Chapter 5

LINEAR INEQUALITIES AND LINEAR PROGRAMMING

EXERCISE 5.1

Q.1: Graph the solution set of each of the following linear inequality in xy-plane.

(i) $2x + y \leq 6$ **(ii)** $3x + 7y \geq 21$ **(iii)** $3x - 2y \geq 6$

(iv) $5x - 4y \leq 20$ **(v)** $2x + 1 \geq 0$ **(vi)** $3y - 4 \leq 0$

Solution:

(i) $2x + y \leq 6$

The associated equation is

$2x + y = 6$ ...... (1)

x-intercept

Put $y = 0$ in eq. (1)

$2x + 0 = 6$

$x = \frac{6}{2} = 3$

$\therefore$ Point is (3, 0)

y-intercept

Put $x = 0$ in eq. (1)

$2(0) + y = 6$

$y = 6$

$\therefore$ Point is (0, 6)

Test Point

Put (0, 0) in

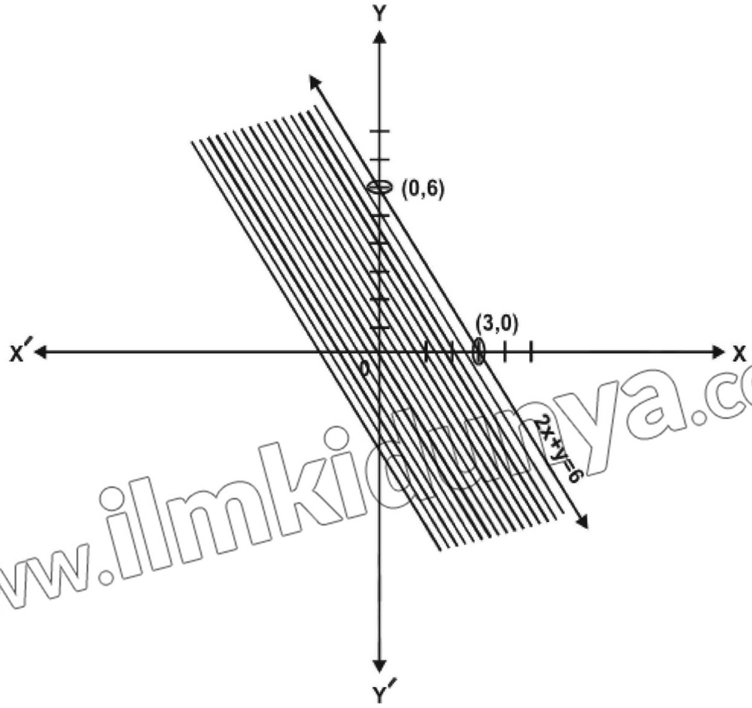
$2x + y < 6$

$2(0) + 0 < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 6$ will be towards the origin side.



(ii) $3x + 7y \geq 21$

The associated equation is

$3x + 7y = 21$ .... (1)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$3x + 7(0) = 21$

$3x = 21$

$x = \frac{21}{3} = 7$

$\therefore$ Point is (7, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 7y = 21$

$7y = 21$

$y = \frac{21}{7} = 3$

$\therefore$ Point is (0, 3)

<u>Test Point</u>

Put (0, 0) in

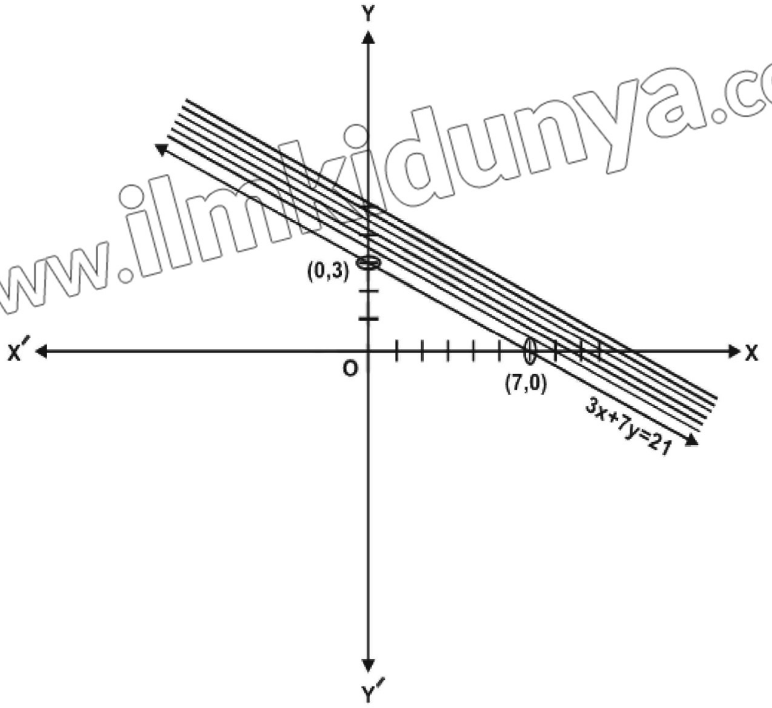
$3x + 7y > 21$

$3(0) + 7(0) > 21$

$0 > 21$

Which is false.

$\therefore$ Graph of an inequality $3x + 7y \geq 21$ will not be towards the origin side.



(iii) $\mathbf{3x - 2y \geq 6}$

The associated equation is

$3x - 2y = 6$ ....(1)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$3x - 2(0) = 6$

$3x = 6$

$$x = \frac{6}{3} = 2$$

$\therefore$ Point is (2, 0)

y-intercept

Put $x = 0$ in eq. (1)

$3(0) - 2y = 6$

$-2y = 6$

$y = \frac{6}{-2} = -3$

$\therefore$ Point is $(0, -3)$

Test Point

Put $(0, 0)$ in

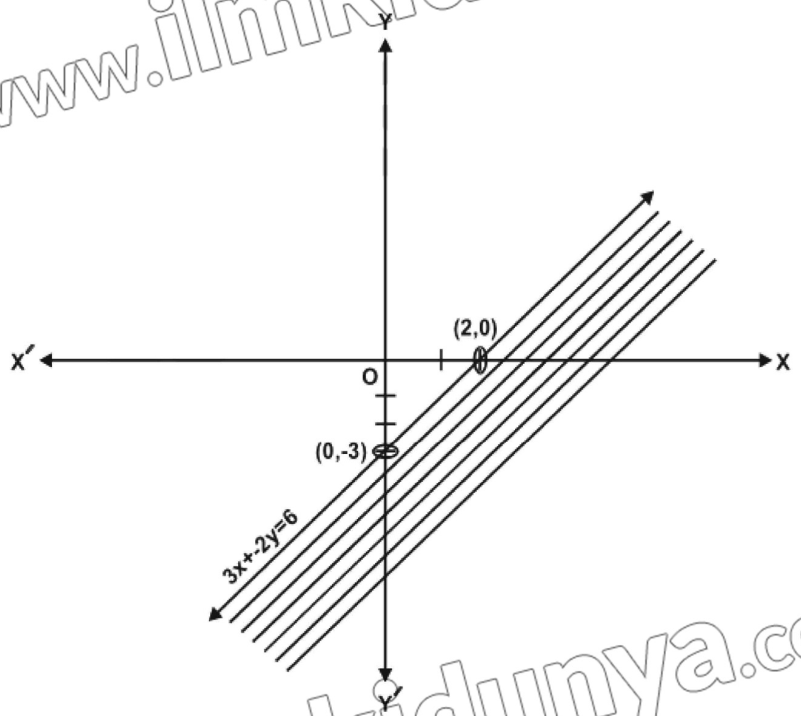
$3x - 2y > 6$

$3(0) + 2(0) > 6$

$0 > 6$

Which is false.

$\therefore$ Graph of an inequality $3x - 2y \geq 6$ will not be towards the origin side.



(iv) $5x - 4y \leq 20$

The associated equation is

$5x - 4y = 20$ .... (1)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$5x - 4(0) = 20$$
$$5x = 20$$
$$x = \frac{20}{5} = 4$$

∴ Point is (4, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$$5(0) - 4y = 20$$
$$-4y = 20$$
$$y = \frac{20}{-4} = -5$$

∴ Point is (0, – 5)

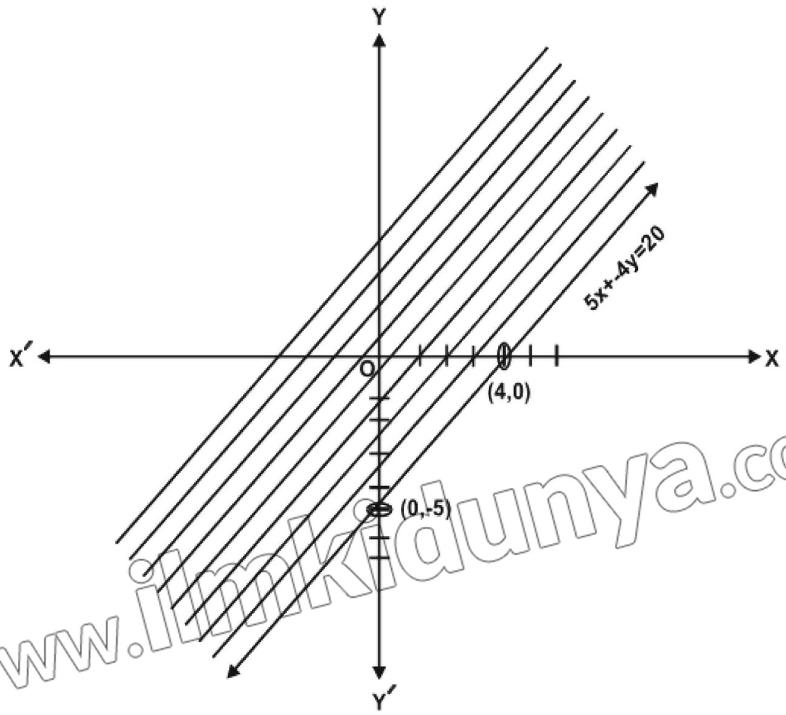
<u>Test Point</u>

Put (0, 0) in

$$5x - 4y < 20$$
$$5(0) - 4(0) < 20$$
$$0 < 20$$

Which is true.

∴ Graph of an inequality $5x - 4y \leq 20$ will be towards the origin side.



(v) $2x + 1 \geq 0$

The associated equation is

$2x + 1 = 0$

$2x = -1$

$x = \frac{-1}{2}$

Put $x = 0$ in

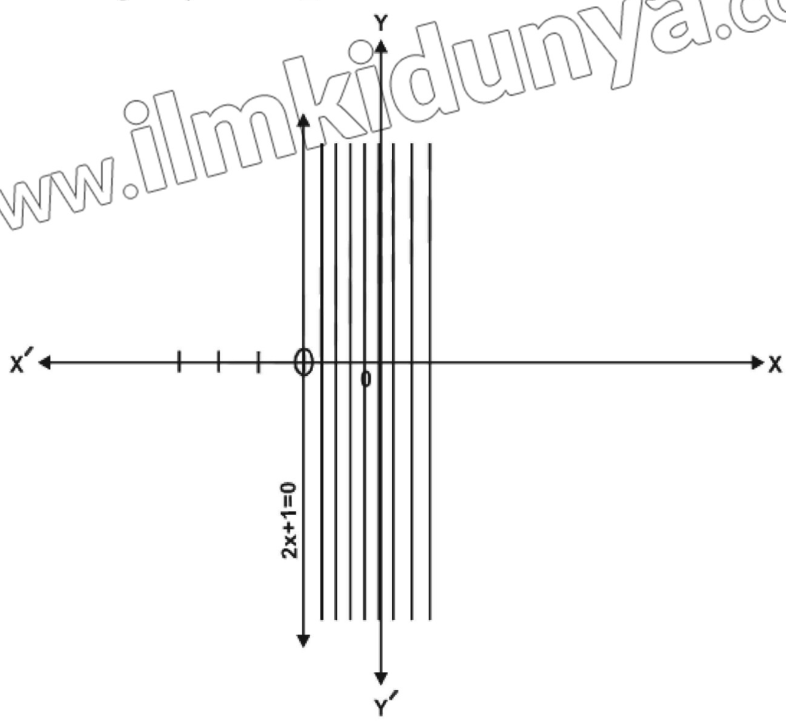
$2x + 1 > 0$

$2(0) + 1 > 0$

$1 > 0$

Which is true.

$\therefore$ Graph of an inequality $2x + 1 \geq 0$ will be towards the origin side.



(vi) $3y - 4 \leq 0$

The associated equation is

$3y - 4 = 0$

$3y = 4$

$y = \frac{4}{3}$

Put $y = 0$ in

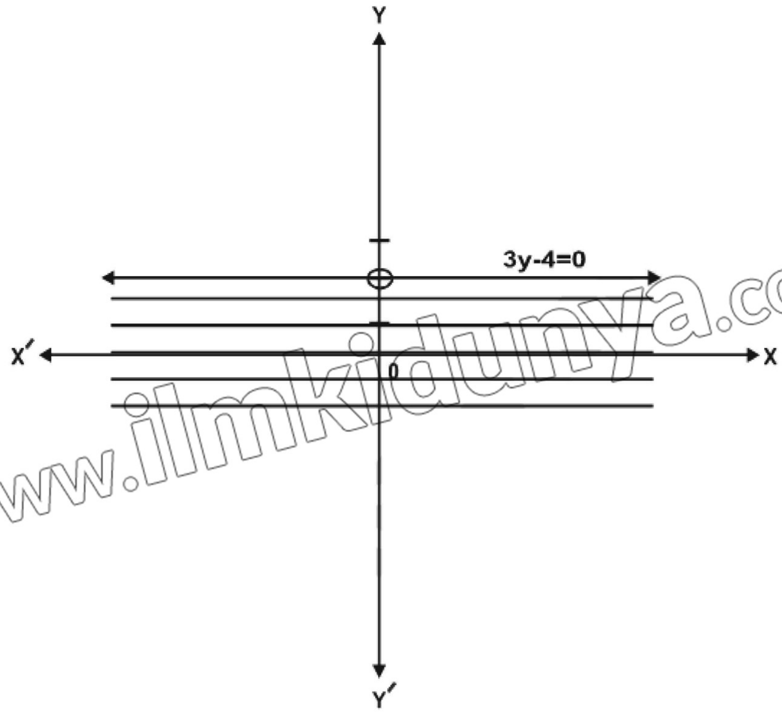
$3y - 4 < 0$

$3(0) - 4 < 0$

$-4 < 0$

Which is true.

$\therefore$ Graph of an inequality $3y - 4 \leq 0$ will be towards the origin side.



Q.2: Indicate the solution set of the following systems of linear inequalities by shading.

(i) $2x - 3y \leq 6$, $2x + 3y \leq 12$ **(ii)** $x + y \geq 5$, $x - y \leq 1$ **(iii)** $3x + 7y \geq 21$, $x - y \leq 2$

(iv) $4x - 3y \leq 12$

$x \geq \frac{-3}{2}$ **(Lhr. Board 2011) (Guj. Board 2008)**

(v) $3x + 7y \geq 21$ **(Lhr. Board 2011)**

$y \leq 4$

Solution:

(i) $2x - 3y \leq 6$

$2x + 3y \leq 12$

The associated equations are

$2x - 3y = 6$ ...... (1)

$2x + 3y = 12$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$2x - 3(0) = 6$

$2x = 6$

$x = \frac{6}{2} = 3$

$\therefore$ Point is (3, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$2(0) - 3y = 6$

$-3y = 6$

$y = \frac{6}{-3} = -2$

$\therefore$ Point is (0, – 2)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$2x + 3(0) = 12$

$2x = 12$

$x = \frac{12}{2} = 6$

$\therefore$ Point is (6, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$2(0) + 3y = 12$

$3y = 12$

$y = \frac{12}{3} = 4$

$\therefore$ Point is (0, 4)

<u>Test Point</u>

Put (0, 0) in

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $2x - 3y \leq 6$ will be towards the origin side.

Put (0, 0) in

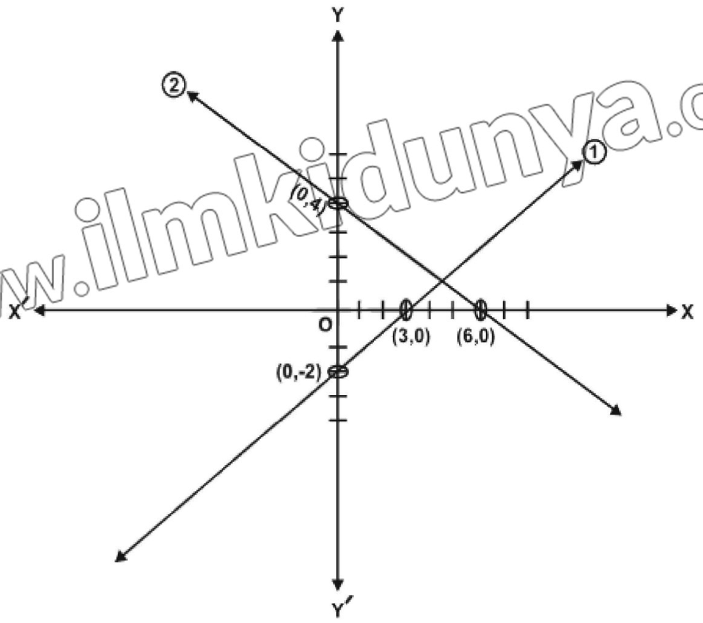
$2x + 3y < 12$

$2(0) - 3(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 12$ will be towards the origin side.



(ii) $\mathbf{x + y \geq 5}$

$\mathbf{x - y \leq 1}$

The associated equations are

$x + y = 5$ ...... (1)

$x - y = 1$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$x + 0 = 5$

$x = 5$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$0 + y = 5$

$y = 5$

$\therefore$ Point is (0, 5)

x-intercept

Put $y = 0$ in eq. (2)

$x - 0 = 1$

$x = 1$

$\therefore$ Point is (1, 0)

y-intercept

Put $x = 0$ in eq. (2)

$0 - y = 1$

$y = -1$

$\therefore$ Point is (0, – 1)

Test Point

Put (0, 0) in

$x + y > 5$

$0 + 0 > 5$

$0 > 5$

Which is false.

$\therefore$ Graph of an inequality $x + y \geq 5$ will not be towards the origin side.

Put (0, 0) in

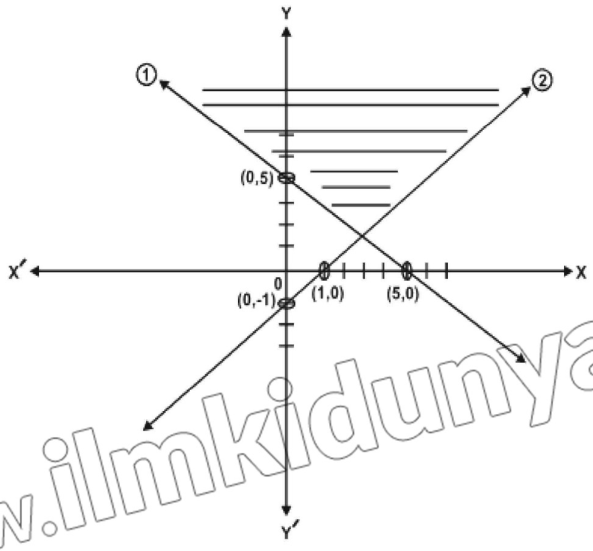
$x - y < 1$

$0 - 0 < 1$

$0 < 1$

Which is true.

$\therefore$ Graph of an inequality $x - y \leq 1$ will be towards the origin side.



(iii) $3x + 7y \geq 21$

$x - y \leq 2$

The associated equations are

$3x + 7y = 21$ ..... (1)

$x - y = 2$ ..... (2)

x-intercept

Put $y = 0$ in eq. (1)

$3x + 7(0) = 21$

$3x = 21$

$x = \frac{21}{3} = 7$

$\therefore$ Point is (7, 0)

y-intercept

Put $x = 0$ in eq. (1)

$3(0) + 7y = 21$

$7y = 21$

$y = \frac{21}{7} = 3$

$\therefore$ Point is (0, 3)

x-intercept

Put $y = 0$ in eq. (2)

$x - 0 = 2$

$x = 2$

$\therefore$ Point is (2, 0)

y-intercept

Put $x = 0$ in eq. (2)

$0 - y = 2$

$y = -2$

$\therefore$ Point is (0, – 2)

Test Point

Put (0, 0) in

$3x + 7y > 21$

$3(0) + 7(0) \quad > \quad 21$

$0 > 21$

Which is false.

$\therefore$ Graph of an inequality $3x + 7y \geq 21$ will not be towards the origin side.

