UNIT 4

Functions and Graphs

Students' Learning Outcomes

After completing this unit, the students will be able to:

- Explain operations on functions and compositions of functions.
- Define inverse functions and demonstrate their domain and range with examples.
- Formulate composite functions as defined by $(f\circ g)(x) = f(g(x))$.
- Apply concepts from functions to real-world problems (such as finance, transportation and sales).
- Plot graphs of absolute valued functions.
- Solve absolute value linear equations and inequalities in one variable and express the solution as a range of values on a number line.
- Apply concepts of absolute valued functions to real-world problems (such as to calculate energy wave, magnitude and distance).

INTRODUCTION

This unit explores the foundational and applied concepts of functions, beginning with operations on functions and compositions of functions, enabling students to combine multiple functions and understand their interactions. Students will define inverse functions, determine their domains and ranges and illustrate these with clear examples. The unit also emphasizes the formulation and interpretation of composite functions. Students will apply function concepts to real-life contexts such as finance, transportation and sales, making mathematics practical and relevant. Additionally, the unit covers absolute value functions graphing them, solving linear equations and inequalities involving absolute values and expressing solutions on a number line.

4.1 Function

> **History!**
> Gottfried Wilhelm Leibniz is credited with first introducing the word "function" in a mathematical context. He used the term in 1673 to describe a quantity related to a curve, such as its coordinates or slope.

If X and Y are two non-empty sets, then a function from a set X to a set Y is a rule that assigns each element of set X exactly one element in set Y. It is written as $f: X \rightarrow Y$, where the set X is called the domain and the set Y is called the co-domain of function f. While the range is the subset of the co-domain that contains only the actual output values produced by the function f.

If $f(a) = 1$, then 1 is called image of a under f and a is called a pre-image of 1.

For example, $f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4$

We can see that all elements in X have only one image in Y. Therefore, f is a function.

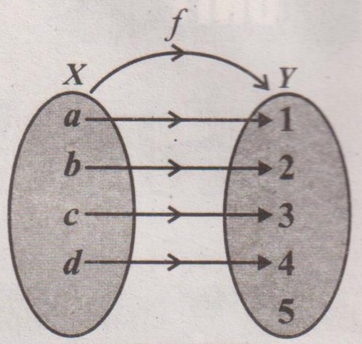


Figure 4.1

Domain $X = \{a, b, c, d\}$

Co-domain $Y = \{1, 2, 3, 4, 5\}$

Range of $f = \{1, 2, 3, 4\}$

The symbol $f(x)$ is often used to briefly denote a function of x. If $y = f(x)$, then by substituting a succession numerical value for 'x', we can obtain a corresponding succession values for y which stands for the values of the function. Hence in this connection it is sometimes conventional to call 'x' the independent variable and 'y' the dependent variable.

Remember!
Range is always a subset of the co-domain.

Example 1 If $f(x) = x^2 + 3x + 1$, then evaluate

(i) $f(0)$ (ii) $f(-2)$ (iii) $f(a)$ (iv) $f(x+1)$

Solution Given that $f(x) = x^2 + 3x + 1$

(i) $f(0) = (0)^2 + 3(0) + 1 = 1$

(ii) $f(-2) = (-2)^2 + 3(-2) + 1$
$= 4 - 6 + 1 = -1$

(iii) $f(a) = a^2 + 3a + 1$

(iv) $f(x+1) = (x+1)^2 + 3(x+1) + 1$
$= x^2 + 2x + 1 + 3x + 3 + 1$
$= x^2 + 5x + 5$

Skilled Practice!
If $f(x) = 2x^2 + 5x$, then find $f(x+2)$.

Example 2 Find the domain and range of $f(x) = (x+1)^2$.

Solution As $f(0) = (0+1)^2 = 1$; $f(4) = (4+1)^2 = 25$; $f(-10) = (-10+1)^2 = 81$;

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2} + 1\right)^2 = \frac{9}{4}$; $f(\sqrt{3}) = (\sqrt{3} + 1)^2 = 7.46$; $f(-1) = 0$; so on

For every real number x, $f(x) = (x+1)^2$ is a non-negative real number. So,

Domain f = set of all real numbers ; Range f = set of all non-negative real numbers.

4.1.1 Operations on Functions

Operations on functions involves performing algebraic operations (addition, subtraction, multiplication and division)

a Addition of Functions

If $f(x)$ and $g(x)$ are two functions, then their sum is written as:

$$(f+g)(x)=f(x)+g(x)$$

Example 3 Let $f(x)=2x+3$ and $g(x)=x^2+x+1$. Add $f(x)$ and $g(x)$.

