs of Trigonometric Functions

The signs depend on the quadrant in which terminal side of the angle lies.

Consider a circle having radius r and centre at point O.



In **1st Quadrant**: $x > 0, y > 0$

$$\sin\theta = \frac{y}{r} > 0, \cos\theta = \frac{x}{r} > 0, \tan\theta = \frac{y}{x} > 0,$$

$$\text{cosec}\,\theta = \frac{r}{y} > 0, \sec\theta = \frac{r}{x} > 0, \cot\theta = \frac{x}{y} > 0$$

Hence, in the 1st quadrant all trigonometric functions are positive.

In **2nd Quadrant**: $x < 0, y > 0$

$$\sin\theta = \frac{y}{r} > 0, \cos\theta = \frac{-x}{r} < 0, \tan\theta = \frac{y}{-x} < 0, \operatorname{cosec}\theta = \frac{r}{y} > 0, \sec\theta = \frac{r}{-x} < 0, \cot\theta = \frac{-x}{y} < 0$$

Hence, in the 2nd quadrant sin and cosec functions are positive and all other trigonometric functions are negative.

In **3rd Quadrant**: $x < 0, y < 0$

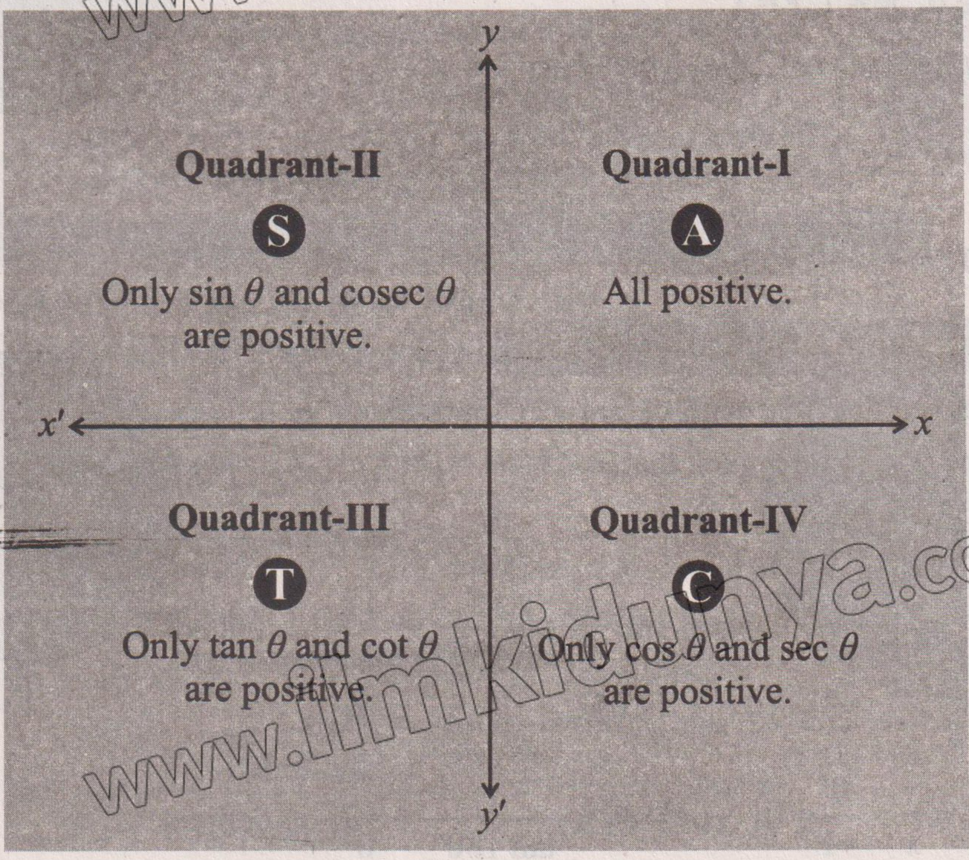
$$\sin\theta = \frac{-y}{r} < 0, \cos\theta = \frac{-x}{r} < 0, \tan\theta = \frac{-y}{-x} > 0, \operatorname{cosec}\theta = \frac{r}{-y} < 0, \sec\theta = \frac{r}{-x} < 0, \cot\theta = \frac{-x}{-y} > 0$$

Hence, in the 3rd quadrant tan and cot functions are positive and all other trigonometric functions are negative.

In **4th Quadrant**: $x > 0, y < 0$

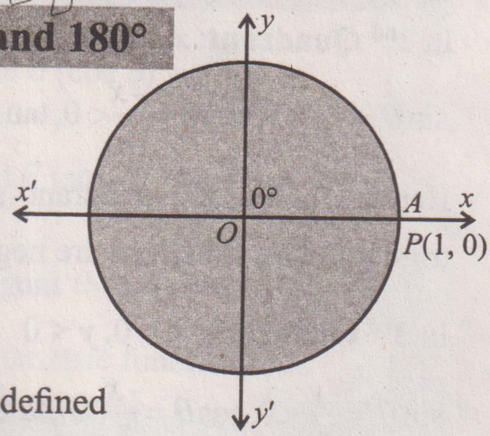
$$\sin\theta = \frac{-y}{r} < 0, \cos\theta = \frac{x}{r} > 0, \tan\theta = \frac{-y}{x} < 0, \operatorname{cosec}\theta = \frac{r}{-y} < 0, \sec\theta = \frac{r}{x} > 0, \cot\theta = \frac{x}{-y} < 0$$

Hence, in the 4th quadrant cos and sec functions are positive and all other trigonometric functions are negative.



7.1.3 Trigonometric Ratios of 0°, 90° and 180°

i Trigonometric Ratios of 0°



Consider a unit circle with centre at O and take any point $P(x, y)$ on it such that $m\angle AOP = 0°$.

