Unit 15 ALTERNATING CURRENT

Major Concepts (27 PERIODS)

- Alternating current (A.C.)
- Instantaneous, peak and rms values of A.C.
- Phase, phase lag and phase lead in A.C.
- A.C. through a resistor
- A.C. through a capacitor
- A.C. through an inductor
- Impedance
- RC series circuit
- RL series circuit
- Power in A.C. circuits
- Resonant circuits
- Electrocardiography
- Principle of metal detectors
- Maxwell's equations and electromagnetic waves (descriptive treatment)

Conceptual Linkage

This chapter is built on Electricity Physics X ICT Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

- describe the terms time period, frequency, instantaneous peak value and root mean square value of an alternating current and voltage.
- represent a sinusoidally alternating current or voltage by an equation of the form $x = x_0 \sin (t$.
- describe the phase of A.C. and how phase lags and leads in A.C. Circuits.
- identify inductors as important components of A.C. circuits termed as chokes (devices which present a high resistance to alternating current).
- explain the flow of A.C. through resistors, capacitors and inductors.
- apply the knowledge to calculate the reactances of capacitors and inductors.
- describe impedance as vector summation of resistances and reactances.
- construct phasor diagrams and carry out calculations on circuits including resistive and reactive components in series.
- solve the problems using the formulae of A.C. Power.

- explain resonance in an A.C. circuit and carry out calculations using the resonant frequency formulae.
- describe that maximum power is transferred when the impedances of source and load match to each other.
- describe the qualitative treatment of Maxwell's equations and production of electromagnetic waves.
- become familiar with electromagnetic spectrum (ranging from radiowaves to (γ rays).
- identify that light is a part of a continuous spectrum of electromagnetic waves all of which travel in vacuum with same speed.
- describe that the information can be transmitted by radiowaves.
- identify that the microwaves of a certain frequency cause heating when absorbed by water and cause burns when absorbed by body tissues.
- describe that ultra violet radiation can be produced by special lamps and that prolonged exposure to the Sun may cause skin cancer from ultra violet radiation.

INTRODUCTION

We have studied in the previous unit that an A.C. generator produces a current or voltage which varies periodically with time and is known as alternating current or alternating voltage. The alternating current has more advantages over the direct current. For example, it can be easily transformed into higher or lower voltage, it can be transmitted over long distances, reliable and can be produced at very low cost. Due to these advantages, the A.C. sources are used to power the circuits of our homes, offices, markets, industries, farms etc. In this unit, we will study the behavior of A.C. circuit, when a resistor, capacitor or an inductor is connected with the A.C. source. We will also study the combined effect of resistor, capacitor and inductor when they are connected either in series or parallel with the A.C. source.

On the other hand, we will discuss the generation, transmission and reception of electromagnetic waves. The electromagnetic waves are composed of fluctuating electric and magnetic fields. They have different forms which are being used for different purposes. For example, electromagnetic waves in the form of visible light enable us to view the world around us, infrared waves warm our environment, radio waves connect all the countries of the world through communication system in terms of video and audio signals, x-rays allow us to perceive not only the structures hidden inside our bodies, but also explore the structure of various elements. Similarly, these electromagnetic waves help us in the observation and study of solar system, stars, galaxy and other heavenly bodies.

15.1 ALTERNATING CURRENT (A.C.)

We have studied that when a rectangular coil is rotated in a uniform magnetic field with a constant angular velocity 'ω', an e.m.f. is induced in the coil. Consequently, there is a flow of current in the output of the coil. As the rotation of the coil is uniform so, its output current varies periodically both in positive and negative directions after an equal interval of time. This current is called alternating current (A.C.). Graphically, the wave shape of A.C. is shown in Fig.15.1(a). Mathematically, this alternating current can be expressed in terms of sinusoidal wave because of the continuous variation in its magnitude and direction with respect to time. That is,

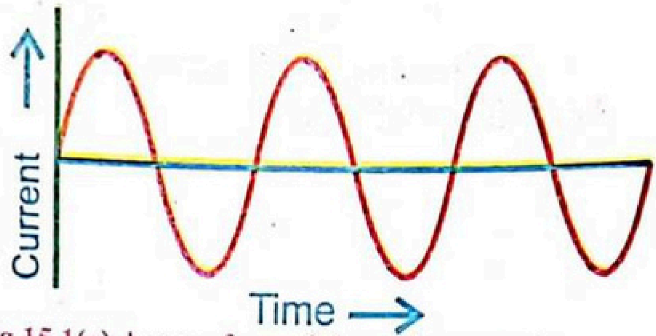


Fig.15.1(a) A wave form of alternating current

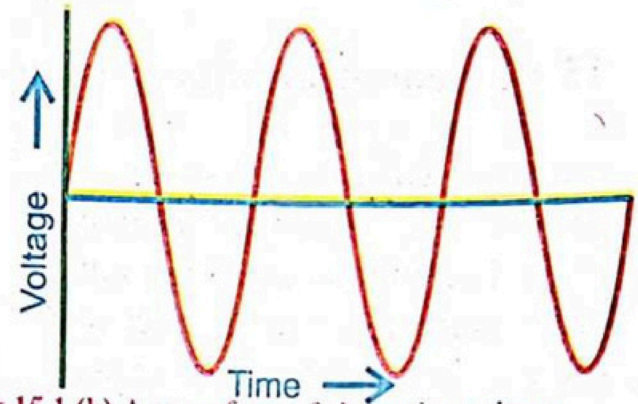


Fig.15.1 (b) A wave form of alternating voltage

POINT TO PONDER
Why 220V of A.C. is more dangerous than 220V of D.C.?

$$i = I_o \sin \omega t$$

But $\omega = 2\pi f$

$$i = I_o \sin 2\pi ft \quad ......(15.1)$$

where 'f' is the frequency of the alternating current and it is related with the number of rotations of the coil. Thus, the frequency of A.C. is defined as the number of wave cycles of A.C. in one second. The reciprocal of the frequency is termed as the time period of A.C.. i.e., $T = \frac{1}{f}$. This is a time in which one cycle of A.C. is completed. Similarly, 'i' is the instantaneous value of current of A.C. at any instant of time 't' and 'I_o' is the maximum current of A.C.. 'I_o' is also known as either positive or negative peak value of current. Similarly, the alternating voltage can also be expressed as,

$$v = V_o \sin \omega t \quad ......(15.2)$$

Graphical representation of alternating voltage is shown in Fig.15.1(b).

Example 15.1

An alternating current of frequency 50Hz has peak value of 70A. Calculate the instantaneous value of current after 0.0015s.

Solution:

$$f = 50Hz$$

peak current(I_0) = 70A

t = 0.0015s

instantaneous current(i) = ?

According to the Eq.15.1

> **DO YOU KNOW**
> An oscilloscope is very versatile piece of equipment which is being used to display A.C. wave form, heartbeats

$$i = I_0 \sin 2\pi f t$$
$$i = 70A \sin 2(3.14)(50Hz)(0.0015s)$$
$$i = 70 \sin (0.471)$$
$$i = 70(0.00822)$$
$$i = 0.58A$$

15.1.1 Root mean square value

One cycle of alternating current or voltage consists of half positive cycle and half negative cycle. Therefore, the average value of the current or voltage over one cycle is zero and it cannot be used to specify an alternating current or voltage. To overcome this we use a power, because power is expressed in terms of i^2 and v^2. Where the magnitude of i^2 and v^2 are always positive even for their negative values. Graphically the square value of square of alternating current is shown in Fig.15.2. Where i^2varies from 0 to I_o^2 and its average value is given as;

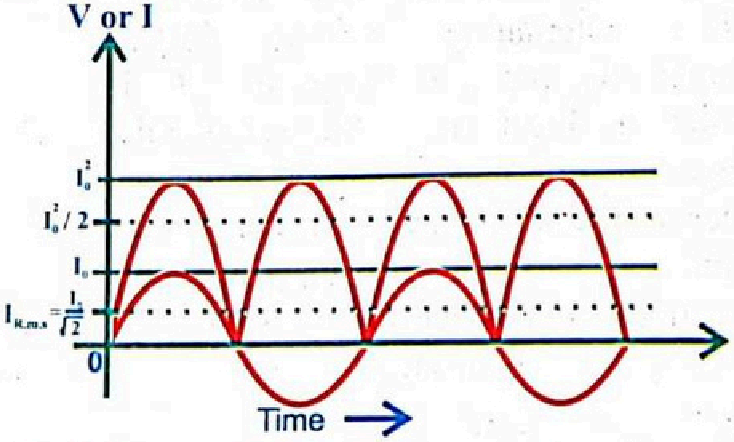


Fig.15.2 Square of the alternating current which varies with time from 0 to I_0^2 and its magnitude remains positive for complete cycle.

$$<i^2> = \frac{0 + I_o^2}{2}$$
$$<i^2> = \frac{1}{2} I_o^2 \quad ......(15.3)$$

> **DO YOU KNOW**
> The average value of an alternating current is zero but its square root of the average square value is not zero.

This result shows that the average value of A.C. is never zero. Similarly, the average of square of alternating voltage is given by;

$$<v^2> = \frac{1}{2} V_o^2 \quad ......(15.4)$$

In case of A.C. source, average power dissipation in a resistor is given by

$$<p> = <i^2> R \quad ......15.5(a)$$

Similarly, power dissipation in terms of voltage is

$$<p> = \frac{<v^2>}{R} \quad ......15.5(b)$$

Now in case of D.C. source, the power dissipation is given by;

$$P = I^2R \quad ......15.6(a)$$

and $$P = \frac{V^2}{R} \quad ......15.6(b)$$

The value of the direct current that dissipates in the same resistor or produces heat at the same rate as the mean rate of heat produces by the alternating current is known as root mean square value of A.C.. It is represented by $I_{r.m.s}$ and it can be calculated by comparing Eq.15.5(a) with Eq.15.6(a)

$$I^2 = <i^2>$$

Taking square root on both sides

$$I_{r.m.s.} = \sqrt{<i^2>}$$

$$I_{r.m.s.} = \sqrt{\frac{1}{2}I_o^2} = \frac{I_0}{\sqrt{2}} = 0.707\ I_o \quad ......(15.7)$$

Similarly, $$V_{r.m.s.} = \sqrt{\frac{1}{2}V_o^2} = \frac{V_0}{\sqrt{2}} = 0.707V_o \quad ......(15.8)$$

The root mean square value of alternating current and root mean square value of alternating voltage are also known as effective current and effective voltage.

Example 15.2

The instantaneous value of current is represented by the equation $i = 25\sin 100\pi t$. Compute its frequency, maximum and rms values of current.

Solution:

We have

$$i = 25\sin 100\pi t$$

According to the Eq.15.1

$$i = I_o \sin 2\pi f\, t$$

By comparing these two equations, we get:

$$2f = 100\text{Hz}$$

$$f = 50\text{Hz}$$

$$I_o = 25\text{A}$$

$$I_{rms} = 0.707I_o$$

$$I_{rms} = (0.707)(25)$$

$$I_{rms} = 17.7\text{A}$$

15.1.2 Phase of A.C.

We have studied that there is a continuous periodic variation of A.C. with time. Therefore, its instantaneous value at time 't' is given by:

$$i = I_o \sin \omega t \;......15.9(a)$$

$$i = I_o \sin \theta \;......15.9(b)$$

where 'θ' is the angle which specifies the instantaneous value of alternating current or voltage and it is known as phase angle or simply phase, and it depends upon time t. The instantaneous value of A.C. with respect to phase angle can further be studied graphically as well. At time t = 0, i.e., $\omega t = \theta = 0$ then current 'i' is also zero. The value of angle θ at t = 0 is known as initial phase of A.C. The instantaneous value of current is also zero when angle θ has value of π, 2π, 3π, 4π,... these are shown in Fig.15.3(a). Similarly, the instantaneous value of current 'i' is maximum positive called peak values when phase angle 'θ' is $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$ ...... and the current 'i' maximum negative when phase angle 'θ' is $\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$, $\frac{15\pi}{2}$ .... These values of current are also shown in Fig.15.3(a). The same wave shape as for current will be obtained for instantaneous value of voltage with respect to phase angle 'θ'. It is shown in Fig.15.3(b).