Solution

$$f(x)=2x+3$$
$$g(x)=x^2+x+1$$
$$(f+g)(x)=f(x)+g(x)$$
$$=(2x+3)+(x^2+x+1)=x^2+3x+4$$

b Subtraction of Functions

If $f(x)$ and $g(x)$ are two functions, then their difference is written as:

$$(f-g)(x)=f(x)-g(x)$$

Example 4 Let $f(x)=5x+2$, $g(x)=x^2-3$. Find $f(x)-g(x)$

Solution

$$f(x)=5x+2$$
$$g(x)=x^2-3$$
$$(f-g)(x)=f(x)-g(x)$$
$$=(5x+2)-(x^2-3)$$
$$=5x+2-x^2+3=-x^2+5x+5$$

c Multiplication of Functions

If $f(x)$ and $g(x)$ are two functions, then their product is written as:
$(f\cdot g)(x)=f(x)\cdot g(x)$

Example 5 Let $f(x)=x+1$, $g(x)=2x-3$, find $f(x)\cdot g(x)$.

Solution

$$f(x)=x+1$$
$$g(x)=2x-3$$
$$(f\cdot g)(x)=f(x)\cdot g(x)$$
$$=(x+1)(2x-3)=2x^2-3x+2x-3$$
$$=2x^2-x-3$$

d Division of Functions

If $f(x)$ and $g(x)$ are two functions, then their quotient is written as:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{for } g(x) \neq 0$$

Example 6 Let $f(x) = x^2 - 4$, $g(x) = x + 2$. Find $\left(\frac{f}{g}\right)(x)$, where $x \neq -2$

Solution

$$f(x) = x^2 - 4$$

$$g(x) = x + 2$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$= \frac{x^2 - 4}{x + 2}$$

$$= \frac{(x+2)(x-2)}{(x+2)} = x - 2$$

4.1.2 Composition of Functions

Composition of function is a process of combining two or more functions to produce a new single function.

The composition operator "o" takes two functions f and g and results a new function. Let us have a look at its definition.

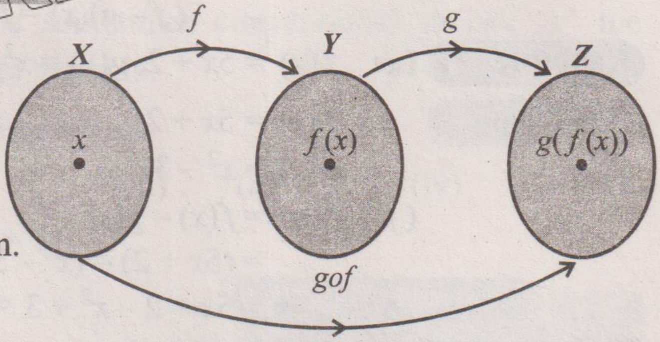
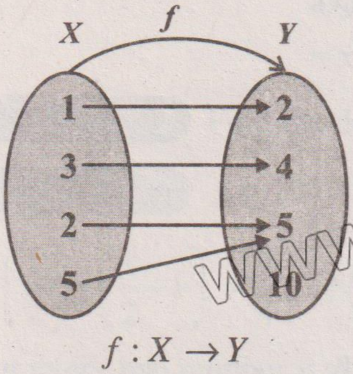


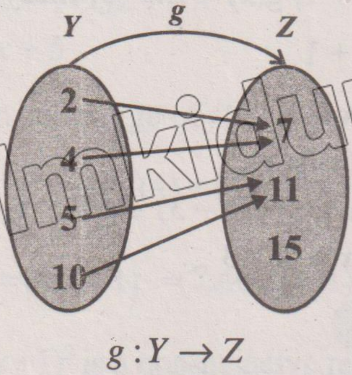
Figure 4.2

Let $f: X \to Y$ and $g: Y \to Z$ be two functions. Then the composition of f and g denoted by gof is defined as the function $gof: X \to Z$ given by $gof(x) = g(f(x))$



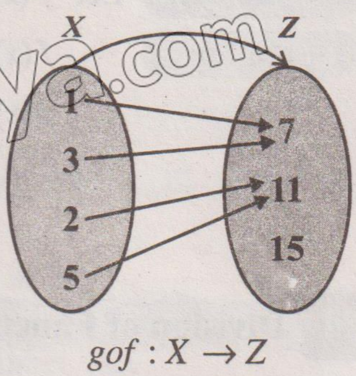
$f: X \to Y$

Figure 4.3



$g: Y \to Z$

Figure 4.4



$gof: X \to Z$

Figure 4.5

Example 7 If $f(x) = 2x + 3$, $g(x) = x^2$, then find

(i) $(fog)(x)$ **(ii)** $(gof)(x)$ **(iii)** $(fof)(x)$ **(iv)** $(gog)(x)$

Solution $f(x) = 2x + 3, g(x) = x^2$

(i)
$$(fog)(x) = f(g(x)) = f(x^2) = 2x^2 + 3$$

(ii)
$$(gof)(x) = g(f(x)) = g(2x+3)$$
$$(gof)(x) = (2x+3)^2 = 4x^2 + 12x + 9$$

(iii)
$$(fof)(x) = f(f(x)) = f(2x+3) = 2(2x+3)+3 = 4x+9$$

(iv)
$$(gog)(x) = g(g(x)) = g(x^2)$$
$$(gog)(x) = (x^2)^2 = x^4$$

> **Remember!**
> In general,
> $(fog)(x) \neq (gof)(x)$

> **One-to-One Function**
> A function f is said to be one-to-one (or injective) if each input maps to a unique output.
> **Onto Function**
> A function is called onto (or surjective) if every element in the codomain has at least one pre-image in the domain.
> **Bijective Function**
> A function is called bijective if it is both one-to-one and onto.

