UNIT 6

Vectors in Plane

Students' Learning Outcomes

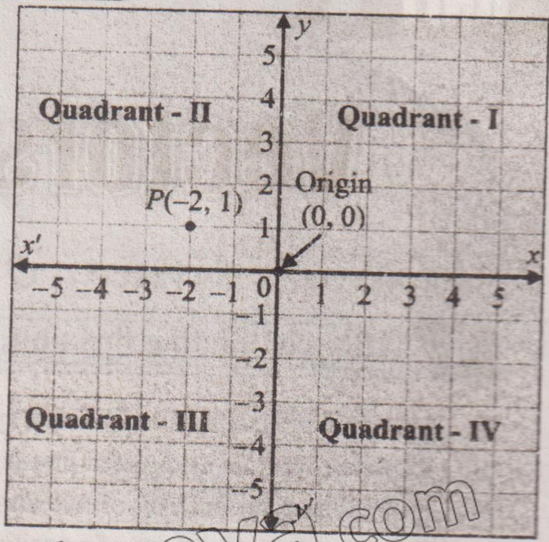
After completing this unit, the students will be able to:

- Introduce a rectangular coordinate system in the plane.
- Represent vectors as directed line segment.
- Express a vector in terms of two non-zero and non-parallel coplanar vectors.
- Express a vector in terms of a position vector.
- Express translation by a vector.
- Find the magnitude of a vector.
- Add and subtract vectors.
- Multiply a vector by a scalar.
- Solve geometrical problems involving the use of vectors.
- Apply concepts from geometrical problems involving the use of vectors (such as parallel and perpendicular lines in geometrical shapes, vector projectile motion, crosswinds aviation, military usage and designing roller coasters).

INTRODUCTION

This unit introduces the foundational concepts of vectors and the rectangular coordinate system in the plane, providing a powerful mathematical tool to describe and analyze movement, direction and position. Students will learn how to represent vectors as directed line segments and express them using position vectors or combinations of two non-zero, non-parallel coplanar vectors. The unit covers essential operations such as finding the magnitude of a vector, vector addition and subtraction and scalar multiplication, enabling students to solve both abstract and practical problems. Emphasis is placed on applying vector concepts to real-world scenarios, including geometrical analysis, projectile motion, navigation in crosswinds, military applications and engineering designs like roller coasters, equipping students with the tools to interpret and model physical phenomena using vectors.

6.1 Rectangular Coordinate Plane

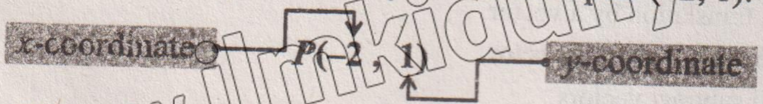


A rectangular coordinate plane is formed by the intersection of a horizontal number line called the **axis of *x*** or shortly ***x*-axis** and a vertical number line called the **axis of *y*** or shortly ***y*-axis.**

The axes meet at a point called the **origin** and divide the coordinate plane into four **quadrants.**

Points in a coordinate plane are represented by **ordered pairs.** The first element is the ***x*-coordinate** and the second element is the ***y*-coordinate.**

In the given figure, point P is represented by the ordered pair $(-2, 1)$.

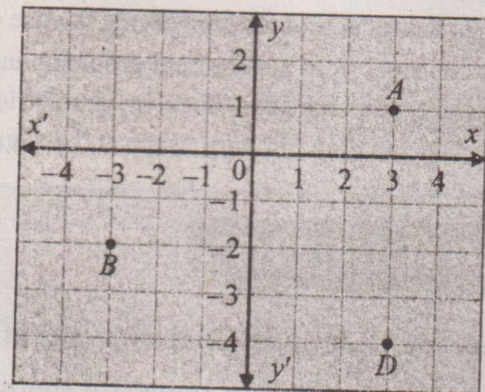


Example 1 Write the coordinates of the following points from the given figure:

(i) A

(ii) B

(iii) D



Solution

(i) Point A is 3 units to the right of the origin and 1 unit up. So, the x-coordinate is 3 and the y-coordinate is 1. The coordinates of A are $(3, 1)$.

(ii) Point B is 3 units to the left of the origin and 2 units down. So, the x-coordinate is -3 and the y-coordinate is -2. The coordinates of B are $(-3, -2)$.

(iii) Point D is 3 units to the right of the origin and 4 units down. So, the x-coordinate is 3 and the y-coordinate is -4. The coordinates of D are $(3, -4)$.

6.2 Scalars and Vectors

There are two types of physical quantities that are used in Physics and Mathematics.

Scalar: A scalar is a quantity having magnitude but no direction. Mass, time, volume and power are the examples of scalar quantities.

Vector: A vector is a quantity having both magnitude and direction. Displacement, velocity, acceleration, force and momentum are examples of vector quantities.

6.2.1 Representation of a Vector as Directed Line Segment

A vector has a tail and a tip. Consider the diagram in Figure 6.1.

The tail point A is called the **initial point** and the tip point B is called the **terminal point** of the vector $\overrightarrow{AB}$. The initial point of a vector is also taken as **origin** of the vector.

We can also represent a vector with $\underline{a}$, $\vec{a}$ or $\boldsymbol{a}$.

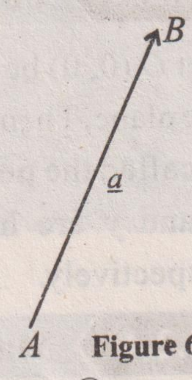


Figure 6.1

(i) A vector that has the same magnitude as the given vector but opposite direction is called **negative of a vector** as shown in Figure 6.2.

