CHAPTER 15
ELECTROMAGNETIC INDUCTION

MULTIPLE CHOICE

1. **A changing magnetic field produces around itself an induced**
   A magnetic field
   B electric field
   C electromotive force
   D gravitational field

2. **The magnitude of induced emf in a circuit depends upon:**
   A Change of flux
   B Rate of change of flux
   C Maximum flux
   D Minimum flux

3. **The direction of induced emf or current is found by the use of:**
   A Faraday's law
   B Amperes law
   C Lenz's law
   D None

4. **Lenz's law is in accordance with the law of conservation of:**
   A Momentum
   B Angular momentum
   C Energy
   D Mass

5. **The process in which a change of current in one coil causes an induced emf in another coil nearby it is called:**
   A Self induction
   B Mutual Induction
   C Henry effect
   D None

6. **The energy stored in an inductor is**
   A $\frac{1}{2} L I^2$
   B $L I^2$
   C $2 L I^2$
   D $\frac{L I^2}{4}$

7. **The transformer works on the base of**
   A mutual induction
   B 1st law of motion
   C 3rd law of motion
   D none of these

8. **The principle of A.C. generator is**
   A mutual induction
   B self induction
   C E M induction
   D All of these

9. **A transformer steps 220 volt to 40 volts if the secondary turns are 40 and primary turns are**
   A 20
   B 40
   C 120
   D 220

10. **Induced emf in a coil according to Faraday's Law is expressed as**
    A $E = N \frac{\Delta t}{\Delta \varphi}$
    B $-\frac{\Delta \varphi}{N \Delta t}$
    C $E = -N \frac{\Delta \varphi}{\Delta t}$
    D $E = \frac{\Delta t}{N \Delta \varphi}$

11. **The magnetic flux linked with a circuit when a unit current flows through it is known as**
    A Induced current
    B Induced emf
    C Coefficient of self inductance of a circuit
    D none of these

12. **The mutual inductance of coils does not depend upon:**
    A Stiffness of coil
    B Material of coils
    C Geometry of coil
    D Area of cross section of coils

13. **Henry is the unit of:**
    A. Self Inductance
    C. Both A & B
    B. Mutual inductance
    D. Magnetic flux

14. **The energy stored in magnetic field is given by**
    A. $L I^2$
    B. $\frac{1}{2} L^2 I$
    C. $I L^2$
    D. $\frac{1}{2} L I^2$

15. **The motional emf developed in a conductor depends upon:**
    A. Length
    B. Magnetic field
    C. Orientation
    D. All

16. **The effective value of A.C. voltage or current is:**
    A. $\sqrt{2}$ Times V^0
    B. $\frac{1}{\sqrt{2}}$ of V_0
    C. $\frac{1}{\sqrt{3}}$ of V_0
    D. None

17. **When the back emf in a circuit is zero, it draws**
    A. Zero Current
    C. Minimum Current
    B. Steady Average current
    D. Maximum Current

18. **The component in a generator which consumes electrical energy is called**
    A. Capacitor
    C. Split rings
    B. Load
    D. Commutator

19. **If we make the magnetic field stronger the value of induced current is**
    A. decreased
    C. vanished
    B. increased
    D. kept constant

20. **Alternating current is converted into direct current by:**
    A. Motor
    C. Rectifier
    B. Dynamo
    D. Transformer

21. **For electroplating we use:**
    A. D.C. source
    B. A.C. source
    C. Both A.C. & D.C
    D. None

22. **Which of the following is scalar quantity:**
    A. Flux density
    C. Induced emf
    B. Magnetic field
    D. Magnetic field intensity

23. **The induced current can be increased by:**
    A. Using strong magnetic field
    C. Replacing loop by coil of many turns
    B. Moving loop faster
    D. All

24. **One meter long rod is moving at 30° through a magnetic field of 1 T. If the velocity of rod is 1 m/s then induced emf in the rod will be:**
    A. 0.5 V
    B. 0.6 V
    C. 1 V
    D. 0.2 V

25. **Alternating current generators use**
    A. Coiled rings
    C. Slip rings
    B. Split rings
    D. Solenoid

26. **An inductor may store energy in**
    A. its magnetic field
    C. a neighbouring circuit
    B. its electric field
    D. its coils

27. **The coil connected with battery and rheostat in the experiment of mutual inductance in called:**
    A. Primary
    C. Loop
    B. Secondary
    D. Variable coil

28. In step down transformer the number of turns in the primary is written as:

    A. $N_p < N_s$ B $N_p = N_s$ C $N_p > N_s$ D None

29. The loss of energy over A.C. cycle is called:

    A. Magnetization loss   B De-magnetization loss

    C. Energy loss   D Hysteresis loss

30. The ratio of average induced emf to the rate of change of current in the coil is called

    A. Mutual inductance   B. Self inductance

    C. self induction   D. mutual induction

Answers:

1. B	2. B	3. C	4. C	5. B	6. A	7. A
8. C	9. D	10. C	11. C	12. A	13. C	14. D
15. D	16. B	17. D	18. B	19. B	20. C	21. C
22. A	23. D	24. A	25. C	26. A	27. A	28. C
29. D	30. B					

SHORT & LONG QUESTIONS

Q1: Describe the historical background of electromagnetic induction?

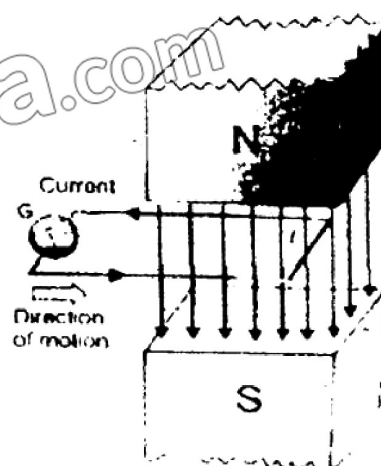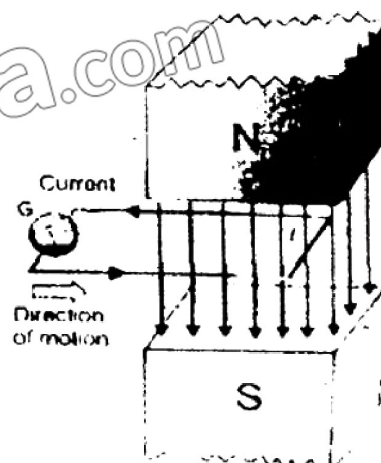**Ans:** **Historical background of electromagnetic induction:**

i. As soon as Oersted discovered that electric currents produce magnetic fields, many scientists began to look for the reverse effect, that is, to cause an electric current by means of a magnetic field.

ii. In 1831 Michael Faraday in England and at the same time Joseph Henry in USA observed that an emf is set up in a conductor when it moves across a magnetic field. If the moving conductor was connected to a sensitive galvanometer, it would show an electric current flowing through the circuit as long as the conductor is kept moving in the magnetic field.

iii. The emf produced in the conductor is called induced emf, and the current generated is called the induced current. This phenomenon is known as electromagnetic induction.

Q2: Discuss the different ways to produce induced emf and induced current?

Ans: **Induced emf and induced current by the magnetic field of a permanent magnet:**

i. Consider a straight piece of wire of length ℓ placed in the magnetic field of a permanent magnet. The wire is connected to a sensitive galvanometer. This forms a closed path or loop without any battery In the beginning when the loop is at rest in the magnetic field, no current is shown by the galvanometer.

ii. If we move the loop from left to right, the length ℓ of the wire is dragged across the magnetic field and a current flow through the loop. As we stop moving the

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C. self induction   D. mutual induction

Answers:

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8. C	9. D	10. C	11. C	12. A	13. C	14. D
15. D	16. B	17. D	18. B	19. B	20. C	21. C
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ii. If we move the loop from left to right, the length ℓ of the wire is dragged across the magnetic field and a current flow through the loop. As we stop moving the

loop. current also stops

iii. Now. if we move the loop in opposite direction, current also reverses its direction. This is indicated by the deflection of the galvanometer in opposite direction. Thus induced emf and induced current will produced.

Q3: On what factors induced current and emf depends?

Ans: The induced current depends upon the speed with which conductor moves and upon the resistance of the loop.

If we change the resistance of the loop by inserting different resistors in the loop, and move it in the magnetic field with the same speed every time, we find that the product of induced current I and the resistance R of the loop remains constant, i.e.,

$$I \times R = \text{constant} = \text{induced emf}$$

This constant is the induced emf. The induced emf leads to an induced current when the circuit is closed.

Methods to increase the current:

The current can be increased by

i) Using a stronger magnetic field:
Stronger is the magnetic field larger is the induced emf.

ii) Moving the loop faster:
It can be concluded that it is the relative motion of the loop and the magnet that causes the induced emf

iii) Replacing the loop by a coil of many turns
Induced emf $\propto$ No. of turns of coil

iv) Rate of change of flux:
The greater the rate of change of flux. the larger is the induced emf.

**Q4: Illustrate the different methods for the production of emf in a loop?**

Ans: There are some other methods described below in which an emf is induced in a loop by producing a change of magnetic flux through it. ·
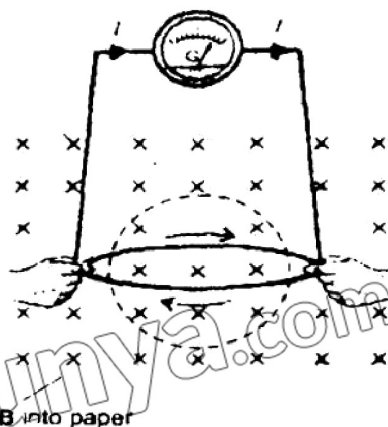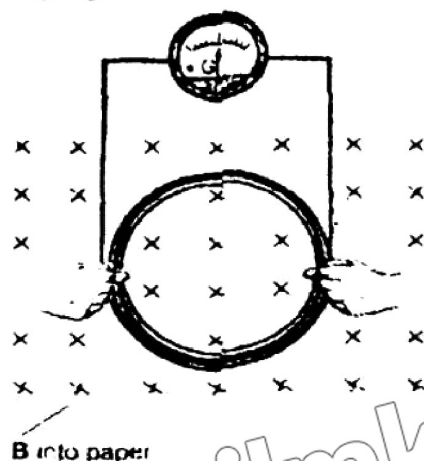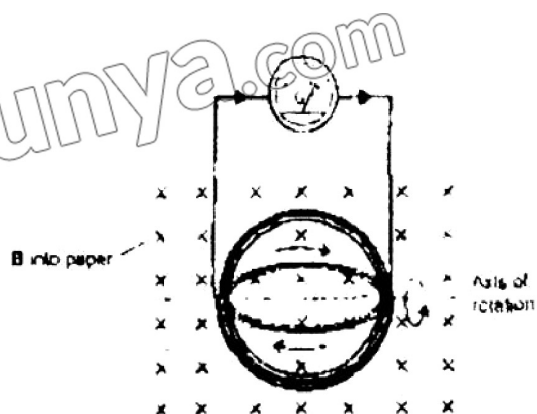
Method 1:

Fig. shows a bar magnet and a coil of wire to which a galvanometer is connected. When there is no relative motion between the magnet and the coil, the galvanometer indicates no current in the circuit. As soon as the bar magnet is moved towards the coil, a current appears in it. As the magnet is moved, the magnetic flux through the coil changes and this changing flux produces the induced current in the coil When the magnet moves away from the coil. a current is again induced but now in opposite direction. The current would also be induced if the magnets were held stationary and the coil is moved.

Method 2:

There is another method in which the current is induced in a coil by changing the area of the coil in a constant magnetic field Fig. shows that no current is induced in the
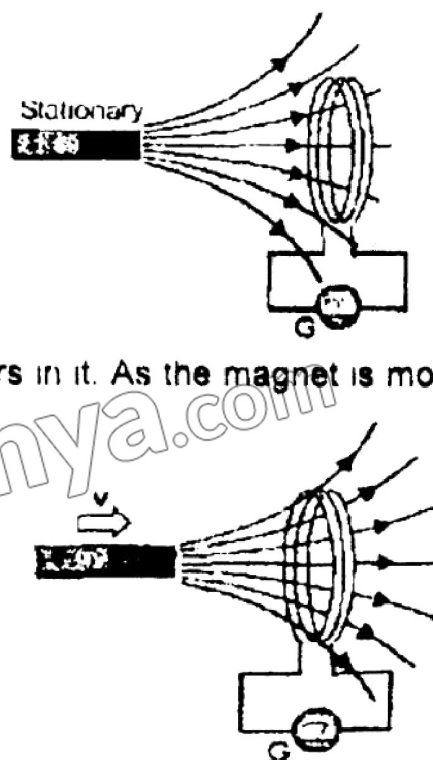
coil of constant area placed in a constant magnetic field. However, when the coil is being distorted so as to reduce its area, an induced emf and hence current appears. The current vanishes when the area is no longer changing. If the distorted coil is brought to its original circular shape thereby increasing the area, an oppositely directed current is induced which lasts as long as the area is changing.

B into paper

B into paper

Method 3:

An induced current can also be generated when a coil of constant area is rotated in a constant magnetic field. Here, also, the magnetic flux through the coil changes. This is the basic principle used in electric generators.