As $P(x, y)$ lies on x-axis,

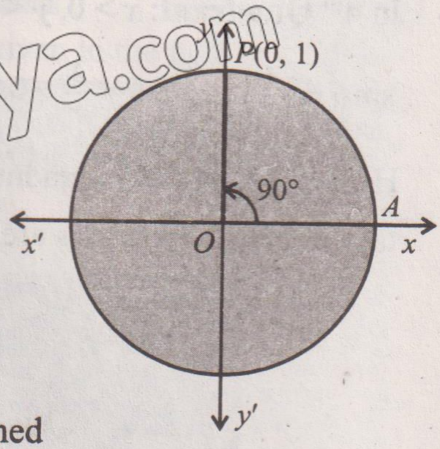
So, $x = 1,\ y = 0,\ r = 1$

$$\sin 0° = \frac{y}{r} = \frac{0}{1} = 0 \quad , \quad \operatorname{cosec} 0° = \frac{1}{\sin 0°} = \frac{1}{0} = \text{undefined}$$

$$\cos 0° = \frac{x}{r} = \frac{1}{1} = 1 \quad , \quad \sec 0° = \frac{1}{\cos 0°} = \frac{1}{1} = 1$$

$$\tan 0° = \frac{y}{x} = \frac{0}{1} = 0 \quad , \quad \cot 0° = \frac{1}{\tan 0°} = \frac{1}{0} = \text{undefined}$$

ii Trigonometric Ratios of 90°



Consider a unit circle with centre at O and take any point $P(x, y)$ on it such that $m\angle AOP = 90°$.

As $P(x, y)$ lies on y-axis,

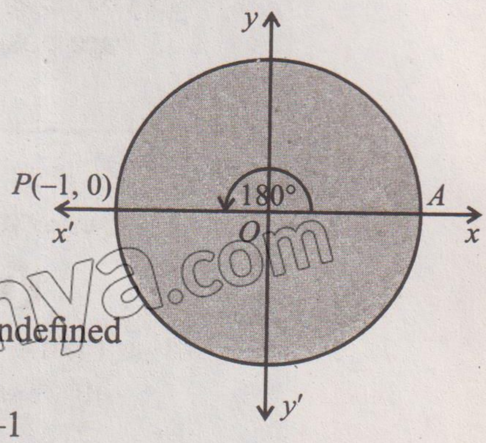
So, $x = 0,\ y = 1,\ r = 1$

$$\sin 90° = \frac{y}{r} = \frac{1}{1} = 1, \quad \operatorname{cosec} 90° = \frac{1}{\sin 90°} = \frac{1}{1} = 1$$

$$\cos 90° = \frac{x}{r} = \frac{0}{1} = 0, \quad \sec 90° = \frac{1}{\cos 90°} = \frac{1}{0} = \text{undefined}$$

$$\tan 90° = \frac{y}{x} = \frac{1}{0} = \text{undefined} \quad , \quad \cot 90° = \frac{0}{1} = 0$$

iii Trigonometric Ratios of 180°



Consider a unit circle with centre at O and take any point $P(x, y)$ on it such that $m\angle AOP = 180°$.

As $P(x, y)$ lies on x'-axis.

so, $x = -1,\ y = 0,\ r = 1$

$$\sin 180° = \frac{y}{r} = \frac{0}{1} = 0, \quad \operatorname{cosec} 180° = \frac{1}{\sin 180°} = \frac{1}{0} = \text{undefined}$$

$$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1, \quad \sec 180° = \frac{1}{\cos 180°} = \frac{1}{-1} = -1$$

$$\tan 180° = \frac{y}{x} = \frac{0}{-1} = 0, \quad \cot 180° = \frac{1}{\tan 180°} = \frac{-1}{0} = \text{undefined}$$

7.1.4 Extending sine and cosine Functions to Angles Between 90° and 180°

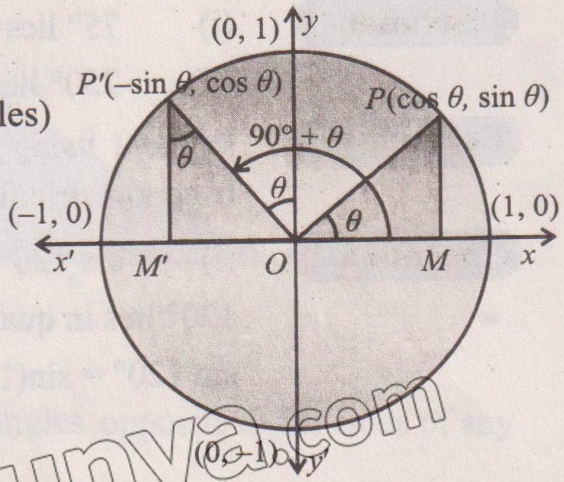
a Trigonometric Ratios of an Angle of the Form (90° + θ)

Consider a unit circle having radius 1 with centre at $O(0, 0)$ in the Cartesian coordinate system. $P(\cos\theta, \sin\theta)$ be any point in quadrant I on the circumference of the circle, where $m\angle MOP = \theta = m\angle YOP'$

$m\angle MOP' = (90° + \theta)$ and $m\angle POP' = 90°$

so, we have $m\angle OP'M' = m\angle YOP'$ (Alternate angles)

That is, the angle $(90° + \theta)$ is obtained by rotating the angle θ counterclockwise by 90°. The coordinates of P' will be $(-\sin\theta, \cos\theta)$, i.e. the new x-coordinate of P' becomes the negative of the y-coordinate of the P and new y-coordinate of P' becomes the x-coordinate of the P.



