Compute the theoretical probabilities and find its mean and variance.

3.27 Fit a binomial distribution of the following and compute the expected/theoretical frequencies:

X	0	1	2	3	4	5	6
f	13	70	137	210	145	56	9

3.28 Define hypergeometric experiment, hypergeometric random variable and hypergeometric probability distribution.

3.29 Find the mean and variance of hypergeometric distribution.

3.30 Construct the hypergeometric probability distribution for the number of black balls among 5 balls drawn at random form a box containing 4 white and 7 black balls. Find the mean and variance of this distribution and compare these with the mean and variance of the hypergeometric probability distribution.

3.31 In a manufacturing company 35 employee use touch screen mobile set and 15 have push button sets. Eight employees are selected randomly without replacement. Find the probability that exactly 5 will be using touch screen mobile.

3.32 Four cards are drawn randomly from a well-shuffled deck of 52 playing cards. Calculate the probability that two will be diamond cards.

Unit - 4

Special Continuous Probability Distributions

After studying this unit, the students will be able to

- Define a continuous uniform probability distribution and continuous uniform probability density distribution.
- Find mean, variance and standard deviation of a continuous uniform probability distribution.
- Solve real life problems using continuous uniform probability distribution.
- Define a Normal probability distribution, Normal probability density function, Normal cumulative distribution function, standard normal random variable, standard Normal distribution, standard Normal probability density function and a standard Normal cumulative distribution function.
- Describe the properties of a Normal probability distribution
- Find the ordinates of the standard normal curve using the table of the ordinates of the standard normal curve.
- Find the probabilities for the standard normal random variable using the table of the standard Normal distribution function.
- Inversely use the standard Normal distribution table to determine the value of (i) standard normal random variable corresponding to a given value of the standard Normal cumulative distribution function, (ii) a normal random variable corresponding to a given value of a Normal cumulative distribution function and (iii) parameters of a normal random variable.
- Describe the Normal distribution as a limit of frequency distribution.
- Solve real life problems using Normal probability distribution.

4.1 Introduction to continuous probability distributions

As you know that it is difficult to locate a specified value on a continuous random variable, that is why point probability in continuous case is always equal to zero and we compute probabilities for interval of values. In this unit we consider only continuous uniform distribution and the Normal distribution.

4.1.1 Continuous uniform or rectangular probability distribution

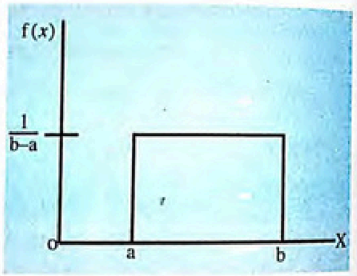
Like discrete case, continuous uniform distribution is the simplest probability distribution. It is used in the situations where the probability density function $f(x)$ remains constant over the entire range of the variable.

4.1.2 Definition of continuous uniform probability density function

The distribution of a continuous random variable X with pdf

$$f(x)=\begin{cases}\dfrac{1}{b-a}, & a\le x\le b\\ 0 & otherwise\end{cases}$$

is called continuous uniform probability distribution or probability density function and the variable X is called continuous uniform random variable. This distribution has two parameters, a and b. This distribution is also known as rectangular distribution because its graph is like a rectangle given as.



4.1.3 Properties of continuous uniform probability distribution

(i) Mean of continuous uniform probability distribution

By definition

$$\text{Mean} = E(X)=\int_x x f(x)\,dx$$

$$=\int_a^b x\frac{1}{b-a}dx$$

$$=\frac{1}{b-a}\int_a^b x\,dx$$

$$=\frac{1}{b-a}\left(\frac{x^2}{2}\Big|_a^b\right)$$

$$=\frac{1}{b-a}\left(\frac{b^2-a^2}{2}\right)$$

$$=\frac{1}{b-a}\left\{\frac{(b-a)(b+a)}{2}\right\}$$

$$=\frac{a+b}{2}$$

(ii) Variance of continuous uniform probability distribution

By definition

$$\text{Variance} = V(X) = E(X^2) - [E(X)]^2 \qquad \text{(i)}$$

$$\text{As } E(X) = \frac{a+b}{2} \qquad \text{(ii)}$$

$$E(X^2)=\int_x x^2 f(x)\,dx$$

$$=\int_a^b x^2\frac{1}{b-a}dx$$

$$=\frac{1}{b-a}\int_a^b x^2dx=\frac{1}{b-a}\frac{x^3}{3}\Big|_a^b=\frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right]$$

$$=\frac{1}{b-a}\frac{(b-a)(b^2+ab+a^2)}{3}$$

$$=\frac{b^2+ab+a^2}{3} \qquad \text{(iii)}$$

Put equation (ii) and equation (iii) in equation (i)

$$V(X)=\frac{b^2+ab+a^2}{3}-\frac{(a+b)^2}{4}$$

$$=\frac{4(b^2+ab+a^2)-3(a^2+b^2+2ab)}{12}$$

$$=\frac{4b^2+4ab+4a^2-3a^2-3b^2-6ab}{12}$$

$$=\frac{b^2+a^2-2ab}{12}$$

$$V(X)=\frac{(b-a)^2}{12}$$

(iii) Standard deviation of continuous uniform probability distribution:

By definition

$$S.D(X)=\sqrt{V(X)}=\sqrt{\frac{(b-a)^2}{12}}=\frac{b-a}{\sqrt{12}}$$

Example 4.1

If X has a uniform distribution over the interval (2, 4), find (i) $P(2\le X\le 3)$ (ii) $P(3\le X\le 4)$

Solution:

Given $a=2$ and $b=4$, so the pdf of uniform distribution in this case is given by

$$f(x)=\begin{cases}\frac{1}{2} & ,2\le x\le 4\\ 0 & ,otherwise\end{cases}$$

Now

(i) $$P(2\le X\le 3)=\int_2^3\frac{1}{2}dx=\frac{1}{2}\int_2^3 dx$$

$$=\frac{1}{2}\left[x\Big|_2^3\right]=\frac{1}{2}(3-2)=\frac{1}{2}(1)=\frac{1}{2}$$

(ii) $$P(3\le X\le 4)=\int_3^4\frac{1}{2}dx=\frac{1}{2}\int_3^4 dx=\frac{1}{2}\left[x\Big|_3^4\right]$$

$$=\frac{1}{2}[4-3]=\frac{1}{2}(1)=\frac{1}{2}$$

Example 4.2

Let $X\sim U(-1,3)$. Find mean, variance and standard deviation for this continuous uniform random variable.