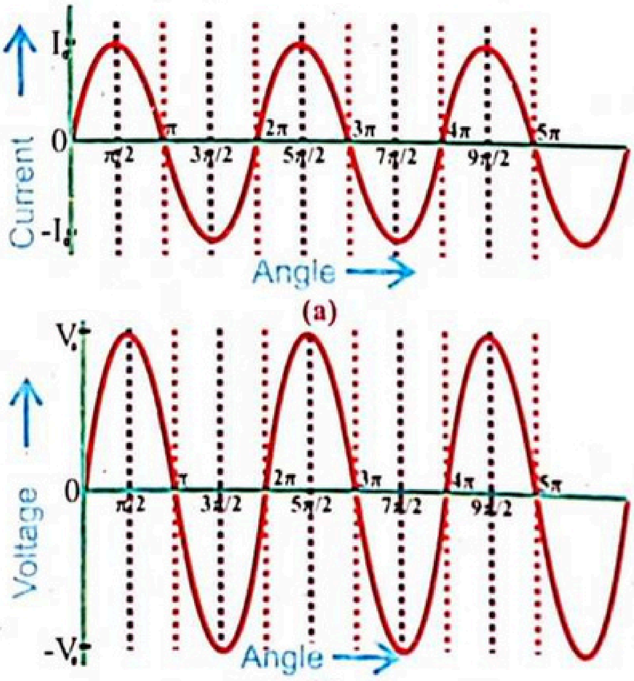


Fig.15.3(a) A graphical representation of instantaneous current with respect to phase angle 'θ' (b) A graphical representation of instantaneous voltage with respect to phase angle θ

15.1.3 Phase lag and phase lead

We know that when a coil (armature) of A.C. generator is rotated in a uniform magnetic field, then we have an induced emf (voltage) as well as a current in form of sinusoidal waves because

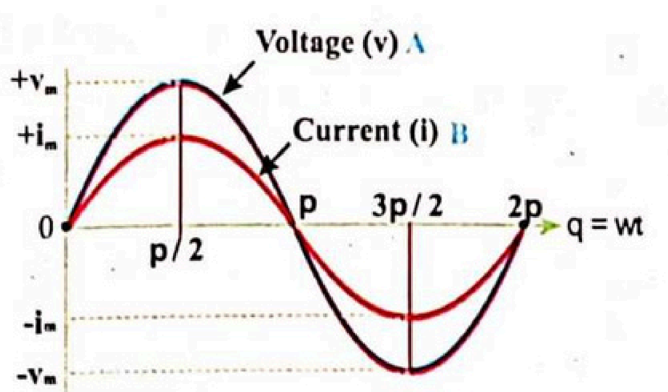


Fig.15.4 Graphical representation of voltage and current which are in phase.

both quantities are varying sinusoidally with the same angular frequency (ω). Now when both quantities reach their minimum (zero) and maximum values at the same time or the values of their initial phase are 0° at time, t = 0 then voltage and current are said to be in phase with each other. Graphically they are shown in Fig. 15.4.

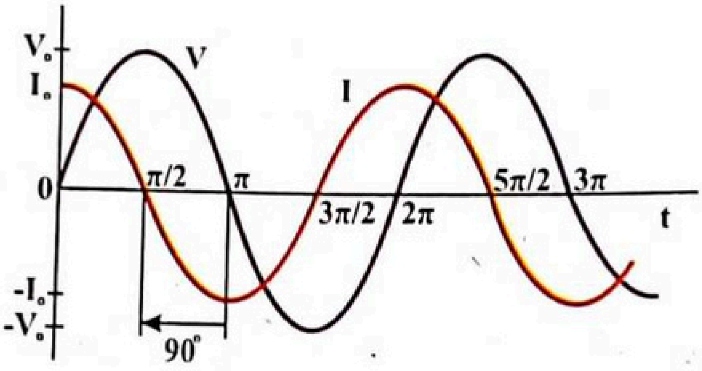


Fig.15.5 Current is leading the voltage by an angle 'e' (e = 90°)

Sometime, both quantities i.e. voltage and current do not reach their minimum and maximum values simultaneously. For example, if at time, t = 0 the value of initial phase of one quantity is 0° whereas, the initial phase of the other quantity greater than 0° then this shows that voltage and current are out of phase. In case of out of phase there are two possibilities.

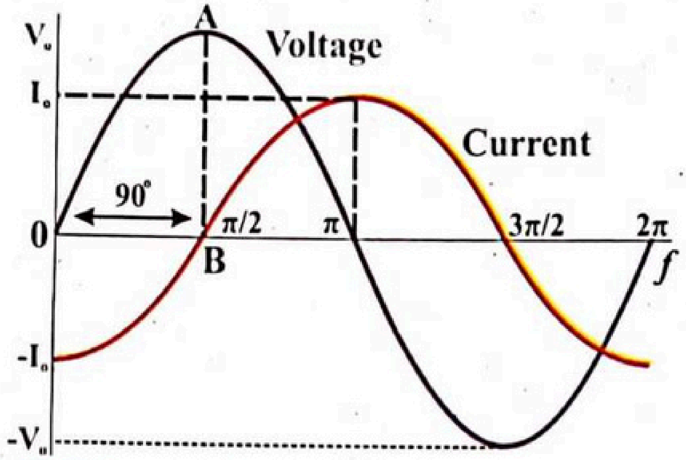


Fig.15.6 Current is lagging behind the voltage by an angle 'e' (e = 90°)

I. Let the initial phase of v is 0° but the initial phase of current i does not 0° and it had a phase of 0° earlier then it is called leading of current. Graphically, the leading of current by voltage by an angle 90° is shown in Fig.15.5.

II. Similarly, if the initial phase of voltage is 0° but current will has its phase of 0° later then it is called lagging of current. Graphically, the lagging of current behind voltage by an angle 90° is shown in Fig.15.6.

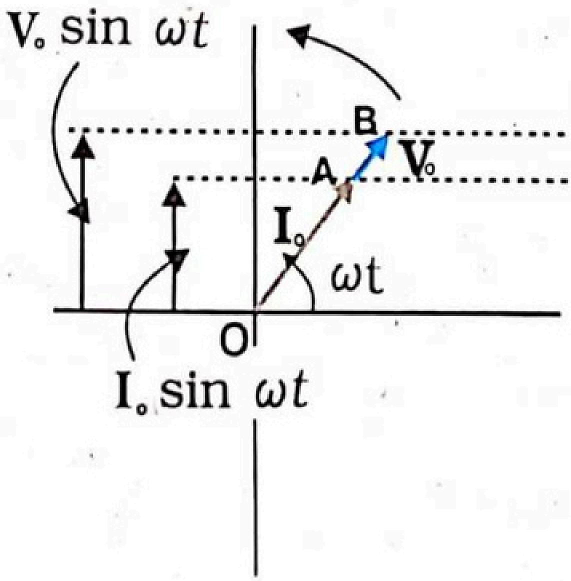


Fig.15.7 Phasor diagram for current and voltage.

The alternating current and voltage that vary sinusoidally can further be explained by a phasor diagram. A phasor is a vector which rotates about the origin with angular frequency ω. The diagram which contains such rotating vector is called phasor diagram. Let alternating current and voltage are represented by two phasors OA and OB as shown in Fig.15.7. The length of these phasors are equal to the peak value of current (I_0) and voltage (V_0) respectively. While, the projection of these phasors onto the vertical axis are equal to instantaneous value of current ($I_0 \sin\omega t$) and voltage ($V_0 \sin\omega t$) respectively. Current and voltage in terms of phasor diagram can be studied under three cases.

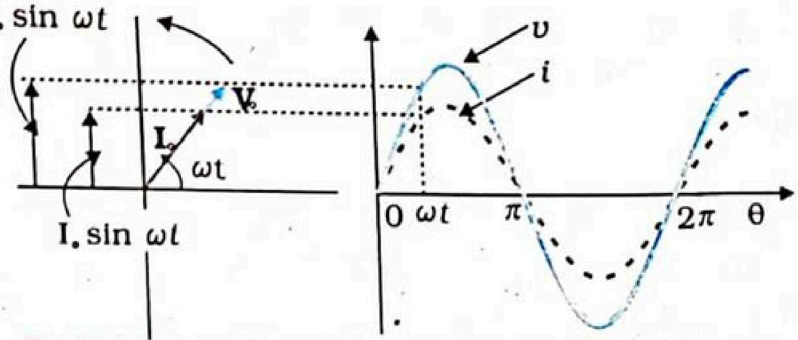


Fig.15.8 Phasor diagram for current and voltage which are in phase.

a. If voltage and current are in phase then both have same initial phase ($\theta = \omega t$) as shown in Fig.15.8. Mathematically, they are expressed as:

$$i = I_0 \sin\omega t \qquad ......(15.10)(a)$$

$$v = V_0 \sin\omega t \qquad ......(15.10)(b)$$

If $\omega t = \theta = 0$ then $i = 0$ and $v = 0$.

Similarly, if $\omega t = \theta = \frac{\pi}{2}$

then $i = I_0$ and $v = V_0$.

These results show that v and i are in phase.

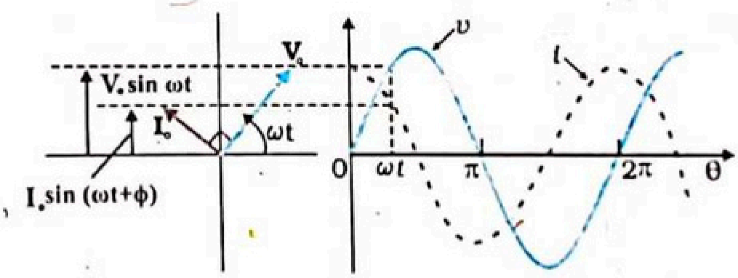


Fig.15.9 Phasor diagram for voltage and current where current is leading by voltage by an angle 'e' (e = 90°)

b. If the initial phase of voltage is 0° at time, $t = 0$ but initial phase of current is positive at the same time then mathematically they are expressed as,

$$v = V_0 \sin\omega t \qquad ...15.11(a)$$

$$i = I_0 \sin(\omega t + \phi) ...15.11(b)$$

At time t = 0, the value of voltage 'v' is zero but current 'i' has some positive value equal to '$I_0 \sin\phi$'. This shows that 'v' and 'i' differ by phase angle ϕ. Because i had its zero value earlier by an angle 'ϕ' of 90° then 'v' as shown in Fig.15.9. Thus the current is leading the voltage by an angle ϕ of 90°.

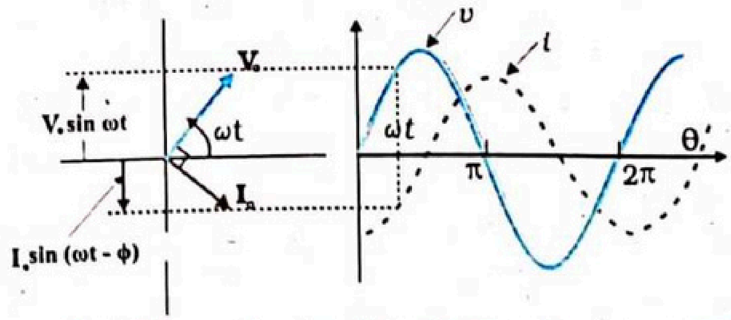


Fig.15.10 Phasor diagram for voltage and current where current is lagging behind voltage by an angle 'e' (e = 90°).

c. If initial phase of voltage is zero at time, $t = 0$ but initial phase of current is negative at the same time then mathematically they are expressed as;

$$v = V_o \sin \omega t \quad \ldots 15.12(a)$$

$$i = I_o \sin(\omega t - \phi) \ldots 15.12(b)$$

at time $t = 0$, the value of 'v' is zero but current i has some negative value equals to '$-I_o \sin\phi$'. This shows that 'v' & 'i' are not in phase, where current 'i' is lagging behind the voltage by an angle ϕ of 90° as show in Fig.15.10.

15.2 A.C. CIRCUIT

An A.C.-circuit is an electrical network which is powered by an A.C. source and the elements such as resistors, capacitors and inductors are connected in series or parallel across it. We will study the behaviour of alternating current and voltage in each element.

15.3 A.C. THROUGH A RESISTOR

When a resistor of resistance 'R' is connected to an A.C. source as shown in Fig.15.11 then there is voltage drop across it. The instantaneous value of voltage is given by

$$v = V_o \sin \omega t \ldots\ldots (15.13)$$

According to Ohm's law

$$v = iR$$

$$i = \frac{v}{R}$$

$$i = \frac{V_o}{R} \sin \omega t$$

$$i = I_o \sin \omega t \ldots\ldots (15.14)$$

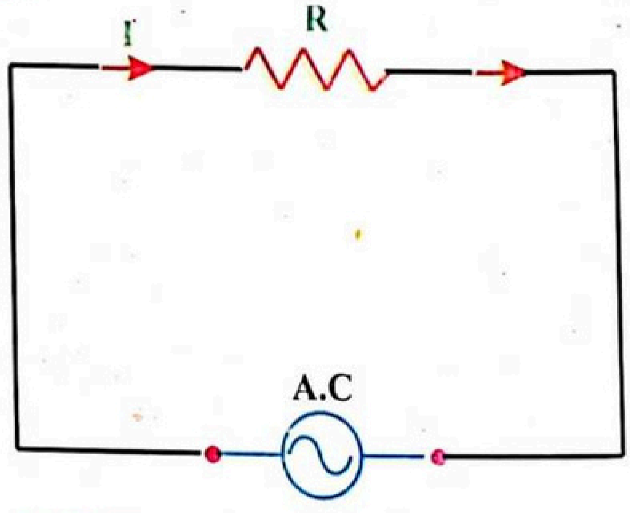


Fig.15.11 A resistor R in an A.C. circuit.

Eq.15.13 and Eq.15.14 show that the voltage and current are in phase with each other as shown in Fig.15.12(a). i.e., If $\theta = 0$ then $I = 0$ and $v = 0$.

Similarly, if $\theta = \frac{\pi}{2}$ then $i = I_o$ and $v = V_o$

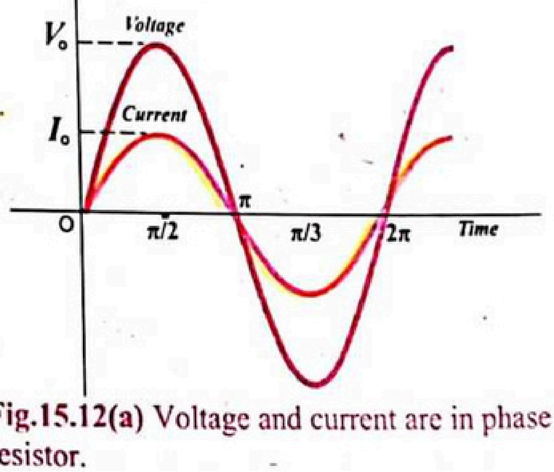


Fig.15.12(a) Voltage and current are in phase in a resistor.