As we know that in unit circle, cosine and sine of any angle are represented by x-coordinate and y-coordinate respectively. Therefore,

$\cos(90° + \theta) = x$-coordinate of $(90° + \theta) = -\sin\theta$

$\sin(90° + \theta) = y$-coordinate of $(90° + \theta) = \cos\theta$

Hence,
$\sin(90° + \theta) = \cos\theta$, $\text{cosec}(90° + \theta) = \sec\theta$
$\cos(90° + \theta) = -\sin\theta$, $\sec(90° + \theta) = -\text{cosec}\,\theta$
$\tan(90° + \theta) = -\cot\theta$, $\cot(90° + \theta) = -\tan\theta$

Note
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$

b Trigonometric Ratios of an Angle of the Form (180° – θ)

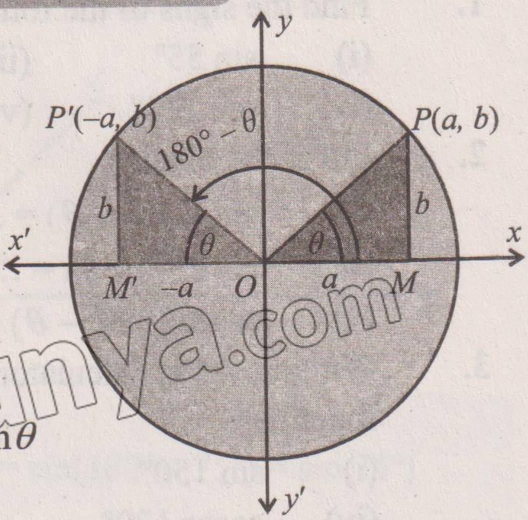
Let $m\angle MOP = \theta$ and $m\angle MOP' = (180° - \theta)$

Here ΔPOM and $\Delta P'OM'$ are congruent, such that

$m\overline{OP'} = m\overline{OP}$, $m\overline{OM'} = m\overline{OM}$ and $m\overline{P'M'} = m\overline{PM}$

In ΔPOM, we have

$$\sin\theta = \frac{b}{m\overline{OP}} \text{ and } \cos\theta = \frac{a}{m\overline{OP}}$$



In $\Delta P'OM'$, we have

$$\sin(180° - \theta) = \frac{y-\text{coordinate of } P'}{m\overline{OP'}} = \frac{b}{m\overline{OP}} = \sin\theta$$

$$\cos(180° - \theta) = \frac{x-\text{coordinate of } P'}{m\overline{OP'}} = \frac{-a}{m\overline{OP}} = -\cos\theta$$

Hence, $\sin(180° - \theta) = \sin\theta$ | $\text{cosec}(180° - \theta) = \text{cosec}\,\theta$
$\cos(180° - \theta) = -\cos\theta$ | $\sec(180° - \theta) = -\sec\theta$
$\tan(180° - \theta) = -\tan\theta$ | $\cot(180° - \theta) = -\cot\theta$

Example 1 Find the signs of the following:
(i) $\cos 75°$ (ii) $\cot 250°$

Solution (i) 75° lies in first quadrant, where cos is positive.
(ii) 250° lies in third quadrant, where cot is positive.

Example 2 Without using calculator, find the exact values of the following trigonometric functions: (i) $\sin 120°$ (ii) $\cot 150°$

Solution (i) $\sin 120°$
120° lies in quadrant II, where sin is positive.
$\therefore$ $\sin 120° = \sin(180° - 60°) = \sin 60°$ $\because \sin(180° - \theta) = \sin\theta$
$= \frac{\sqrt{3}}{2}$

(ii) $\cot 150°$
150° lies in quadrant II, where cot is negative.
$\therefore$ $\cot 150° = \cot(180° - 30°) = -\cot 30°$ $\because \cot(180° - \theta) = -\cot\theta$
$= -\sqrt{3}$

EXERCISE 7.1

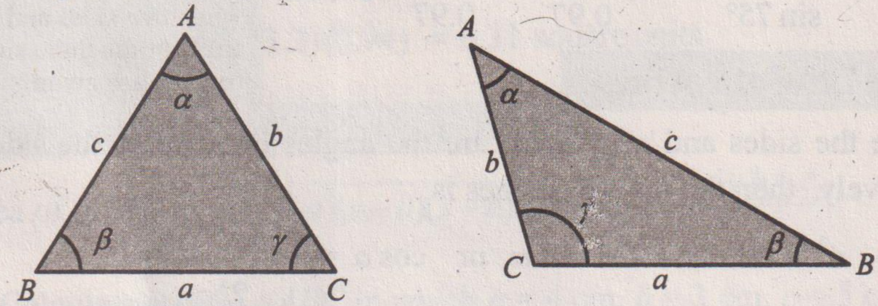
1. Find the signs of the following:
(i) $\sin 55°$ (ii) $\cos 145°$ (iii) $\tan 111°$
(iv) $\sec 179°$ (v) $\text{cosec}\, 88°$ (vi) $\cot 14°$

2. Fill in the blanks:
(i) $\tan(180° - \theta) = \ldots \tan\theta$ (ii) $\sin(180° - \theta) = \ldots \sin\theta$
(iii) $\tan(90° + \theta) = \ldots \cot\theta$ (iv) $\cos(90° + \theta) = \ldots \sin\theta$
(v) $\text{cosec}(180° - \theta) = \ldots \text{cosec}\,\theta$ (vi) $\sec(90° - \theta) = \ldots \text{cosec}\,\theta$

3. Without using calculator, find the exact values of the following trigonometric functions:
(i) $\sin 150°$ (ii) $\tan 150°$ (iii) $\sec 150°$
(iv) $\text{cosec}\, 120°$ (v) $\cos 120°$ (vi) $\cot 120°$
(vii) $\sin 135°$ (viii) $\sec 135°$ (ix) $\cot 135°$

7.2 Oblique Triangles

An oblique triangle is any triangle that does not contain a right angle. It can be either acute triangle (all angles less than 90°) or obtuse triangle (one angle greater than 90°). Following triangles are oblique triangles:



7.2.1 The Law of sines

If a, b and c are the sides and α, β and γ are the angles opposite to the sides of any triangle respectively, then the law of sines is:

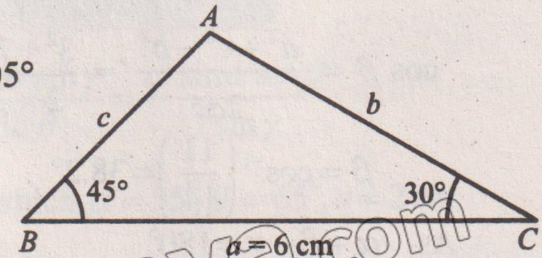
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

The law of sines is a relationship between the angles and the sides of a triangle. While solving a triangle, the law of sines can be used to solve a triangle, if two angles and one side which is opposite to one of given angles, are given.

