Unit 18

DAWN OF MODERN PHYSICS

Major Concepts (24 PERIODS)

- Special theory of relativity
- Quantum theory of radiation
- Photoelectric effect
- Compton's effect
- Pair production and pair annihilation
- Wave nature of particles
- Electron microscope
- Uncertainty principle

Conceptual Linkage

This chapter is built on Planck's quantum theory Chemistry XI
Resolving power, Magnifying power of microscope Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- distinguish between inertial and non-inertial frames of reference.
- describe the significance of Einstein's assumption of the constancy of the speed of light.
- identify that if c is constant then space and time become relative.
- explain qualitatively and quantitatively the consequence of special relativity in relation to:
  - the relativity of simultaneity
  - the equivalence between mass and energy
  - length contraction
  - time dilation
  - mass increase
- explain the implications of mass increase, time dilation and length contraction for space travel.
- describe the concept of black body radiation.
- describe how energy is distributed over the wavelength range for several values of source temperature.
- describe the Planck's hypothesis that radiation emitted and absorbed by the walls of a black body cavity is quantized.
- elaborate the particle nature of electromagnetic radiation.
- describe the phenomenon of photoelectric effect.
- solve problems and analyze information using: $E = hf$ and $c = f\lambda$.

- identify data sources, gather, process and present information to summarize the use of the photoelectric effect in solar cells & photocells.
- describe the confirmation of de Broglie's proposal by Davisson and Germer experiment in which the diffraction of electrons by the surface layers of a crystal lattice was observed.
- describe the impact of de Broglie's proposal that any kind of particle has both wave and particle properties.
- explain the particle model of light in terms of photons with particular energy and frequency.
- describe Compton effect qualitatively.
- explain the phenomena of pair production and pair annihilation.
- explain how the very short wavelength of electrons, and the ability to use electrons and magnetic fields to focus them, allows electron microscope to achieve very high resolution:.
- describe uncertainty principle.

INTRODUCTION

Classical physics is the physics that evolved before the 20th century. It consists of Newton's laws of motion, gravitational laws, laws of thermodynamics, kinetic theory, Maxwell's theory of electromagnetic wave and so many others. Due to all these achievements, many scientists felt that most of the great discoveries in physics had been made. The post-Newtonian concepts in the world of physics brought a revolution in the field of physics which is known modern physics. Modern physics means physics based on the two major breakthroughs of the early 20th century which are relativity and quantum mechanics. In 1900, Max Planck introduced the concept of the quantum theory and it leads to understanding the distribution of black body radiation. Later on, Einstein also explained the photoelectric effect on the basis of Planck's quantization. Bohr in 1913, introduced his model of the hydrogen atom. In 1923, Compton confirmed experimentally the particle nature of light by scattering X-rays with electrons. In 1923, de-Broglie proposed the wave nature of particles and this notion was proved experimentally by Davisson and Germer in 1927. Heisenberg and Schrodinger further explained analytically the dual nature of matter.

Even though the physics that was developed during the 20th century has led to a multitude of important technological achievements, the story is still incomplete. Discoveries will continue to evolve during our lifetimes and beyond that, and many of these discoveries will refine our understanding of the nature of universe around us. It is still a marvelous time to be alive. In this unit, we will explain some important theories of modern physics, such as special theory of relativity and its consequences, black body radiation and Planck's law, photoelectric effect and Compton effect, De-

Broglie wave equation and Davisson-Germer experiment, pair production and annihilation of matter and Heisenberg's uncertainty principle.

18.1 RELATIVE MOTION

Our moving earth appears at rest with respect to the geostationary satellite, while appears in motion with respect to the moon. This example provides the principle of relative rest and relative motion. The experimental facts show that observations of all the events would have different meaning to different observers. It can further be explained by another example of a boy who is standing in a train which is moving with uniform velocity and is holding a ball in his hand. The ball appears stationary for an observer A, who is also standing in the train. But the same ball appears moving with the train for another observer B who is standing on the ground outside the train. Now if the boy throws the ball vertically straight upward, it will come back straight downward. This will be observed by the observer A as shown in Fig.18.1(a). But the observer B who is standing outside the train will observe that the ball is moving along the parabolic path as shown in Fig.18.1(b). Although the two observers disagree on the path followed by the ball, but both agree over the motion of the ball obeys the law of gravity and Newton's laws of motion, and they also agree on how long the ball is in the air. By

> ***POINT TO PONDER***
> Let you are standing at the roof the building and you are observed by two observer one is at the moon and other is at the geostationary satellite. Who will observe that you are in motion?

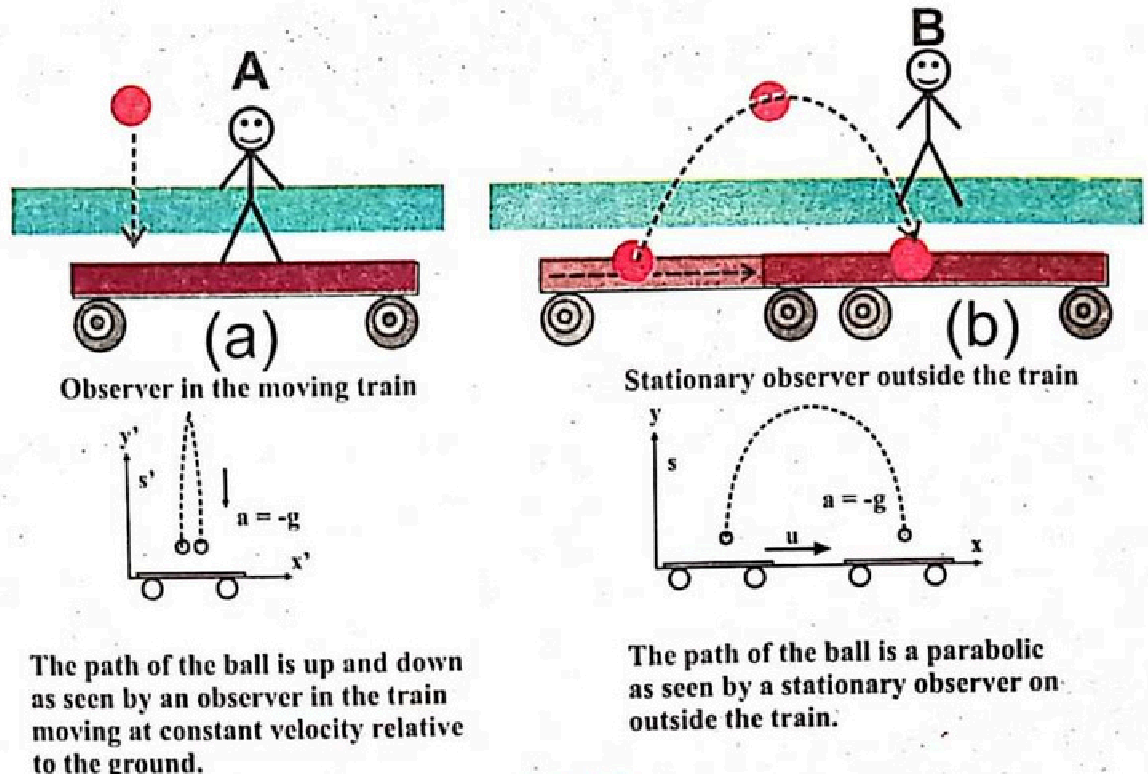


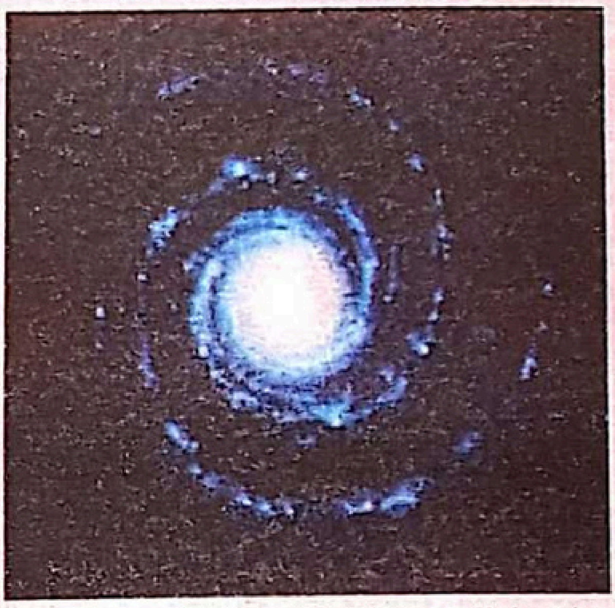
Fig.18.1

summing up these observations, it is concluded that all the motions are relative. There is no absolute rest or no absolute motion.

18.2 FRAME OF REFERENCE

In daily life, when we observe an event i.e., physical happening or measure position, or change in position of a body then we use some relative points such as east, west, north and south which are lying on the earth's surface. These points are called reference points and the earth is a frame of reference. In physics we use Cartesian coordinates as reference points for measurements. The set of coordinates with respect to which the measurements or observations are made is called frame of reference. There are two kinds of frame of reference.

INTERESTED INFORMATION



From the earth frame of reference, light takes 25,000 years to travel from the center of our milky way galaxy to our solar system. From the frame of reference of a high speed space ship flying outward from the galactic center toward earth, the trip takes less time. If a frame of reference could be attached to the light itself, the travel time could be reduced to zero.

I Inertial frame of reference

A frame of reference which is at rest or moving with uniform velocity is called inertial frame of reference. e.g., earth is taken almost as an inertial frame of reference. In inertial frame of reference, the acceleration of the body is zero and law of inertia remains valid in it. That is, if a body is at rest it will be remain at rest and if a body is moving with uniform velocity it will be continue its uniform motion unless an unbalanced force produces an acceleration in it. According to the principle of Galilean relativity, all the laws of physics must be the same in all inertial frames of reference.

II Non-inertial frame of reference

A frame of reference which is accelerating, or decelerating is known as non-inertial frame of reference. It may be pointed out that Newton's laws of motion are not valid in non-inertial frame of reference.

18.3 SPECIAL THEORY OF RELATIVITY

The Special Theory of relativity was proposed by Albert Einstein in 1905. This theory has brought a revolution in the field of the science. The scientists completely changed their notion about Ether (a hypothetical medium), speed of light, space and time coordinates and so many other. The special theory of relativity is based on the following two postulates.

1. All the laws of physics are same in all inertial frames of reference.
2. The speed of light is a universal constant. That is, the value of speed of light ($3 \times 10^8\ ms^{-1}$) remains the same in all inertial frames of reference.

The first postulate is also called the principle of relativity and it states that all the laws of physics are equally applicable in all frames of reference, if the relative motion between them is uniform. For example, let any kind of experiment such as mechanical, thermal, optical or electrical is performed in a laboratory at rest. Then according to Einstein's principle of relativity, the same result will be obtained when the same experiments are performed in a laboratory moving with uniform speed. Hence there are no preferred inertial frame of reference.

> **FOR YOUR INFORMATION**
>
> Special relativity is "special" in the sense that it deals with uniformly moving reference frames – once that are not accelerated. General relativity is "general" and deals also with accelerating reference frames. The general theory of relativity presents a new theory of gravity.

The second postulate is contrary to the Ether theory. Most of earlier physicists believed that the light travel through a hypothetical medium called Ether and the speed of light relative to ether is different in different direction. But according to Einstein, the speed of light in vacuum is the same in all inertial frames of reference. For example, let two observers measure speed of light in vacuum, such that one observer is at rest with respect to the source of light and the other is moving away from it with uniform velocity. According to Einstein's principle of relativity, the result of both observers will be the same.

