UNIT 5

Algebraic Fractions

Students' Learning Outcomes

After completing this unit, the students will be able to:

- Describe rational expressions.
- Factorize and simplify rational expressions.
- Demonstrate manipulation of algebraic fractions.
- Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).
- Solve different types of equations reducible to quadratic form of the type:
  - Exponential Equation
  - Reciprocal Equation
  - $ax^{2n} + bx^n + c = 0;\ a \neq 0, n = 2$
  - $(x+a)(x+b)(x+c)(x+d) = k$, where $a + b = c + d$
- Apply the concept of rational equations (limited to numerators and denominators that are monomials, binomials or trinomials) to real-world problems (such as the amount of work a person can do in certain amount of time, rates and work).

INTRODUCTION

This unit introduces the concept of rational expressions and equations, focusing on how to describe, factorize and simplify them effectively. Students will explore the manipulation of algebraic fractions and perform various operations on rational expressions involving monomials, binomials and trinomials. The unit also covers solving complex equations that are reducible to quadratic form, including exponential and reciprocal equations. Finally, students will apply these mathematical concepts to solve real-world problems involving work rates, shared tasks and time-related scenarios.

5.1 Algebraic Fraction

In Mathematics, an algebraic fraction is a fraction where the numerator, denominator or both contains algebraic expressions.

For example, $\frac{3}{x}, \frac{2x}{5y}, \frac{2x+9}{10x}, \frac{x^2+8}{x^2+x+3}$ are all algebraic fractions. The denominator is never zero.

5.1.1 Rational Expression

A rational expression is the qoutient $\frac{P(x)}{Q(x)}$, where $Q(x) \neq 0$, of two polynomials $P(x)$ and $Q(x)$. For example, $\frac{5}{x-2}, \frac{x+5}{5}, x^2+5x+4, \frac{x+1}{x+2}$ are all rational algebraic expressions.

5.1.2 Simplification of Rational Expressions

Simplification of a rational expression means reducing the fraction to its lowest form by cancelling out the common factors of the numerator and the denominators.

Example 1 Reduce to lowest form $\frac{24a^3c^2x^2}{18a^3x^2 - 12a^2x^3}$

Solution

$$\frac{24a^3c^2x^2}{18a^3x^2 - 12a^2x^3}$$

$$= \frac{24a^3c^2x^2}{6a^2x^2(3a-2x)} = \frac{4a^{3-2}c^2x^{2-2}}{3a-2x}$$

$$= \frac{4ac^2}{3a-2x}$$

> **Note**
> Factorise where possible because there might be a factor that will be cancelled.

Example 2 Reduce to lowest form $\frac{6x^2 - 8xy}{9xy - 12y^2}$

Solution

$$\frac{6x^2 - 8xy}{9xy - 12y^2}$$

$$= \frac{2x(3x-4y)}{3y(3x-4y)} = \frac{2x}{3y}$$

EXERCISE 5.1

1. Reduce the following rational expressions to lowest forms:

(i) $\frac{3a^2 - 6ab}{2a^2b - 4ab^2}$ (ii) $\frac{abx + bx^2}{acx + cx^2}$ (iii) $\frac{ac}{a^2x^2 - ax}$

(iv) $\frac{15a^2b^2c}{100(a^3 + a^2b)}$ (v) $\frac{4x^2 - 9y^2}{4x^2 + 6xy}$ (vi) $\frac{20(x^3 - y^3)}{5x^2 + 5xy + 5y^2}$

(vii) $\frac{x(2a^2 - 3ax)}{a(4a^2x - 9x^3)}$ (viii) $\frac{x^2 - 5x}{x^2 - 4x - 5}$ (ix) $\frac{3x^2 + 6x}{x^2 + 4x + 4}$

(x) $\frac{x^2+xy-2y^2}{x^3-y^3}$ (xi) $\frac{2x^2+17x+21}{3x^2+26x+35}$

5.2 Manipulation of Rational Expressions

Definition:

Manipulation is to apply operations like addition, subtraction, multiplication and division while maintaining the equality or value of the expression.