Example 3 In a triangle ABC, $\beta = 45°$, $\gamma = 30°$, $a = 6$ cm, calculate b and c.

Solution As we know that

$$\alpha + \beta + \gamma = 180°$$
$$\alpha = 180° - \beta - \gamma$$
$$= 180° - 45° - 30° = 105°$$



Now, by the law of sines

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 105°} = \frac{b}{\sin 45°} = \frac{c}{\sin 30°}$$

$$\frac{a}{\sin 75°} = \frac{b}{\sin 45°} = \frac{c}{\sin 30°} \quad [\sin 105° = \sin(180° - 75°) = \sin 75°]$$

Taking $\frac{a}{\sin 75°} = \frac{b}{\sin 45°}$

$$b = \frac{a \sin 45°}{\sin 75°} = \frac{6\times(0.71)}{0.97} = \frac{4.26}{0.97} = 4.4 \text{ cm}$$

Now, $\frac{a}{\sin 75°} = \frac{c}{\sin 30°}$

$$c = \frac{a\times \sin 30°}{\sin 75°} = \frac{6\times(0.5)}{0.97} = \frac{3.0}{0.97} = 3.1 \text{ cm}$$

Skilled Practice!

Solve ΔABC, in which $\alpha = 77°, \beta = 66°, a = 2$ cm

Challenge!

Can we use law of sines, when two sides and included angle or the three sides of a triangle are given?

7.2.2 The Law of cosines

If a, b and c are the sides and α, β and γ are the angles opposite to the sides of any triangle respectively, then the law of cosines is:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad \text{or} \quad \cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$b^2 = c^2 + a^2 - 2ca\cos\beta \quad \text{or} \quad \cos\beta = \frac{c^2+a^2-b^2}{2ca}$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \quad \text{or} \quad \cos\gamma = \frac{a^2+b^2-c^2}{2ab}$$

Example 4 In ΔABC, $a = 3, b = 5, c = 7$. Find the values of α, β and γ.

Solution Given that $a = 3, b = 5, c = 7$

By using the law of cosines, we have

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc} = \frac{5^2+7^2-3^2}{2\times 5\times 7} = \frac{25+49-9}{70} = \frac{13}{14}$$

$$\alpha = \cos^{-1}\left(\frac{13}{14}\right) = 21.8°$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac} = \frac{3^2+7^2-5^2}{2\times 3\times 7} = \frac{9+49-25}{42} = \frac{11}{14}$$

$$\beta = \cos^{-1}\left(\frac{11}{14}\right) = 38.2°$$

As $\alpha+\beta+\gamma = 180°$

$$\gamma = 180° - \alpha - \beta = 180° - 21.8° - 38.2° = 120°$$

7.2.3 Area of a Triangle

a When two sides and their included angle of a triangle are given, then

$$\text{Area of } \Delta ABC = \frac{1}{2}bc\sin\alpha = \frac{1}{2}ac\sin\beta = \frac{1}{2}ab\sin\gamma$$

Example 5 Calculate area of the triangle ABC in which $a = 4.2$, $b = 3.2$ and $\gamma = 70°$.

Solution $a = 4.2,\ b = 3.2,\ \gamma = 70°$

$$\text{Area of } \Delta ABC = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(4.2)(3.2)\sin 70°$$

$$= \frac{1}{2}(4.2)(3.2)(0.94) = 6.31 \text{ square units}$$

b When three sides of a triangle are given, then

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{a+b+c}{2}$$

Example 6 Calculate area of ΔABC in which $a = 4$ cm, $b = 3$ cm, $c = 5$ cm.

Solution $a = 4 \text{ cm},\ b = 3 \text{ cm},\ c = 5 \text{ cm}$

$$s = \frac{a+b+c}{2} = \frac{4+3+5}{2} = \frac{12}{2} = 6$$

$$s-a = 6-4 = 2$$

$$s-b = 6-3 = 3$$

$$s-c = 6-5 = 1$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{6(2)(3)(1)}$$

$$= \sqrt{36}$$

$$= 6 \text{ cm}^2$$

Skilled Practice!

Find area of ΔABC, in which

(i) $a = 8$ cm, $b = 9$ cm, $c = 10$ cm

(ii) $a = 6.1$ cm, $b = 2.3$ cm, $\gamma = 80°$

c When two angles and one side of a triangle are given, then

$$\text{Area of } \Delta ABC = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha} = \frac{b^2 \sin\alpha \sin\gamma}{2\sin\beta} = \frac{c^2 \sin\alpha \sin\beta}{2\sin\gamma}$$

Example 7 Calculate area of triangle ABC in which $\alpha = 35°, \beta = 65°, a = 2$ cm.

Solution As we know that

$$\alpha + \beta + \gamma = 180°$$

$$35° + 65° + \gamma = 180°$$

$$\gamma = 180° - 100°$$

$$= 80°$$

Skilled Practice!