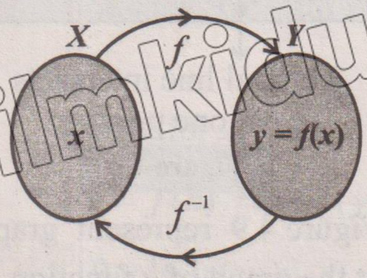
4.1.3 Inverse Function

A function f takes an input x from a set X and maps it to an output y in a set Y. i.e.,

$$f : X \to Y,\ f(x) = y$$

Then the inverse function of f denoted by f^{-1}, reverses this process. It takes y as input and gives x as output:

$$f^{-1} : Y \to X \quad , \quad f^{-1}(y) = x$$



Domain of f = Range of f^{-1}

Range of f = Domain of f^{-1}

> **Note**
> If $y = f(x)$ then $f^{-1}(y) = x$ is called inverse of $f(x)$

Let us understand it with the help of an example:

Consider $X = \{1, 2, 3\}$, $Y = \{a, b, c\}$

and $f : \{(1, a), (2, b), (3, c)\}$, then $f^{-1} : \{(a, 1), (b, 2), (c, 3)\}$

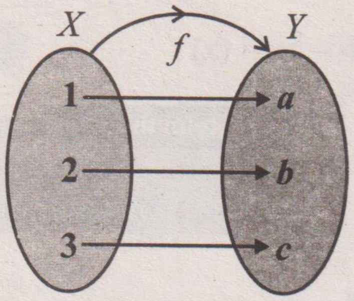


Figure 4.6

$f(1) = a$

$f(2) = b$

$f(3) = c$

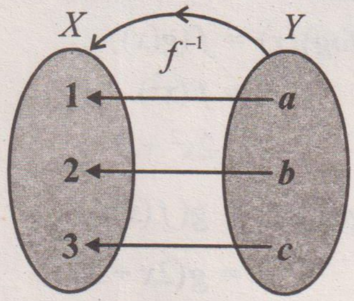


Figure 4.7

$f^{-1}(a) = 1$

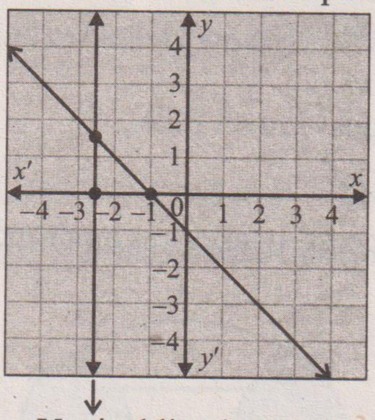
$f^{-1}(b) = 2$

$f^{-1}(c) = 3$

It is important to note that a function has an inverse if and only if it is one-to-one and onto (bijective).

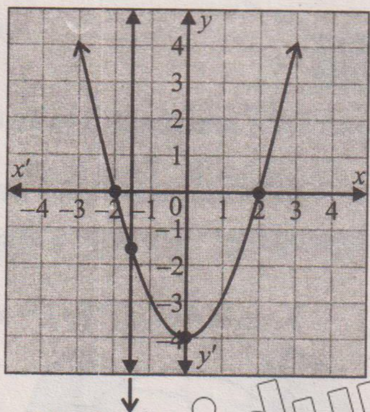
4.1.4 Vertical Line Test

A curve/line drawn in a graph represents a function, if every vertical line intersects the curve/line at most one point.



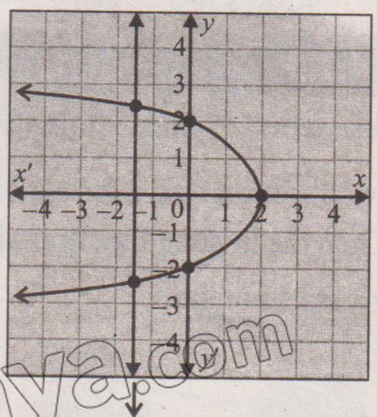
Vertical line intersects at one point

Figure 4.8



Vertical line intersects at one point

Figure 4.9



Vertical line intersects at two points

Figure 4.10

Graphs of Figure 4.8 and Figure 4.9 represent graphs of functions and graph of Figure 4.10 does not represent the graph of a function.

