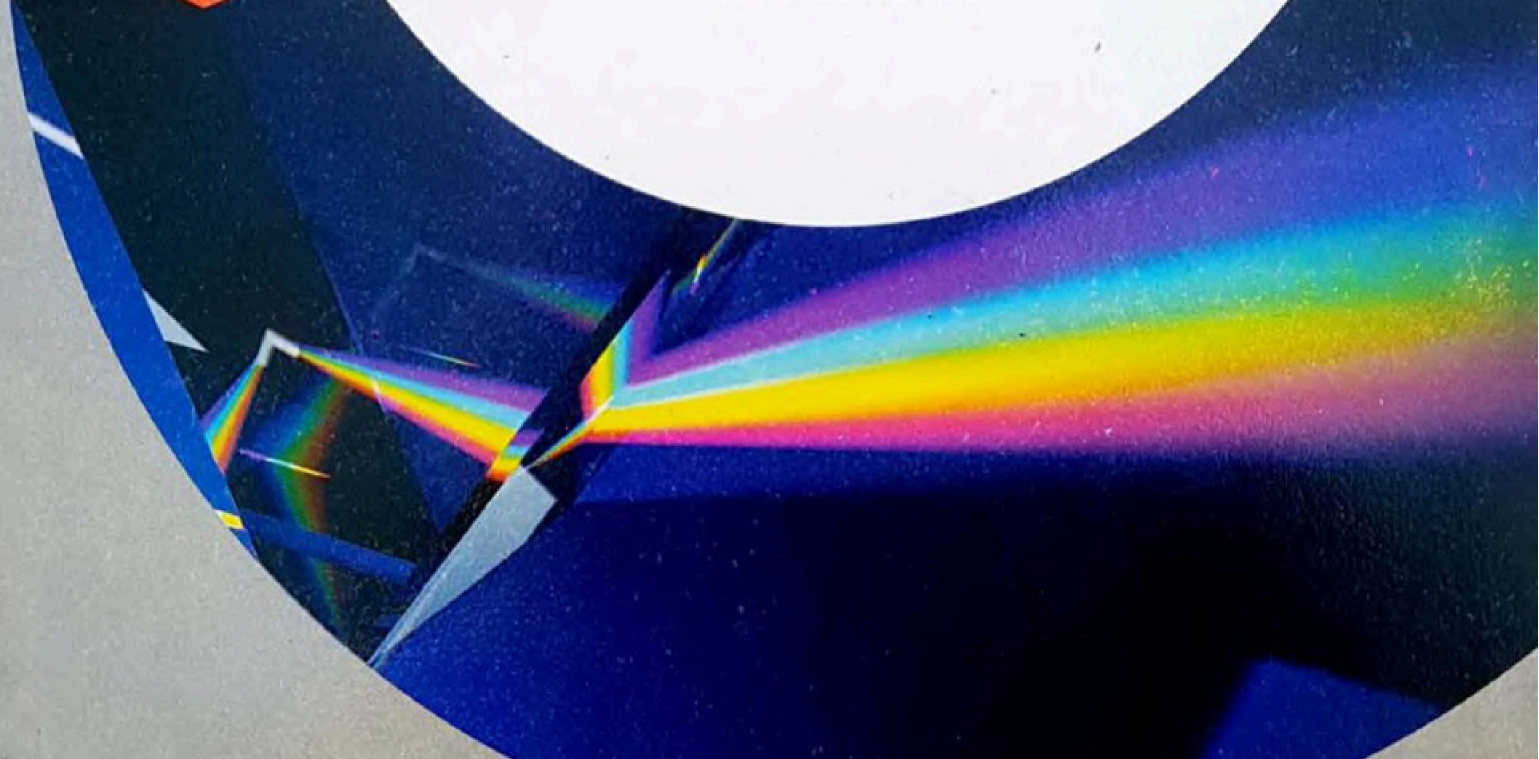
A TEXTBOOK OF

PHYSICS XII



Balochistan Textbook Board, Quetta.

بسم اللہ الرحمن الرحیم

شروع اللہ کے نام سے جو بڑا مہربان نہایت رحم کرنے والا ہے

A TEXTBOOK OF

PHYSICS XII

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Syed Arif Shah Sherazi

Chairman

Balochistan Textbook Board, Quetta.

Ph: 081-2470501

btbb_quetta@yahoo.com

Author

Mr. Ghulam Rabbani
Associate Professor,
Govt: Postgraduate Science College Quetta

Ms. Shumaila Fazal
Assistant Professor
Govt: Girls Postgraduate College Quetta Cantt:

Editor

Dr. Hamdullah Khan Tareen (Ph.D Physics)
Professor,
Department of Physics, BUITEMS Quetta.

Dr. Syed Muzamil Bokhari
Assistant Professor, (Department of Physics)
University of Balochistan

Internal Review Committee

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Govt: Postgraduate Science College, Quetta

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Lecturer
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Mr. Abdul Shakoor (Coordinator)
Subject Specialist
Balochistan Text Book Board Quetta

Provincial Review Committee

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Associate Professor
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Govt: Postgraduate Degree College, Quetta

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Sardar Bahadur Khan Women's University

Dr. Gulab Khan
Deputy Director (Curriculum)
Bureau of Curriculum and Extension Center

Supervision: **Din Muhammad** (Subject Specialist) BTBB Quetta.

Prepared by: New College Publication, Quetta

Graphics: Farrukh Iqbal / Shoaib Sahil

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CONTENTS

Standard abbreviations and symbols used in text

Symbol	Meaning	Symbol	Meaning
A	ampere	Hz	hertz
A°	Angstrom	J	joule
α	temperature co-efficient	K	kelvin
B	magnetic field	K	boltzmann's constant
B	magnetic field density	L	inductance
C	coulomb	m_o	rest mass of electron
C	capacitance	N	north pole
c	speed of light	N	n-type semi conductor
°C	degree celsius	P	dipole moment
Cal	calorie	P	p-type semi conductor
Ci	curie	P	primary coil
D	debye	p	instantaneous power
E	electric field	ρ	resistivity
E	battery (cell)	q	charge
ε	induced e.m.f	Q	total charge
ε_o	permittivity of free space	R	resistance
ε_r	relative permittivity	R_h	rydberg constant
e^-	electron	r	internal resistance
e^+	positron	S	south pole
ev	electron volt	S	secondary coil
F	force	s	second
F	farad	Sv	sievert
F°	degree fahrenheit	T	tesla
f	frequency	T	time period
G	gauss	$T_{1/2}$	half life
G	galvanometer	τ	time constant
G	conductance	τ	torque
σ	conductivity	u	atomic mass unit
σ	charge density	U	potential
I	direct current	v	velocity
i	alternating current	V	potential difference (volt)
H	henry	ν	alternating voltage
H	magnetizing force	Wb	weber
h	planck's constant	ω	angular frequency
λ	wavelength	Z	Impedance
λ	decay constant	ϕ	electric flux
η	efficiency	ϕ_B	magnetic flux
γ	photon	μ_o	permeability of free space

Unit 11

ELECTROSTATICS

Major Concepts (21 PERIODS)	Conceptual Linkage
• Force between charges in different media • Electric field • Electric field of various charge configurations • Electric field due to a dipole • Electric flux • Gauss's law and its applications • Electric potential • Capacitors • Energy stored in a capacitor	This chapter is built on Electrostatics Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

- state Coulomb's law and explain that force between two point charges is reduced in a medium other than free space using Coulomb's law.
- derive the expression $E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$ for the magnitude of the electric field at a distance 'r' from a point charge 'q'.
- describe the concept of an electric field as an example of a field of force.
- define electric field strength as force per unit positive charge.
- solve problems and analyze information using $E = F/q$.
- solve problems involving the use of the expression $E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$
- calculate the magnitude and direction of the electric field at a point due to two charges with the same or opposite signs.
- sketch the electric field lines for two point charges of equal magnitude with same or opposite signs.
- describe the concept of electric dipole.
- define and explain electric flux.
- describe electric flux through a surface enclosing a charge.

- state and explain Gauss's law.
- describe and draw the electric field due to an infinite size conducting plate of positive or negative charge.
- sketch the electric field produced by a hollow spherical charged conductor.
- sketch the electric field between and near the edges of two infinite size oppositely charged parallel plates.
- define electric potential at a point in terms of the work done in bringing unit positive charge from infinity to that point.
- define the unit of potential.
- solve problems by using the expression $V = \frac{W}{q}$.
- describe that the electric field at a point is given by the negative of potential gradient at that point.
- solve problems by using the expression $E = \frac{V}{d}$.
- derive an expression for electric potential at a point due to a point charge.
- calculate the potential in the field of a point charge using the equation $V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}$.
- define and become familiar with the use of electron volt.
- define capacitance and the farad and solve problems by using C=Q/V.
- describe the functions of capacitors in simple circuits.
- solve problems using formula for capacitors in series and in parallel.
- explain polarization of dielectric of a capacitor.
- demonstrate charging and discharging of a capacitor through a resistance.
- prove that energy stored in a capacitor is W=1/2QV and hence $W=1/2CV^2$.