Put (0, 0) in

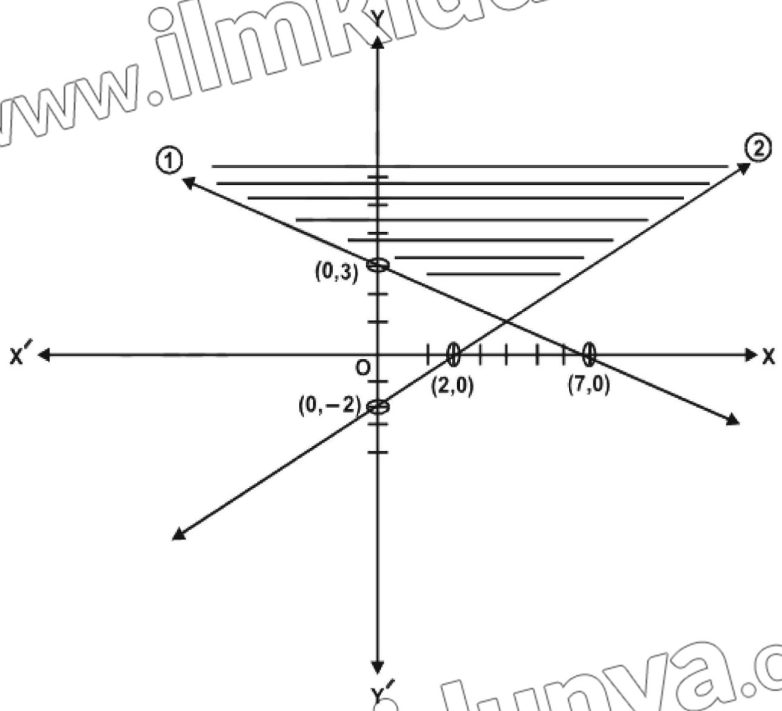
$x - y \quad < \quad 2$

$0 - 0 \quad < \quad 2$

$0 \quad < \quad 2$

Which is true.

$\therefore$ Graph of an inequality $x - y \leq 2$ will be towards the origin side.



(iv) $\mathbf{4x - 3y \leq 12}$

$\mathbf{x \geq \frac{-3}{2}}$

The associated equations are

$4x - 3y \quad = \quad 12 \qquad \text{..... (1)}$

$x = \frac{-3}{2}$ ..... (2)

x-intercept

Put $y = 0$ in eq. (1)

$4x - 3(0) = 12$

$4x = 12$

$x = \frac{21}{4} = 3$

∴ Point is (3, 0)

y-intercept

Put $x = 0$ in eq. (1)

$4(0) - 3y = 12$

$-3y = 12$

$y = \frac{12}{-3} = -4$

∴ Point is (0, −4)

Test Point

Put (0, 0) in

$4x - 3y < 12$

$4(0) - 3(0) < 12$

$0 < 12$

Which is true.

∴ Graph of an inequality $4x - 3y \leq 12$ will be towards the origin side.

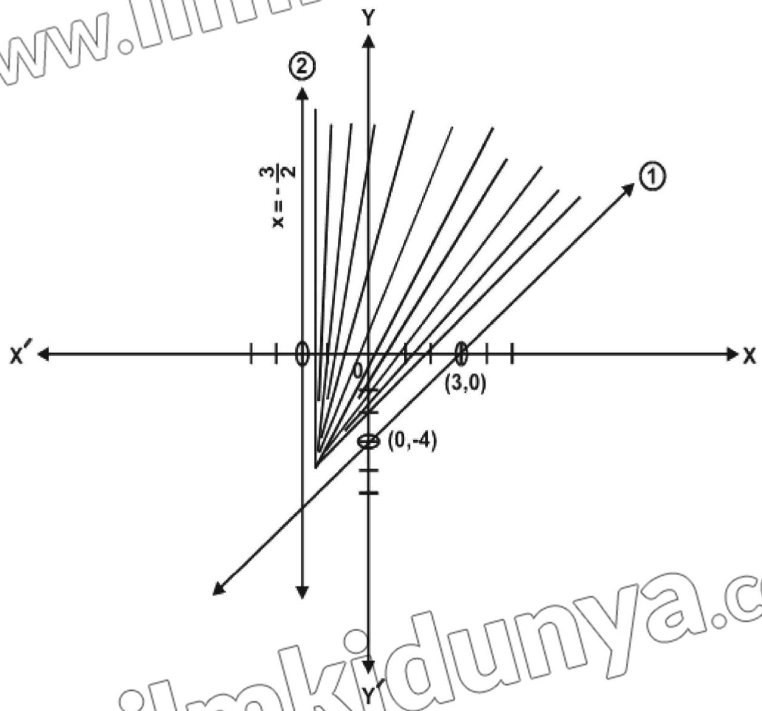
Put $x = 0$ in

$x > \frac{-3}{2}$

$0 > \frac{-3}{2}$

Which is true.

∴ Graph of an inequality $x \geq \frac{-3}{2}$ will be towards the origin side.



(v) $\mathbf{3x + 7y \geq 21}$

$\mathbf{y \leq 4}$

The associated equations are

$3x + 7y = 21$ ...... (1)

$y = 4$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$3x + 7(0) = 21$

$3x = 21$

$x = \frac{21}{3} = 7$

$\therefore$ Point is (7, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 7y = 21$

$7y = 21$

$y = \frac{21}{7} = 3$

$\therefore$ Point is (0, 3)

<u>Test Point</u>

Put (0, 0) in

$3x + 7y > 21$

$3(0) + 7(0) > 21$

$0 > 21$

Which is false.

$\therefore$ Graph of an inequality $3x + 7y \geq 21$ will not be towards the origin side.

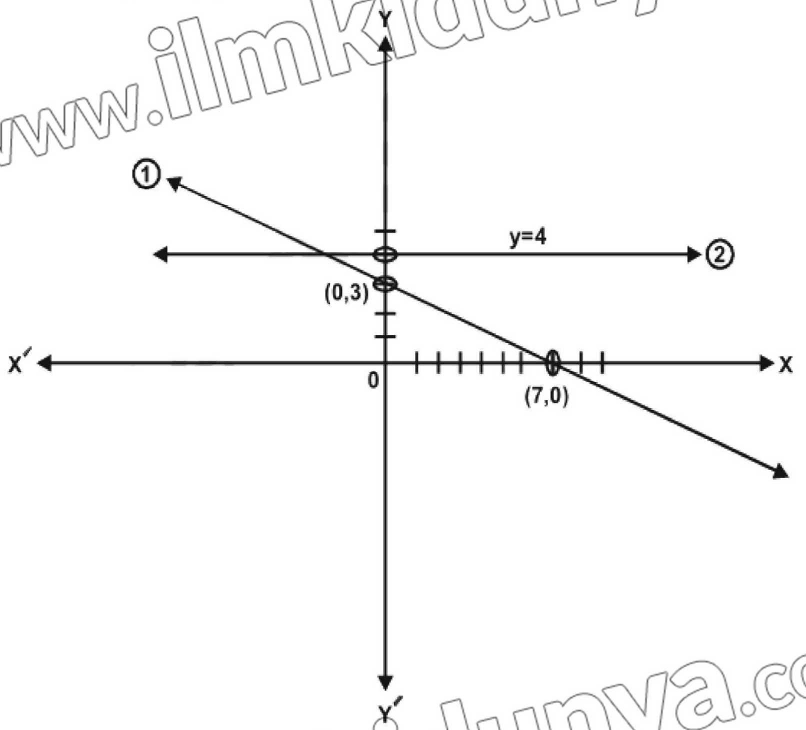
Put $y = 0$ in

$y < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $y \leq 4$ will be towards the origin side.



Q.3: Indicate the solution region of the following systems of linear inequalities by shading.

(i)	**$2x - 3y \leq 6$**	**(ii)**	**$x + y \leq 5$**	**(iii)**	**$x + y \geq 5$**
	$2x + 3y \leq 12$		**$y - 2x \leq 2$**		**$x - y \geq 1$**
	$y \geq 0$		**$x \geq 0$**		**$y \geq 0$**

(iv) $3x + 7y \leq 21$
$x - y \leq 2$
$x \geq 0$

(v) $3x + 7y \leq 21$
$x - y \leq 2$
$y \geq 0$

(vi) $3x + 7y \leq 21$
$2x - y \geq -3$
$x \geq 0$

Solution:

(i) $2x - 3y \leq 6$ **(Lhr. Board 2007)**
$2x + 3y \leq 12$
$y \geq 0$

The associated equations are

$2x - 3y = 6$ ...... (1)

$2x + 3y = 12$ ...... (2)

x-intercept

Put $y = 0$ in eq. (1)

$2x - 3(0) = 6$

$2x = 6$

$x = \frac{6}{2} = 3$

$\therefore$ Point is (3, 0)

y-intercept

Put $x = 0$ in eq. (1)

$2(0) - 3y = 6$

$-3y = 6$

$y = \frac{6}{-3} = -2$

$\therefore$ Point is (0, – 2)

x-intercept

Put $y = 0$ in eq. (2)

$2x + 3(0) = 12$

$2x = 12$

$x = \frac{12}{2} = 6$

$\therefore$ Point is (6, 0)

y-intercept

Put $x = 0$ in eq. (2)

$2(0) + 3y = 12$

$3y = 12$

$y = \frac{12}{3} = 4$

$\therefore$ Point is (0, 4)

<u>Test Point</u>

Put (0, 0) in

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $2x - 3y \leq 6$ will be towards the origin side.

Put (0, 0) in

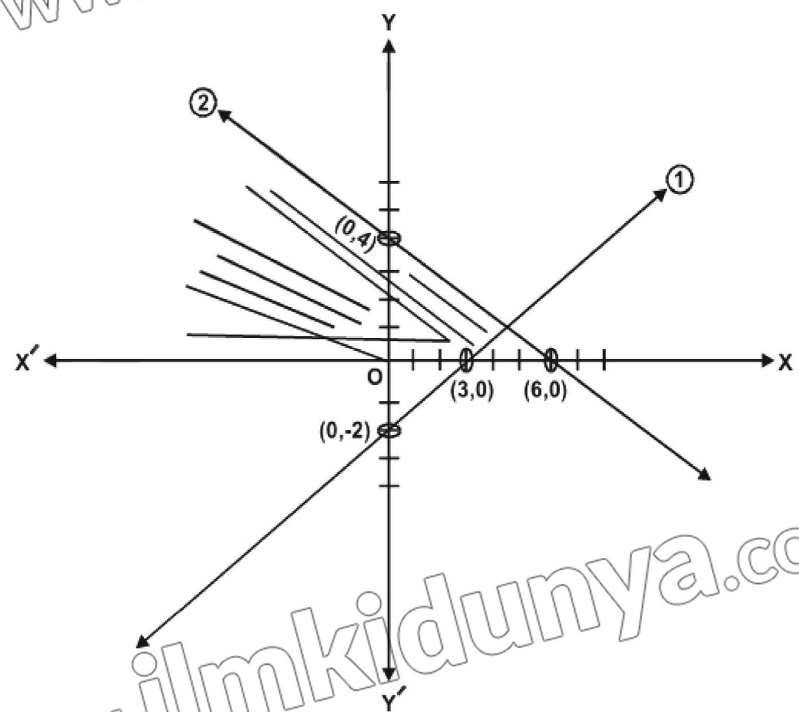
$2x + 3y < 12$

$2(0) + 3(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 12$ will be towards the origin side.



(ii) $\mathbf{x + y \leq 5}$

$\mathbf{y - 2x \leq 2}$

$\mathbf{x \geq 0}$

The associated equations are

$x + y = 5$ ...... (1)

$y - 2x = 2$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$x + 0 = 5$

$x = 5$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$0 + y = 5$

$y = 5$

$\therefore$ Point is (0, 5)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$0 - 2x = 2$

$$x = \frac{2}{-2} = -1$$

$\therefore$ Point is (– 1, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$y - 2(0) = 2$

$y = 2$

$\therefore$ Point is (0, 2)

<u>Test Point</u>

Put (0, 0) in

$x + y < 5$

$0 + 0 < 5$

$0 < 5$

Which is true.

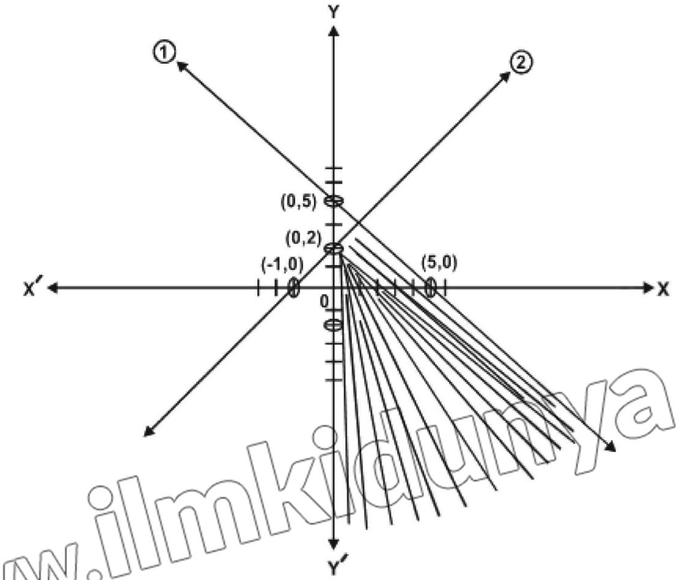
$\therefore$ Graph of an inequality $x + y \leq 5$ will towards the origin side.

Put (0, 0) in

$y - 2x < 2$
$0 - 2(0) < 2$
$0 < 2$

Which is true.

$\therefore$ Graph of an inequality $y - 2x \leq 2$ will towards the origin side.



(iii) $\mathbf{x + y \geq 5}$

$\mathbf{x - y \geq 1}$

$\mathbf{y \geq 0}$

The associated equations are

$x + y = 5$ .... (1)

$x - y = 1$ .... (2)

<u>x–intercept</u>

Put $y = 0$ in eq. (1)

$x + 0 = 5$

$x = 5$

$\therefore$ Point is (5, 0)

<u>y–intercept</u>

Put $x = 0$ in eq. (1)

$0 + y = 5$

$y = 5$

$\therefore$ Point is (0, 5)

x–intercept

Put $y = 0$ in eq. (2)

$x - 0 = 1$

$x = 1$

∴ Point is (1, 0)

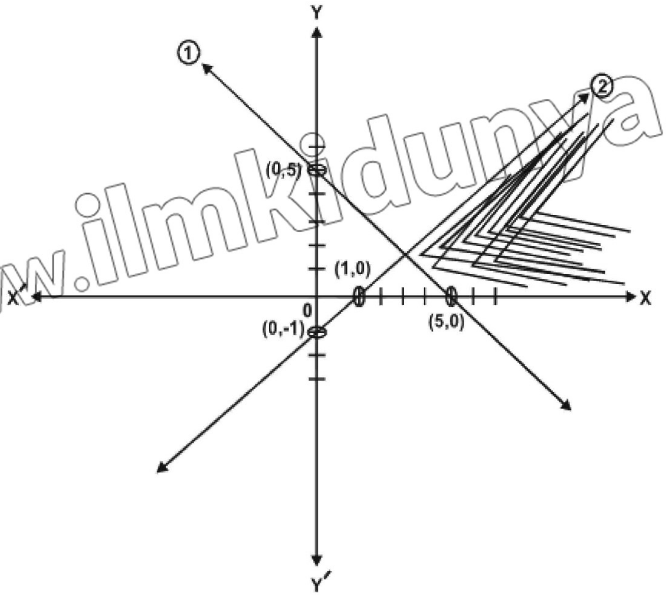
y–intercept

Put $x = 0$ in eq. (2)

$0 - y = 1$

$y = -1$

∴ Point is (0, – 1)



Test Point

Put (0, 0) in

$x + y > 5$

$0 + 0 > 5$

$0 > 5$

Which is false.

∴ Graph of an inequality $x + y \geq 5$ will not be towards the origin side.

Put (0, 0) in

$x - y > 1$

$0 - 0 > 1$

$0 > 1$

Which is false.

$\therefore$ Graph of an inequality $x - y \geq 1$ will not be towards the origin side.

(iv) $\mathbf{3x + 7y \leq 21}$

$\mathbf{x - y \leq 2}$

$\mathbf{x \geq 0}$

The associated equations are

$3x + 7y = 21$ ...... (1)

$x - y = 2$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$3x + 7(0) = 21$

$3x = 21$

$x = \frac{21}{3} = 7$

$\therefore$ Point is (7, 0)

<u>y–intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 7y = 21$

$7y = 21$

$y = \frac{21}{7} = 3$

$\therefore$ Point is (0, 3)

<u>x–intercept</u>

Put $y = 0$ in eq. (2)

$x - 0 = 2$

$x = 2$

$\therefore$ Point is (2, 0)

<u>y–intercept</u>

Put $x = 0$ in eq. (2)

$0 - y = 2$

$y = -2$

$\therefore$ Point is (0, –2)

<u>Test Point</u>

Put (0, 0) in

$3x + 7y < 21$

$3(0) + 7(0) < 21$

$0 < 21$

Which is true.

$\therefore$ Graph of an inequality $3x + 7y \leq 21$ will be towards the origin side.

Put (0, 0) in

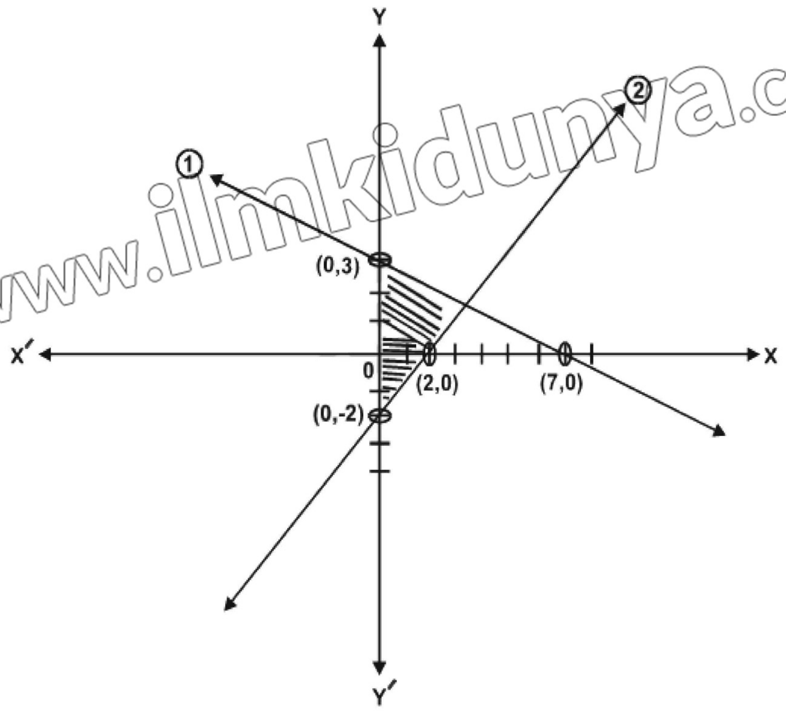
$x - y < 2$

$0 - 0 < 2$

$0 < 2$

Which is true.

$\therefore$ Graph of an inequality $x - y \leq 2$ will be towards the origin side.



(v) **$3x + 7y \leq 21$** **(Gujranwala Board 2007)**

$x - y \leq 2$

$y \geq 0$

The associated equations are

$3x + 7y = 21$ ...... (1)

$x - y = 2$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$3x + 7(0) = 21$$
$$3x = 21$$
$$x = \frac{21}{3} = 7$$

$\therefore$ Point is (7, 0)

<u>y–intercept</u>

Put $x = 0$ in eq. (1)

$$3(0) + 7y = 21$$
$$7y = 21$$
$$y = \frac{21}{7} = 3$$

$\therefore$ Point is (0, 3)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$$x - 0 = 2$$
$$x = 2$$

$\therefore$ Point is (2, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$$0 - y = 2$$
$$y = -2$$

$\therefore$ Point is (0, – 2)

<u>Test Point</u>

Put (0, 0) in

$$3x + 7y < 21$$
$$3(0) + 7(0) < 21$$
$$0 < 21$$

Which is true.

$\therefore$ Graph of an inequality $3x + 7y \leq 21$ will be towards the origin side.

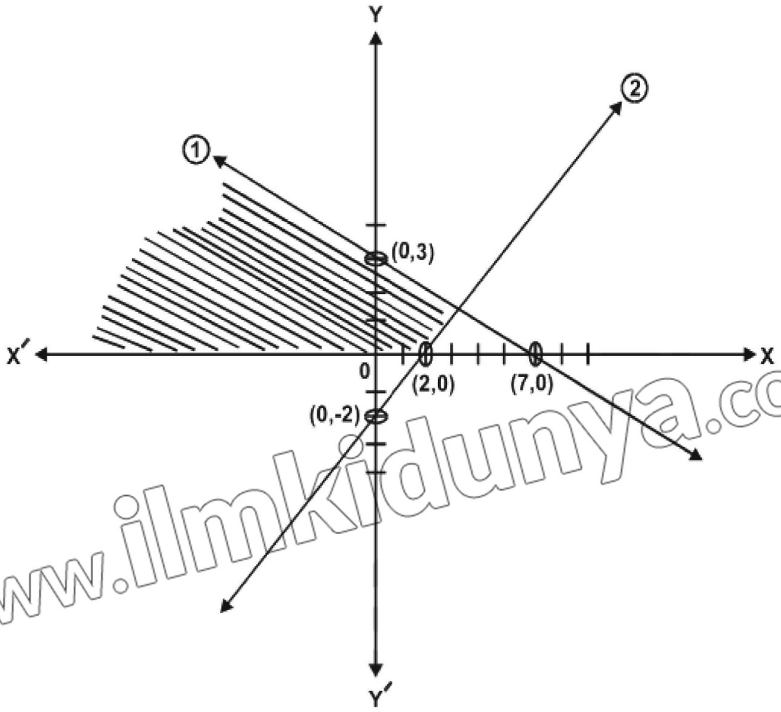
Put (0, 0) in

$$x - y < 2$$
$$0 - 0 < 2$$

$0 < 2$

Which is true.

$\therefore$ Graph of an inequality $x - y \leq 2$ will be towards the origin side.