> **POINT TO PONDER**
>
> If you were travelling in a space ship at a speed of C/2 relative to earth and you fired a laser beam in the direction of the spaceship's motion. What will be the light speed from the laser relative to the earth?

18.4 CONSEQUENCES OF SPECIAL THEORY OF RELATIVITY

Einstein's special theory of relativity has some very important results which are related to the base physical quantities such as length, time and mass. According to the special theory of relativity, the results of measurement of these quantities are different in different frame of reference. Therefore, we measure the distance between two points, the time interval of an event and mass of a body based on the consequence of special theory of relativity.

The relativity of simultaneity

The concept of simultaneity gives an interesting result of Einstein's second postulate. i.e., the speed of light is a universal constant. It is stated as, **when two**

events that are simultaneous in one frame of reference need not be simultaneous in a second frame of reference that moving relative to the first. It is explained by an example. Consider a light source suspended by the exact centre of the roof of the compartment of a train. When the light is switched on, it spreads out in all direction with speed 'c'. An observer inside the compartment observes that light reaches the front wall of the compartment at the same time it reaches the back wall as shown in Fig. 18.2(a). This occurs whether the train is at rest or moving at constant speed. Here the two events of falling of light on the back wall and falling on the front wall occur simultaneously for the observer inside the compartment.

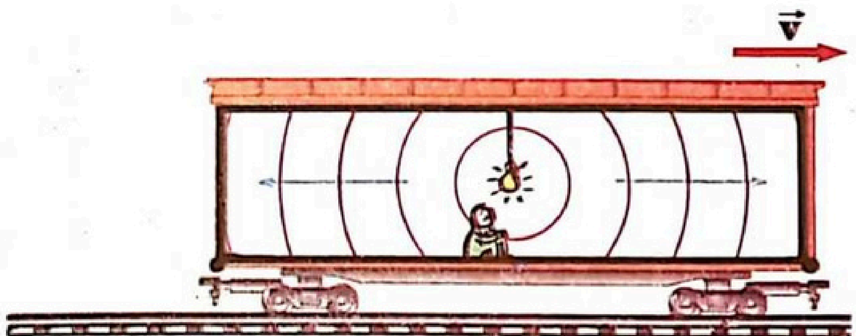


Fig.18.2(a) With respect to the observer who travels inside the compartment, light from the source travels equal distance to both walls of the compartment and therefore strikes both walls

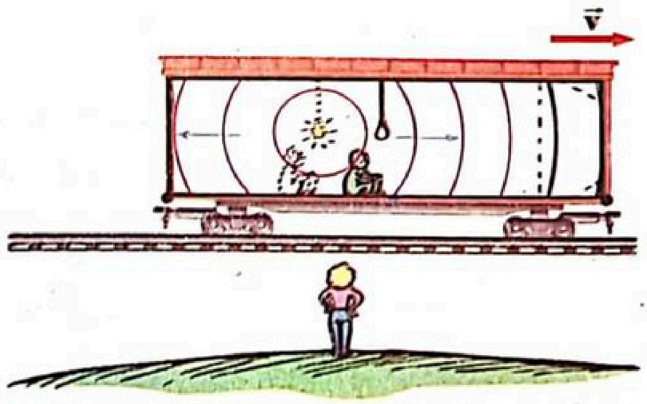


Fig.18.2(b) the events of light striking the front and back walls of the compartment are not simultaneous with respect to an observer who is standing on the ground.

On the other hand, if the same two events are observed by another observer who is standing on the ground (at rest) and outside the compartment, then these two events are not simultaneous. As light travel away from the source, this observer sees the train moves forward, so the back wall of the compartment moves toward the beam while the front moves away from it. The beam going to the back of the compartment, therefore, has a shorter distance to travel than the beam going forward, as shown in Fig. 18.2(b). Thus this outside observer sees the event of light falling the back wall of the compartment before seeing the event of light falling the front wall of the compartment. Similarly, another observer in another moving train that passes in the opposite direction would report that the light reaches the front wall of the compartment first.

We should not wonder which observer is right concerning the two events. According to Einstein's theory of relativity, both observers are correct, because simultaneously is not absolute and there is no preferred inertial frame of reference, but the two observers must find that light travel at the same speed.

Length contraction

When both ends of length of an object are measured simultaneously by an observer at rest relative to the object, then such length of the object is called its rest or proper length. According to special theory of relativity, the length of an object measured in a frame of reference which is moving with respect to the object is always less than the rest length or proper length. This effect is known as length contraction. The length contraction takes place only along the direction of motion, this effect is negligible at ordinary speeds, no such effects may be observed perpendicular to the direction of motion.

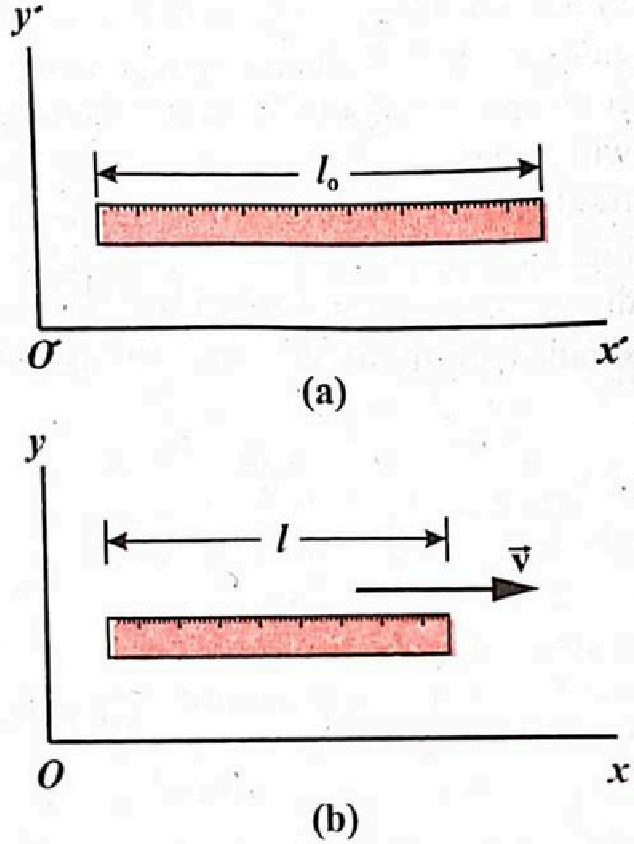


Fig.18.3 Length contraction of the body in a moving frame of reference.

Consider, the length of an object (rod) between two point which is measured by two observers, such that one observer is at rest relative to the object while the other one is moving with very high velocity 'v' with respect to the object. If the length measured by the rest observer is 'ℓ_0' and the length measured by the moving observer is 'ℓ' then according to the relativistic mechanics,

$$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}} \quad ......(18.1)$$

If the moving observer is also at rest, i.e., $v = 0$ then $\ell = \ell_0$, that is, the results of the both observers are the same. But if there is a relative motion of the observer with velocity 'v', the length ℓ is less than the proper length ℓ_0. This result shows that a moving body has a decrease in its length along the direction of motion as shown in Fig.18.3.

Time dilation

When a time interval between two events is measured by an observer who is at rest then it is called a proper time. Now according the special theory of relativity time is not absolute quantity but the time interval between the two same events would be greater than the proper time which is measured by another observer who is moving at very high-speed relative to the place of event. This effect is called time dilation i.e., to dilate means to become larger and it is explained as:

Consider two events which occurred at the same place. The time interval between these two events measured by two observers, such that one observer is at rest frame and he noted the proper time interval as 't_0' while, the other observer in other frame of reference is moving with uniform velocity v relative to the rest frame and he noted the time interval between the two events is 't'. Thus, according to the relativistic mechanics

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ......(18.2)$$

> ***POINT TO PONDER***
>
> If a rod of length 1m moves with a very high speed, will you observe its length less or greater than 1m?

> ***POINT TO PONDER***
>
> Two identical constructed clocks are synchronized. One is attached with a wall of a moving spaceship while the other remains on earth. Which clock runs more fast?

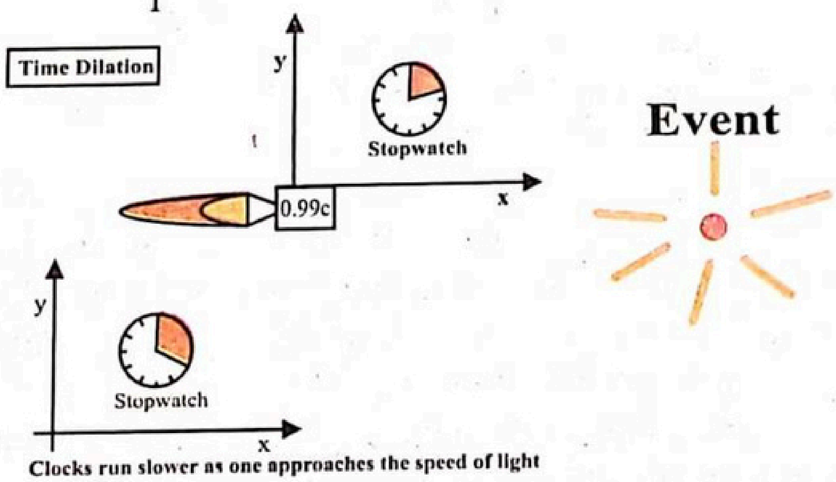


Fig.18.4 Different time interval between two events observed by two observers in two different frames of reference.

If the moving observer is also at rest. i.e., $v = 0$ then $t = t_o$. But if there is a relative motion between the two observers and the denominator of equation 18.2 is less than one thus time 't' is always greater than the proper time 't_o' as shown in Fig.18.4.

> ***DO YOU KNOW***
>
> At constant distance, time and speed are inversely proportional to each other.
>
> i.e. $\text{speed} \propto \frac{1}{\text{time}}$
>
> It means greater is the speed lesser is the time taken.

Mass variation.

Like length contraction and time dilation, mass of a body is also not absolute rather it is a varying quantity. It depends upon the speed of frame of reference or frame of observers. It is explained as: when a body is at rest, its measured mass is called its rest mass or proper mass. Now when the body starts motion at very high speed, then according to the special theory of relativity, the mass of the moving body will be increased as shown in Fig.18.5. This is called mass variation effect. Let m_0 be the rest mass of a body and m be the mass of the same body which is moving with velocity 'v' then the mass variation is given as

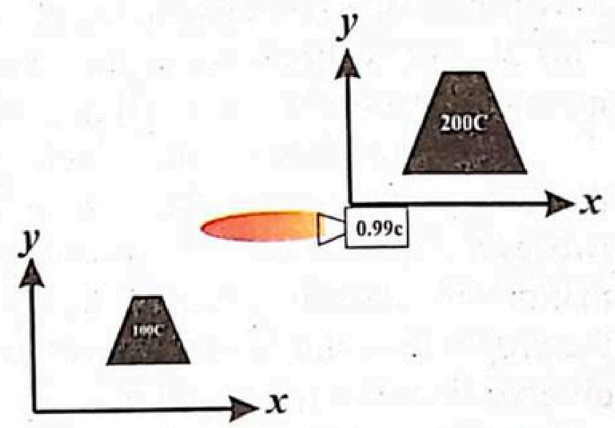


Fig.18.5 Mass dilation in a moving frame of reference.