Solution:

Here $a=-1, b=3$

Now

$$E(X)=\frac{a+b}{2}=\frac{-1+3}{2}=\frac{2}{2}=1$$

$$V(X)=\frac{(b-a)^2}{12}=\frac{[3-(-1)]^2}{12}=\frac{(3+1)^2}{12}=1.333$$

$$S.D(X)=\sqrt{1.333}=1.155$$

4.2 Normal distribution

Normal distribution is the most common and useful amongst all known distribution. It is considered as the cornerstone of the modern statistical theory. The reason of importance is due to the facts that:

i. Many natural phenomena like age, weight, light, I.Q, grade, temperature, income etc. tend to be approximately normal.
ii. Most of the discrete distributions such as Binomial, Poisson, etc. tend to Normal distribution as sample size increases i.e. $n\to\infty$.
iii. For a sample of size $n\ge 30$, the distribution is considered as normal.
iv. Many variables which are not normally distributed can be normalized through suitable transformations.

v. When Normal distribution is shown on a graph paper, it gives a bell-shaped curve called Normal curve which is generally taken as a standard for comparison.

4.2.1 Definition of Normal probability distribution

A random variable X is said to follow a Normal distribution if its probability density function is

$$f(x)=\frac{1}{\sqrt{2\pi}\,\sigma}\,e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2},\ -\infty\le X\le\infty,\ -\infty\le\mu\le\infty,\ 0\le\sigma\le\infty$$

Where

X = Normal random variable.

$f(x)$ = The height of the curve corresponding to a given value of X.

μ = mean of Normal distribution.

σ = standard deviation of Normal distribution

π = a constant approximately equal to $\frac{22}{7} = 3.1429$

e = a constant approximately equal to 2.7183.

This distribution has two parameters, μ and σ^2. Normal distribution is simply written as $X \sim N(\mu,\sigma^2)$.

4.2.2 Properties of Normal distribution

i) The total probability within its range $-\infty$ to $+\infty$ is always equal to one i.e.

$$\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx=1$$

ii) The Normal distribution is symmetrical about the mean and is a bell-shaped.

iii) Mean = Median = Mode.

iv) The Normal curve is unimodal.

v) The first and third quartiles are equidistant from the centre, or Mean μ.

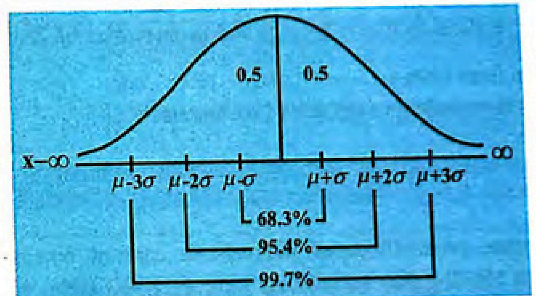
vi) Q.D = $\frac{2}{3}\sigma$ and M.D = $\frac{4}{5}\sigma$

vii) All odd order moments about mean i.e. $\mu_1, \mu_3, \ldots$ are zero.

viii) Measures of skewness, $\beta_1 = 0$ or $\gamma_1 = 0$

ix) Measures of kurtosis, $\beta_2 = 3$ or $\gamma_2 = 0$

x) For Normal distribution, out of the total observation, 68.3 % lies within the limits $(\mu \pm \sigma)$, 95.4 % lies within the limits $(\mu \pm 2\sigma)$ and 99.7 % lies within the limits $(\mu \pm 3\sigma)$. Graphically this statement is shown as



4.2.3 Standard normal variable

Normal random variable X is transformed by subtracting its mean from it and the difference is divided by its standard deviation and is called standard Normal variable, usually denoted by Z, that is

$$Z=\frac{X-\mu}{\sigma}$$

Possible values of Z are also form $-\infty$ to $+\infty$.

4.2.4 Definition of standard Normal probability distribution

The probability density function of the standard normal random variable Z defined as

$$f(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}, \qquad -\infty \le z \le \infty$$

is called standard Normal probability distribution.

Total probability under the density is equal to one i.e. $\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz=1$. It is simply written as $Z \sim N(0,1)$. The reason for using standard Normal probability distribution in place of Normal probability distribution is that:

(i) To easily calculate the probabilities of Normal distribution i.e. $P[x_1 < X < x_2] = P[z_1 < Z < z_2]$, and probabilities of Z can directly be taken from table 4.1.

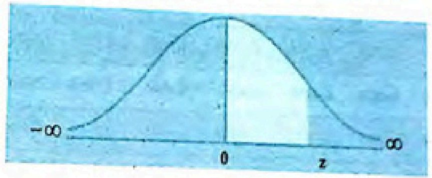
(ii) Z is independent of the unit of measurement.

4.2.5 Area (probability) under the standard normal curve

To compute probabilities of intervals in case of continuous random variables, it is a blessing that statisticians have designed tables in which areas (probabilities) have been compiled. In case of Normal distribution such a table is available for standard Normal probability distribution.

Table 4.1: Areas under the standard Normal curve from 0 to z

The entries in this table are the probabilities that random variable having the standard Normal distribution takes on a value between 0 and z (the shaded area in the figure). For negative values of Z, areas are found by symmetry.



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1106	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3990	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441

1.6	0.4452	0.4463	0.4474	0.4484	0.4498	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4719	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.7981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5	0.5	0.5	0.5	0.52	0.5	0.5	0.5	0.5	0.5

4.2.6 Use of the area table

The table of areas of the standard Normal distribution gives probabilities for standard Normal variable Z between 0 (its mean) and a specified value, say z. Therefore, Z-value must always be rounded up to two decimal points as required for reading the table. Locate the first two values in the stub and third value in the box lead. Because of the symmetry property of the Normal distribution the probability (area) between 0 and a positive Z-value must be exactly the same as the probability between a negative Z-value and 0 as long as the Z-value on both sides is of the same magnitude, that is

$$P[0<Z<1.2]=P[-1.2<Z<0]$$

Example 4.3

If Z has a standard Normal distribution, find:

a. $P[-\infty<Z<\infty]$

b. $P[0<Z<\infty]$

c. $P[-\infty<Z<0]$

d. $P[0<Z<2.63]$

e. $P[-1.45<Z<0]$

f. $P[1<Z<1.5]$

g. $P[Z\geq 1.64]$

h. $P[-1.96<Z<-1.06]$

i. $P[Z<-1.64]$

j. $P[-1.7<Z<1.25]$

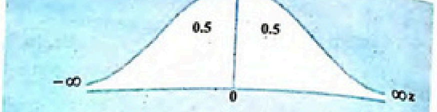
k. $P[Z<-2.46]$

l. $P[Z<2.11]$

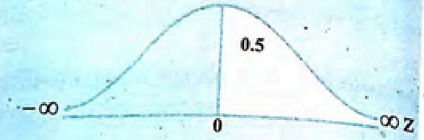
Solution:

This example is a practice on how to take the required probability from the areas table. It is quite simple and interesting. First draw standard Normal curve for each case, shade the area in which you are interested and then take the probability directly from table 4.1.