(vi) $3x + 7y \leq 21$ **(Gujranwala Board 2006)**

$2x - y \geq -3$

$x \geq 0$

The associated equations are

$3x + 7y = 21$ ...... (1)

$2x - y = -3$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$3x + 7(0) = 21$

$3x = 21$

$x = \frac{21}{3} = 7$

$\therefore$ Point is $(7, 0)$

<u>y–intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 7y = 21$

$7y = 21$

$y = \frac{21}{7} = 3$

$\therefore$ Point is (0, 3)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$2x - 0 = -3$

$x = \frac{-3}{2}$

$\therefore$ Point is $\left(\frac{-3}{2}, 0\right)$

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$2(0) - y = -3$

$-y = -3$

$y = 3$

$\therefore$ Point is (0, 3)

<u>Test Point</u>

Put (0, 0) in

$3x + 7y < 21$

$3(0) + 7(0) < 21$

$0 < 21$

Which is true.

$\therefore$ Graph of an inequality $3x + 7y \leq 21$ will be towards the origin side.

Put (0, 0) in

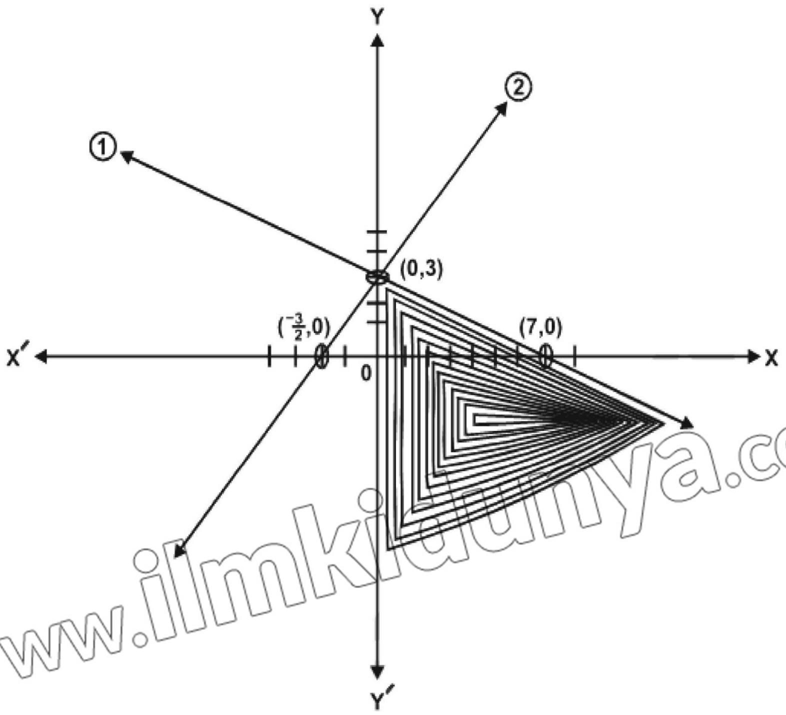
$2x - y > -3$

$2(0) - 0 > -3$

$0 > -3$

Which is true.

$\therefore$ Graph of an inequality $2x - y \geq -3$ will be towards the origin side.



Q.4: Graph the solution region of the following system of linear inequalities and find the corner points in each case.

(i) $2x - 3y \leq 6$, $2x + 3y \leq 12$, $x \geq 0$

(ii) $x + y \leq 5$, $-2x + y \leq 2$, $y \geq 0$

(iii) $3x + 7y \leq 21$, $2x - y \leq -3$, $y \geq 0$

(iv) $3x + 2y \geq 6$, $x + 3y \leq 6$, $y \geq 0$

(v) $5x + 7y \leq 35$, $-x + 3y \leq 3$, $x \geq 0$

(vi) $5x + 7y \leq 35$, $x - 2y \leq 2$, $x \geq 0$

Solution:

(i) $2x - 3y \leq 6$

$2x + 3y \leq 12$

$x \geq 0$

The associated equations are

$2x - 3y = 6$ ...... (1)

$2x + 3y = 12$ ...... (2)

x-intercept

Put $y = 0$ in eq. (1)

$2x - 3(0) = 6$

$2x = 6$

$x = \frac{6}{2} = 3$

$\therefore$ Point is (3, 0)

y-intercept

Put $x = 0$ in eq. (1)

$2(0) - 3y = 6$

$-3y = 6$

$y = \frac{6}{-3} = -2$

$\therefore$ Point is (0, – 2)

x-intercept

Put $y = 0$ in eq. (2)

$2x + 3(0) = 12$

$x = 12$

$x = \frac{12}{2} = 6$

$\therefore$ Point is (6, 0)

y-intercept

Put $x = 0$ in eq. (2)

$2(0) + 3y = 12$

$3y = 12$

$y = \frac{12}{3} = 4$

$\therefore$ Point is (0, 4)

Test Point

Put (0, 0) in

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $2x - 3y \leq 6$ will be towards the origin side.

Put (0, 0) in

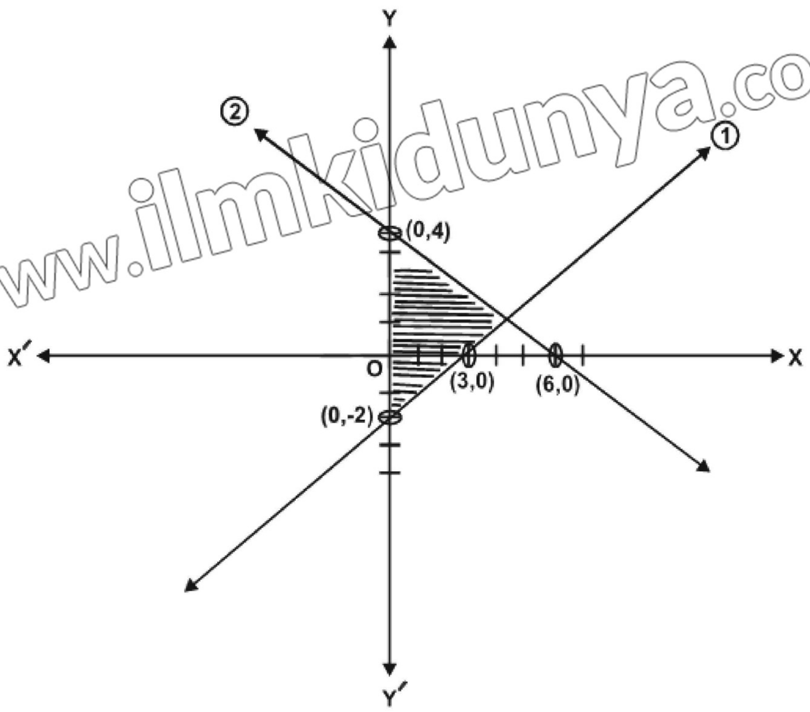
$2x + 3y < 12$

$2(0) + 3(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 12$ will be towards the origin side.



To find the intersection of both the lines solving eq. (1) & eq. (2)

Adding eq. (1) and eq. (2)

$2x - 3y = 6$

$\underline{2x + 3y = 12}$

$4x = 18$

$x = \frac{18}{4} = \frac{9}{2}$

Put $x = \frac{9}{2}$ in eq. (1)

$$2\left(\frac{9}{2}\right) - 3y = 6$$

$$9 - 3y = 6$$

$$y = \frac{8}{3} = 1$$

$\therefore$ Point $\left(\frac{9}{2}, 1\right)$

So the corner points are $(0, -2)$, $\left(\frac{9}{2}, 1\right)(0, 4)$

(ii) $\mathbf{x + y \le 5}$

$\mathbf{-2x + y \le 2}$

$\mathbf{y \ge 0}$

The associated equations are

$x + y = 5$ ...... (1)

$y - 2x = 2$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$x + 0 = 5$$

$$x = 5$$

$\therefore$ Point is $(5, 0)$

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$$0 + y = 5$$

$$y = 5$$

$\therefore$ Point is $(0, 5)$

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$$0 - 2x = 2$$

$$x = \frac{2}{-2} = -1$$

$\therefore$ Point is $(-1, 0)$

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$y - 2(0) = 2$

$y = 2$

∴ Point is (0, 2)

Test Point

Put (0, 0) in

$x + y < 5$

$0 + 0 < 5$

$0 < 5$

Which is true.

∴ Graph of an inequality $x + y \leq 5$ will towards the origin side.

Put (0, 0) in

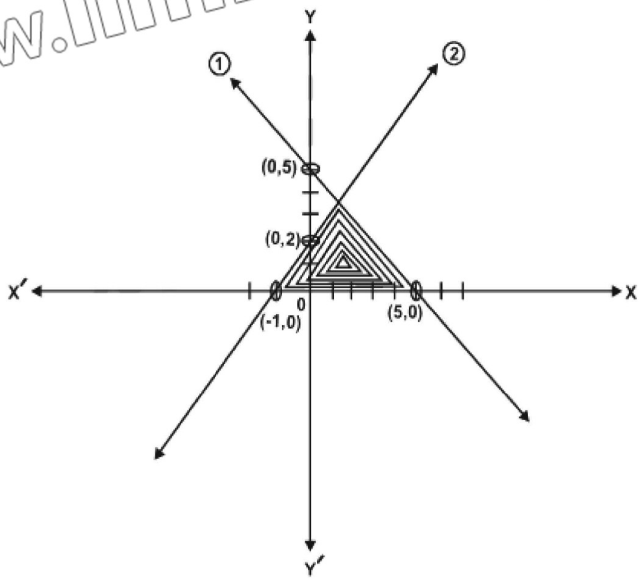
$y - 2x < 2$

$0 - 2(0) < 2$

$0 < 2$

Which is true.

∴ Graph of an inequality $y - 2x \leq 2$ will towards the origin side.



To find the intersection of both the lines solving eq. (1) & eq. (2).

Equation (1) – Eq. (2), we get

$$\begin{array}{rcl} x + y & = & 5 \\ -2x \pm y & = & -2 \\ \hline 3x & = & 3 \end{array}$$

$$x = \frac{3}{3} = 1$$

Put $x = 1$ in eq. (1)

$1 + y = 5$

$y = 5 - 1 = 4$

$\therefore$ Point (1, 4)

So the corner points are (– 1, 0), (5, 0), (1, 4)

(iii) $\mathbf{3x + 7y \le 21}$

$\mathbf{2x - y \le -3}$

$\mathbf{y \ge 0}$

The associated equations are

$3x + 7y = 21$ ...... (1)

$2x - y = -3$ ...... (2)

x-intercept

Put $y = 0$ in eq. (1)

$3x + 7(0) = 21$

$3x = 21$

$x = \frac{21}{3} = 7$

$\therefore$ Point is (7, 0)

y–intercept

Put $x = 0$ in eq. (1)

$3(0) + 7y = 21$

$7y = 21$

$y = \frac{21}{7} = 3$

$\therefore$ Point is (0, 3)

x-intercept

Put $y = 0$ in eq. (2)

$2x - 0 = -3$

$x = \frac{-3}{2}$

$\therefore$ Point is $\left(\frac{-3}{2}, 0\right)$

y-intercept

Put $x = 0$ in eq. (2)

$2(0) - y = -3$

$-y = -3$

$y = 3$

$\therefore$ Point is (0, 3)

<u>Test Point</u>

Put (0, 0) in

$3x + 7y < 21$

$3(0) + 7(0) < 21$

$0 < 21$

Which is true.

$\therefore$ Graph of an inequality $3x + 7y \leq 21$ will not be towards the origin side.

Put (0, 0) in

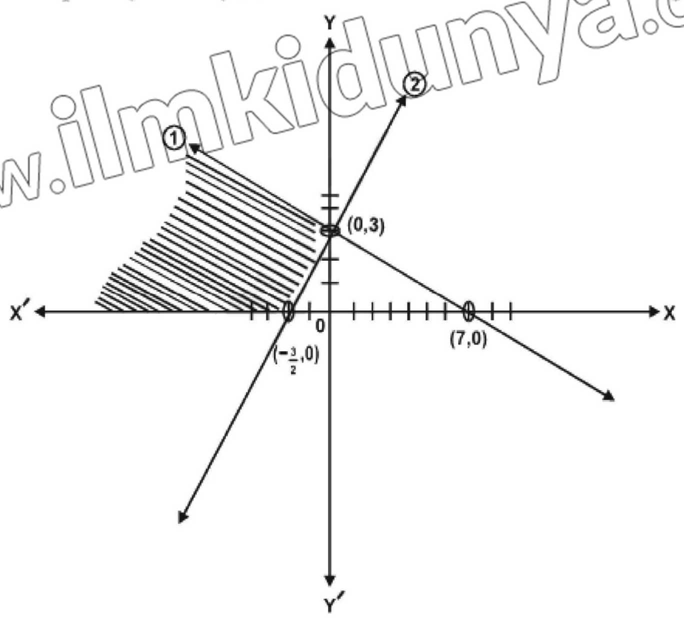
$2x - y < -3$

$2(0) - 0 < -3$

$0 < -3$

Which is false.

$\therefore$ Graph of an inequality $2x - y \leq -3$ will not be towards the origin side.



So the corner points are $\left(\frac{-3}{2}, 0\right)(0, 3)$

(iv) $3x + 2y \geq 6$

$x + 3y \leq 6$

$y \geq 0$

The associated equations are

$3x + 2y = 6$ ...... (1)

$x + 3y = 6$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$3x + 2(0) = 6$

$3x = 6$

$x = \frac{6}{3} = 2$

$\therefore$ Point is (2, 0)

<u>y–intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 2y = 6$

$y = \frac{6}{2} = 3$

$\therefore$ Point is (0, 3)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$x + 3(0) = 6$

$x = 6$

$\therefore$ Point is (6, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 + 3y = 6$

$y = \frac{6}{3}$

$y = 2$

$\therefore$ Point is (0, 2)

<u>Test Point</u>

Put (0, 0) in

$3x + 2y > 6$

$3(0) + 2(0) > 6$

$0 < 6$

Which is false.

$\therefore$ Graph of an inequality $3x + 2y \geq 6$ will not be towards the origin side.

Put (0, 0) in

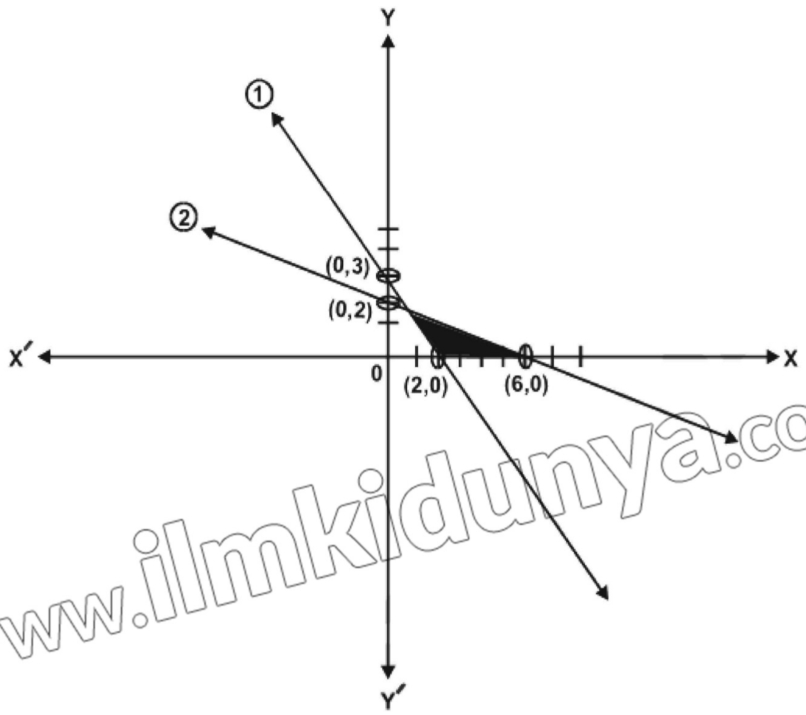
$x + 3y < 6$

$0 - 3(0) < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $x + 3y \leq 6$ will be towards the origin side.



To find the intersection of both the equations solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 3, we get

$$\begin{array}{r} 3x + 2y = \quad 6 \\ \underline{-3x \pm 9y = -18} \\ -7y = -12 \end{array}$$

$$y = \frac{12}{7}$$

Put $y = \frac{12}{7}$ in eq. (2)

$$x + 3\left(\frac{12}{7}\right) = 6$$

$$x + \frac{36}{7} = 6$$

$$x = 6 - \frac{36}{7}$$

$$x = \frac{42-36}{7}$$

$$= \frac{6}{7}$$

$\therefore$ Point $\left(\frac{6}{7}, \frac{12}{7}\right)$

So the corner points are (2, 0), (6, 0), $\left(\frac{6}{7}, \frac{12}{7}\right)$

(v) $\mathbf{5x + 7y \le 35}$

$\mathbf{-x + 3y \le 3}$

$\mathbf{x \ge 0}$

The associated equations are

$5x + 7y = 35$ ...... (1)

$-x + 3y = 3$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$5x + 7(0) = 35$

$5x = 35$

$x = \frac{35}{5} = 7$

$\therefore$ Point is (7, 0)

<u>y–intercept</u>

Put $x = 0$ in eq. (1)

$5(0) + 7y = 35$

$y = \frac{35}{7} = 5$

$\therefore$ Point is (0, 5)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$-x + 3(0) = 3$

$-x = 3$

$x = -3$

$\therefore$ Point is (– 3, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$-0 + 3y = 3$

$y = \frac{3}{3} = 1$

∴ Point is (0, 1)

<u>Test Point</u>

Put (0, 0) in

$5x + 7y < 35$

$5(0) + 7(0) < 35$

$0 < 35$

Which is true.

∴ Graph of an inequality $5x + 7y \leq 35$ will be towards the origin side.

Put (0, 0) in

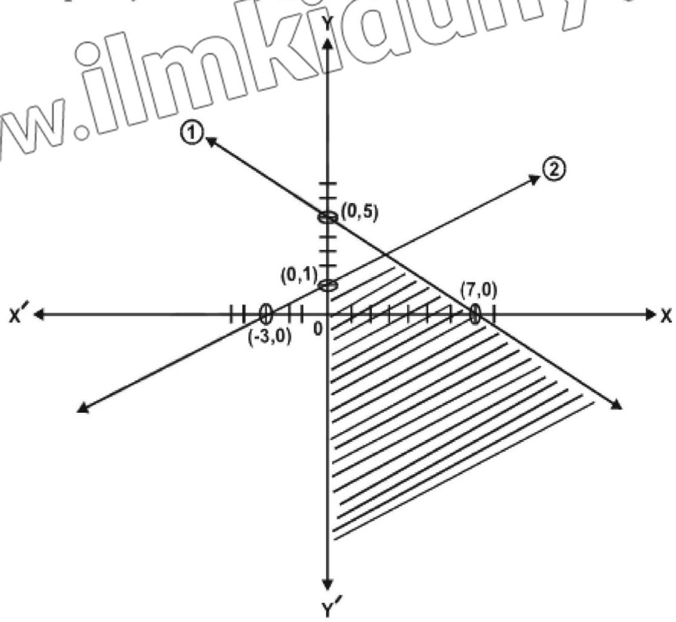
$-x + 3y < 3$

$-0 + 3(0) < 3$

$0 < 3$

Which is true.

∴ Graph of an inequality $-x + 3y \leq 6$ will be towards the origin side.



To find the intersection of both the equations solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 5, we get

$$\begin{aligned} 5x + 7y &= 35 \\ -5x + 15y &= 15 \\ \hline 22y &= 50 \\ y &= \frac{50}{22} = \frac{25}{11} \end{aligned}$$

Put $y = \frac{25}{11}$ in eq. (2)

$-x + 3\left(\frac{25}{11}\right) = 3$

$\frac{75}{11} - 3 = x$

$x = \frac{42}{11}$

∴ Point $\left(\frac{42}{11}, \frac{25}{11}\right)$

So the corner points are (0,1), $\left(\frac{42}{11}, \frac{25}{11}\right)$

(vi) $\mathbf{5x + 7y \leq 35}$

$\mathbf{x - 2y \leq 2}$

$\mathbf{x \geq 0}$

The associated equations are

$5x + 7y = 35$ ...... (1)

$x - 2y = 2$ ...... (2)

x-intercept

Put $y = 0$ in eq. (1)

$5x + 7(0) = 35$

$x = \frac{35}{5} = 7$

∴ Point is (7, 0)

y-intercept

Put $x = 0$ in eq. (1)

$5(0) + 7y = 35$

$x = \frac{35}{7} = 5$

∴ Point is (0, 5)

x-intercept

Put $y = 0$ in eq. (2)

$x - 2(0) = 2$

$x = 2$

∴ Point is (2, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 - 2y = 2$

$y = \dfrac{2}{-2} = -1$

∴ Point is (0, – 1)

<u>Test Point</u>

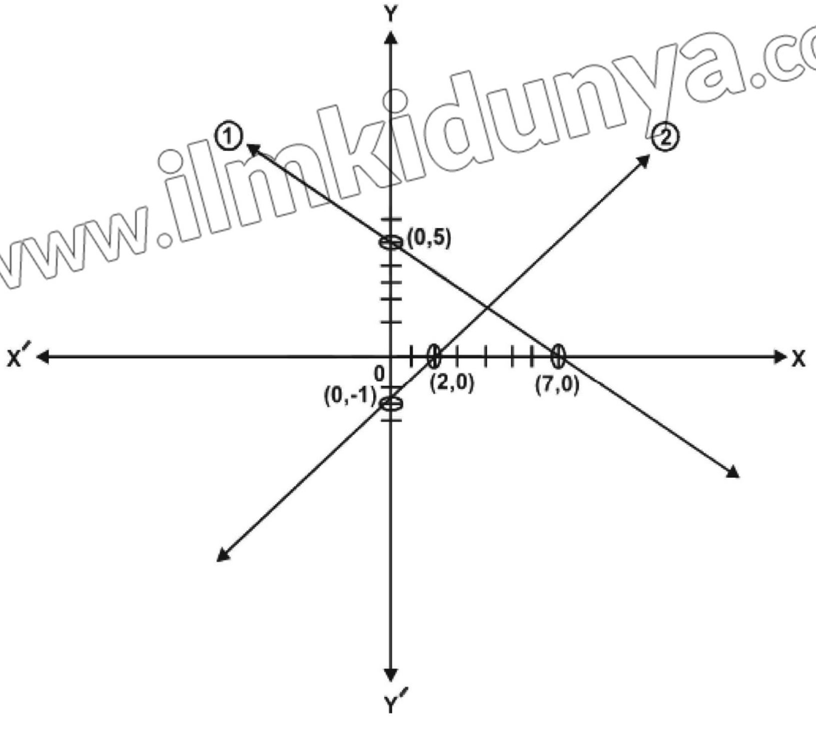
Put (0, 0) in

$5x + 7y < 35$

$5(0) + 7(0) < 35$

$0 < 35$

Which is true.