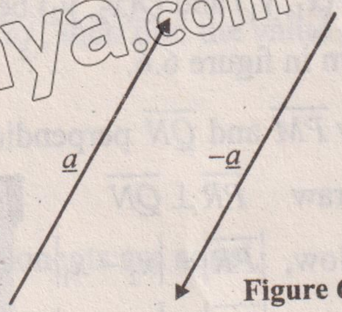


Figure 6.2

(ii) Two vectors $\underline{a}$ and $\underline{b}$ are **equal** if they have the same magnitude and direction regardless of the position of their initial points. Thus $\underline{a} = \underline{b}$ in Figure 6.3.

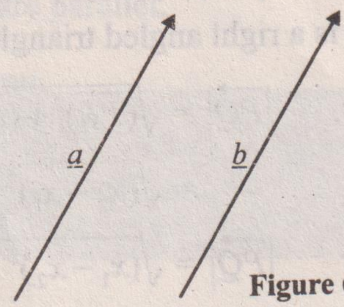


Figure 6.3

(iii) A vector whose initial and terminal points are coincident is called a **zero or null vector.** Zero vector has zero magnitude and it is parallel to every vector.

6.2.2 Vectors and Coordinates

When a Cartesian coordinate system is added to a vector diagram, it becomes possible to assign coordinates to both the initial and terminal points (tail and head) of each vector. This allows vectors to be expressed in terms of their coordinates, which can then be used effectively in calculations. To simplify this

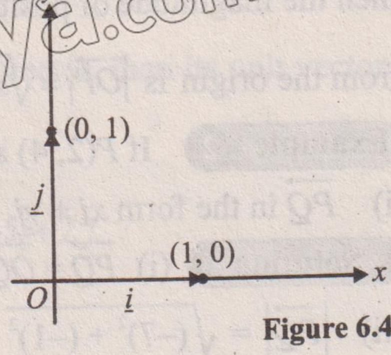


Figure 6.4

process, we introduce two special vectors along the coordinate axes: $\underline{i}$ along the x-axis and $\underline{j}$ along the y-axis. Both vectors originate at the origin and point in the positive direction of their respective axes.

6.2.3 Position Vectors

Let $O(0, 0)$ be the origin and $P(x, y)$ be any point in the plane. Then the vector $\overrightarrow{OP} = (x-0)\underline{i} + (y-0)\underline{j} = x\underline{i} + y\underline{j}$ is called the **position vector** of P with respect to O. x and y are horizontal and vertical components respectively.

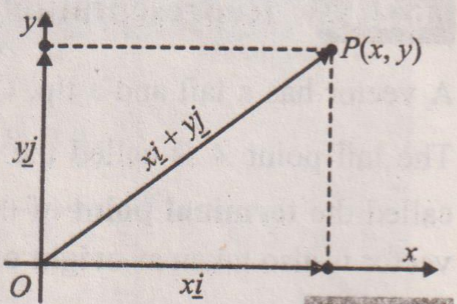


Figure 6.5

6.2.4 Magnitude of a Vector

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the given points as shown in figure 6.6.

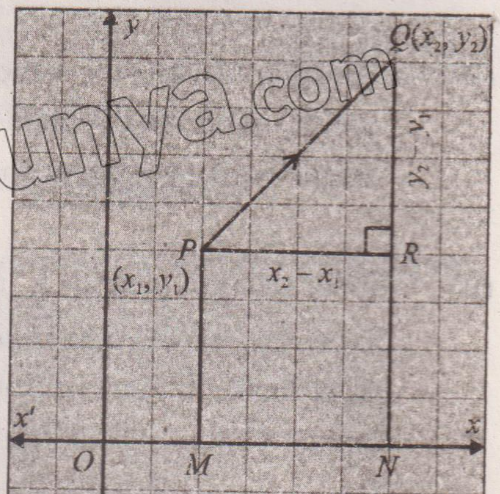


Figure 6.6

Draw $\overline{PM}$ and $\overline{QN}$ perpendiculars on the x-axis.

Draw $\overline{PR} \perp \overline{QN}$

> **Note**
> Magnitude of a vector is also known as norm or length of a vector.

Now, $\left|\overline{PR}\right| = \left|x_2 - x_1\right|$

$\left|\overline{QR}\right| = \left|y_2 - y_1\right|$

PRQ is a right angled triangle.

$$\therefore \quad \left|\overrightarrow{PQ}\right| = \sqrt{(\overline{PR})^2 + (\overline{QR})^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{or} \quad \left|\overrightarrow{PQ}\right| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

If $P(x, y)$ be any point in the plane as shown in Figure 6.7, then the magnitude of position vector $\overrightarrow{OP} = x\underline{i} + y\underline{j}$ from the origin is $\left|\overrightarrow{OP}\right| = \sqrt{x^2 + y^2}$.