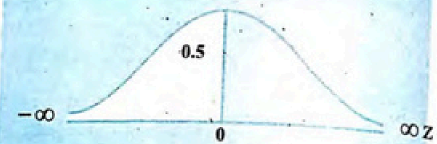
a. $P[-\infty < Z < \infty] = 1$



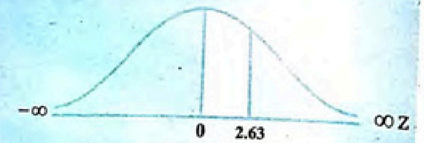
b. $P[0 < Z < \infty] = 0.5$



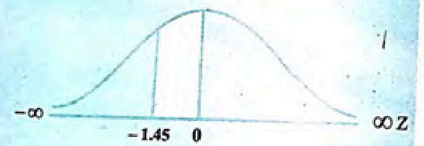
c. $P[-\infty < Z < 0] = 0.5$



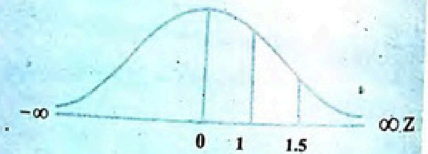
d. $P[0 < Z < 2.63] = 0.4957$



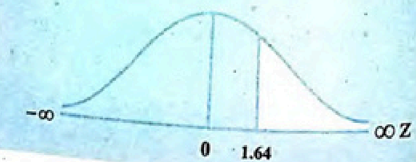
e. $P[-1.45 < Z < 0] = 0.4265$



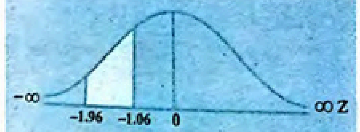
f. $P[1 < Z < 1.5]$
$= P[0 < Z < 1.5] - P[0 < Z < 1]$
$= 0.4332 - 0.3413 = 0.0919$



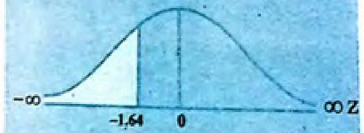
g. $P[Z \geq 1.64]$
$= P[0 < Z < \infty] - P[0 < Z < 1.64]$
$= 0.5 - 0.4498 = 0.0502$



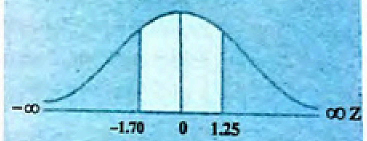
h. $P[-1.96 < Z < -1.06]$
$= P[-1.96 < Z < 0] - P[-1.06 < Z < 0]$
$= 0.4750 - 0.3554 = 0.1196$



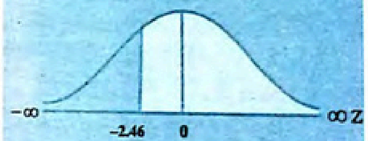
i. $P[Z < -1.64]$
$= P[0 < Z < \infty] - P[-1.64 < Z < 0]$
$= 0.5 - 0.4498 = 0.0502$



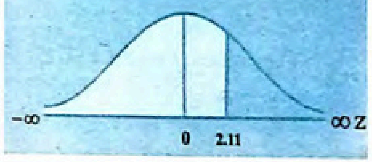
j. $P[-1.70 < Z < 1.25]$
$= P[-1.70 < Z < 0] + P[0 < Z < 1.25]$
$= 0.4554 + 0.3944 = 0.8498$



k. $P[Z > -2.46]$
$= P[-2.46 < Z < 0] + P[0 < Z < \infty]$
$= 0.4931 + 0.5 = 0.9931$



l. $P[Z < 2.11]$
$= P[-\infty < Z < 0] + P[0 < Z < 2.11]$
$= 0.5 + 0.4826 = 0.9826$



4.2.7 Inverse use of table of areas under standard normal curve

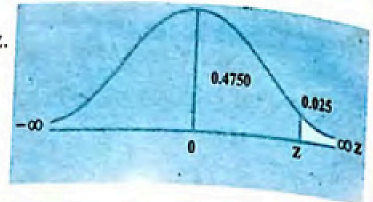
Hope you will be able to determine the interval probability for the standard normal variable Z from the area table. Now we try to reverse the process and determine a value or values of Z corresponding to a given area (probability) from the same areas table. This is called inverse use of the area table.

Example 4.4

If $P[Z \geq z]=0.025$, find the value of z.

Solution:

i) Sketch the given statement graphically as



ii) From the figure we see that the unknown value z is a positive value because it lies to the right of 0 (the mean) and probability above it is equal to 0.025.

iii) As we know that probability from 0 to ∞ is equal to 0.5, so the probability for 0 to z will definitely be equal to 0.5 − 0.025 = 0.4750, i.e. $P[0 \leq Z \leq z]=0.4750$

iv) Now search the figure 0.4750 in the body of the area table to determine the corresponding value of Z. From table 4.1, we observe that the figure 0.4750 corresponds to 1.96. Hence $z = 1.96$ and the area above this value is equal to 0.025.

- Note that if the value we are searching for is not available in the table, then we may pick the nearest value to find Z-values.

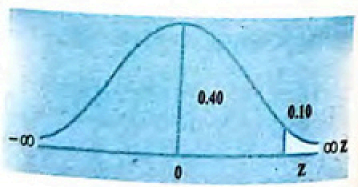
Example 4.5

Determine z when $P[Z \geq z]=0.10$

Solution:

The probability $P[Z \geq z]=0.10$ is graphically shown as:

As $P[Z \geq z] = 0.10 \Rightarrow P[0 < Z < z] = 0.5-0.10 = 0.40$. In the area table 4.1 we observe that exactly 0.40 is not available we consider the closest area 0.3997, which corresponds to a Z-value 1.28. Hence $z = 1.28$ and area above it is equal to 0.10 or 10%.

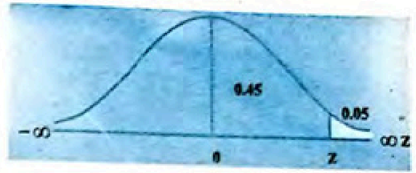


Example 4.6

Let $Z \sim N(0,1)$. If $P[Z \geq z]=0.5$, what will be the value of z?

Solution:

Draw the sketch:



Since $P[Z \geq z] = 0.05$, therefore, $P[0 < Z < z] = 0.5 \; 0.05 \; 0.45$. In the area table we can see that 0.45 is lying between 0.4495 and 0.4505 whose corresponding Z-values are 1.64 and 1.65 respectively. The Z-value in this case is given as $z = \frac{1.64+1.65}{2} = 1.645$.