∴ Graph of an inequality $5x + 7y \leq 35$ will be towards the origin side.

Put (0, 0) in

$x - 2y < 2$

$0 - 2(0) < 2$

$0 < 2$

Which is true.

∴ Graph of an inequality $x - 2y \leq 4$ will be towards the origin side.

To find the intersection of both is the equations solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 5, we get

$$\begin{array}{rcl} 5x + 7y &=& 35 \\ -5x \mp 10y &=& -10 \\ \hline 17y &=& 25 \end{array}$$

$$y = \frac{25}{17}$$

Put $y = \frac{25}{17}$ in eq. (2), we get

$$x - 2\left(\frac{25}{17}\right) = 2$$

$$x - \frac{50}{17} = 2$$

$$x = 2 + \frac{50}{17}$$

$$x = \frac{34 + 50}{17}$$

$$x = \frac{84}{17}$$

$$\therefore \text{ Point}\left(\frac{84}{17}, \frac{25}{17}\right)$$

So the corner points are $\left(\frac{84}{17}, \frac{25}{17}\right)$, (0, 5), (0, – 2)

Q.5: Graph the solution region of the following system of linear inequalities by shading.

(i)	**$3x - 4y \le 12$**	**(ii)**	**$3x - 4y \le 12$**
	$3x + 2y \ge 3$		**$x + 2y \le 6$**
	$x + 2y \le 9$		**$x + y \ge 1$**
(iii)	**$2x + y \le 4$**	**(iv)**	**$2x + y \le 10$**
	$2x - 3y \ge 12$		**$x + y \le 7$**
	$x + 2y \le 6$		**$-2x + y \le 4$**
(v)	**$2x + 3y \le 18$**	**(vi)**	**$3x - 2y \ge 3$**
	$2x + y \le 10$		**$x + 4y \le 12$**
	$-2x + y \le 2$		**$3x + y \le 12$**

Solution:

(i) $\mathbf{3x - 4y \le 12}$

$\mathbf{3x + 2y \ge 3}$

$\mathbf{x + 2y \le 9}$

The associated equations are

$3x - 4y = 12$ .... (1)

$3x + 2y = 3$ .... (2)

$x + 2y = 9$ .... (3)

x-intercept

Put y = 0 in eqs. (1), (2) and (3)

$3x - 4(0) = 12$

$3x = 12$

$x = \frac{12}{3} = 4$

$\therefore$ Point is (4, 0)

$3x + 2(0) = 3$

$3x = 3$

$x = \frac{3}{3} = 1$

$\therefore$ Point is (1, 0)

$x + 2(0) = 9$

$x = 9$

$\therefore$ Point is (9, 0)

y-intercept

Put x = 0 in eqs. (1), (2) and (3)

$3(0) - 4y = 12$

$y = \frac{12}{-4} = -3$

$\therefore$ Point is (0, – 3)

$3(0) + 2y = 3$

$y = \frac{3}{2}$

$\therefore$ Point is $\left(0, \frac{3}{2}\right)$

$0 + 2y = 9$

$y = \frac{9}{2}$

$\therefore$ Point is $\left(0, \frac{9}{2}\right)$

Test Point

Put (0, 0) in

$3x - 4y < 12$

$3(0) - 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $3x - 4y \le 12$ will be towards the origin side.

Put (0, 0) in

$3x + 2y > 3$

$3(0) + 2(0) > 3$

$0 > 3$

which is false.

∴ Graph of an inequality $3x + 2y \geq 3$ will not be towards the origin side.

Put (0, 0) in

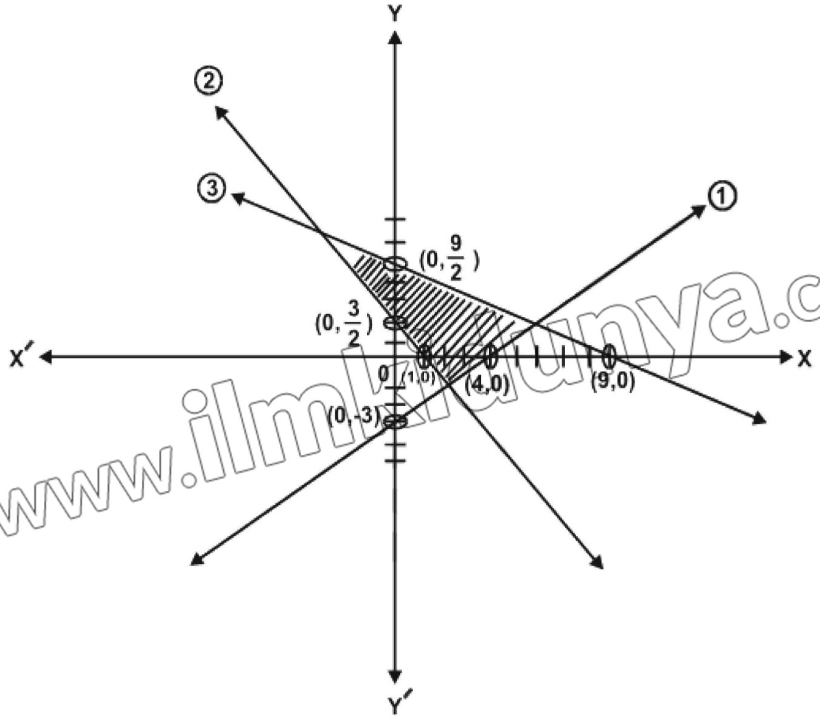
$x + 2y < 9$

$0 + 2(0) < 9$

$0 < 9$

Which is true.

∴ Graph of an inequality $x + 2y \leq 9$ will be towards the origin side.



(ii) $\mathbf{3x - 4y \leq 12}$

$\mathbf{x + 2y \leq 6}$

$\mathbf{x + y \geq 1}$

The associated equations are

$3x - 4y = 12$ .... (1)

$x + 2y = 6$ .... (2)

$x + y = 1$ .... (3)

<u>x-intercept</u>

Put $y = 0$ in equations (1), (2) and (3)

$3x - 4(0) = 12$	$x + 2(0) = 6$	$x + 0 = 1$
$3x = 12$	$x = 6$	$x = 1$

$x = \frac{12}{3} = 4$

$\therefore$ Point is (4, 0)

$\therefore$ Point is (6, 0)

$\therefore$ Point is (1, 0)

y-intercept

Put x = 0 in equations (1), (2) and (3)

$3(0) - 4y = 12$

$y = \frac{12}{-4} = -3$

$\therefore$ Point is (0, – 3)

$0 + 2y = 6$

$y = \frac{6}{2} = 3$

$\therefore$ Point is (0, 3)

$0 + y = 1$

$y = 1$

$\therefore$ Point is (0, 1)

Test Point

Put (0, 0) in

$3x - 4y < 12$

$3(0) - 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $3x - 4y \leq 12$ will be towards the origin side.

Put (0, 0) in

$x + 2y < 6$

$0 + 2(0) < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $x + 2y \leq 6$ will not be towards the origin side.

Put (0, 0) in

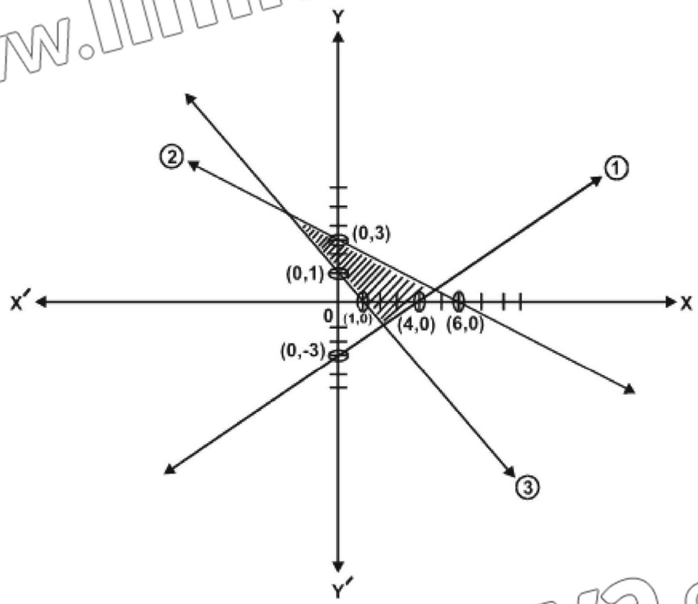
$x + y > 1$

$0 + 0 > 1$

$0 > 1$

Which is false.

$\therefore$ Graph of an inequality $x + y \geq 1$ will not be towards the origin side.



(iii) $\mathbf{2x + y \leq 4}$

$\mathbf{2x - 3y \geq 12}$

$\mathbf{x + 2y \leq 6}$

The associated equations are

$2x + y = 4$ .... (1)

$2x - 3y = 12$ .... (2)

$x + 2y = 6$ .... (3)

<u>x-intercept</u>

Put y = 0 in equations (1), (2) and (3)

$2x + 0 = 4$	$2x - 3(0) = 12$	$x + 2(0) = 6$
$x = \frac{4}{2} = 2$	$2x = 12$	$x = 6$
$\therefore$ Point is (2, 0)	$x = \frac{12}{2} = 6$	$\therefore$ Point is (6, 0)
	$\therefore$ Point is (6, 0)	

<u>y-intercept</u>

Put x = 0 in equations (1), (2) and (3)

$2(0) + y = 4$	$2(0) - 3y = 12$	$0 + 2y = 6$
$y = 4$	$y = \frac{12}{-3} = -4$	$y = \frac{6}{2} = 3$
$\therefore$ Point is (0, 4)	$\therefore$ Point is (0, – 4)	$\therefore$ Point is (0, 3)

Test Point

Put (0, 0) in

$2x + y < 4$

$2(0) + 0 < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 4$ will be towards the origin side.

Put (0, 0) in

$2x - 3y > 12$

$2(0) - 3(0) > 12$

$0 > 12$

Which is false.

$\therefore$ Graph of an inequality $2x - 3y \geq 12$ will not be towards the origin side.

Put (0, 0) in

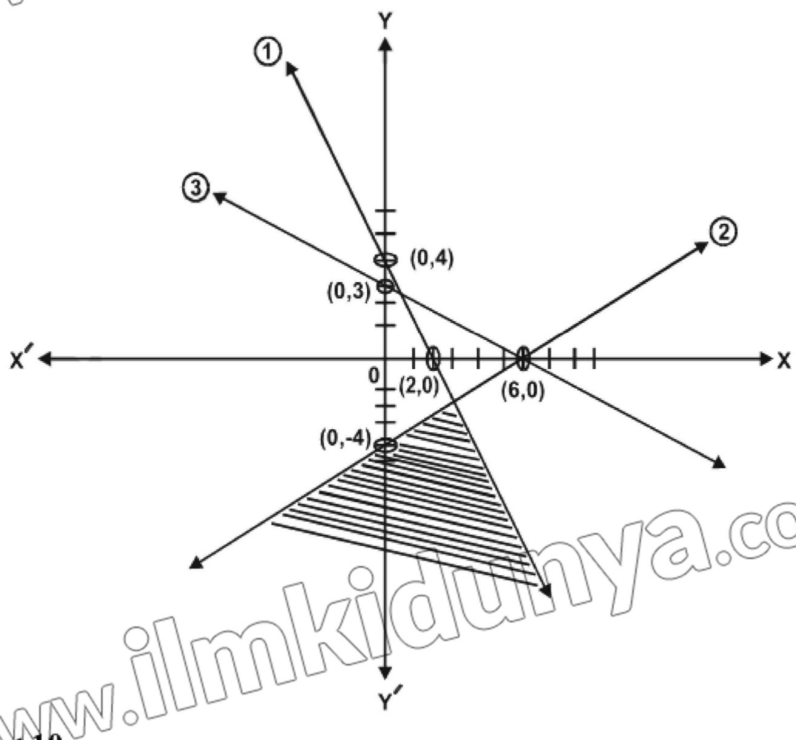
$x + 2y < 6$

$0 + 2(0) < 6$

$0 < 6$

Which is true.

$\therefore$ Graph of an inequality $x + 2y \leq 6$ will be towards the origin side.



(iv) $\mathbf{2x + y \leq 10}$

$\mathbf{x + y \leq 7}$

$-2x + y \leq 4$

The associated equations are

$2x + y = 10$ .... (1)

$x + y = 7$ .... (2)

$-2x + y = 4$ .... (3)

<u>x-intercept</u>

Put y = 0 in equations (1), (2) and (3)

$2x + 0 = 10$	$x + 0 = 7$	$-2x + 0 = 4$
$x = \frac{10}{2} = 5$	$x = 7$	$x = \frac{4}{-2} = -2$
∴ Point is (5, 0)	∴ Point is (7, 0)	∴ Point is (– 2, 0)

<u>y-intercept</u>

Put x = 0 in equations (1), (2) and (3)

$2(0) + y = 10$	$0 + y = 7$	$-2(0) + y = 4$
$y = 10$	$y = 7$	$y = 4$
∴ Point is (0, 10)	∴ Point is (0, 7)	∴ Point is (0, 4)

<u>**Test Point**</u>

Put (0, 0) in

$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

∴ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.

Put (0, 0) in

$x + y < 7$

$0 + 0 < 7$

$0 < 7$

Which is true.

∴ Graph of an inequality $x + y \leq 7$ will be towards the origin side.

Put (0, 0) in

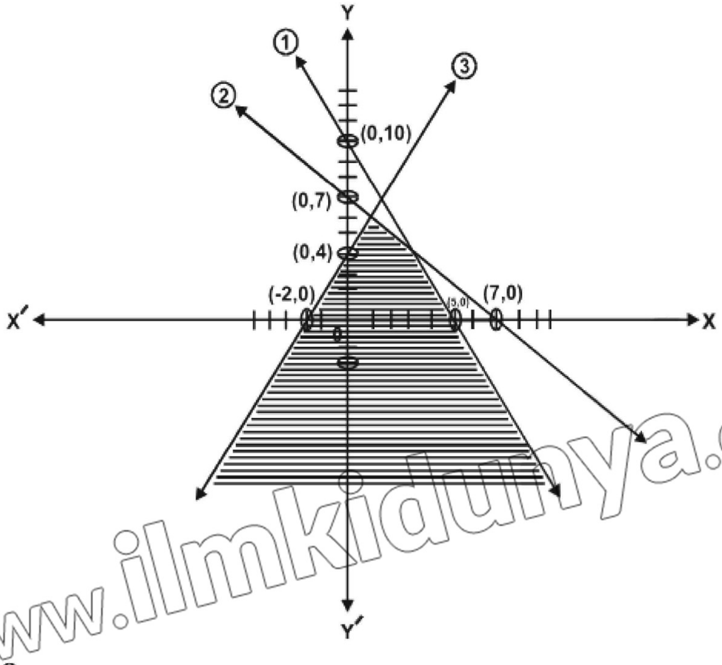
$-2x + y < 4$

$-2(0) + 0 < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $-2x + y \leq 4$ will be towards the origin side.



(v) $\mathbf{2x + 3y \leq 18}$

$\mathbf{2x + y \leq 10}$

$\mathbf{-2x + y \leq 2}$

The associated equations are

$2x + 3y = 18 \quad .... (1)$

$2x + y = 10 \quad .... (2)$

$-2x + y = 2 \quad .... (3)$

<u>x-intercept</u>

Put y = 0 in equations (1), (2) and (3)

$2x + 3(0) = 18$

$2x = 18$

$x = \frac{18}{2} = 9$

$\therefore$ Point is (9, 0)

$2x + 0 = 10$

$2x = 10$

$x = \frac{10}{2} = 5$

$\therefore$ Point is (5, 0)

$-2x + 0 = 2$

$x = \frac{2}{-2} = -1$

$\therefore$ Point is (– 1, 0)

<u>y-intercept</u>

Put x = 0 in equations (1), (2) and (3)

$2(0) + 3y = 18$
$3y = 18$
$y = \frac{18}{3} = 6$
$\therefore$ Point is (0, 6)

$2(0) + y = 10$
$y = 10$
$\therefore$ Point is (0, 10)

$-2(0) + y = 2$
$y = 2$
$\therefore$ Point is (0, 2)

<u>Test Point</u>

Put (0, 0) in

$2x + 3y < 18$

$2(0) + 3(0) < 18$

$0 < 18$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 18$ will be towards the origin side.

Put (0, 0) in

$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.

Put (0, 0) in

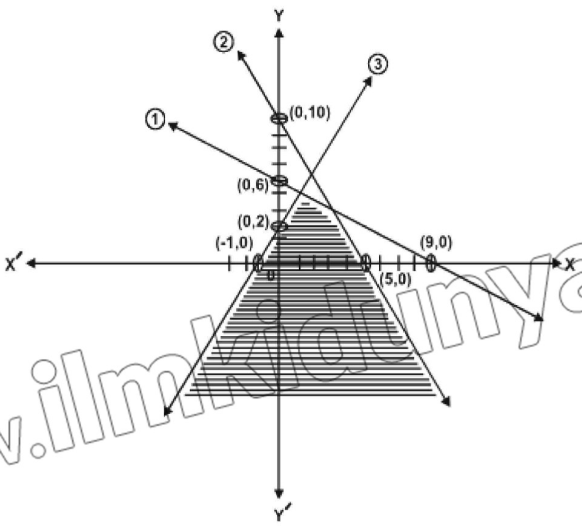
$-2x + y < 2$

$-2(0) + 0 < 2$

$0 < 2$

Which is true.

$\therefore$ Graph of an inequality $-2x + y \leq 2$ will be towards the origin side.



(vi) $\mathbf{3x - 2y \geq 3}$

$\mathbf{x + 4y \leq 12}$

$\mathbf{3x + y \leq 12}$

The associated equations are

$3x - 2y = 3$ .... (1)

$x + 4y = 12$ .... (2)

$3x + y = 12$ .... (3)

x-intercept

Put y = 0 in equations (1), (2) and (3)

$3x - 2(0) = 3$

$3x = 3$

$x = \frac{3}{3} = 1$

∴ Point is (1, 0)

$x + 4(0) = 12$

$x = 12$

∴ Point is (12, 0)

$3x + 0 = 12$

$x = \frac{12}{3} = 4$

∴ Point is (4, 0)

y-intercept

Put x = 0 in equations (1), (2) and (3)

$3(0) - 2y = 3$

$y = \frac{3}{-2}$

∴ Point is $\left(0, \frac{-3}{2}\right)$

$0 + 4y = 12$

$y = \frac{12}{4} = 3$

∴ Point is (0, 3)

$3(0) + y = 12$

$y = 12$

∴ Point is (0, 12)

Test Point

Put (0, 0) in

$3x - 2y > 3$

$3(0) - 2(0) > 3$

$0 > 3$

Which is false.

∴ Graph of an inequality $3x - 2y \geq 3$ will not be towards the origin side.

Put (0, 0) in

$x + 4y < 12$

$0 + 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.

Put $(0, 0)$ in

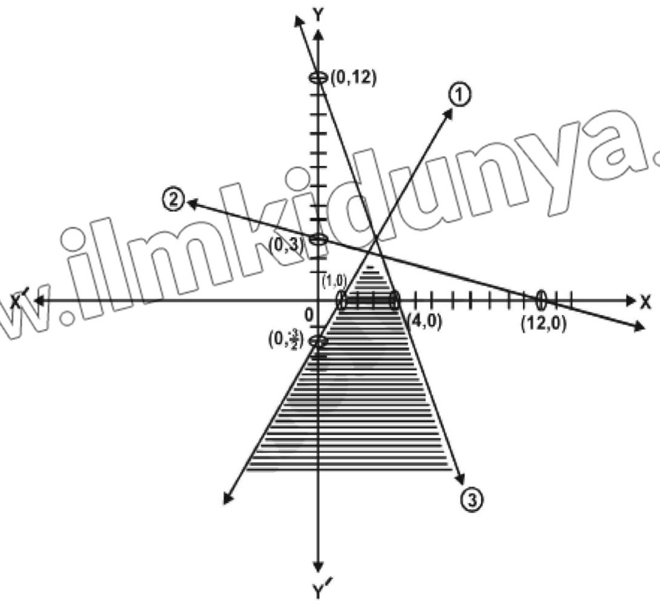
$3x + y < 12$

$3(0) + 0 < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $3x + y \leq 12$ will be towards the origin side.