4.1.5 Horizontal Line Test

If every horizontal line intersects the curve at most one point, then the graph is a one-to-one function.

The vertical line intersects the graph of $f(x) = x^2$ at only one point, so it is the graph of a function. It fails in horizontal test, so it is not a one to one function.

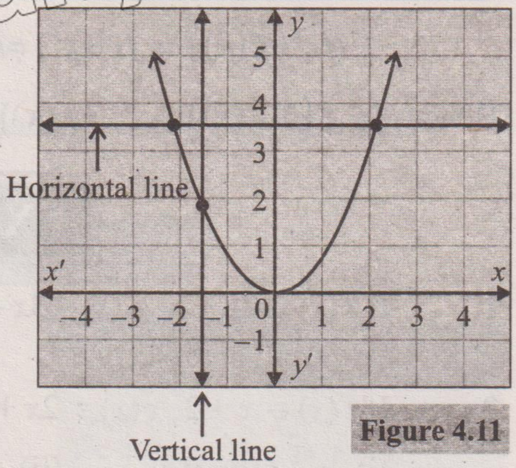


Figure 4.11

Example 8 Find the inverse of $f(x) = 2x + 3$. Also find domain and range of $f^{-1}(x)$.

Solution

$$f(x) = 2x + 3$$

$$y = 2x + 3 \qquad \text{[Replace } f(x) \text{ with } y]$$

$$y - 3 = 2x$$

$$\therefore \quad x = \frac{y-3}{2} \qquad \text{[Solve for } x]$$

$$f^{-1}(y) = \frac{y-3}{2} \qquad [\because \; y = f(x) \Rightarrow f^{-1}(y) = x]$$

$$f^{-1}(x) = \frac{x-3}{2} \qquad \text{[Replacing } y \text{ by } x]$$

$$\begin{cases} \text{Domain } f(x): & \text{All real numbers} \\ \text{Range } f(x): & \text{All real numbers} \end{cases}$$

$$\begin{cases} \text{Domain } f^{-1}(x): & \text{All real numbers} \\ \text{Range } f^{-1}(x): & \text{All real numbers} \end{cases}$$

Example 9 If $f(x) = x^3$, find $f^{-1}(x)$. Also verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

Solution

$$f(x) = x^3$$

$$y = x^3 \qquad \text{[Replace } f(x) \text{ with } y]$$

$$x^3 = y$$

$$x = y^{\frac{1}{3}} \qquad \text{[Solve for } x]$$

$$f^{-1}(y) = y^{\frac{1}{3}} \qquad [y = f(x) \Rightarrow f^{-1}(y) = x]$$

$$f^{-1}(x) = x^{\frac{1}{3}} \qquad \text{[Replacing } y \text{ by } x]$$

$$f\left(f^{-1}(x)\right) = f(x^{\frac{1}{3}}) = (x^{\frac{1}{3}})^3 = x$$

Challenge!

If $f(x) = -\frac{1}{4}x + 2$, then verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

$$f^{-1}(f(x)) = f^{-1}(x^3) = (x^3)^{\frac{1}{3}} = x$$

Hence $\quad f\left(f^{-1}(x)\right) = f^{-1}(f(x)) = x$

EXERCISE 4.1

1. If $f(x) = 2x + 5$, $g(x) = 3x - 2$, then find
   (i) $f(x) + g(x)$ (ii) $f(x) - g(x)$
2. If $f(x) = x + 2$, $g(x) = 2x + 4$, then find
   (i) $f(x) \cdot g(x)$ (ii) $g(x) \cdot f(x)$ (iii) $\dfrac{f(x)}{g(x)}$ (iv) $\dfrac{g(x)}{f(x)}$
3. For the functions f and g, find
   (a) $(fog)(x)$ (b) $(gof)(x)$ (c) $(fof)(x)$ (d) $(gog)(x)$
   Where,
   (i) $f(x) = 2x + 3$; $g(x) = x^3$ (ii) $f(x) = \dfrac{2}{x}, x \neq 0$; $g(x) = 2x^2 - 1$
   (iii) $f(x) = 2x - 1$; $g(x) = \dfrac{x+1}{2}$
4. Find the value of k, such that $(fog)(x) = (gof)(x)$, where $f(x) = 3x + 2$, $g(x) = 6x - k$.
5. Given that $f(x) = 3x + 2$ and $g(x) = 2x + 3$. Find
   (i) $g(f(4))$ (ii) $f\left(f(3)\right)$ (iii) $f\left(g(-2)\right)$
6. Find $f^{-1}(x)$ in each of the following:
   (i) $f(x) = 2x - 3$ (ii) $f(x) = 4x^3 - 1$
   (iii) $f(x) = \sqrt{x-1}, x \geq 1$ (iv) $f(x) = \dfrac{x+1}{3x-2}, x \neq \dfrac{2}{3}$
7. The functions f and g are defined such that $f(x) = 4x + 2$ and $g(x) = 6x - 18$.
   (i) Find $f^{-1}(x)$ and $g^{-1}(x)$. (ii) Find x if $f^{-1}(x) = g^{-1}(x)$.
8. Verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$
   (i) $f(x) = x - 6$ (ii) $f(x) = 7x - 4$
   (iii) $f(x) = \dfrac{x-3}{4}$ (iv) $f(x) = \dfrac{x-4}{x+2}, x \neq -2$
9. Without finding $f^{-1}(x)$, find domain and range of $f^{-1}(x)$:
   (i) $f(x) = 12x - 3$ (ii) $f(x) = \dfrac{1}{2}x + 8$