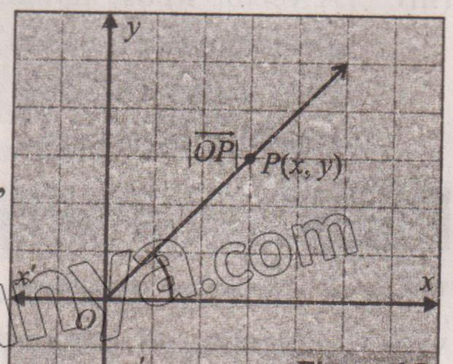


Figure 6.7

Example 2 If $P(2, 4)$ and $Q(-5, 3)$, then find

(i) $\overrightarrow{PQ}$ in the form $x\underline{i} + y\underline{j}$ (ii) $\left|\overrightarrow{PQ}\right|$

Solution (i) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (-5-2)\underline{i} + (3-4)\underline{j} = -7\underline{i} - \underline{j}$

(ii) $\left|\overrightarrow{PQ}\right| = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

> **Skilled Practice!**
> If $P(4, 7)$ and $Q(5, 10)$, then find the magnitude of the vector $\overrightarrow{PQ}$.

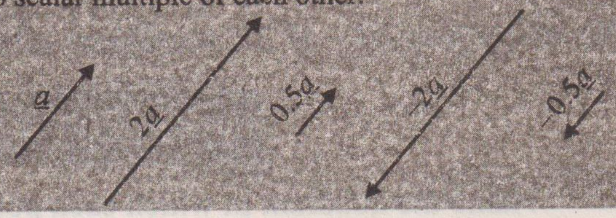
6.2.5 Scalar Multiplication of a Vector

If $\underline{a}$ is a vector and λ is a scalar (a number), then multiplication of a vector $\underline{a}$ with scalar λ is scalar multiplication and it is written as $\lambda \underline{a}$.

$\lambda\underline{a}$ is a new vector that has its magnitude scaled by $|\lambda|$.

Note

Two vectors are said to be parallel, if they are non zero scalar multiple of each other.



(i) If $\lambda > 0$, then both $\underline{a}$ and $\lambda \underline{a}$ are in the same direction.

(ii) If $\lambda < 0$, then both $\underline{a}$ and $\lambda \underline{a}$ are in opposite directions.

Example 3 If $\underline{a} = 3\underline{b}$, where $\underline{a} = 9\underline{i} - k\,\underline{j}$ and $\underline{b} = 3\underline{i} - 12\,\underline{j}$, then find the value of k.

Solution Given that $\underline{a} = 3\underline{b}$

$$9\underline{i} - k\underline{j} = 3(3\underline{i} - 12\underline{j})$$
$$9\underline{i} - k\underline{j} = 9\underline{i} - 36\underline{j}$$

Two vectors are equal, if their horizontal and vertical components are equal.

$$\therefore \quad k = 36$$

Example 4 Show that $\underline{a} = 5\underline{i} - 2\underline{j}$ and $\underline{b} = -10\underline{i} + 4\underline{j}$ are parallel.

Solution Given that:

$$\underline{a} = 5\underline{i} - 2\underline{j}$$
$$\underline{b} = -10\underline{i} + 4\underline{j}$$
$$\underline{b} = -2(5\underline{i} - 2\underline{j})$$
$$\underline{b} = -2\underline{a}$$

Note

(i) If $\underline{a} = \underline{b}$, then $\underline{a}$ and $\underline{b}$ have the same direction and $|\underline{a}| = |\underline{b}|$.

(ii) If $\underline{a}$ is the zero vector, then $|\underline{a}| = 0$

This shows that $\underline{a}$ and $\underline{b}$ are parallel with $\lambda = -2$

6.2.6 Unit Vector

A unit vector is a vector whose magnitude is 1. If $\underline{a}$ is any vector, then its unit vector is represented as $\hat{\underline{a}}$ (read as $\underline{a}$ hat). It is defined as $\hat{\underline{a}} = \dfrac{\underline{a}}{|\underline{a}|}$.

Example 5 Find a unit vector in the direction of $\underline{a} = -5\underline{i} + 2\,\underline{j}$.

Solution Given that $\underline{a} = -5\underline{i} + 2\,\underline{j}$

$$|\underline{a}| = \sqrt{(-5)^2 + (2)^2}$$

$$= \sqrt{25+4} = \sqrt{29}$$

$$\hat{\underline{a}} = \frac{\underline{a}}{|\underline{a}|} = \frac{-5\underline{i}+2\underline{j}}{\sqrt{29}}$$

$$= \frac{-5}{\sqrt{29}}\underline{i} + \frac{2}{\sqrt{29}}\underline{j}$$

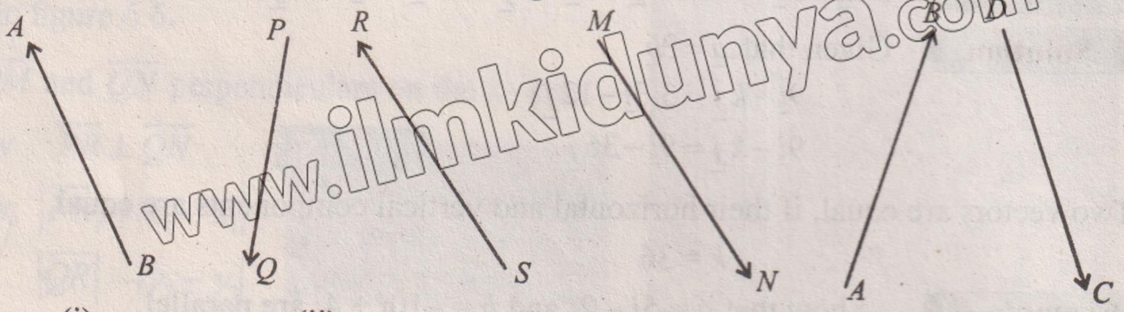
Challenge!

(i) Can a zero vector be a unit vector? Explain.

(ii) If $\underline{a}$ is a unit vector, then what is the value of $|\underline{a}|$?