INTRODUCTION

Electrostatics is a branch of physics in which we study about the charges at rest. In this unit, we will not only discuss the behaviour of charged particles at rest but also introduce laws governing it. For example, Coulomb's law which explains the electrostatic force of attraction or repulsion between two stationary point charges separated by some distance. Here a general question arises, how is the electrostatic

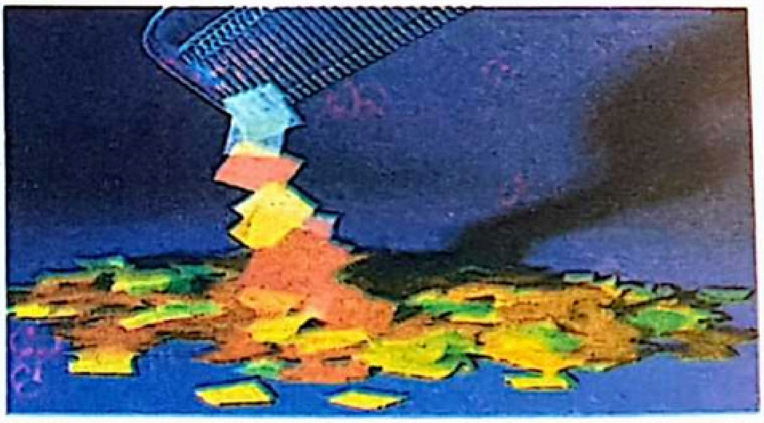
> ***FOR YOUR INFORMATION***
> Sometime a static electricity causes a big problem at gasoline filling station, where the spark can ignite gasoline vapours and make fire. A good safety rule is to touch metal and discharge static charge from your body before you fuel.

force exerted when the charged particles are separated? This "action at a distance" force can be explained by using the electric field theory, i.e. every charge q produces an electric field around it. Now if another charge q_o is brought near to q, then the electric field of q exerts a force on q_o. In the same way, we will also develop the concept of electric field in terms of electric lines of force. These electric lines of force can be studied by using Gauss's law. We will also study these electric lines of force passing through different conducting media called flux. All these are very useful in the understanding of various electrostatic phenomena. Similarly, we will explain the electric potential gained by a charged particle due to work done on it in an electric field between a pair of parallel and opposite charged plates. In the last section of this unit, we will introduce the most important device capacitor and its capacitance in the absence and presence of dielectrics.

11.1 CHARGE AND ITS PROPERTIES

Like mass, electric charge is the intrinsic property of some particles such as electrons etc. The presence of charge causes it to exert or experience a force when it is placed in an electric or magnetic field. Charge is a basic property of matter which is responsible for all electric and magnetic interactions.

It is a common experience that when we comb our hair by a plastic comb, then it may attract the nearby small pieces of paper. It is therefore, the comb has gained charges through the rubbing process. The charge which is created by rubbing two objects against each other is called static charge. There are two kinds of charges, which were named as negative and positive by Benjamin Franklin (1706-1790). It is explained by a



A charged comb attracts small pieces of paper.

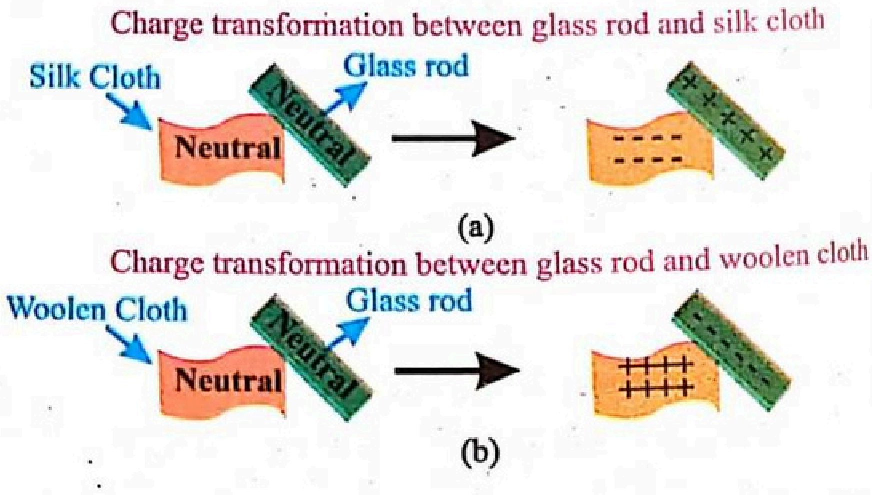


Fig.11.1(a) Transfer of negative charges from glass rod to silk cloth under the process of rubbing (b) transfer of charges from woolen cloth to plastic rod.

simple experiment. When a glass rod is rubbed with silk cloth, electrons are transferred from the glass rod to the silk cloth. The silk cloth now has an excess of electrons and hence becomes negatively charged. Whereas, the glass rod which has deficiency of electrons becomes positively charged as shown in Fig.11.1(a). Similarly, when a hard plastic rod is rubbed with woolen cloth then the wool becomes positively charged whereas, the plastic rod becomes negatively charged. It is shown in Fig. 11.1(b).

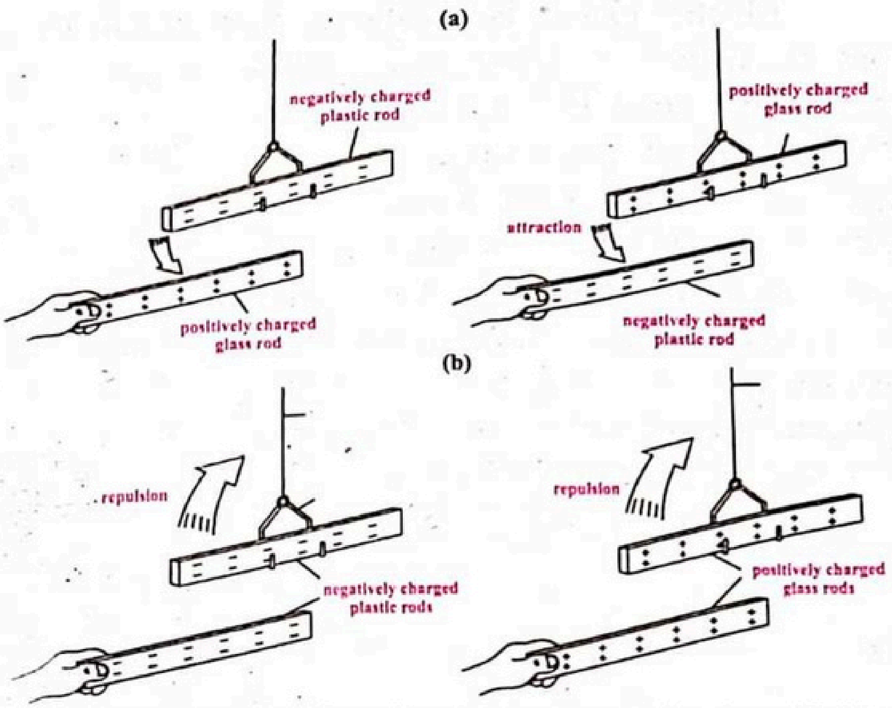


Fig.11.2(a) Unlike charged rods attract to each other. **(b)** like charged rods repel to each other.

If the positively charged glass rod is brought near to the negatively charged suspended plastic rod then it will attract the plastic rod as shown in Fig. 11.2(a). Similarly, when the negatively charged plastic rod is brought near to the suspended plastic rod which is also negatively charged then they will repel each other as shown in Fig.11.2(b) The same result of repulsion will be observed when two positively charged glass rods are brought near each other. Based on these observations, we have two most important results. First, like charges repel each other and unlike charges attract each other. Secondly, if one material gains some charges, the same magnitude of charges is lost by the other material during the rubbing process. This is known as law of conservation of charges, which states that **"electric charge can neither be created nor be destroyed or the total charges of an isolated system remains constant."**

DO YOU KNOW?

A charge is transferred from one body to another by electrons.

POINT TO PONDER

If an object transfers its charges to another object, will its mass be affected?

POINT TO PONDER

If 2×10^9 electrons are transferred from a glass rod to a piece of silk by rubbing, what will be the net charges on both objects?

FOR YOUR INFORMATION

Like energy and momentum, charge is also a conserved and quantized quantity.