(iii) $f(x) = \dfrac{x}{1+x}, x \neq -1$ (iv) $f(x) = \sqrt{x-2}, x \geq 2$

10. Given that $f(x) = x^2 + 9$ and $g(x) = x + 21$. Find the values of a such that: $f(a) = g(a)$

4.2 Graphs of Absolute Valued Functions

An absolute valued function is a type of function that includes an algebraic expression enclosed within absolute value bars. Its general form is $f(x) = |x|$. The absolute value of a number represents its distance from zero on the number line and is always non-negative.

Remember!

The graphs of all other absolute value functions are variations of the parent function $f(x) = |x|$ obtained through transformations.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

The graph of $f(x) = |x|$ has V-shape and is symmetric with respected to the y-axis. The point where the graph changes direction is called the vertex (0, 0).

The vertex form of an absolute value function is $g(x) = a|x - h| + k$, where $a \neq 0$ and (h, k) is the vertex. Its graph is symmetric about the line $x = h$.

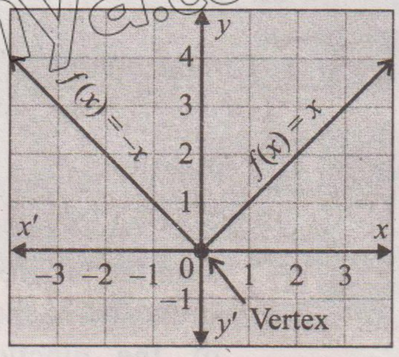


Figure 4.12

Example 10 Plot the graph of the following functions:

(i) $f(x) = |x| + 2$ (ii) $g(x) = |x - 3|$

Solution (i) $f(x) = |x| + 2$

Step I Make a table of values.

x	−4	−2	0	2	4...
$f(x)$	6	4	2	4	6...

Step II Plot the ordered pairs.

Step III Join the ordered pairs and draw the V-shaped graph.

(ii) $g(x) = |x - 3|$

Step I Make a table of values.

x	−4	−2	0	3	5...
$g(x)$	7	5	3	0	2...

Step II Plot the ordered pairs.

Step III Join the ordered pairs and draw the V-shaped graph.

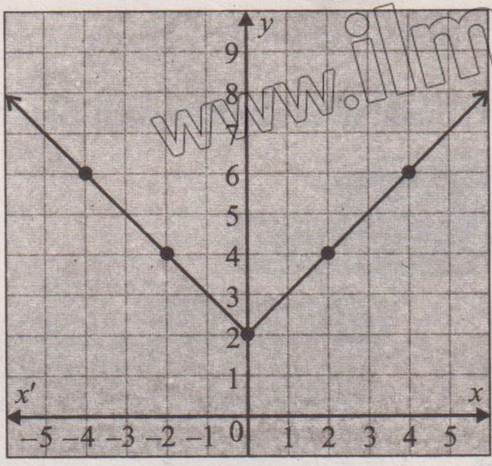


Figure 4.13

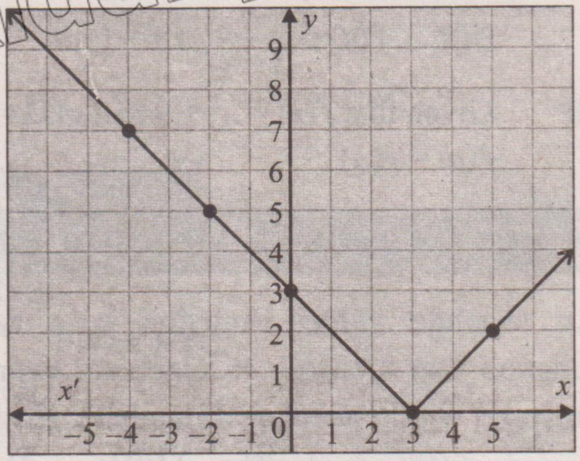


Figure 4.14

Example 11 Plot the graph of $f(x) = 2|x+1| - 3$.