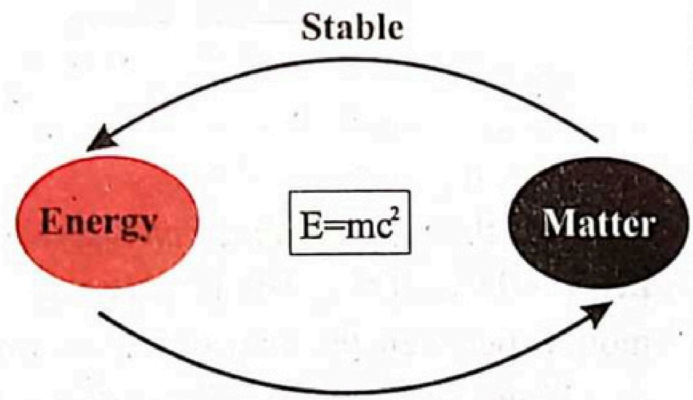
$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad ...(18.3)$$

FOR YOUR INFORMATION
The global positioning system (GPS) takes account of the time dilation of orbiting atomic clocks. Otherwise, our GPS receiver would badly miss our location.

This relation shows that increase in mass of a body depends upon its speed. If the speed of mass is one-tenth of speed of light (0.1c), the increase in mass is only 0.5%. Similarly, the increase in mass of the body will be 100% if the body has a speed nine-tenth (0.9c) of the speed of light. But, in our daily life, we deal with extremely small speeds compared to the speed of light and hence $m = m_0$. This is the reason why Newton's laws of motion are valid in everyday life. However, in case of atomic particles which are moving with speeds comparable with speed of light, the relativistic effect can be observed evidently, and their experimental results cannot be explained without taking Einstein's equations into account.

Mass-Energy Relation

According to special theory of relativity, mass and energy are not two different quantities, but they are interconvertible to each other. That is, the mass 'm' of a moving body can be converted into energy 'E' and vice versa.



Mass and energy are convertible to each other.

Mass-Energy Equivalence
1kg $\leftrightarrow$ 8.988×10^{16}J
1u $\leftrightarrow$ 931.5 MeV_0
1eV_0 $\leftrightarrow$ 1.074×10^{-9}u

Such conversion can be studied under the following mass-energy relation:

$$E = mc^2 \quad ......(18.4)$$

Similarly, if the body is at rest then its energy is called rest mass energy E_0 and its value is given by

$$E_0 = m_0c^2 \quad ......(18.5)$$

According to the mass-dilation effect, change in mass causes of change in energy, since $E > E_0$, so

$$E - E_0 = mc^2 - m_0c^2$$

$$\Delta E = (m - m_0)c^2$$

$$\Delta E = \Delta mc^2 \quad ......(18.6)$$

This shows that a small variation in mass cause an enormous amount of change in energy, because the value of c^2 is very large. It can be observed experimentally in a nuclear power plant which produces energy by fission of uranium, it involves the conversion of a small amount of the mass of the uranium into large amount of energy.

INTERESTED INFORMATION

Saying that a power plant delivers 90 million megajoules of energy to its consumers is equivalent to saying that it delivers 1 gram of energy to its consumer because mass and energy are equivalent.

Example 18.1

A rocket of proper length 50m is moving with a speed of 0.13c. Then how much will it appear to be shortened to an observer on earth?

Solution:

Proper length = $\ell_0 = 50$m

Speed of rocket= $v = 0.13c$

Speed of light = $c = 3 \times 10^8$ ms^{-1}

Shortened length = $\ell = ?$

According to the relation of length contraction

$$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\ell = 50\sqrt{1-\frac{(0.13c)^2}{c^2}}$$

$$\ell = 50\sqrt{1-0.0169}$$

$$\ell = 49.5m$$

Shortening the length of the rocket

$$\Delta\ell = \ell_0 - \ell$$

$$\Delta\ell = 50m - 49.5m$$

$$\Delta\ell = 0.5m$$

Example 18.2

An atom will decay in 2×10^{-6} s. What will be the decay time as measured by an observer in a laboratory if the atom is moving with a speed of 0.8c.

Solution:

Time of decay when the atom is rest = $t_o = 2 \times 10^{-6}s$

Time of decay when the atom is moving = t = ?

Speed of atom = 0.8c

According to time dilation equation

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{2\times10^{-6}}{\sqrt{1-\frac{(0.8c)^2}{c^2}}} = \frac{2\times10^{-6}}{\sqrt{1-0.64}}$$

$$t = \frac{2\times10^{-6}}{\sqrt{0.36}} = \frac{2\times10^{-6}}{0.6}$$

$$t = 3.33 \times 10^{-6}s$$

Example 18.3

At what speed the mass of the body will become twice the rest mass value?

Solution:

Rest mass of the body = m_o

Mass of the moving body = m

Let us assume that at a speed of 'v', the mass of the body will be doubled. i.e.,

$$m = 2m_0$$

According to mass variation relation

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$2m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$2\sqrt{1-\frac{v^2}{c^2}} = 1$$

Squaring both sides

$$4\left(1-\frac{v^2}{c^2}\right) = 1$$

$$4 = \frac{4v^2}{c^2} + 1$$

$$\frac{4v^2}{c^2} = 3$$

$$v^2 = \frac{3}{4}c^2 \quad \Rightarrow v = 0.866c = 0.866 \times 3 \times 10^8 \, m\,s^{-1}$$

$$v = 2.6 \times 10^8 \, m\,s^{-1}$$

18.5 BLACKBODY RADIATION

When a solid body is heated, the body glows and emits radiation. This emission of radiation depends upon temperature only. That is, when the temperature increases, the body becomes red then yellow then white. The wavelengths of their corresponding emitted radiations are decreased which are lying in the range from infrared to ultraviolet. For example, at low temperature, the emitted radiation has long wavelength, while at high temperature, the emitted radiation has short wavelength.

FOR YOUR INFORMATION

Volcanic lava emits light and is a very good example of a blackbody radiation.

The analysis show that when these radiations fall on a body, some of them may be absorbed and some of them reflected. So, we could not study well the distribution of radiation from a hot body. We consider **an ideal body that absorbs and emits the radiations of all wavelength falling on it. Such body is called Blackbody.** Blackbody is perfect absorber for all incident radiations and also a perfect emitter or radiator of all kind of radiations. When the black body is heated, it emits the radiations of all wavelengths depending upon its temperature. Thus, a black body is a perfect absorber as well as a perfect emitter of radiation. Practically, a perfect black body does not exist, but it can be constructed by making a very small hole in one of the walls of a hollow body called cavity as shown in Fig.18.6. The inner walls of the cavity are blackened with carbon soot to make them as good absorber and also as a reflector. The radiation that enters through the small hole will be trapped inside the cavity. If this body is heated to a certain temperature, it will emit radiations of all wavelengths which is called blackbody radiation. The nature of emission of radiation depends upon temperature only. That is, when the temperature of the blackbody is increased, the radiated energy emitted from the blackbody is also increased. Graphically the experimental data for the distribution of energy of blackbody radiation i.e. intensity versus wavelength are shown in Fig.18.7. These results show that wavelength of radiated energy varies with temperature and shifts towards the shorter values as the temperature of the cavity increases.

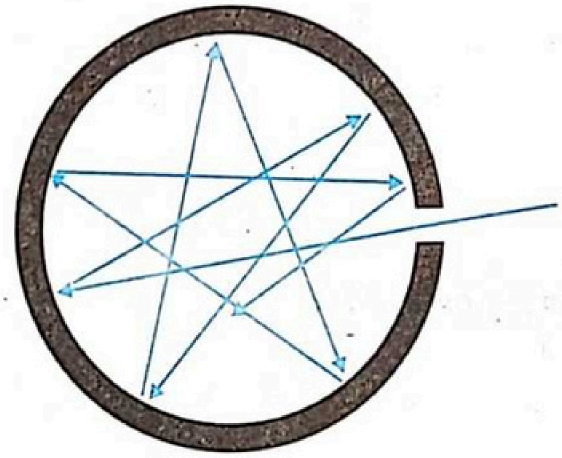


Fig.18.6 A hole in the wall of a hollow sphere is a good approximation of a black body.

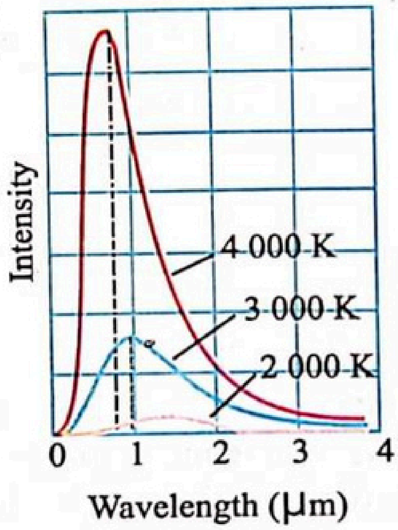


Fig.18.7 Graph between intensity of blackbody radiation and its wavelength. The energy increase with increasing temperature and hence decrease in its wavelength

> ***POINT TO PONDER***
> An incandescent light bulb is connected to a dimmer switch. When the bulb operates at full power, it appears white, but as it is dimmed it looks more and more red. Explain?

Several attempts were made to explain the distribution and characteristics of these radiations. The four most important among them are given by:

Stefan-Boltzmann law

This law states that the total energy per second per unit area (i.e. total radiant heat power) under the curve of the radiation is directly proportional to the fourth power of its absolute temperature

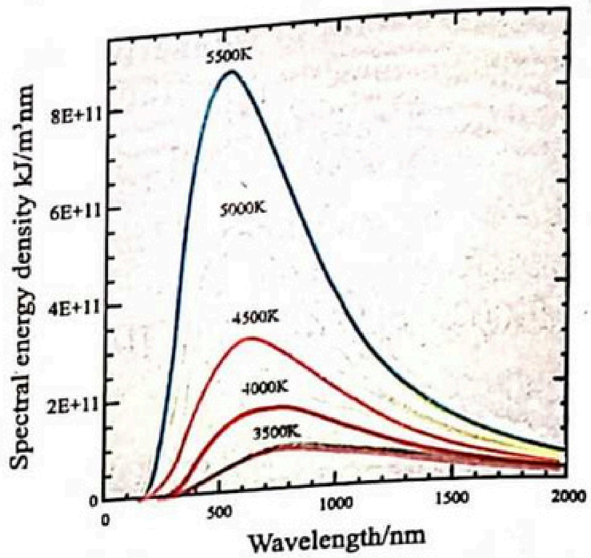
$$E \propto T^4$$

$$E = \sigma T^4 \quad ......(18.7)$$

where 'σ' is Stefan's constant and its value is $5.67 \times 10^{-8}\ wm^{-2}K^{-4}$.

Wien's displacement law

According to Wien's displacement law, the wavelength with maximum intensity in the radiation emitted from a blackbody is inversely proportional to the temperature of the blackbody. That is, with increasing temperature the peak of the distribution shifts to shorter wave lengths;



The peak wavelengths of radiations decreasing with increasing temperature.

$$\lambda_m = \frac{1}{T}$$

$$\lambda_m T = \text{Constant} \quad ...(18.8)$$

where the value of Wien's constant is 2.9×10^{-3}mK

Rayleigh-Jean's law

Rayleigh and Jean explained the distribution of radiation based on kinetic theory. They assumed that the radiation inside the cavity consists of standing waves. These standing waves are due to the oscillation of radiation from one wall to the other wall of the cavity. Based on this argument, they derived the following relation.

$$E = \frac{8\pi kT}{\lambda^4} \quad ......(18.9)$$

where 'k' is the Boltzmann's constant and its value is $1.3807 \times 10^{-23}\ JK^{-1}$.