EXERCISE 6.1

1. Name the quadrant in which each point lies.
   (i) (4, 3) (ii) (5, −4) (iii) (−6,2) (iv) (−4, −4)
2. Plot the following points on the coordinate plane:
   (i) $A(3, -3)$ (ii) $B(-3, 3)$ (iii) $C(5, 7)$ (iv) $D(-2, -4)$
3. Name the tail and tip of the following vectors:



(i) (ii) (iii) (iv) (v) (vi)

4. Write the vector AB in the form of $x\underline{i} + y\underline{j}$:
   (i) $A(1, -7), B(-2, 4)$ (ii) $A(8, 9), B(12, 3)$
5. Find the magnitude of the $\underline{a}$:
   (i) $\underline{a} = -3\underline{i} + 2\underline{j}$ (ii) $\underline{a} = 4\underline{i} - 3\underline{j}$ (iii) $\underline{a} = \frac{1}{2}\underline{i} + \frac{3}{2}\underline{j}$
6. Find a unit vector in the direction of the vector given below:
   (i) $\underline{a} = -4\underline{i} + 5\underline{j}$ (ii) $\underline{a} = 6\underline{i} + 8\underline{j}$
   (iii) $\underline{a} = \frac{1}{\sqrt{6}}\underline{i} + \frac{1}{\sqrt{6}}\underline{j}$ (iv) $\underline{a} = \frac{1}{2}\underline{i} - \frac{3}{4}\underline{j}$
7. If $\underline{a} = 5\underline{i} - 7\underline{j}$, $\underline{b} = -\underline{i} - \underline{j}$ and $\underline{c} = 2\underline{i} + 3\underline{j}$, then find unit vector parallel to $\underline{a} + \underline{b} - 3\underline{c}$.
8. If $\underline{a} = 3\underline{i} - \underline{j}$, $\underline{b} = -2\underline{i} + 4\underline{j}$ and $\underline{c} = \underline{i} + 2\underline{j}$, then find unit vector parallel to $3\underline{a} - 2\underline{c} + 4\underline{b}$.

9. Which of the following vectors are parallel?

(i) $\underline{a} = 6\underline{i} + \underline{j},\ \underline{b} = 12\underline{i} + 2\underline{j}$ (ii) $\underline{a} = -2\underline{i} + 3\underline{j},\ \underline{b} = 6\underline{i} - 9\underline{j}$

(iii) $\underline{a} = 5\underline{i} - 4\underline{j},\ \underline{b} = 6\underline{i} - 3\underline{j}$ (iv) $\underline{a} = 3\underline{i} - 7\underline{j},\ \underline{b} = 6\underline{i} - 14\underline{j}$

10. Find a vector thrice in length of $3\underline{i} - 2\underline{j}$, but opposite in direction.

11. Find two vectors that are double in magnitude of $3\underline{i} - 5\underline{j}$, one in the same direction of it and other in its opposite direction.

6.3 Addition of Vectors

(i) Triangle Law of Addition:

If two vectors $\underline{u}$ and $\underline{v}$ are represented by the two adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of a triangle such that the terminal point of $\underline{u}$ coincides with the initial point of $\underline{v}$, then the third side $\overrightarrow{AC}$ of the triangle gives the vector sum $\underline{u} + \underline{v}$ that is:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \quad \Rightarrow \quad \underline{u} + \underline{v} = \overrightarrow{AC}$$

By head to tail rule

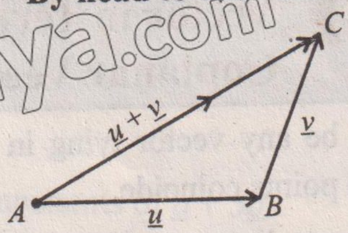


Figure 6.9

(ii) Parallelogram Law of Addition:

If two vectors $\underline{u}$ and $\underline{v}$ are represented by two adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ of a parallelogram as shown in the Figure 6.10, then diagonal $\overrightarrow{AD}$ gives the sum or resultant of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC}$

$\overrightarrow{AD} = \underline{u} + \underline{v}$

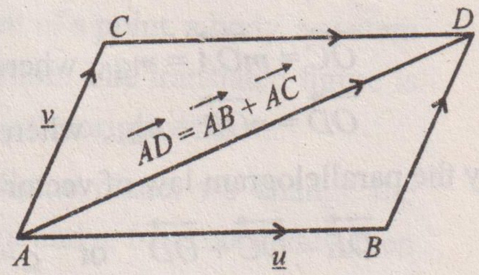


Figure 6.10

Example 6 If $\underline{a} = 2\underline{i} + 3\underline{j}$, $\underline{b} = 4\underline{i} - 6\underline{j}$, then find $\underline{a} + \underline{b}$.

Solution $\underline{a} + \underline{b} = (2\underline{i} + 3\underline{j}) + (4\underline{i} - 6\underline{j})$

$= (2+4)\underline{i} + (3-6)\underline{j}$

$= 6\underline{i} - 3\underline{j}$

Skilled Practice!

If $\underline{a} = 10\underline{i} - \frac{1}{2}\underline{j}$, $\underline{b} = \frac{3}{4}\underline{i} - \frac{7}{6}\underline{j}$, then find $\underline{a} + \underline{b}$.

6.4 Difference of Two Vectors

The vector subtraction of two vectors $\underline{a}$ and $\underline{b}$ is represented by $\underline{a}-\underline{b}$ and defined as $\underline{a}+(-\underline{b})$.