Solution $f(x) = 2|x+1| - 3$

Step I Make a table of values.

x	-2	-1	0	$0.5...$
$f(x)$	-1	-3	-1	$0 ...$

Step II Plot the ordered pairs.

Step III Join the ordered pairs and draw the V-shaped graph of the function. Notice that the vertex of the graph of f is $(-1, -3)$ and the graph is symmetric about $x = -1$.

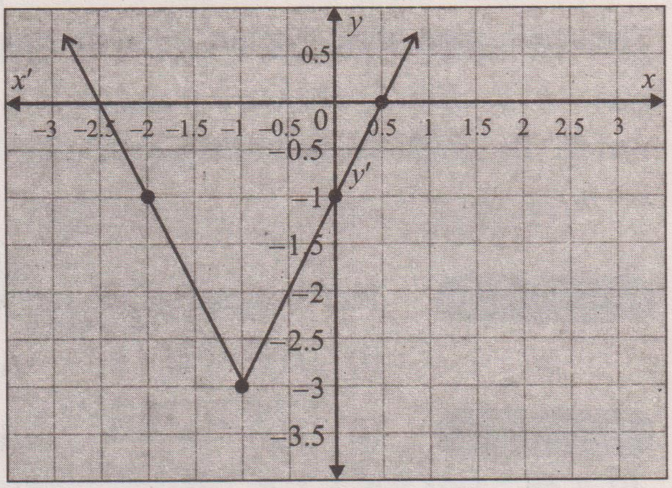


Figure 4.15

4.3 Solution of Absolute Valued Linear Equation in One Variable

An absolute value linear equation is an equation in which the absolute value is applied to a linear expression.

Example 12 Solve $2|x-3| = 16$ and express the solution on number line.

Solution

$2|x-3| = 16$

$|x-3| = 8$

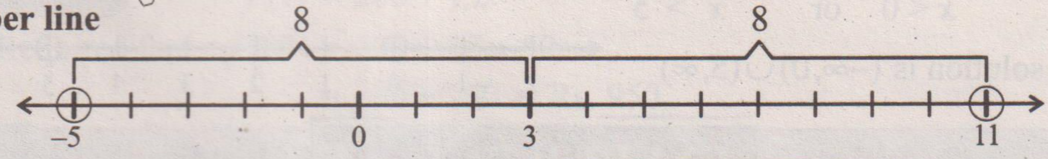
$\pm(x-3) = 8$ or $x - 3 = \pm 8$

Now $x - 3 = 8$ or $x - 3 = -8$

$x = 11$ or $x = -5$

Solution set = $\{11, -5\}$

Number line



4.4 Solution of Absolute Value Linear Inequality in One Variable

An absolute value linear inequality in one variable is an inequality that involves the absolute value of a linear expression with only one variable. It has forms such as:

$$|ax+b|<0,\ |ax+b|>0\ ,\ |ax+b|\geq 0\ ,\ |ax+b|\leq 0$$

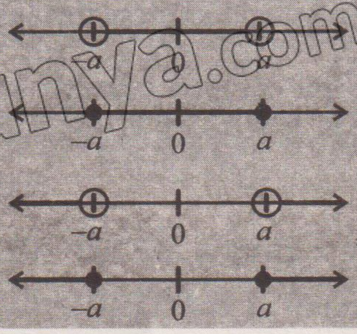
Note

(i) $-a < x < a$ is equivalent to $(-a, a)$

(ii) $-a \leq x \leq a$ is equivalent to $[-a, a]$

(iii) $x < -a$ or $x > a$ is equivalent to $(-\infty, -a) \cup (a, \infty)$

(iv) $x \leq -a$ or $x \geq a$ is equivalent to $(-\infty, -a] \cup [a, \infty)$



Example 13 Solve the inequality $|x-1| \geq 5$ and express the solution on number line.

Solution

$$|x-1| \geq 5$$

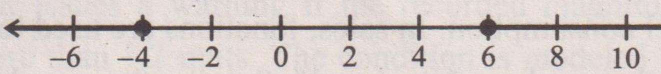
$$\pm(x-1) \geq 5$$

$-(x-1) \geq 5$ or $x - 1 \geq 5$

$x - 1 \leq -5$ or $x - 1 \geq 5$

$x \leq -4$ or $x \geq 6$

The solution is $(-\infty, -4] \cup [6, \infty)$



Example 14 Express the solution of $40|2x-5|+1 > 201$ on number line.