Max Planck's law

Wien's displacement law and Rayleigh-Jean's law both explain only single aspect of the energy distribution along the experimental intensity and wavelength curves. Because, Wien's formula can apply for those radiations which have short wavelengths while Rayleigh-Jean's formula for radiations having long wavelengths as shown in Fig.18.8. None of these laws is able to explain the entire experimentally observed curve.

To overcome this problem, Max Planck in the year 1900 developed a formula for distribution of energy that fitted well into the experimental curve for the entire range of wavelengths. His formula for the distribution of energy based on his quantum theory. According to his theory, the absorption and emission of radiation from a black body is in the form of energy packets which he termed as 'quanta'. The energy of each quanta is directly proportional to the frequency of the radiation, i.e.,

$$E \propto f$$

$$E = hf \quad ......(18.10a)$$

where 'h' is known as Planck's constant. Its value is 6.63×10^{-34} Js.

But $f\lambda = c$ therefore eq. 18.10a becomes

$$E = \frac{hc}{\lambda} \quad ......(18.10b)$$

Later, Einstein extended Planck's idea that the energy is absorbed or emitted in discrete packets which are integral part of all kind of electromagnetic radiations know as photon. The energy of each photon is 'hf' and its momentum can be calculated as

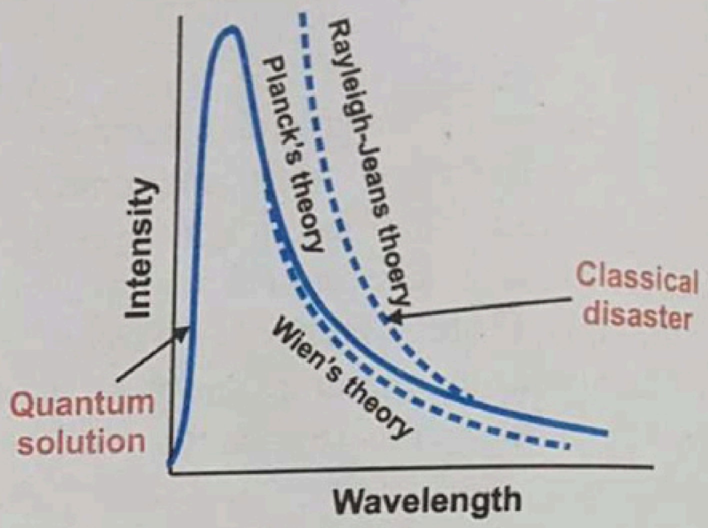


Fig.18.8. Comparison the experimental curves among Wien's, Rayleigh-Jean's and Planck's laws.

Since $$E = hf = mc^2$$

So $$mc = \frac{hf}{c}$$

But the product of mass 'm' and velocity (velocity of light 'c') is called momentum p. Thus,

$$p = \frac{hf}{c}$$

But $f\lambda = c$ therefore

$$p = \frac{h}{\lambda} \quad ......(18.11)$$

> *POINT TO PONDER*
> A particle with non-zero mass can never move faster than the speed of light. Is there also upper limit on its momentum and kinetic energy?

Example 18.4

What is the wavelength of the radiation having maximum intensity when it is emitted from a blackbody at temperature 57°C.

Solution:

Wavelength $(\lambda_m) = ?$

Temperature (T) = 57°C = 330K

Wien's constant = 2.9×10^{-3} mK

According to Wien's formula

$$\lambda_m T = \text{Constant}$$

$$\lambda_m = \frac{\text{constant}}{T}$$

$$\lambda_m = \frac{2.9 \times 10^{-3}\,\text{mK}}{330\text{K}} = 8.8 \times 10^{-6}\text{m} = 8.8\mu\text{m}$$

Example 18.5

Compute the energy of photon of infrared light of wavelength 1240nm.

Solution:

Energy of photon (E) = ?

Wavelength of infrared light (λ) = 1240nm = 1240×10^{-9}m = 1.24×10^{-6} m

Planck constant (h) = 6.63×10^{-34}Js

The energy of the photon is given by

$$\therefore \quad E = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34}\,\text{Js} \times 3 \times 10^{8}\,\text{ms}^{-1}}{1.24 \times 10^{-6}\,\text{m}}$$

$$E = 1.6 \times 10^{-19}\text{J}$$

As $\quad 1.6 \times 10^{-19}\text{J} = 1\text{eV}$

$$\therefore \quad E = 1\text{eV}$$

The energy of given infrared light is 1eV.

18.6 THE PARTICLE THEORY OF ELECTROMAGNETIC RADIATIONS

In the previous class, we have verified the wave nature of light by interference and diffraction phenomenas. In this section, we are going to explain the particle nature of electromagnetic radiations by means of the following three phenomenas:

I Photoelectric effect

II Compton effect

III Pair production

18.7 PHOTOELECTRIC EFFECT

Heinrich Hertz made the first observation of the photoelectric effect in 1887. He observed that when ultraviolet light falls on a metal surface (zinc, cadmium, magnesium etc), then there is emission of electrons from the metal surface. This phenomenon is known as the photoelectric effect and the emitted electrons are called photo-electrons. The photoelectric effect is also possible by visible light when the target material is alkali metal such as sodium, potassium, calcium etc.

A schematic diagram of a photoelectric effect is shown in Fig.18.9. It consists of photosensitive metallic plate P and collector 'C' which are enclosed in an evacuated glass tube. The plate is connected to negative terminal of the battery called cathode. The second electrode called collector or anode is connected to positive terminal of the battery through a galvanometer G. When light of certain frequency falls on the metallic plate, then there is emission of photo electrons from the plate. These photoelectrons are attracted by the anode to constitute an electric current called photo electric current which is detected by the galvanometer. The current becomes zero as soon as the light is switched off. This confirms that the light energy is converted into the electrical energy under the process of photoelectric effect.

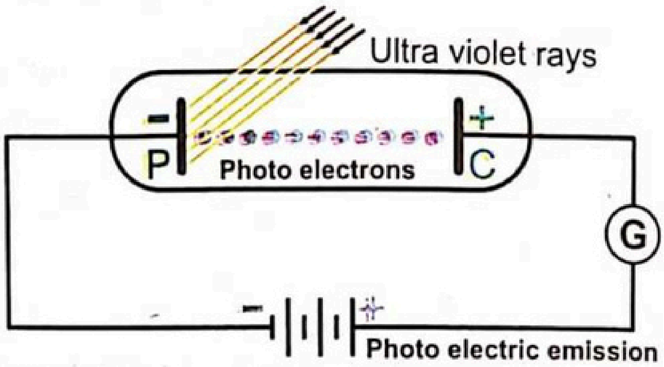


Fig.18.9. An experimental circuit diagram for photo electric effect.

Maximum kinetic energy of photoelectrons

In the experimental arrangement of photoelectric effect, anode C always collects the photoelectrons, because, it is connected to the positive terminal of the battery. When the terminals of the battery are reversed as shown in Fig.18.10, then 'C' becomes negative. In this condition, the photoelectrons are repelled by the 'C', as a result the photoelectric current starts decreasing. When the negative potential is further increased then at certain negative potential, the photoelectric current becomes zero. At this stage, no electron reaches at the anode C. This specific negative potential is called stopping potential V and it is related to the kinetic energy of photo electrons. That is,

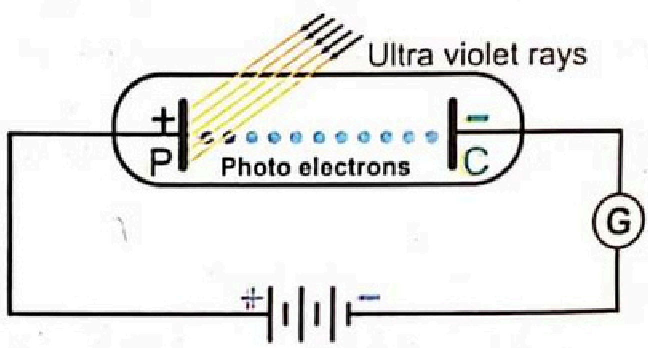


Fig.18.10. A reverse circuit diagram a photo electric effect in order to calculate the kinetic energy of photo electrons.

$$\frac{1}{2}mv^2 = V_0e \;\ldots\ldots(18.12)$$

This is the fundamental equation of photoelectric effect.

Experimental observations or laws of photoelectric effect

The experimental observations lead different laws based upon the photoelectric effect. All these laws are summarized as:

I Threshold frequency and work function

The emission of electrons will not take place if the frequency of incident radiation or photon is below a certain minimum frequency known as threshold frequency. In other words, the minimum frequency of the incident light is required for the emission of electrons from the metal surface. The threshold frequency is denoted by f_0. The threshold frequency has a different value for different metals. The minimum energy required to eject an electron from photo emissive surface is called the work function. The work functions of different metals are given in table 18.1. If the energy of incident light is above the work function appears as the kinetic energy of photoelectrons.

II Spontaneous emission of photoelectron

When the frequency of the incident light is equal or greater than the threshold value then there will be spontaneous emission of photo electron. The time of emission is 10^{-18}s.

Table 18.1

Element	Work function (eV)
Aluminum	4.3
Carbon	5.0
Copper	4.7
Gold	5.1
Iron	4.7
Nickel	5.1
Potassium	2.3
Silicon	4.8
Silver	4.3
Sodium	2.7
Tungsten	4.6

III The photoelectric current depends upon the intensity of light

The analysis shows that the photoelectric current is directly proportional to the intensity of incident light provided that frequency if incident photon is above f_0. That is, brighter incident light will cause more emission of electrons from the metal surface and hence a large current flow in the circuit. It is noticed that the intensity of light does not increase the kinetic energy of photoelectrons. In other word, kinetic energy of photoelectrons is independent of the intensity of the incident light.

IV The kinetic energy of photoelectrons depends upon the frequency of light

The maximum kinetic energy of photoelectrons is directly proportional to the frequency of the incident light. Furthermore, the stopping potential of the photoelectrons also depends upon the frequency of the incident light.

18.7.2 Einstein's photoelectric effect equation

Einstein explained the photoelectric effect on the basis of quantum theory of light. According to Planck's quantum theory, light is absorbed or emitted in form of energy packet called quanta or photon. The energy of each photon is *hf*. When light falls on a metal surface then the energy of photon is absorbed by an electron of the metal surface. Thus according to Einstein, a part of this energy is used to liberate or emit the electron from the metal surface called work function ϕ and the remaining part of the incident energy is converted into the kinetic energy of the photoelectron. That is,

Energy of incident photon = work function + K.E. of photoelectrons

$$hf = \phi + \frac{1}{2}mv^2$$

$$hf = hf' + eV_0$$

$$eV_0 = hf - hf' \quad ......(18.13)$$

This is known as Einstein's equation of photo electric effect. If we plot a graph between frequency of the incident photons and kinetic energy of photoelectrons then we get a straight line as shown in Fig.18.11.