Method 4:

A very interesting method to induce current in a coil involves by producing a change of magnetic flux in a nearby coil.
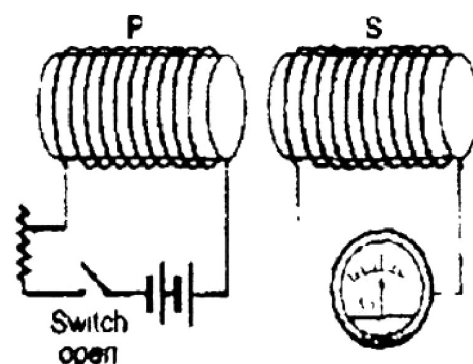

Fig. shows two coils placed side by side. The coil P is connected in series with a battery, a rheostat and a switch, while the other coil S is connected to a galvanometer only. Since there is no battery in the coil S, one might expect that the current through it will always be zero. Now, if the switch of the coil P is suddenly closed, a momentary current is induced in coil S. This is indicated by the galvanometer, which suddenly deflects and then returns to zero. No induced current exists in coil S as long as the current flows steadily in the coil P. An oppositely directed current is induced in the coil S at the instant the switch of coil P is opened. Actually, the current in P grows from zero to its maximum value just after the switch is closed. The current comes down to zero when the switch is opened. Due to change in
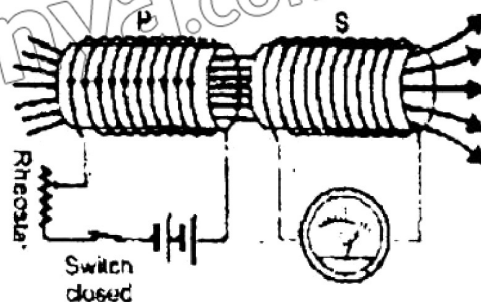
current. the magnetic flux associated with the coil P changes momentarily This changing flux is also linked with the coil S that causes the induced current in it. Current in coil P can also be changed with the help of rheostat

Method 5:

It is also possible to link the changing magnetic flux with a coil by using an electromagnet instead of a permanent magnet The coil is placed in the magnetic field of an electromagnet.

Both the coil and the electromagnet are stationary The magnetic flux through the coil is changed by changing the current of the electromagnet, thus producing the induced current in the coil.
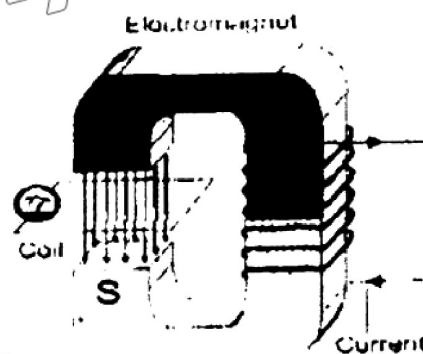
Q5: Define motional emf. Derive a relation for the magnitude of motional emf?

Ans: Motional emf:

The emf induced by the motion of a conductor across a magnetic field is called motional emf

Derivation of motional emf:

i. Consider a conducting rod of length L placed on two parallel metal rails separated by a distance L

ii. A galvanometer is connected between the ends c and d of the rails. This forms a complete conducting loop abcda

iii. A uniform magnetic field B is applied directed into the page.

iv. Initially when the rod is stationary, galvanometer indicates no current in the loop.

If the rod is pulled to the right with constant velocity v, the galvanometer indicates a current flowing through the loop Obviously, the current is induced due to the motion of the conducting rod across the magnetic field. The moving rod is acting as a source of emf
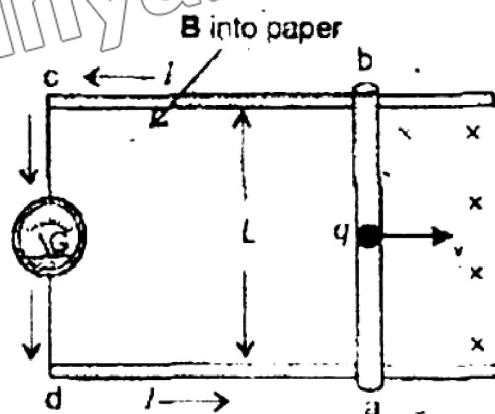
$$\varepsilon = V_b - V_a = \Delta V$$

v. When the rod moves, a charge q within the rod also moves with the same velocity v in the magnetic field B and experiences a force given by

$$F = qv \times B$$

The magnitude of the force is

$$F = q \, v \, B \sin\theta$$

Since angle θ between v and B is 90°, so

$$F = q \, vB$$

vi. Applying the right hand rule. we see that F is directed from a to b in the rod As a result the charge migrates to the top end of the conductor As more and more of the charges migrate concentration of the charge is produced at the top b and deficiency of charges at the bottom a This redistribution of charge sets up an electrostatic field E directed from b to a

vii. The electrostatic force on the charge is $F_e = qE$ directed from b to a. The system quickly reaches an equilibrium state in which these two forces on the charge are balanced. If E_0 is the electric intensity in this state then

$$qE_0 = qvB$$
$$E_0 = vB \qquad (1)$$

viii. The motional emf ε will be equal to the potential difference $\Delta V = V_L - V_a$ between the two ends of the moving conductor in this equilibrium state. The gradient of potential will be given by $\Delta V / L$. As the electric intensity is given by the negative of the gradient therefore,

$$E_0 = -\frac{\Delta V}{L} \qquad (2)$$

or $\qquad \Delta V = -LE_0 = -(LvB)$

ix. The motional emf:

$$\varepsilon = \Delta V = -LvB \qquad (3)$$

This is the magnitude of motional emf.

x. If the angle between v and B is θ, then

$$\varepsilon = -vBL \sin\theta \quad \dots\dots \quad (4)$$

Due to induced emf positive charges would flow along the path abcda, therefore the induced current is anticlockwise in the diagram.

Q6: This heater operates on the principle of electromagnetic induction. The water in the metal pot is boiling whereas that in the glass pot is not. Even the glass top of the heater is cool to touch. The coil just beneath the top carries ac current that produces changing magnetic flux. Flux linking with pots induces emf in them. Current is generated in the metal pot that heats up the water, but no current flows through the glass pan, why?

Ans: Current is generated in the metal pot because metals are good conductor of heat and electricity due to free electrons. The drag force provides the energy for the induced current to flow. This energy is the source of induced current.

No current flows through the glass pan because it is bad conductor of heat and electricity due to the absence of free electrons.

Q7: Define Faraday's law and establish the relationship between Faraday's law and induced emf?

Ans: Faraday's law of electromagnetic Induction:

"The average emf induced in a conducting coil of N loops is equal to the negative of the rate at which the magnetic flux through the coil is changing with time"

Faraday's law and induced emf:

Let the conducting rod L moves from position 1 to position 2 in a small interval of time Δt. The distance travelled by the rod in time Δt is $x_2 - x_1 = \Delta x$

Since the rod is moving with constant velocity v, therefore

$$v = \frac{\Delta x}{\Delta t} \qquad (1)$$

Since

$$\mathcal{E} = -vBL = -\frac{\Delta x}{\Delta t} BL \quad \ldots (2)$$

As the rod moves through the distance Δx, the increase in the area of loop is given by $\Delta A = \Delta x.L$. This increases the flux through the loop by $\Delta\phi = \Delta A.B = \Delta x.L.B$. Putting $\Delta x.L.B = \Delta\phi$ in Eq 2 we get

$$\mathcal{E} = -\frac{\Delta\phi}{\Delta t} \quad (3)$$

If there is a coil of N loops instead of a single loop, then the induced emf will become N times, i.e.,

$$\mathcal{E} = -N\frac{\Delta\phi}{\Delta t} \quad \ldots (4)$$

Although the above expression is derived on the basis of motional emf, but it is true in general. This conclusion was first arrived at by Faraday, so this is known as Faraday's law of electromagnetic induction.

Note:

The minus sign indicates that the direction of the induced emf is such that it opposes the change in flux.

Q8: **A cooper ring passes through a rectangle region where a constant magnetic field is directed into the page. What do you guess about the current in the ring at the positions 2, 3 and 4?**

Ans: i. Since the position **2** lies outside the magnetic field therefore current will be zero. ($B = \mu_0 n I$)

ii. Since the position **3** lies inside the magnetic field therefore current will be maximum. ($B = \mu_0 n I$)

iii. Since the position **4** lies outside the magnetic field therefore current will be zero. ($B = \mu_0 n I$)

Q9: **Define Lenz's law. How Lenz's law helps to find the direction of induced emf? Describe the relationship between Lenz's law and law of conservation of energy?**

Ans: **Lenz's law and direction of induced emf:**

The direction of the induced current is always so as to oppose the change which causes the current". This statement is known as Lenz's law.

He found that the polarity of an induced emf always leads to an induced current that opposes, through the magnetic field of the induced current, the change inducing the emf. The rule is known as Lenz's law.

Note:

The Lenz's law refers to induced currents and not to induced emf, which means that we can apply it directly to close conducting loops or coils. However, if the loop is not closed we can imagine as if it were closed, and then from the direction of induced current, we can find the direction of the induced emf.
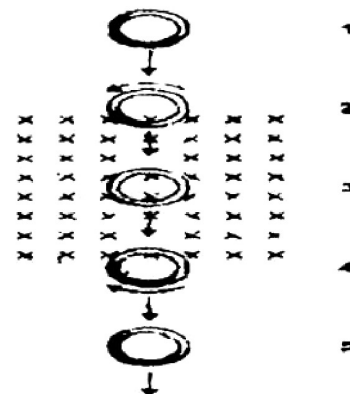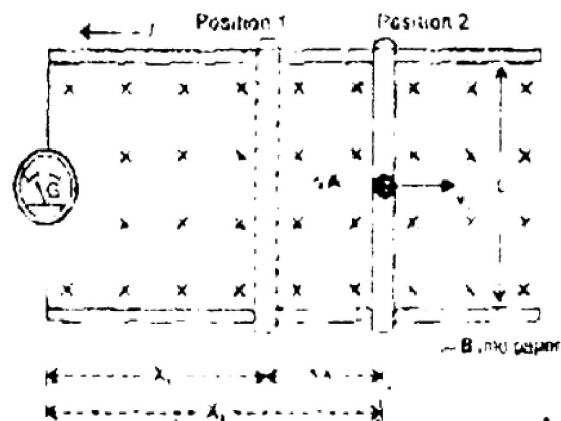
Application of Lenz's law to the point:

i. Let us apply the Lenz's law to the coil in which current is induced by the movement of a bar magnet. We know that a current carrying coil produces a magnetic field similar to that of a bar magnet.

ii. One face of the coil acts as the North Pole whiles the other one as the South Pole.

iii. If the coil is to oppose the motion of the bar magnet, the face of the coil towards the magnet must become a north pole.
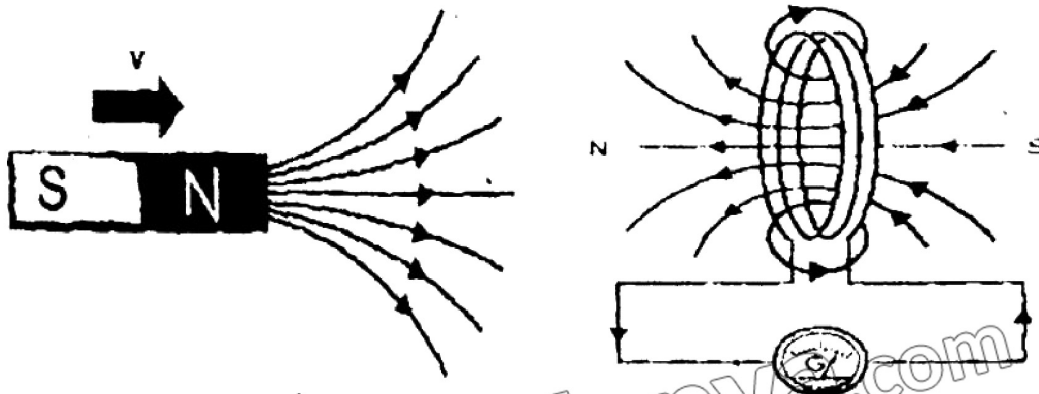
The two north poles will then repel each other.

iv. The right hand rule applied to the coil suggests that the induced current must be anticlockwise as seen from the side of the bar magnet.

v. **Application of "Push" and "Pull":**

According to Lenz's law the "push" of the magnet is the "change" that produces the induced current, and the current acts to oppose the push. On the other hand if we pull the magnet away from the coil the induced current will oppose the "pull" by creating a south pole on the face of coil towards the bar magnet.

Lenz's law and law of conservation of energy:

The Lenz's law is also a statement of law of conservation of energy that can be conveniently applied to the circuits involving induced currents. To understand this, consider once again the experiment in Fig.

i. When the rod moves towards right, emf is induced in it and an induced current flow through the loop in the anti-clockwise direction. Since the current carrying rod is moving in the magnetic field, it experiences a magnetic force F_m having the magnitude $F_m = IL \sin 90°$

ii. By right hand rule the direction of F_m is opposite to that of v, so it tends to stop the rod.