EXERCISE 5.2

Q.4: Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i) $2x - 3y \leq 6$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$

(ii) $x + y \leq 5$, $-2x + y \leq 2$, $x \geq 0$, $y \geq o$

(iii) $x + y \leq 5$, $-2x + y \geq 2$, $x \geq 0$, $y \geq 0$

(iv) $3x + 7y \leq 21$, $x - y \leq 3$, $x \geq 0$, $y \geq 0$

(v) $3x + 2y \geq 6$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$

(vi) $5x + 7y \leq 35$, $x - 2y \leq 4$, $x \geq 0$, $y \geq 0$

Solution:

(i) $\mathbf{2x - 3y \leq 6}$ **(Lhr. Board 2005)**

$\mathbf{2x + 3y \leq 12}$

$\mathbf{x \geq 0 \quad , \quad y \geq 0}$

The associated equations are

$2x - 3y = 6 \quad ...... (1)$

$2x + 3y = 12 \quad ...... (2)$

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$2x - 3(0) = 6$

$2x = 6$

$x = \frac{6}{2} = 3$

$\therefore$ Point is (3, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$2(0) - 3y = 6$

$-3y = 6$

$y = \frac{6}{-3} = -2$

$\therefore$ Point is (0, – 2)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$2x + 3(0) = 12$

$x = 12$

$x = \frac{12}{2} = 6$

$\therefore$ Point is (6, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$2(0) + 3y = 12$

$3y = 12$

$y = \frac{12}{3} = 4$

∴ Point is (0, 4)

<u>Test Point</u>

Put (0, 0) in

$2x - 3y < 6$

$2(0) - 3(0) < 6$

$0 < 6$

Which is true.

∴ Graph of an inequality $2x - 3y \le 6$ will be towards the origin side.

Put (0, 0) in

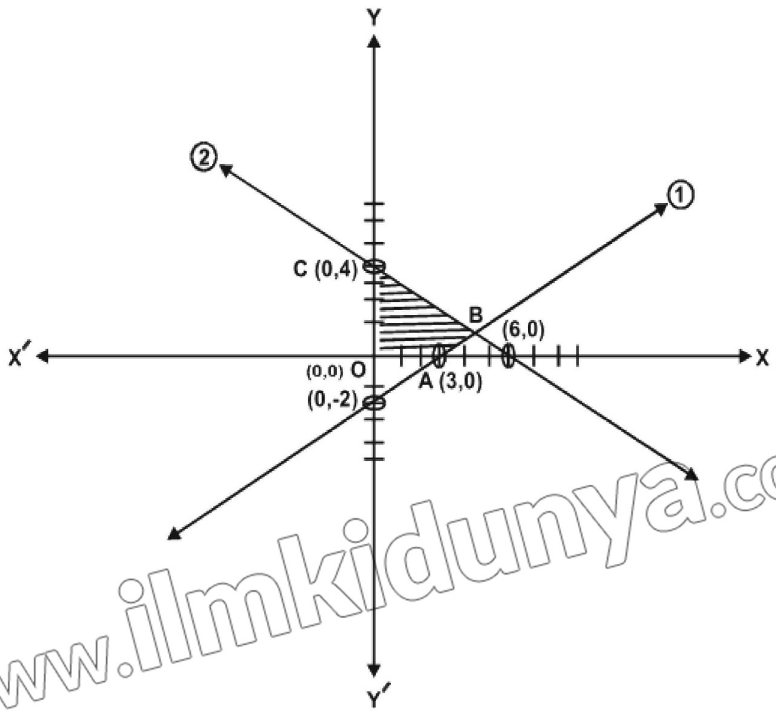
$2x + 3y < 12$

$2(0) + 3(0) < 12$

$0 < 12$

Which is true.

∴ Graph of an inequality $2x + 3y \le 12$ will be towards the origin side.



∴ OABC is the feasible solution region so corner points are

O(0, 0), A(3, 0), C(0, 4)

To find B solving eq. (1) & eq. (2)

Adding eq. (1) & eq. (2)

$2x - 3y = 6$

$\underline{2x + 3y = 12}$

$4x = 18$

$x = \frac{18}{4} = \frac{9}{2}$

Put

$x = \frac{9}{2}$ in eq. (1)

$2\left(\frac{9}{2}\right) - 3y = 6$

$9 - 6 = 3y$

$y = \frac{3}{3} = 1$

$\therefore$ $B\left(\frac{9}{2}, 1\right)$

(ii) $\mathbf{x + y \leq 5}$

$\mathbf{-2x + y \leq 2}$

$\mathbf{x \geq 0}$, $\mathbf{y \geq 0}$

The associated equations are

$x + y = 5$ ...... (1)

$y - 2x = 2$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$x + 0 = 5$

$x = 5$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$0 + y = 5$

$y = 5$

∴ Point is (0, 5)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$0 - 2x = 2$

$x = \frac{2}{-2} = -1$

∴ Point is (– 1, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$y - 2(0) = 2$

$y = 2$

∴ Point is (0, 2)

<u>Test Point</u>

Put (0, 0) in

$x + y < 5$

$0 + 0 < 5$

$0 < 5$

Which is true.

∴ Graph of an inequality $x + y \le 5$ will towards the origin side.

Put (0, 0) in

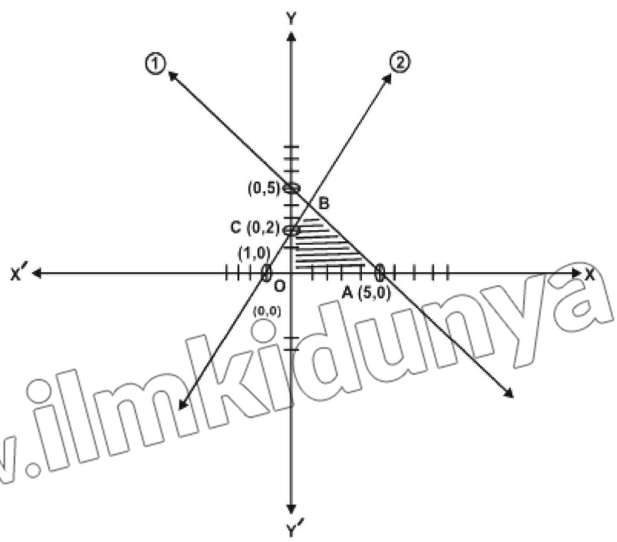
$y - 2x < 2$

$0 - 2(0) < 2$

$0 < 2$

Which is true.

∴ Graph of an inequality $y - 2x \le 2$ will towards the origin side.



∴ OABC is the feasible solution region so corner points are

O(0, 0), A(5, 0), C(0, 2)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) we get

$$\begin{array}{rcl} x + y &=& 5 \\ \mp 2x \pm y &=& _{-} 2 \\ \hline 3x &=& 3 \end{array}$$

$$x = \frac{3}{3} = 1$$

Put

$x = 1$ in eq. (1)

$1 + y = 5$

$y = 5 - 1 = 4$

$\therefore$ B (1, 4)

(iii) $\mathbf{x + y \leq 5}$

$\mathbf{-2x + y \geq 2}$

$\mathbf{x \geq 0 \quad , \quad y \geq 0}$

The associated equations are

$x + y = 5$ ..... (1)

$-2x + y = 2$ ..... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$x + 0 = 5$

$x = 5$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$0 + y = 5$

$y = 5$

$\therefore$ Point is (0, 5)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$-2x + 0 = 2$

$x = \frac{2}{-2} = -1$

∴ Point is (– 1, 0)

<u>y-intercept</u>

Put x = 0 in eq. (2)

$-2(0) + y = 2$

$y = 2$

∴ Point is (0, 2)

<u>Test Point</u>

Put (0, 0) in

$x + y < 5$

$0 + 0 < 5$

$0 < 5$

Which is true.

∴ Graph of an inequality $x + y \le 5$ will be towards the origin side.

Put (0, 0) in

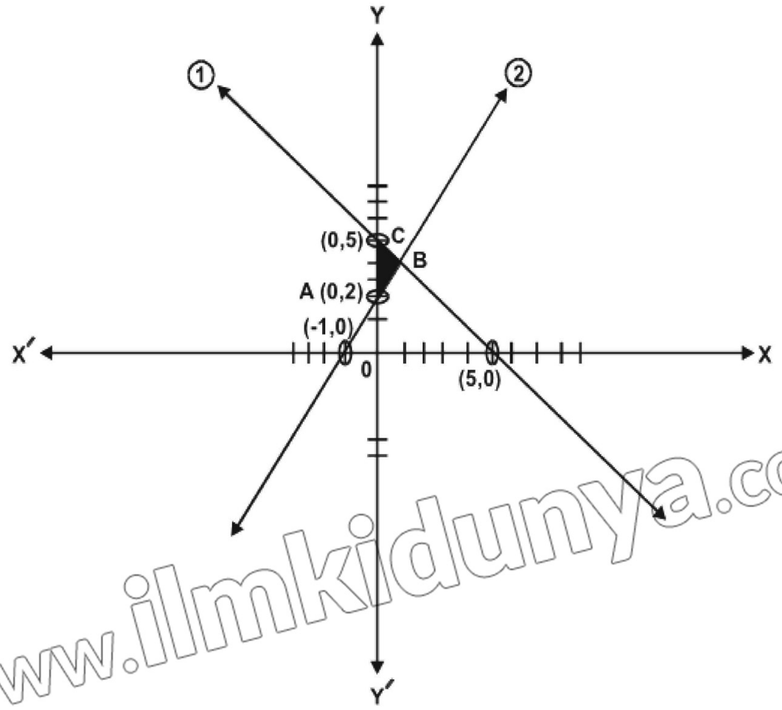
$-2x + y > 2$

$-2(0) + 0 > 2$

$0 > 2$

Which is false.

∴ Graph of an inequality $-2x + y \ge 2$ will not be towards the origin side.



∴ ABC is the feasible solution region. So corner points are A (0, 2), C (0, 5). To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2), we get

$$x + y = 5$$
$$\mp 2x \pm y = 2$$
$$3x = 3$$
$$x = \frac{3}{3} = 1$$

Put $x = 1$ in eq. (1)

$$1 + y = 5$$
$$y = 5 - 1 = 4$$

∴ B (1, 4)

(iv) $\mathbf{3x + 7y \leq 21}$

$\mathbf{x - y \leq 3}$

$\mathbf{x \geq 0 \;,\; y \geq 0}$

The associated equations are

$$3x + 7y = 21 \quad \text{...... (1)}$$
$$x - y = 3 \quad \text{...... (2)}$$

x-intercept

Put $y = 0$ in eq. (1)

$$3x + 7(0) = 21$$
$$3x = 21$$
$$x = \frac{21}{3} = 7$$

∴ Point is (7, 0)

y-intercept

Put $x = 0$ in eq. (1)

$$3(0) + 7y = 21$$
$$y = \frac{21}{7} = 3$$

∴ Point is (0, 3)

x-intercept

Put $y = 0$ in eq. (2)

$x - 0 = 3$

$x = 3$

$\therefore$ Point is (3, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 - y = 3$

$y = -3$

$\therefore$ Point is (0, – 3)

<u>Test Point</u>

Put (0, 0) in

$3x + 7y < 21$

$3(0) + 7(0) < 21$

$0 < 21$

Which is true.

$\therefore$ Graph of an inequality $3x + 7y \leq 21$ will be towards the origin side.

Put (0, 0) in

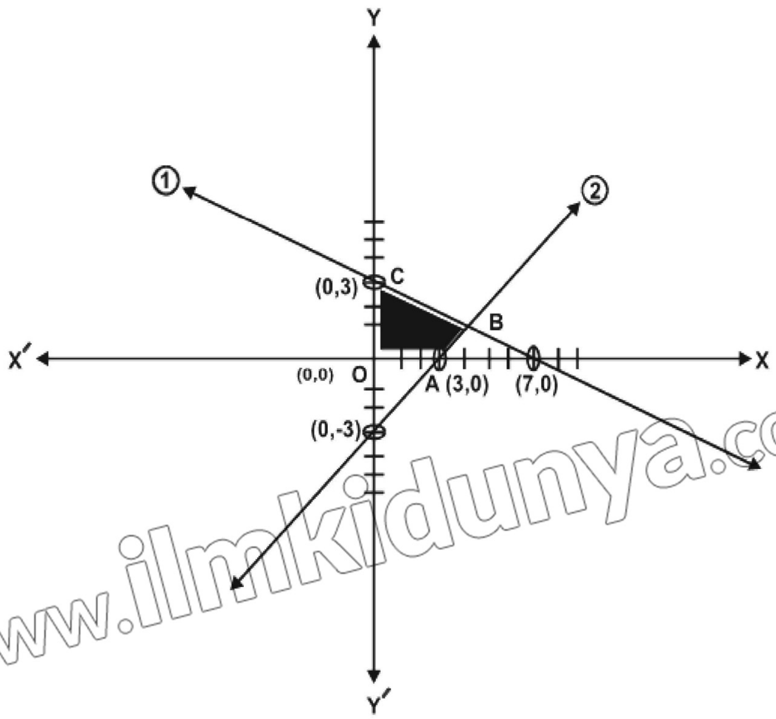
$x - y < 3$

$0 - 0 < 3$

$0 < 3$

Which is true.

$\therefore$ Graph of an inequality $x - y \leq 3$ will be towards the origin side.



∴ OABC is the feasible solution region so corner points are

O(0, 0), A(3, 0), C(0, 3)

To find B solving eq. (1) & eq. (2)

Eq. (1) + Eq. (2) × 7, we get

$$3x + 7y = 21$$
$$\underline{7x - 7y = 21}$$
$$10x = 42$$
$$x = \frac{42}{10} = \frac{21}{5}$$

Put $x = \frac{21}{5}$ in eq. (2)

$$\frac{21}{5} - y = 3$$
$$\frac{21}{5} - 3 = y$$
$$y = \frac{21 - 15}{5}$$
$$y = \frac{6}{5}$$

∴ $B\left(\frac{21}{5}, \frac{6}{5}\right)$

(v) $\mathbf{3x + 2y \geq 6}$

$\mathbf{x + y \leq 4}$

$\mathbf{x \geq 0 \ , \ y \geq 0}$

The associated equations are

$3x + 2y = 6$ .... (1)

$x + y = 4$ .... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$$3x + 2(0) = 6$$
$$x = \frac{6}{3} = 2$$

∴ Point is (2, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 2y = 6$

$y = \frac{6}{2} = 3$

$\therefore$ Point is (0, 3)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$x + 0 = 4$

$x = 4$

$\therefore$ Point is (4, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 + y = 4$

$y = 4$

$\therefore$ Point is (0, 4)

<u>Test Point</u>

Put (0, 0) in

$3x + 2y > 6$

$3(0) + 2(0) > 6$

$0 > 6$

Which is false.

$\therefore$ Graph of an inequality $3x + 2y \geq 6$ will not be towards the origin side.

Put (0, 0) in

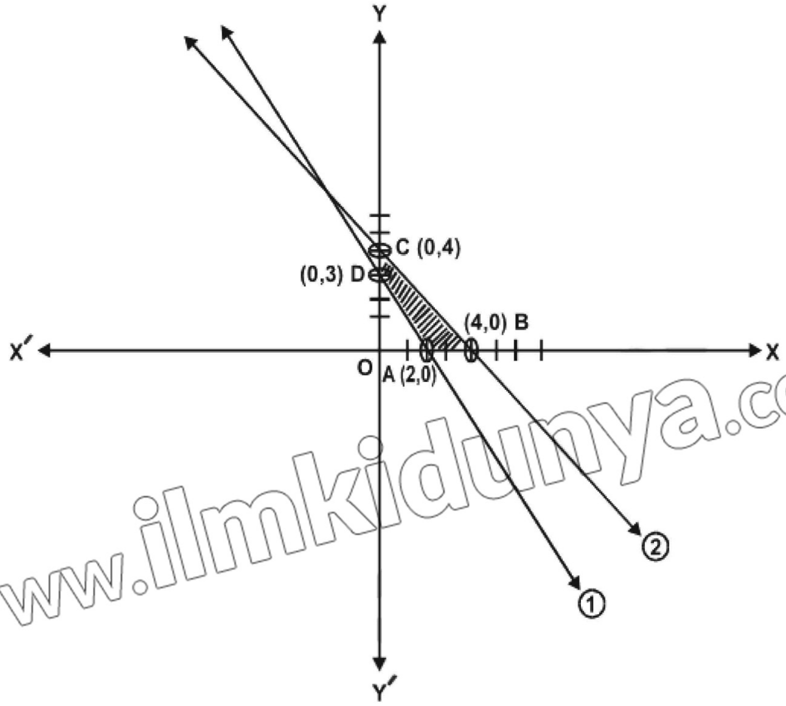
$x + y < 4$

$0 + 0 < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $x + y \leq 4$ will be towards the origin side.



$\therefore$ ABCD is the feasible solution region so corner points are

A (2, 0), B (4, 0), C(0, 4), D (0, 3)

(vi) $5x + 7y \leq 35$

$x - 2y \leq 4$

$x \geq 0 \;,\; y \geq 0$

The associated equations are

$5x + 7y = 35$ .... (1)

$x - 2y = 4$ .... (2)

x-intercept

Put $y = 0$ in eq. (1)

$5x + 7(0) = 35$

$$x = \frac{35}{5} = 7$$

$\therefore$ Point is (7, 0)

y-intercept

Put $x = 0$ in eq. (1)

$5(0) + 7y = 35$

$y = \frac{35}{7} = 5$

$\therefore$ Point is (0, 5)

x-intercept

Put $y = 0$ in eq. (2)

$x - 2(0) = 4$

$x = 4$

$\therefore$ Point is (4, 0)

y-intercept

Put $x = 0$ in eq. (2)

$0 - 2y = 4$

$y = \frac{4}{-2} = -2$

$\therefore$ Point is (0, – 2)

Test Point

Put (0, 0) in

$5x + 7y < 35$

$5(0) + 7(0) < 35$

$0 < 35$

Which is true.

$\therefore$ Graph of an inequality $5x + 7y \leq 35$ will be towards the origin side.

Put (0, 0) in

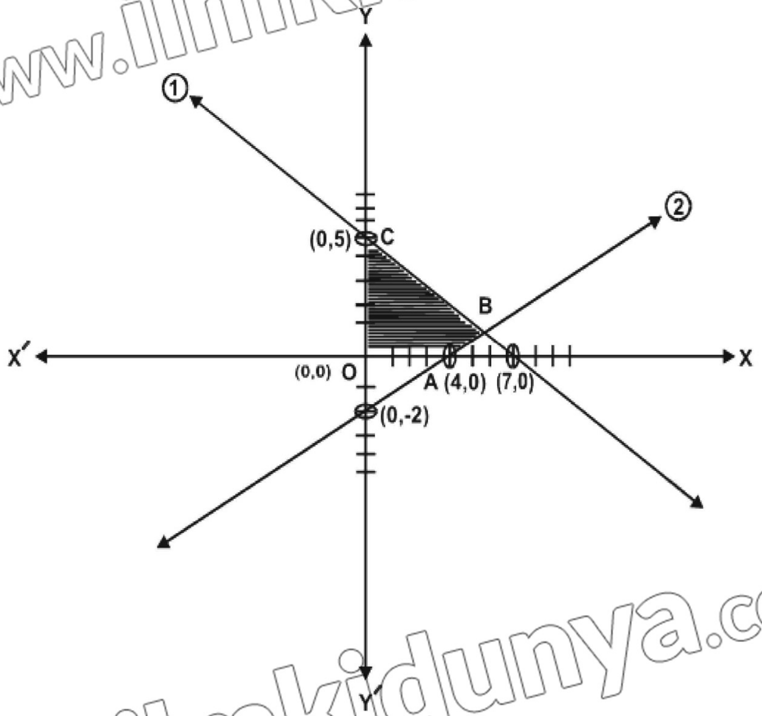
$x - 2y < 4$

$0 - 2(0) < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $x - 2y \leq 4$ will be towards the origin.



$\therefore$ OABC is the feasible solution region so corner points are

O (0, 0), A (4, 0), C (0, 5)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 5, we get

$$\begin{aligned} 5x + 7y &= 35 \\ -\ 5x \mp 10y &= -20 \\ \hline 17y &= 15 \\ y &= \frac{15}{17} \end{aligned}$$

Put $y = \frac{15}{17}$ in eq. (2)

$$x - 2\left(\frac{15}{17}\right) = 4$$

$$x - \frac{30}{17} = 4$$

$$x = 4 + \frac{30}{17}$$

$$x = \frac{68 + 30}{17}$$

$x = \frac{98}{17}$

$\therefore \quad B = \left(\frac{98}{17}, \frac{15}{17}\right)$

Q.2: Graph the feasible region of the following system of linear inequalities and find the corner points in each case.