5.2.1 Addition and Subtraction of Rational Expressions with Unlike Denominators

Steps:

(i) Determine the Least Common Multiple (LCM) of the denominators.

(ii) Rewrite each fraction as an equivalent fraction with the LCM obtained in step (i)

(iii) Follow the same step for doing addition or subtraction of rational expression with like denominators.

Skilled Practice!

Simplify:

(i) $\frac{4a}{a^2-1}-\frac{a+1}{a-1}$

(ii) $\frac{a^3}{a-b}-\frac{b^3}{b-a}$

(iii) $\frac{4-x^2}{x}-\frac{x+2}{x+2}+5$

Example 3 Simplify: $\frac{x-2y}{4}+\frac{x+y}{6}-\frac{2x-y}{15}$

Solution LCM of 4, 6 and 15 = 60

$$\frac{x-2y}{4}+\frac{x+y}{6}-\frac{2x-y}{15}$$

$$=\frac{15(x-2y)+10(x+y)-4(2x-y)}{60}$$

$$=\frac{15x-30y+10x+10y-8x+4y}{60}=\frac{17x-16y}{60}$$

Example 4 Simplify: $\frac{4}{x^2-2xy-3y^2}+\frac{1}{x^2+3xy+2y^2}$

Solution

$$\frac{4}{x^2-2xy-3y^2}+\frac{1}{x^2+3xy+2y^2}$$

$$=\frac{4}{x^2+xy-3xy-3y^2}+\frac{1}{x^2+2xy+xy+2y^2}$$

$$=\frac{4}{x(x+y)-3y(x+y)}+\frac{1}{x(x+2y)+y(x+2y)}$$

$$=\frac{4}{(x-3y)(x+y)}+\frac{1}{(x+y)(x+2y)}$$

$$=\frac{4(x+2y)+1(x-3y)}{(x-3y)(x+y)(x+2y)}$$ (Taking LCM)

$$= \frac{4x+8y+x-3y}{(x-3y)(x+y)(x+2y)} = \frac{5x+5y}{(x-3y)(x+y)(x+2y)}$$

$$= \frac{5(x+y)}{(x-3y)(x+y)(x+2y)} = \frac{5}{(x-3y)(x+2y)}$$

Example 5 Simplify: $\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$

Solution

$$\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$$

$$= \frac{1}{(x-2)(x-3)} + \frac{1}{(x-2)(x-1)} - \frac{1}{(x-5)(x-3)}$$

$$= \frac{(x-1)(x-5)+(x-3)(x-5)-(x-1)(x-2)}{(x-1)(x-2)(x-3)(x-5)} \quad \text{(Taking LCM)}$$

$$= \frac{(x^2-6x+5)+(x^2-8x+15)-(x^2-3x+2)}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-6x+5+x^2-8x+15-x^2+3x-2}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{x^2-11x+18}{(x-1)(x-2)(x-3)(x-5)}$$

$$= \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)} = \frac{x-9}{(x-1)(x-3)(x-5)}$$

EXERCISE 5.2

1. Simplify:

(i) $\frac{3}{x-y} + \frac{1}{y-x}$ (ii) $\frac{4x}{x^2-3x+2} - \frac{4}{1-x} - \frac{5}{x-2}$

(iii) $\frac{x+y}{12x-6y} + \frac{x-y}{18x-9y}$ (iv) $\frac{2}{x+2y} - \frac{x-6y}{x^2-4y^2}$

(v) $\frac{1}{x+1} - \frac{2}{x+2} - \frac{2x+3}{x^2+3x+2}$ (vi) $\frac{5}{x^2+x-6} - \frac{1}{2x^2-7x+6}$

(vii) $\frac{2x}{x+y} - \frac{y}{x-y} - \frac{2y^2}{x^2-y^2}$ (viii) $\frac{7}{2x^2-x-6} - \frac{8}{3x^2-4x-4}$

(ix) $\dfrac{x+2}{x^2-x-12}-\dfrac{x}{x^2+6x+9}$

(x) $\dfrac{1}{x^2-4x+3}+\dfrac{1}{x^2-3x+2}-\dfrac{1}{x^2-5x+6}$ (xi) $\dfrac{x^3}{x-y}+\dfrac{y^3}{y-x}$