Electric charge is a scalar quantity and its SI unit is coulomb 'C'. As we know that electron is a sub-atomic particle of an atom, it carries negative charge of magnitude 1.6×10^{-19} C. A proton is also a sub-atomic particle of an atom and it carries positive charge of magnitude 1.6×10^{-19} C. This shows that electron and proton both have charges of same magnitude but opposite in nature. One coulomb corresponds to the amount of charge on 6.25×10^{18} electrons. As an atom contains equal number of protons and electrons, so it is electrically neutral at normal condition. It has been observed that the charges are transferred from one body to another body in discrete form. That is, a body may have charges of value $\pm 1e, \pm 2e, \pm 3e \ldots$ respectively but not $\pm 0.5e, \pm 1.5e, \pm 2.1e$ ... etc. where e is charge of an electron or proton.

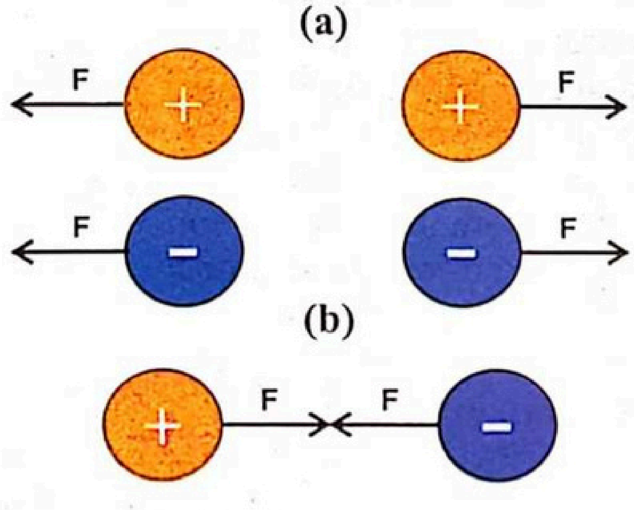


Fig.11.3(a) repulsive force between two like point charges **(b)** attractive force between two unlike point charges.

11.2 COULOMB'S LAW

As we have discussed that the like charges repel and unlike charges attract each other with a force called electrostatic force of repulsion and attraction respectively. This force was first quantitatively studied by a French military engineer Charles Coulomb under different conditions and formulated a law in 1785 A.D. known as Coulomb's Law, which is stated as:

The magnitude of electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance between them.

> ***POINT TO PONDER***
> Does an electrostatic force exist between a charged and an uncharged bodies?

Mathematically, it is expressed as:

Let two point charges q_1 and q_2 which are separated by a distance 'r' as shown in Fig.11.3. According to Coulomb's law the electrostatic force of attraction or repulsion between them is given as:

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

> ***DO YOU KNOW?***
> Both electrostatic force between charges and gravitational forces between masses obey inverse square law.

By combining these two relations,

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k\frac{q_1 q_2}{r^2} \quad ......(11.1)$$

where ' k ' is constant of proportionality known as Coulomb's or electrostatic constant. Its value depends upon the nature of the medium between the charges. If the medium between the two point charges is free space then the value of ' k ' in terms of SI units is 9×10^9 Nm^2/C^2 and it can be calculated by the following relation:

$$k = \frac{1}{4\pi\varepsilon_o} \quad ......(11.2)$$

Putting the value of electrostatic constant 'k' from Eq. 11.2 in Eq. 11.1

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \quad ......(11.3)$$

where ' ε_o ' is known as permittivity of free space and its value is 8.85×10^{-12} $C^2N^{-1}m^{-2}$.

> ***POINT TO PONDER***
>
> What would be the magnitude of electrostatic force between two point charges, when the separation between them is doubled?

As force is a vector quantity and the line of action of electrostatic force is along the position vector ' $\vec{r}$ ', so eq. 11.3 can be expressed in term of the unit vector of $\hat{r}$ as;

$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r} \quad ......(11.4)$$

Eq. (11.4) is the vector form of Coulomb's law.

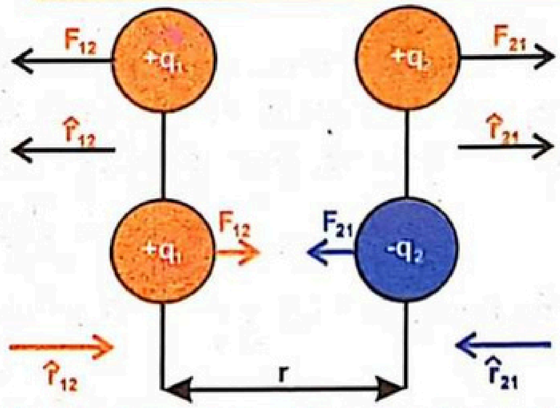


Fig.11.4 The charges q_1 and q_2 exert forces each other which are same in magnitude but in opposite direction.

The electrostatic force between two point charges is a mutual force, i.e. both charges exert the forces on each other, which are same in magnitude but in opposite directions. Let $\vec{F}_{12}$ to be the force which is exerted by q_1 on q_2 along the line joining the two point charges represented by unit vector $\hat{r}_{12}$ as shown in Fig.11.4. Similarly, $\vec{F}_{21}$ be the force which is exerted by q_2 on q_1 along the line joining the two points charges represented by unit vector $(-\hat{r}_{21})$. The negative sign shows that $\hat{r}_{21}$ in opposite direction with respect to $\hat{r}_{12}$. Thus by using Eq. 11.4 the two forces $\vec{F}_{12}$ and $\vec{F}_{21}$ can be expressed as:

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{12} \quad ......(11.5)$$

Similarly,

$$\vec{F}_{21} = \frac{-1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{21} \quad ......(11.6)$$

Comparing eq. 11.5 and eq. 11.6

$$\vec{F}_{12} = -\vec{F}_{21} \quad ......(11.7)$$

Equation 11.3 shows that the electrostatic force not only depends upon the magnitude of charges and distance between the charges but also depends upon the medium between the charges. If the medium between the charges is an insulating material which is known as dielectric, then we introduce a relative permittivity ε_r which is defined as the ratio between the permittivity of insulating medium ε to the permittivity of free space ε_o, that is:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o} \quad ......(11.8)$$

Table 11.1
The values of relative permittivity for different materials for static electric field

Material	Relative Permittivity (ε_r)
Vacuum	1.0000
Air	1.0006
PTFE, FEP (Teflon)	2.0
Polypropylene	2.20 to 2.28
Polystyrene	2.4 to 3.2
Wood (Oak)	3.3
Bakelite	3.5 to 6.0
Wood (Maple)	4.4
Glass	4.9 to 7.5
Wood (Birch)	5.2
Glass-Bonded Mica	6.3 to 9.3
Porcelain, Steatite	6.5

Relative permittivity is also known as dielectric constant and its value is always greater than one (as shown in table 11.1). For static electricity, the electrostatic force ' F′ ' between point charges for insulating medium is given as

$$\vec{F'} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}\hat{r}$$

Using Equation 11.6 $\varepsilon = \varepsilon_o\varepsilon_r$. Therefore

$$\vec{F'} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^2}\hat{r} \quad ...(11.9)$$

By comparing Eq. 11.4 and 11.7 we have

$$\vec{F'} = \frac{1}{\varepsilon_r}\vec{F} \quad ......(11.10)$$

Since, $\frac{1}{\varepsilon_r} < 1$

So, $\vec{F'} < \vec{F}$

This result shows that the electrostatic force of attraction or repulsion between two point charges is greater, if the medium between them is a free space.

Example 11.1

How many electrons constitute a charge of 2C.

Solution:

number of electrons (n) = ?

total charge = Q = 2C

charge on an electron = e = 1.6×10^{-19}C

charge on 'n' electrons = Q = ne

$$n = \frac{Q}{e}$$

$$n = \frac{2C}{1.6 \times 10^{-19} C}$$

$$n = 1.25 \times 10^{19} \text{ electrons}$$

Example 11.2

The Hydrogen atom consists of a single electron which is orbiting around the nucleus and contains a single proton in its nucleus. Calculate the ratio between electrostatic force and gravitational force between electron and proton in hydrogen atom.