(i) $2x + y \le 10$, $x + 4y \le 12$, $x + 2y \le 10$, $x \ge 0, y \ge 0$

(ii) $2x + 3y \le 18$, $2x + y \le 10$, $x + 4y \le 12$, $x \ge 0, y \ge 0$

(iii) $2x + 3y \le 18$, $x + 4y \le 12$, $3x + y \le 12$, $x \ge 0, y \ge 0$

(iv) $x + 2y \le 14$, $3x + 4y \le 36$, $2x + y \le 10$, $x \ge 0, y \ge 0$

(v) $x + 3y \le 15$, $2x + y \le 12$, $4x + 3y \le 24$, $x \ge 0, y \ge 0$

(vi) $2x + y \le 20$, $8x + 15y \le 120$, $x + y \le 11$, $x \ge 0, y \ge 0$

Solution:

(i) $2x + y \le 10$

$x + 4y \le 12$

$x + 2y \le 10$

$x \ge 0, y \ge 0$

The associated eqs. are

$2x + y = 10 \quad ...... (1)$

$x + 4y = 12 \quad ...... (2)$

$x + 2y = 10 \quad ...... (3)$

<u>x-intercept</u>

Put y = 0 in eqs. (1), (2) and (3)

$2x + 0 = 10$	$x + 4(0) = 12$	$x + 2(0) = 10$
$2x = 10$	$x = 12$	$x = 10$
$x = \frac{10}{2} = 5$	$\therefore$ Point is (12, 0)	$\therefore$ Point is (10, 0)
$\therefore$ Point is (5, 0)		

<u>y-intercept</u>

Put x = 0 in eqs. (1), (2) and (3)

$2(0) + y = 10$	$0 + 4y = 12$	$0 + 2y = 10$
$y = 10$	$4y = 12$	$2y = 10$

∴ Point is (0, 10)

$y = \frac{12}{4} = 3$

∴ Point is (0, 3)

$y = \frac{10}{2} = 5$

∴ Point is (0, 5)

<u>Test Point</u>

Put (0, 0)

$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

∴ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.

Put (0, 0) in

$x + 4y < 12$

$0 + 4(0) < 12$

$0 < 12$

Which is true.

∴ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.

Put (0, 0) in

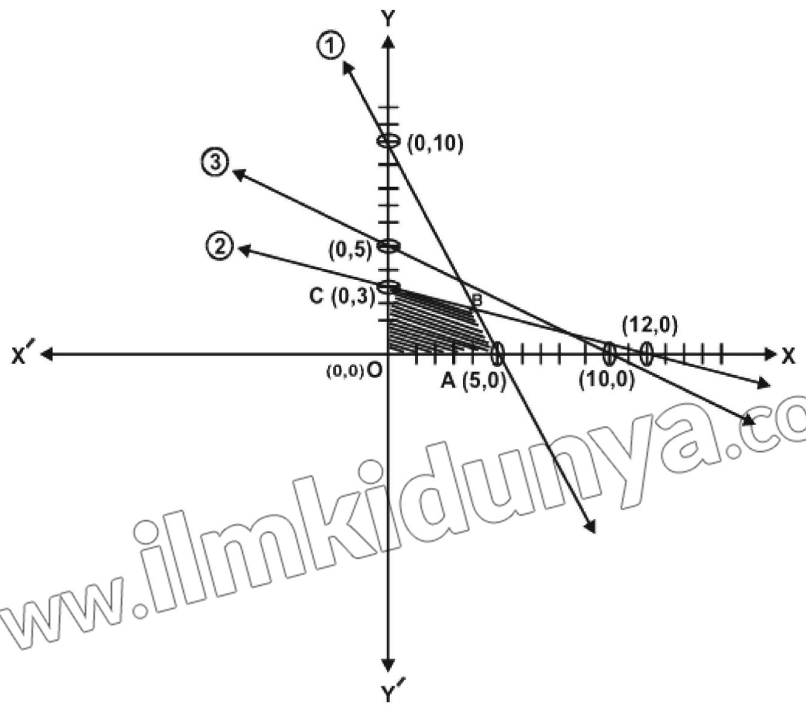
$x + 2y < 10$

$0 + 2(0) < 10$

$0 < 10$

Which is true.

∴ Graph of an inequality $x + 2y \leq 10$ will be towards the origin side.



∴ OABC is the feasible solution region so the corner points are

O (0, 0), A (5, 0), C (0, 3)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 2, we get

$$\begin{aligned} 2x + y &= 10 \\ -\ 2x \pm 8y &= -24 \\ \hline -7y &= -14 \\ y &= \frac{14}{7} = 2 \end{aligned}$$

Put y = 2 in eq. (2)

$$\begin{aligned} x + 4(2) &= 12 \\ x + 8 &= 12 \\ x &= 12 - 8 = 4 \end{aligned}$$

∴ B = (4, 2)

(ii) $2x + 3y \leq 18$ **(Guj. Board 2005) (Lhr. Board 2008)**

$2x + y \leq 10$

$x + 4y \leq 12$

$x \geq 0$, $y \geq 0$

The associated equations are

$2x + 3y = 18$ .... (1)

$2x + y = 10$ .... (2)

$x + 4y = 12$ .... (3)

<u>x-intercept</u>

Put y = 0 in eqs. (1), (2) and (3)

$$\begin{aligned} 2x + 3(0) &= 18 \\ 2x &= 18 \\ x &= \frac{18}{2} = 9 \end{aligned}$$

∴ Point is (9, 0)

$$\begin{aligned} 2x + 0 &= 10 \\ 2x &= 10 \\ x &= \frac{10}{2} = 5 \end{aligned}$$

∴ Point is (5, 0)

$$\begin{aligned} x + 4(0) &= 12 \\ x &= 12 \end{aligned}$$

∴ Point is (12, 0)

y-intercept

Put $x = 0$ in eqs. (1), (2) and (3)

$2(0) + 3y = 18$

$3y = 18$

$y = \frac{18}{3} = 6$

$\therefore$ Point is (0, 6)

$2(0) + y = 10$

$y = 10$

$\therefore$ Point is (0, 10)

$0 + 4y = 12$

$4y = 12$

$y = \frac{12}{4} = 3$

$\therefore$ Point is (0, 3)

Test Point

Put (0, 0)

$2x + 3y < 18$

$2(0) + 3(0) < 18$

$0 < 18$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 18$ will be towards the origin side.

Put (0, 0) in

$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.

Put (0, 0) in

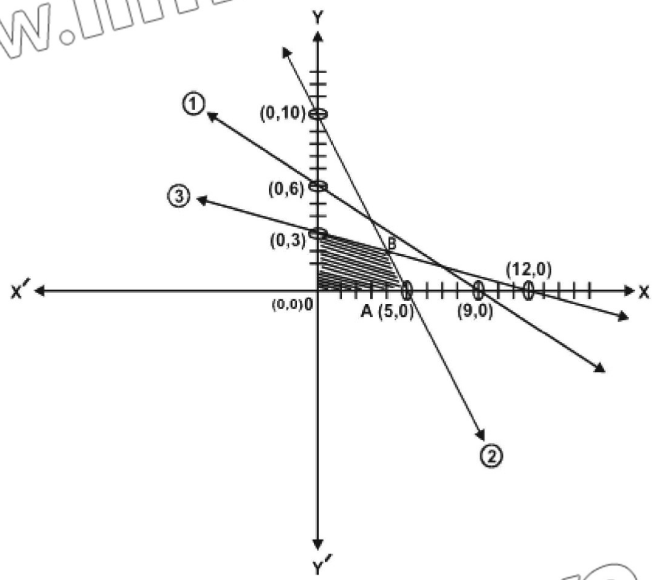
$x + 4y < 12$

$0 + 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (5, 0), C (0, 3)

To find B solving eq. (2) & eq. (3)

Eq. (2) – Eq. (3) × 2, we get

$$\begin{aligned} 2x + y &= 10 \\ -\ 2x \pm 8y &= -24 \\ \hline -7y &= -14 \\ y &= \frac{-14}{-7} = 2 \end{aligned}$$

Put $y = 2$ in eq. (3)

$$\begin{aligned} x + 4(2) &= 12 \\ x + 8 &= 12 \\ x &= 12 - 8 = 4 \end{aligned}$$

$\therefore$ $B = (4, 2)$

(iii) $\mathbf{2x + 3y \leq 18}$

$\mathbf{x + 4y \leq 12}$

$\mathbf{3x + y \leq 12}$

$\mathbf{x \geq 0 \quad , \quad y \geq 0}$

The associated equations are

$2x + 3y = 18$ .... (1)

$x + 4y = 12$ .... (2)

$3x + y = 12$ .... (3)

<u>x-intercept</u>

Put $y = 0$ in eqs. (1), (2) and (3)

$2x + 3(0) = 18$

$2x = 18$

$x = \frac{18}{2} = 9$

$\therefore$ Point is (9, 0)

$x + 4(0) = 12$

$x = 12$

$\therefore$ Point is (12, 0)

$3x + 0 = 12$

$3x = 12$

$x = \frac{12}{3} = 4$

$\therefore$ Point is (4, 0)

<u>y-intercept</u>

Put $x = 0$ in eqs. (1), (2) and (3)

$2(0) + 3y = 18$

$3y = 18$

$x = \frac{18}{3} = 6$

$\therefore$ Point is (0, 6)

$0 + 4y = 12$

$y = \frac{12}{4} = 3$

$\therefore$ Point is (0, 3)

$3(0) + y = 12$

$y = 12$

$\therefore$ Point is (0, 12)

<u>Test Point</u>

Put (0, 0) in

$2x + 3y < 18$

$2(0) + 3(0) < 18$

$0 < 18$

Which is true.

$\therefore$ Graph of an inequality $2x + 3y \leq 18$ will be towards the origin side.

Put (0, 0) in

$x + 4y < 12$

$0 + 4(0) < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $x + 4y \leq 12$ will be towards the origin side.

Put (0, 0) in

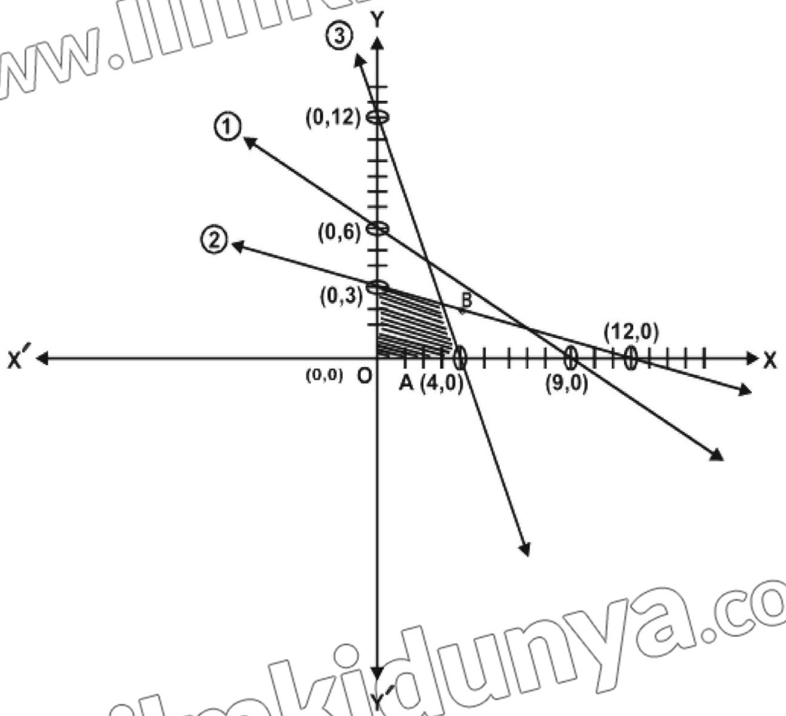
$3x + y < 12$

$3(0) + 0 < 12$

$0 < 12$

Which is true.

$\therefore$ Graph of an inequality $3x + y \leq 12$ will be towards the origin side.



Y
③
(0,12)
①
(0,6)
②
(0,3)
B
(12,0)
X′
X
(0,0)
O
A
(4,0)
(9,0)
Y′

$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (4, 0), C (0, 3)

To find B solving eq. (2) & eq. (3)

Eq. (2) × 3 – Eq. (3), we get

$$\begin{array}{rcl} 3x + 12y &=& 36 \\ -\ 3x \pm\ \ y &=& -12 \\ \hline 11y &=& 24 \end{array}$$

$$y = \frac{24}{11}$$

Put $y = \frac{24}{11}$ in eq. (3)

$$3x + \frac{24}{11} = 12$$

$$3x = 12 - \frac{24}{11}$$

$$3x = \frac{132 - 24}{11}$$

$$x = \frac{108}{33} = \frac{36}{11}$$

$$\therefore \quad B\left(\frac{36}{11}, \frac{24}{11}\right)$$

(iv) $\mathbf{x + 2y \le 14}$

$\mathbf{3x + 4y \le 36}$

$\mathbf{2x + y \le 10}$

$\mathbf{x \ge 0 \quad , \quad y \ge 0}$

The associated equations are

$x + 2y = 14$ .... (1)

$3x + 4y = 36$ .... (2)

$2x + y = 10$ .... (3)

x-intercept

Put y = 0 in eqs. (1), (2) and (3)

$x + 2(0) = 14$

$x = 14$

$\therefore$ Point is (14, 0)

$3x + 4(0) = 36$

$3x = 36$

$x = \frac{36}{3} = 12$

$\therefore$ Point is (12, 0)

$2x + 0 = 10$

$2x = 10$

$x = \frac{10}{2} = 5$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eqs. (1), (2) and (3)

$0 + 2y = 14$

$y = \frac{14}{2} = 7$

$\therefore$ Point is (0, 7)

$3(0) + 4y = 36$

$4y = 36$

$x = \frac{36}{4} = 9$

$\therefore$ Point is (0, 9)

$2(0) + y = 10$

$y = 10$

$\therefore$ Point is (0, 10)

<u>Test Point</u>

Put (0, 0) in

$x + 2y < 14$

$0 + 2(0) < 14$

$0 < 14$

Which is true.

$\therefore$ Graph of an inequality $x + 2y \leq 14$ will be towards the origin side.

Put (0, 0) in

$3x + 4y < 36$

$3(0) + 4(0) < 36$

$0 < 36$

Which is true.

$\therefore$ Graph of an inequality $3x + 4y \leq 36$ will be towards the origin side.

Put (0, 0) in

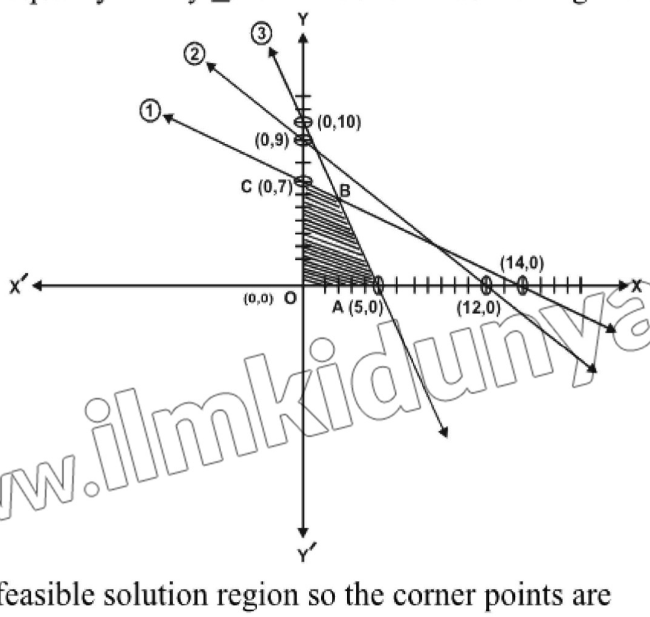
$2x + y < 10$

$2(0) + 0 < 10$

$0 < 10$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 10$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (5, 0), C (0, 7)

To find B solving eq. (1) & eq. (3)

Eq. (1) × 2 – Eq. (3), we get

$$\begin{array}{rcl} 2x + 4y &=& 28 \\ -\ 2x \pm y &=& -10 \\ \hline 3y &=& 18 \\ y &=& \frac{18}{3} = 6 \end{array}$$

Put $y = 6$ in eq. (1)

$$\begin{aligned} x + 2(6) &= 14 \\ x + 12 &= 14 \\ x &= 14 - 12 \\ x &= 2 \end{aligned}$$

$\therefore$ B (2, 6)

(v) $\mathbf{x + 3y \leq 15}$

$\mathbf{2x + y \leq 12}$

$\mathbf{4x + 3y \leq 24}$

$\mathbf{x \geq 0, \quad y \geq 0}$

The associated equations are

$x + 3y = 15$ .... (1)

$2x + y = 12$ .... (2)

$4x + 3y = 24$ .... (3)

<u>x-intercept</u>

Put $y = 0$ in eqs. (1), (2) and (3)

$$\begin{aligned} x + 3(0) &= 15 \\ x &= 15 \end{aligned}$$

$\therefore$ Point is (15, 0)

$$\begin{aligned} 2x + 0 &= 12 \\ 2x &= 12 \\ x &= \frac{12}{2} = 6 \end{aligned}$$

$\therefore$ Point is (6, 0)

$$\begin{aligned} 4x + 3(0) &= 24 \\ 4x &= 24 \\ x &= \frac{24}{4} = 6 \end{aligned}$$

$\therefore$ Point is (6, 0)

<u>y-intercept</u>

Put $x = 0$ in eqs. (1), (2) and (3)

$$\begin{aligned} 0 + 3y &= 15 \\ y &= \frac{15}{3} = 5 \end{aligned}$$

$\therefore$ Point is (0, 5)

$$\begin{aligned} 2(0) + y &= 12 \\ y &= 12 \end{aligned}$$

$\therefore$ Point is (0, 12)

$$\begin{aligned} 4(0) + 3y &= 24 \\ 3y &= 24 \\ y &= \frac{24}{3} = 8 \end{aligned}$$

$\therefore$ Point is (0, 8)

Test Point

Put (0, 0) in

$x + 3y < 15$

$0 + 3(0) < 15$

$0 < 15$

Which is true.

∴ Graph of an inequality $x + 3y \le 15$ will be towards the origin side.

Put (0, 0) in

$2x + y < 12$

$2(0) + 0 < 12$

$0 < 12$

Which is true.

∴ Graph of an inequality $2x + y \le 12$ will be towards the origin side.

Put (0, 0) in

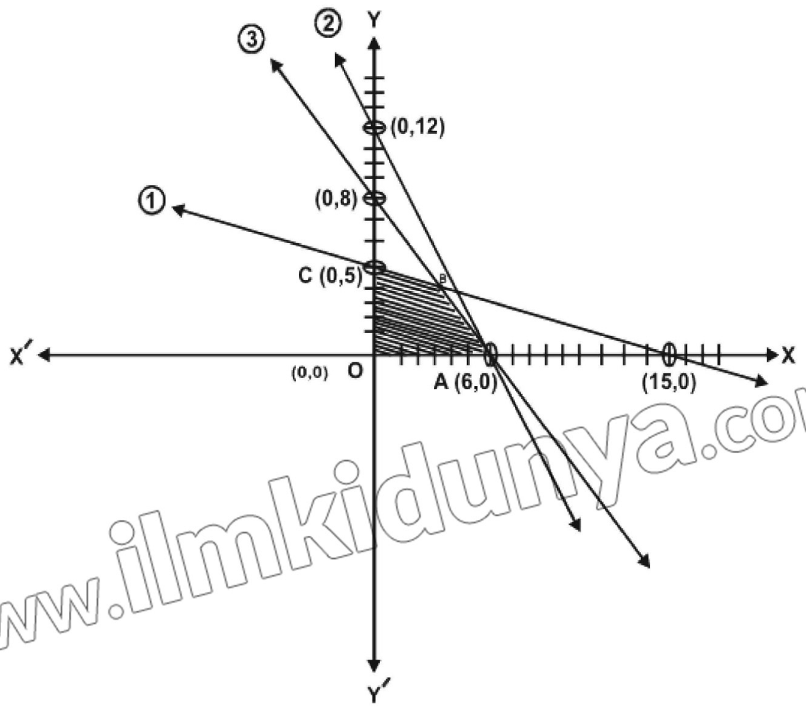
$4x + 3y < 24$

$4(0) + 3(0) < 24$

$0 < 24$

Which is true.

∴ Graph of an inequality $4x + 3y \le 24$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region so the corner points are

O (0, 0), A (6, 0), C (0, 5)

To find B solving eq. (1) & eq. (3)

Eq. (1) – Eq. (3), we get

$$\begin{array}{rcl} x + 3y &=& 15 \\ -\ 4x \pm 3y &=& -24 \\ \hline -3x &=& -9 \\ y &=& \frac{-9}{-3} = 3 \end{array}$$

$$\begin{aligned} \text{Put } x &= 3 \text{ in eq. (1)} \\ 3 + 3y &= 15 \\ 3y &= 15 - 3 \\ 3y &= 12 \\ y &= \frac{12}{3} = 4 \end{aligned}$$

$\therefore$ B (3, 4)

(vi) $\mathbf{2x + y \leq 20}$

$\mathbf{8x + 15y \leq 120}$

$\mathbf{x + y \leq 11}$

$\mathbf{x \geq 0, \quad y \geq 0}$

The associated equations are

$$\begin{aligned} 2x + y &= 20 \quad \text{.......... (1)} \\ 8x + 15y &= 120 \quad \text{.......... (2)} \\ x + y &= 11 \quad \text{.......... (3)} \end{aligned}$$

x-intercept

Put $y = 0$ in eqs. (1), (2) and (3)

$2x + 0 = 20$	$8x + 15(0) = 120$	$x + 0 = 11$
$2x = 20$	$8x = 120$	$x = 11$
$x = \frac{20}{2} = 10$	$x = \frac{120}{8} = 15$	$\therefore$ Point is (11, 0)
$\therefore$ Point is (10, 0)	$\therefore$ Point is (15, 0)	

y-intercept

Put $x = 0$ in eqs. (1), (2) and (3)

$2(0) + y = 20$	$8(0) + 15y = 120$	$0 + y = 11$
$y = 20$	$15y = 120$	$y = 11$
$\therefore$ Point is (0, 20)		$\therefore$ Point is (0, 11)

$$y = \frac{120}{15} = 8$$

$\therefore$ Point is (0, 8)

Test Point

Put (0, 0) in

$2x + y < 20$

$2(0) + 0 < 20$

$0 < 20$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 20$ will be towards the origin side.