Calculate area of ΔABC, in which

$\beta = 50°$, $\gamma = 40°$, $c = 10$ cm

$$\text{Area of } \Delta ABC = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$$

$$= \frac{(2)^2 \sin 65^\circ \sin 80^\circ}{2 \sin 35^\circ}$$

$$= \frac{4(0.91)(0.98)}{2(0.57)} = 3.13 \text{ cm}^2$$

EXERCISE 7.2

1. Solve the triangle ABC, in which

(i) $a = 6.1$ cm, $b = 8.4$ cm, $\alpha = 42^\circ$

(ii) $a = 12.2$ cm, $c = 15.8$ cm, $\gamma = 50^\circ$

(iii) $b = 5.2$ cm, $c = 5$ cm, $\gamma = 48^\circ$ (iv) $b = 4.8$ cm, $a = 4$ cm, $\beta = 71^\circ$

(v) $\beta = 70^\circ$, $b = 8$cm, $\alpha = 100^\circ$ (vi) $a = 6$ cm, $\alpha = 55^\circ$, $\gamma = 60^\circ$

(vii) $c = 7$ cm, $\beta = 34^\circ$, $\gamma = 64^\circ$ (viii) $b = 12$ cm, $\alpha = 92^\circ$, $\beta = 77^\circ$

2. Calculate area of each triangle ABC.

(i) $a = 7$ cm, $b = 8$ cm, $\gamma = 38^\circ$ (ii) $a = 11$ cm, $c = 14$ cm, $\beta = 51^\circ$

(iii) $b = 3$ cm, $c = 9$ cm, $\alpha = 78^\circ$ (iv) $a = 10$ cm, $\alpha = 62^\circ$, $\beta = 69^\circ$

(v) $c = 4$ cm, $\beta = 36^\circ$, $\gamma = 80^\circ$ (vi) $c = 6.6$ cm, $\alpha = 23^\circ$, $\gamma = 89^\circ$

(vii) $a = 5.3$ cm, $b = 4.7$ cm, $c = 8.2$ cm (viii) $a = 6.12$ cm, $b = 8.34$ cm, $c = 7.12$ cm

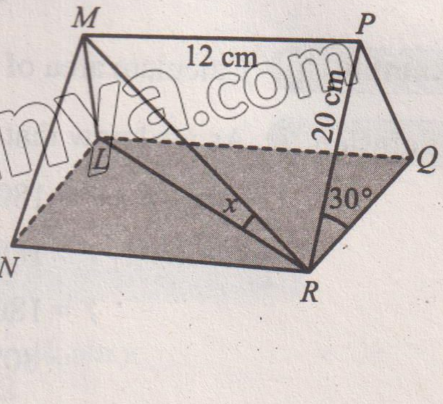
7.3 Solve Simple Trigonometric Problems in Three Dimensions

Example 8 The diagram shows a prism. Triangle PQR is a cross section of the prism, where

$m\overline{PR} = 20$ cm, $m\overline{MP} = 12$ cm,

$m\angle PRQ = 30^\circ$ and $m\angle PQR = 90^\circ$.

Calculate the size of the angle that the line MR makes with the plane $RQLN$.



Solution In right angled triangle PQR, we have

$$\frac{m\overline{PQ}}{20} = \sin 30^\circ$$

$$m\overline{PQ} = 20 \times \frac{1}{2}$$
$$= 10 \text{ cm}$$

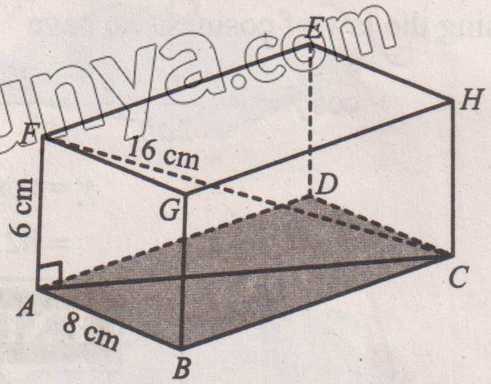
In right angled triangle *MPR*, we have

$$m\overline{MR} = \sqrt{(m\overline{PR})^2 + (m\overline{MP})^2} = \sqrt{(20)^2 + (12)^2} = \sqrt{400 + 144} = \sqrt{544}$$
$$= 23.3238 \text{ cm}$$

In right angled triangle *MLR*, we have

$$\sin x = \frac{\text{opp.}}{\text{hyp.}} = \frac{10}{23.3238} = 0.4287$$
$$\therefore \quad x = \sin^{-1}(0.4287)$$
$$= 25.4°$$

Example 9 The diagram shows a cuboid *ABCDEFGH* in which $m\overline{AB} = 8\,cm$, $m\overline{AF} = 6\,cm$ and $m\overline{FC} = 16\,cm$. Find the length of $\overline{BC}$.



Solution In right angled triangle *AFC*, we have

$$m\overline{AC} = \sqrt{(m\overline{FC})^2 - (m\overline{AF})^2} = \sqrt{(16)^2 - (6)^2}$$
$$= \sqrt{256 - 36} = \sqrt{220} = 14.83 \text{ cm}$$

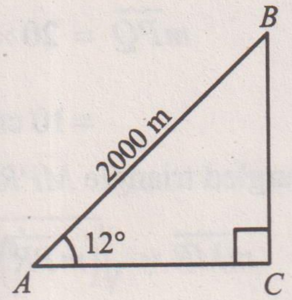
In right angled triangle *ABC*, we have

$$m\overline{BC} = \sqrt{(m\overline{AC})^2 - (m\overline{AB})^2} = \sqrt{(14.83)^2 - (8)^2}$$
$$= \sqrt{220 - 64}$$
$$= \sqrt{156} = 12.5 \text{ cm}$$

7.4 Concept of Trigonometry in Real Life

Example 10 A plane takes off at an angle of elevation of 12° and travels 2000 metres along this path. How high is the plane after traveling this distance?

Solution In triangle ABC,



$$\sin 12^\circ = \frac{m\overline{BC}}{2000}$$

$$(0.2079)2000 = m\overline{BC}$$

$$m\overline{BC} = 415.8 \text{ m}$$

The plane is approximately 415.8 m above the ground.

Example 11 In a sound lab, two microphones are placed $c = 3$ m apart and a sound source is $a = 2$ m from one and $b = 2.5$ m from the other. Find the angle between the sound paths a and b.