Solution:

Charge on an electron = $q_1 = 1.6 \times 10^{-19}$C

Charge on a proton = $q_2 = 1.6 \times 10^{-19}$C

Mass of electron = $m_1 = 9.11 \times 10^{-31}$ kg

Mass of proton = $m_2 = 1.67 \times 10^{-27}$ kg

Coulomb's constant (k) = 9×10^9 Nm^2 c^{-2}

Gravitational constant (G) = 6.673×10^{-11} Nm^2 kg^{-2}

Distance between electron and proton = r

Now, the electrostatic force between electron & proton is given as

$$F_e = k\frac{q_1 q_2}{r^2} \ldots\ldots(i)$$

Similarly, the gravitational force between electron & proton is given as

$$(F_g) = G\frac{m_1 m_2}{r^2} \ldots\ldots(ii)$$

Dividing eq. (i) by eq. (ii)

$$\frac{F_e}{F_g} = \frac{k\frac{q_1q_2}{r^2}}{G\frac{m_1m_2}{r^2}}$$

$$\frac{F_e}{F_g} = \frac{kq_1q_2}{Gm_1m_2}$$

$$\frac{F_e}{F_g} = \frac{\left(9\times10^{9}\,Nm^2c^{-2}\right)\left(1.6\times10^{-19}C\right)\left(1.6\times10^{-19}C\right)}{\left(6.673\times10^{-11}\,Nm^2kg^{-2}\right)\left(9.11\times10^{-31}kg\right)\left(1.67\times10^{-27}kg\right)}$$

$$\frac{F_e}{F_g} = \frac{2.304\times10^{-28}}{1.014\times10^{-67}}$$

$$\frac{F_e}{F_g} = 2.27\times10^{39}$$

POINT TO PONDER

Does an electric field exist in empty space?

DO YOU KNOW?

There is no electric field inside the conductor.

The result shows that the electrostatic force between an electron and a proton is extremely large i.e. 2.27×10^{39} times greater than the gravitational force between them for the given same distance.

11.3 ELECTRIC FIELD AND ELECTRIC FIELD INTENSITY

In Coulomb's law, we have studied the electrostatic force of attraction and repulsion between every two point charges. According to field theory, the electrostatic force of attraction and repulsion is transmitted from one charged body to another through a field called electrostatic field, which exists in the region around every electric charge. Electric field is defined as, **the region or space around a charged body in which another charged body can experience force of attraction or repulsion.**

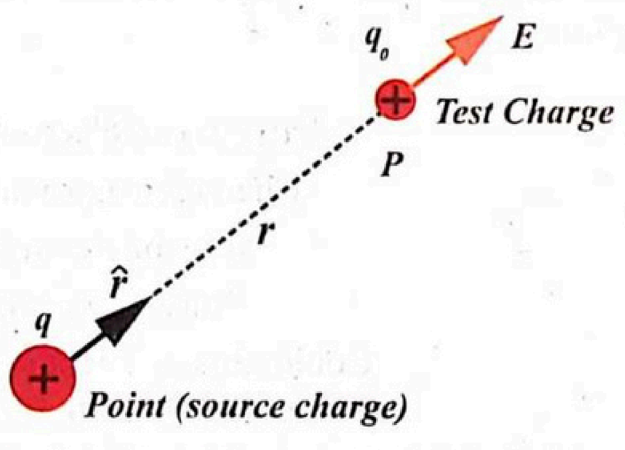


Fig.11.5 Electric field intensity E due to a force by a point charge q on a test charge q_0.

The charge which has a considerable electric field around it, is known as a point charge. It may be positive or negative. Usually, the point charge is being considered as a source charge. Similarly, the charge which has negligible field is known as test charge. It is always taken as a unit positive charge and it is being used to determine the strength of the field of the point charge. The source charge has a very large value of charge as compared with test charge. We assume that test charge moves in the electric field of source charge without causing any appreciable change in its field.

The strength of the electric field at any point is known as electric field intensity or electric intensity. Electric field strength is a vector quantity and it has both magnitude and direction. It is explained as:

Let an electric field is produced by a positive point charge 'q' in the space around it. In order to examine whether, the field exists in space, we introduce a unit positive test charge q_o. As the test charge is very very small in magnitude and it does not affect or disturb the field of the point charge so the electrostatic force is exerted by a point (source) charge on a test charge as shown in Fig.11.5 and it is known as electric intensity. It is represented by $\vec{E}$ and is also defined as a force per unit positive charge q_0 i.e.,

$$\vec{E} = \frac{\vec{F}}{q_o} \quad ......(11.11)$$

Electric intensity is a vector quantity. Its direction is the same as that of the force. It is directed outwards from positive source charge and directed inwards for a negative source charge. The SI unit of electric intensity is NC^{-1}.

11.4 ELECTRIC INTENSITY DUE TO A POINT CHARGE

Consider an electric field due to a point charge q as shown in Fig.11.7. To determine the strength of this field at point 'p' at a distance 'r' from surface of point charge 'q', we place a test charge 'q_o' at that point. Now according to Coulomb's law, the electrostatic force between them is given as

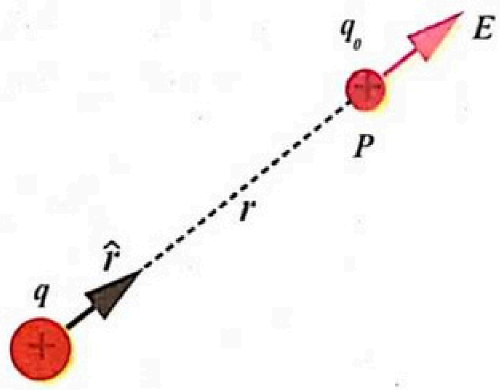


Fig.11.7 Electric intensity due to a point charge q at point 'P' at a distance r from q.

$$F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

$$\frac{F}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

But $$E = \frac{F}{q_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad ......(11.12)$$

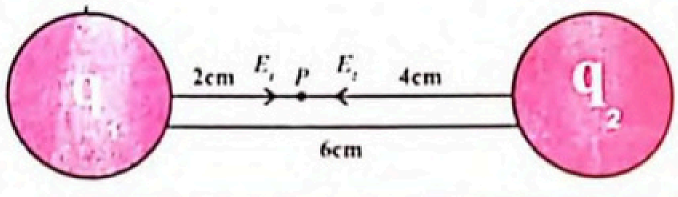
As $\vec{E}$ is a vector quantity, its direction is same as that of the force. So, eq. 11.12 can be expressed in vector form as

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} \quad ...(11.13)$$

This results shows that electric intensity depends upon magnitude of point charge, distance and medium.

Example 11.3

Two charges of the magnitude 15×10^{-9}C and 30×10^{-9}C are separated by a distance 6cm. Find the intensity of electric field at a point which is at a distance 2cm from the first and 4cm from the second charge respectively.



> *FOR YOUR INFORMATION*
> If the charges on a conductor are at rest then the conductor is in static equilibrium.

Solution:

$$\because q_1 = 15 \times 10^{-9}C$$
$$q_2 = 30 \times 10^{-9}C$$

Distance between q_1 & $q_2 = r = 6cm = 0.06m$

Distance between q_1 & $P = r_1 = 2cm = 0.02m$

Distance between q_2 & $P = r_2 = 4cm = 0.04m$

1. Electric intensity E_1 due to q_1 at point P and at distance $r_1 = 0.02m$ from q_1 is given as

$$E_1 = k\frac{q_1}{r_1^2} = \frac{9\times10^9\times15\times10^{-9}}{(0.02)^2} = \frac{135}{0.0004}$$
$$E_1 = 3.375 \times 10^5 NC^{-1}$$

2. Electric intensity E_2 due to q_2 at point P and at a distance $r_2 = 0.04m$ from q_2 is given as

$$E_2 = k\frac{q_2}{r_2^2} = \frac{9\times10^9\times30\times10^{-9}}{(0.04)^2} = \frac{270}{0.0016}$$
$$E_2 = 1.69 \times 10^5 NC^{-1}$$

The resultant intensity 'E' at point P is given by

$$E = E_1 - E_2$$

$$E = 3.375 \times 10^5 - 1.69 \times 10^5$$

$E = 1.685 \times 10^5\ NC^{-1}$ which is directed towards the larger charge i.e., q_2.

11.5 ELECTRIC FIELD LINES:

We have discussed that an electric field is a region around a point charge where an electrostatic force acts on another charged body placed in such region. The electric field is represented by imaginary lines which was introduced by Michael Faraday and these are called electric lines of force. **An electric line of force is the path along which a small positive test charge, placed in an electric field of a point charge can move if free to do so.** The lines of force of an electric field may be straight or curved but its direction is always from positive to negative charged body. It is explained under the following different field patterns.