2. Subtract $\dfrac{1}{x^2+2}$ from $\dfrac{2x^3+x^2+3}{(x^2+2)^2}$

5.2.2 Multiplication and Division of Rational Expressions

a Multiplication

Example 6 Simplify: $\dfrac{x^2-5x+6}{x^2-25}\times\dfrac{x^2+5x}{2x-6}$

Solution

$$\frac{x^2-5x+6}{x^2-25}\times\frac{x^2+5x}{2x-6}=\frac{x^2-2x-3x+6}{x^2-5^2}\times\frac{x(x+5)}{2(x-3)}$$

$$=\frac{(x-2)(x-3)}{(x-5)(x+5)}\times\frac{x(x+5)}{2(x-3)}=\frac{x(x-2)}{2(x-5)}$$

Example 7 Simplify: $\dfrac{3a^2-6ab}{6ab}\times\dfrac{10b^2}{4ab-8b^2}$

Solution

$$\frac{3a^2-6ab}{6ab}\times\frac{10b^2}{4ab-8b^2}$$

$$=\frac{3a(a-2b)}{6ab}\times\frac{10b^2}{4b(a-2b)}$$

$$=\frac{30}{24}=\frac{5}{4}$$

Skilled Practice!

Simplify:

(i) $\dfrac{32x^2y^2z^5}{3x^3y^3}\cdot\dfrac{243x^6}{x^4y^2z}$

(ii) $\dfrac{5a^2}{4a-8}\cdot\dfrac{6a-12}{10a}$

b Division

To divide by a rational expression, multiply by its reciprocal as in multiplication.

Example 8 $\dfrac{2a^2-2c^2}{15a^2c^2}\div\dfrac{3a+3c}{5ac}$

Solution

$$\frac{2a^2-2c^2}{15a^2c^2}\div\frac{3a+3c}{5ac}$$

$$=\frac{2a^2-2c^2}{15a^2c^2}\times\frac{5ac}{3a+3c}$$

$$=\frac{2(a^2-c^2)}{15a^2c^2}\times\frac{5ac}{3(a+c)}$$

$$= \frac{2(a+c)(a-c)}{15a^2c^2} \times \frac{5ac}{3(a+c)} = \frac{2(a-c)}{9ac}$$

Example 9 Simplify: $\dfrac{x^3-a^3}{x^2-2bx+b^2} \times \dfrac{x^2-bx-cx+bc}{ax-x^2} \div \dfrac{x^2-cx}{x-b}$

Solution $\dfrac{x^3-a^3}{x^2-2bx+b^2} \times \dfrac{x^2-bx-cx+bc}{ax-x^2} \div \dfrac{x^2-cx}{x-b}$

$$= \frac{x^3-a^3}{x^2-2bx+b^2} \times \frac{x^2-bx-cx+bc}{ax-x^2} \times \frac{x-b}{x^2-cx}$$

$$= \frac{(x-a)(x^2+ax+a^2)}{(x-b)(x-b)} \times \frac{(x-b)(x-c)}{x(a-x)} \times \frac{x-b}{x(x-c)}$$

$$= -(a-x)(x^2+ax+a^2) \times \frac{1}{x(a-x)} \times \frac{1}{x}$$

$$= \frac{-(x^2+ax+a^2)}{x^2}$$

EXERCISE 5.3

1. Simplify:

(i) $\dfrac{24lm}{5l+10m} \times \dfrac{5l^2-20m^2}{16mn}$ (ii) $\dfrac{x^2-3x+2}{x^2+3x-4} \times \dfrac{2x^2+8x}{3x+6}$

(iii) $\dfrac{a^2-4b^2}{a^2+2ba} \times \dfrac{2a^2+10ab}{a^2+3ab-10b^2}$ (iv) $\dfrac{(a+b)^2-c^2}{(a+c)^2-b^2} \times \dfrac{a^2-(b-c)^2}{(a+c)^2-c^2}$