Solution Here $a = 2$ m, $b = 2.5$ m, $c = 3$ m

Using the law of cosines, we have

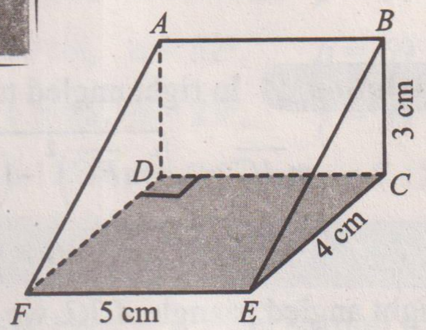
$$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2^2 + (2.5)^2 - 3^2}{2\times 2\times 2.5} = \frac{4 + 6.25 - 9}{10} = 0.125$$

$$\gamma = \cos^{-1}(0.125)$$

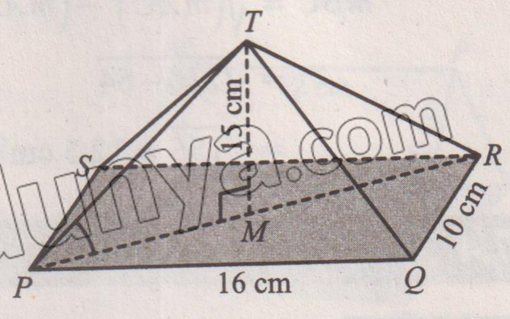
$$= 82.8^\circ$$

EXERCISE 7.3

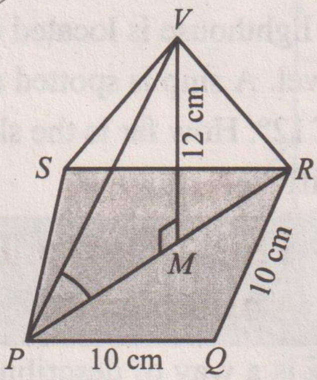
1. In the triangular prism, find
   (i) the length $\overline{CF}$.
   (ii) the length $\overline{BF}$.
   (iii) the angle BFC, correct to one decimal place.



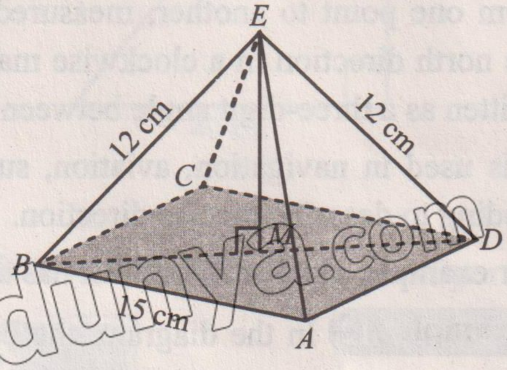
2. The diagram shows a triangular pyramid with a horizontal rectangular base $PQRS$, in which $m\overline{PQ} = 16$ cm, $m\overline{QR} = 10$ cm. M is the midpoint of the line PR. The vertex, T, is vertically above M and $m\overline{MT} = 15$ cm. Calculate the size of the angle between $\overline{TP}$ and the base $PQRS$. Give your answer correct to 1 decimal place.



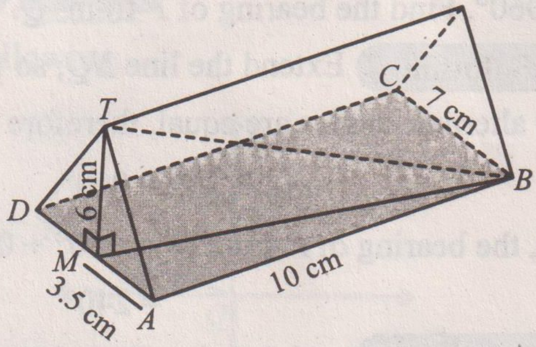
3. The diagram shows a pyramid. The base, $PQRS$, is a horizontal square of side 10 cm. The vertex, V, is vertically above the midpoint, M and $m\overline{VM} = 12$ cm. Calculate the size of angle VPM.



4. $ABCDE$ is a square based pyramid, in which $m\overline{AE} = m\overline{BE} = m\overline{CE} = m\overline{DE} = 12$ cm and $m\overline{AB} = 15$ cm. Calculate the size of angle DEB. Give your answer in degree (whole numbers).



5. The diagram shows a triangular prism with a horizontal rectangular base $ABCD$. $m\overline{AB} = 10$ cm, $m\overline{BC} = 7$ cm, M is the midpoint of AD. The vertex T is vertically above M and $m\overline{MT} = 6$ cm. Calculate the size of the angle between $\overline{TB}$ and the base.



6. In an isometric game, the camera is placed at a 45° angle from the ground. If the player is 10 units in front and 10 units above, what is the direct line of sight distance?
7. A surveyor spots the top of a tower at an elevation angle of 28°. He is standing 60 metres from the base. Find the height of the tower.
8. A boat crosses a river 80 m wide, at a 60° angle to the current. What distance does the boat actually travel?
9. A listener hears a sound from two speakers. One is 6 m directly ahead and the other is at 30° to the side, 6 m away. Find the distance between the speakers.
10. A 10 m ladder leans against a wall making an angle of 75° with the ground. How high does it reach up the wall?

11. A lighthouse is located on a cliff 80 m above sea level. A ship is spotted at an angle of depression of 12°. How far is the ship from the base of the cliff?

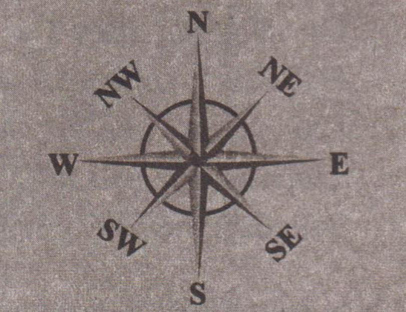
7.5 Three Figure Bearing and its Applications

A bearing is a way of describing the direction or angle from one point to another, measured in degrees from the north direction in a clockwise manner. It is always written as a three-digit angle between 000° and 360°.

It is used in navigation, aviation, surveying and map reading to describe precise direction.

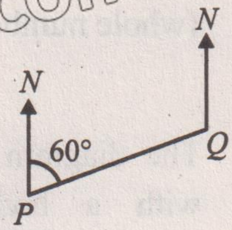
For example, a plane flying east has a bearing of 090°.