Put (0, 0) in

$8x + 15y < 120$

$8(0) + 15(0) < 120$

$0 < 120$

Which is true.

$\therefore$ Graph of an inequality $8x + 15y \leq 120$ will be towards the origin side.

Put (0, 0) in

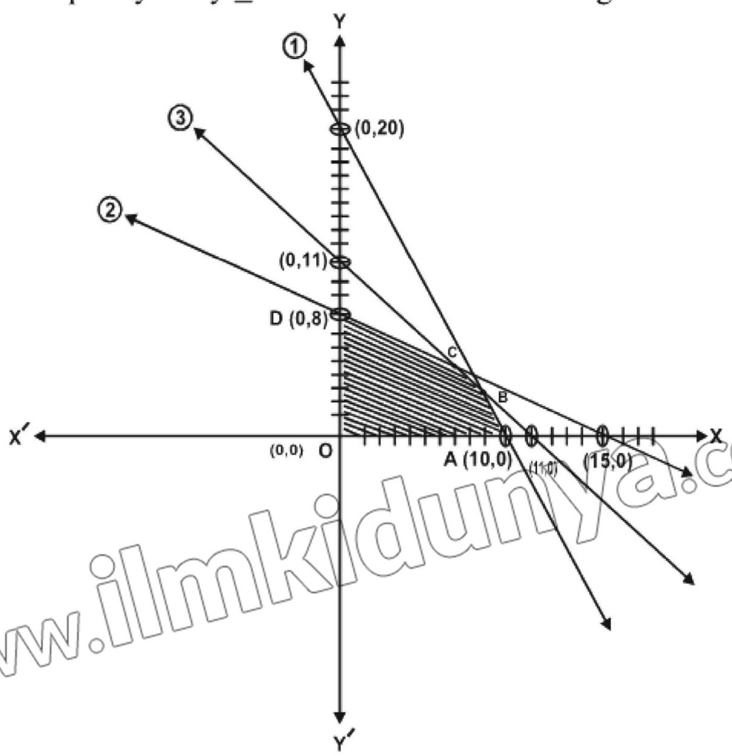
$x + y < 11$

$0 + 0 < 11$

$0 < 11$

Which is true.

$\therefore$ Graph of an inequality $x + y \leq 11$ will be towards the origin side.



$\therefore$ OABCD is the feasible solution region so the corner points are

O (0, 0), A (10, 0), D (0, 8)

To find B solving eq. (1) & eq. (3)

Eq. (1) – Eq. (3), we get

$$\begin{array}{rcl} 2x + y &=& 20 \\ -\; x \pm y &=& -11 \\ \hline x &=& 9 \end{array}$$

Put $x = 9$ in eq. (3)

$$9 + y = 11$$

$$y = 11 - 9$$

$$y = 2$$

$\therefore$ B (9, 2)

To find C solving eq. (2) & eq. (3)

Eq. (2) – Eq. (3) × 8, we get

$$\begin{array}{rcl} 8x + 15y &=& 120 \\ -\; 8x \pm 8y &=& -88 \\ \hline 7y &=& 32 \end{array}$$

$$y = \frac{32}{7}$$

Put $y = \frac{32}{7}$ in eq. (3)

$$x + \frac{32}{7} = 11$$

$$x = 11 - \frac{32}{7}$$

$$= \frac{77 - 32}{7}$$

$$= \frac{45}{7}$$

$\therefore$ $C\left(\frac{45}{7}, \frac{32}{7}\right)$

EXERCISE 5.3

Q.1 Maximize $f(x, y) = 2x + 5y$ (Lhr. Board 2007)

Subject to the constraints

$2y - x \le 8$; $x - y \le 4$; $x \ge 0$; $y \ge 0$

Solution:

The associated equations are

$2y - x = 8$ ...... (1)

$x - y = 4$ ...... (2)

x-intercept

Put $y = 0$ in eq. (1)

$2(0) - x = 8$

$x = -8$

$\therefore$ Point is $(-8, 0)$

y-intercept

Put $x = 0$ in eq. (1)

$2y - 0 = 8$

$2y = 8$

$y = \frac{8}{2} = 4$

$\therefore$ Point is $(0, 4)$

x-intercept

Put $y = 0$ in eq. (2)

$x - 0 = 4$

$x = 4$

$\therefore$ Point is $(4, 0)$

y-intercept

Put $x = 0$ in eq. (2)

$0 - y = 4$

$y = -4$

$\therefore$ Point is $(0, -4)$

Test Point

Put $(0, 0)$ in

$2y - x < 8$

$2(0) - 0 < 8$

$0 < 8$

Which is true.

∴ Graph of an inequality $2y - x < 8$ will be towards the origin side.

Put (0, 0) in

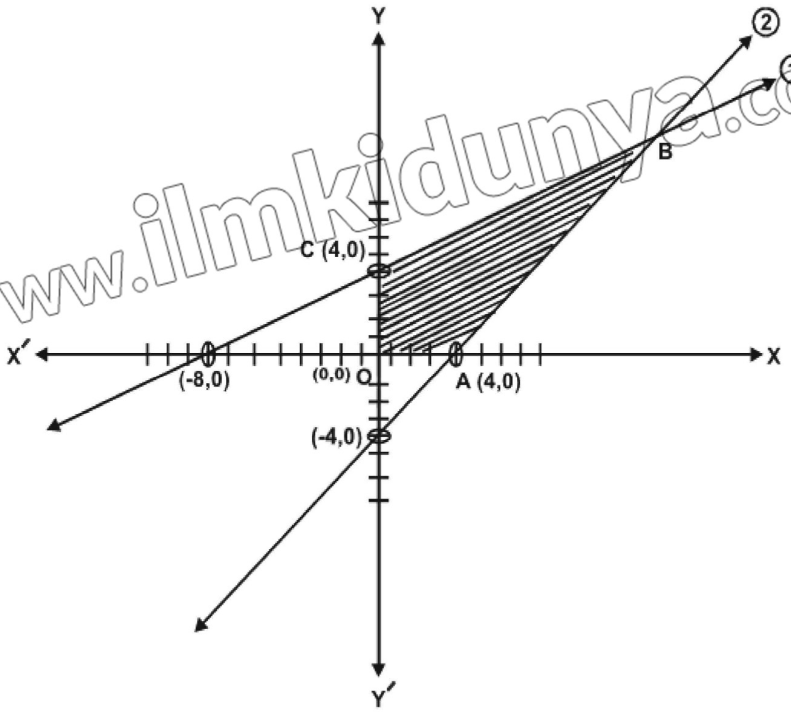
$x - y < 4$

$0 - 0 < 4$

$0 < 4$

Which is true.

∴ Graph of an inequality $x - y \leq 4$ will be towards the origin side.



∴ OABC is the feasible solution region so corner points are

O (0, 0), A (4, 0), C (0, 4)

To find B solving eq. (1) & eq. (2)

Adding eq. (1) & eq. (2)

$$\begin{aligned} 2y - x &= 8 \\ \underline{x - y} &\underline{= 4} \\ y &= 12 \end{aligned}$$

Put $y = 12$ in eq. (2)

$x - 12 = 4$

$x = 4 + 12 = 16$

$\therefore$ B (16, 12)

Now

$f(x, y) = 2x + 5y$ .... (3)

Put O (0, 0) in eq. (3)

$f(0, 0) = 2(0) + 5(0) = 0$

Put A (4, 0) in eq. (3)

$f(4, 0) = 2(4) + 5(0) = 8 + 0 = 8$

Put B (16, 12) in eq. (3)

$f(16, 12) = 2(16) + 5(12) = 32 + 60 = 92$

Put C (0, 4) in eq. (3)

$f(0, 4) = 2(0) + 5(4) = 20$

The maximum value of $f(x, y) = 2x + 5y$ is 92 at the corner point B (16, 12).

Q.2 Maximize $f(x, y) = x + 3y$ (Lhr. Board 2006) (Guj. Board 2007, 2008)
Subject to the constraints
$2x + 5y \leq 30$; $5x + 4y \leq 20$; $x \geq 0$; $y \geq 0$

Solution:

The associated equation are

$2x + 5y = 30$ ...... (1)

$5x + 4y = 20$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$2(x) + 5(0) = 30$

$2x = 30$

$$x = \frac{30}{2} = 15$$

$\therefore$ Point is (15, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$2(0) + 5y = 30$

$5y = 30$

$$y = \frac{30}{5} = 6$$

$\therefore$ Point is (0, 6)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$5x + 4(0) = 20$

$5x = 20$

$x = \frac{20}{5} = 4$

$\therefore$ Point is (4, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$5(0) + 4y = 20$

$y = \frac{20}{4} = 5$

$\therefore$ Point is (0, 5)

<u>Test Point</u>

Put (0, 0) in

$2x + 5y < 30$

$2(0) - 5(0) < 30$

$0 < 30$

Which is true.

$\therefore$ Graph of an inequality $2x + 5y \leq 30$ will be towards the origin side.

Put (0, 0) in

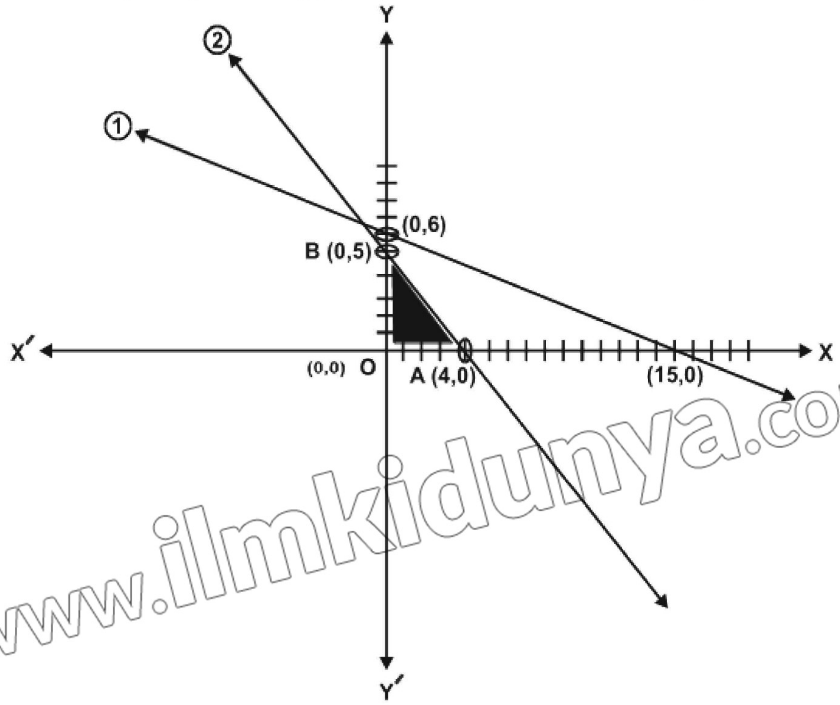
$5x + 4y < 20$

$5(0) + 4(0) < 20$

$0 < 20$

Which is true.

$\therefore$ Graph of an inequality $5x + 4y \leq 20$ will be towards the origin side.



$\therefore$ OAB is the feasible solution region so corner points are

O (0, 0), A (4, 0), B (0, 5)

$f(x, y) = x + 3y$ .... (3)

Put O (0, 0) in eq. (3)

$f(0, 0) = 0 + 3(0) = 0$

Put A (4, 0) in eq. (3)

$f(4, 0) = 4 + 3(0) = 4$

Put A (0, 5) in eq. (3)

$f(0, 5) = 0 + 3(5) = 15$

The maximum value of f (x, y) = x + 3y is 15 at corner point B (0, 5).

Q.3 Maximize Z = 2x + 3y

Subject to the constraints

$3x + 4y \le 12$; $2x + y \le 4$; $4x - y \le 4$; $x \ge 0$; $y \ge 0$

Solution:

The associated eqs. are

$3x + 4y = 12$ ...... (1)

$2x + y = 4$ ...... (2)

$4x - y = 4$ .... (3)

<u>x-intercept</u>

Put $y = 0$ in eqs. (1), (2) and (3)

$3x + 4(0) = 12$ $3x = 12$ $x = \frac{12}{3} = 4$ $\therefore$ Point is (4, 0)	$2x + 0 = 4$ $x = \frac{4}{2} = 2$ $\therefore$ Point is (2, 0)	$4x - 0 = 4$ $x = \frac{4}{4}$ $x = 1$ $\therefore$ Point is (1, 0)

<u>y-intercept</u>

Put $x = 0$ in eqs. (1), (2) and (3)

$3(0) + 4y = 12$ $y = \frac{12}{4}$ $y = 3$ $\therefore$ Point is (0, 3)	$2(0) = 4$ $y = 4$ $\therefore$ Point is (0, 4)	$4(0) - y = 4$ $y = -4$ $\therefore$ Point is (0, – 4)

<u>Test Point</u>

Put (0, 0) in

$3x + 4y < 12$

$3(0) + 4(0) \leq 12$

$0 \leq 12$

Which is true.

$\therefore$ Graph of an inequality $3x + 4y \leq 12$ will be towards the origin side.

Put (0, 0) in

$2x + y < 4$

$2(0) + 0 < 4$

$0 < 4$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 4$ will be towards the origin side.

Put (0, 0) in

$4x - y < 4$

$4(0) - 0 < 4$

$4(0) - 0 < 4$

$0 < 4$

which is true

$\therefore$ Graph of an inequality $4x - y \leq 4$ will be towards the origin side.

$\therefore$ OABCD is the feasible solution region so corner points are

O (0, 0), A (1, 0), D (0, 3)

To find B solving eq. (2) and eq. (3)

Eq. (2) – Eq. (3), we get

$$2x + y = 4$$

$$\underline{4x - y = 4}$$

$$6x = 8$$

$$x = \frac{8}{6} = \frac{4}{3}$$

Put $x = \frac{4}{3}$ in eq. (2)

$$2\left(\frac{4}{3}\right) + y = 4$$

$$y = 4 - \frac{8}{3}$$

$$= \frac{12 - 8}{3} = \frac{4}{3}$$

$\therefore$ $B\left(\frac{4}{3}, \frac{4}{3}\right)$

To find C solving eq. (1) and eq. (2)

Eq. (1) – Eq. (2) × 4, we get

$$\begin{aligned} 3x + 4y &= 12 \\ 8x + 4y &= 16 \\ - \quad - \quad &\quad - \\ \hline -5x &= -4 \\ x &= \frac{-4}{-5} \end{aligned}$$

Put $x = \frac{4}{5}$ in eq. (2)

$$2\left(\frac{4}{5}\right) + y = 4$$

$$y = 4 - \frac{8}{5}$$

$$= \frac{20 - 8}{5}$$

$$= \frac{12}{5}$$

$\therefore \quad C\left(\frac{4}{5}, \frac{8}{5}\right)$

$z = 2x + 3y$ .... (3)

Put O (0, 0) in eq. (3)

$z = 2(0) + 3(0) = 0$

Put A (1, 0) in eq. (3)

$z = 2(1) + 3(0) = 2$

Put $B\left(\frac{4}{3}, \frac{4}{3}\right)$ in eq. (3)

$$z = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right)$$

$$= \frac{8}{3} + \frac{12}{3}$$

$$= \frac{20}{3}$$

Put $C\left(\frac{4}{5}, \frac{12}{5}\right)$ in eq. (3)

$$z = 2\left(\frac{4}{5}\right) + 3\left(\frac{12}{5}\right)$$

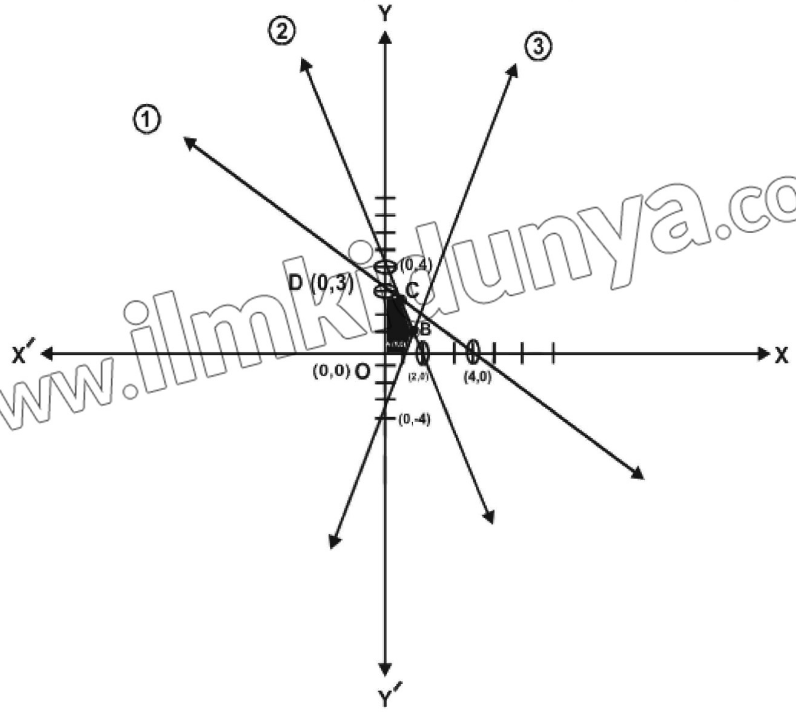
$$= \frac{8}{5} + \frac{36}{5}$$

$$= \frac{44}{5}$$

Put D (0, 3) in eq. (3)

$z = 2(0) + 3(3) = 9$

The maximum value of $z = 2x + 3y$ is 9 at corner point D (0, 3).



Q.4 Minimize $z = 2x + y$

Subject to the constraints

$x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$ (Guj. Board 2005)

Solution:

The associated eqs. are

$x + y = 3$ ...... (1)

$7x + 5y = 35$ ...... (2)

x-intercept

Put $y = 0$ in eq. (1)

$x + 0 = 3$

$x = 3$

$\therefore$ Point is (3, 0)

y-intercept

Put $x = 0$ in eq. (1)

$0 + y = 3$

$y = 3$

$\therefore$ Point is (0, 3)

x-intercept

Put $y = 0$ in eq. (2)

$7x + 5(0) = 35$

$7x = 35$

$x = \frac{35}{7} = 5$

$\therefore$ Point is (5, 0)

y-intercept

Put $x = 0$ in eq. (2)

$7(0) + 5y = 35$

$y = \frac{35}{5} = 7$

$\therefore$ Point is (0, 7)

Test Point

Put (0, 0) in

$x + y > 3$

$0 + 0 > 3$

$0 > 3$

Which is false.

$\therefore$ Graph of an inequality $x + y \geq 3$ will not towards the origin side.

Put (0, 0) in

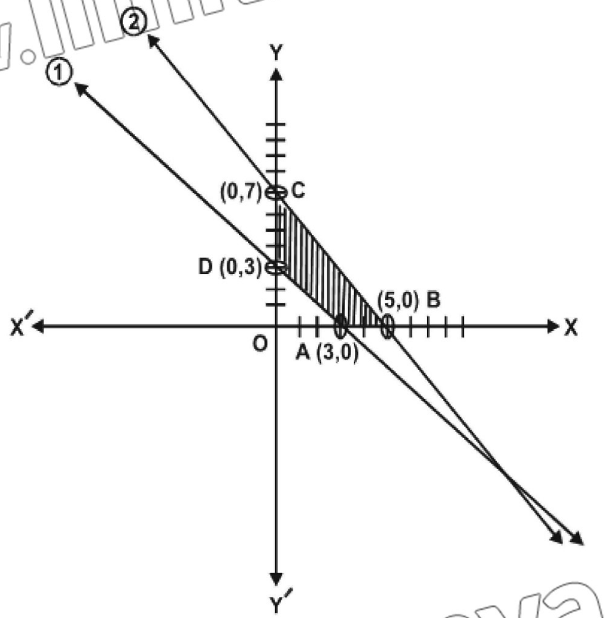
$7x + 5y < 35$

$7(0) + 5(0) < 35$

$0 < 35$

Which is true.

$\therefore$ Graph of an inequality $7x + 5y \leq 35$ will be towards the origin side.



$\therefore$ ABCD is the feasible solution region so corner points are

A (3, 0), B (5, 0), C (0, 7), D (0, 3)

$z = 2x + y$ .... (3)

Put A (3, 0) in eq. (3)

$z = 2(3) + 0 = 6$

Put B (5, 0) in eq. (3)

$z = 2(5) + 0 = 10$

Put C (0, 7) in eq. (3)

$z = 2(0) + 7 = 7$

Put D (0, 3) in eq. (3)

$z = 2(0) + 3 = 3$

The minimum value of $z = 2x + y$ is 3 at corner point D (0, 3).