(v) $\dfrac{x^3-8}{x^2-4} \div \dfrac{x^2+2x+4}{x^2+4x+4}$ (vi) $\dfrac{(a^2+ab)^2}{(a^2-ab)^2} \div \left(\dfrac{a+b}{a-b}\right)^2$

(vii) $\dfrac{x^2-x-6}{x^2-x-20} \div \left\{\dfrac{x^3-3x^2}{x^2+4x} \times \dfrac{x^3-5x^2-14x}{x^2-12x+35}\right\}$

(viii) $\dfrac{3x^2-3xy}{10x^2+10xy-20y^2} \times \dfrac{5xy+10y^2}{6x^2+6xy+6y^2} \div \dfrac{2x^2-2y^2}{4x^3-4y^3}$

(ix) $\dfrac{2x^2-98}{x^3-125} \times \dfrac{x^2-3x-10}{3x-21} \div \dfrac{x^2+5x-14}{x^2+5x+25}$

(x) $\dfrac{x^2-2x}{2x+6} \times \dfrac{x^2+x-6}{x^2-5x} \times \dfrac{6x-30}{x^2-2x}$

5.3 Solution of Equations Reducible to Quadratic Equations

Some algebraic equations are not in standard quadratic form, but they can be reduced or transformed into a quadratic equation by suitable substitution. Such type of equations are called equations reducible to quadratic equations.

Type (i) $ax^{2n} + bx^n + c = 0;\ \ a \neq 0, n = 2$

Example 10 Solve: $2x^4 - 11x^2 + 12 = 0$

Solution

$$2x^4 - 11x^2 + 12 = 0$$

Let $x^2 = y$, then $x^4 = y^2$, so that

$$2y^2 - 11y + 12 = 0$$

$$2y^2 - 3y - 8y + 12 = 0$$

$$y(2y-3) - 4(2y-3) = 0$$

$$(2y-3)(y-4) = 0$$

$$2y - 3 = 0, \quad y - 4 = 0$$

$$y = \frac{3}{2}, \quad y = 4$$

Put $x^2 = y$

$$\therefore \quad x^2 = \frac{3}{2}, \quad x^2 = 4$$

$$x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}, \quad x = \pm 2 \Rightarrow S.S = \left\{\pm 2, \pm\frac{\sqrt{6}}{2}\right\}$$

Type (ii) **Exponential Equation**

Equations in which variable occurs in exponents are called exponential equations.

For example, $2^x - 3\cdot 2^{2x} = -56$

Example 11 Solve: $4^x - 3\cdot 2^{x+3} + 128 = 0$

Solution

$$4^x - 3\cdot 2^{x+3} + 128 = 0$$

The equation can be written as

$$(2^2)^x - 3\cdot 2^x\cdot 2^3 + 128 = 0$$

or $\quad 2^{2x} - 24\cdot 2^x + 128 = 0 \quad$ ...(i)

Let $\quad 2^x = y$, then $2^{2x} = y^2$

$\therefore$ (i) becomes

$$y^2 - 24y + 128 = 0$$

$$\Rightarrow \quad y^2 - 16y - 8y + 128 = 0$$

$$y(y-16) - 8(y-16) = 0$$

$$(y-16)(y-8) = 0$$

$$y - 16 = 0, \; y - 8 = 0$$

$$y = 16, \; y = 8$$

But $\quad y = 2^x$

$$2^x = 16 \quad , \quad 2^x = 8$$

$$2^x = 2^4 \quad , \quad 2^x = 2^3$$

$$x = 4 \quad , \quad x = 3$$

$\therefore$ Solution set $= \{4, 3\}$

Type (iii) Reciprocal Equation

An equation which remains unchanged when variable is replaced by its reciprocal.