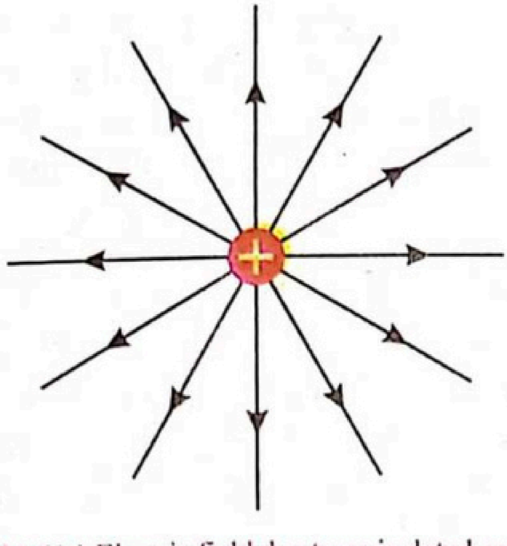


Fig.11.6(a) Electric field due to an isolated positive charge (+q) and the direction of lines are outwards.

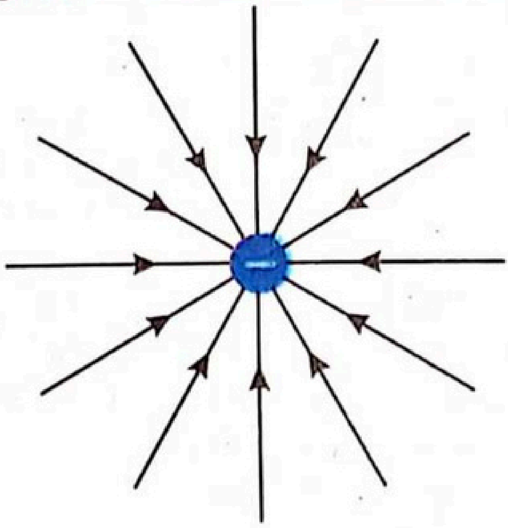


Fig.11.6(b) Electric field due to an isolated negative charge (−q) and the direction of lines are inwards.

When a field is produced by an isolated positive charge (+q) then the lines of force are directed radially outward from the charge in all directions as shown in Fig.11.6(a). It is therefore, when we introduce a positive test charge in this filed, it would be repelled by a point charge in outward direction.

Similarly, when a field is due to an isolated negative charge $(-q)$, then the lines of force of the field are directed radially inward as shown in Fig.11.6(b). If we place a positive test charge in this field, it would be attracted by the point charge in inward direction.

If the field is due to two charges of same magnitude, but one is positive and other is negative then the lines of force start from positive charge surface and terminate at the negative charge surface, as shown in Fig.11.6(c). In this case, we can observe both straight and curved lines. The direction of field at any point along a curved line can be determined by drawing a tangent at that point.

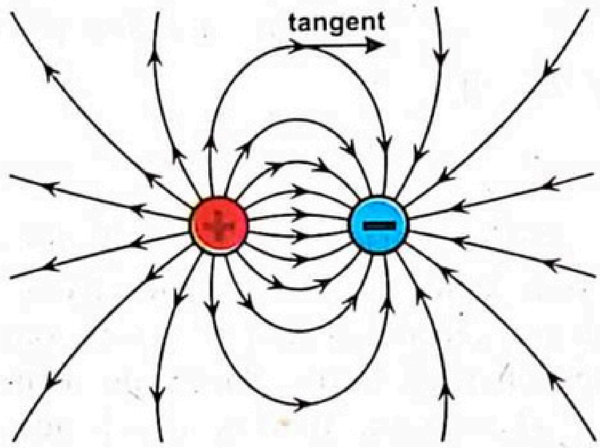


Fig.11.6(c) The electric field pattern due to two charges of same magnitude but having opposite

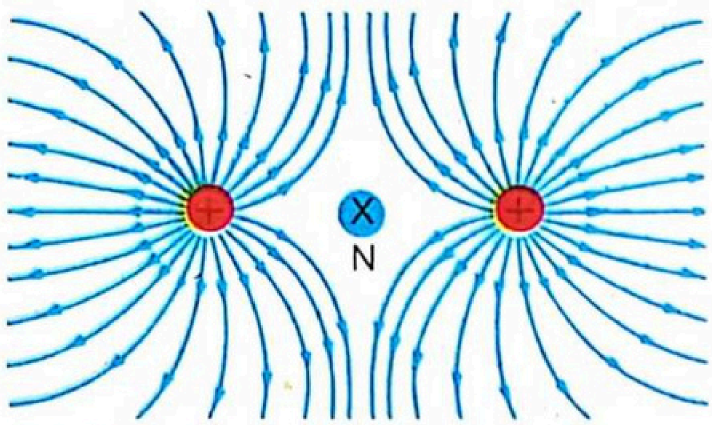


Fig.11.6(d) The electric field pattern due to two charges of same magnitude and having same signs.

The Fig.11.6(d) shows the field pattern due to two positive charges of same magnitude. In this case the lines of force are repelled to one another. Again, we can observe both straight and curved lines. The strength of the field at the middle point between the two charges is zero. This point is called neutral point of the field.

From the above discussion, we have some important results which are summarized as:

1. The electric lines of force initiate from positive charge surface and terminate at negative charge surface.
2. The number of lines drawn is directly proportional to the magnitude of the charge.
3. Curved lines show non-uniform electric field while straight and parallel lines indicate uniform electric field.
4. The tangent at any point on a curved line gives the direction of the field.
5. The lines of force do not exist inside the charged body.
6. The lines of force are closer when the field is stronger, and the lines are farther apart when the field is weaker.
7.. Two lines of force never cross each other. If they cross then at the point of intersection, electric intensity would have more than one direction but only one magnitude and it is not possible.

11.6 ELECTRIC DIPOLE

A pair of charges of same magnitude, but one is positive and other is negative separated by a small distance is known as electric dipole. For example, many molecules such as, H_2O, HCl etc. behave as electric dipoles in their normal states. The charges are distributed in these molecules in such a way that the centre of positive charge does not coincide with centre of negative charge, as a result, one end of the molecule is positively charged and the other is negatively charged as shown in Fig. 11.7.

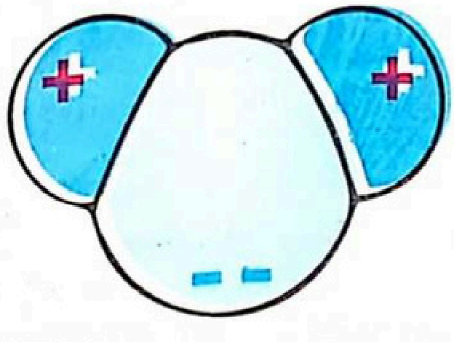


Fig.11.7.A molecule of H_2O behaves as an electric

Let an electric dipole which consists of two charges of equal magnitude but opposite nature or signs (q and −q) separated by a small distance 'd' as shown in Fig.11.8. In electric dipole, the resultant charge is equal to zero, but its corresponding field exists, because the charges are placed at some distance. The line joining the charges is known as dipole axis. It is a vector and its direction is from negative charge to positive charge.

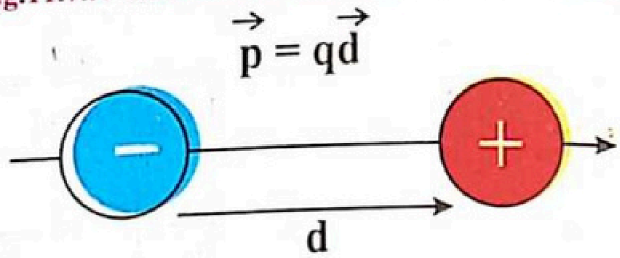


Fig.11.8 Electric dipole due to two point charges of same magnitude but having opposite signs separated by a distance d.

Now the product of magnitude of either of the charge and the distance of separation between the two charges is known as dipole moment. It is represented by $\vec{p}$ and it is expressed as

$$\vec{P} = q\vec{d} \quad ......(11.14)$$

Dipole moment is a vector quantity. Its direction is along the dipole axis, i.e., from negative charge to positive charge. The SI unit of a dipole moment is C m, which is a larger unit. However, a smaller unit used for dipole moment is debye (D).

11.6.1 Electric field due to a dipole

Let an electric field due to a dipole which consists of two charges of equal magnitude, but of opposite nature or signs and are separated by a distance 2d from each other. Let we calculate the strength of this field at point 'C' at a distance 'r' from centre 'O' of the dipole. Such that C is at a distance r – d from the positive charge +q and at a distance r + d from the negative charge −q as shown in Fig.11.9.

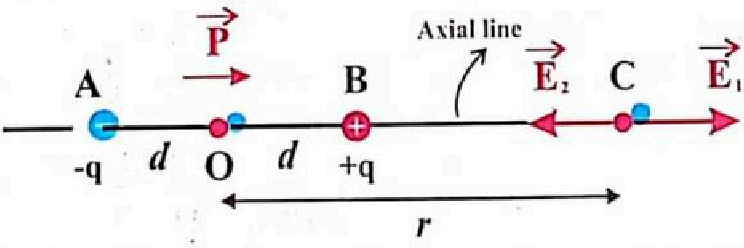


Fig.11.9 Resultant Electric field strength at point C due to a dipole.