Q.5 Maximize the function defined as f (x, y) = 2x + 3y subject to the constraints

$2x + y \leq 8$; $x + 2y \leq 14$; $x \geq 0$; $y \geq 0$ (Lhr. Board 2009)

Solution:

The associated eqs. are

$2x + y = 8$ ...... (1)

$x + 2y = 14$ ...... (2)

x-intercept

Put $y = 0$ in eq. (1)

$2x + 0 = 8$

$2x = 8$

$x = \frac{8}{2} = 4$

$\therefore$ Point is (4, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$2(0) + y = 8$

$y = 8$

$\therefore$ Point is (0, 8)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$x + 2(0) = 14$

$x = 14$

$\therefore$ Point is (14, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 + 2y = 14$

$y = \frac{14}{2} = 7$

$\therefore$ Point is (0, 7)

<u>Test Point</u>

Put (0, 0) in

$2x + y < 8$

$2(0) + 0 < 8$

$0 < 8$

Which is true.

$\therefore$ Graph of an inequality $2x + y \leq 8$ will be towards the origin side.

Put (0, 0) in

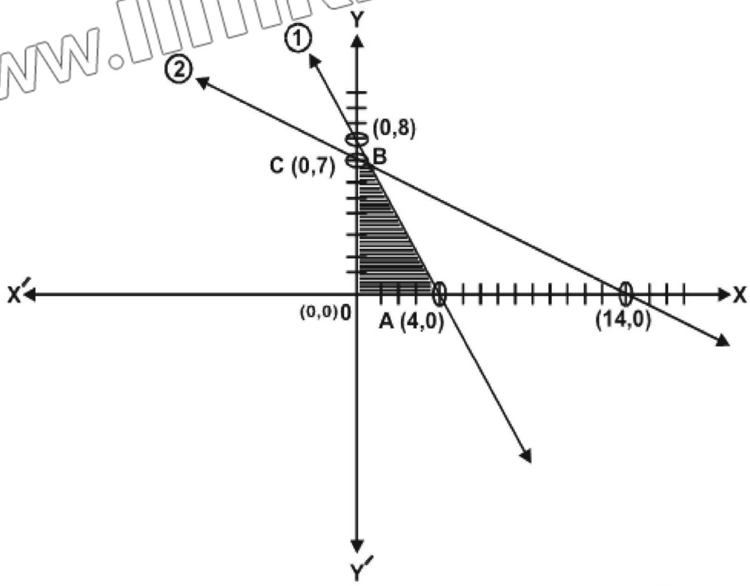
$x + 2y < 14$

$0 + 2(0) < 14$

$0 < 14$

Which is true.

$\therefore$ Graph of an inequality $x + 2y \leq 14$ will be towards the origin side.



$\therefore$ OABC is the feasible solution region. So corner points are

O (0, 0), A (4, 0), C (0, 7)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 2, we get

$$\begin{array}{rcl} 2x + y & = & 8 \\ -2x \pm 4y & = & -28 \\ \hline -3y & = & -20 \end{array}$$

$$y = \frac{20}{3}$$

Put $y = \frac{20}{3}$ in eq. (2)

$$x + 2\left(\frac{20}{3}\right) = 14$$

$$x + \frac{40}{3} = 14$$

$$x = 14 - \frac{40}{3}$$

$$x = \frac{42 - 40}{3}$$

$$x = \frac{2}{3}$$

$$\therefore \quad B\left(\frac{2}{3}, \frac{20}{3}\right)$$

$f(x, y) = 2x + 3y$ .... (3)

Put O (0, 0) in eq. (3)

$f(0, 0) = 2(0) + 3(0) = 0$

Put A (4, 0) in eq. (3)

$f(4, 0) = 2(4) + 3(0) = 8$

Put $B\left(\frac{2}{3}, \frac{20}{3}\right)$ in eq. (3)

$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right)$

$= \frac{4}{3} + \frac{60}{3}$

$= \frac{4 + 60}{3} = \frac{64}{3}$

Put C (0, 7) in eq. (3)

$f(0, 7) = 2(0) + 3(7) = 21$

The maximum value of $f(x, y) = 2x + 3y$ is $\frac{64}{3}$ at corner point $B\left(\frac{2}{3}, \frac{20}{3}\right)$.

Q.6: Minimize $z = 3x + y$ subject to the constraints

$3x + 5y \geq 15$; $x + 6y \geq 9$; $x \geq 0$; $y \geq 0$ (Lhr. 2005, 2011)

Solution:

The associated eqs. are

$3x + 5y = 15$ ...... (1)

$x + 6y = 9$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$3x + 5(0) = 15$

$3x = 15$

$x = \frac{15}{3} = 5$

$\therefore$ Point is (5, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$3(0) + 5y = 15$

$5y = 15$

$y = \frac{15}{5} = 3$

$\therefore$ Point is (0, 3)

x-intercept

Put $y = 0$ in eq. (2)

$x + 3(0) = 9$

$x = 9$

$\therefore$ Point is (9, 0)

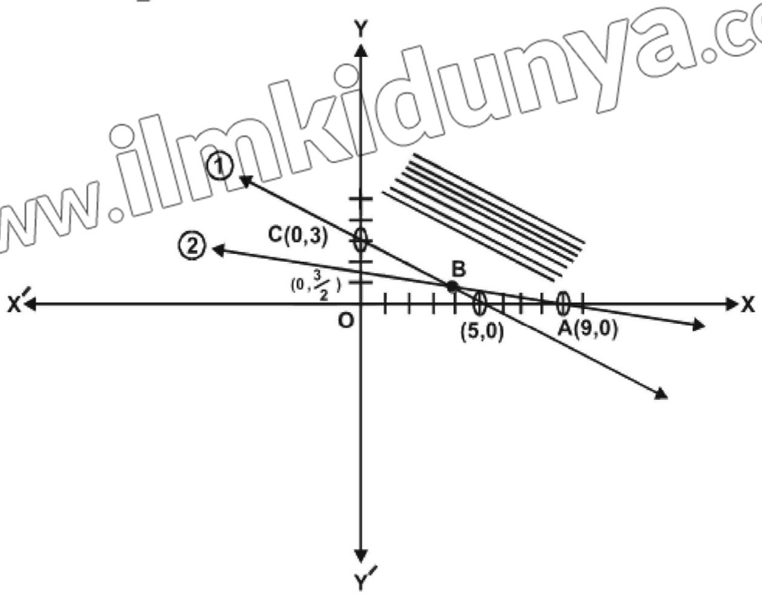
y-intercept

Put $x = 0$ in eq. (2)

$0 + 6y = 9$

$y = \frac{3}{2} = 3$

$\therefore$ Point is $(0, \frac{3}{2})$



Test Point

Put (0, 0) in

$3x + 5y > 15$

$3(0) + 5(0) > 15$

$0 > 15$

Which is false.

$\therefore$ Graph of an inequality $3x + 5y \geq 15$ will not be towards the origin side.

Put (0, 0) in

$x + 6y > 9$

$0 + 6(0) > 9$

$0 > 9$

Which is true.

$\therefore$ Graph of an inequality $x + 3y \leq 9$ will not be towards the origin side.

$\therefore$ ABC is the feasible solution region. So corner points are

A (9, 0), C (0, 3)

$z = 3x + y \qquad (3)$

Put A (9, 0) in eq. (3)

$z = 3(9) + 0 = 27$

Put $B\left(\frac{45}{13}, \frac{12}{13}\right)$ in eq. (3)

$z = 3\left(\frac{45}{13}\right) + \frac{12}{13}$

$z = \frac{135}{13} + \frac{12}{13} = \frac{147}{13}$

Put C (0, 3) in eq. (3)

$z = 3(0) + 3 = 3$

To find B solving eq. (1) & eq. (2)

eq. (1) – eq. × 3, we get

$$\begin{aligned} 3x + 5y &= 15 \\ 3x + 18y &= 27 \\ - \quad - \quad &\quad - \\ \hline -13y &= -12 \end{aligned}$$

$y = \frac{12}{13}$

Put $y = \frac{12}{13}$ in eq. (2)

$x + 6\left(\frac{12}{13}\right) = 9$

$x + \frac{72}{13} = 9$

$x = 9 - \frac{72}{13}$

$x = \frac{117 - 72}{13}$

$= \frac{45}{13}$

$\therefore B\left(\frac{45}{13}, \frac{12}{13}\right)$

The minimum value of $z = 3x + y$ is 3 at corner point C (0, 3).

Q.7: Each unit of food x costs Rs. 25 and contains 2 units of protein and 4 units of iron while each unit of food Y costs Rs. 30 and contains 3 units of protein and 2 units of iron. Each animal must receive at least 12 units of protein and 16 units of iron each day. How many units of each food should be fed to each animal at the smallest possible cost?

Solution:

Let x be the unit of food X and y be the unit of food Y.

Minimize $f(x, y) = 25x + 30y$

Subject to the constraints

$2x + 3y \geq 12$

$4x + 2y \geq 16$

$x \geq 0 \quad , \; y \geq 0$

The associated eqs. are

$2x + 3y = 12 \quad \text{.......} (1)$

$4x + 2y = 16 \quad \text{.......} (2)$

x-intercept

Put $y = 0$ in eq. (1)

$2x + 3(0) = 12$

$2x = 12$

$x = \frac{12}{2} = 6$

$\therefore$ Point is (6, 0)

y-intercept

Put $x = 0$ in eq. (1)

$2(0) + 3y = 12$

$3y = 12$

$y = \frac{12}{3} = 4$

$\therefore$ Point is (0, 4)

x-intercept

Put $y = 0$ in eq. (2)

$4x + 2(0) = 16$

$4x = 16$

$x = \frac{16}{4} = 4$

$\therefore$ Point is (4, 0)

y-intercept

Put $x = 0$ in eq. (2)

$4(0) + 2y = 16$

$2y = 16$

$y = \frac{16}{2} = 8$

$\therefore$ Point is (0, 8)

Test Point

Put (0, 0) in

$2x + 3y > 12$

$2(0) + 3(0) > 12$

$0 > 12$

Which is false.

$\therefore$ Graph of an inequality $2x + 3y \geq 12$ will not be towards the origin side.

Put (0, 0) in

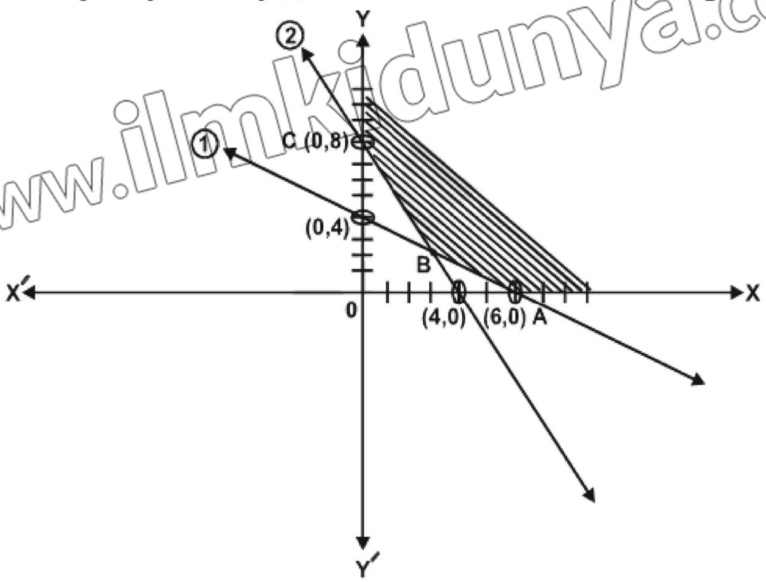
$4x + 2y > 16$

$4(0) + 2(0) > 16$

$0 > 16$

Which is false.

$\therefore$ Graph of an inequality $4x + 2y \geq 16$ will not be towards the origin side.



$\therefore$ ABC is the feasible solution region. So corner points are

A (6, 0), C (0, 8)

To find B solving eq. (1) & eq. (2)

Eq. (1) × 2 – Eq. (2), we get

$$\begin{aligned} 4x + 6y &= 24 \\ -4x \pm 2y &= -16 \\ \hline 4y &= 8 \\ y &= \frac{8}{4} = 2 \end{aligned}$$

Put $y = 2$ in eq. (1)

$2x + 3(2) = 12$

$2x + 6 = 12$

$2x = 12 - 6$

$2x = 6$

$x = \frac{6}{2} = 3$

$\therefore$ B (3, 2)

$f(x, y) = 25x + 30y$ .... (3)

Put A (6, 0) in eq. (3)

$f(6, 0) = 25(6) + 30(0) = 150$

Put B (3, 2) in eq. (3)

$f(3, 2) = 25(3) + 30(2) = 75 + 60 = 135$

Put C (0, 8) in eq. (3)

$f(0, 8) = 25(0) + 30(8) = 240$

The smallest cost of $f(x, y) = 25x + 30y$ is 135 at corner point B (3, 2)

Q.8: A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space at most for 20 items. A fan costs him Rs. 360 and a sewing machine costs Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit?

Solution:

Let x be the number of fans and y be the number of sewing machines.

Maximize $f(x, y) = 22x + 18y$

Subject to the constraints

$360x + 240y \leq 5760$

$x + y \leq 20$

$x \geq 0 \quad , \quad y \geq 0$

The associated eqs. are

$360x + 240y = 5760$ ...... (1)

$x + y = 20$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$360x + 240(0) = 5760$

$x = \dfrac{5760}{360} = 16$

$\therefore$ Point is (16, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$360(0) + 240y = 5760$

$y = \dfrac{5760}{240} = 24$

$\therefore$ Point is (0, 24)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$x + 0 = 20$

$x = 20$

$\therefore$ Point is (20, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (2)

$0 + y = 20$

$y = 20$

$\therefore$ Point is (0, 20)

<u>Test Point</u>

Put (0, 0) in

$360x + 240y < 5760$

$360(0) + 240(0) < 5760$

$0 < 5760$

Which is true.

$\therefore$ Graph of an inequality $360x + 240y \leq 5760$ will be towards the origin side.

Put (0, 0) in

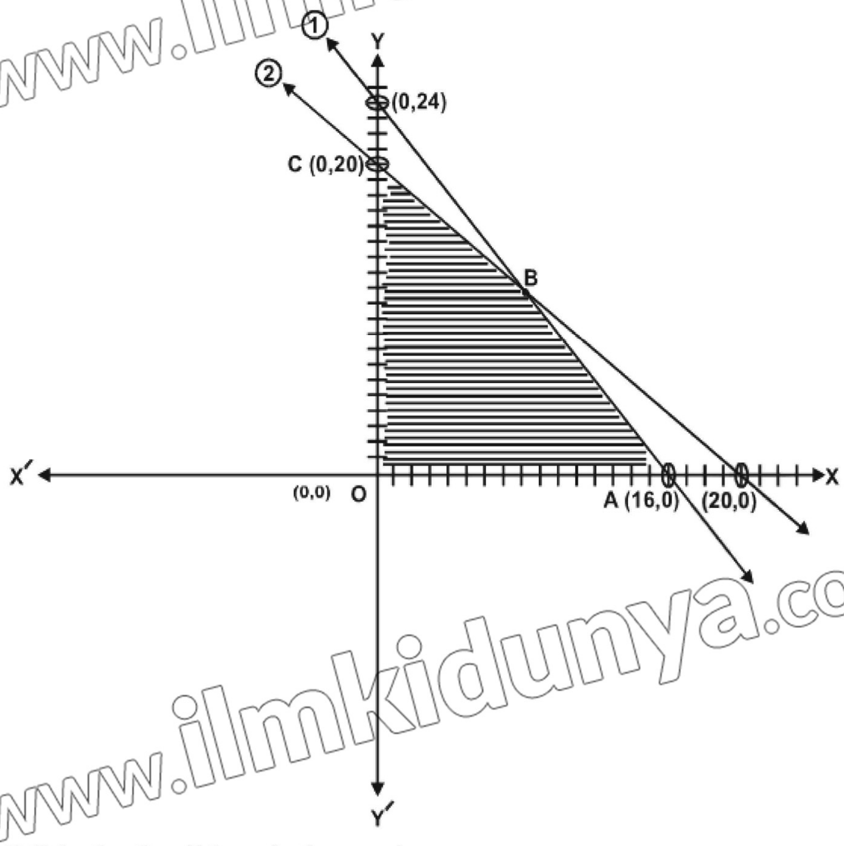
$x + y < 20$

$0 + 0 < 20$

$0 < 20$

Which is true.

$\therefore$ Graph of an inequality $x + y \leq 20$ will be towards the origin side.



∴ OABC is the feasible solution region.

So corner points are

O (0, 0), A (16, 0), C (0, 20)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 240, we get

$$360x + 240y = 5760$$

$$\underline{-240x \pm 240y = -4800}$$

$$120x = 8$$

$$x = \frac{690}{120} = 8$$

Put $x = 8$ in eq. (2)

$$8 + y = 20$$

$$y = 20 - 8 = 12$$

∴ B (8, 12)

$f(x, y) = 22x + 18y$ .... (3)

Put O (0, 0) in eq. (3)

$$f(0, 0) = 22(0) + 18(0) = 0$$

Put A (16, 0) in eq. (3)

$f(16, 0) = 22(16) + 18(0) = 352$

Put B (8, 12) in eq. (3)

$f(8, 12) = 22(8) + 18(12) = 176 + 216 = 392$

Put C (0, 20) in eq. (3)

$f(0, 20) = 22(0) + 18(20) = 360$

The maximum profit of (x, y) = 22x + 18 y is 392 at corner point B (8, 12).

Q.9: A machine can produce product A by using 2 units of chemical and 1 unit of a compound or can produce product B by using 1 unit of chemical and 2 units of the compound. Only 800 units of chemical and 1000 units of the compound are available. The profits per unit of A and B are Rs. 30 and Rs. 20 respectively, maximize the profit function.

Solution:

Let x be the units of product A and y be the units of product B.

Maximize $f(x, y) = 30x + 20y$

Subject to the constraints

$2x + y \leq 800$

$x + 2y \leq 1000$

$x \geq 0 \quad , \quad y \geq 0$

The associated eqs. are

$2x + y = 800$ ...... (1)

$x + 2y = 1000$ ...... (2)

<u>x-intercept</u>

Put $y = 0$ in eq. (1)

$2x + 0 = 800$

$$x = \frac{800}{2} = 400$$

$\therefore$ Point is (400, 0)

<u>y-intercept</u>

Put $x = 0$ in eq. (1)

$2(0) + y = 800$

$y = 800$

$\therefore$ Point is (0, 800)

<u>x-intercept</u>

Put $y = 0$ in eq. (2)

$x + 2(0) = 1000$

$x = 1000$

∴ Point is (1000, 0)

y-intercept

Put $x = 0$ in eq. (2)

$$0 + 2y = 1000$$

$$y = \frac{1000}{2} = 500$$

∴ Point is (0, 500)

Test Point

Put (0, 0) in

$2x + y < 800$

$2(0) + 0 < 800$

$0 < 800$

Which is true.

∴ Graph of an inequality $2x + y \leq 800$ will be towards the origin side.

Put (0, 0) in

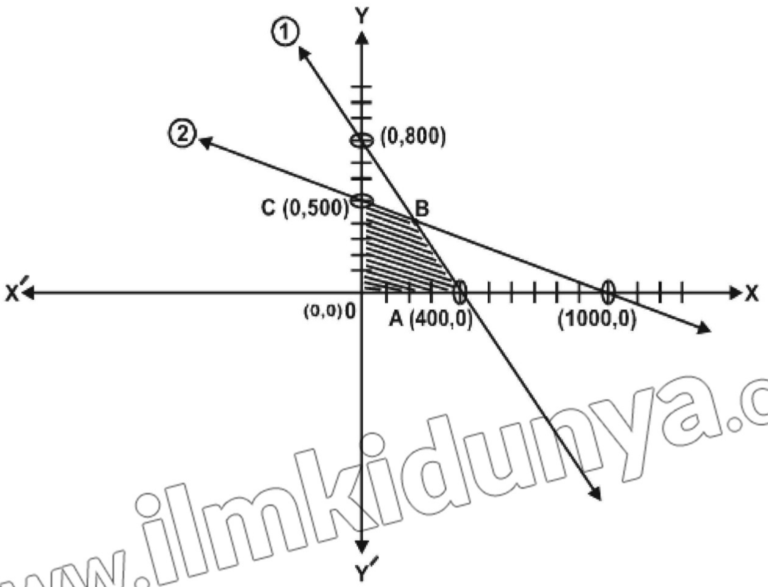
$x + 2y < 1000$

$0 + 2(0) < 1000$

$0 < 1000$

Which is true.

∴ Graph of an inequality $x + 2y \leq 1000$ will be towards the origin side.



OABC is the feasible solution region. So corner points are

O (0, 0), A (400, 0), C (0, 500)

To find B solving eq. (1) & eq. (2)

Eq. (1) – Eq. (2) × 2, we get

$$\begin{array}{rcl} 2x + y &=& 800 \\ -2x \pm 4y &=& -2000 \\ \hline -3y &=& -1200 \end{array}$$

$$y = \frac{1200}{3} = 400$$

Put $y = 400$ in eq. (2)

$$x + 2(400) = 1000$$

$$x + 800 = 1000$$

$$x = 1000 - 800$$

$$x = 200$$

$\therefore$ B (200, 400)

$f(x, y) = 30x + 20y$ .... (3)

Put O (0, 0) in eq. (3)

$f(0, 0) = 30(0) + 20(0) = 0$

Put A (400, 0) in eq. (3)

$f(400, 0) = 30(400) + 20(0) = 12000$

Put B (200, 400) in eq. (3)

$f(200, 400) = 30(200) + 20(400)$

$= 6000 + 8000 = 14000$

Put C (0, 500) in eq. (3)

$f(0, 500) = 30(0) + 20(500) = 10000$

The maximum profit of $f(x, y) = 30x + 20y$ is 14000 at corner point B (200, 400).