Hence $P[Z \geq 1.645]=0.05$

4.2.8 Application of area table to any Normal distribution

After learning the use of areas table and its inverse use, now you would be able to compute probabilities for any Normal random variable X by first converting X to Z by the formula $Z = \frac{X-\mu}{\sigma}$ and then use the table of areas for the standard Normal distribution to obtain the desired probabilities.

Example 4.7

If a Normal distribution has mean 40 and standard deviation 5, find the probabilities for the values of X specified as i) $P(X \geq 44)$, ii) $P(X \leq 25)$, iii) $P(32 < X < 50)$.

Solution:

In order to find the probabilities, we first standardize X by subtracting its mean 40 and dividing the difference by standard deviation 5 to get Z and then find probabilities through area table as follow:

(i) $P[X \geq 44]$

$= P\left[\frac{X-\mu}{\sigma} \geq \frac{44-\mu}{\sigma}\right]$

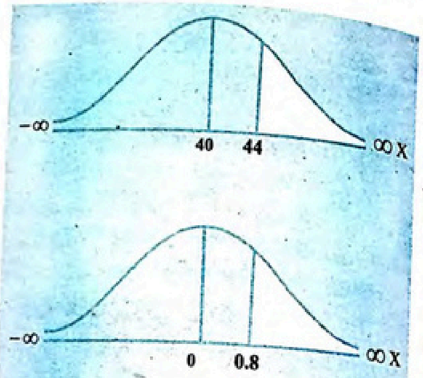
$= P\left[Z \geq \frac{44-40}{5}\right]$

$= P\left[Z \geq \frac{4}{5}\right]$

$= P[Z \geq 0.8]$

$= P[0 < Z < \infty] - P[0 < Z < 0.80]$

$= 0.5 - 0.2881 = 0.2119$



(ii) $P(X \leq 25)$

$= P\left[\frac{X-\mu}{\sigma} \leq \frac{25-\mu}{\sigma}\right]$

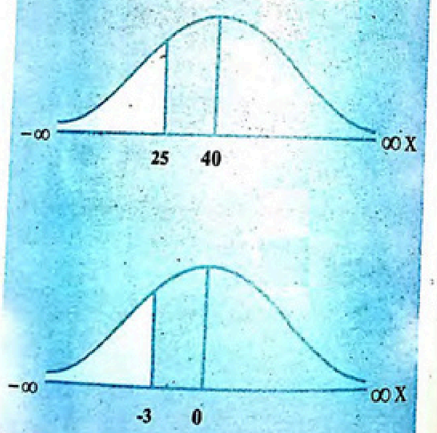
$= P\left[Z \leq \frac{25-40}{5}\right]$

$= P\left[Z \leq \frac{-15}{5}\right]$

$= P[Z < -3]$

$= [-\infty < Z < 0] - P[-3 < Z < 0]$

$= 0.5 - 0.4987 = 0.0013$



(iii) $P(32 \leq X \leq 50)$

$= P\left[\frac{32-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{50-\mu}{\sigma}\right]$

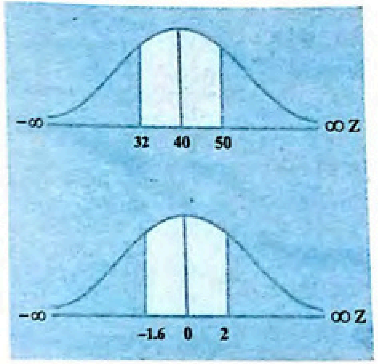
$= P\left[\frac{32-40}{5} < Z < \frac{50-40}{5}\right]$

$= P\left[\frac{-8}{5} < Z < \frac{10}{5}\right]$

$= P[-1.6 < Z < 2]$

$= P[-1.6 < Z < 0] + P(0 < Z < 2)$

$= 0.4452 + 0.4772 = 0.9224$



Example 4.8

Suppose the ages at time of onset of a certain disease are approximately normally distributed with a mean of 11 years and standard deviation of 3 years. A child has just come down with disease. What is the probability that the child is i) between the ages of 8 and 14 years? ii) over 10 years of age? iii) under 12 years?

Solution:

Given $\mu = 11$ years, $\sigma = 3$ years

i) $P[8 < X < 14]$

$= P\left[\frac{8-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{14-\mu}{\sigma}\right]$

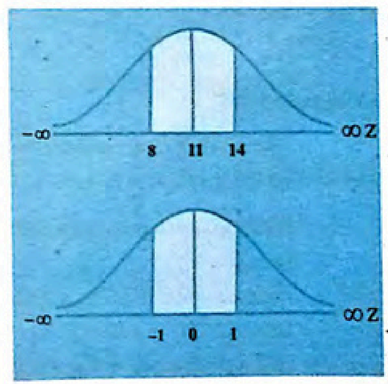
$= P\left[\frac{8-11}{3} < Z < \frac{14-11}{3}\right]$

$= P\left[\frac{-3}{3} < Z < \frac{3}{3}\right]$

$= P[-1 < Z < 1]$

$= P[-1 < Z < 0] + P[0 < Z < 1]$

$= 0.3413 + 0.3413 = 0.6826$



ii) $P[X>10]$

$=P\left[\frac{X-\mu}{\sigma}>\frac{10-\mu}{\sigma}\right]$

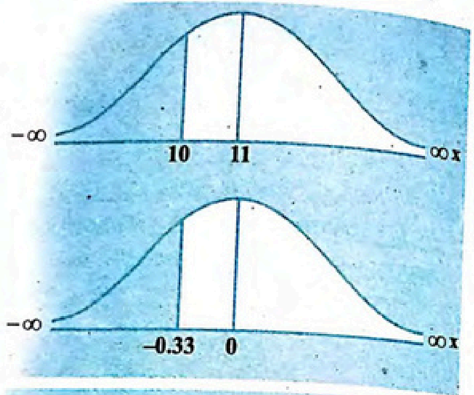
$=P\left[Z>\frac{10-11}{3}\right]$

$=P\left[Z>\frac{-1}{3}\right]$

$=P[Z>-0.33]$

$=P[-0.33<Z<0]+P[0<Z<\infty]$

$=0.1293+0.5=0.6293$



iii) $P\ [X<12]$

$=P\left[\frac{X-\mu}{\sigma}<\frac{12-\mu}{\sigma}\right]$

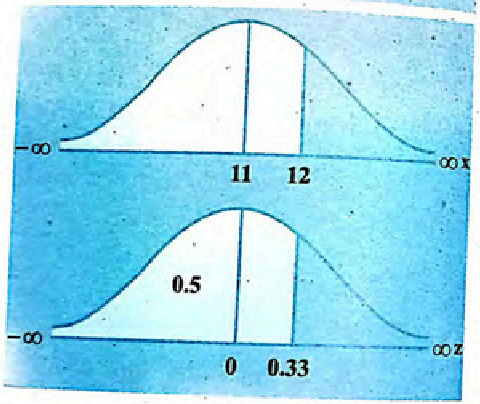
$=P\left[Z<\frac{12-11}{3}\right]$

$=P[Z<0.33]$

$=P[-\infty<Z<0]+P[0<Z<0.33]$

$=0.5\ +\ 0.1293$

$=0.6293$



Example 4.9

The sucrose concentration in a population is normally distributed with a mean = 65 mg and S.D = 25 mg.

i. What proportion of the population is greater than sucrose concentration of 85 mg?

ii. What proportion of the population is less than sucrose concentration of 45 mg?

iii. What proportion of the population are lies between 45 and 85 mg?