For example, $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) + 4 = 0$

Example 12 Solve: $3\left(x^2 + \frac{1}{x^2}\right) + 4\left(x + \frac{1}{x}\right) - 14 = 0$

Solution

$$3\left(x^2 + \frac{1}{x^2}\right) + 4\left(x + \frac{1}{x}\right) - 14 = 0$$

Let $\quad x + \frac{1}{x} = y \quad$ ...(i)

So $\quad \left(x + \frac{1}{x}\right)^2 = y^2$

$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = y^2$

or $\quad x^2 + \frac{1}{x^2} = y^2 - 2$

The given equation becomes

$$3(y^2 - 2) + 4y - 14 = 0$$

$$3y^2 - 6 + 4y - 14 = 0$$

$$3y^2 + 4y - 20 = 0$$

$$3y^2 + 10y - 6y - 20 = 0$$

$$y(3y + 10) - 2(3y + 10) = 0$$

$$(3y + 10)(y - 2) = 0$$

$$3y + 10 = 0 \quad , \quad y - 2 = 0$$

$$y = -\frac{10}{3},\ y = 2$$

Put $y = -\frac{10}{3}$ in (i), we get

$$x + \frac{1}{x} = -\frac{10}{3}$$

$$3x^2 + 3 = -10x \quad \text{[Multiplying by } 3x\text{]}$$

$$3x^2 + 10x + 3 = 0$$

$$3x^2 + 9x + x + 3 = 0$$

$$3x(x + 3) + 1(x + 3) = 0$$

$$(x + 3)(3x + 1) = 0$$

$$x + 3 = 0,\ 3x + 1 = 0$$

$$x = -3,\ -\frac{1}{3}$$

Now, put $y = 2$ in (i), we get

$$x + \frac{1}{x} = 2 \quad \text{[Multiplying by } x\text{]}$$

$$x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0 \quad \text{[Transposing]}$$

$$(x - 1)^2 = 0$$

$$x - 1 = 0$$

$$\Rightarrow \quad x = 1$$

$$\text{Solution set} = \left\{1, -3, -\frac{1}{3}\right\}$$

Type (iv) **$(x + a)(x + b)(x + c)(x + d) = k$, where $a + b = c + d$ and k is any constant.**

Example 13 Solve: $(x + 2)(x + 3)(x - 5)(x - 6) = -12$

Solution Here $2 - 5 = 3 - 6$

Re-arrange the factors, we get

$$[(x + 2)(x - 5)][(x + 3)(x - 6)] = -12$$

$$(x^2 - 3x - 10)(x^2 - 3x - 18) = -12 \quad \text{...(i)}$$

Let $\quad x^2 - 3x = y \quad$ ...(ii)

(i) becomes: $(y-10)(y-18)=-12$

$$y^2-28y+180=-12$$

$$y^2-28y+192=0 \quad \text{[Transposing]}$$

$$y^2-12y-16y+192=0$$

$$y(y-12)-16(y-12)=0$$

$$(y-12)(y-16)=0$$

$$y-12=0, y-16=0$$

$$y=12, 16$$

Put $y=12$ in (ii), we get

$$x^2-3x=12 \quad \text{or}$$

$$x^2-3x-12=0$$

$$x=\frac{3\pm\sqrt{(-3)^2-4\times1\times(-12)}}{2(1)}$$

$$=\frac{3\pm\sqrt{57}}{2}$$

Now put $y=16$ in (ii), we get

$$x^2-3x=16$$

$$x^2-3x-16=0$$

$$x=\frac{3\pm\sqrt{(-3)^2-4\times1\times(-16)}}{2(1)}=\frac{3\pm\sqrt{73}}{2}$$

$\therefore$ Solution set $=\left\{\frac{3\pm\sqrt{57}}{2}, \frac{3\pm\sqrt{73}}{2}\right\}$

Skilled Practice!

Solve:

$(x+1)(x+2)(x-5)(x-6)=144$

EXERCISE 5.4

1. Solve the following equations:

(i)	$5x^4-19x^2+12=0$	(ii)	$4x^4-27x^2+18=0$
(iii)	$5x^4-22x^2+8=0$	(iv)	$4\cdot2^{2x+1}-9\cdot2^x+1=0$
(v)	$4^{1+x}+4^{1-x}-10=0$	(vi)	$3^x+3^{3-x}-12=0$
(vii)	$5^{1+3x}+5^{2-3x}-126=0$	(viii)	$\left(x^2+\frac{1}{x^2}\right)+2\left(x+\frac{1}{x}\right)-33=0$

(ix) $2\left(x^2+\frac{1}{x^2}\right)-25\left(x-\frac{1}{x}\right)-17=0$ (x) $2x^4-5x^3-14x^2-5x+2=0$

(xi) $(x+1)(x+2)(x-4)(x-5)+8=0$ (xii) $(x+3)(x+4)(x+5)(x+6)=-1$

5.4 Real World Problems

Example 14 A tradesman buys a number of articles for Rs. 300. Four of them are broken in transit but by selling the remaining at a profit of Rs. 1.25 each, he gains Rs.350 altogether. How many articles did he buy?