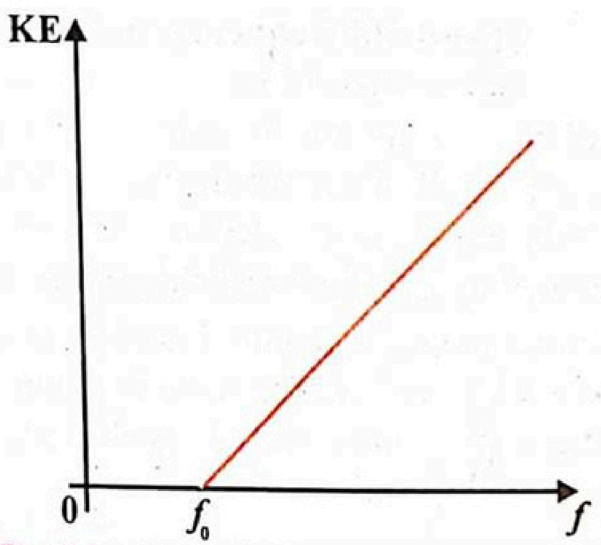


Fig.18.11. A straight line graph between kinetic energy of photo electrons and the frequency of the incident photons

18.8 PHOTOCELL

A photocell is a device that converts light energy into electrical energy. It works on the principle of photoelectric effect. The photocell consists of two electrodes, emitter and collector which are enclosed in an evacuated glass tube. The metal of emitter or cathode used must be either sodium or potassium, because these metals are very sensitive to visible light. However, if the surface of the cathode is coated with cesium then it emits electrons even for infrared light. The collector or anode is in the form of a straight rod whereas the cathode is curved to receive the maximum incident light as shown in Fig.18.12.

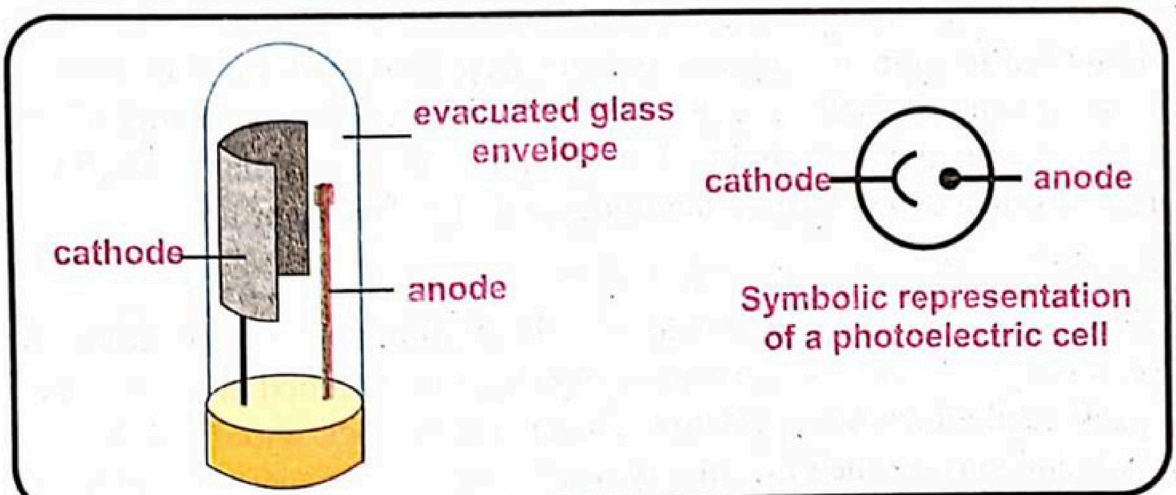


Fig.18.12. A photo cell with its symbol.

When light from a source is incident on the cathode, photoelectrons are emitted from it. These electrons are attracted by the positive anode to constitute an electric current in the external circuit. The magnitude of the current can be increased by increasing the intensity of light above threshold frequency. If the light beam is switched off, then the flow of current also stops.

There are number of applications of a photocell, some of them are listed below:

1. Counting systems
2. Security systems
3. Automatic door opening/closing
4. Automatic street lighting system
5. Soundtrack of movies
6. Burglar alarm system
7. Measurement the temperature of stars

> *POINT TO PONDER*
> Rest mass of a photon is zero. Is its momentum also zero?

18.9 SOLAR CELL

Like photocell, **a solar cell is also an electrical device which converts sun's light energy directly into electrical energy**. A crystalline silicon solar cell consists of N-type and P-type semiconductors, sandwiched in between two metal contacts that are responsible for conducting electricity out of such a device as shown in Fig.18.13(a). An anti-refractive coating is also placed on the top of the metal contact and N-type in order to absorb maximum solar energy by the cell. Finally, a thin glass sheet is placed on the top of the cell to protect it from weather and mechanical shock.

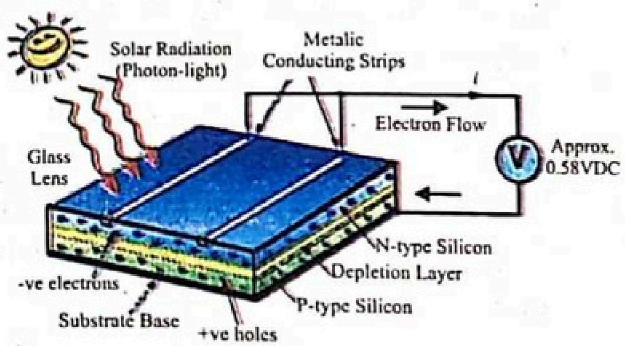


Fig.18.13(a) Schematic diagram of a solar cell with its various parts..

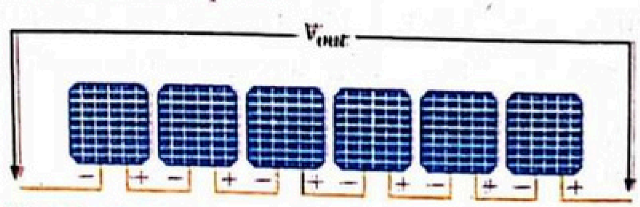


Fig.18.13(b). A series connection of solar cells panel.

When sunlight falls on the PN-junction of the solar cell then this light has sufficient energy to knock an electron out of the valance band. The electron becomes a free electron and leaves a hole in the valance band, creating an electron-hole pair. This free electron can be easily accelerated by the electric field existing naturally at the PN-junction towards the N-type and hole towards the P-type. In this way, electrons accumulated in the N-type, creating negative charges and holes accumulated in the P-type, creating positive charges. Thus an electric potential is developed between N-type and P-type contacts.

Practically, a single solar cell can produce a potential difference of 0.6V. To produce higher voltage, multiple solar cells are connected in series called solar panels as shown in Fig.18.13(b). Solar panels are generally available in 12V, 24V, 36V and 48V.

A solar cell technology is a wonderful development in the field of alternative energy sources. In the present age, its application can be observed in every walk of life, but some of these are given by:

INTERESTING INFORMATION

Noor Solar Power Plant is the world's largest solar power plant which consist of 7400 solar panels and it occupy 25000 hectares land. It is located in the Sahara Desert. The project has a 580-megawatt capacity and is expected to provide electricity for over 1 million people.

I. It is being used in the remote areas where access to the main power grid is impossible.
II. It is used to power the solar energy driven vehicles, such as: car, auto etc.
III. It is being used in boats, submarines etc. as a source of electricity.
IV. It is used in spacecraft, artificial satellite and space station to provide electricity.

Example 18.6

Electrons with a maximum kinetic energy of 3eV are ejected from a metal surface by ultraviolet light of wavelength 150nm. Determine the work function of the metal, the threshold wavelength of the metal and the negative potential required to stop the emission of electrons.

Solution:

$$\text{K.E of electrons} = 3\text{eV} = 3\left(1.6\times10^{-19}\,\text{J}\right) \qquad \therefore 1\text{eV} = 1.6\times10^{-19}\,\text{J}$$

$$= 4.8\times10^{-19}\,\text{J}$$

$$\text{Wavelength } (\lambda) = 150\text{nm} = 1.5\times10^{-7}\,\text{m}$$

$$\text{Speed of light} = c = 3\times10^{8}\,\text{m s}^{-1}$$

$$\text{Planck's constant} = h = 6.63\times10^{-34}\,\text{J s}$$

(a) Work function $(\phi) = ?$

(b) Threshold wavelength $(\lambda_0) = ?$

(c) Stopping potential $(V_0) = ?$

According to the equation of photoelectric effect

$$hf = \phi + \frac{1}{2}mv^2$$

$$\phi = \frac{hc}{\lambda} - \frac{1}{2}mv^2$$

$$\phi = \frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,ms^{-1}}{1.5\times10^{-7}\,m} - 4.8\times10^{-19}\,J$$

$$\phi = 13.26\times10^{-19} - 4.8\times10^{-19}$$

$$\phi = 8.46\times10^{-19}J = 5.28eV$$

By the definition of work function

$$\phi = hf_0 = \frac{hc}{\lambda_o}$$

$$\lambda_o = \frac{hc}{\phi} = \frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,ms^{-1}}{8.46\times10^{-19}\,J}$$

$$\lambda_o = 2.35\times10^{-7}m = 235nm$$

According to the basic equation of photoelectric effect:

$$\frac{1}{2}mv^2 = eV_o$$

$$V_o = \frac{\frac{1}{2}mv^2}{e} = \frac{4.8\times10^{-19}\,J}{1.6\times10^{-19}\,C}$$

$$V_o = 3 \text{ volts}$$

18.10 PARTICLE (PHOTON) MODEL OF LIGHT

In quantum theory, we have studied that light consists of a small packet of energy. This packet of energy was named as 'photon' by Einstein in 1905. According to Einstein, a photon behaves as a particle, it travels with speed of light 'c' and it has both energy as well as momentum. Thus Einstein defined light in terms of photon which is called 'photon (particle) theory of light'. The particle nature of light has been observed in the Compton effect and practically it has been proved in Davisson and Germer experiment.

Salient features of photon

i. A photon behaves as a particle whose rest mass is zero and it travels with speed of light 3×10^8 m s^{-1}. In other words, a photon exists as long as it is moving. It ceases to exist when it comes to rest.

ii. Photons are electrically neutral and are not deflected in the presence of electric and magnetic fields.

iii. The energy of a photon is given as:

$$E = hf = \frac{hc}{\lambda} \quad \therefore f\lambda = c$$

This shows that the energy of photon depends upon frequency. (or wavelength)

iv. Momentum of photon is given as:

As $$E = hf = mc^2$$

So $$mc = \frac{hf}{c}$$

$$p = \frac{hf}{c}$$

v. Rest mass of a photon can be calculated by using Einstein's mass variation equation:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_0 = m\sqrt{1-\frac{v^2}{c^2}}$$

Since photon is moving with speed of light 'c' so $v = c$

$$m_0 = 0$$

This shows that rest mass of a photon is zero.

18.11 COMPTON EFFECT

The experiment by Arthur H. Compton is another justification of the particle theory of light. The experimental setup consists of a beam of x-rays of wavelength λ_0 which is incident on a block of graphite. This incident x-ray photon is scattered at some angle from initial direction. Compton observed that the wavelength 'λ' of the scattering x-ray is slightly longer than the wavelength of the incident x-ray. It means that the energy of the scattering x-rays is also lower than the energy of the incident x-ray photon. The increase in wavelength

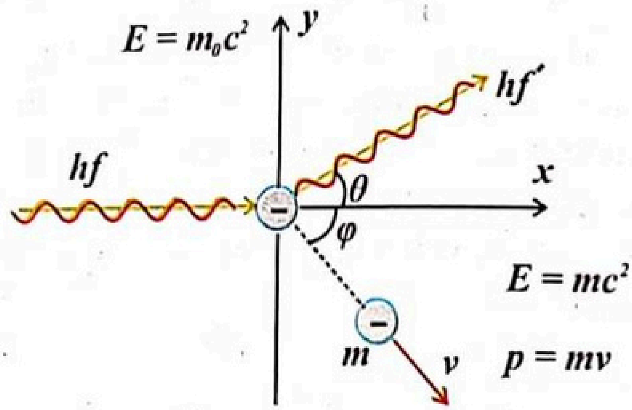


Fig.18.14. A schematic diagram for Compton effect where the elastic collision occurs between a x-ray photon and an electron at rest.

or decrease in energy of x-rays when they interact with matter or are scattered is known as Compton effect. The change in wavelength $\Delta\lambda$ between them is termed as Compton's shift.