Solution:

Given $\mu = 65$ mg

σ=25 mg

i) $P[X>85]$

$=P\left[\frac{X-\mu}{\sigma}>\frac{85-\mu}{\sigma}\right]$

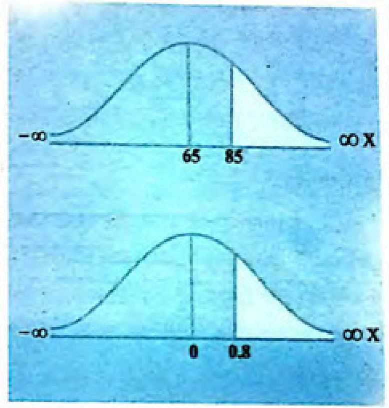
$=P\left[Z>\frac{85-65}{25}\right]$

$=P\left[Z>\frac{20}{25}\right]$

$=P[Z>0.80]$

$=P[0<Z<\infty]-P[0<Z<0.80]$

$=0.5-0.2881=0.2119=21.19\%$



ii) $P[X<45]$

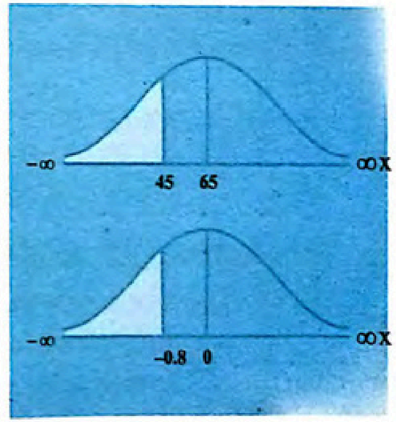
$=P\left[\frac{X-\mu}{\sigma}<\frac{45-\mu}{\sigma}\right]$

$=P\left[Z<\frac{45-65}{25}\right]$

$=P[Z<-0.8]$

$=P[-\infty<Z<0]-P[-0.80<Z<0]$

$=0.5-0.2881=0.2119=21.19\%$



iii) $P[45 < X < 85]$

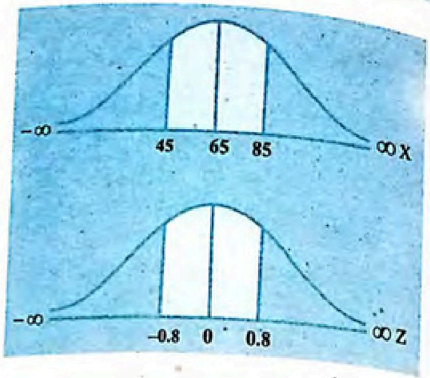
$= P\left[\frac{45-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{85-\mu}{\sigma}\right]$

$= P\left[\frac{45-65}{25} < Z < \frac{85-65}{25}\right]$

$= P[-0.80 < Z < 0.80]$

$= 2P[0 < Z < 0.80]$

$= 2(0.2881) = 0.5762 = 57.62\%$



Example 4.10

Suppose the yearling trout in a lake have lengths that are approximately normally distributed, about a mean $\mu = 9.5''$ with a standard deviation $\sigma = 1.4''$.
(a) What percent of the trout caught have length over $12''$? (b) If 80 percent of the trout caught have length greater than x, find x.

Solution:

(a) We need to find

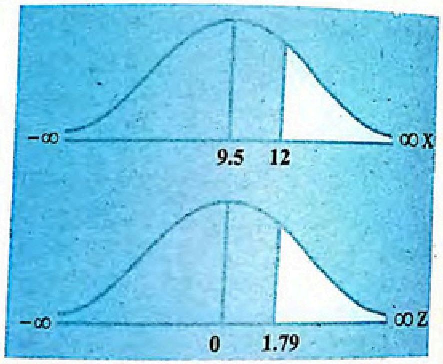
$P[X > 12]$

$= P\left[\frac{X-\mu}{\sigma} > \frac{12-9.5}{1.4}\right]$

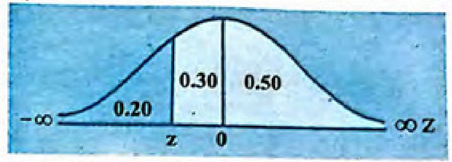
$= P\left[Z > \frac{2.5}{1.4}\right] = P[Z > 1.79]$

$= P[0 < Z < \infty] - P[0 < Z < 1.79]$

$= 0.5 - 0.4633 = 0.0367 = 3.67\% \quad 4\%$



(b) We need X-value above which 80% observation will lie.
Mathematically this statement can be written as $P[X > x] = 0.80$ which in standard units is equal to $P[Z > z] = 0.80$, shown in the following figure.



We see that area below z is equal to 0.20. Search this value 0.20 in the area table 4.1, it corresponds $Z = -0.52$ (minus sign is used because Z value lies to the left of 0).

As $Z = \frac{X-\mu}{\sigma}$ $\Rightarrow X = \mu + Z\sigma = 9.5 + (-0.52)\,1.4 = 9.5 - 0.728 = 8.8$
$X = 8.8$

Hence $P[X > 8.8''] = 0.80 = 80\%$. It means that 80 percent of the trout caught from the lake have length greater than $x = 8.8''$.

Example 4.11

The Peshawar Municipal Corporation installs 10,000 electric lamps in the streets of Peshawar. Average life of these lamps is 1850 hours with a standard deviation of 200 hours. How many lamps may be expected to burn for more than 2000 hours?