Solution Suppose he buys x articles for Rs. 300

$\therefore$ Each cost = Rs. $\frac{300}{x}$

He sold $(x-4)$ articles for Rs. 350

Each sold for Rs. $\frac{350}{x-4}$

Now, $\frac{350}{x-4}-\frac{300}{x}=1.25$

$$\frac{350}{x-4}-\frac{300}{x}=\frac{125}{100}=\frac{5}{4}$$

Multiplying both sides by $4x(x-4)$, we get

$$350(4x)-300(x-4)(4)=5x(x-4)$$

$$1400x-1200x+4800=5x^2-20x$$

$$5x^2-20x+1200x-1400x-4800=0$$

$$5x^2-220x-4800=0 \quad \text{(Dividing by 5)}$$

$$x^2-44x-960=0$$

$$x^2-60x+16x-960=0$$

$$x(x-60)+16(x-60)=0$$

$$(x-60)(x+16)=0$$

$$x-60=0,\ x+16=0$$

$$\therefore \quad x=60,\ -16$$

Negative value is rejected.

Number of articles purchased = 60

> **Challenge!**
> Amna bought 50 kg of fruits consisting of mangoes and grapes. She paid twice as much per kg for the mango as she did for the grapes. If Amna bought Rs. 18000 worth of mangoes and Rs. 6000 worth grapes, then how many kg's of each fruit did she buy?

Example 15 A train is scheduled to cover a distance of 120 km at a certain average speed, owing to a service checkup this average speed is reduced by 5 km/h, and in consequence the journey takes 20 minutes more than the scheduled time. What was the scheduled speed?

Solution Let x km/h be scheduled speed;

then scheduled time for 120 km $= \frac{120}{x}$ hrs

$$20 \text{ minute} = \frac{20}{60} \text{ hrs} = \frac{1}{3} \text{ hrs}$$

But $(x-5)$ km/h is actual speed: therefore, actual time for 120 km $= \frac{120}{x-5}$

$$\therefore \quad \frac{120}{x-5} - \frac{120}{x} = \frac{1}{3}$$

Multiply each term by $3x(x-5)$

$$360x - 360(x-5) = x(x-5)$$

$$x^2 - 5x - 1800 = 0$$

$$(x-45)(x+40) = 0$$

$$x - 45 = 0, x + 40 = 0$$

$$\therefore \quad x = 45, \; x = -40$$

Negative value is rejected.

The scheduled speed is 45 km/h.

EXERCISE 5.5

1. A train travels a distance of 240 km at a uniform rate, if it had finished 4 km an hour slower, it would have taken 2 hours more over the journey. Find its rate of travelling.
2. Arshia and Ibraheem complete a job together in 4 hours. If Arshia takes 6 hours, then find how much time will Ibraheem take?
3. One pipe fills water in 5 hours, another in 8 hours. Second pipe closed after 2 hours. Find the total time.
4. Huria can complete a project in 12 hours by working alone. If Abdul Hadi join her and they finish it together in 5 hours, how long would it take Abdul Hadi to do the project alone?
5. Two cars start from opposite towns and head towards each other. The distance between them is 240 km. One travels at x km/h and the other at $(x+10)$ km/h. They meet after 2 hours. Find the speed of each car.
6. Rashid can paint a house in 6 days, but if he gets a helper he can do it in 4 days. How long would it take the helper to paint the house alone?