An external dragging force equal to F_m in magnitude but opposite in direction must be applied to keep the rod moving with constant velocity. This dragging force provides the energy for the induced current to flow. This energy is the source of induced current, thus electromagnetic induction is exactly according to law of conservation of energy.
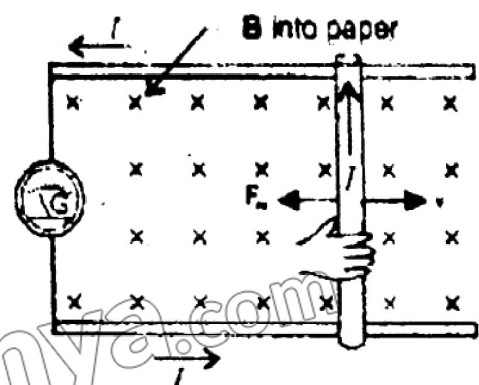
iii. Limitation of Lenz's law:

The Lenz's law forbids the induced current directed clockwise in this case. because the force F_m would be then, in the direction of **v** that would accelerate the rod towards right.

This in turn would induce a stronger current, the magnetic field due to it also increases and the magnetic force increases further. Thus the motion of the wire is accelerated more and more. Starting with a minute quantity of energy, we obtain an ever increasing kinetic energy of motion apparently from nowhere. Consequently the process becomes **self-perpetuating** which is against the law of conservation of energy.
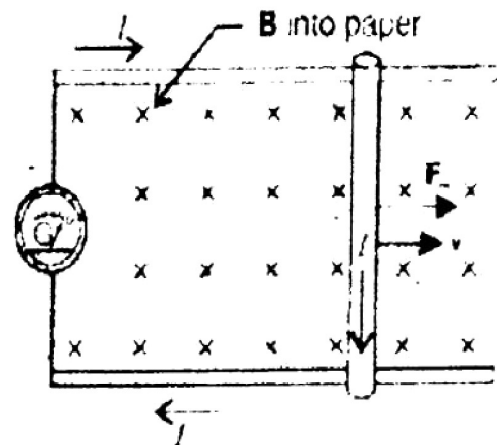
Q10: Define mutual induction. Explain how induced emf is produced in the secondary coil. What factors does mutual induction depends upon?
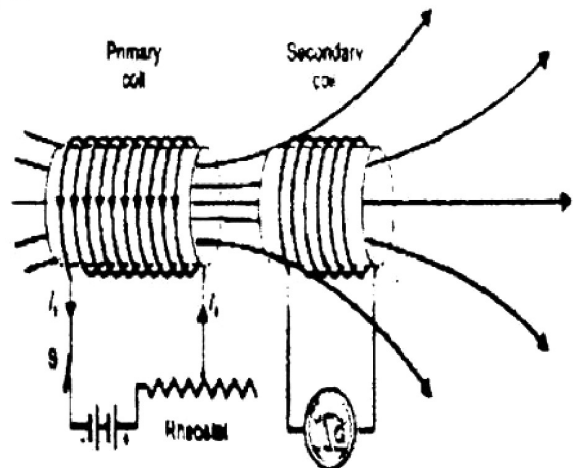
Ans: Mutual induction:

"The phenomenon in which a changing current in one coil induces an emf in another coil is called the mutual induction."

Derivation of mutual induction:

i. Consider two coils placed close to each other

One coil connected with a battery through a switch and a rheostat is called the "primary" and the other one connected to the galvanometer is called the "secondary".

ii. If the current in the primary is changed by varying the resistance of the rheostat, the magnetic flux in the surrounding region changes. Since the secondary coil is in the magnetic field of the primary, the changing flux also links with the secondary. This causes an induced emf in the secondary.

iii. According to Faraday's law, the emf induced in the secondary coil is proportional to the rate of change of flux $\frac{\Delta \phi_s}{\Delta t}$ passing through it, i.e.,

$$\mathcal{E}_s = -N_s \frac{\Delta \phi_s}{\Delta t}$$

Where N_s is the number of turns in the secondary coil.

iv. Mutual inductance:

Let the flux passing through one loop of the secondary coil be ϕ_s. Net flux passing through the coil of N_s loops is $N_s \phi_s$. As this flux is proportional to the magnetic field produced by the current I_p in the primary and the magnetic field itself is proportional to I_p, therefore

$$N_s \phi_s \propto I_p$$

$$N_s \phi_s = M I_p \qquad \dots\dots (1)$$

Where $M = \dfrac{N_s \phi_s}{i_p}$ is proportionality constant called the mutual inductance of the two coils. It depends upon the number of turns of the coils, their area of cross-section, their closeness together and the nature of the core material upon which the two coils are wound.

v. By Faraday's law the emf induced in the secondary coil is given by the rate of change of flux through the secondary

$$\varepsilon_s = -N_s \frac{\Delta \phi_s}{\Delta t} = -\frac{\Delta N_s \phi_s}{\Delta t}$$

Putting $N_s\phi_s = Mi_p$ from Eq.1

$$\varepsilon_s = -\frac{\Delta(Mi_P)}{\Delta t}$$

$$\varepsilon_s = -M\frac{\Delta i_p}{\Delta t} \quad \ldots\ldots\ldots\ldots (2)$$

The Eq.2 shows that the emf induced in the secondary is proportional to the time rate of change of current in the primary.

Note:

The negative sign in Eq.2 indicates the fact that the induced emf is in such a direction that it opposes the change of current in the primary coil. While finding out the value of M from Eq.2, negative sign is ignored. Thus M may be defined as the ratio of average emf induced in the secondary to the time rate of change of current in the primary.

$$M = \frac{\varepsilon_s}{\frac{\Delta i_p}{\Delta t}}$$

Unit of mutual induction:

The SI unit for the mutual inductance M is Vs A^{-1}, which is called as henry (H) after Joseph Henry.

Henry:

"One henry is the mutual inductance of the pair of coils in which the rate of change of current of one ampere per second in the primary causes an induced emf of one volt in the secondary."

Q11: Define self induction. Derive a relation for the magnitude of self induction?

Ans: Self induction:

"The phenomenon in which a changing current in a coil induces an emf in itself is called self induction."

Explanation:

According to Faraday's law, the change of flux through a coil by any means induces an emf in it. In all the examples we have discussed so far, induced emf was produced by a changing magnetic flux from some external source. But the change of flux through a coil may also be due to a change of current in the coil itself. Such phenomenon is known as self induction.
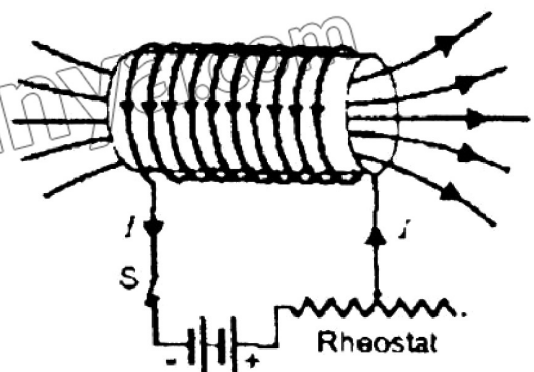
Derivation of self induction:

i. Consider the circuit shown in Fig. A coil is connected in series with a battery and a rheostat.

ii. Magnetic flux is produced through the coil due to current in it. If the current is changed by varying the rheostat quickly, magnetic flux through the coil changes that causes an induced emf in the coil. Such an emf is called as self induced emf.

iii. **Self inductance:**

If the flux through one loop of the coil be ϕ, then the total flux through the coil of N turns would be $N\phi$. As ϕ is proportional to the magnetic field which is in turn proportional to the current I, therefore

$$N\phi \propto I$$
$$N\phi = LI \qquad (1)$$

Where $L = \dfrac{N\phi}{I}$ is the constant of proportionality called the self inductance of the coil. It depends upon the number of turns of the coil, its area of cross-section and the core material. By winding the coil around a ferromagnetic (iron) core, the magnetic flux and hence the inductance can be increased significantly relative to that for an air core.

iv. By Faraday's Law, emf induced in the coil is

$$\varepsilon_L = -N\frac{\Delta\phi}{\Delta t} = -\frac{\Delta(N\phi)}{\Delta t}$$

Putting $\qquad N\phi = LI$ from Eq 1

$$\varepsilon_L = -\frac{\Delta(LI)}{\Delta t}$$

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t} \qquad (2)$$

The Eq 2 shows that the self induced emf in a coil is proportional to the time rate of change of current in the coil.

v. **Self inductance (L):**

Self inductance L of a coil may be defined as the ratio of the emf to the rate of change of current in the coil.

Unit of Self inductance (L):

The unit of L is also henry (H)

Back emf: $\quad \varepsilon_L = -L\dfrac{\Delta I}{\Delta t}$

The negative sign in Eq 2 indicates that the self induced emf must oppose the change that produced it. That is why the self induced emf is sometimes called as back emf. This is exactly in accord with the Lenz's law.

Note:

If the current is increased, the induced emf will be opposite to that of battery and if the current is decreased the induced emf will aid, rather than opposing the battery.

Inductor:

Because of their self-inductance, coils of wire are known as inductors, and are widely used in electronics. In alternating current, inductors behave like resistors.

Q12: Derive a relation for energy stored in an inductor and energy density?

Ans: **Energy stored in an inductor:**

As in case of a capacitor energy is stored in the electric field between the plates likewise in an inductor, energy is stored in the magnetic field.

Derivation of energy stored in an inductor:

i. Consider a coil connected to a battery and a switch in series. When the switch is turned on voltage V is applied across the ends of the coil and current through it raises from zero to its maximum value I.

ii. Due to change of current, an emf is induced, which is opposite to that of battery. Work is done by the battery to move charges against the induced emf.

iii. Work done by the battery in moving a small charge Δq is

$$W = \Delta q\, \mathcal{E}_L \quad \ldots\ldots \ (1)$$

where $\mathcal{E}_L$ is the magnitude of induced emf, given by

$$\mathcal{E}_L = L\frac{\Delta I}{\Delta t}$$

Putting the value of $\mathcal{E}_L$ in Eq. 1 we get

$$W = \Delta q.L\frac{\Delta I}{\Delta t} = \frac{\Delta q}{\Delta t}.L\Delta I \quad (2)$$

iv. Total work done in establishing the current from 0 to I is found by inserting for $\frac{\Delta q}{\Delta t}$, the average current and the value of ΔI

$$\text{Average current} = \frac{\Delta q}{\Delta t} = \frac{0+I}{2} = \frac{1}{2}I$$

$$\text{Change in current} = \Delta I = I - 0 = I$$

$$\text{Total work} = W = (\tfrac{1}{2}I)L I = \frac{1}{2}I^2$$

This work is stored as potential energy in the inductor. Hence the energy stored in an inductor is

$$U_m = \frac{1}{2}L I^2 \quad \ldots\ldots\ldots \ (3)$$

Energy stored in an inductor (U_m) and magnetic field (B):

Therefore, Eq. 3 can be expressed in terms of the magnetic field B of a solenoid which has n turns per unit length and area of cross section A. The magnetic field strength inside it is $B = \mu_0 n I$. Since flux through the coil is

$$\phi = BA$$

$$\phi = \mu_0 n I \quad \ldots\ldots \ (4)$$

Substituting the value of ϕ in Eq.

$$N\phi = LI \quad \text{or} \quad L = \frac{N\phi}{I}$$
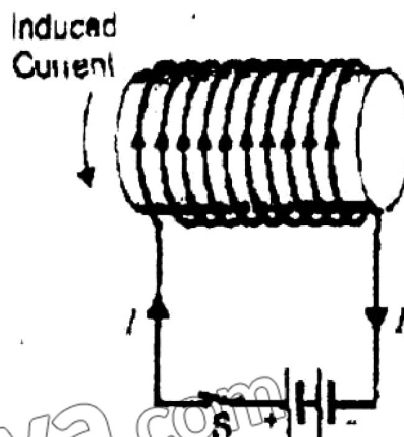
we have

$$L = N\frac{\mu_0 n I A}{I} = N \mu_0 nA$$

If ℓ is the length of solenoid, then putting $N = n\ell$ in above Eq. we get the self inductance of the solenoid as

$$L = (n\ell)\mu_0 nA$$

$$L = \mu_0 n^2 A \ell \quad \ldots\ldots \ (5)$$

Substituting for L, the Eq 3 becomes,

$$U_m = \frac{1}{2}(\mu_0 n^2 A \ell)I^2 \ldots\ldots \ (6)$$

Since B for a solenoid is given by

$$B = \mu_0 n I \qquad or \qquad I = \frac{B}{\mu_0 n}$$

Substituting for I, Eq.6 becomes

$$U_m = \frac{1}{2}(\mu_0 n^2 A \ell)\left(\frac{B}{\mu_0 n}\right)^2$$

$$U_m = \frac{1}{2}\frac{B^2}{\mu_0}(A \ell) \qquad \ldots\ldots\ldots (7)$$

Energy density:

Energy density can be defined as the energy stored per unit volume inside the solenoid, so dividing Eq.7 by the volume $(A\ell)$, we get energy density,

$$u_m = \frac{1}{2}\frac{B^2}{\mu_0}$$

Q13: What is alternating current generator? Describe its principle, construction and working? Also derive a relation for induced emf and current?

Ans: Alternating current generator (A.C. generator):

"A current generator is a device that converts mechanical energy into electrical energy."

Principle of A.C. generator:

The principle of an electric generator is based on Faraday's law of electromagnetic induction.

Working of A.C. generator:

When a coil is rotated in a magnetic field by some mechanical means, magnetic flux through the coil changes, and consequently an emf is induced in the coil.

If the generator is connected to an external circuit, an electric current is the output of the generator.

(Front view)

Construction of A.C. generator:
Rectangular loop of wire:

Let a rectangular loop of wire of area A be placed in a uniform magnetic field B. The loop is rotated about z-axis through its centre at constant angular velocity ω.