Solution:

We first find probability and then multiply by the total number of lamps to compute the required expected frequency as

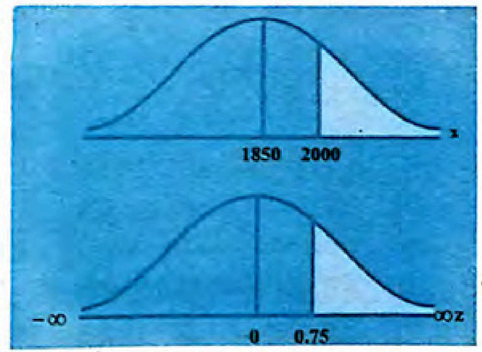
$P[X > 2000]$

$= P\left[\frac{X-\mu}{\sigma} > \frac{2000-\mu}{\sigma}\right]$

$= P\left[Z > \frac{2000-1850}{200}\right]$

$= P\left[Z > \frac{150}{200}\right]$

$= P[Z > 0.75]$



$$= P[0 < Z < \infty] - P[0 < Z < 0.75]$$

$$= 0.5 - 0.2734 = 0.2266$$

Thus, the number of lamps that is expected to burn for more than 2000 hours is

$N f(x) = 10,000\ (0.2266) = 2266.$

Example 4.12

In a statistics examination, the mean score of students was 78 marks and the standard deviation was 10 marks.

a) Determine the standard scores of students receiving marks:

i) 70 ii) 83 iii) 92.

b) Find the marks corresponding to the standard scores:

i) −1 ii) 1.6.

Solution:

Given $\mu = 78$, $\sigma=10$

(a) X-values are given and we need to find Z-values:

i) $Z=\dfrac{X-\mu}{\sigma}=\dfrac{70-78}{10}=\dfrac{-8}{10}=-0.8$

ii) $Z=\dfrac{X-\mu}{\sigma}=\dfrac{83-78}{10}=\dfrac{5}{10}=0.5$

iii) $Z=\dfrac{X-\mu}{\sigma}=\dfrac{92-78}{10}=\dfrac{14}{10}=1.4$

(b) Z-values are given and X-values are required, we know that

$$Z=\frac{X-\mu}{\sigma}, \Rightarrow X=\mu+z\,\sigma$$

(i) $X = 78 + (-1)10 = 78-10 = 68$

(ii) $X = 78 + (1.6)10 = 78+16 = 94$

Example 4.13

Two students A and B were informed that they received standard scores of −1 and 1.6 respectively on a multiple choice examination in Mathematics. If their marks are 68 and 94 respectively, find the mean and standard deviation of the examination marks.

Solution:

We know that, $Z=\dfrac{X-\mu}{\sigma} \Rightarrow X=\mu+Z\,\sigma$

Putting values for student A $68=\mu+(-1)\,\sigma$

$68=\mu-\sigma$ (i)

student B $94=\mu+1.6\,\sigma$ (ii)

Subtracting equation (i) from equation (ii)

$26=2.6\,\sigma$

$\sigma=\dfrac{26}{2.6}=10$

Putting value of σ in eq (1) we get

$68=\mu-10 \Rightarrow \mu=78$

Therefore the mean and standard deviation are μ=78 and σ=10.

4.2.9 Ordinates of the standard Normal distribution

Ordinates mean heights of the standard normal curve corresponding to a specified Z-value. It is denoted by $f(z)$ and is obtained by using the function

$$f(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}, \quad -\infty \le Z \le \infty$$

Fisher and Yates have computed ordinates for different positive values of z and presented them in tabular form as shown in table 4.2. It is important to note that the ordinates at negative values of Z equal to the ordinates at positive values of Z due to unique property of symmetry of the Normal distribution.

TABLE 4.2: Ordinates y or $f(z)$ of the standard Normal curve at z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.3989	0.3989	0.3989	0.3988	0.3986	0.3984	0.3982	0.3980	0.3977	0.3973
0.1	0.3970	0.3965	0.3961	0.3956	0.3951	0.3945	0.3939	0.3932	0.3925	0.3918
0.2	0.3910	0.3902	0.3894	0.3885	0.3876	0.3867	0.3857	0.3847	0.3836	0.3825
0.3	0.3814	0.3802	0.3790	0.3778	0.3765	0.3752	0.3739	0.3725	0.3812	0.3697
0.4	0.3683	0.3668	0.3653	0.3637	0.3621	0.3605	0.3589	0.3572	0.3555	0.3538
0.5	0.3521	0.3503	0.3485	0.34674	0.3448	0.3429	0.3410	0.3391	0.33742	0.3352
0.6	0.3332	0.3312	0.3292	0.3271	0.3251	0.3230	0.3209	0.3187	0.3166	0.3144
0.7	0.3123	0.3101	0.3079	0.3056	0.3034	0.3011	0.2989	0.2966	0.2943	0.2920
0.8	0.2897	0.2874	0.2850	0.2827	0.2803	0.2780	0.2756	0.2732	0.2709	0.2685
0.9	0.2661	0.2637	0.2613	0.2589	0.2565	0.2541	0.2516	0.2492	0.2468	0.2444
1.0	0.2420	0.2396	0.2371	0.2347	0.2323	0.2299	0.2275	0.2251	0.2227	0.22203
1.1	0.2179	0.2155	0.2131	0.2107	0.2083	0.2059	0.2036	0.2012	0.1989	0.1965
1.2	0.1942	0.1919	0.1895	0.1872	0.1849	0.1826	0.1804	0.1781	0.1758	0.1736
1.3	0.1714	0.1691	0.1669	0.1647	0.1626	0.1604	0.1582	0.1561	0.1539	0.1518
1.4	0.1497	0.1476	0.1456	0.1435	0.1415	0.1394	0.1374	0.1354	0.1334	0.1315
1.5	0.1295	0.1276	0.1257	0.1238	0.1219	0.1200	0.1182	0.1163	0.1145	0.1127
1.6	0.1109	0.1092	0.1074	0.1057	0.10401	0.1023	0.1006	0.0989	0.0373	0.0957
1.7	0.0940	0.0925	0.0909	0.0893	0.0878	0.0863	0.0848	0.0833	0.0818	0.0804
1.8	0.0790	0.0775	0.0761	0.0748	0.0734	0.0721	0.0707	0.0694	0.0681	0.0669
1.9	0.0656	0.0644	0.0632	0.0620	0.0608	0.0596	0.0584	0.0573	0.0562	0.0551

2.0	0.0540	0.0529	0.0519	0.0508	0.0489	0.0488	0.0478	0.0468	0.0459	0.0449
2.1	0.0440	0.0431	0.0422	0.0413	0.0404	0.0396	0.0387	0.0379	0.0371	0.0363
2.2	0.0355	0.0347	0.0339	0.0332	0.0325	0.0317	0.0310	0.0303	0.0297	0.0290
2.3	0.0283	0.0277	0.0270	0.0264	0.0258	0.0252	0.0246	0.0241	0.0235	0.0229
2.4	0.0224	0.0219	0.0213	0.0208	0.0203	0.0198	0.0194	0.0189	0.0184	0.0180
2.5	0.0175	0.0171	0.0167	0.0163	0.0158	0.0154	0.0151	0.0147	0.0143	0.0139
2.6	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
2.7	0.0104	0.0101	0.0099	0.0096	0.0093	0.0091	0.0083	0.0086	0.0084	0.0081
2.8	0.0079	0.0077	0.0075	0.0073	0.0071	0.0069	0.0067	0.0065	0.0063	0.00614
2.9	0.0060	0.0058	0.0056	0.0055	0.0053	0.0051	0.0050	0.0048	0.0047	0.0046
3.0	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035	0.0034
3.1	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0025	0.0025
3.2	0.0024	0.0023	0.0022	0.0022	0.0021	0.0020	0.0020	0.0019	0.0018	0.0018
3.3	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013	0.0013
3.4	0.0012	0.0012	0.0012	0.0011	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009
3.5	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006
3.6	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004
3.7	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003
3.8	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002
3.9	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001

- **Use of ordinates table**

Ordinates of the standard normal curve given in table 4.2, as usual require Z-value $Z = \frac{X - \mu}{\sigma}$ rounded up to two decimal points.