These results show that when a resistor is connected with A.C. source then v and i are in phase and it is shown in phasor diagram 15.12(b).

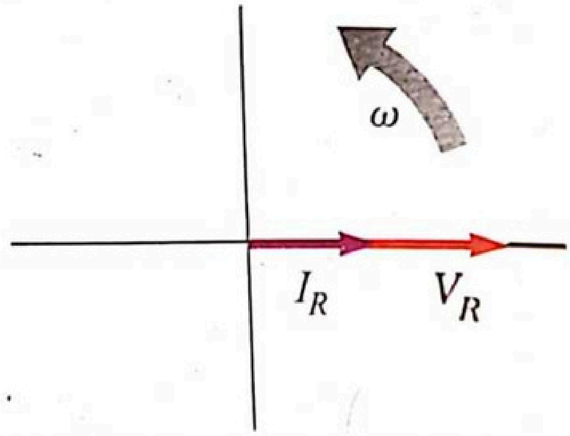


Fig.15.12(b) Phasor diagram which shows v and i are in phase.

Power dissipation

The instantaneous value of power in an A.C. circuit which contains a resistor only is given by

$$p = vi$$

$$p = V_o I_o \sin^2 \omega t \quad ......(15.15)$$

In case of alternating current or alternating voltage, we use average dissipation by taking limits from 0° to $\frac{\pi}{2}$.

If $\omega t = \theta = 0$

Then
$$p = V_o I_o \sin^2 0$$
$$p = 0$$

> ***FOR YOUR INFORMATION***
> In an A.C. circuit, power is always measured in terms of rms values of current and voltage.

Similarly, if $\omega t = \theta = \frac{\pi}{2}$

$$p = V_o I_o \sin^2 \frac{\pi}{2}$$

$$p = V_o I_o$$

Thus, the average power is given by

$$< p > = \frac{0 + V_o I_o}{2}$$

$$< p > = \frac{V_o I_o}{2}$$

$$< p > = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}}$$

$$< p > = V_{r.m.s} I_{r.m.s} \quad ......(15.16)$$

This shows that the average power dissipation in a pure resistive circuit is equal to the product of r.m.s. value of voltage and current.

Example 15.2

A voltage $V = 60\sin\omega t$ is applied across a resistor of 20Ω in A.C. circuit. Calculate the root mean square value of current in the circuit?

Solution:

$$V_o = 60\text{volt}$$

$$I_{r.m.s.} = ?$$

As

$$V_{r.m.s.} = \frac{V_0}{\sqrt{2}} = \frac{60}{\sqrt{2}}$$

$$V_{r.m.s.} = 42.4V$$

$$V_{r.m.s.} = I_{r.m.s.}R$$

$$I_{r.m.s.} = \frac{V_{r.m.s}}{R}$$

$$I_{r.m.s.} = \frac{42.4V}{20\Omega}$$

$$I_{r.m.s.} = 2.12A$$

15.4 A.C. THROUGH A CAPACITOR

Let a capacitor of capacitance 'C' is connected to A.C. source as shown in Fig.15.13. There is alternating voltage drop across the capacitor and the instantaneous value of this voltage is given by

$$v = V_o \sin\omega t \quad ......(15.17)$$

The amount of charge stored in the capacitor due to instantaneous voltage is given by

$$q = Cv$$

$$q = CV_0 \sin\omega t$$

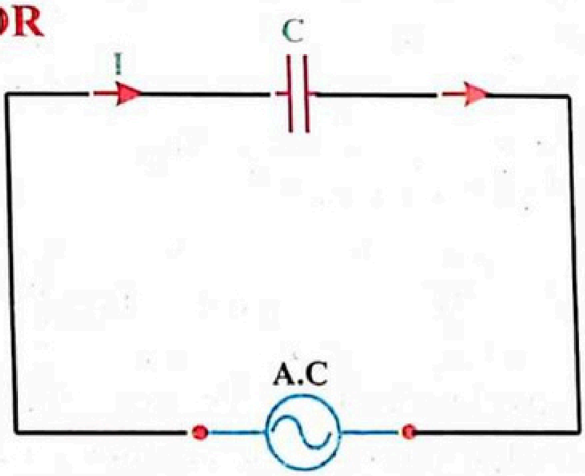


Fig.15.13 A capacitor C in an A.C. circuit.

We know that current is defined as the rate of flow of charges, therefore instantaneous value of current through capacitor is given by

$$i = \frac{\Delta q}{\Delta t}$$

$$i = \frac{\Delta}{\Delta t}\left(CV_0 \sin\omega t\right)$$

$$i = CV_0 \frac{\Delta}{\Delta t}\sin\omega t$$

The mathematical solution of $\frac{\Delta}{\Delta t}(\sin\omega t)$ is $\omega\cos\omega t$. So,

$$i = \omega C V_0 \cos\omega t$$

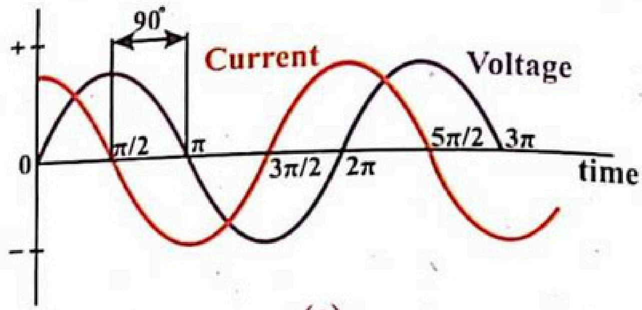
But $\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

And $\omega C V_0 = X_C V_0 = I_0$

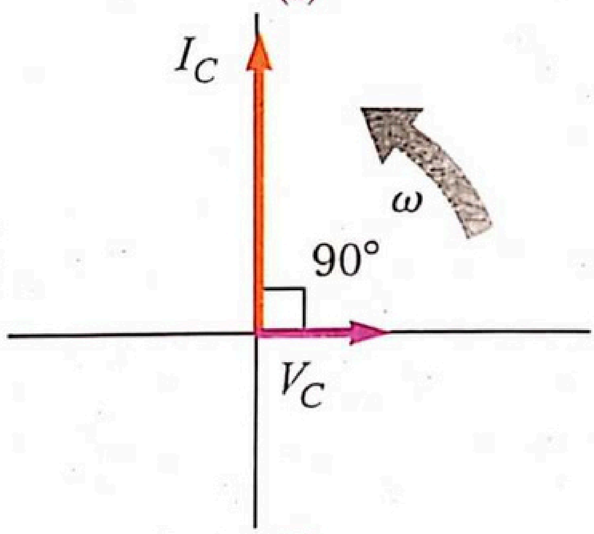
where X_C is the resistance (more precisely capacitive reactance) of capacitor. Thus,

$$i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad \ldots(15.18)$$

Eq.15.17 and Eq.15.18 give us that v and i are out of phase in A.C. circuit contains a capacitor as shown in Fig.15.14(a), that is, at $t = 0$, then $v = 0$ and $i = I_0$. Similarly, at $t = T/4$; $v = V_0$ and $i = 0$, and at $t = T$ $v = 0$ again and $i = I_0$. This shows that current is leading the voltage by 90°. It is also shown in the phasor diagram 15.14(b).



(a)



(b)

Fig.15.14(a) Current and voltage are out phase in a capacitor (b) current leading the voltage by 90°.

Capacitive reactance

The opposition offered by a capacitor to the flow of A.C. is known as capacitive reactance. It is defined in terms of the ratio of root mean square values of voltage V_{rms} to the root mean square values of current $I_{r.m.s.}$. It is represented by X_c and is measured in Ohm.

$$X_c = \frac{V_{rms}}{I_{rms}}$$

$$X_c = \frac{\left(V_0/\sqrt{2}\right)}{\left(I_0/\sqrt{2}\right)}$$

$$X_c = \frac{V_0}{I_0}$$

But $I_0 = \omega C V_0$

> **DO YOU KNOW**
>
> In an A.C. circuit, the capacitive reactance X_C of a capacitor is infinite ohms for D.C. At the opposite extreme, the X_C of a capacitor is zero ohms for A.C. Therefore, a capacitor allows (filters) A.C. through it but blocks D.C.

$$X_c = \frac{V_0}{\omega C V_0} = \frac{1}{\omega C}$$

$$X_c = \frac{1}{2\pi f C} \qquad \text{......(15.19)}$$

This shows that capacitive reactance is inversely proportional of both frequency of current and capacitance of the capacitor. In case of A.C., frequency f is large, while capacitive reactance X_C is small. In case of D.C., f is zero so X_C is infinity theoretically. In practice X_C has an extremely large value for D.C. source. It is concluded that a capacitor allows the A.C. and blocks the D.C. in an A.C. electric circuit.

Power dissipation

The instantaneous value of power dissipation in a capacitive circuit is given by

$$p = vi$$

$$p = V_o I_o \sin\omega t \cos\omega t$$

$$p = \frac{V_0 I_0}{2}(2\sin\omega t \cos\omega t)$$

$$= \frac{V_0 I_0}{2}\sin 2\omega t$$

> **POINT TO PONDER**
> What are the reactances of capacitor and inductor, when A.C. source is applied across them?

If we integrate the above expression with respect to ωt between the limits 0 and 2π the final answer is zero.

Thus, the average power dissipation in a capacitive A.C. circuit over a complete cycle is zero.

Example 15.2

A capacitor of capacitance $100\mu F$ is connected to an alternating potential difference of 12volts and frequency 50Hz. Calculate the reactance of the capacitor and current in the circuit.

Solution:

$$C = 100\mu F = 100\times10^{-6} F = 1\times10^{-4} F$$

$$V_0 = 12V$$

$$f = 50Hz$$

$$X_C = ?$$

$$I = ?$$

As $$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2(3.14)(50Hz)(1\times10^{-4}F)}$$

$$X_C = \frac{1}{3.14\times10^{-2}}$$

$$X_C = 31.8\Omega$$

$$V = IX_C$$

$$I = \frac{V}{X_C}$$

$$I = \frac{12V}{31.8\Omega}$$

$$I = 0.377A$$

15.5 A.C. THROUGH AN INDUCTOR

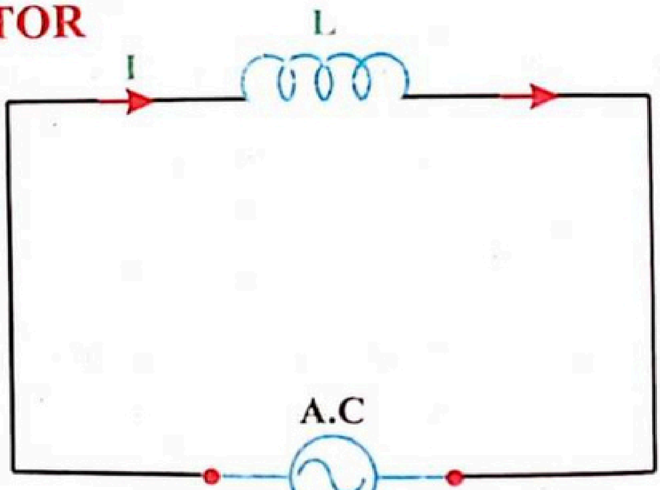


Fig.15.15 An inductor in A.C. circuit.

An inductor is usually a coil in the form of a solenoid of inductance 'L'. When it is connected to an A.C. source as shown in Fig.15.15, then there is a flow of alternating current through it and the instantaneous value of current is given by

$$i = I_0 \sin\omega t \;......(15.20)$$

The growth of this current causes an induced e.m.f. in the inductor whose direction is opposite to the applied e.m.f.. Thus, the magnitude of induced emf in the inductor is given by

$$v = L\frac{\Delta I}{\Delta t}$$

$$v = L\frac{\Delta}{\Delta t}I_0\sin\omega t$$

$$v = I_0 L\frac{\Delta}{\Delta t}\sin\omega t$$

> ***DO YOU KNOW***
> The inductive reactance X_L of an inductor is infinite ohms for A.C. Conversely, the reactance X_L of an inductor is zero ohms for A.C. Therefore, an inductor allows (filters) D.C. and blocks A.C.

The mathematical solution of $\frac{\Delta}{\Delta t}(\sin\omega t)$ is $\omega\cos\omega t$. So,

$$v = \omega I_0 L\cos\omega t$$

But $\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

and $\omega I_o L = I_o X_L = V_o$

Where X_L is the resistance of inductor

Thus, $v = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ ...(15.22)

Eq.15.21 and 15.22 show that current and voltage are out of phase in an A.C. circuit contains an inductor as shown in Fig.15.16(a). That is, at time, $t = 0$, $i = 0$ and $V = V_0$ and at time, $t = \frac{T}{4}$, $i = I_0$ and $V = 0$. The voltage is leading the current by 90° in an inductive A.C. circuit. It is shown in phasor diagram 15.16(b).