The field due to positive charge +q at point 'C' at a distance r – d is given by

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} \quad ......(11.15)$$

Similarly, the field due to negative charge $-q$ at point 'C' and at a distance r + d is given by

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{-q}{(r+d)^2} \quad ......(11.16)$$

The resultant field at point C is given by;

$$E = E_1 + E_2$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{(r+d)^2-(r-d)^2}{(r-d)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{4rd}{\left(r^2-d^2\right)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{4rd}{\left(r^2\left(1-\frac{d^2}{r^2}\right)\right)^2}\right)$$

As $\frac{d^2}{r^2}$ is very small so it can be neglected.

$$E = \frac{1}{4\pi\varepsilon_o}\frac{2dq(2r)}{r^4(1-0)}$$

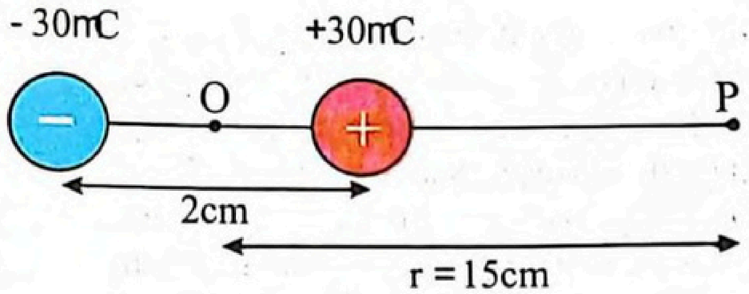
But $(2d)(q) = P$ (Dipole moment)

$$E = \frac{1}{2\pi\varepsilon_o}\frac{P}{r^3} \quad ......(11.17)$$

This is the electric field intensity at a point due to a dipole, which depends upon dipole moment, distance and medium.

Example 11.4

An electric dipole consists of two charges $+30\mu C$ and $-30\mu C$ separated by a distance of 2cm. Calculate the electric field intensity at a point P on the axial line at a distance of 15cm from the mid-point of the dipole.



Solution:

We have,

$$\text{Charge, } q_1 = +30\mu C = 30\times10^{-6}C$$

$$\text{Charge, } q_2 = -30\mu C = -30\times10^{-6}C$$

$$\text{Distance between dipole} = d = 2cm = 0.02m$$

$$\text{Electric field intensity at point } P = E = ?$$

$$\text{Distance between mid-point of dipole and point } P = OP = r = 15cm = 0.15m$$

$$\text{Electrostatic constant} = k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 Nm^2C^{-2}$$

By definition of electric intensity due to a dipole

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3}$$

But $P = qd$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{r^3}$$

$$E = \frac{9\times10^9 Nm^2C^{-2}\,2\left(30\times10^{-6}C\times0.02m\right)}{(0.15m)^3}$$

$$E = \frac{10.8\times10^3}{0.003375}$$

$$E = 3200\times10^3$$

$$E = 3.2\times10^6 NC^{-1}$$

11.7 ELECTRIC FLUX

We have studied that an electric field is represented by electric lines of force, which originate from a positively charged surface and terminate at a negatively charged surface. Now when an object having an area is placed in an electric field such that the direction of the area is parallel to the direction of electric field as shown in Fig.11.10 then the electric lines of force pass through it and is called electric flux. The word flux comes from a Latin word which means to flow or penetration of field lines through some surface. **Thus, the electric flux is defined as the number of electric lines of force passing through a certain area held perpendicular to the field.**

It is usually represented by a Greek letter ϕ and it depends upon the strength of the field (Electric intensity), the size of the surface area it passes through, and on how the area is oriented with respect to the field. That is, stronger the electric field $\vec{E}$, larger will be the electric flux ϕ. Similarly, the larger the area, the more field lines pass through it, the greater the flux.

Based on these results, the electric flux is defined as the product of electric intensity E and vector area A.

$$\phi = EA \quad \ldots(11.18)$$

Now if the area is not held perpendicular but it makes an angle 'θ' with the field $\vec{E}$ as shown in Fig.11.11. In this case we take the base component A_x of area $\overline{A}$ which is parallel to the

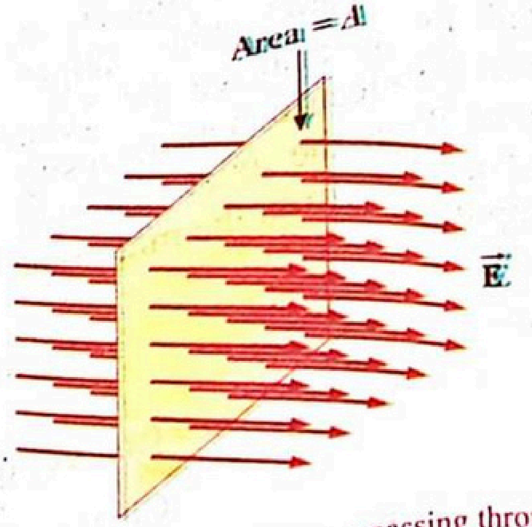


Fig.11.10 Electric lines of forces passing through a surface area perpendicular to the field.

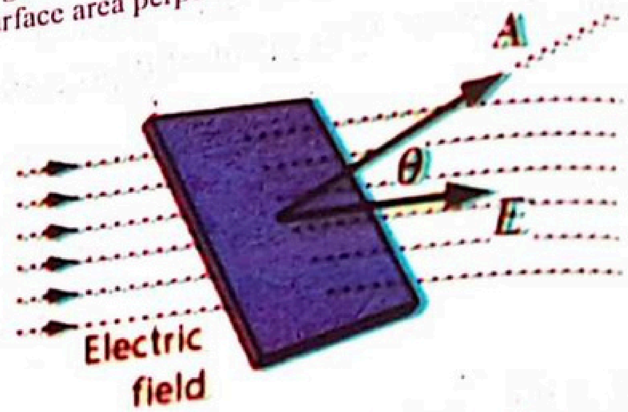


Fig.11.11 Lines of force passing through an area and the direction of area is at an angle 'θ' with the

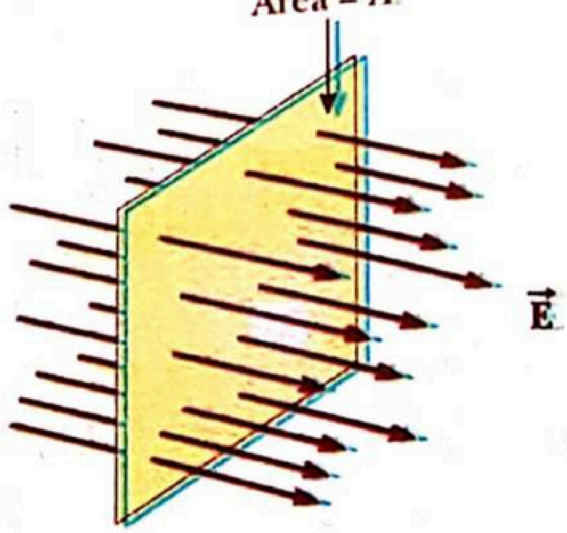


Fig.11.12 Angle θ between E and A is zero and the flux is maximum.

electric lines of force and thus Eq.11.18 becomes

$$\phi = EA_x$$

$$\phi = E\,A\cos\theta \quad ......(11.18)$$

$$\phi = \overline{E}\cdot\overline{A} \quad ......(11.19)$$

This is an electric flux in terms of the scalar product of $\vec{E}$ and $\overline{A}$. Thus Electric flux is a scalar quantity and its SI unit is Nm^2C^{-1}. The flux can be studied under the following two cases:

Case-I: When area is held perpendicular to $\vec{E}$ such that the vector area $\overline{A}$ becomes parallel to the direction of electric lines of force and angle 'θ' between them is zero as shown in Fig.11.12. Then Eq.11.18 becomes

$$\phi = EA\cos 0^\circ$$

$$\phi = EA$$

This is the maximum electric flux.