If $\underline{a}=x_1\underline{i}+y_1\underline{j},\ \underline{b}=x_2\underline{i}+y_2\underline{j}$

Then $\underline{a}+(-\underline{b})=(x_1\underline{i}+y_1\underline{j})+(-x_2\underline{i}-y_2\underline{j})$

$=(x_1-x_2)\underline{i}+(y_1-y_2)\underline{j}$

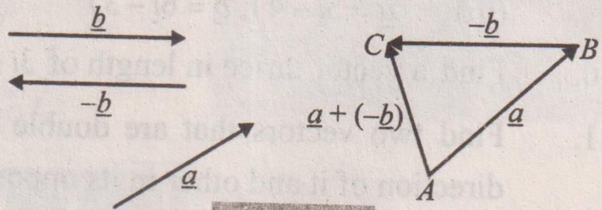


Figure 6.11

Example 7 If $\underline{a}=6\underline{i}+13\underline{j}$ and $\underline{b}=-4\underline{i}+7\underline{j}$, then find $\underline{a}-\underline{b}$.

Solution $\underline{a}-\underline{b}=(6\underline{i}+13\underline{j})-(-4\underline{i}+7\underline{j})$

$=(6+4)\underline{i}+(13-7)\underline{j}=10\underline{i}+6\underline{j}$

6.5 Vector in Terms of Two Non-Zero and Non-Parallel Coplanar Vectors

Let $\underline{c}$ be any vector lying in the plane of $\underline{a}$ and $\underline{b}$. Draw $\underline{a}$, $\underline{b}$ and $\underline{c}$ such that their initial points coincide.

Construct lines parallel to the vectors $\underline{a}$ and $\underline{b}$ from the terminal point E of $\underline{c}$. Complete the parallelogram $ODEC$ by extension of $\underline{a}$ and $\underline{b}$ if necessary.

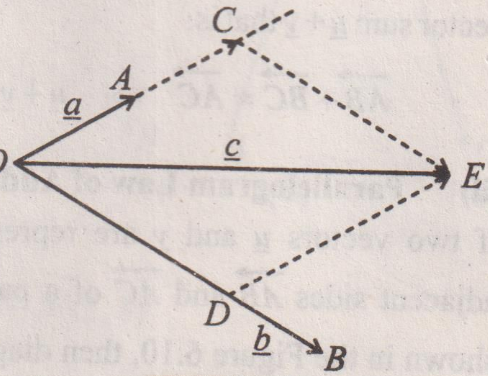


Figure 6.12

$\overrightarrow{OC}=m\overrightarrow{OA}=m\underline{a}$, where m is a scalar.

$\overrightarrow{OD}=n\overrightarrow{OB}=n\underline{b}$, where n is a scalar.

By the parallelogram law of vector addition

$\overrightarrow{OE}=\overrightarrow{OC}+\overrightarrow{OD}$ or $\underline{c}=m\underline{a}+n\underline{b}$

Where $m\underline{a}$ and $n\underline{b}$ are vector components of $\underline{c}$.

Remember!
Two or more vectors are said to be **coplanar vectors** if they lie in the same plane.

Example 8 If $\underline{a}=2\underline{i}-3\underline{j}$ and $\underline{b}=3\underline{i}+4\underline{j}$, then find the vector components of $3\underline{a}+4\underline{b}$.

Solution $3\underline{a}+4\underline{b}$

$=3(2\underline{i}-3\underline{j})+4(3\underline{i}+4\underline{j})$

$=6\underline{i}-9\underline{j}+12\underline{i}+16\underline{j}=18\underline{i}+7\underline{j}$

Thus, the vector components of $3\underline{a}+4\underline{b}$ are $18\underline{i}$ and $7\underline{j}$.

EXERCISE 6.2

1. If $\underline{a} = 7\underline{i} - 3\underline{j}$ and $\underline{b} = \underline{i} + 5\underline{j}$, then find the following vectors:

   (i) $\underline{a} + \underline{b}$ (ii) $\underline{a} + 3\underline{b}$ (iii) $3\underline{a} + \frac{1}{2}\underline{b}$

   (iv) $\underline{b} - \underline{a}$ (v) $4\underline{b} - 5\underline{a}$ (vi) $\frac{3}{2}\underline{a} - \underline{b}$

2. If $\underline{a} = 6\underline{i} - \underline{j}$, $\underline{b} = \underline{i} + 5\underline{j}$ and $\underline{c} = 3\underline{i} + 5\underline{j}$, then find the magnitudes of the following vectors:

   (i) $\underline{b} - \underline{c}$ (ii) $\underline{a} - 2\underline{b} + \underline{c}$ (iii) $\underline{c} - \underline{b} - \underline{a}$

3. Find the values of x and y in each of the following equations:

   (i) $(x\underline{i} + y\underline{j}) + (2\underline{i} + 3\underline{j}) = 7\underline{i} + 6\underline{j}$ (ii) $(x\underline{i} - 5\underline{j}) + (3\underline{i} + 5\underline{j}) = -8\underline{i} + y\underline{j}$

   (iii) $(y\underline{i} + 3\underline{j}) + (-5\underline{i} + 2x\underline{j}) = 9\underline{i} + 7\underline{j}$

4. If $\underline{a} = \underline{i} + 3\underline{j}$, $\underline{c} = 2\underline{i} + \underline{j}$ and $\underline{a} + 2\underline{b} = \underline{c}$, then find $|\underline{b}|$.