7. Ishmal runs 10 km in $\frac{2x}{x+4}$ hours. If her average speed is 8 km/h, find the value of x.

8. Fahad runs 600 m at a certain pace, and then doubling his pace, does another 600 m. If he took $2\frac{1}{2}$ to cover the distance 1200 m, find the pace he started at, in metres per seconds.

9. A cyclist travels 30 km at a certain speed. If the speed had been 5 km/h faster, the journey would have taken 1 hour less. Find the speed.

REVIEW EXERCISE 5

1. Four possible answers are given for the following questions. Choose the correct answer:

(i) The expression $\frac{2x-1}{x^2+4}$ is:

(a) a polynomial (b) an algebraic fraction
(c) a numerical fraction (d) an equation

(ii) Lowest form of $\frac{18a^2bc^2}{12ac}$ is:

(a) $\frac{9abc}{6}$ (b) $\frac{3abc}{2}$ (c) $\frac{18b}{12}$ (d) $\frac{3}{2}$

(iii) Lowest form of $\frac{15xyz}{10}$ is:

(a) $\frac{3xyz}{2}$ (b) $\frac{3}{2}$ (c) xyz (d) $\frac{xyz}{2}$

(iv) Simplified form of $\frac{3x+1}{5}+\frac{2x+3}{5}$ is:

(a) $\frac{5x+4}{10}$ (b) $\frac{5x+4}{5}$ (c) $\frac{5x+4}{25}$ (d) $5x+4$

(v) Simplified form of $\left(\frac{x^2-y^2}{x+y}\right)(x-y)$ is:

(a) $(x+y)^2$ (b) x^2+y^2 (c) $x-y$ (d) $(x-y)^2$

(vi) $(x^3-y^3)\div(x-y)$ in simplified form is:

(a) x^2-xy+y^2 (b) x^2+xy+y^2 (c) x^2-y^2 (d) x^2+y^2

(vii) An equation of the form $\frac{5}{x}+\frac{1-x}{3}=\frac{1}{6x}$ is called:

(a) radical equation (b) reciprocal equation
(c) fractional equation (d) exponential equation

(viii) An equation of the form $5^x+64\cdot5^{-x}-20=0$ is called:

(a) radical equation (b) exponential equation
(c) reciprocal equation (d) fractional equation

(ix) Roots of $y^2-24y+128=0$ are:

(a) 8, –16 (b) 8, 16 (c) –8, 16 (d) –8, –16

(x) Linear factors of $3x^2+10x+3=0$ are:

(a) $(x+3)$, $(3x+1)$ (b) $(x+3)$, $(3x-1)$
(c) $(x-3)$, $(3x+1)$ (d) $(x-3)$, $(3x-1)$

2. Reduce the following to the lowest form:

(i) $\frac{x^2-7x+12}{x^2-6x+9}$ (ii) $\frac{x^2+3x-18}{7x-21}$

3. Simplify:

(i) $\frac{5}{x^2+x-6}-\frac{1}{2x^2-7x+6}$ (ii) $\frac{x^3-27}{x^2-9}\div\frac{x^2+3x+9}{x^2+6x+9}$

(iii) $\frac{a^2-(b-c)^2}{(a+b)^2-c^2}\times\frac{a^2-(b+c)^2}{(a-b)^2-c^2}$

4. Solve the following equations:

(i) $x^4-16x^2+63=0$ (ii) $4x^4-16x^2+15=0$

(iii) $3^{2x}-12\cdot3^x+27=0$ (iv) $\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)-2=0$

(v) $\left(x^2+\frac{1}{x^2}\right)+\left(x+\frac{1}{x}\right)=4$ (vi) $(x+9)(x-3)(x-7)(x+5)=385$

(vii) $(x-1)(x-2)(x+5)(x-8)=-360$

5. Ahmad takes 2 hours to paint 50 glasses. Faiza takes 2 hours to paint 45 glasses. Working together, how long should it take them to paint 150 glasses?

6. A tap can fill a tank in 6 hours. Another tap can empty it in 9 hours. If both taps are opened together, how long will it take to fill the empty tank?

7. Maham bought a certain number of toys for Rs. 300. If each toy had cost Rs. 5 less, she could have bought two more for the same amount. How many toys did she buy?