Case-II: When surface area held parallel to the electric field such that the vector area becomes perpendicular to the direction of electric lines of force and angle 'θ' between them is 90^0 as shown in Fig.11.13. Then Eq.11.16 becomes

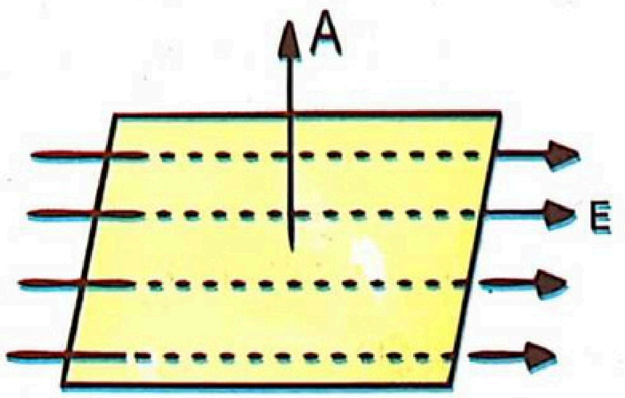


Fig.11.13 Angle θ between E and A is 90^0 and the flux is minimum.

$$\phi = EA\cos 90^\circ$$

$$\phi = 0$$

This is the minimum electric flux or zero flux.

11.8 ELECTRIC FLUX THROUGH A SURFACE ENCLOSING A CHARGE

Consider a point charge q which is enclosed by spherical shape surface of radius 'r' as shown in Fig.11.14. Since the charge is at the centre of the sphere so the electric intensity 'E' at the whole spherical surface remains same (constant). To calculate the total flux due to this point charge, we divide the whole spherical surface of the sphere into 'n' number of small and equal patches of

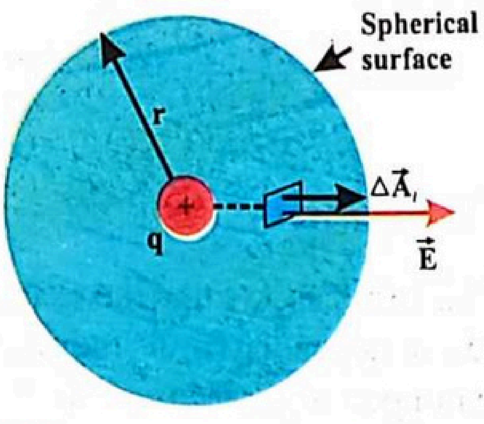


Fig.11.14 A point charge q at the center of the sphere.

area $(\Delta A_1, \Delta A_2, \Delta A_3, \cdots, \Delta A_n)$. It is to be noted that the radius of the sphere is always perpendicular to each small patch. Therefore, the direction of vector area through each patch remains parallel to the direction of intensity $\vec{E}$ and angle 'θ' between them is zero.

Thus, the flux ϕ_1 through small patch of area ΔA_1 is given by

$$\phi_1 = \vec{E} \cdot \Delta\vec{A}_1$$

$$\phi_1 = E\Delta A_1 \cos 0^0$$

$$= E\Delta A_1$$

Similarly, the flux ϕ_2 through ΔA_2 is given by

$$\phi_2 = \vec{E} \cdot \Delta\vec{A}_2 = E\Delta A_2$$

$$\phi_3 = \vec{E} \cdot \Delta\vec{A}_3 = E\Delta A_3$$

.

.

.

$$\phi_n = \vec{E} \cdot \Delta\vec{A}_n = E\Delta A_n$$

POINT TO PONDER

The electric lines of force that can pass through an area, is named as electric flux. Can the same lines of force pass through a volume?

Hence, the total flux

$$\phi = \phi_1 + \phi_2 + \phi_3 \cdots \phi_n$$

$$\phi = E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + \cdots + E\Delta A_n$$

$$\phi = E(\Delta A_1 + \Delta A_2 + \Delta A_3 + \cdots + \Delta A_n)$$

$$\phi = \frac{q}{4\pi\varepsilon_o r^2}\sum_{i=1}^{n}\Delta A_i$$

But surface area of sphere = $\sum \Delta A_i = 4\pi r^2$

$$\phi = \frac{q}{4\pi\varepsilon_o r^2} \cdot 4\pi r^2$$

$$\phi = \frac{q}{\varepsilon_o} \quad \ldots\ldots(11.20)$$

This result shows that the electric flux due to a point charge through the surface of sphere is independent of its radius. It means, the flux through a closed surface does not depend upon a geometrical shape but it depends upon magnitude of charge and medium used.

11.9 GAUSS'S LAW

We have studied in the previous section that the flux due to a point charge at the centre of the sphere is q/ε_o. This shows that the flux is independent of the radius. Therefore, this result can be extended for 'n' number of point charges $q_1, q_2, q_3 \ldots q_n$ which are arbitrarily distributed in a closed surface of non-geometrical or non-uniform shape as shown Fig.11.15. To calculate the total flux due to all the given point charges, we enclose each point charge in a small imaginary sphere within the boundary of the closed surface as shown in Fig.11.16. The surface of this imaginary sphere enclosing the charges is called gaussian surface.

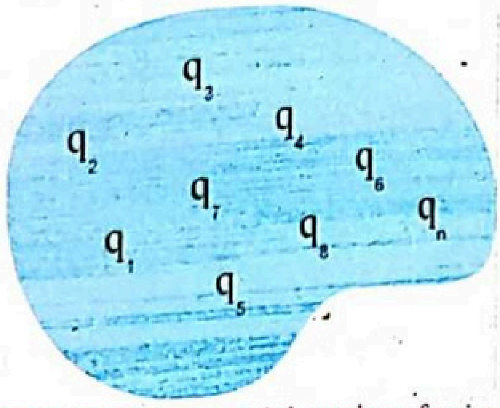


Fig.11.15 There are 'n' number of point charges enclosed by a surface of non-geometrical shape.

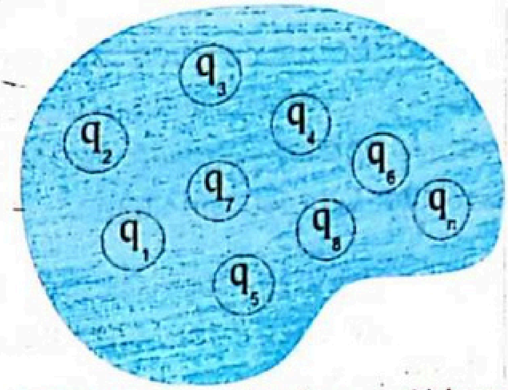


Fig.11.16 The point charges which are enclosed in small spheres.

Thus, the flux ϕ_1 due to a point charge q_1 which is enclosed by a small sphere is given by

$$\phi_1 = \frac{q_1}{\varepsilon_o}$$

Similarly, the flux ϕ_2 due to a point charge q_2 is given by

$$\phi_2 = \frac{q_2}{\varepsilon_o}$$

and $$\phi_3 = \frac{q_3}{\varepsilon_o}$$

$$\vdots$$

$$\phi_n = \frac{q_n}{\varepsilon_o}$$

Total flux can be obtained by adding above equations i.e.,

$$\phi = \phi_1 + \phi_2 + \phi_3 + \cdots + \phi_n$$

$$\phi = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + \frac{q_3}{\varepsilon_o} + \cdots + \frac{q_n}{\varepsilon_o}$$

$$\phi = \frac{1}{\varepsilon_o}(q_1 + q_2 + q_3 + \cdots + q_n)$$

$$\phi = \frac{1}{\varepsilon_o} \text{ (total charge enclosed in the surface)}$$

$$\phi = \frac{1}{\varepsilon_o} (Q) \quad \ldots\ldots(11.21)$$

This is the mathematical form of Gauss's law and it is stated as '**the total flux through a closed surface is equal to $1/\varepsilon_o$ times the total charge enclosed by the surface**'.

11.10 APPLICATIONS OF GAUSS'S LAW

Gauss's law can be applied to calculate the electric intensity due to the different configuration of the charges. Though the charges must be enclosed by a surface called 'Gaussian Surface', but it is not necessary that the surface should have a regular geometrical shape.

(a) Electric field due to an infinite charged sheet

Consider a thin infinite charged sheet of uniform surface charge density σ which is given by

$$\sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{A} \quad \ldots\ldots (11.22)$$

To calculate the electric field $\overline{E}$ due to this charged sheet at point 'p' near the sheet, we pass a cylinder of cross-section area 'A' through the sheet. This closed cylinder acts as a Gaussian surface as shown in Fig.11.17.

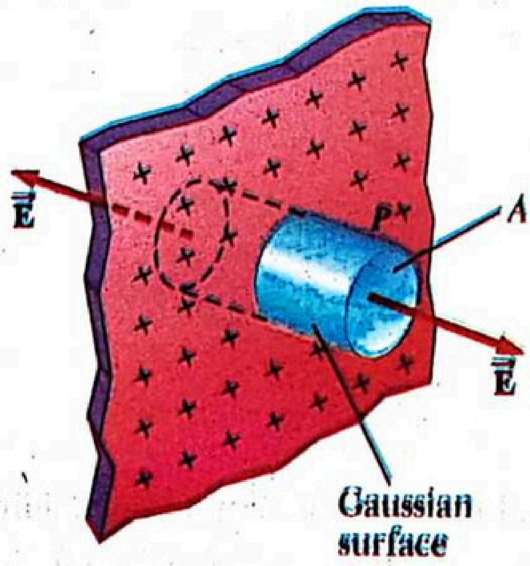


Fig.11.17 A Gaussian-Surface inform of a cylinder passed through a sheet of positive charge.