Slip rings:

One end of the loop is attached to a metal ring R and the other end to the ring R' These rings called the slip rings are concentric with the axis of the loop and rotate with it.

(Top view)

Carbon brushes:

Rings R, R' slide against stationary carbon brushes to which external circuit is connected.

Calculation of induced emf in the loop:

I.   To calculate the induced emf in the loop, consider its position while it is rotating anticlockwise. The figure shows the top view of the coil.
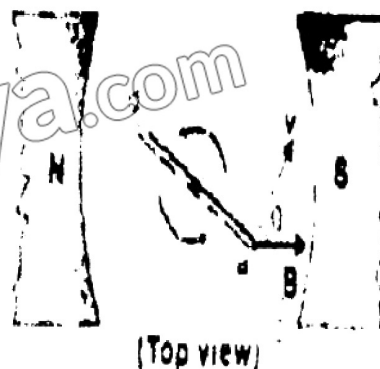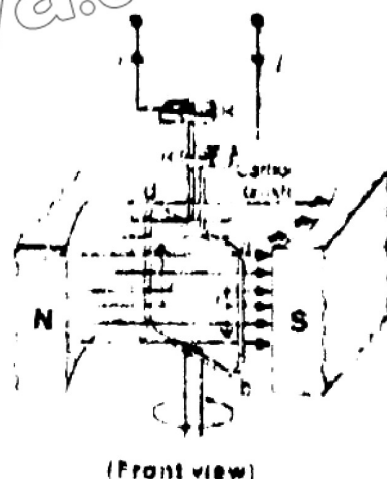
ii. The vertical side ab of the loop is moving with velocity v in the magnetic field B. If the angle between v and B be θ, the motional emf induced in the side ab has the magnitude.

$$\varepsilon_{ab} = vBL \sin\theta$$

The direction of induced current in the wire ab is the same as that of force F experienced by the charges in the wire, i.e., from top to the bottom.

iii. The same amount of emf is induced in the side cd but the direction of current is from bottom to the top. Therefore

$$\varepsilon_{cd} = vBL \sin\theta$$

iv. The net contribution to emf by sides bc and da is zero because the force acting on the charges inside bc and da is not along the wire. Thus

$$\varepsilon_{bc} = \varepsilon_{da} = 0$$

v. Since both the emfs in the sides ab and cd drive current in the same direction around the loop, the total emf in the loop is

$$\varepsilon = \varepsilon_{ab} + \varepsilon_{cd}$$
$$\varepsilon = vBL \sin\theta + vBL \sin\theta = 2vBL \sin\theta$$

vi. If the loop is replaced by a coil of N turns, the total emf in the coil will be,

$$\varepsilon = 2NvBL \cdot \sin\theta \quad\dots\dots\quad (1)$$

vii. The linear speed v of the vertical wire is related to the angular speed ω by the relation

$$v = \omega r$$

where r is the distance of the vertical wires from the centre of the coil. Substituting ωr for v in Eq. (1), We get

$$\varepsilon = 2N(\omega r)BL \sin\theta$$
$$\varepsilon = N\omega(2rL)B \sin\theta$$
$$\varepsilon = N\omega AB\sin\theta \quad\dots\dots\quad (2)$$

where A = 2rL = area of the coil

As the angular displacement θ = ωt, so the Eq. 2 becomes

$$\varepsilon = N\omega AB \sin(\omega t). \quad (3)$$

viii. Maximum value of emf (ε_0):

Eq.3 shows that the induced emf varies sinusoidally with time. It has the maximum value ε_0 when sin (ωt) is equal to 1. Thus

$$\varepsilon_0 = N\omega AB\dots\dots\quad (4)$$

The Eq. 3 can be written as,

$$\varepsilon = \varepsilon_0 \sin(\omega t) \quad (5)$$

ix. Induced current:

If R is the resistance of the coil, then by Ohm's law, induced current in the coil will be

$$I = \frac{\varepsilon}{R} = \frac{\varepsilon_0 \sin(\omega t)}{R} = \frac{\varepsilon_0}{R} \sin(\omega t)$$

or

$$I = I_0 \sin(\omega t)\dots\dots\quad (6)$$

where I_0 is maximum current.

x. Relation of induced emf and induced current with frequency:

Angular speed ω of the coil is related to its frequency of rotation f as.

$$\omega = 2\pi f$$
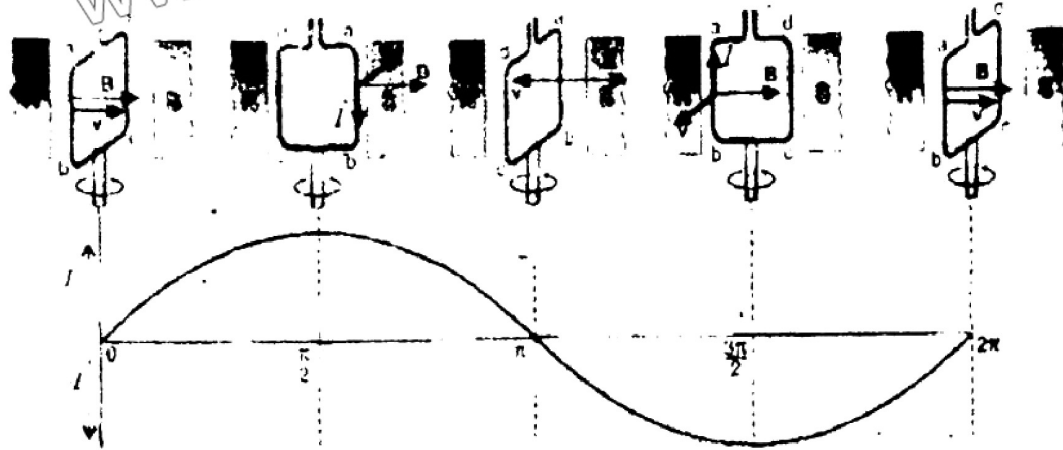
The Eqs. 5 and 6 can be written as

$$\varepsilon = \varepsilon_0 \sin(2\pi f t) \quad\dots\dots\quad (7)$$

$$I = I_o \sin(2\pi f t) \qquad \qquad (8)$$

Eq. 8 indicates the variation of current as a function of $\theta = 2\pi f t$.

Graphical representation for the current and different positions of coil during one loop:

Fig. shows the graph for the current corresponding to different positions of one loop of the coil.

Case 1:

When the angle between v & B is $\theta = 0°$, the plane of the loop is perpendicular to B, current is zero.

Case 2:

As θ increases, current also increases and at $\theta = 90° = \pi/2$ rad, the loop is parallel to B, current is maximum, directed along abcda.

Case 3:

On further increase in θ current decreases, and at $\theta = 180° = \pi$ rad the current becomes zero as the loop is again perpendicular to B.

Case 4:

For $180° < \theta < 270°$ current increases but reverses its direction as is clear from the figure. Current is now directed along dcbad. At $\theta = 270° = 3\pi/2$ rad, current is maximum in the reverse direction as the loop is parallel to B.

Case 5:

At $\theta = 360° = 2\pi$ rad, one rotation is completed, the loop is perpendicular to B and the current decreases to zero. After one rotation the cycle repeats itself.

Note:

The current alternates in direction once in one cycle. Therefore, such a current is called the alternating current. It reverses its direction f times per second.

Q14: What is armature?

Ans: Armature:

In actual practice a number of coils are wound around an iron cylinder which is rotated in the magnetic field. This assembly is called an armature. The magnetic field is usually provided by an electromagnet. Armature is rotated by a fuel engine or a turbine run by a waterfall. In some commercial generators, field magnet is rotated around a stationary armature.

Q15: Describe its principle, construction and working of D.C. generator?

OR

Differentiate between A.C. and D.C. generator?

Ans: D.C. generator:

D C generator is the device which converts mechanical energy into electrical energy in the form of direct current.

Principle of D.C. generator:

The principle of D C generator is based upon the fact that when magnetic flux changes through a coil and emf is induced in it

Limitations of Alternating current generator:

Alternating current generators are not suitable for many applications, for example, to run a D.C. Motor In 1834 William Sturgeon invented a simple device called a commutator that prevents the direction of current from changing.

Difference between D.C. generator and A.C generator:

I. A D C generator is similar to the A.C generator in construction with the difference that "slip rings" are replaced by "split rings" The "split rings" are two halves of a ring that act as a commutator

Fig shows the "split rings" A and A' attached to the two ends of the coil that rotates in the magnetic field When the current in the coil is zero and is about to change direction, the split rings also change the contacts with the carbon brushes BB' In this way the output from BB' remains in the same direction, although the current is not constant in magnitude
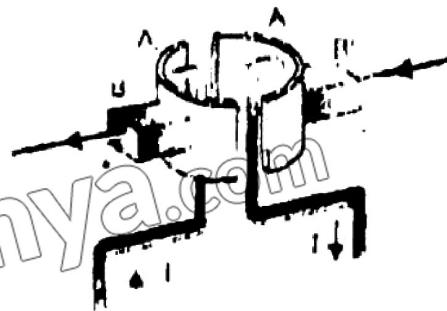
II. The curve of the current is shown in Fig. It is similar to a **sine curve** with the lower half inverted
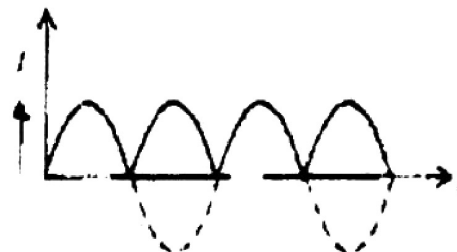
III. The fluctuations of the output can be significantly reduced by using many coils rather than a single one

IV. Multiple coils are wound around a cylindrical core to form the armature Each coil is connected to a separate commutator and the output of every coil is tapped only as it reaches its peak emf. Thus the emf in the outer circuit is almost constant.

Q16: Explain the phenomenon of back motor effect in generators?

Ans: Back motor effect in generators:

I. The generator supplies current to the external circuit The devices in the circuit that consume electrical energy are known as the "load". The greater the load the larger the current is supplied by the generator.

II. When the circuit is open the generator does not supply electrical energy, and a very little force is needed to rotate the coil.

III. As soon as the circuit is closed, a current is drawn through the coil.

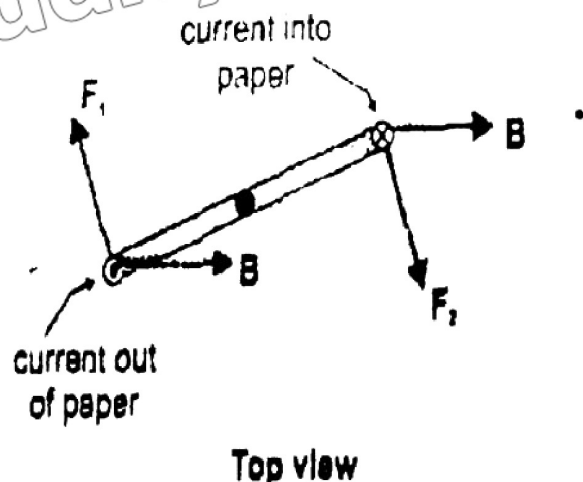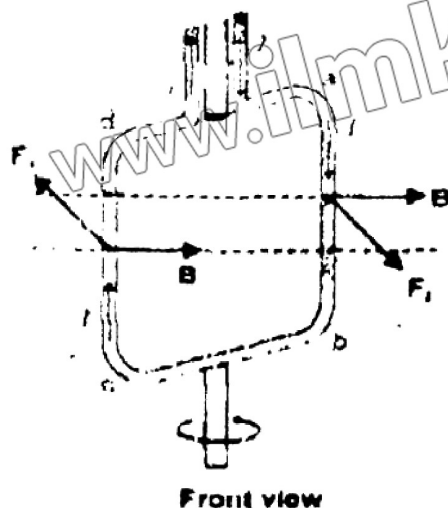IV. The magnetic field exerts force on the current carrying coil

Front view   **Top view**

Fig. shows the forces acting on the coil. Force F_1 is acting on the left side of the coil whereas an equal but opposite force F_2 acts on the right side of the coil. These forces are such that they produce a counter torque that opposes the rotational motion of the coil. This effect is sometimes referred to as back motor effect in the generators.

v. The larger the current drawn, the greater is the counter torque produced. That means more mechanical energy is required to keep the coil rotating with constant angular speed. This is in agreement with the law of conservation of energy The energy consumed by the "load" must come from the "energy source" used to drive the turbine.

Q17: Describe its principle, construction and working of D.C. motor?

Ans: D.C. motor:

A motor is a device which converts electrical energy into mechanical energy.