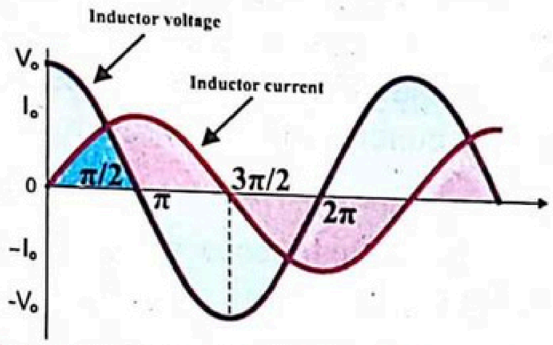


Fig.15.16(a) current and voltage are out of phase in an inductor

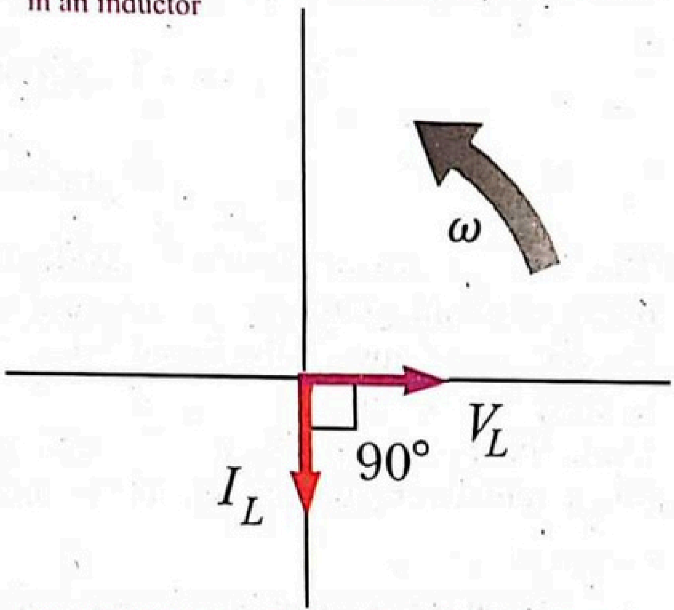


Fig.15.16(b) current lagging behind voltage by 90^0.

Inductive reactance

The opposition offered by an inductor to the flow of A.C. is known as inductive reactance. It is represented by X_L and it is measured in Ohm. The inductive reactance can be calculated by using Ohm's law.

$$X_L = \frac{V_{rms}}{I_{rms}}$$

$$X_L = \frac{\left(V_0/\sqrt{2}\right)}{\left(I_0/\sqrt{2}\right)}$$

$$X_L = \frac{V_o}{I_o}$$

But $$V_o = \omega L I_o$$

$$X_L = \frac{\omega L I_o}{I_o}$$

$$X_L = 2\pi f L \quad ......(15.23)$$

This shows that inductive reactance is directly proportional to both frequency of current and the inductance of the inductor. In case of D.C., $f = 0$, so that $X_L = 0$, while in case of A.C., frequency is large, so X_L is also large.
Thus, we conclude that an inductor allows the D.C. but blocks the A.C..

Power dissipation

The instantaneous value of power dissipation in an inductive A.C. circuit is given by

$$p = vi$$
$$p = V_o I_o \sin\omega t \cos\omega t$$
$$p = \frac{V_0 I_0}{2} 2\sin\omega t \cos\omega t$$
$$= \frac{V_0 I_0}{2} \sin 2\omega t$$

The average power over one cycle can be obtained by integrating above expression with respect to ωt between the limits 0 and 2π and final answer is zero.
Thus, the power dissipation in an inductive A.C. circuit over a complete cycle is zero.

> ***POINT TO PONDER***
> What are the reactances of capacitor and inductor, when D.C. source is applied across them?

Example 15.3

What is the potential difference across an inductor of inductance 15H when an alternating current of 15mA, frequency 50Hz flows through it?

Solution:

$X_L = ?$

$L = 15H$

$I_0 = 15mA = 15 \times 10^{-3}A$

$f = 50Hz$

$V = i X_L$

$V = 2\pi f L i$

$V = 2(3.14)(50Hz)(15H)(15\times10^{-3}A)$

$V = 70.65V$

15.6 IMPEDANCE

We know that a resistor offers opposition to the flow of current in a circuit. On the other hand, a capacitor and an inductor in an A.C. circuit also offer some opposition to the flow of A.C. Their opposition is called reactances which are represented by X_C and X_L respectively. Now let an A.C. circuit consists of a resistor, a capacitor and an inductor in series as shown in Fig.15.17(a) then the combined effect of resistance of a resistor and reactances of a capacitor and an inductor is termed as impedance. It is represented by Z and it is measured in Ohm. Impedance can be determined in terms of ratio between voltage to current that is,

$$Z = \frac{v_z}{i_z} \quad ......(15.24)$$

Since all the three components are in series so the current i_z through each component is the same, but the voltage drop across each component is different. The resultant voltage v_z can be calculated with help of phasor diagram as shown in Fig.15.17(b) and Fig.15.17(c).

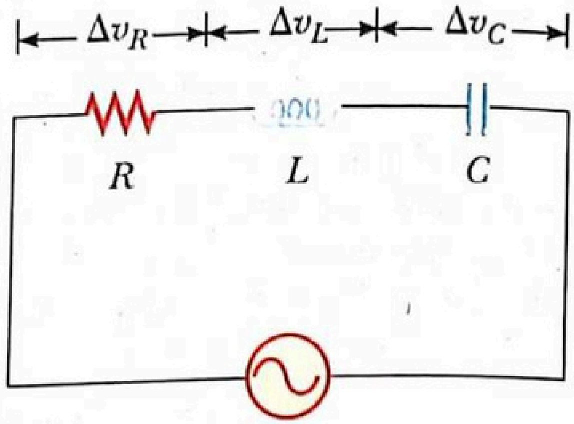
$$v^2 = v_R^2 + (v_L - v_C)^2$$

$$v = \sqrt{i_Z^2 R^2 + i_Z^2 (X_L - X_C)^2}$$

$$v = i_Z \sqrt{R^2 + (X_L + X_C)^2}$$

As $Z = \dfrac{v_Z}{i_Z}$

$$Z = \frac{i_Z \sqrt{R^2 + (X_L - X_C)^2}}{i_Z}$$



15.17(a) RLC-series A.C. circuit

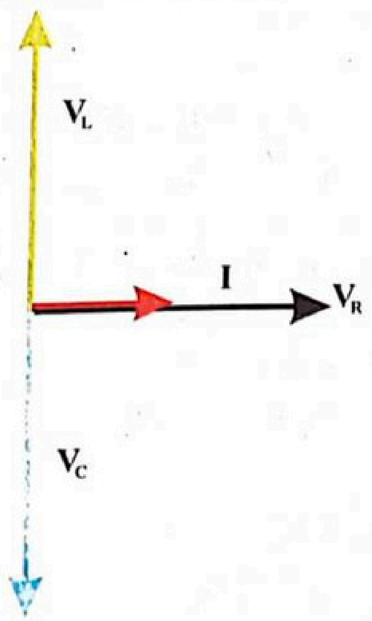


Fig.15.17(b) Phasor Diagram for RCL circuit

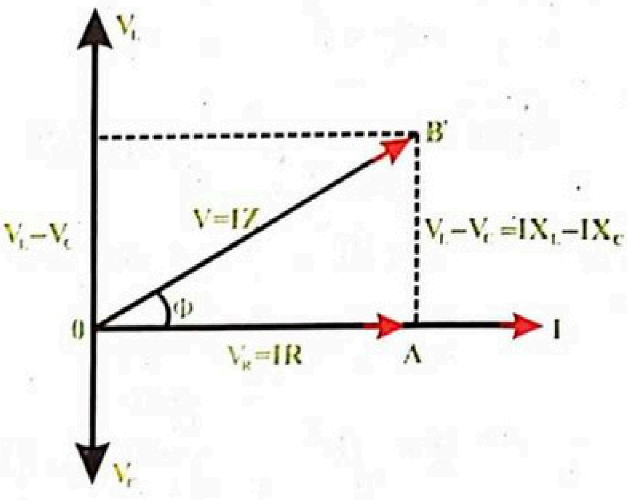


Fig.15.17(c) Resultant voltage in RCL circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{......(15.25)}$$

This is the impedance Z of RLC-series circuit.

15.7 A.C. THROUGH R-C SERIES CIRCUIT

Consider a resistor and a capacitor which are connected in series to an A.C. source as shown in Fig.15.18. As the resistor and the capacitor are in series so the current in the circuit is same at its each point and at any instant. Thus, the instantaneous value of the current is given by

$$i = I_o \sin \omega t \quad \text{......(15.26)}$$

Now the voltage can be calculated with the help of phasor diagram. As we know that the voltage drop 'v_R' across 'R' is iR and it is in phase with i. So the phasor for v_R is parallel to i as shown in Fig.15.19. On the other hand, voltage (V_C) across the capacitor is lagging behind the current by 90^0. Thus, the resultant potential difference 'v' across resistor 'R' and capacitor 'C' is equal to the vector sum of v_R and v_C.

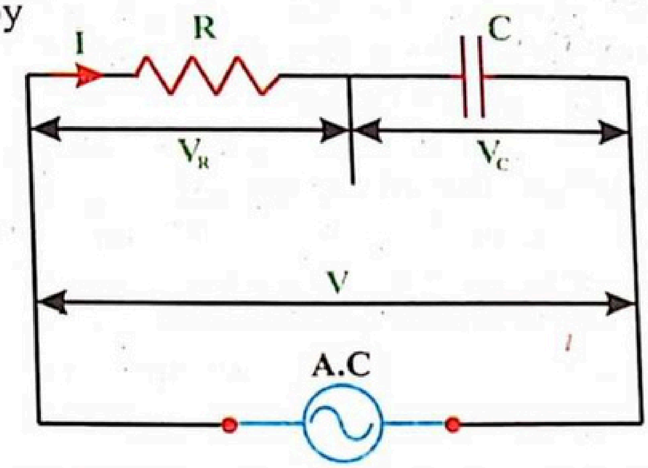


Fig.15.18 A resistor and capacitor in series in an A.C. circuit

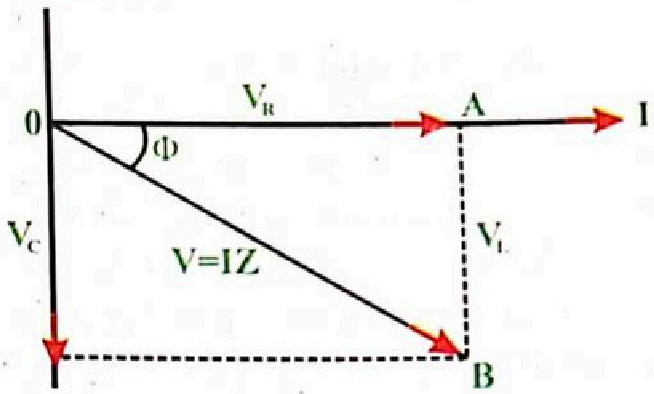


Fig.15.19 A phasor diagram for RC series circuit.

$$v^2 = v_R^2 + v_C^2$$

$$v = \sqrt{i^2R^2 + i^2X_C^2}$$

$$v = i\sqrt{R^2 + \frac{1}{(\omega C)^2}}$$

$$v = i\sqrt{R^2 + \frac{1}{(2\pi f C)^2}} \quad \text{......(15.27)}$$

The Impedance of RC-series circuit is given by

$$Z = \frac{v}{i}$$

$$Z = \sqrt{R^2 + \frac{1}{(2\pi f C)^2}} \quad \text{......(15.28)}$$

Fig.15.19 shows that the resultant voltage 'v' of the circuit lags behind the current i by an angle ϕ called phase difference between v and i and its value is given by

$$\tan\phi = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right)$$

$$\phi = \tan^{-1}\left(\frac{iX_C}{iR}\right)$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

$$\phi = \tan^{-1}\left(\frac{1}{2\pi fCR}\right) \quad ...(15.29)$$

> ***FOR YOUR INFORMATION***
> The average power delivered by a generator in a RLC series circuit has maximum value when the inductive reactance equals to the capacitive reactance.

Example 15.4

An alternating current of 2mA with angular frequency 100rad s^{-1} flows through a resistor of $9k\Omega$ and a capacitor of $0.4\mu F$ connected in series. Calculate potential difference across R and C, resultant potential difference and impedance in the circuit.

Solution:

$$i = 2mA = 2 \times 10^{-3}A$$

$$\omega = 100 \text{ rad s}^{-1}$$

$$R = 9K\Omega = 9\times10^{3}\Omega$$

$$C = 0.4\mu F = 0.4\times10^{-6}F = 4\times10^{-7}F$$

$$V_R = ?$$

$$V_C = ?$$

$$V = ?$$

$$Z = ?$$

$$V_R = iR = (2 \times 10^{-3}A)(9 \times 10^{3}\Omega)$$

$$V_R = 18V$$

$$V_C = iX_C = \frac{i}{\omega C}$$

$$V_C = \frac{2\times10^{-3}A}{(100\text{rad s}^{-1})(4\times10^{-7}F)}$$

$$v_C = 50V$$

$$v = \sqrt{v_R^2 + v_C^2} = \sqrt{(18)^2 + (50)^2}$$

$$= \sqrt{324 + 2500}$$

$$v = \sqrt{2824}$$

$$v = 53V$$

$$Z = \frac{v}{i}$$

$$Z = \frac{53V}{2 \times 10^{-3} A}$$

$$Z = 2.65 \times 10^4 \Omega = 26.5k\Omega$$

15.8 A.C. THROUGH R-L SERIES CIRCUIT

Consider a resistor and an inductor which are connected in series to an A.C. source as shown in Fig.15.20. The current flowing through 'R' and 'C' is same at any instant. Thus, the instantaneous value of current is given by;

$$i = I_0 \sin \omega t$$

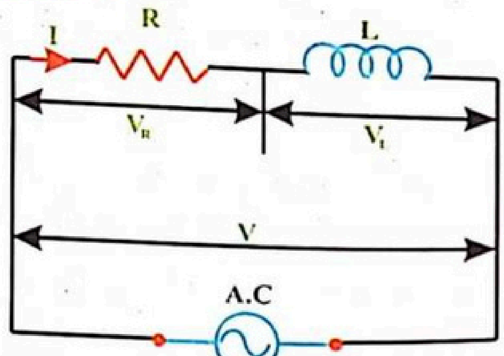


Fig.15.20 A resistor and an inductor in series in an A.C. circuit.