Based on quantum theory, Compton assumed that x-rays consist of photons and these photons behave like particles. Now the collision of a single photon with an electron at rest is like a collision between two billiard balls. As x-ray photon carries both energy (hf) and momentum ($\frac{hf}{c}$). Hence during each interaction incident X-ray photon must transfers some of its energy and momentum to the electron. So, these two quantities must be conserved in an elastic collision between photon and electron at rest, where photon transfers some of its energy and momentum to the electron.

The schematic diagram of the elastic collision between a x-ray photon and an electron is shown in Fig.18.14. If 'θ' and 'ϕ' be the scattering angles of photon and electron respectively after collision then according to law of conservation of energy.

energy of a system before collision = energy of a system after collision

$$(E)_{Photon} + (E)_e = (E)_{Photon} + (E)_e$$

$$hf + m_0c^2 = hf' + mc^2$$

$$hf - hf' = (m - m_0)c^2 \quad ......(18.14)$$

Similarly, according to law conservation of momentum along x-axis

$$\frac{hf}{c} + 0 = \frac{hf'}{c}\cos\theta + mv\cos\phi \quad ......(18.15)$$

Conservation of momentum along y-axis

$$0 + 0 = \frac{hf'}{c}\sin\theta - mv\sin\phi \quad ......(18.16)$$

By simplifying Eq.18.14, Eq.18.15 and Eq.18.16 we get the following result:

$$\frac{1}{f'} - \frac{1}{f} = \frac{h}{m_0c^2}(1 - \cos\theta) \quad ......(18.17)$$

This equation shows that the photon after collision scattered at angel 'θ' with respect to its incident direction. If it is not scattered, then the value of 'θ' would be zero and hence the value of right hand side of Eq.18.17 would be zero. But angle θ has some finite value. Therefore, right hand side of Eq.18.17 is greater than zero. i.e.,

$$\frac{1}{f'} - \frac{1}{f} > 0$$

$$f > f'$$

This condition is known as Compton Effect.

POINT TO PONDER

Why does a photon that has been scattered from an electron, initially at rest have a longer wavelength than the incident photon?

Compton shift in wavelength

As
$$c = f\lambda$$

$$\frac{1}{f} = \frac{\lambda}{c}$$

and
$$\frac{1}{f'} = \frac{\lambda'}{c}$$

Therefore, Eq.18.17 becomes

$$\frac{\lambda'}{c} - \frac{\lambda}{c} = \frac{h}{m_0c^2}(1-\cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{h}{m_0c}(1-\cos\theta) \quad \text{......(18.18)}$$

This is known as Compton shift in wavelength and its value depends upon the scattering angle of photon.

Example 18.7

A photon with a wavelength 0.4nm strikes with an electron having rest mass 9.11×10^{-31}kg. If the x-rays are scattered at angle of 45° after the collision, then what is the wavelength of the scattering x-rays?

Solution:

Wavelength of incident photon = $\lambda = 0.4$nm $= 0.4 \times 10^{-9}$m

Angle $\theta = 45^0$

Mass of electron $= 9.11 \times 10^{-31}$kg

Wavelength scattered photon or x-rays $= \lambda' = ?$

Planck's constant $= h = 6.63 \times 10^{-34}$ Js

speed of light $= c = 3 \times 10^8 ms^{-1}$

According to Compton shift in wavelength

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_0c}(1-\cos\theta)$$

$$\lambda' = 0.4\times10^{-9}\,m + \frac{6.63\times10^{-34}\,Js}{9.11\times10^{-31}\,kg\times3\times10^{8}\,ms^{-1}}(1-\cos45^\circ)$$

$$\lambda' = 0.4\times10^{-9} + 0.243\times10^{-11}(1-0.707)$$

$$\lambda' = 0.4\times10^{-9} + 0.243\times10^{-11}(0.293)$$

$$\lambda' = 0.4\times10^{-9} + 0.07\times10^{-11}$$

$$\lambda' = 0.4\times10^{-9} + 0.0007\times10^{-9}$$

$$\lambda' = 0.4007\times10^{-9}\,m$$

$$\lambda' = 0.4007\ nm$$

18.12 PAIR PRODUCTION

Pair production implies the creation of an elementary particle and its antiparticle. **A process in which a high energetic gamma ray photon disappears by producing a pair i.e., an electron and a positron when it passes close to a heavy nucleus.** A positron is the antiparticle of an electron. It has same mass as that of an electron but carries opposite charge i.e. +e. In pair production energy, momentum and even charges are conserved.

Pair production process confirms the particle theory of electromagnetic radiations, because, it is a direct conversion of radiant energy into matter. Hence this process is also known as materialization of energy.

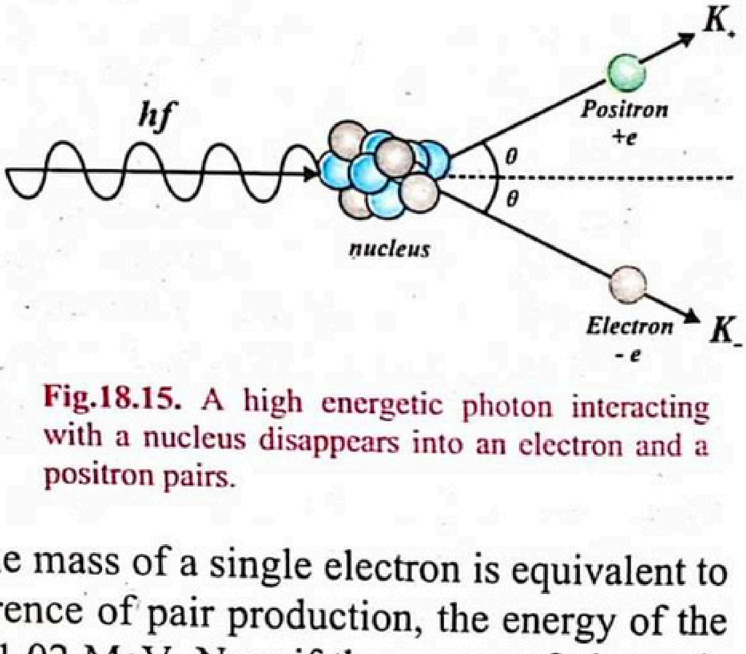


Fig.18.15. A high energetic photon interacting with a nucleus disappears into an electron and a positron pairs.

The schematic diagram of pair production is shown in Fig.18.15. The incident photon's energy hf must be equal or greater than rest mass energy of two electrons. According to Einstein's Energy-mass equation $E = mc^2$, the mass of a single electron is equivalent to 0.51 MeV of energy. Thus, for the occurrence of pair production, the energy of the incident photon must be at least equal to 1.02 MeV. Now if the energy of photon is greater than 1.02 MeV, called threshold, 1.02 MeV energy is used for pair production and the surplus energy would appear as kinetic energies of an electron and a positron. On the basis of this principle, we can develop a mathematical relation for a pair production as

Energy of incident photon = required energy for pair production + $K.E_{-e}$ + $K.E_{+e}$

$$hf = 2m_0c^2 + K.E_{-e} + K.E_{+e} \quad \ldots\ldots(18.18)$$

This is the basic equation for pair production.

Annihilation of matter

A process in which a particle and its antiparticle disappear during their interaction, as a result two gamma ray photons are produced is known as Annihilation of matter. It is the inverse process of pair production. For instance, when an electron and a positron are combined, they annihilate each other and give rise to two gamma ray photons as shown in Fig.18.16, which are moving in opposite directions. The energy of each gamma-ray photon is 0.51 MeV which is equal to the rest mass energy of electron or positron i.e., m_oc^2. Like in pair production, the energy and momentum are also conserved in annihilation of matter.

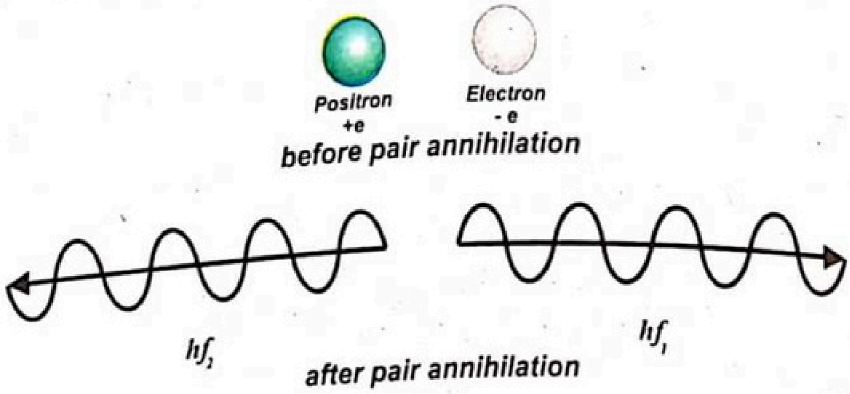


Fig.18.16. Combination of electron and positron produced two gamma ray protons in the process of annihilation of matter.

Besides of electron and positron, the process of annihilation of other anti-particles can also take place. For example, annihilation of proton and antiproton, lepton and antilepton, quark and antiquark and so on.

18.13 WAVE NATURE OF PARTICLES

It has been verified experimentally that light has dual nature or characteristics. As the results of some experiments reveal that it behaves as a particle while some other experiments reveal the wave nature of light. It must be noted that both aspects of nature of light cannot observed at the same time. Photoelectric effect, Compton effect and pair production verify the particle nature of electromagnetic radiations.

Subsequent to the confirmation of dual nature of light, Louis de Broglie in 1924 proposed that just as light has both wave-like and particle-like properties, electrons also have wave-like properties, i.e., any kind of matter has wave like properties. De Broglie proposed that 'a wave is always associated with every particle of matter of mass m moving with velocity 'v'. In this connection he proposed a mathematical relation for the photon, which is expressed as;

$$\text{Momentum of particle} = mv \quad ......(18.19)$$

$$\text{Momentum of photon} = \frac{hf}{c}$$

But $\quad c = f\lambda$

And $$\frac{1}{\lambda} = \frac{f}{c}$$

$\therefore$ Momentum of photon $= \frac{h}{\lambda}$ ...(18.20)

$\therefore$ Comparing Eq.18.19 and Eq.18.20

$$mv = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv} \quad ...(18.21)$$

> ***POINT TO PONDER***
> If an electron and a proton have the same de-Broglie wavelength. Which particle has higher speed?

This is known as de-Broglie wave equation and it shows that, if a particle of mass 'm' moving with velocity v then it has wavelength λ associated with particle. This equation illustrates that wave like properties cannot be observed for too heavy particles. If the mass of an object is very small, like sub-atomic particles such as, electron and is moving until high velocity then the wave like properties can be detected experimentally.