Principle of D.C. motor:

A wire carrying current placed in a magnetic field experiences a force. This is the basic principle of an electric motor

Construction of D.C. motor:

In construction a D C motor is similar to a D C generator, having three basic components

I)   **a magnetic field,**
II)   **a commutator**
III)   **an armature**

Difference between generator and D.C. motor:

In the generator the armature is rotated in the magnetic field and current is the output In the D C motor current passes through the armature that rotates in the magnetic field In the D C motor the brushes are connected to a D C supply or battery

Working of D.C. motor:

I. When current flows through the armature coil, the force on the conductors produces a torque that rotates the armature The amount of this torque depends upon the current, the strength of the magnetic field, the area of the coil and the number of turns of the coil
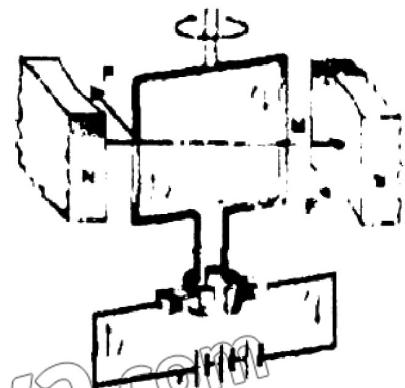
ii. If the current in the coil were all the time in the same direction, the torque on it would be reversed after each half revolution. But at this moment, commutator reverses the direction of current that keeps the torque always in the same sense.

iii. **Creation of jerk in the smooth running of motor:**

A little problem arises due to the use of commutator. that is, the torque vanishes each time the current changes its direction. This creates jerks in the smooth running of the armature. However the problem is overcome by using more than one coil wrapped around a soft-iron core. This results in producing a more steady torque.

iv. **Function of field coils in motor:**

The magnetic field in the motor is provided by a permanent magnet or an electromagnet. The windings of the electromagnet are usually called the field coils. The field coils may be in series or in parallel to the armature coils.

Q18: Discuss back emf effect in motors?

Ans: Back emf effect in motors:

i. A motor is just like a generator running in reverse. When the coil of the motor rotates across the magnetic field by the applied potential difference V, an emf ε is induced in it. The induced emf is in such a direction that opposes the emf running the motor. **Due to this reason the induced emf is called back emf of the motor.** The magnitude of the back emf increases with the speed of motor.

ii. Since V and ε are opposite in polarity, the net emf in the circuit is $V - \varepsilon$. If R is the resistance of the coil and I is the current drawn by the motor, then by Ohm's law

$$I = \frac{V - \varepsilon}{R} \quad \text{or} \quad V = \varepsilon + IR \qquad (1)$$

iii. **Back emf and starting of motor:**

When the motor is just started, back emf is almost zero and hence a large current passes through the coil. As the motor speeds up, the back emf increases and the current becomes smaller and smaller.

However, the current is sufficient to provide torque on the coil to drive the load and to overcome losses due to friction.

v. **Burn out of motor:**

If the motor is overloaded, it slows down. Consequently, the back emf decreases and allows the motor to draw more current. If the motor is overloaded beyond its limits, the current could be so high that it may burn out the motor.

Q19: Describe its principle, construction and working of transformer?

Ans: Transformer:

A transformer is an electrical device used to change a given alternating emf into a larger or smaller alternating emf.

Principle of transformer:

It works on the principle of mutual induction between two coils.

Construction of transformer:

The transformer consists of two coils of copper, electrically insulated from each other wound on the same iron core.

Primary coil:

The coil to which A C power is supplied is called primary coil

Secondary coil:

The coil from which power is delivered to the circuit is called the secondary coil

Note:

It should be noted that there is no electrical connection between the two coils but they are magnetically linked

The instantaneous value of the self induced emf:

Suppose that an alternating emf is applied to the primary. If at some instant (the flux in the primary is changing at the rate of $\frac{\Delta\phi}{\Delta t}$ then there will be back emf induced in the primary which will oppose the applied voltage. The instantaneous value of the self induced emf is given by

$$\text{Self induced emf} = -N_P \left[\frac{\Delta\phi}{\Delta t}\right]$$

Back emf:

If the resistance of the coil is negligible then the back emf is equal and opposite to applied voltage V_P.

$$V_p = -\text{back emf} = N_p\left[\frac{\Delta\phi}{\Delta t}\right] \qquad (1)$$

Where N_p is the number of turns in the primary.

Magnitude of the induced emf across the secondary coil:

Assuming the flux through the primary also passes through the secondary, i.e., the two coils are tightly coupled, the rate of change of flux in the secondary will also be $\Delta\phi/\Delta t$ and the magnitude of the induced emf across the secondary is given by

$$V_s = N_s\left[\frac{\Delta\phi}{\Delta t}\right]\ldots\ldots \quad (2)$$

Where N_s is the number of turns in the secondary coil.

Step up transformer

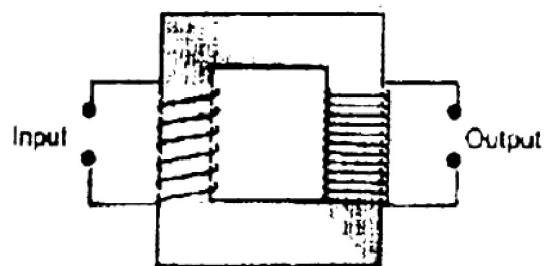### Equation of transformer:

Dividing Eq.2 by Eq. 1, we get

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \ldots\ldots\ldots\ldots (3)$$
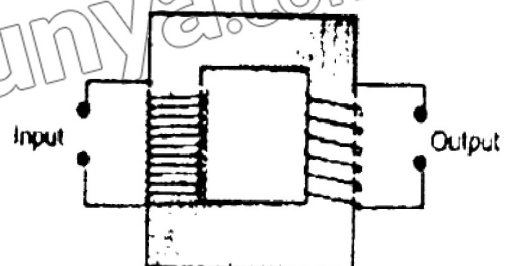
Step up transformer:

If $N_S > N_p$ then according to Eq. 3, $V_S > V_p$, such a transformer in which voltage across secondary is greater than the primary voltage is called a step up transformer.

Step down transformer:

Similarly if $N_S < N_p$, i.e., the number of turns in the secondary is less than the number in primary, then $V_S < V_p$, such transformer in which voltage across secondary is less than the primary voltage is called a step down transformer.

Step down transformer

Q20: Explain the transmission of power by transformers?

Ans: **i.** For an ideal case the power input to the primary is nearly equal to the power output from the secondary i.e.,

Power input = Power output

$$V_p I_p = V_s I_s \dots \dots (1)$$

$$\frac{V_S}{V_P} = \frac{I_s}{I_P} \quad \dots \dots \dots (2)$$

I_p is the current in the primary and I_p in the secondary

ii. The currents are thus inversely proportional to the respective voltages. Therefore, in a step up transformer when the voltage across the secondary is raised, the value of current is reduced. This is the principle behind its use in the electric supply network where transformer increases the voltage and reduces the current so that it can be transmitted over long distances without much power loss.

iii. **Power loss due to heating effect:**

When current I passes through a resistance R, the power loss due to heating effect is $I^2 R$. Suppose R is the resistance of transmission line

iv. **Minimization of loss during transmission:**

a. In order to minimize the loss during transmission, it is not possible to reduce R because it requires the use of thick copper wire which becomes highly uneconomical. The purpose is well served by reducing I.

b. At the generating power station the voltage is stepped up to several thousand of volts and power is transmitted at low current to long distances without much loss.

c. **Function of step down transformer:**

Step down transformers then decrease the voltage to a safe value at the end of line where the consumer of electric power is located. Inside a house a transformer may be used to step down the voltage from 250 volts to 9 volts for ringing bell or operating a transistor radio. Transformers with several secondaries are used in television and radio receivers where several different voltages are required.

Q21: Discuss the main causes of power loss in transformer?

Ans: **Causes of power loss in transformer:**

Only in an ideal transformer the output power is nearly equal to the input power. But in an actual transformer, this in not the case. The output is always less than input due to power losses. There are two main causes of power loss.

i. eddy currents   ii. magnetic hysteresis.

Eddy currents:

a. In order to enhance the magnetic flux, the primary and secondary coils of the transformer are wound on soft iron core

b. The flux generated by the coils also passes through the core. As magnetic flux changes through a solid conductor. induced currents are set up in closed paths in the body of the conductor. These induced currents are set up in a direction
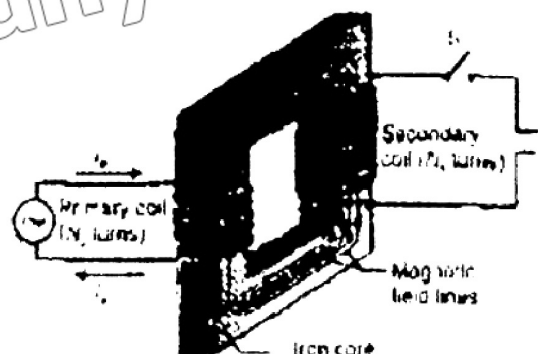
perpendicular to the flux and are known as eddy currents.

c. It results in power dissipation and heating of the core material.

Elimination of eddy currents:

In order to minimize the power loss due to flow of these currents, the core is laminated with insulation in between the layers of laminations which stops the flow of eddy currents

Magnetic hysteresis:

Hysterises loss is the energy expended to magnetize and demagnetize the core material in each cycle of the A.C.

Due to these power losses, a transformer is far from being an ideal. Its output power is always less than its input power.

Q22: What is meant by efficiency of a transformer? Describe the steps in order to improve the efficiency of transformer?

Ans: The efficiency of a transformer is defined as the ratio between output power and input power

$$E = \frac{Output\ power}{input\ power} \times 100$$

Improvement in the efficiency of transformer:

i. In order to improve the efficiency, care should be exercised, to minimize all the power losses.

ii. For example core should be assembled from the laminated sheets of a material whose hysteresis loop area is very small.

iii. The insulation between lamination sheets should be perfect so as to stop the flow of eddy currents.

iv. The resistance of the primary and secondary coils should be kept to a minimum.

v. As power transfer from primary to secondary takes place through flux linkages, so the primary and secondary coils should be wound in such a way that flux coupling between them is maximum.

Q23: Draw a symbol of transformer?
Ans:
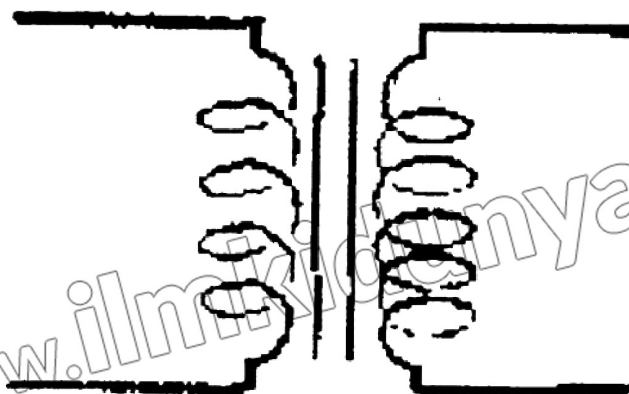
Transformer symbol

SUMMARY

1. An emf is set up in a conductor when it moves across a magnetic field. It is called an induced emf.

2. The emf induced by the motion of a conductor across a magnetic field is called motional emf.

3. Magnitude of motional emf in a rod of length L moving with velocity v across a magnetic field of strength B making θ with it is $\varepsilon = vBL \sin \theta$

4. Faraday's law states that the emf induced in a conducting coil of N loops is equal to the negative of the rate at which the magnetic flux through the coil is changing with time.

5. The Lenz's law states that the direction of the induced current is always so as to oppose the change which causes the current.

6. The phenomenon in which a changing current in one coil induces an emf in another coil is called the mutual induction.

7. One Henry is the mutual inductance of that pair of coils in which a change of current of one ampere per second in the primary causes an induced emf of one volt in the secondary.

8. The phenomenon in which a changing current in a coil induces an emf in itself is called self induction.

9. A current generator is a device that converts mechanical energy into electrical energy.

10. The emf produced in a generators $\varepsilon = N\omega AB \sin (\omega t)$ or $\varepsilon = \varepsilon_0 \sin (2\pi ft)$.

11. A motor is a device which converts electrical energy into mechanical energy.

12. The induced emf in a motor opposes the emf running the motor. This induced emf is called the back emf of the motor.

SOLUTION OF EXERCISE

15.1. Does the induced emf in a circuit depend on the resistance of the circuit? Does the induced current in a circuit depend on the resistance of the circuit?

Ans: According to Faraday's law induced emf is $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$ This relation shows that induced emf does not depend upon the resistance of the circuit.

According to Ohm's law $I = \frac{\varepsilon}{R}$ This relation shows that induced current does depend upon the resistance of the circuit.

15.2. A square loop of wire is moving through a uniform magnetic field. The normal to the loop is oriented parallel to the magnetic field. Is an emf induced in the circuit? Give a reason for your answer.

Ans: Since $\varepsilon = vBL \sin \theta$

There will be no emf induced in the loop. Because $\theta = 0$.

$$\varepsilon = vBL \sin \theta = vBL \sin 0^0 = 0$$

SUMMARY

1. An emf is set up in a conductor when it moves across a magnetic field. It is called an induced emf.

2. The emf induced by the motion of a conductor across a magnetic field is called motional emf.

3. Magnitude of motional emf in a rod of length L moving with velocity v across a magnetic field of strength B making θ with it is $\varepsilon = vBL \sin \theta$.

4. Faraday's law states that the emf induced in a conducting coil of N loops is equal to the negative of the rate at which the magnetic flux through the coil is changing with time.

5. The Lenz's law states that the direction of the induced current is always so as to oppose the change which causes the current.

6. The phenomenon in which a changing current in one coil induces an emf in another coil is called the mutual induction.

7. One Henry is the mutual inductance of that pair of coils in which a change of current of one ampere per second in the primary causes an induced emf of one volt in the secondary.