As R and C are in series, so the voltage drop across each component is different and resultant voltage can be calculated with the help of phasor diagram. In a resistor 'v_R' in phase with I. So, the phasor for 'v_R' is parallel to the current i as shown in Fig.15.21. Similarly, in an inductor, 'v_L' leads the current by $\pi/2$ as shown in the phasor diagram. Thus 'v' be the resultant of v_R & v_L and it is equal to their vector sum, that is

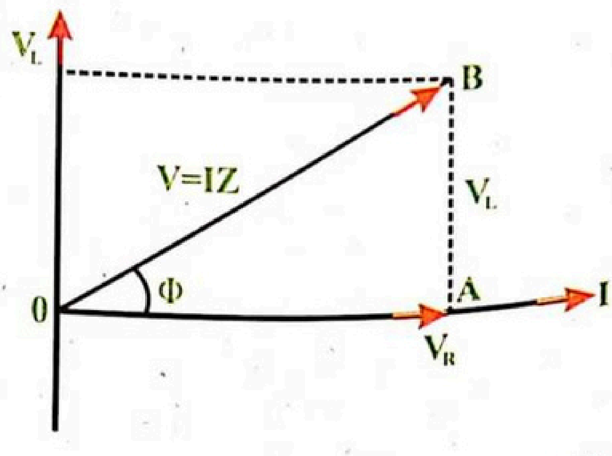


Fig.15.21 A phasor diagram for RL-series circuit.

$$v^2 = v_R^2 + v_L^2$$

$$v = \sqrt{i^2R^2 + i^2L^2}$$

$$v = i\sqrt{R^2 + (\omega L)^2}$$

$$v = i\sqrt{R^2 + (2\pi fL)^2} \quad ...(15.30)$$

Impedance of RL-series circuit is given by

$$Z = \frac{v}{i}$$

$$Z = \sqrt{R^2 + (2\pi fL)^2} \quad ...(15.31)$$

As the resultant voltage 'v' in RL-series circuit leads the current by an angle 'ϕ', so its value is given by

$$\phi = \tan^{-1}\left(\frac{v_L}{v_R}\right)$$

$$\phi = \tan^{-1}\left(\frac{iX_L}{iR}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{2\pi fL}{R}\right) \quad ......(15.32)$$

Example 15.5

An inductor of inductance 0.3H is connected in series with resistor of resistance 6Ω. If the voltage drop across resistor is found to be 12V, then calculate current drawn from the source of frequency 50Hz (ii) the phase angle between V & I.

Solution:

$$L = 0.3H$$
$$R = 6\Omega$$
$$v_R = 12V$$
$$f = 50Hz$$
$$i = ?$$
$$\phi = ?$$

$$\text{Current in the circuit} = i = \frac{v_R}{R}$$

$$i = \frac{12V}{6\Omega} = 2A$$

Reactance of coil (X_L) = $2\pi fL$

$$X_L = 2(3.14)(50Hz)(0.3H)$$

$$X_L = 94.2\,\Omega$$

Voltage across $L(v_L) = iX_L$

$$v_L = (2A)(94.2\,\Omega)$$

$$v_L = 188.4\ V$$

Supplied Voltage $(v) = \sqrt{v_R^2 + v_L^2}$

$$v = \sqrt{(12V)^2 + (188.4V)^2}$$

$$v = 188.78V$$

$$\phi = \tan^{-1}\frac{2\pi fL}{R}$$

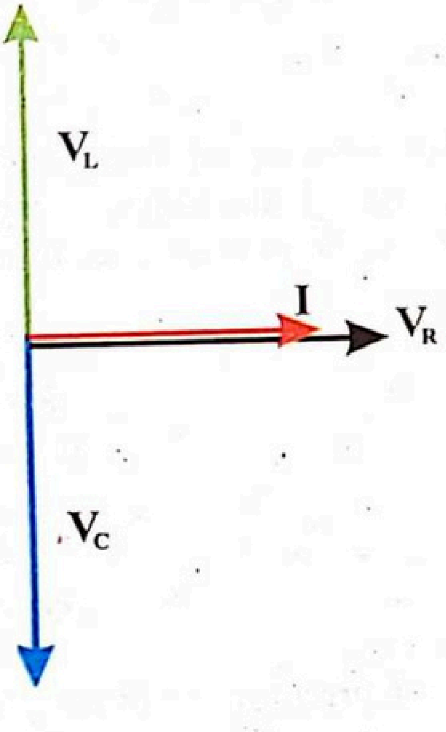
$$\phi = \tan^{-1}\frac{2(3.14)(50Hz)(0.3H)}{6\Omega}$$

$$\phi = \tan^{-1}(15.7)$$

$$\phi = 86^{o}$$

15.9 POWER IN AN A.C. CIRCUIT

We have already discussed in the previous sections that the power dissipation in a pure capacitor and a pure inductor is zero. We knows that, a pure capacitor stores the energy in terms of electrical potential energy and a pure inductor in terms of magnetic potential energy. However, there is a power dissipation in a resistor and its value is given by



$$P = i^2R$$

But according to Ohm's law

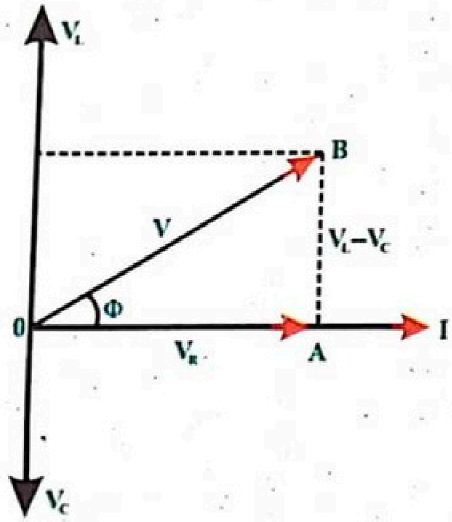
$$R = \frac{v_R}{i}$$

$$P = iv_R \quad ......(15.33)$$

In RLC-series circuit, the current is the same at each point of the circuit, but the voltage drop across each component is different. It can be explained with the help of phasor diagram as shown in Fig.15.14(a) and Fig.15.14(b). Where v_R in phase with I but v_L leads i by $\pi/2$ and v_C lags i by $\pi/2$.

Thus $\frac{v_R}{v} = \cos\phi$

$v_R = v\cos\phi$



Put it in Eq.15.33

$$p = vi\cos\phi \quad ...(15.34)$$

This is known as a true power in RLC circuit and it shows that a maximum power will be dissipated when $\phi = 0$ and 'v' and 'i' are in phase with each other. It is possible only in a resistor. Thus one can say that **the power dissipation in a resistor is called true power.** However, **the power dissipation in an impedance is called apparent power (vi).**

Power factor

Power factor of an A.C. circuit is defined as the ratio of true power to apparent power

$$\text{Power factor} = \frac{\text{true power}}{\text{Apparent power}}$$

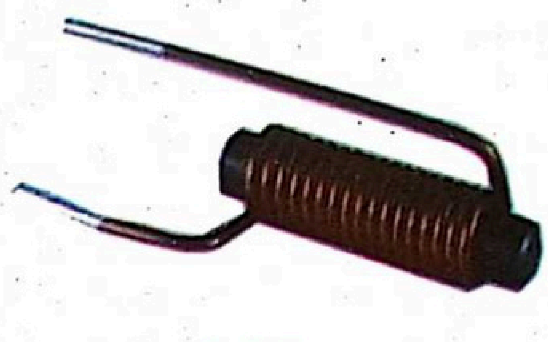
$$= \frac{vi\cos\phi}{vi}$$

$$\text{Power factor} = \cos\phi$$

15.10 CHOKE COIL

A choke coil is an inductor that presents a relatively high impedance in order to control the current in an A.C. circuit.

A choke coil consists of an insulated thick copper wire wound closely in large number of turns over a soft iron laminated core as shown in Fig.15.22. Since the wire of the coil is of thick copper, so its resistance is very small. However, due to the large number of turns, the inductance of the choke coil is very large. Thus, its power loss is extremely small. For example, a



An Inductor

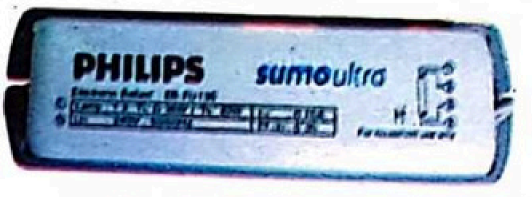


Fig.15.22 A choke coil.

fluorescent tube light requires 120V for its fluorescence, but our applied voltage is 220V. If we use a resistor to control the extra current due to potential difference of 220V, a lot of power would be dissipated by resistor in the form of heat. To overcome this problem, a choke coil is connected in series with the tube or any other electric device. It offers the reactance $(X_L = 2\pi fL)$ to the flow of current. The power dissipation of a choke coil is almost zero, but it controls to extra current in the circuit.

The impedance of choke coil will be:

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{R^2 + (2\pi fL)^2}$$

In case of D.C., f = 0, then $X_L = 0$ and Z = R, this shows that a choke coil cannot be used to control D.C. because its resistance is negligible, but it is used to control A.C. only.

15.11 RESONANCE IN RLC-SERIES CIRCUIT

Consider a resistor, a capacitor and an inductor which are connected in series across the source of A.C. of constant voltage but adjustable angular frequency 'ω' as shown in Fig.15.23. As the components are in series, so there is a same current in the circuit. That is, $i = \frac{V}{Z}$, where 'Z' is the impedance of the RLC-series circuit and it depends upon the frequency. For example, if the frequency increases, then the inductive reactance $X_L = \omega L$ increases while the capacitive reactance $X_C = \frac{1}{\omega C}$ decreases. If the frequency decreases, then we have the reverse results. In between these lower and higher frequencies, there is always a frequency f_0 at which both reactances become same but in opposite direction. So both are cancelled to each other. As a result we have minimum impedance (Z) and maximum current (i). Thus, the frequency f_0 at which the impedance (Z) has its smallest value and the current amplitude reaches at its maximum value is called resonance frequency. Its value can be calculated as;

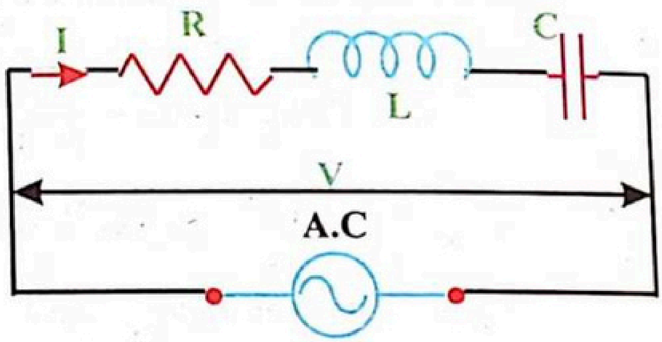


Fig.15.23 A resistor, a capacitor and an inductor connected in series in an A.C. circuit.

The resonance occurs when

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

or $$2\pi f_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \quad \ldots(15.35)$$

This is resonance frequency. If we plot a graph between current and frequency then we have a resonance curve as shown in Fig.15.24. The graph shows that the amplitude of the current reaches its maximum value at the resonance frequency f_o.

POINT TO PONDER

If resistance remain same but capacitance and inductance are doubled, how will the resonance frequency change?

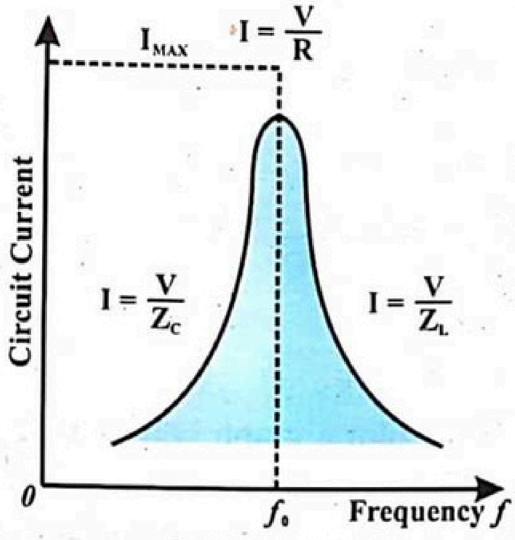


Fig.15.24 A graph between current and frequency of an A.C. in RLC-series circuit.