Solution First isolate the absolute value expression on one side of the inequality.

$$40|2x-5|+1 > 201$$

$$40|2x-5| > 200$$

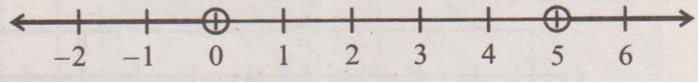
$$\frac{40|2x-5|}{40} > \frac{200}{40} \quad \text{(Divide each side by 40)}$$

$$|2x-5| > 5$$

$$\pm(2x-5) > 5$$

$$-(2x-5)>5 \quad \text{or} \quad 2x-5>5$$
$$2x-5<-5 \quad \text{or} \quad 2x-5>5$$
$$2x<0 \quad \text{or} \quad 2x>10$$
$$x<0 \quad \text{or} \quad x>5$$

The solution is $(-\infty, 0) \cup (5, \infty)$



EXERCISE 4.2

1. Plot graph for the following absolute valued functions:
   (i) $f(x) = |x-2|$ (ii) $f(x) = 3|x+3| - 4$
   (iii) $f(x) = 5|x|$ (iv) $f(x) = |x+2| + 3$
   (v) $f(x) = 2|x+4| - 3$ (vi) $f(x) = 2|x+1| - 6$
2. Solve and express the solution on number line:
   (i) $|x-2| = 6$ (ii) $|2x+1| = 7$ (iii) $|4x-9| = 3$ (iv) $|7-2x| = 1$
3. Solve and express the solution on number line:
   (i) $|5x+8| \leq 3$ (ii) $|4x-12| \leq 0$
   (iii) $|3-4x| > 0$ (iv) $1 - 2\left|\frac{3}{2}x - 5\right| > -3$
   (v) $|3x-2| \leq 12$ (vi) $|1-2x| > 5$

4.5 Solving Real-Life Problems Involving Function

Functions are widely used to model and solve real-life problems in areas such as finance, transportation and sales. In finance, functions can represent income, expenses, interest or profit over time. In transportation, functions can describe the relationship between distance, speed and time, helping to calculate travel times or fuel consumption. In sales, functions are used to model revenue, cost and profit based on the number of items sold. Solving these problems involves identifying the relevant variables, expressing their relationship as a function or equation and using algebraic methods to find unknown values, make decisions or predict outcomes.

Example 15 Let $E(n) = 1{,}500n$ is the total earning (in Rs.) function, where n is the number of days. If labourer works 10 days, find his earning.

Solution Given that: $E(n) = 1{,}500n$

If the labourer works for 10 days, his earning will be $E(10) = 1{,}500 \times 10$
$= \text{Rs. } 15{,}000$

Example 16 Let $f(x) = 200 + 15x$ is a function of charging taxi fare (in Rs.), where x represents the number of kilometres. Find proposed fare for a distance of 50 km.

Solution $f(x) = 200 + 15x$

Required fare $= f(50) = 200 + 15 \times 50$

$= 200 + 750 = \text{Rs. } 950$

4.6 Solving Real-Life Problems Involving Absolute Valued Functions

Absolute value functions are essential for solving real-life problems where distance, magnitude or fluctuations are involved, regardless of direction. In energy wave analysis, such as sound or seismic waves, absolute value functions model the amplitude or strength of a wave. In physics and engineering, magnitude is often expressed using absolute values to describe the size of a force, charge or vector, independent of direction. In distance-related problems, such as the location of an object from a reference point, absolute value functions measure how far one quantity is from another, ensuring the result is always non-negative. These functions help describe situations with symmetry or two-sided variation.

Example 17 A sensitive device operates best at $220V$ but can tolerate a variation of up to $5V$. The condition is modeled as: $|V - 220| \leq 5$. What is the range of acceptable voltages?

Solution $|V - 220| \leq 5$

or $-5 \leq V - 220 \leq 5$

or $-5 + 220 \leq V \leq 5 + 220$ or $215 \leq V \leq 225$

So, acceptable voltage is between $215V$ and $225V$.

Example 18 An earthquake sensor issues a warning if the recorded magnitude differs from a safe level of 4.5 by more than 1.2 units. The condition is modeled by $|M - 4.5| > 1.2$.

For which values of magnitude M will the sensor issue a warning?

Solution $|M - 4.5| > 1.2$

$\pm(M - 4.5) > 1.2$

$-(M - 4.5) > 1.2$ or $M - 4.5 > 1.2$

$M - 4.5 < -1.2$ or $M - 4.5 > 1.2$

$M < 3.3$ or $M > 5.7$

Hence, the sensor issues a warning when $M < 3.3$ or $M > 5.7$.