Davisson and Germer experiment

According to De-Broglie's hypothesis and his equation, the mass of an electron is small enough to exhibit the wave like properties, i.e., the electrons which are particles have also wave nature. This idea was confirmed experimentally in 1927 by Davisson and Germer, when they observed that scattering of electrons from crystals which act like a three-dimensional diffractions grating and shows a diffraction pattern. Their experimental setup consists of the electrons from a source (Filament)

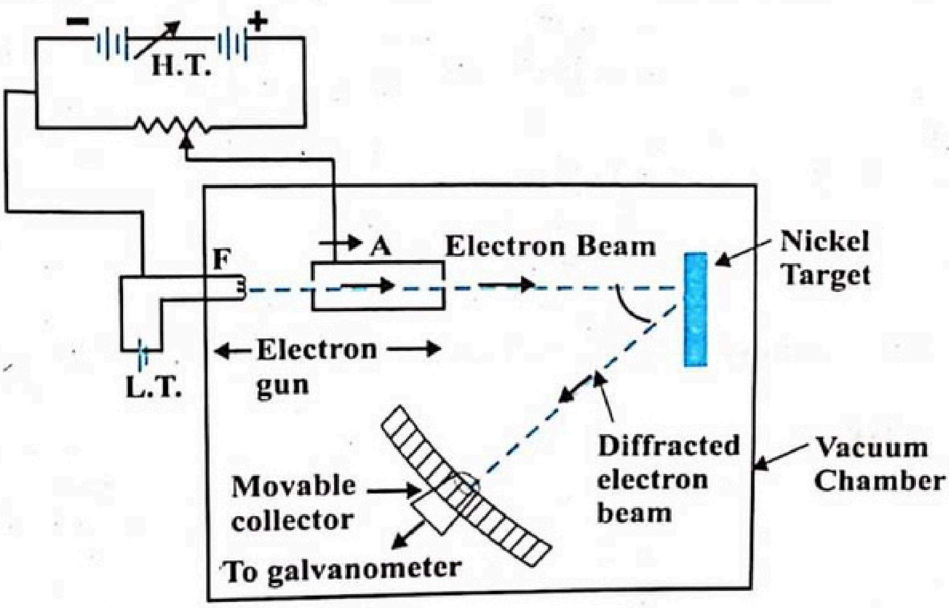


Fig.18.17. The experimental arrangement of Davisson and Germer experiment.

which are accelerated by a potential-difference V. A beam of these accelerated electrons emerges from the anode 'A' and is allowed to fall on a target which is a nickel crystal as shown in Fig.18.17. The incident electrons are scattered in different direction which are detected by a detector which moves along a circular scale. It is also observed that the intensity of scattered electrons at different angles is different and as a result, we have a diffraction pattern as shown in Fig.18.18.

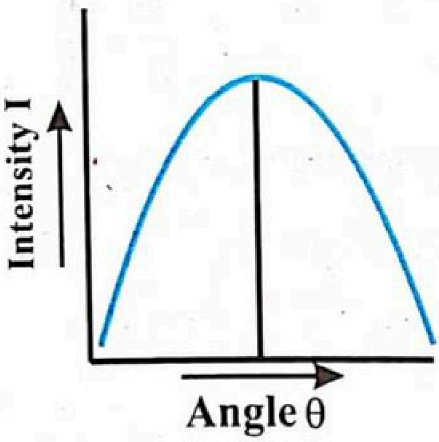


Fig.18.18. The intensity of the diffracted electrons at different angles.

The result proved that the reflected electrons have been diffracted by the crystal planes. Thus, the electrons which are particles demonstrates wave nature i.e., due to their diffraction from the crystal. The dual nature of electrons can be further verified mathematically, by using the experimental and theoretical data. The analysis show that the electrons are accelerated at 54eV and the intensity of the first order (m = 1) of the diffraction pattern is maximum at angle 50°, the spacing (d) between two adjacent planes of the Nickel crystal is 0.9×10^{-10} m. Using the data in the following two relations.

Wave nature

According to equation of diffraction from a surface of a crystal (Bragg's law)

$$m\lambda = 2d\sin\theta$$

$$(1)(\lambda) = 2(0.91\times10^{-10}\text{m})\sin(90^\circ - 25^\circ)$$

$$\lambda = 1.82\sin 65$$

$$\lambda = 1.65\times10^{-10}\text{m} \quad \ldots\ldots(18.22)$$

Particle nature

According to de-Broglie wave equation

$$\lambda = \frac{h}{mv}$$

According to the equation of photoelectric effect

$$\frac{1}{2}mv^2 = eV_o$$

$$v = \sqrt{\frac{2eV_o}{m}}$$

$$\therefore \quad \lambda = \frac{h}{m\sqrt{\frac{2eV_o}{m}}} = \frac{h}{\sqrt{2eV_o m}}$$

$$\lambda = \frac{6.63\times10^{-34}\,Js}{\sqrt{2(1.6\times10^{-19}\,C)(54V)(9.1\times10^{-31}\,kg)}}$$

$$\lambda = 1.66\times10^{-10}\,m \ldots\ldots(18.23)$$

These two results given by Eq.18.22 and 18.23 have verified the De-Broglie hypothesis.

Example 18.8

Calculate the de-Broglie wavelength of an electron that has been accelerated through a potential difference of 9kV.

Solution:

de-Broglie wavelength $(\lambda) = ?$

Mass of an electron $(m) = 9.1 \times 10^{-31}$Kg

Charge on an electron $(e) = 1.6 \times 10^{-19}$C

Potential difference $(V_o) = 9kV = 9 \times 10^3$ V

Planck's constant $(h) = 6.63 \times 10^{-34}$J s

According to de-Broglie wave equation

$$\lambda = \frac{h}{mv}$$

But $\frac{1}{2}mv^2 = eV_o$

$$v = \sqrt{\frac{2eV_0}{m}}$$

$$\therefore \quad \lambda = \frac{h}{m\sqrt{\frac{2eV_o}{m}}}$$

$$\lambda = \frac{h}{\sqrt{2meV_0}}$$

$$\lambda = \frac{6.63\times10^{-34}\,Js}{\sqrt{2(9.1\times10^{-31}\,kg)(1.6\times10^{-19}\,C)(9\times10^{3}\,V)}}$$

$$\lambda = \frac{6.625\times10^{-34}}{5.12\times10^{-47}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{5.12 \times 10^{-23}}$$

$$\lambda = 1.3 \times 10^{-11} m$$

18.14 ELECTRON MICROSCOPE

An electron microscope is useful device to obtain high resolution images of extremely small biological and non-biological specimens. It uses a beam of accelerated electrons as a source of illumination instead of light, i.e., it relies on the wave nature of electrons. In electron microscope, the electrons are accelerated by applying high potential difference from 30kV to several mega volts. Such high voltage produces a high kinetic energy beam of electron of shorter wavelength. Typically, the wavelength of electrons is about 100 times smaller than that of the visible light. Due to shorter wavelength of electrons, the resolving power and magnifying power of electron microscope is about one thousand times that of the optical microscope. The beam of electrons is controlled by applied electric and magnetic fields, the electrons diverging from a small region are brought to convergence by these electric and magnetic fields.

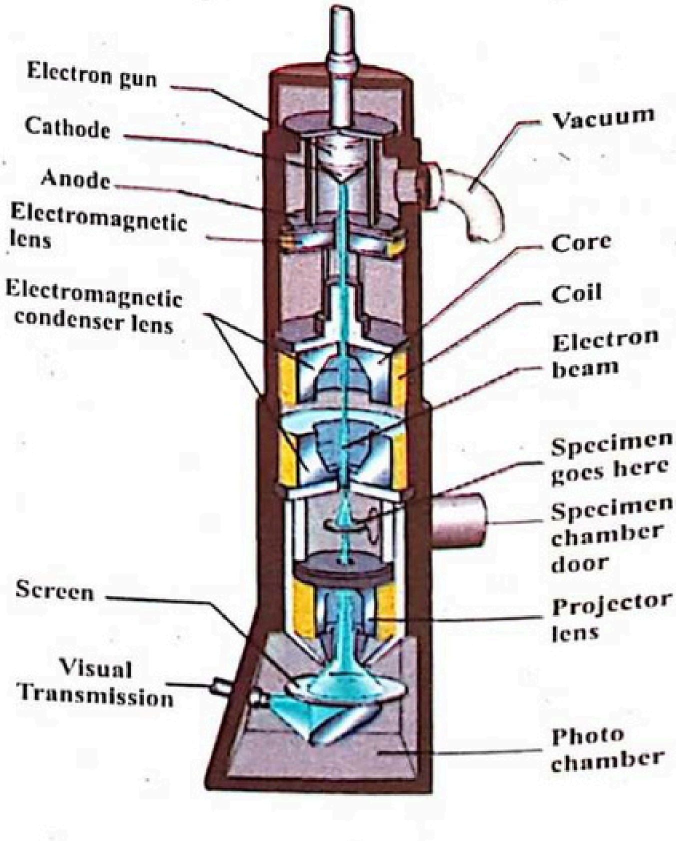


Fig.18.19. A schematic diagram of an Electron microscope

A schematic diagram of electron microscope is shown in Fig.18.19. The beam of electrons is usually focused by magnetic conducting lens and has an energy of 50-100 keV. It is directed onto the whole area of the sample under investigation and the electrons emerging are focused by second magnetic conducting lens onto a florescent screen. The screen must be florescent, otherwise, the obtained image would not be visible.

There are two main types of electron microscope the scanning electron microscope (SEM) and transmission electron microscope (TEM).

18.15 UNCERTAINTY PRINCIPLE

The fact mentioned by Werner Heisenberg in 1927 and it is stated that, **'it is impossible to know both the exact position and exact momentum of an object at**

the same time accurately'. This is known as Heisenberg's uncertainty principle and it is further explained as under:

To locate the position, speed and energy of a particles we must look at it, using a beam of light. Since light has also wave nature, so we should determine the position of the particle only within one wavelength of the used/applied light. Similarly, to reduce the uncertainty in the position of the particle, we must use the light of shorter wavelength because the light of shorter wavelength increases the accuracy of the position by its large resolution. If 'λ' be the wavelength of light which is being used to locate the particle moving along x-axis, then the uncertainty in the position measurement is given by

$$\Delta x \approx \lambda \qquad ......(18.23)$$

On the other hand, if the wavelength of the light is shorter then it would disturb more the momentum of the particle, as a result there is more uncertainty in the measurement of momentum of the particle. In order to reduce uncertainty in its momentum, we should use light of longer wavelength. If the photon of the applied light is transferred its momentum $\left(p = \frac{\lambda}{h}\right)$ to the particle then its momentum would be changed. Thus, the uncertainty in the momentum of the particle Δp is given by

$$\Delta p \approx \frac{h}{\lambda} \quad ......(18.24)$$

These two relations (18.23) and (18.24) show that if we use the light of shorter wavelength then the accuracy in the measurement of position will be increased and its uncertainty in momentum will be decreased.

Likewise, by using light of long wavelength, the accuracy in the measurement of momentum will be increased while its uncertainty in position will be decreased. A general relation can be obtained by multiplying these two uncertainties i.e.,

$$(\Delta x)(\Delta p) = h \quad ......(18.25)$$

This is the mathematical form of Heisenberg's uncertainty principle. It shows that the product of uncertainty in the simultaneous determination of Δx. and Δp is equal to the Planck's constant 'h'. Planck's constant h is so small that the limitations imposed by the uncertainty principle are significant only in the realm of the atom. Hence Heisenberg's uncertainty principle provides a useful tool, not just a negative statement.