Example 4.14

Find the ordinates of the standard normal curve at i) $Z = 0.25$ ii) $Z = 3.18$ iii) $Z = -1.64$

Solution:

i) Look at table 4.1 and search 0.2 in the z-column, go in the body of the table across this value up to the column headed by 0.05. The desired ordinate is 0.3867.

ii) Similarly ordinate at $Z = 3.18$ is equal to 0.0025

iii) Ordinate at $Z = -1.64$ = ordinate at $Z = 1.64 = 0.1040$ (By symmetry property).

4.2.10 Fitting of Normal distribution to observed frequency distribution

The fitting of Normal distribution to observe data means to compute theoretical/expected frequencies corresponding to the observed frequencies. This can be done by three methods, the cumulative standard normal probabilities method, the area method and the ordinate method. Here we consider only the ordinate method because it is simple to understand and comparatively less laborious.

♦ Fitting of Normal distribution by ordinate method

This method involves the following steps:

(i) Compute $\bar{x}$ and S from the observed frequency distribution to estimate the unknown parameters μ and σ.

(ii) Calculate $Z = \dfrac{x-\bar{x}}{S}$ from the mid-points (x).

(iii) Find ordinates corresponding to Z from ordinate table 4.2.

(iv) Multiply the ordinates by $\left(\dfrac{n\,c}{S}\right)$, to get the expected frequencies, where n is total frequency and c is the size of class interval.

Example 4.15

Fit a Normal distribution to the following table which shows the weight measurements of 60 male workers of a factory.

Weights (kg)	50–54	55–59	60–64	65–69	70–74	75–79	80–84	85–89
Number of workers	2	4	6	12	16	12	6	2

Solution:

Weights	f	x	fx	fx^2	$z=\dfrac{x-70.833}{8.029}$	$f(z)$	$e_f=\dfrac{nc}{s}f(z)=37.37f(z)$
50–54	2	52	104	5408	–2.35	0.0252	37.37(0.0252) = 1
55–59	4	57	228	12996	–1.72	0.0909	3.4
60–64	6	62	372	23064	–1.10	0.2179	8.2
65–69	12	67	804	53868	–0.48	0.3555	13.3
70–74	16	72	1152	82944	0.15	0.3945	15
75–79	12	77	924	71148	0.77	0.2966	11.1
80–84	6	82	492	40344	1.39	0.1518	6
85–89	2	87	174	15138	2.01	0.0529	2
Total	60	-	4250	304910	-	-	60

Here $n = \Sigma f = 60$

$c = 5$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{4250}{60} = 70.833 \text{ kg}$$

$$S = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{304910}{60} - \left(\frac{4250}{60}\right)^2} = \sqrt{5081.83 - 5017.36}$$

$$= \sqrt{64.472} = 8.029$$

As μ and σ are unknown, therefore, $Z = \frac{x - \bar{x}}{S} = \frac{x - 70.833}{8,029}$

Put values of x (mid-points) one by one to get z-values as shown above in the table. Ordinates $f(z)$ is obtained from table 4.2. The factor $\frac{n\,c}{s} = \frac{60\ (5)}{8.029} = 37.37$

Key points

- A density function defined as $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ is the pdf of continuous uniform probability distribution.
- Uniform distribution has two parameters, a and b.
- The range of Normal distribution is from $-\infty$ to $+\infty$.
- For Normal distribution the total probability within its range is always equal to one.
- Normal distribution has two parameters, μ and σ.
- For Normal distribution Mean = Median = Mode.
- For Normal distribution M.D $= \frac{4}{5}\sigma$
- For Normal distribution out of the total observations 68.3 % lies within the limits $\mu \pm \sigma$, 95.4 % lies within the limits $\mu \pm 2\sigma$ and 99.7 % lies within the limits $\mu \pm 3\sigma$.
- $Z = \frac{X - \mu}{\sigma}$ (standard Z-score)
- $P[0 < Z < 1.2] = P[-1.2 < Z < 0]$
- Ordinate mean (height) of the standard normal curve corresponding to a specified z-value. It is denoted by $f(z)$
- Point probability in continuous case is always equal to zero.
- Continuous probability distributions give interval probability.
- When Normal distribution is shown on a graph paper, it gives a bell-shaped curve called normal curve which is generally taken as a standard for comparison.
- For Normal distribution $\beta_1 = 0$ and $\beta_2 = 3$.

Exercise

4.1 Read the following statements carefully and indicate which statement is true or false.

i. If X is a continuous uniform random variable $U(a, b)$ then

$$f(x) = \frac{1}{a-b}, \quad a < x < b.$$

ii. For a Normal distribution, the mean is always lies between the median and the mode.

iii. The right and left tails of the normal curve extend indefinitely, never touching the horizontal axis.

iv. The tail will be on the right hand side of a Normal distribution for any positive z-score.

v. Mean = Median = Mode for Normal distribution.

vi. The mean and the S.D of the Standard Normal distribution is 0 and 1 respectively.

vii. The Normal curve is symmetric around the standard deviation.

viii. For Normal distribution about 99.7% observations lies within the limits $\mu \pm 3\sigma$.

ix. All even order moments of Normal distribution are zero.

x. In case of Normal distribution the two quartiles Q_1 and Q_3 are equidistant from the Centre.

4.2 Fill in the suitable word in the blanks.

i. If X has a continuous random variable over the interval (2, 6), the mean of the distribution is equal to________

ii. If $X \sim N(\mu, \sigma^2)$, then standard normal variable Z is distributed as ________.

iii. The maximum height of the normal curve lies at the point__________.

iv. For a Normal distribution, the mean deviation from mean is ________.

v. For a Normal distribution, the odd order moments are equal to ________.

vi. The value of skewness = kurtosis = 0 for ________ distribution.

vii. If for a Normal distribution mean = 10, mode = 10, the value of median =?

viii. The normal curve is symmetric around the __________.

ix. The standard normal curve is symmetric around the_________.

x. The interval ($\mu \pm 2\sigma$) under the normal curve always cover about ____ % of the area.