Do you know?



Cardinal directions are the four main points on a compass: North, South, East and West.
Ordinal directions are the four inter-cardinal points of a compass: NE, SE, SW, NW

Example 12 In the diagram, the bearing of Q from P is 060°. Find the bearing of P from Q.



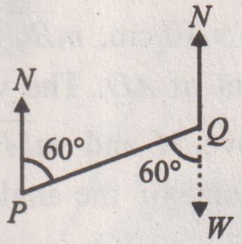
Solution Extend the line NQ, so that angle $NQW = 180°$

As alternate angles are equal, therefore

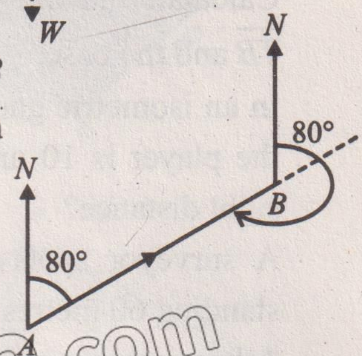
$$m\angle PQW = 060°$$

So, the bearing of P from Q = $180° + 060°$

$= 240°$



Example 13 Abdullah sails a ship from point A to point B on a bearing of 080°. What bearing should he follow to return from point B back to point A?



Solution

The diagram shows the journey from A to B.

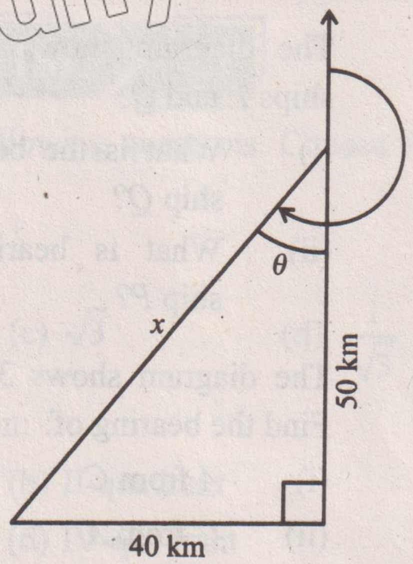
Extend the line of the journey and we get an angle of 80° at B.

Bearing of A from B = $080° + 180°$

$= 260°$

Example 14 An aeroplane departs from an airport and travels 40 km due east, followed by 50 km due north. The pilot then returns to the airport along the shortest possible route. What is the bearing and the distance of the shortest route?

Solution



To find θ

$$\tan\theta = \frac{40}{50} = 0.8$$

$$\theta = \tan^{-1}(0.8)$$

$$= 38.66^\circ$$

Bearing $= 180^\circ + 38.66^\circ = 218.66^\circ$

Suppose x is the shortest route

$$x^2 = 40^2 + 50^2$$

$$= 1600 + 2500$$

$$= 4100$$

$$x = \sqrt{4100}$$

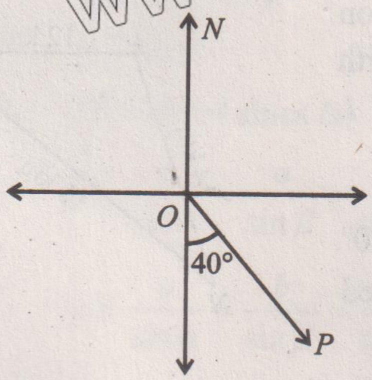
$$= 64.03\,\text{km}$$

The pilot will fly on a bearing of 218.66° for 64.03 km to return to the airport.

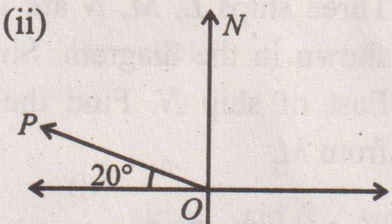
EXERCISE 7.4

1. Find bearing of point P in each of the following:

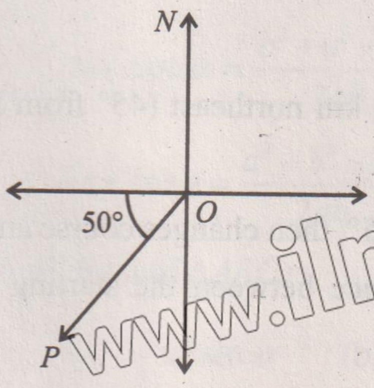
(i)



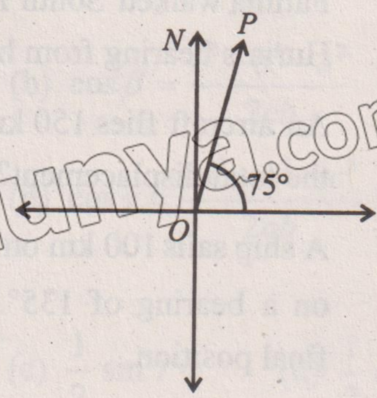
(ii)



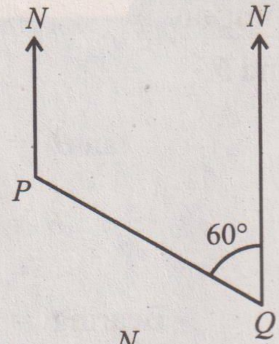
(iii)



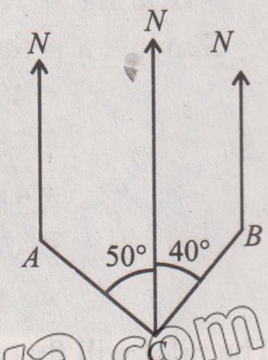
(iv)



2. The diagram shows the positions of two ships P and Q.
   (i) What is the bearing of ship P from ship Q?
   (ii) What is bearing of ship Q from ship P?