In the same way, the uncertainty principle can also be expressed in terms of uncertainty in the simultaneous determination of energy (ΔE) and time (Δt) as;

$$\Delta E \, \Delta t = h \qquad ......(18.26)$$

The quantity $h/2\pi$ appears often in modern physics because it turns out to be the basic unit of angular momentum and it is equal to '$\hbar$' which reduces Planck's constant. Therefore, in terms of '$\hbar$' eq. 18.25 and eq. 18.26 cannot hold more equality. Thus, these two equations now can be expressed as:

$$\Delta x \Delta P > \hbar \qquad ......(18.27)$$

$$\Delta E \Delta t > \hbar \qquad ......(18.28)$$

Example 18.9

An electron is found in a sphere of size 1.25×10^{-10}m which is the order of size of the atom. Estimate the velocity of the electron in the sphere along x-rays.

Solution:

Position of the electron = diameter of the sphere

$$\Delta x = 1.25 \times 10^{-10} m$$

Mass of electron = 9.11×10^{-31}kg

Planck's constant = h = 6.63×10^{-34}J-s

Velocity of electron = ?

According to uncertainty principle

$$\Delta p \Delta x = h$$

$$\Delta p = \frac{h}{\Delta x}$$

$$\Delta p = \frac{6.63 \times 10^{-34} Js}{1.25 \times 10^{-10} m}$$

$$\Delta p = 5.30 \times 10^{-24} kg\ ms^{-1}$$

But

$$\Delta P_x = mv_x$$

$$V_x = \frac{\Delta p_x}{m}$$

$$V_x = \frac{5.30 \times 10^{-24} kgms^{-1}}{9.11 \times 10^{-31} kg}$$

$$V_x = 5.82 \times 10^6 ms^{-1}$$

Example 18.10

The life-time of an electron in an excited state is measured to be 5×10^{-7}s to an accuracy of 0.003%. Find the minimum uncertainty in determining the energy in this time.

Solution:

Lifetime of electron in excited state= 5×10^{-7}s

The uncertainty in time t is 0.003% = $\Delta t = 5 \times 10^{-7} s \times 0.003\%$

$$\Delta t = 5\times10^{-7}s\times3\times10^{-5}$$
$$\Delta t = 1.5\times10^{-11}s$$
$$\text{Planck's constant} = h = 6.63\times10^{-34}\,Js$$

To calculate the uncertainty in the determination of energy, we use the uncertainty principle

$$\Delta E\cdot\Delta t = h$$
$$\Delta E = \frac{h}{\Delta t}$$
$$\Delta E = \frac{6.63\times10^{-34}\,Js}{1.5\times10^{-11}s}$$
$$\Delta E = 4.42\times10^{-23}J$$

SUMMARY

- **Frame of Reference:** The set of coordinate system with respect to which observations or measurement are made.
- **Inertial frame of reference:** A frame of reference which is either at rest or moving with uniform velocity, i.e., non-accelerated frame of reference.
- **Non-inertial frame of reference:** It is an accelerated frame of reference.
- **Special Theory of Relativity:** Special theory of relativity is based upon the following postulates.
    - All the laws of physics are same applicable in all inertial frame of reference.
    - Speed of light is a universal constant.
- **Consequences of special theory of relativity:** Special theory of relativity has some important consequences such as; length contraction, time dilation, mass variation and energy-mass relation ($E = mc^2$).
- **Blackbody and Blackbody Radiation:** A perfect blackbody is one which absorbs radiations of all wavelength incident on it. When blackbody is heated then it emits all the radiation known as black body radiations.
- **Stefan Boltzmann's Law:** The energy per second per unit area is directly proportional to the fourth power of absolute temperature.
- **Wien's Law:** The wavelength having maximum intensity in the emitted radiation spectrum is inversely proportional to the temperature.
- **Max Planck's Law:** Energy exchange takes place in the form of energy packet called quanta and the energy of each quanta is directly proportional to its frequency ($E = hf$).

- **Photo electric effect:** When light of suitable frequency falls on a metal surface then emission of electrons from the metal surface take place. These electrons are called photo electrons and this phenomenon is known as photo electric effect.
- **Photocell:** It is a device which converts the light energy into the electrical energy.
- **Solar Cell:** It is a device which converts sun's light energy into electrical energy.
- **Photon:** It is a small packet of light energy and it behaves as a particle moving with speed of light.
- **Compton Effect:** A.H. Compton studied the scattering of x-ray photon from electrons in a carbon target. He observed that x-ray photons scattered by the target have a longer wavelength than the wavelength of incident photons. The increase in scattered x-ray photon wavelength resulting from the transfer of energy is known as Compton effect.
- **Pair Production:** A process in which a high energy gamma ray photon is converted into a pair of electron and a positron is called pair production.
- **Annihilation of matter:** A process in which particle moving in opposite direction and its anti-particle disappear releasing energy in the form of two γ-rays photon is known as annihilation of matter.
- **De-Broglie wave equation:** According to de-Broglie's postulate when a particle is moving with velocity 'v' then it has some wavelength 'λ' associated with it.

  i.e., $\lambda = \frac{h}{mv}$.
- **Electron microscope:** It is a device which has much higher magnification and resolution: power than an optical microscope. An electron microscope is a device to obtain high resolution: and magnification of extremely small specimens. It uses a beam of accelerated electrons as a source of illumination instead of light.
- **Uncertainty Principle:** It is impossible to determine simultaneously position and momentum of a particle with perfect accuracy.

EXERCISE

O Select the appropriate option of the following questions.

1. Inertial frame of reference is one which satisfies

(a) Newton's Theory (b) Einstein Theory

(c) Hertz Theory (d) Special theory of relativity

2. Non-inertial frame of reference has

(a) Zero acceleration (b) Zero velocity

(c) Uniform velocity (d) Variable velocity

3. If the source of light is moving towards the observer, then the speed of light received by the observer will be
(a) Decreased (b) Increased (c) Remain same (d) Maximum
4. The relativistic length of an moving object will be
(a) Remain same (b) Decreased (c) Increased (d) Doubled
5. If the rest mass of a particle is zero, then its speed is
(a) Equal to speed light (b) Less than speed of light
(c) Greater than speed of light (d) Not comparable with speed of light
6. Blackbody radiation depends upon
(a) Pressure (b) Volume (c) Temperature (d) Density
7. If the temperature of black body is doubled then the emitted energy from it will be increased
(a) Doubled (b) Four time (c) Eight time (d) Sixteen time
8. The dimension of a Planck's constant is
(a) $[MLT^{-1}]$ (b) $[ML^2T^{-1}]$ (c) $[ML^2T^{-2}]$ (d) $[ML^2T^2]$
9. Who did observe 1st time the photoelectric effect?
(a) Maxwell (b) Hertz (c) Einstein (d) Heisenberg
10. Photo electric effect depends upon the photon's
(a) Pressure (b) Temperature (c) Intensity (d) Frequency
11. A photon can transfer its energy into an electron, it was first explained by
(a) Maxwell (b) Hertz (c) Einstein (d) Bohr
12. The momentum of a photon is
(a) mv (b) mc (c) hf (d) $\frac{hf}{c}$
13. A change in energy of a photon occurs when it collides with an electron at rest is known as
(a) Photoelectric effect (b) Compton effect
(c) Pair production (d) Annihilation
14. Which phenomenon does not verify the particle nature of light
(a) Photoelectric effect (b) Compton effect
(c) Pair production (d) diffraction
15. The antiparticle of electron is
(a) Neutron (b) Proton (c) Photon (d) Positron
16. Davisson and Germer proved experimentally the wave nature of particle under the phenomenon of
(a) Reflection (b) Refraction (c) Interference (d) Diffraction
17. In the electron positron pair production, the speed of electron is
(a) Zero (b) Less than speed of positron
(c) Equals to speed of positron (d) Greater than speed of positron

18. If the energy of the used light is high, then the momentum of the investigated particle has
(a) Less uncertainty (b) High uncertainty
(c) High accuracy (d) Equal uncertainty

19. For small uncertainty in the measurement of position of a particle, the wavelength of the incident light should be
(a) Small (b) Large
(c) Average (d) Does not depend of wavelength

SHORT QUESTIONS

1. How inertial frame of reference is different from non-inertial frame of reference?
2. State the postulates of special theory of relativity.
3. Mention the important results of special theory of relativity.
4. Under what condition, a particle can move with a speed of light?
5. Give the formula to convert the mass of a particle into energy.
6. Does a perfect black body exist? If yes, then give an example.
7. How can you construct a blackbody?
8. How did Max Planck solve the dilemma of distribution of energy by blackbody?
9. How can you calculate the K.E of photoelectrons?
10. What is the difference between work function and threshold energy?
11. Name the metals which emit the photoelectrons for visible light.
12. What do you know about the Einstein's equation for photo electric effect?
13. What is the function of photocell?
14. How does Compton effect verify the wave theory of light?
15. What is meant by the Compton shift in wavelength?
16. What should be the minimum value of energy of photon to induce the pair production?
17. What is the difference between pair production and annihilation of matter?
18. What do you know about the de-Broglie wavelength?
19. At what angle the intensity of diffraction electrons is maximum?
20. Why position and momentum of an electron cannot be measured simultaneously with perfect accuracy?

COMPREHENSIVE QUESTIONS

1. What do you know about the relative motion? Explain the relative motion with examples.
2. Distinguish between inertial and non-inertial frame of reference.
3. State and explain special theory of relativity with its consequences.

4. What is blackbody radiation? Explain the distribution of energy from the black body under various laws.
5. Define photoelectric effect and derive its fundamental equation.
6. State and explain Einstein's equation of photoelectric effect and different laws of photoelectric effect.
7. What do you know about the Compton effect? How does it verify the particle nature of light by Compton theory.
8. Describe pair production and annihilation of matter.
9. What is the de-Broglie hypothesis? How such hypothesis was verified experimentally by Davisson and Germer.
10. State and explain electron microscope with its function and its working principle.
11. What do you know about Heisenberg uncertainty principle? Express uncertainty principle under two mathematical relations.

NUMERICAL PROBLEMS

1. How fast a rocket has to go for its length to be contracted to 80% of its rest length? **(1.8×10^8 m s^{-1})**
2. The period of a second pendulum is measured to be 2s in an inertial frame of reference of the pendulum. What is its period measured by an observer moving with a speed of 0.9c with respect to the pendulum's frame of reference? **(4.6s)**
3. Calculate the variation in the mass of a moving object with a speed of 0.85c. **($1.9m_0$)**
4. What is the energy of a photon of a blue light of wavelength 450nm (in joules and in eV). **(4.41×10^{-19}J, 2.76eV)**
5. Calculate the wavelength of light in which the photons have an energy of 650eV. **(1.9nm)**
6. Determine the maximum K.E of photoelectrons ejected from a potassium surface by ultraviolet radiation of wavelength 200nm. If the work function of the potassium surface is 2.8eV, calculate the stopping potential. **(3.4eV, 3.4V)**
7. With what speed will the fastest photoelectrons be emitted from a surface whose work function is 2eV, when the surface is illuminated with a light of wavelength 4×10^{-7}m? **(6×10^5 ms^{-1})**
8. Consider a photon that scatters from an electron at rest. If the Compton's wavelength shift is observed to be triple the wavelength of the incident photon and if the photon scatters at 60^0 then calculate the wavelength of the incident photon. **(4.05×10^{-13}m)**
9. Determine the de-Broglie wavelength of an electron accelerated from rest through a potential difference of 3.3kV volt. **(0.38 nm)**

10. An electron-positron pair, each with a K.E of 220KeV is produced by a photon. Calculate the energy and wavelength of the photon. **(1.46 MeV, 8.5×10^{-13}m)**

11. The speed of an electron is measured to be $4 \times 10^4 ms^{-1}$ to an accuracy of 0.002%. Find the minimum uncertainty in determining the position of this electron. **(0.92mm)**