4.3 Select one correct alternative out of the given ones.

i. The random variable of the Normal distribution is:

(a) discrete (b) continuous (c) positive

ii. If the distribution follows Normal then Mean = Median = Mode

(a) true (b) false (c) impossible

iii. If X is a uniform variable over the interval (5, 10), then the mean of x is

(a) 5 (b) 7.5 (c) 10 (d) 15

iv. Use of the standard normal variable Z instead of normal variable X

(a) complicates the calculation of normal probabilities.

(b) simplifies the calculation of normal probabilities.

(c) Does not make any difference.

(d) gives wrong answer.

v. The total area under a Normal distribution curve to the left of the mean is always equal to

(a) 1 (b) 0 (c) 0.5 (d) 0.9

vi. A normal curve with a small standard deviation will be

(a) positively skewed (b) more spread out

(c) less spread out (d) platy kurtic

vii. The tail of the Normal distribution

(a) meet the horizontal axis at Z = 3.0

(b) cross the horizontal axis at Z = 4.0

(c) never meet or cross the horizontal axis.

(d) are asymmetric

viii. The area under the normal curve within two standard deviation of the mean is

(a) 68.3% (b) 95.4%

(c) 99.7% (d) 99.99%

ix. For a Normal distribution $\mu = 40$, $\sigma = 8$. The value of Z for $X = 52$ is

(a) 2.00 (b) −1.75 (c) 0.80 (d) 1.50

x. An approximate relation between M.D about mean and S.D of Normal distribution is

(a) 5M.D = 4S.D (b) 4M.D = 5S.D

(c) 3M.D = 3S.D (d) 3M.D = 2S.D

4.4 (a) Define continuous uniform probability distribution.

(b) Find mean and variance of the continuous uniform distribution.

4.5 Describe Normal distribution and throw light on the importance of Normal distribution.

4.6 Define Normal distribution. Write down its properties.

4.7 Define standard normal variable and standard Normal distribution. What is the role of this distribution?

4.8 If Z is a standard normal variable, calculate:

(i) $P[Z > 1.60]$
(ii) $P[1.60 < Z < 2.30]$
(iii) $P[-1.64 < Z < -1.02]$
(iv) $P[0 < Z < 1.96]$
(v) $P[-1.96 < Z < 1.96]$
(vi) $P[-1.50 < Z < 0.67]$
(vii) $P[Z < -2.50]$

4.9 Find the proportion of a Normal distribution that corresponds to each of the following sections:

(i) $Z < 0.25$ (ii) $Z > 0.80$ (iii) $Z < -1.50$ (iv) $Z > -0.75$

4.10 Let $Z \sim N(0, 1)$. Find the area under the normal curve in the following cases:

(i) to the right of 2.63 (ii) to the left of −1.45

(iii) between 2.27 and 3.02 (iv) between −1.96 and −1.06

(v) between −2.65 and 2.09 (vi) below 2.17

4.11 Determine the Z-value in the following statements:

(i) $P[Z > z] = 0.005$ (ii) $P[Z > z] = 0.1075$ (iii) $P[Z > z] = 0.9599$

4.12 For a Normal distribution, find the Z-score location that divides the distribution as follows:

(i) separate the top 20% from the rest.

(ii) separate the top 60% from the rest.

(iii) separate the middle 70% from the rest.

4.13 Suppose the distribution of X is normal having mean 46 and standard deviation 4. Compute the following probabilities:

(i) $P[X > 50]$ (ii) $P[X < 38]$ (iii) $P[45 \le X \le 49]$

4.14 A Normal distribution has mean = 100 and variance = 225. Find the following probabilities:

(i) $P[X \ge 76]$ (ii) $P[X \ge 124]$ (ii) $P[X \le 92.5]$

(iv) $P[X \le 107.5]$ (v) $P[91 \le X \le 127]$ (vi) $P[112 \le X \le 128.5]$

4.15 The average seasonal rainfall in certain country is 16 inches with a standard deviation of 4 inches. What is the probability that in a year the rainfall in the country will be between 20 inches and 24 inches?

4.16 In a certain book, the frequency distribution of the number of words per page may be taken as approximately normally with mean 400 and SD 25. If a page is chosen at random, what is the probability that the number of words lies between 415 and 450?

4.17 The heights of a certain population of corn plants follow a Normal distribution with mean 145cm and standard deviation 22cm. What percentage of the plant heights are

(i) 100 cm or more, (ii) between 150 and 180 cm (ii) 180cm or more

4.18 The number of calories in a salad on the lunch menu is normally distributed with mean 200 and SD 5. Find the probability that the salad you select will contain

(i) more than 208 calories (ii) between 190 and 200 calories

4.19 A sales tax officer has reported the average sales of the 500 firms that he has to deal with during a year amount to 72,000 with a SD of 20000. Assuming that the sales in these firms are normally distributed, find.

(i) the number of firms whose sales are over 80000 and

(ii) the number of firms whose sales are likely to range between 60000 and 80000.

4.20 The mean height of 1000 students at a certain college is 165 cm and SD is 10cm. Find the number of students whose height is

(i) less than 172 cm (ii) between 159 and 178 cm and

(iii) more than 173.2 cm.

4.21 The mean wage of a certain group of workers working in a factory is Rs 285 with a standard deviation of Rs 50. Find the percentage of workers get above 200 rupees.

4.22 Assume the mean height of soldiers to be 69 inches with a variance of 9 inches. How many soldiers in a regiment of 1000 would you expect to be over six feet tall?

4.23 If $Z \sim N(0,1)$. Find (i) $P(-1 < Z < 1]$, (ii) ordinate of the normal curve for $Z = 2.25$

4.24 A Normal distribution has mean 1 and $\sigma = 3$, find $P[|X| \leq 2]$

4.25 Find the ordinates of the normal curve at

(i) $Z = 2.52$ (ii) $Z = -0.23$ (iii) -1.81

4.26 Find (i) Mean (ii) Standard deviation on an examination in which marks of 90 and 98 correspondents to standard scores of -0.5 and 1.3 respectively.

4.27 Ina Normal distribution the mean is 20 and the standard deviation is 5. What is the approximate value of the mean deviation?

4.28 The mean deviation of a Normal distribution is 16. Find the approximate value of its standard deviation.

4.29 The length of left middle fingers of 1000 criminals is given below:

Length of finger	9.8	10.1	10.4	10.7	11.0	11.3	11.6	11.9	12.2
No. of criminals	4	30	106	206	272	219	120	37	6

Fit a normal distribution to this data by the ordinate method.

4.30 Fit a Normal distribution to the following data.

Classes	40-44	45-49	50-54	55-59	60-64	65-69	70-74
Frequency	9	20	45	55	43	17	11