15.12 RESONANCE IN LC-PARALLEL CIRCUIT

Consider an inductor of inductance 'L' which is connected in parallel with a capacitor of capacitance C across an A.C. source of voltage 'v' and has adjustable angular frequency 'ω' as shown in Fig.15.25. The voltage drop across each component is the same. But the current through 'L' is i_L and through C is i_C. The phase difference between i_L and i_C is 180°, so $i = i_L - i_C$. If the frequency increases, the inductive reactance $X_L = \omega L$ also increases and the current i_L decreases while the capacitive reactance $X_C = \frac{1}{\omega c}$ decreases hence current i_C increases. Similarly, if the frequency decreases, we will observe the inverse result. The experiments show that there is a certain frequency between the lower and the higher frequencies at which $i_L = i_C$ and $i = 0$. Such current

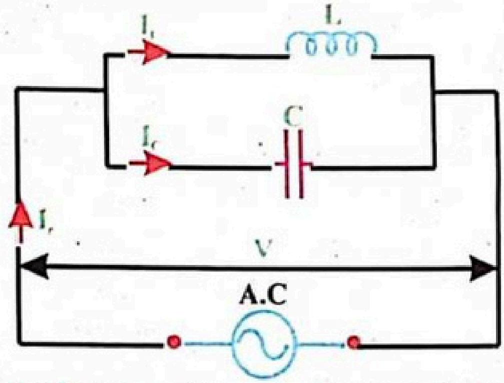


Fig.15.25 A capacitor and an inductor in parallel in an A.C. circuit.

with minimum amplitude is obtained at certain frequency
frequency f_o for LC parallel circuit. Its value can be calculatec
When the resonance occurs then

$$i_L - i_C = 0$$

$$i_L = i_C$$

$$\frac{V}{X_L} = \frac{V}{X_C}$$

$$X_L = X_C$$

$$2\pi f_o L = \frac{1}{2\pi f_o C}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

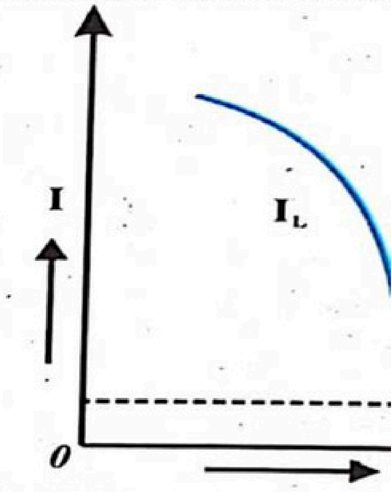


Fig.15.26 A graph
of A.C. in LC-parallel c

If we plot a graph between current and frequency then
curve which is inverse as that of the curve obtained in R
Fig.15.26, we have observed that at resonance frequency 'f_o',

15.13 METAL DETECTOR

The metal detector comprises of a resonating circuit and is used to detect the presence of metal nearby, hidden metals or metallic objects, buried metal underground etc. It consists of two LC oscillatory circuits A and B which are connected across the beat frequency amplifier circuit as shown in Fig.15.27. Each oscillator contains an inductor coil parallel to a capacitor to form a LC parallel circuit. A metal detector is based on the principle that the presence of metal within the range of the coil, changes its inductance L which in turn causes a change in the resonance frequency of the LC-circuit. At the normal condition, that is, when there is no metal near by the coil 'B' called search coil. The frequency of the both oscillators A and B is the same, i.e., adjusted at same resonance frequency, hence they produce zero beat When the search

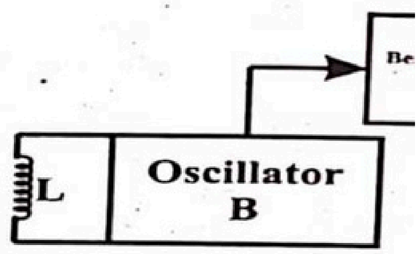
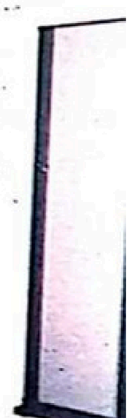


Fig.15.27 A circuit



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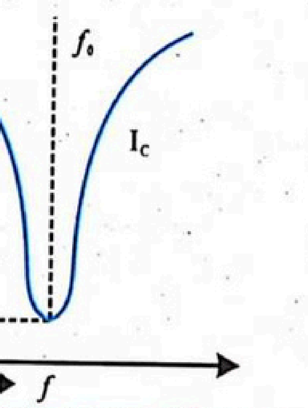
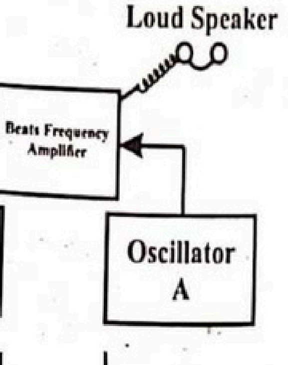


tween current and frequency

n we have a resonance
RLC-series circuit. In
, the current is zero.



coil approaches to the a metallic object, its inductance 'L' decreases and the frequency of the oscillator 'B' increases. So, there is difference in frequencies between the two oscillators resulting in a beat phenomenon and the loud speaker attached to the amplifier circuit is sounded as an alarm. The metal detectors are used not only for security purpose but also to detect buried and hidden metal objects. In the same way, metal detectors are used in mining and other scientific research.

15.14 MAXIMUM POWER TRANSFER IN AN A.C. CIRCUIT

In unit 12, we have studied that a D.C. source transfers its maximum power to an electric network when the internal resistance of source equals to the load resistance. This is named as "maximum power transfer theorem". Such theorem also applicable to an A.C. network which is explained as under:

Consider an A.C. circuit having a load impedance 'Z_L' which is connected across the A.C. source of impedance 'Z_g' as shown in Fig. 15.28. According to maximum power transfer theorem, the maximum power is transferred from source to load when the impedance of source Z_g and impedance of load Z_L are matching to each other, i.e.,

$$Z_L = Z_g \quad \ldots\ldots(15.36)$$

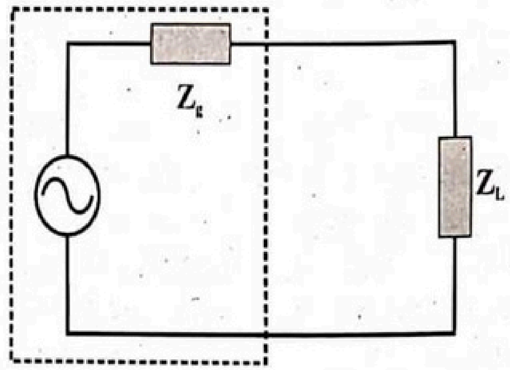


Fig. 15.28 An A.C. circuit in which the impedance of the source matches to impedance of load and source delivers its maximum power to the load.

Eq. 15.36 holds in case of a communication system where a signal transfers with its maximum power between transmitting antenna and receiving antenna when the impedances of the transmitter and the receiver match to each other. However, in case of power transmission system or any other electrical network, their efficiency in terms of maximum power transfer is about 50%, because there is large load resistance across the source.

> **FOR YOUR INFORMATION**
> To get the maximum radiated power from an antenna in a communication system, the radiation resistance of the antenna must match the output resistance of the transmitter.

15.15 ELECTROMAGNETIC WAVES

The waves which compose of oscillating electric and magnetic fields at right angle to each other also perpendicular to the direction of their motion through space are called Electromagnetic waves, these waves do not require any medium for their propagation.

Based on Gauss's law, Faraday's and Ampere's laws, British Physicist James Clerk Maxwell in 1865 showed a closed relationship between electric and magnetic fields. He predicted that electric and magnetic fields can move through space as waves. He explained this mutual interaction between the two fields in the form of a set of four mathematical equations which are called Maxwell equations. These equations may be summarized as: a time varying or changing magnetic field produces a changing electric field. This changing electric field will in turn produce a changing magnetic field and so on.

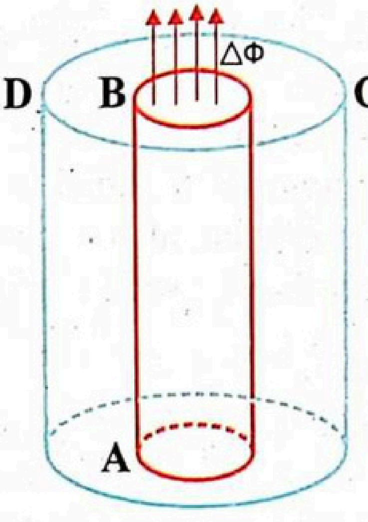


Fig.15.29(a) Changing of magnetic flux in the region AB induce an electric field in its surrounding.

To explain the Maxwell's hypothesis, we consider a conducting rod AB which is connected to an alternating voltage source. The charges are accelerated through rod such that a changing magnetic field produces around it in the region CD as shown in Fig.15.29(a). This changing magnetic field again set up a changing electric field and so on. This shows that each field generates the other and as a result the whole package of electric and magnetic fields start motion in forward direction through space. Such a combineed motion of electric and magnetic field is known as electromagnetic waves which consists of electric and magnetic field perpendicular to each other as shown in Fig.15.29(b). These waves travel in space with the speed of light 'c'. Its numerical value $3 \times 10^8 \mathrm{m\ s^{-1}}$ can be calculated by the following equation

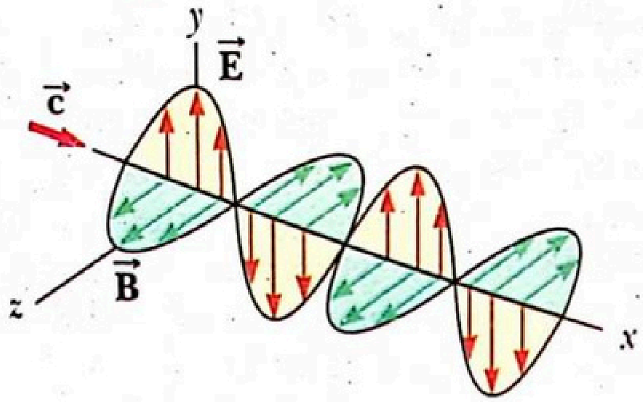


Fig.15.29(b) Propagation of electromagnetic wave that consists of electric and magnetic fields perpendicular to each other.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \ldots\ldots(15.37)$$

where μ_0 and ε_0 are permeability and permittivity of free space respectively.

Electromagnetic wave has many forms such as radio waves, microwaves, infrared waves, visible light, ultraviolet light, X-rays and Gamma-rays. All these electromagnetic waves travel through free space with the same speed equal to speed of light 'c', their frequency and wave length can be study by the following relation.

$$c = f\lambda \quad \ldots\ldots(15.38)$$

15.15.1 The spectrum of electromagnetic waves

The speed of all the electromagnetic waves is same but their frequencies and wavelengths are different. Therefore, the electromagnetic waves can be classified on the basis of their frequencies and wavelength. The orderly classification of electromagnetic wave with respect to their frequencies and wavelengths is known as the electromagnetic spectrum. Graphically, a complete spectrum of electromagnetic waves is shown in Fig.15.30. Now all these classes of electromagnetic waves are explained as:

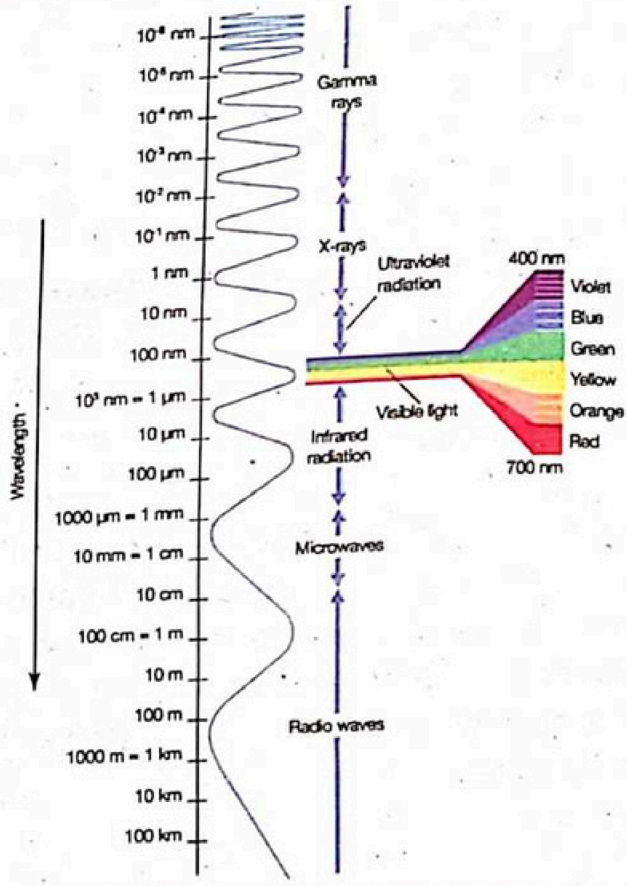


Fig.15.30 Electromagnetic waves spectrum in terms of wavelength λ and frequency ranges f.

Radio waves

Radio waves are those electromagnetic waves which have the longest wavelength. Typically, the wavelength of radio waves is longer than 1mm. They are produced by the electrical oscillation in the LC-circuits and used in the global communication system with different frequencies. e.g., AM (Amplitude Modulation) uses waves with frequency from 530KHz to 1170KHz, while FM (Frequency Modulation) radio broadcasts are at frequency from 88MHz to 108MHz and TV broadcasts use frequency from 54MHz to 890MHz. Radio waves are also used for the communication system of cellular phones with frequency from 300MHz to 3000MHz.