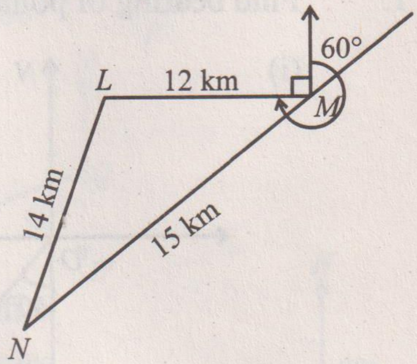


3. The diagram shows 3 places A, B and C. Find the bearing of:
   (i) A from C
   (ii) C from A
   (iii) C from B
   (iv) B from C



4. Abdul Hadi walks 100 m North and then 300 m East.
   (i) How far is he from his starting position?
   (ii) On what bearing should he walk to get back to his starting position?

5. Three ships L, M, N are in the position shown in the diagram. Ship M is North East of ship N. Find the bearing of L from M.



6. A ship sails 20 km on a bearing of 120°. How far to the South has the ship moved from its original position?

7. Fatima walked South for 5.5 km and then turned West for 1.3 km. Calculate Huria's bearing from her starting point.

8. An aircraft flies 150 km east, then 100 km northeast (45° from East). What is the total displacement?

9. A ship sails 100 km on a bearing of 045°, then changes course and sails 120 km on a bearing of 135°. Find the distance between the starting point and the final position.

REVIEW EXERCISE 7

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) What is the value of cot 60°?

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\dfrac{1}{\sqrt{3}}$

(ii) All trigonometric ratios are positive in:

(a) I-quadrant (b) II-quadrant
(c) III-quadrant (d) IV-quadrant

(iii) cosec θ is positive in:

(a) I & III-quadrants (b) II & IV-quadrants
(c) I & II-quadrants (d) I & IV-quadrants

(iv) $\sin(90° + \theta) =$

(a) $\sin\theta$ (b) $-\sin\theta$ (c) $\cos\theta$ (d) $-\cos\theta$

(v) $\tan(180° - \theta) =$

(a) $\tan\theta$ (b) $\cot\theta$ (c) $-\cot\theta$ (d) $-\tan\theta$

(vi) The law of sines is:

(a) $\dfrac{b}{\sin\alpha} = \dfrac{a}{\sin\beta} = \dfrac{c}{\sin\gamma}$ (b) $\dfrac{a}{\sin\alpha} = \dfrac{c}{\sin\beta} = \dfrac{b}{\sin\gamma}$

(c) $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$ (d) $\dfrac{a}{\sin\alpha} = \dfrac{a}{\sin\beta} = \dfrac{a}{\sin\gamma}$

(vii) The law of cosines is:

(a) $\cos\alpha = \dfrac{b^2 + a^2 - c^2}{2ab}$ (b) $\cos\beta = \dfrac{c^2 + b^2 - a^2}{2cb}$

(c) $\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$ (d) $\cos\gamma = \dfrac{ca^2 + b^2 - c^2}{2ab}$

(viii) Area of ΔABC =

(a) $\dfrac{1}{2}ab\sin\alpha$ (b) $\dfrac{1}{2}bc\sin\beta$ (c) $\dfrac{1}{2}\sin\gamma$ (d) $\dfrac{1}{2}ac\sin\beta$

(ix) Bearing is measured from:

(a) East (b) West (c) North (d) South

(x) Bearing is written as a:

(a) 1 figure (b) 2 figures (c) 3 figures (d) 4 figures

2. Calculate the area of ΔABC, in which

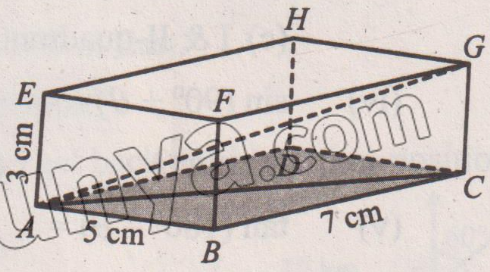
(i) $a = 4$ cm, $b = 6$ cm, $c = 8$ cm (ii) $b = 2.1$ cm, $c = 5$ cm, $\gamma = 45^\circ$

(iii) $c = 3.1$ cm, $\gamma = 44^\circ$, $\alpha = 36^\circ$

3. Solve the triangle ABC, in which

(i) $a = 5.4$ cm, $b = 3.4$ cm, $\alpha = 49^\circ$ (ii) $\alpha = 32^\circ$, $\gamma = 48^\circ$, $c = 81$ cm

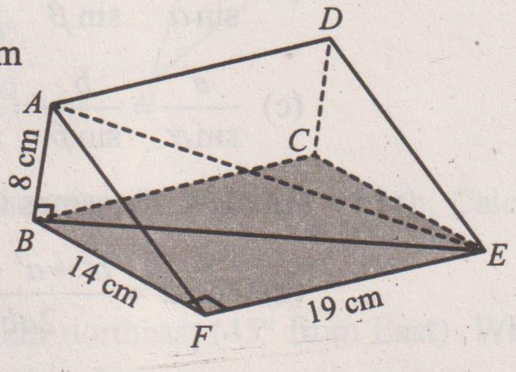
4. The diagram shows a cuboid $ABCDEFGH$ in which $m\overline{AB} = 5$ cm, $m\overline{BC} = 7$ cm and $m\overline{AE} = 3$ cm.



(i) Calculate the length of $\overline{AG}$. Give your answer correct to 3 significant figures.

(ii) Calculate the size of the angle between $\overline{AG}$ and the plane $ABCD$. Give your answer correct to 1 decimal place.

5. The diagram shows a triangular prism $ABCDEF$ in which $m\overline{AB} = 8$ cm, $m\overline{BF} = 14$ cm and $m\overline{EF} = 19$ cm.



(i) Calculate the distance between A and F.

(ii) Calculate the angle between $\overline{AF}$ and the plane $BCEF$.

6. Hashim walks 4 km due North, then turns and walks 3km due East. What is the bearing from the starting point to his final position?

7. A pilot flies 200 km on a bearing of 045°, then turns and flies 150 km on a bearing of 135°. How far is the plane from its original position?