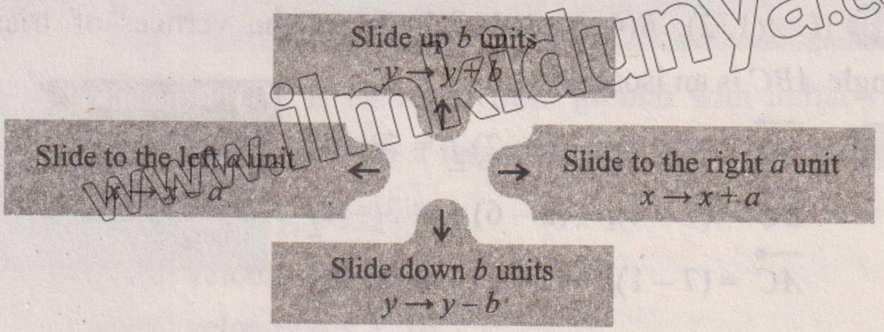
5. If $\underline{a} = -2\underline{i} + 7\underline{j}$ and $\underline{b} = 3\underline{i} - 5\underline{j}$, then find the vector components of $\underline{a} + 5\underline{b}$.

6. If $5\underline{i} - 3\underline{j} = m(\underline{i} - 10\underline{j}) + n(4\underline{i} - 3\underline{j})$, then find the values of m and n.

6.6 Translation by a Vector

A translation vector is a vector that describes the movement of a point, a body, a system from one position to another without rotation or deformation. The translated figure is called the image of original figure. The image is congruent to the original figure.

To translate a figure with translation vector in a coordinate plane, we change the coordinates of its points. If $\underline{v} = a\underline{i} + b\underline{j}$ is the traslation vector and a, b are constants, then we can use the guidelines below:



Let us translate point $P(2, 1)$ by translation vectors $2\underline{j}$, $3\underline{i}$, $-3\underline{j}$ and $-5\underline{i}$.

$$P_1 = (2\underline{i} + \underline{j}) + (0\underline{i} + 2\underline{j}) = 2\underline{i} + 3\underline{j}$$

$$P_2 = (2\underline{i} + \underline{j}) + (3\underline{i} + 0\underline{j}) = 5\underline{i} + \underline{j}$$

$$P_3 = (2\underline{i} + \underline{j}) + (0\underline{i} - 3\underline{j}) = 2\underline{i} - 2\underline{j}$$

$$P_4 = (2\underline{i} + \underline{j}) + (-5\underline{i} + 0\underline{j}) = -3\underline{i} + \underline{j}$$

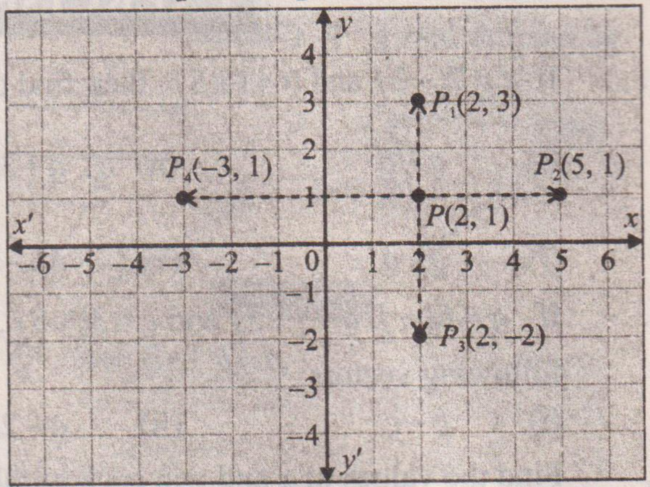


Figure 6.13

Example 9 Plot $A(-4, 2)$, $B(-1, 2)$ and $C(-3, 4)$ to form a triangle ABC. Also translate ΔABC to $\Delta A'B'C'$ by the translation vector $7\underline{i} + 2\underline{j}$.

Solution

New vertex $A' = (-4\underline{i} + 2\underline{j}) + (7\underline{i} + 2\underline{j}) = 3\underline{i} + 4\underline{j} = (3, 4)$

New vertex $B' = (-\underline{i} + 2\underline{j}) + (7\underline{i} + 2\underline{j}) = 6\underline{i} + 4\underline{j} = (6, 4)$

New vertex $C' = (-3\underline{i} + 4\underline{j}) + (7\underline{i} + 2\underline{j}) = 4\underline{i} + 6\underline{j} = (4, 6)$

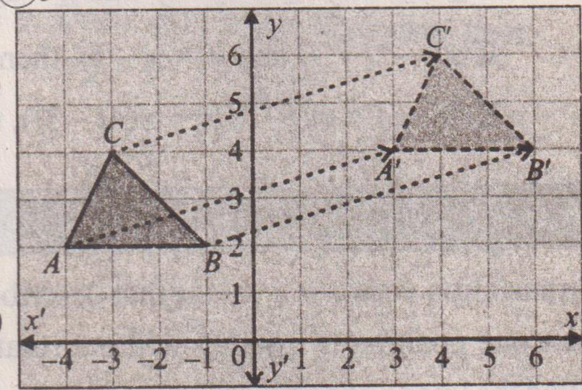


Figure 6.14

6.7 Solution and Application of Geometric Problems Involving Vectors

Example 10 If $A(1, 2)$, $B(4, 6)$ and $C(7, 2)$ are the vertices of triangle. Check whether triangle ABC is an isosceles.