Microwaves

Microwaves are also generated by the LC oscillating circuit. They have wavelengths ranging from 1mm to 30cm. They are used in the radar as well as in aircraft navigation system. Similarly, the short wavelength microwaves are also used for study of atomic and molecular properties of matter. Microwave oven is a useful application of the microwaves.

Heating and burning effect of microwaves

Water is a good absorber of microwaves because the water molecules are polar and the positive and negative charges of the molecules are attracted in opposite direction. As a result, the molecules start oscillation. The frequency of rotation of water molecules is about 2.5GHz. Now if microwaves of this frequency are allowed to fall on water, the molecules of water absorb these radiations and hence water gets heated up, such heat energy is spread out in the whole region due to the rotation of molecules of water. A microwave oven is used for heating or cooking the food under this principle. Food with any moisture (i.e. free of water) cannot be heated in a microwave oven.

On the other hand, microwave radiation can heat the body living tissue in the same way that it heats the food. Therefore, prolong exposure of microwaves cause burning of body tissues.

Infrared waves

Infrared radiation IR, sometimes called infrared light or heat waves, is electromagnetic radiation with wavelengths longer than those of visible light i.e. its wavelength is lying between 700 nm and 1 mm. The large amount of infrared waves are generated by the sun, the sun gives off half of its total energy as infrared. Infrared waves have also some interesting scientific applications in vibrational spectroscopy. For example infrared is being used as short range signal wireless system between a device and a remote control.

Visible light

This is the most important part of the electromagnetic waves spectrum emitted by the sun, because it is the only radiation which is detected by the human eye. It is produced by the transitions of electrons in the atoms. The wavelengths of visible light are ranging from violet (4×10^{-7}m) to red (7×10^{-7}m).

Ultraviolet light

Ultraviolet light has a wavelength range from 4×10^{-7}m to 6×10^{-10}m. Their most important source is the sun. But they are also produced by carbon-arc lamp, electric spark, discharge tube, mercury vapor lamp, hot bodies etc. Most of the ultraviolet light from the sun is absorbed by ozone (O_3) molecules in the earth's upper atmosphere, or stratosphere. This is fortunate, otherwise prolong exposure to ultraviolet light has harmful effects on human. Such as sun burns as well as skin cancer.

Ultraviolet lamp

A ultraviolet lamp is a device that produce electromagnetic wave in the wavelength between those of visible light and x-rays. It consist of a glass tube contains a small amount of mercury. When the potential difference is applied across the tube, the molecules of the mercury are excited and they emit ultraviolet light as shown in Fig.15.31. The wavelength of emitted ultraviolet light depends upon the pressure inside the tube. The ultraviolet light emitted by the lamp is being used for the following purpose:

Fig.15.31 Excited molecules of mercury inside the tube emit ultraviolet light.

- Ultraviolet light produce vitamin D in the skin, therefore it is useful for Cystic fibrosis (CF) and Short Bowel Syndrome (SBS) patients, who suffer from vitamin D deficiency
- It is being used in killing and eliminating viruses, bacteria, microbes and other harmful organisms.
- It is also used in hospitals as well as drug and food industries to sterilize the equipments.

X-ray

X-rays are electromagnetic radiation with wavelength ranging from 10^{-8}m to 10^{-12}m. They are produced when fast moving beam of electrons strikes on the metal target. X-rays are used as diagnostic tool and also used as a treatment for certain forms of cancer. Care must be taken to avoid unnecessary exposure because X-rays can destroy living tissues and organisms. X-rays are also used in the study of crystal structure.

Gamma rays

Gamma rays have the shortest wave-lengths and their wavelength ranges from 10^{-10}m to less than 10^{-14}m. Gamma rays are highly penetrating. Therefore, they are used to destroy cancerous cells in humans. They are also used to conduct nuclear reactions. Gamma rays are emitted by radioactive nuclei in radioactive decay such as, $^{60}C_0$ and $^{137}C_S$. These rays are also emitted during the nuclear reactions.

15.16 PRINCIPLE OF GENERATION, TRANSMISSION AND RECEPTION OF ELECTROMAGNETIC WAVES

We have discussed the Maxwell's electromagnetic field theory that electromagnetic waves are generated when either an electric or a magnetic field is changing with time and these waves are capable of traveling through space. Similarly, we have also observed that an electric field due to a charged particle at rest or moving with constant velocity does not radiate in space because the magnetic flux is not changing in these cases. But the electric field only radiates through a certain region of space, when the charged particle is accelerated. Based on this principle,

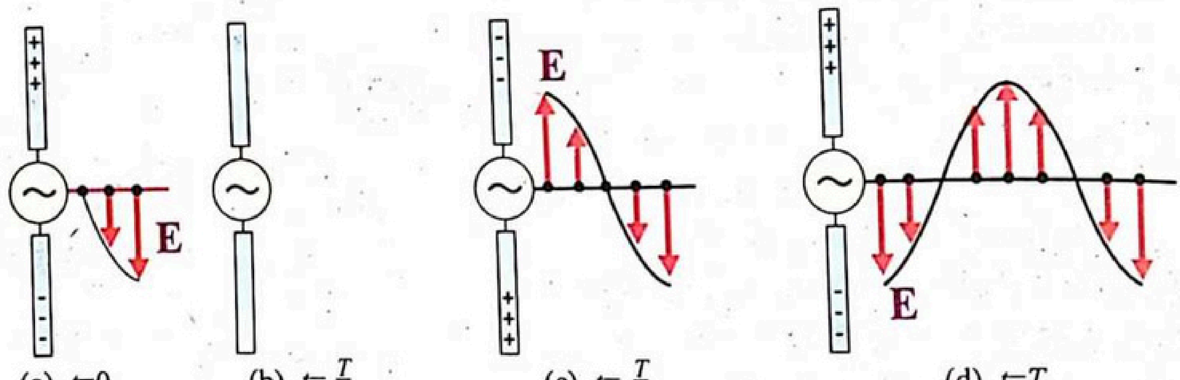


Fig.15.32 (a) An arrangement for generation and transmission of electromagnetic waves by a dipole antenna.

electromagnetic waves can be generated in space by using a radio wave transmitting dipole antenna.

A dipole-antenna consists of a long wire which is powered by A.C. source of frequency 'f' and time period T. One pole of the antenna is in space while the other is grounded as shown in Fig.15.32(a). As the source is alternating so the charges vary periodically at the ends of antenna. Now we study such variation of the charges on antenna for one cycle of A.C.

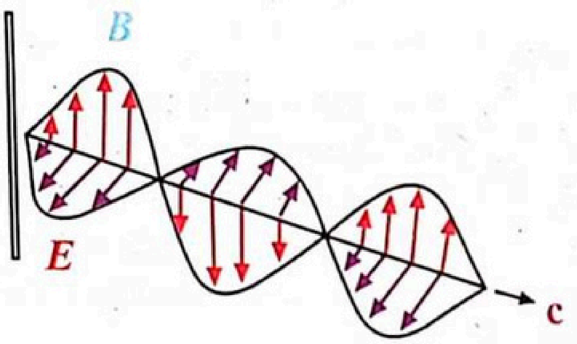


Fig.15.32(b) electric lines and magnetic lines of force perpendicular to each other.

At time t = 0; the upper part of the antenna is positively charged and the lower is negatively charged. The direction of $\overline{E}$ is downward. At $t=\frac{T}{4}$; the charges are neutralized, and $\overline{E}$ is zero. At $t=\frac{T}{2}$; upper part is negatively charged and lower is positively charged and the direction of $\overline{E}$ is upward. At $t = T$; $\overline{E}$ again comes at its initial position. For the next cycle, the same process of variation of charges on the antenna is repeated. On the other hand, the oscillation of these charges produces a current in the antenna, as a result a magnetic field is also generated. Now these two fields are out of phase to each other

> ***FOR YOUR INFORMATION***
>
> A resonant LC circuit generally has hundreds or thousands of signals present at its input, but only one is selected to be present at its output. e.g., the antenna for an FM receiver intercepts the signals of many different FM broadcast stations, but by turning an LC circuit to resonance, the listener can select only the station he would like listen to.

and the lines of force of $\overline{E}$ are perpendicular to the lines of force of $\overline{B}$ as shown in Fig.15.32(b). In this way, the electromagnetic waves travel through space away from the antenna with the speed of light and its frequency depends upon the frequency of the alternating voltage source.

After transmission the electromagnetic waves from the transmitting antenna, these waves can be received by an LC-parallel electrical circuit where L is inductor and C is variable capacitor. A long wire called receiving antenna is connected with this LC-parallel circuit as shown in Fig.15.33. Now when the electromagnetic waves fall on the receiving antenna, the electrons in the antenna start oscillation due to the oscillating electric field of the waves. As a result, an alternating voltage induced across the antenna. The frequency of this voltage is the same as that of the waves intercepting the antenna. There are a number of electromagnetic waves that travel through space with their own frequencies. But the receiving antenna can receive one of them at a time. It takes place only when we adjust the value of the variable capacitor such that the natural frequency $\left(f = \frac{1}{2\pi\sqrt{LC}}\right)$ of the LC-circuit is same to the frequency of the incoming waves from the transmitting station. Thus due to the resonance phenomena, the LC-circuit will respond to electromagnetic waves of particular wanted frequency and reject all the other unwanted frequencies. Such arrangement is used in the tuning circuit of a radio.

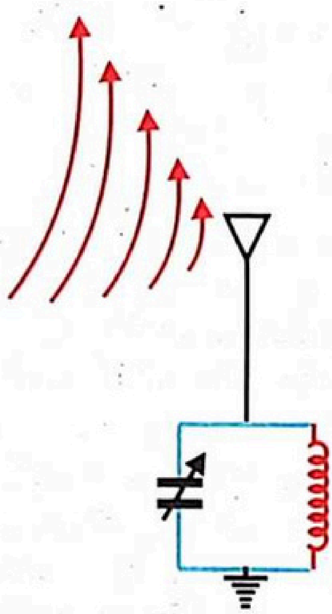


Fig.15.33 Receiving antenna with LC-circuit

15.17 TRANSMISSION AND RECEPTION OF INFORMATION

The process of transmission of information by a radio wave is shown in Fig. 15.34(a). Our information or message which is to be transmitted in the form of sound or picture, where sound has frequency ranging from 20Hz to 20KHz with speed 334ms^{-1} in air at 0^{0}C and it is a non-electrical signal. Therefore, microphone converts sound signal into the electrical signal and then it is fed to audio amplifier for raising the strength of the signal. This audio signal cannot be radiated out from the transmitting antenna directly. For this, a very high frequency wave called radio wave is produced by oscillator. The radio carrier wave is also known as carrier signal. The amplitude of radio wave is constant and they have high frequencies from 30kHz to 300000kHz with a speed of 3×10^8 m s^{-1}. Now the audio signal is super imposed on the radio frequency wave by using modulator. This process is called modulation. The modulation wave is finally transmitted in free space by the transmitting antenna.

When, when the modulated transmitted wave falls on the receiving antenna as shown in Fig.15.34(b), the receiving signal is first amplified by the tuned amplifier and then the modulator accepts the relevant audio signal and reject the unwanted radio frequency wave. This process by which the radio waves and audio waves are separated is known as demodulation. Finally, the audio signal is amplified by the audio amplifier and then fed to the loud speaker for producing the replica of original sound waves.

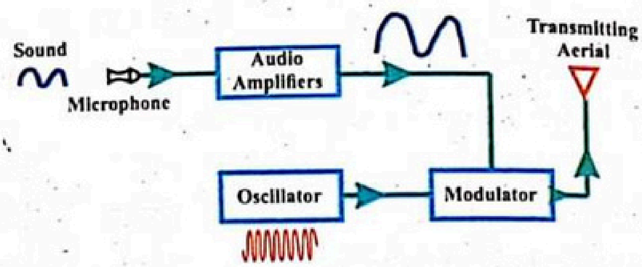


Fig.15.34(a) A process in which the information in terms of audio frequency wave is transmitted by a radio frequency wave.

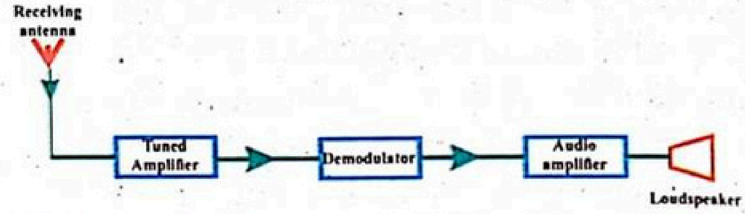


Fig.15.34(b) A process in which the information audio frequency signal is received under various stages.

15.18 ELECTROCARDIOGRAPHY (E.C.G.)

The instrument that records the voltage pulses associated with heartbeats is called electrocardiography and the pattern recorded by such instrument on a graph paper or computer is called an electrocardiogram. It is explained as under;

The heart is a muscular organ made up mostly of cardiac muscles. Now we will study an individual muscle cell in its resting state. There are negative ions on the inner surface of the membrane of the cell, while positive ions on the outer surface as shown in Fig.15.35(a). This ions distribution across the membrane of the cell causes of electric potential.