8. The phenomenon in which a changing current in a coil induces an emf in itself is called self induction.

9. A current generator is a device that converts mechanical energy into electrical energy.

10. The emf produced in a generators $\varepsilon = N\omega AB \sin(\omega t)$ or $\varepsilon = \varepsilon_0 \sin(2\pi ft)$.

11. A motor is a device which converts electrical energy into mechanical energy.

12. The induced emf in a motor opposes the emf running the motor. This induced emf is called the back emf of the motor.

SOLUTION OF EXERCISE

15.1. Does the induced emf in a circuit depend on the resistance of the circuit? Does the induced current in a circuit depend on the resistance of the circuit?

Ans: According to Faraday's law induced emf is $\varepsilon = -N \frac{\Delta\phi}{\Delta t}$. This relation shows that induced emf does not depend upon the resistance of the circuit.

According to Ohm's law $I = \frac{\varepsilon}{R}$. This relation shows that induced current does depend upon the resistance of the circuit.

15.2. A square loop of wire is moving through a uniform magnetic field. The normal to the loop is oriented parallel to the magnetic field. Is an emf induced in the circuit? Give a reason for your answer.

Ans: Since $\varepsilon = vBL \sin \theta$

There will be no emf induced in the loop. Because $\theta = 0$.

$$\varepsilon = vBL \sin \theta = vBL \sin 0^0 = 0$$

It is due to the fact that the rate of change of magnetic flux through the loop will be zero if it is moved. But it must not be rotated as rotation about an axis perpendicular to the field will change magnetic flux and induce emf.

15.3. A light metallic ring is released from above into a vertical bar magnet (figure). Viewed from above, does the current flow clockwise or anticlockwise?

Ans: i. According to **Lenz's law** the direction of induced current will oppose the motion of ring towards the bar magnet. Thus N-pole must be produced towards the face of the ring opposite to the N-pole of the bar magnet.

Right hand rule shows that this magnetic field will be produced only if induced current flows through the ring in clockwise direction.
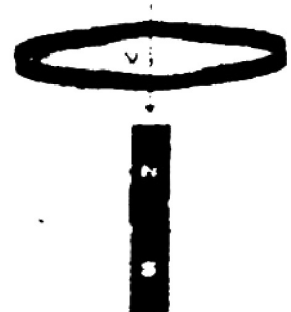
15.4. What is the direction of current through resistor R in figure? When switch S is (a) closed (b) opened (figure)?
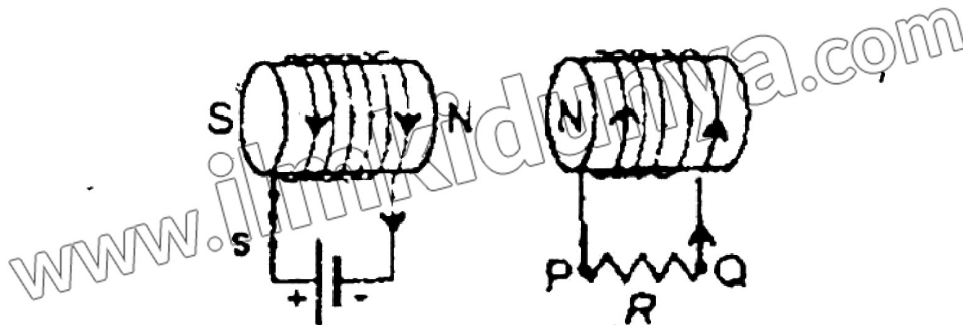
Ans: (a) When the switch 'S' is closed, then the current in the circuit increases from zero to maximum. According to Lenz's law, the direction of induced current through 'R' is anti-clockwise.

(b) When the switch 'S' is opened then the current in the circuit decreases. According to Lenz's law, the direction of induced current through 'R' is clockwise.
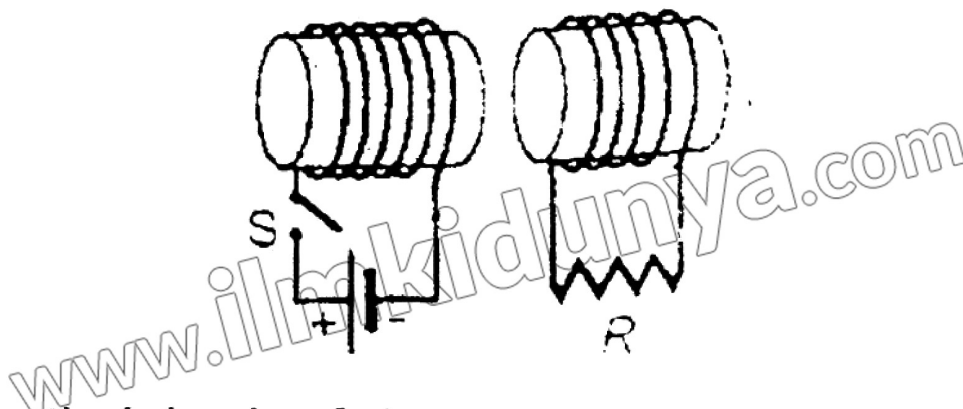
15.5. Does the induced emf always act to decrease the magnetic flux through the circuit?

Ans: No. According to Lenz's law, it always acts to maintain the flux by opposing any change in it. If the flux increases, it will tend to decrease it and vice versa.

15.6. When the switch in the circuit is closed a current is established in the coil and the metal ring jumps upwards. Why? Describe what would happen to the ring if the battery polarity were reversed? (Figure)

Ans: When switch 'S' is closed in the coil, then a current produces a magnetic field in the coil which changes from zero to maximum. Due to this change an induced current is produce in the metal ring. This induced current produces its own magnetic field which is opposite to the magnetic field of coil (Lenz's law). Both the fields repel each other so the metal ring jumps upward.

Same will be the process when the battery polarity is reversed
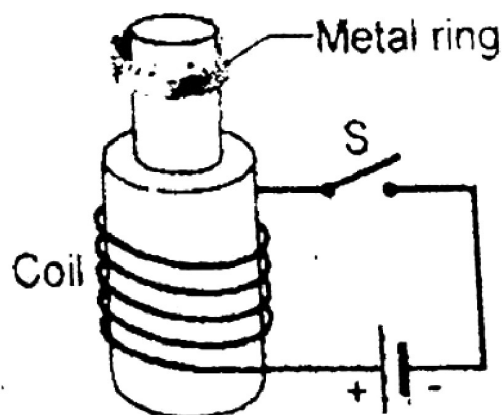
15.7. The figure shows a coil of wire in the XY plane with a magnetic field directed along the Y-axis. Around which of the three coordinate axes the coil should be rotated in order to generate an emf and a current in a coil?

Ans: The induced emf in the coil is given by

$$\varepsilon = N\omega A B \sin\theta$$

This relation shows that in order to generate an emf and a current in he coil, it must be rotated about x-axis because vector area of the loop is along z-axis and $\vec{B}$ is along y-axis.
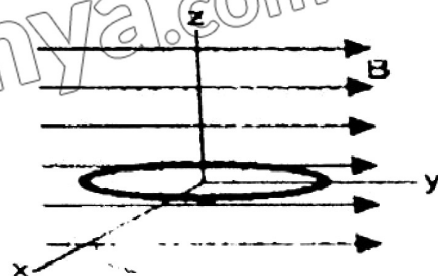
15.8. How would you position a flat loop of wire in a changing magnetic field so that there is no emf induced in the loop?

Ans: The induced emf in the loop of wire is given by

$$\varepsilon = N\omega A B \sin\theta$$

When the loop is placed in the magnetic field with its normal parallel to the field, then $\theta = 0^c$

So from Eq.

$$\varepsilon = N\omega A B \sin 0^0$$

Hence no emf is induced in the loop.

15.9. In a certain region the earth's magnetic field points vertically down. When a plane flies due north, which wingtip is positively charged?

Ans: When a plane flies through the magnetic field of earth along north, the electrons in the wing experience a magnetic force, $\vec{F} = -e(\vec{V} \times \vec{B})$, from right to left wing. Thus the west wing of the plane is positively charged

15.10. Show that ε and $\dfrac{\Delta\phi}{\Delta t}$ have the same units.

Ans: We know that, $\varepsilon = \dfrac{W}{q}$

Unit of $\varepsilon = \dfrac{Joule}{Coulomb} = $ Volt

Now, $\dfrac{\Delta\phi}{\Delta t} = \dfrac{B\Delta A}{\Delta t} = \dfrac{NA^{-1}m^{-1}\times m^2}{s} = \dfrac{Nm}{s\times A} = \dfrac{Joule}{Coulomb} = Volt$

Hence ε and $\dfrac{\Delta\phi}{\Delta t}$ have the same units

15.11. When an electric motor such as an electric drill is being used, does it also act as a generator? If so, what is the consequence of this?

Ans: Yes, an electric motor acts as a generator running in reverse.

When coil of the motor rotates, an induced emf is produced in it. This is called back emf of the motor. It opposes the rotation of the coil. When motor speeds up, back emf increases.

15.12. Can a D.C. motor be converted into a D.C. generator? What changes are required to be done?

Ans: Yes D.C. motor can be turned into D.C. generator. The following changes are required to do:

i.   The magnetic field should be provided by permanent magnet.

ii.   There should be an arrangement to rotate the coil (armature).

15.13. Is it possible to change both area of the loop and the magnetic field passing through the loop and still not have an induced emf in the loop?

Ans: Yes, No flux will change. We just have to maintain the flux i.e. $\phi = BA\cos\theta$. Hence, if we increase the area, A, an equal decrease in magnetic filed, B can compensate it by keeping the product unchanged and vice versa.

15.14. Can an electric motor be used to drive an electric generator with the output from the generator being used to operate the motor?

Ans: No, it is not possible because it would be against law of conservation of energy as it will become a self operating machine. In order to satisfy this law, in any combination of devices, if there is a consumer of energy then there must also be some source of energy present in the same combination.

15.15. A suspended magnet is oscillating freely in a horizontal plane. The oscillations are strongly damped when a metal plate is placed under the magnet. Explain why this occurs?

Ans: The oscillations are damped due to the emf induced, in the metal plate, which according to Lenz's law, produces a current that by its own magnetic field tends to stop the oscillations of the magnet which are responsible for the induced emf.

15.16. Four unmarked wires emerge from a transformer, what steps would you take to determine the turns ratio?

Ans: First of all we check the continuity of pairs by using ohm-metre and separate the pairs of primary and secondary coils of transformer. Measure the primary voltage V_p and secondary voltage V_s by a voltmeter and then find the turn ratio by using the transformer equation.

$$\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p}$$

15.17. (a) Can a step-up transformer increases the power level?

(b)   In a transformer, there is no transfer of charge from the primary to the secondary. How is then power transferred?

Ans: **(a)** A step-up transformer cannot increase the power level it increase the voltage and lowers the current If voltage increase then current will decrease but power (P = IV) will remain constant

(b) In a transformer power is transferred from its primary coil to its secondary coil by changing magnetic flux passing from the primary towards the secondary coil.

15.18. **When the primary of a transformer is connected to a.c mains the current in it.**

(a) **is very small if the secondary circuit is open , but**

(b) **increases when the secondary circuit is closed. Explain these facts.**

Ans: **(a)** When the secondary circuit is open then there is no load to consume electric power or output power used is zero

Output power = $V_s I_s$ = 0

Since output power of a transformer is smaller than input power Hence the primary coil of the transformer draws a very small current from a.c.

(b) When the secondary circuit is closed then output power increases

input power = output power

$I_p V_s = I_s V_s$

The transformer will draw large current draw from the a c mains to increase the primary power because greater current is needed in primary coil to equalize power in the secondary coil

SOLUTION OF EXAMPLES

Example 15.1: **A metal rod of length 25 cm is moving at a speed of 0.5 ms^{-1} in a direction perpendicular to a 0.25 T magnetic field. Find the emf produced in the rod.**

Solution: Speed of rod = v = 0.5 ms^{-1}

Length of rod = L = 25 cm = 0.25 m

Magnetic flux density = B = 0.25 T = 0.25 N A^{-1}W^{-1}

Induced emf = ε = ?

Since ε = v B L sinθ

ε = 0.5 × 0.25 × 0.25 × sin 90^0 = 3.13 × 10^{-2} × 1 JC^{-1} (∵JC^{-1} = V)

ε = 3.13 × 10^{-} V

Example 15.2: **A loop of wire is placed in a uniform magnetic field that is perpendicular to the plane of the loop. The strength of the magnetic field is 0.6 T.**

The area of the loop beings to shrink at a constant rate of $\frac{\Delta A}{\Delta t}$ = 0.8 ms^{-1}. What is the magnitude of emf induced in the loop while it is magnitude of emf induced in the loop while it is shrinking?

Solution: Rate of change of area $\frac{\Delta A}{\Delta t}$ = 0.8m^2 s^{-1}

Magnetic flux density = B = 0.6 T

Magnetic flux density = B = 0.6 N A^{-1}m^{-1} (∵T = N A^{-1}m^{-1})

Number of turns = N = 1

Induced emf = ε = ?