Solution

$$\overrightarrow{AB} = (4-1)\underline{i} + (6-2)\underline{j} = 3\underline{i} + 4\underline{j}$$

$$\overrightarrow{BC} = (7-4)\underline{i} + (2-6)\underline{j} = 3\underline{i} - 4\underline{j}$$

$$\overrightarrow{AC} = (7-1)\underline{i} + (2-2)\underline{j} = 6\underline{i} - 0\underline{j}$$

Remember!

A triangle having two sides equal in length is called an isosceles triangle.

Now $|\overrightarrow{AB}| = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$|\overrightarrow{BC}| = \sqrt{(3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Shows $|\overrightarrow{AB}| = |\overrightarrow{BC}| = 5$

So, ΔABC is an isosceles.

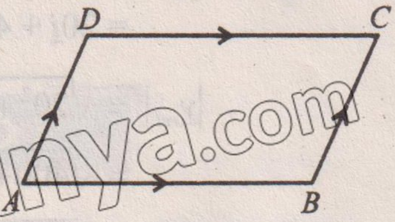
Example 11 Use vector method, to show that the points $A(1, 2)$, $B(4, 6)$, $C(7, 4)$ and $D(4, 0)$ form a parallelogram.

Solution $\overrightarrow{AB} = (4-1)\underline{i} + (6-2)\underline{j} = 3\underline{i} + 4\underline{j}$

$\overrightarrow{DC} = (7-4)\underline{i} + (4-0)\underline{j} = 3\underline{i} + 4\underline{j}$

$\overrightarrow{BC} = (7-4)\underline{i} + (4-6)\underline{j} = 3\underline{i} - 2\underline{j}$

$\overrightarrow{AD} = (4-1)\underline{i} + (0-2)\underline{j} = 3\underline{i} - 2\underline{j}$



$\therefore$ $\overrightarrow{AB} \parallel \overrightarrow{DC}$ and $\overrightarrow{BC} \parallel \overrightarrow{AD}$

Shows the given points form a parallelogram.

Note

Equations of motions are formulas used to describe the movement of objects under uniform acceleration. They help us to calculate the relationship between initial velocity ($\underline{v}_i$), final velocity ($\underline{v}_f$), acceleration ($\underline{a}$), time (t) and displacement ($\underline{s}$). These are very useful in problems involving moving cars, falling objects or thrown balls.

First Equation of Motion $\underline{v}_f = \underline{v}_i + \underline{a}t$

It tells us the final velocity ($\underline{v}_f$) after time t when an object starts with initial velocity $\underline{v}_i$ and moves with acceleration $\underline{a}$.

Second Equation of Motion $\underline{s} = \underline{v}_i t + \frac{1}{2}\underline{a}t^2$

It tells us the distance ($\underline{s}$) travelled by the object in time t.

Third Equation of Motion $\underline{v}_f^2 = \underline{v}_i^2 + 2\underline{a}\underline{s}$

It relates the initial and final velocity, acceleration and displacement.

Example 12 A projectile is launched from ground with initial velocity vector $12\underline{i} + 16\underline{j}$. After how long does it hit a target located 24 metres away horizontally?

Solution Given that $\underline{v}_i = 12\underline{i} + 16\underline{j}$

$\therefore$ Horizontal velocity $= v_x = 12$

Vertical velocity $= v_y = 16$

Horizontal velocity is constant, therefore $x = v_x t$

$$t = \frac{x}{v_x} = \frac{24}{12} = 2\,s$$

Note

Equation for horizontal motion is $\underline{x} = \underline{v}_x t$

Equation for vertical motion is $\underline{y} = \underline{v}_y t - \frac{1}{2} g t^2$

Example 13 A tank fires a shell with velocity vector $\underline{v}_1 = 30\underline{i} + 40\underline{j}$ at a moving target heading east at $\underline{v}_2 = 10\underline{i} + 0\underline{j}$. Find relative velocity and its magnitude.

Solution Relative velocity = Shell velocity – Target velocity

$$\underline{v}_{rel} = (30 - 10)\underline{i} + (40 - 0)\underline{j}$$

$$= 20\underline{i} + 40\underline{j}$$

$$|\underline{v}_{rel}| = \sqrt{20^2 + 40^2}$$

$$= \sqrt{400 + 1600}$$

$$|\underline{v}_{rel}| = \sqrt{2000} = 44.72\, m/s$$

Note

Relative velocity is the velocity of an object as observed from another moving object. It tells us how fast and in what direction one object is moving relative to another.

EXERCISE 6.3

1. Plot $A(-5, 4)$, $B(-6, 1)$ and $C(-3, 1)$ to form a ΔABC. Also translate ΔABC to $\Delta A'B'C'$ by translation vector $8\underline{i} - 6\underline{j}$.
2. Plot $A(-6, -2)$, $B(-6, -5)$, $C(-3, -5)$ and $D(-3, -2)$ to form a square $ABCD$. Also translate square $ABCD$ to square $A'B'C'D'$ by translation vector $9\underline{i} + 7\underline{j}$.
3. Plot $A(-6, 3)$, $B(-4, 0)$, $C(-2, 3)$ and $D(-4, 4)$ to form a kite $ABCD$. Also translate kite $ABCD$ to kite $A'B'C'D'$ by translation vector $6\underline{i} - 6\underline{j}$.
4. The coordinates of A, B and D are (1,2), (6,3) and (2,8) respectively. Find the coordinates of C by using vector method if $ABCD$ is a parallelogram.
5. In parallelogram $ABCD$, the vectors representing two opposite sides are $\overrightarrow{AB} = 6\underline{i} + 2\underline{j}$, $\overrightarrow{DC} = -6\underline{i} - 2\underline{j}$. Show that the opposite sides are equal in magnitude and parallel.
6. Points $A(1, 2)$, $B(4, 6)$ and $C(7, 2)$ form a triangle. Check whether triangle ABC is an isosceles by using vector magnitude.
7. Use vectors to show that $PQRS$ is a parallelogram, where the points P, Q, R and S have coordinates (1, 2), (5, 2), (7, 6) and (3, 6) respectively.