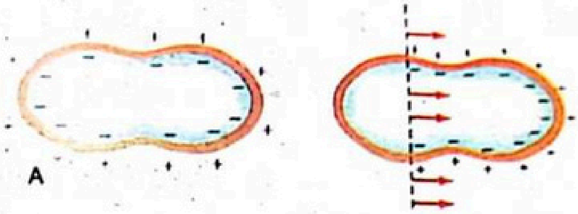


Fig.15.35(a) charge distribution across the membrane of the cell in its resting state (b) charge distribution as the depolarization wave passes.

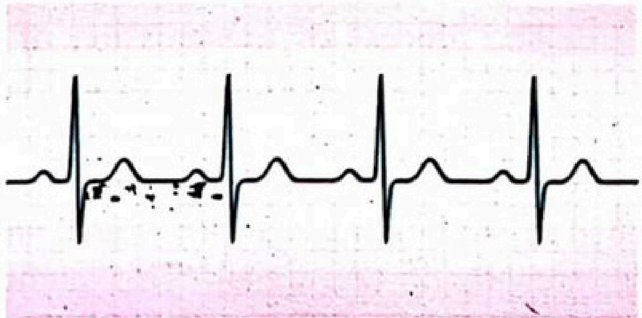


Fig.15.36 Graphical representation of ECG.

The electric impulses that originate in the muscle fibers gradually spread from cell to cell, causing the muscles to contract. The pulse that passes through the muscles cells is called a depolarization wave. The generated impulse will pump the positive ions on the outside of the cell to flow in and neutralize the negative ions on the inside of the cell as shown in Fig.15.35(b). This effect causes of neutralize the potential difference. Once the depolarization wave has passed through an individual heart muscle cell, the cell is polarized again. i.e., it recovers the resting state ions distribution in about 250ms. Thus, the depolarization and polarization of cells in the heart causes potential difference that can be measured using electrode connected to the skin. The potential difference measured by the electrodes is amplified and recorded on a graph paper or a computer as shown in Fig.15.36.

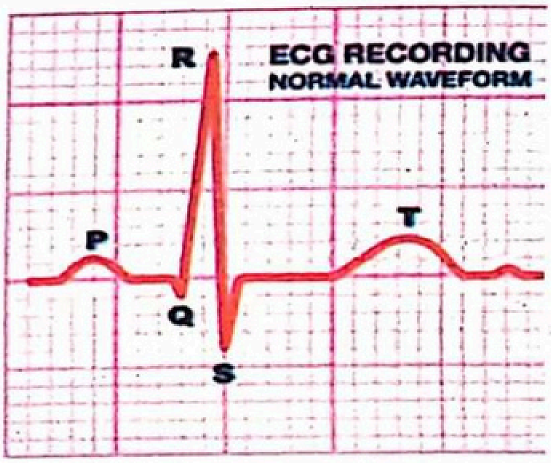


Fig.15.37 (a) The recorded ECG for a normal heart.

Let us study the recorded ECG for one beat of a normal heart as shown in Fig.15.37(a). The pulse 'P' shows the generated pulse before the muscle to contract. Similarly, when the pulse passes through the cell called polarization and it is represented by the pulse QRS. Finally, the T pulse occurs when the cells are polarized again. Similarly, the recorded ECG for an abnormal heart is shown in Fig.15.37(b). The registered result shows that there is no constant relation between the pulses due to the polarization and depolarization. On the other hand, the pulse QRS is wider than the pulse QRS of the normal one.

Fig.15.37(b) The recorded ECG for an abnormal heart.

SUMMARY

- **<u>Alternating Current (A.C.):</u>** A current or voltage which changes periodically with time both in positive and negative direction is called alternating current or voltage. i.e., $i = I_0 \sin \omega t$ or $v = V_0 \sin \omega t$
- **<u>Root Mean Square Value of Current or Voltage:</u>** The R.M.S. value is that effective value of an alternating current or voltage which produces same amount to heat in a resistor as that of produced by D.C.

$$I_{rms} = 0.707\, I_0.$$

- **A.C. Circuit:** A circuit which is powered by A.C. source and a number of components (such as R, L and C) are connected across it is known as A.C.-circuit.
- **Capacitive Reactance:** The opposition offered by a capacitor to the flow of A.C. is known as capacitive reactance $\left(X_C = \frac{1}{\omega C}\right)$.
- **Inductive Reactance:** The opposition offered by an inductor to the flow of A.C. is known as inductive reactance $\left(X_L = \omega L\right)$.
- **Impedance:** The combined resistance of a resistor, capacitor and an inductor in A.C. circuit is known as impedance.
- **Power factor:** The ratio between true power to the apparent power is called power factor, it is equal to $\cos\phi$.
- **Choke coil:** Choke coil is an inductor whose power dissipation is zero and it is being used to control the current in an A.C. circuit.
- **Metal Detector:** It is an electronic instrument which consists of resonant circuit and is used to detect metal, metallic objects, and the buried metals.
- **Maximum Power Transfer Theorem:** An A.C. source transfers its maximum power to load when its impedance equals to the impedance of load.
- **Electromagnetic waves:** Electromagnetic waves are composed of oscillating electric and magnetic fields at right angle to each other and they do not require any medium for their propagation.
- **Modulation:** A process in which audio frequency signal is super imposed on the radio frequency wave.
- **Electrocardiography:** The instrument that records the voltage pulses associated with the action of heart is called electrocardiography.
- **Electrocardiogram:** The recorded heart pulses on a paper is called electrocardiogram.

EXERCISE

Select the best option of the following questions.

1. The mean value of current over one complete cycle of A.C. is
   (a) Zero (b) One (c) I (d) I_0
2. Root mean square value of alternating current is equal to
   (a) 50% of I_0 (b) 50.7% of I_0 (c) 70% of I_0 (d) 70.7% of I_0
3. When the initial phase of A.C. is $\pi/2$ then it will complete its one cycle at the phase of
   (a) π (b) $3\pi/2$ (c) 2π (d) $5\pi/2$

4. Average power dissipation in an A.C. circuit across the resistor is
(a) VI (b) V_oI_o (c) $V_{rms}I_{rms}$ (d) V^2I^2

5. When a capacitor is connected across the A.C. source then
(a) V & I are in phase (b) I lagging behind V
(c) I leading by V (d) V leading by I

6. When the frequency of A.C. is increased then the reactance of the inductor will
(a) Remain the same (b) Decreased
(c) Increased (d) Become negative

7. The power dissipation in LC- circuit is
(a)V.I (b) V_oI_o (c) $V_{rms}I_{rms}$ (d) Zero

8. A capacitor is an A.C. circuit is working at
(a) Any frequency (b) Low frequency
(c) High frequency (d) Zero frequency

9. The unit of impedance is
(a) Ampere (b) Volt (c) Watt (d) Ohm

10. According to the phasor diagram, the phase difference between V_L and V_C is
(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) π (d) 2π

11. Power factor is equal to
(a) VI (b) $VI\cos\varphi$ (c) $\cos\varphi$ (d) $\tan\varphi$

12. In a choke and a resistor series circuit, the A.C. source delivers its maximum power when
(a) $R = 0$ (b) $Z = 0$ (c) $R = Z$ (d) $R > Z$

13. The A.C. source transfers its maximum power to load when
(a) Impedance of source is equal to 0
(b) Impedance of source less than impedance of load
(c) Impedance of source greater than impedance of load
(d) Impedance of source equals to impedance of load

14. In RLC-series circuit the amplitude of the current has its maximum value when
(a) $X_C = 0$ (b) $X_L = 0$ (c) Both zero (d) $X_L = X_C$

15. In RCL-parallel circuit, the resonance phenomenon will be observed when the net current in the circuit is
(a) Increasing (b) Decreasing (c) Reverse (d) Zero

16. The resonance frequency is
(a) $f = 2\pi\sqrt{LC}$ (b) $f = \frac{2\pi}{\sqrt{LC}}$ (c) $f = \frac{1}{2\pi\sqrt{LC}}$ (d) $\frac{\sqrt{LC}}{2\pi}$

17. The propagation of electromagnetic waves was predicted by

(a) Hertz (b) Maxwell (c) Faraday (d) Ampere

18. The phase difference between electric and magnetic line of forces in electromagnetic waves is

(a) 45^0 (b) 90^0 (c) 180^0 (d) 270^0

19. The speed of electromagnetic waves in free space is given by the equation

(a) $c = \frac{1}{\sqrt{\varepsilon\mu}}$ (b) $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ (c) $c = \frac{1}{\sqrt{\varepsilon/\mu}}$ (d) $c = \frac{1}{\sqrt{\varepsilon_0/\mu_0}}$

20. The electromagnetic waves do not transport

(a) Energy (b) Charges (c) Momentum (d) Information

21. Which one of the following waves has the shortest wavelength?

(a) Radio wave (b) Microwave (c) Ultraviolet wave (d) γ-rays wave

22. A process in which audio signal superimpose on radio frequency wave is called

(a) Rectifier (b) Amplifier (c) Modulation (d) Demodulation

SHORT QUESTIONS

1. Why we use I^2 instead of I while taking the mean value of A.C. cycle?
2. Why 220V of A.C. is more effective then 220V of D.C.?
3. At what value, the A.C. and the D.C. become equal?
4. What is the cause of the current leading by voltage in an A.C. circuit?
5. What do you know about the impedance in RLC-circuit of A.C.?
6. What will happen if the frequency of A.C. across the inductor is increased?
7. What do you know about the power factor of A.C. circuit?
8. What is a phasor and phasor diagram?
9. Why a choke cannot control D.C.?
10. Under what condition a source transfers its maximum power to a load?
11. What is the condition of the resonance phenomenon in RLC-series circuit?
12. How does the amplitude of the current becomes zero in LC parallel circuit?
13. What is the working principle of metal detector?
14. How did Maxwell predict the electromagnetic waves?
15. How does transmitting antenna transmit the electromagnetic waves?
16. What do you know about the spectrum of electromagnetic waves?
17. How does the receiving antenna receive the electromagnetic waves?
18. How can you calculate the speed of electromagnetic waves?
19. Distinguish between the process of modulation and demodulation.
20. Distinguish between electrocardiography and electrocardiogram.

COMPREHENSIVE QUESTIONS

1. Explain alternating current and alternating voltage with their mathematical and graphical representation.

2. What do you know about the instantaneous value, root mean square value and peak value of alternating current and alternating voltage?
3. What is phase of A.C.? Explain phase lag and phase lead between alternating current and alternating voltage with the help of phasor diagram.
4. Study alternating current through a resistor and calculate power dissipation in the resistor.
5. Discuss A.C. through a capacitor and power dissipation in it.
6. What are the value of instantaneous current and voltage in an inductor when it is connected across the A.C. source?
7. What is impedance? Calculate the impendence of RCL-series circuit.
8. Calculate impedance and phase angle of RC-series circuit.
9. Discuss the behaviour of A.C. through RL-series circuit.
10. State and explain power in an A.C.-circuit and power factor.
11. What do you know about choke coil, its function and application?
12. State and explain resonance in RLC-series circuit with the resonance frequency.
13. How can resonance be observed in LC-parallel circuit? Also calculate the resonance frequency in LC-parallel circuit.
14. Explain metal detector, its working principle and function.
15. What are electromagnetic waves? Discuss the process of generation, propagation and reception of electromagnetic waves.
16. What do you know about the spectrum of electromagnetic waves? State and explain all classes of electromagnetic waves.
17. Explain electrocardiography with its working principle and its function.

NUMERICAL PROBLEMS

1. The r.m.s. value of alternating current is 5A and its frequency is 50Hz which flows in a circuit through a resistor. Calculate the peak current and the value of the current after 0.002s. **(7A, 4.4A)**
2. An alternating current flows through a resistor of 10Ω and produces heat at the rate of 360W. Calculate the effective value of current and voltage. **(6A, 60V)**
3. A capacitor of capacitance 6μF is connected across the A.C. source of 220V. Calculate the current through the capacitor, if the frequency of the source is 50Hz. **(0.4A)**
4. An inductor of inductance 0.6H is connected across the A.C. source of 220V. Calculate the current through the inductor, if the frequency of the source is 50Hz. **(1.2A)**

3. What is PN junction? How can you form a PN junction?
4. Define biasing of PN junction and discuss forward biased and reverse biased of a PN junction.
5. What is rectification? How can a diode be used as a rectifier? Explain half-wave rectification.
6. State and explain full wave bridge rectification by using four diodes.
7. What is transistor? Explain the operation of NPN transistor?
8. Discuss the three configurations of a transistor.
9. What is amplification? Explain the amplification of common emitter NPN transistor.
10. How can transistor be used as an automatic switch?

NUMERICAL PROBLEMS

1. Calculate the resistance across the PN-junction silicon diode that has 0.5A of current through it with a 0.8V drop across the two terminals of the diode. **(1.6Ω)**
2. How much is the emitter current with 100mA for the collector current and 800μA for base current? **(100.8mA)**
3. A transistor has $\alpha = 0.995$. If the emitter current is 80mA, what is (i) the collector current (ii) the base current (iii) current gain β? **(i) 79.6mA, (ii) 0.4mA, (iii) 199**
4. The current gain β of a transistor is 200. If the base current is 0.2mA. Find (i) the collector current (ii) emitter current (iii) the value of α. **(i) 40mA (ii) 40.2mA (iii) 0.995**
5. The input resistance in the common emitter amplifier circuit of a given transistor is $2k\Omega$. The output resistance of this circuit is $8k\Omega$. What would be the output voltage corresponding to an input voltage 6mV if the current gain is 100? **(2.4V)**