Rate of change of flux $= \frac{\Delta\phi}{\Delta t} = B\frac{\Delta A}{\Delta t}\cos\theta$

or $\frac{\Delta\phi}{\Delta t} = B\frac{\Delta A}{\Delta t}$ $\quad (\because \theta = 0° \text{ and } \cos 0 = 1)$

Magnitude of induced emf can be calculated by applying Faraday's law

$$\varepsilon = N\frac{\Delta\phi}{\Delta t} = NB\frac{\Delta A}{\Delta t}$$

$\varepsilon = 1 \times 0.6 \times 0.8 = 0.48 \text{ JC}^{-1} = 0.48 \text{ V}$

Example 15.3:

An emf of 5.6V is induced in a coil while the current in a nearby coil is decreased from 100 A to 20A in 0.02s. What is the mutual inductance of the two coils? If the secondary has 200 turns, find the change in flux during this interval.

Solution: emf induced in the secondary coil $= \varepsilon_s = 5.6\text{V}$

Change in current in primary coil $= \Delta I_P = 100\text{ A} - 20\text{A}$

Change in current in primary coil $= \Delta I_P = 80\text{A}$

Time interval for the change $= \Delta t = 0.02\text{s}$

Number of turns in the secondary coil $= N_s = 200$

Mutual inductance $= M = ?$

Change in flux $= \Delta\phi = ?$

Using the relation for mutual inductance

$$\varepsilon_s = M\frac{\Delta I_p}{\Delta t}$$

$5.6 = M \times \frac{80}{0.02}$

$M = \frac{5.6 \times 0.02}{80} = 1.4 \times 10^{-3} \text{ Vs A}^{-1}$

According to Faraday's law

$$\varepsilon_s = N_s \frac{\Delta\phi}{\Delta t}$$

$\varepsilon_s \Delta t = N_s \Delta\phi$

$\Delta\phi = \frac{\varepsilon_s \Delta t}{N_s}$

$\Delta\phi = \frac{\varepsilon_s \Delta t}{N_s} = \frac{5.6 \times 0.02}{200} = 5.6 \times 10^{-4} \text{ Wb}$

Example 15.4:

The current in a coil of 1000 turns is changed from 5 A to zero in 0.2s. If an average emf of 50 V is induced during this interval, what is the self-inductance of the coil? What is the flux through each turn of the coil when a current of 6 A is flowing?

Solution: Change in current $= \Delta I = 5\text{A} - 0 = 5\text{A}$

Time interval $= \Delta t = 0.2\text{s}$

emf induced $= \varepsilon = 50\text{V}$

Steady current $= I = 6\text{A}$

No. of turns of coil $= N = 1000$

Self inductance $= L = ?$

Flux through each turn $= \phi = ?$

For self induction, we know that

$$\varepsilon = L\frac{\Delta I}{\Delta t}$$

$$L = \frac{\varepsilon \times \Delta t}{\Delta I}$$

$$L = \frac{50 \times 0.2}{5} = 2\ V\,s\,A^{-1} \qquad (\because V\,s\,A^{-1} = H)$$

$$L = 2\ H$$

Now by using the relation

$$N\phi = LI$$

$$\phi = \frac{LI}{N}$$

$$\phi = \frac{2 \times 6}{1000} = 1.2 \times 10^{-2}\ Wb$$

Example 15.5: A solenoid coil 10.0 cm long has 40 turns per cm. When the switch is closed, the current rises from zero to its maximum value of 5.0 A in 0.01s. Find the energy stored in the magnetic field if the area of cross-section of the solenoid be 28cm².

Solution: Length of solenoid coil $= \ell = 10.0\ cm = 0.1\ m$

No. of turns $= n = 40$ per cm $= 4000\ m^{-1}$

Steady current $= I = 5.0A$

Time interval $= \Delta t = 0.01\ s$

Area of cross section $= A = 28\ cm^2 = 2.8 \times 10^{-3}\ m^2$

Energy stored $= U_m = ?$

First of all, we calculate the value of L:

$$L = \mu_0 n^2 A \ell$$

$$L = 4\pi \times 10^{-7} \times (4000)^2 \times 2.8 \times 10^{-3} \times 0.1$$

$$L = 5.63 \times 10^{-3} = 5.63 \times 10^{-3}\ H \qquad (\because WbA^{-1} = H)$$

Since Energy stored $= U_m = \frac{1}{2} L I^2$

$$U_m = \frac{1}{2} \times 5.63 \times 10^{-3} \times (5)^2 \qquad (\because H = NmA^{-2})$$

$$U_m = 7.04 \times 10^{-2} J$$

Example 15.6: An alternating current generator operating at 50 Hz has a coil of 200 turns. The coil has an area of 120 cm². What should be the magnetic field in which the coil rotates in order to produce an emf of maximum value of 240 volt?

Solution: Frequency of rotation $= f = 50\ Hz$

No. of turns of the coil $= N = 200$

Area of the coil $= A = 120\ cm^2 = 1.2 \times 10^{-2}\ m^2$

Max. value of emf $= \varepsilon_{max} = \varepsilon_0 = 240\ V$

Magnetic flux density $= B = ?$

First of all we will find the angular speed ω:

$$\omega = 2\pi f$$

$$\omega = 2 \times 3.14 \times 50 = 314.3\ rads^{-1}$$

Maximum value of emf is given by the relation

$$\varepsilon_0 = N \omega A B$$

$$\Rightarrow B = \frac{\varepsilon_0}{N\omega A}$$

$$B = \frac{240}{200 \times 314.3 \times 1.2 \times 10^{-2}} = 0.32\ T$$

Example 15.7: A permanent magnet d.c. motor is run by a battery of 24 volts. The coil of the motor has a resistance of 2 ohms. It develops a back emf of 22.5 volts when driving the load at normal speed. What is the

current when motor just starts up? Also find the current when motor is running at normal speed.

Solution:

Operated voltage	= V	= 24 V
Resistance of the coil	= R	= 2Ω
Back emf	= ε	= 22.5 V
Current	= I	=?

When motor just starts up, the back emf ε = 0

Using the relation

$$V = \varepsilon + IR$$

$$24 = 0 + I \times 2$$

$$I = \frac{24}{2} = 12 \text{ V } \Omega^{-1} = 12 \text{ A } \textbf{Ans}$$

When motor runs at normal speed, the back emf ε = 22.5 V Again using the relation

$$V = \varepsilon + IR$$

$$4 = 22.5 + I \times 2$$

$$I = \frac{24 - 22.5}{2} = 0.75 \text{ V } \Omega^{-1} = 0.75 \text{ A } \textbf{Ans}$$

Example 15.8: The turns ratio of a step up transformer is 50. A current of 20A is passed through its primary coil at 220 volts. Obtain the value of the voltage and current in the secondary coil assuming the transformer to be ideal one.

Solution:

Current in primary coil	= I_p	= 20 A
Voltage of primary coil	= V_p	= 200 V
Voltage in secondary coil	= V_s	= ?
Turn ratio	= $\frac{N_s}{N_p}$	= 50

Current in secondary coil = I_s =?

Since

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$\Rightarrow \quad V_s = \frac{N_s}{N_p} V_p$$

$$\frac{V_s}{200} = 50 \qquad\qquad (\because \frac{I_p}{I_s} = \frac{N_s}{N_p} = 50)$$

$$V_s = 220 \times 50 = 1100V$$

Since

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$I_s = \frac{V_p}{V_s} I_p$$

$$I_s = \frac{220}{110} \times 20 = \frac{1}{50} \times 20 = 0.4 \text{ A}$$

SOLUTION OF PROBLEMS

15.1 An emf 0.45 V is induced between the ends of a metal bar moving through a magnetic field of 0.22 T. What field strength would be needed to produce an emf of 1.5V between the ends of the bar, assuming that all other factors remain the same?

Solution:

current when motor just starts up? Also find the current when motor is running at normal speed.

Solution: Operated voltage   = V = 24 V
                  Resistance of the coil   = R = 2Ω
                  Back emf   = ε = 22.5 V
                  Current   = I =?

When motor just starts up, the back emf ε = 0
Using the relation

$$V = \varepsilon + IR$$

$$24 = 0 + I \times 2$$

$$I = \frac{24}{2} = 12 \ V \ \Omega^{-1} = 12 \ A \ \textbf{Ans}$$

When motor runs at normal speed, the back emf ε = 22.5 V Again using the relation
$$V = \varepsilon + IR$$

$$4 = 22.5 + I \times 2$$

$$I = \frac{24 - 22.5}{2} = 0.75 \ V \ \Omega^{-1} = 0.75 \ A \ \textbf{Ans}$$

Example 15.8: The turns ratio of a step up transformer is 50. A current of 20A is passed through its primary coil at 220 volts. Obtain the value of the voltage and current in the secondary coil assuming the transformer to be ideal one.

Solution: Current in primary coil   = I_p =   20 A
                  Voltage of primary coil   = V_p =   200 V
                  Voltage in secondary coil = V_s =   ?
                  Turn ratio   = $\frac{N_s}{N_p}$ = 50

Current in secondary coil = I_s =?

Since
$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$\Rightarrow \qquad V_s = \frac{N_s}{N_p} V_p$$

$$\frac{V_s}{200} = 50 \qquad\qquad\qquad \left(\because \frac{I_p}{I_s} = \frac{N_s}{N_p} = 50\right)$$

$$V_s = 220 \times 50 = 1100V$$

Since
$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

$$I_s = \frac{V_p}{V_s} I_p$$

$$I_s = \frac{220}{110} \times 20 = \frac{1}{50} \times 20 = 0.4 \ A$$

SOLUTION OF PROBLEMS

15.1 An emf 0.45 V is induced between the ends of a metal bar moving through a magnetic field of 0.22 T. What field strength would be needed to produce an emf of 1.5V between the ends of the bar, assuming that all other factors remain the same?

Solution:

Motional emf $= \dfrac{}{} = 0.45$ volts

Motional emf $= \dfrac{}{} = 1.5$ volts

$$\varepsilon_1 = B_1 vL \sin\theta \qquad (1)$$
$$\varepsilon_2 = B_2 vL \sin\theta \qquad (2)$$

By dividing equation (1) by (2) we get

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{B_1 vL \sin\theta}{B_2 vL \sin\theta}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{B_1}{B_2}$$

$$B_2 = \frac{\varepsilon_2 \times B_1}{\varepsilon_1}$$

$$B_2 = \frac{1.5 \times 0.22}{0.45} = 0.73 \text{ T}$$

15.2
The flux density B in a region between the pole faces of a horse-shoe magnet is 0.5 Whm^{-2} directed vertically downward. Find the emf induced in a straight wire 5.0 cm long, perpendicular to B when it is moved in a direction at an angle of 60° with the horizontal with a speed of 100 cm s⁻¹.

Solution:

B = 0.5 T (directed into the paper i.e. vertically downward)

L = 5 cm = 5×10^{-2} m

θ = 60° (with horizontal)

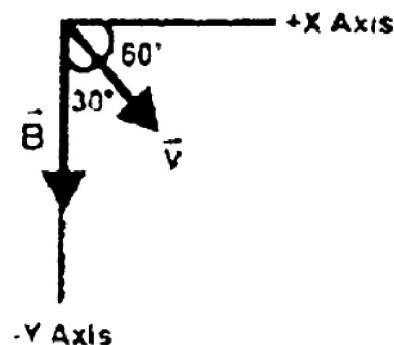θ = 30° (Angle between v and B)

v = 100 cm s⁻¹ = 1.0 ms⁻¹

ε = ?

$$\varepsilon = B vL \sin\theta$$
$$\varepsilon = 0.5 \times 1 \times 0.05 \times \sin(30°)$$
$$= 0.025 \times 0.5 = 0.0125$$
$$\varepsilon = 1.25 \times 10^{-2} \text{ V}$$

15.3
A coil of wire has 10 loops. Each loop has an area of 1.5×10^{-3} m^2. A magnetic field is perpendicular to the surface of each loop at all times. If the magnetic field is changed from 0.05 T to 0.06 T in 0.1s, find the average emf induced in the coil during this time.

Solution: Number of loops = N = 10 loops

A = 1.5×10^{-3} m^2

B_1 = 0.05 T

B_2 = 0.06 T

ΔB = $B_2 - B_1$ = 0.06 - 0.05 = 0.01 T

Δt = 0.1s

ε = ?

According to Faraday's Law of electromagnetic induction, the magnitude of the induced emf is given by

$$\varepsilon = N \frac{\Delta \phi}{\Delta t}$$

As B is always perpendicular to the surface of coil therefore

15.4 **A circular coil has is turns of radius 2 cm each. The plane of the coil lies at 40° to a uniform magnetic field of 0.2 T. If the field is increased to 0.5T in 0.2s, find the magnitude of the induced emf.**

Solution:

Number of turns	= N =	15 turns
r	=	2 cm = 2×10^{-2} m
A	=	πr^2 = $3.142 \times (2 \times 10^{-2}\text{m})^2$
A	=	12.568×10^{-4} m²
θ'	=	40° (The angle between plane and Magnetic field)
θ	=	50° (The angle between vector area and magnetic field)
B_1	=	0.2T
B_2	=	0.5T
ΔB	=	$B_2 - B_1$ = 0.5 T - 0.2 T = 0.3 T
Δt	=	0.2 s
ε	=	?

According to Faraday's Law of electromagnetic induction, the magnitude of the induced emf is given by

$$\varepsilon = N \frac{\Delta \phi}{\Delta t}$$

$$N \frac{A \Delta B \cos 50°}{\Delta t}$$

$$\varepsilon = \frac{...\cos 50°}{...} = 181.78 \times 10^{-4}\,\text{V}$$

$$\varepsilon = 1.8 \times 10^{-2}\,\text{V}$$

15.5 **Two coils are placed side by side. An emf of 0.8v is observed in one coil when the current is changing at the rate of 200 As⁻¹ in the other coil. What is the mutual inductance of the coils?**

Solution:

ε	=	0.8V
$\frac{\Delta I_p}{\Delta t}$	=	200 A s⁻¹
M	=	?