8. Use vectors to show that triangle XYZ is an isosceles, where the points X, Y, Z have the coordinates (0, 0), (2, 0) and (1, 3) respectively.

9. A ball is projected with an initial velocity vector $\underline{v}_0 = 10\underline{i} + 20\underline{j}$. The horizontal component is in the x-direction and gravity is $\underline{g} = 0\underline{i} - 10\underline{j}$. Find the maximum height and horizontal range.

10. A car enters a loop with velocity vector $\underline{v} = 30\underline{j}$ and exits with velocity $\underline{v}' = 30\underline{i}$. What is the change in velocity vector?

11. An aeroplane has airspeed $\underline{v}_p = 20\underline{i}$ and there is a crosswind $\underline{v}_w = 50\underline{j}$. Find the resultant velocity and its magnitude.

REVIEW EXERCISE 6

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) x-axis and y-axis divide a coordinate plane into ________ parts.
(a) one (b) two (c) three (d) four

(ii) $P(4,-4)$ lies in __________ quadrant.
(a) first (b) second (c) third (d) fourth

(iii) A vector having magnitude 1, is called:
(a) equal vector (b) parallel vector
(c) unit vector (d) zero vector

(iv) What is the value of $|3\underline{i}+4\underline{j}|$?
(a) 3 (b) 4 (c) 5 (d) 7

(v) If $\underline{a} = \lambda\underline{b}$, then $\underline{a}$ and $\underline{b}$ are:
(a) equal (b) parallel (c) perpendicular (d) non-parallel

(vi) If $\overrightarrow{OA} = \underline{a}, \overrightarrow{OB} = \underline{b}$, then $\overrightarrow{AB}$ is:
(a) $\underline{b}-\underline{a}$ (b) $\underline{a}-\underline{b}$ (c) $\underline{a}+\underline{b}$ (d) $\underline{b}+\underline{a}$

(vii) Translation vector shows:
(a) deformation (b) rotation (c) movement (d) enlargement

(viii) Sum of two vectors is:
(a) a triangle (b) a vector (c) a scalar (d) a length

(ix) The position vector of point P(3, −2) with respect to O is:
(a) $3\underline{i}+2\underline{j}$ (b) $3\underline{i}-2\underline{j}$ (c) $-3\underline{i}+2\underline{j}$ (d) $2\underline{i}-3\underline{j}$

(x) Vector from point P(3, 4) to origin is:
(a) $3\underline{i}+4\underline{j}$ (b) $-3\underline{i}+4\underline{j}$ (c) $-3\underline{i}-4\underline{j}$ (d) $3\underline{i}-4\underline{j}$

2. Find magnitude of the $\overrightarrow{AB}$:
(i) $A(7, 7), B(-12, 0)$ (ii) $A(9, 3), B(2, 11)$

3. Find a unit vector in the direction of $\underline{a} = \frac{5}{3}\underline{i} + \frac{1}{3}\underline{j}$.

4. If $\underline{a} = 2\underline{i} - \underline{j}$, $\underline{b} = 3\underline{i} + \underline{j}$ and $\underline{c} = 4\underline{i} + \underline{j}$, then find the following vectors:
(i) $5\underline{b} - \underline{a} + \underline{c}$ (ii) $8\underline{a} + \underline{b} + 5\underline{c}$ (iii) $\underline{c} + \underline{b} - 4\underline{a}$

5. Find the values of x and y in the following equation.
$$(2x\underline{i} + y\underline{j}) + (-\underline{i} + 5\underline{j}) = \frac{1}{4}\underline{i} - 8\underline{j}$$

6. Plot $A(-5, 3)$, $B(-2, 3)$ and $C(-4, 5)$ to form a triangle ABC. Also, translate ΔABC to $\Delta A'B'C'$ by translation vector $5\underline{i} - 2\underline{j}$.

7. Use vectors to show that $ABCD$ is a parallelogram, where the points are $A(2, 3)$, $B(6, 3)$, $C(7, 6)$ and $D(3, 6)$.

8. Use vectors to show that triangle ABC is an isosceles triangle, where the points A, B and C have coordinates (1, 2), (4, 6) and (7, 2) respectively.

9. A ball is projected with velocity vector $\underline{v} = 6\underline{i} + 8\underline{j}$. What is the magnitude of velocity?

10. An aircraft is flying due east with an airspeed of 200 km/h. There is a wind blowing due north at 60 km/h. Find the resultant velocity and its magnitude.

Activity

1. Suppose vectors $\underline{a}$ and $\underline{b}$ are equal.
   (i) Can we say they originate from the same point? Why or why not?
   (ii) Do they have equal magnitudes? Explain.
   (iii) Do they have same direction? Why?
2. Suppose vectors $\underline{a}$ and $\underline{b}$ are opposite.
   (i) Can we assume they begin at the same point? Give a reason.
   (ii) Do they have same magnitude? Why?
   (iii) Do they have the same direction? Explain why or why not.