EXERCISE 4.3

1. A function $B(t) = 5{,}000 + 200t$ represents the total balance (in rupees) after t months. What will be the balance after 6 months?
2. A function $f(k) = 150 + 20k$ represents the total fare (in rupees) for k kilometres. How much will the fare be for a 12 kilometres ride?
3. The cost of manufacturing fancy sofa set would be fixed charges Rs. 5500 which is modeled as $f(n) = 5500\,n$, where n is the number of sofa sets. Find the cost of 50 sofa sets.
4. A function $T(d) = \dfrac{d}{60}$ represents the time T in hours to travel a distance d kilometres. How long will it take to travel 180 km?
5. A company charges Rs. 100 for an encoding work. In addition, the company charges Rs. 5 per page of printed output. The model of function $f(x) = 100 + 5x$, where x represents the number of pages printed out. How much will company charge for 55 page encoding and printing work?
6. A chemical reaction is stable at $37°C$. The process must be stopped if the temperature deviates by more than $2.5°C$. The condition is modeled as: $|T - 37°| > 2.5°$, T be the temperature. For what temperature values must the process be stopped?
7. A factory produces metal rods that must be 2.5 metres long, with a tolerance of ± 0.04 metres. An absolute value inequality models this: $|x - 2.5| \le 0.04$. What is the range of acceptable lengths?
8. A machine part must be aligned so that its centre is exactly at 0. If it shifts more than 0.1 mm, the part is rejected. The model is given by $|x| > 0.1$. What positions of the centre cause rejection?

REVIEW EXERCISE 4

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) If $f(x) = \dfrac{5x - 6}{3}$, then what is the value of $f(3)$?

(a) −1 (b) 3 (c) 9 (d) 15

(ii) A function f from X to Y is represented by:

(a) $f: XY$ (b) $f: Y \rightarrow X$ (c) $f: X \rightarrow Y$ (d) $f: \frac{X}{Y}$

(iii) $(fog)(x) =$

(a) $(f+g)(x)$ (b) $(f-g)(x)$ (c) $f(g(x))$ (d) $f(x) \div g(x)$

(iv) If $f(x) = 2x + 3$, $g(x) = x + 1$, then $f(x) + g(x) =$

(a) $3x$ (b) $3x + 4$ (c) 4 (d) $2x^2 + 3$

(v) If $f(x) = 5x + 2$, $h(x) = 2x - 2$, then $f(x) - h(x) =$

(a) $3x$ (b) $5x^2 - 4$ (c) $3x + 4$ (d) $-3x - 4$

(vi) If $f(x) = 3x + 1$, $g(x) = 2x$, then $g(x) \times f(x) =$

(a) $6x + 2x$ (b) $5x^2 + 1$ (c) $x + 1$ (d) $6x^2 + 2x$

(vii) If $f(x) = x^2 - 4$, $g(x) = x + 2, x \neq -2$, then $\frac{f(x)}{g(x)} =$

(a) $\frac{1}{x-2}$ (b) $\frac{1}{x+2}$ (c) $x + 2$ (d) $x - 2$

(viii) What is the shape of the graph of an absolute value function?

(a) U-shaped (b) V-shaped

(c) L-shaped (d) M-shaped

(ix) If a graph represents a function, then every vertical line must intersect it at:

(a) 4 points (b) 3 points (c) 2 points (d) 1 point

(x) If $f(x) = x^3$, then $f(-2) =$

(a) -8 (v) 8 (c) 4 (d) -6

2. If $f(x) = 25 - x^2$ and $g(x) = 5 + x$, then find

(i) $f(x) + g(x)$ (ii) $f(x) - g(x)$ (iii) $f(x) \cdot g(x)$

(iv) $\frac{f(x)}{g(x)}$ (v) $f(7)$ (vi) $g(-8)$

3. If $f(x) = x^3$ and $g(x) = 14 + 2x$, then find

(i) $(fog)(x)$ (ii) $(gof)(x)$

(iii) $(fof)(x)$ (iv) $(gog)(x)$

4. Find $f^{-1}(x)$, if

(i) $f(x) = 9x - 1$ (ii) $f(x) = \dfrac{5}{x-1}, x \neq 1$

(iii) $f(x) = \sqrt{x+5}, x \geq 5$ (iv) $f(x) = \dfrac{3-x}{2}$

5. Plot graph for the following absolute valued functions:

(i) $f(x) = 7|x|$ (ii) $f(x) = |x+6| - 2$

6. Solve and express the solution on number line:

(i) $|3x - 2| = 1$ (ii) $|6x + 1| = 9$

7. Solve and express the solution on number line:

(i) $|7 - 2x| \leq 1$ (ii) $|6x + 18| \leq 24$

8. Given that $f(x) = 3x + 7$, find

(i) $f^{-1}(x)$ (ii) the value of x for which $f(x) = f^{-1}(x)$

9. A company earns a profit of Rs. 100 for each item sold after a fixed monthly expense of Rs. 10,000. A function $P(x) = 100x - 10{,}000$ represents the profit after selling x items. How many items must be sold to break even (profit = 0)?

10. A store offers a 15% discount on a product. A function $D(p) = 0.85p$ gives the selling price after the discount on an original price p. What is the selling price of an item that originally costs Rs. 2,000?

11. A GPS system is considered accurate if the actual position differs from the reported position by no more than 6 metres. If the actual location is at point $x = 100$, the allowed range is defined as: $|x - r| \leq 6$, where r is the reported location. What is the range of acceptable reported locations?