For mutual inductance

$$M = \frac{\varepsilon}{\frac{\Delta I_p}{\Delta t}}$$

$$M = \frac{0.8}{200} = 4 \times 10^{-3}\,\text{H} = 4\,\text{mH}$$

15.6 **A pair of adjacent coils has a mutual inductance of 0.75 H. If the current in the primary changes from 0 to 10 A in 0.025 s, what is the average induced emf in the secondary? What is the change in flux in it if the secondary has 500 turns?**

Solution:

Number of turns = N = 500 turns

ΔI		
Δt		
$\Delta \phi$		

emf induced in the secondary coil is:

$$\varepsilon_s = M \frac{\Delta I_p}{\Delta t}$$

$$\varepsilon_s = 0.75 \times \frac{10}{0.025} = 300 \text{ volts}$$

In case of mutual induction:

$$N\phi = M I_p$$

$$\Rightarrow \quad N \Delta\phi = M \Delta I_p$$

$$\Rightarrow \quad \Delta\phi = \frac{M \Delta I_p}{N}$$

$$\Delta\phi = \frac{0.75 \times 10}{500} = 0.015 = 1.5 \times 10^{-2} \text{ Wb}$$

15.7 A solenoid has 250 turns and its self inductance is 2.4 mH. What is the flux through each turn when the current is 2A? What is the induced emf when the current changes $20As^{-1}$?

Solution:

Number of turns $= N = 250$ turns

$$L = 2.4 \times 10^{-3} H$$
$$I = 2 A$$
$$\frac{\Delta I}{\Delta t} = 20 \text{ As}^{-1}$$
$$\phi = ?$$
$$\varepsilon = ?$$

In case of self induction:

$$N\phi = LI$$

$$\phi = \frac{LI}{N}$$

$$\phi = \frac{2.4 \times 10^{-3} \times 2}{250} = 1.92 \times 10^{-5} \text{ Wb}$$

For self induced emf:

$$\varepsilon = L \frac{\Delta I}{\Delta t}$$

$$\varepsilon = 2.4 \times 10^{-3} \times 20 = 0.048 \text{ V} = 48 \text{ mV}$$

15.8 A solenoid of length 8.0 cm and cross sectional area 0.5 cm² has 520 turns. Find the self inductance of the solenoid when the core is air. If the current in the solenoid increases through 1.5 A in 0.2s, find the magnitude of induced emf in it.

Solution:

$$\ell = 8 \text{ cm} = 8 \times 10^{-2} \text{ m}$$
$$A = 0.5 \text{ cm}^2 = 0.5 \times 10^{-4} \text{ m}^2$$

Number of turns $= N = 520$ turns

$$\Delta I = 1.5 A$$
$$\Delta t = 0.2 s$$

Permeability of free space $= \mu_0 = 4\pi \times 10^{-7} \text{ Wb A}^{-1}\text{m}^{-1}$

$$\varepsilon = ?$$

The formula for self inductance is:

$$L = \mu_0 n^2 i A$$

$$L = \mu_0 \frac{N^2}{\ell} \times A$$

$$L = 4\pi \times 10^{-7} \times \frac{(520)^2}{8 \times 10^{-2}} \times 0.5 \times 10^{-4} = 2.12 \times 10^{-4} \, H$$

The formula for self induced emf is,

$$\varepsilon = L \frac{\Delta I}{\Delta t}$$

$$\varepsilon = 2.12 \times 10^{-4} \times \frac{1.5}{0.2} = 15.9 \times 10^{-4} = 1.6 \times 10^{-3} \, V$$

15.9 When current through a coil changes from 100 mA to 200mA in 0.005s, an induced emf of 40 mV is produced in the coil.

(a) What is the self inductance of the coil?

(b) Find the increase in the energy stored in the coil.

Solution:

I_1 = 100 mA

I_2 = 200 mA

ΔI = 200 mA - 100 mA = 100 mA = 100×10^{-3} A

Δt = 0.005 s

ε = 40×10^{-3} volts.

L = ?

ΔU = ?

For self inductance:

$$L = \frac{\varepsilon}{\frac{\Delta I}{\Delta t}}$$

$$L = \frac{40 \times 10^{-3}}{\left(\frac{100 \times 10^{-3}}{5 \times 10^{-3}}\right)} = 2 \times 10^{-3} \, H$$

Formula for the increase in energy stored in a solenoid is,

$$\Delta U = \frac{1}{2} L (I_2^2 - I_1^2)$$

ΔU = $\frac{1}{2} \times 2 \times 10^{-3} [(200 \times 10^{-3})^2 - (100 \times 10^{-3})^2]$

= $10^{-3} (40000 \times 10^{-6} - 10000 \times 10^{-6})$

= $(40000 - 10000) \times 10^{-9} \, J = 30000 \times 10^{-6} \times 10^{-3} \, J = 0.03 \times 10^{-3}$

ΔU = 0.03 mJ

15.10 Like any field, the earth's magnetic field stores energy. Find the magnetic energy stored in a space where strength of earth's field is 7×10^{-5} T, if the space occupies an area of 10×10^8 m² and has a height of 750m.

Solution:

B = 7×10^{-5} T

A = 10×10^8 m²

L = h = 750 m

Permeability of free space = $\mu_0 = 4\pi \times 10^{-7}$ Wb·A^{-1}m^{-1}

U_m = ?

Magnetic energy stored in magnetic field is,

$$U_m = \frac{1}{2} \frac{B^2}{\mu_0} AL$$

U_m = $\frac{1}{2} \times \frac{(7 \times 10^{-5})^2}{4\pi \times 10^{-7}} \times 10 \times 10^8 \times 750 = 1462.065 \times 10^6 \, J$

U_m = $1.462 \times 10^9 \, J$

15.11 A square coil of side 16 cm has 200 turns and rotates in a uniform magnetic field of magnitude. 0.05T. If the peak emf is 12V, what is the angular velocity of the coil?

Solution:

$$A = 16cm \times 16cm = 256 \ cm^2 = 256 \times 10^{-4} m^2$$

Number of turns $= N = 200$ turns

$$B = 0.05 \ T$$
$$\varepsilon = 12V$$

Angular velocity $= \omega = ?$

For maximum emf induced in a generator is

$$\varepsilon_0 = N\omega AB$$

$$\omega = \frac{\varepsilon}{BNA}$$

$$\omega = \frac{12}{0.05 \times 200 \times 256 \times 10^{-4}} = 46.875 \ rad.s^{-1} = 47 \ rad.s^{-1}$$

15.12 A generator has a rectangular coil consisting of 360 turns. The coil rotates at 420 rev per min in 0.14 T magnetic fields. The peak value of emf produced by the generator is 50 V, if the coil is 5.0 cm wide, find the length of the side of the coil.

Solution:

Number of turns $= N = 360$ turns

$$f = 420 \ rev/min$$

$$\omega = \frac{420 \times 2 \times 3.14}{60} = \frac{2637.6}{60} = 43.96 \ rad \ s^{-1}$$

$$B = 0.14 \ T$$
$$\varepsilon = 50V$$
$$b = 5 \ cm = 0.05 \ m$$
$$L = ?$$

Since $\varepsilon_0 = BNA\omega$

$$\varepsilon_0 = BN(\ell \times b)\omega \qquad (\because A = \ell \times b)$$

$$L = \frac{\varepsilon_0}{BNb\omega}$$

$$L = \frac{50}{0.14 \times 360 \times 43.96 \times 0.05} = \frac{50}{110.77} = 0.5 \ m = 45 \ cm$$

15.13 It is desired to make an A.C. generator that can produce an emf of maximum value 5 kV with 50 Hz frequency. A coil of area 1m² consisting of 200 turns is used as armature. What should be the magnitude of the magnetic field in which the coil rotates?

Solution:

$$\varepsilon_0 = 5 \ KV = 5000 \ V$$
$$f = 50 \ Hz$$
$$A = 1 \ m^2$$

Number of turns $= N = 200$ turns

For maximum induced emf in a generator

$$\varepsilon_0 = N\omega AB$$

$$B = \frac{\varepsilon_0}{N\omega A} = \frac{\varepsilon_0}{N \times 2\pi f \times A} \qquad (\because \omega = 2\pi f)$$

$$B = \frac{}{} = 0.0795 = 0.08 \text{T}$$

15.14 The back emf in a motor is 120 V when the motor is turning at 1680 rev per min. What is the back emf when the motor turns 3360 rev per min?

Solution:

$$\varepsilon_1 = 120V$$

$$\omega_1 = 1680 \text{ rev / min} = \frac{1680 \times 2 \times 3.14}{60} = 175.84 \text{ rad s}$$

$$\omega_2 = 3360 \text{ rev / mm} = \frac{3360 \times 2 \times 3.14}{60} = 351.68 \text{ rad s}$$

$$\varepsilon_2 = ?$$

Since

$$\varepsilon = N\omega AB$$

$$\varepsilon_1 = N\omega_1 AB \qquad (1)$$

$$\varepsilon_2 = N\omega_2 AB \qquad (2)$$

By dividing Eq. 1 by 2

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{BNA\omega_1}{BNA\omega_2}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\omega_1}{\omega_2}$$

$$\varepsilon_2 = \frac{\varepsilon_1 \times \omega_2}{\omega_1}$$

$$\varepsilon_2 = \frac{120 \times 351.68}{175.84} = 240 \text{ V}$$

15.15 A D.C motor operates at 240V and has a resistance of 0.5Ω. When the motor is running at normal speed, the armature current is 15A. Find the back emf in the armature.

Solution:

$$V = 240 \text{ V}$$

$$R = 0.5 \, \Omega$$

$$I = 15 \text{ A}$$

$$\varepsilon = ?$$

back emf in a motor is given by

$$\varepsilon = V - IR$$

$$\varepsilon = 240 - 15 \times 0.5 = (240 - 7.5) \text{ V} = 232.5 \text{ V}$$

15.16 A copper ring has a radius of 4.0 cm and resistance of 1.0 mΩ. A magnetic field is applied over the ring, perpendicular to its plane. If the magnetic field increases from 0.2 T to 0.4 T in a time interval of 5 × 10⁻³ s, what is the current in the ring during this interval?

Solution:

Radius $= r = 4cm = 4 \times 10^{-2}$ m

Resistance $= R = 1 m\Omega = 1 \times 10^{-3} \Omega$

$$B_1 = 0.2 \text{ T}$$

$$B_2 = 0.4 \text{ T}$$

$$\Delta B = B_2 - B_1 = 0.4 - 0.2 = 0.2 \text{ T}$$

$$\Delta t = 5 \times 10^{-3} \text{ s}$$

Number of turns $= N = 1$ (for a ring)

$$I = ?$$

According to Faraday's law of electromagnetic induction

$$\varepsilon = N\frac{\Delta\phi}{\Delta t} = N\frac{BA}{\Delta t}$$

$$\Rightarrow \qquad \varepsilon = N\frac{B \times \pi r^2}{\Delta t}$$

$$\varepsilon = \frac{0.2 \times 3.142 \times 16 \times 10^{-4}}{5 \times 10^{-3}} = 0.201 \text{ volts}$$

By using ohm's law,

$$I = \frac{\varepsilon}{R}$$

$$I \qquad \frac{0.201}{1 \times 10^{-3}} = 0.201 \times 10^3 = 201 \text{ A}$$

15.17 A coil of 10 turns and 35 cm² area is in a perpendicular magnetic field of 1.0 s. The coil is pulled out of the field in 1.0 s. Find the induced emf in the coil as it is pulled out of the field?

Solution: Number of turns = N = 10 turns

$$\begin{aligned}
A &= 35 cm^2 = 35 \times 10^{-4} m^2 \\
B_1 &= 0.5 T \\
B_2 &= 0 T \\
\Delta B &= 0 - 0.5 = -0.5 T \\
\Delta t &= 1.0 s
\end{aligned}$$

Induced emf = ε = ?

For induced emf:

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

$$\varepsilon = -N\frac{\Delta B A}{\Delta t}$$

$$\varepsilon = \frac{10 \times 0.5 \times 35 \times 10^{-4}}{1} = 174 \times 10^{-4} = 1.75 \times 10^{-2} V$$

15.18 An ideal step down transformer is connected on main supply of 240V. It is desired to operate a 12V, 30 w lamps. Find the current in the primary and the transformation ration?

Solution:
$$\begin{aligned}
V_P &= 240 V \\
V_s &= 12 V \\
P_s &= 30 \text{ watts} \\
I_P &= ?
\end{aligned}$$

Transformation ratio = $\frac{N_s}{N_p}$ = ?

For a transformer:

$$P_S = I_S V_S$$

$$30 W = I_s \times 12 V$$

$$I_s = \frac{30}{12} = 2.5 A$$

Since

$$\frac{V_p}{V_s} = \frac{I_s}{I_p}$$

$$\Rightarrow \qquad I_p = \frac{I_s \times V_s}{V_p}$$

$$I_p = \frac{2.5 \times 12}{240} = 0.125 A$$

Also

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

Transformation ratio = $\frac{N_s}{N_p} = \frac{12}{240} = \frac{1}{20}$