Since the cylinder is perpendicular to the plane of the sheet, so the direction of electric field is along the cylinder. Now we determine the electric flux through the two flats and one curved surface of the cylinder.

Flux due to the curved surface of the cylinder is

$$\phi_1 = E\,A_1 \cos\theta$$

where the angle 'θ' between electric intensity E and curved area A is 90^0 so

$$\phi_1 = E A_1 \cos 90^0 = 0$$

Similarly, the flux through the flat surfaces or end faces of the cylinder is given by

$$\phi_2 = E\, A_2 \cos\theta$$

The direction of flat surface of the cylinder is parallel to the electric intensity and angle 'θ' between them is zero

$\therefore$
$$\phi_2 = E\, A_2 \cos 0^0$$
$$\phi_2 = EA_2$$

The flux for the other flat surface of the cylinder is given by;

$$\phi_3 = EA_3$$

Total flux

$$\phi = \phi_1 + \phi_2 + \phi_3$$
$$\phi = 0 + EA_2 + EA_2$$

We know that $A_1 = A_2 = A =$ Area of circular surface of cylinder

$$\phi = 2EA \quad \text{......(11.23)}$$

Now, by definition of Gauss's Law

$$\phi = \frac{Q}{\varepsilon_o}$$

But from Eq.11.22; $Q = \sigma A$

$$\phi = \frac{\sigma A}{\varepsilon_o} \quad \text{......(11.24)}$$

Comparing Eq.11.23 and Eq.11.24

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

In vector form

$$\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{r} \quad \text{......(11.25)}$$

where $\hat{r}$ is a unit vector, perpendicular to the sheet and E is directed away from it. Similarly, if the given sheet is negatively charged then

$$\vec{E} = \frac{-\sigma}{2\varepsilon_o}\hat{r}$$

In this case $\vec{E}$ is directed toward the sheet.

The above results show that the electric field due to a charged sheet depends on its charged density and medium in which the sheet is placed.

(b) Electric field due to two parallel and opposite charged plates

Consider two charged plates of same size with equal and opposite surface charge densities $+\sigma$ and $-\sigma$. The plates are held closed and parallel to each other such that there is a uniform electric field between them.

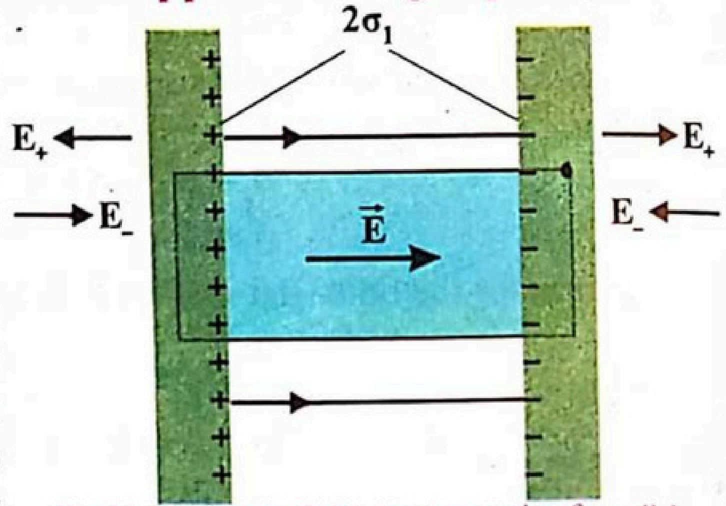


Fig.11.18 Electric field due to a pair of parallel and opposite charged plates.

To calculate the electric field due to these plates by using Gauss's law, we draw a Gaussian surface in the form of a box as shown in Fig.11.18. The electric field outside the plates is zero, because the direction of field (E_+) due to the positive plate is opposite to direction of field (E_-) due to the negative plate, However, there is uniform electric field inside the plates, it is therefore, the magnitude of E_+ and E_- at each point is the same inside the plates. Since each charge plate behaves as a charge sheet, so the field inside the plates due the positive charge plate is given by

$$E_+ = \frac{+\sigma}{2\varepsilon_o} \quad ......(11.26)$$

Similarly, the field inside the plates due to the negative charge plate is given by

$$E_- = -\left(\frac{-\sigma}{2\varepsilon_o}\right) \quad ......(11.27)$$

Negative sign shows that the direction of $\vec{E}_-$ is opposite to the direction of $\vec{E}_+$. Resultant field intensity E between the plates is given by

$$E = E_+ + E_-$$

$$E = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_o}$$

In vector form

$$\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{r} \quad ......(11.28)$$

Where $\hat{r}$ is a unit vector and it shows the direction of $\vec{E}$. i.e., E is directed from positive charged plate to negative charged plate.

(c) Electric field due to the hollow spherical charged conductor

Consider uniformly charged spherical shell of radius 'R'. The electric field due to this charged shell is calculated for two different cases.

I Electric field outside the shell

To calculate the electric field outside the shell, we surrounded it by a closed spherical Gaussian Surface of radius 'r' as shown in Fig.11.19(a) where r > R. Thus, the flux through the Gaussian Surface is given by

$$\phi = \vec{E} \cdot \vec{A}$$

In case of sphere, angle 'θ' between E and A is zero

So
$$\phi = E A \cos 0^{o}$$
$$\phi = EA$$

According to Gauss Law

$$\phi = \frac{q}{\varepsilon_o}$$

$$EA = \frac{q}{\varepsilon_o}$$

Here A = Area of sphere = $4\pi r^2$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad ......(11.29)$$

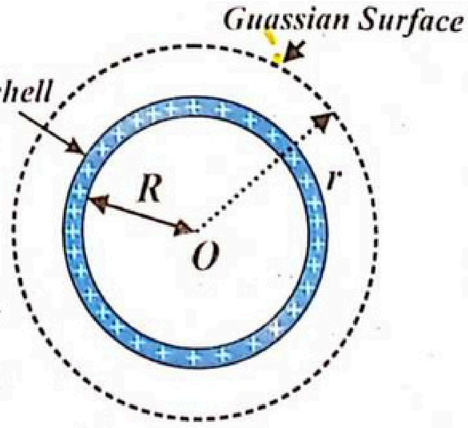


Fig.11.19(a) A uniformly charged spherical shell of radius R surrounded by Gaussian surface of radius r (r > R)

II Electric field inside the shells

Similarly, to calculate the electric field inside the shell, we consider a closed spherical Gaussian surface of radius 'r' and area 'A' inside the shell as shown in Fig.11.19(b) where R > r. Thus, by definition of flux

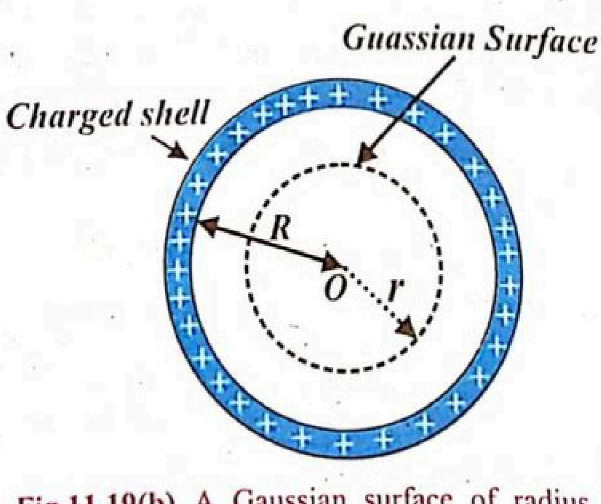


Fig.11.19(b) A Gaussian surface of radius r inside the uniformly charged shell of radius R (R > r)

$$\phi = EA' \quad ......(11.30)$$

But according to Gauss's Law

$$\phi = \frac{q}{\varepsilon_o} = 0 \quad ......(11.31)$$

It may be noted that Gaussian surface does not enclose any charge, therefore electric flux through it is zero. Comparing eq. 11.30 and eq. 11.31

$$E\,A' = 0$$

As $\quad A' \neq 0$

Therefore, $\quad E = 0$

This shows that the electric field inside the charged spherical shell is zero.

11.11 ELECTRIC POTENTIAL

We have studied the force by an electric field on a charged particle in the previous section. Now in this section we study the work done on the charged particle. Consider a uniform electric field due to parallel and opposite charged plates separated by a distance d. The direction of the field is from positive to negative plate. This field exerts a force (qE) on a small positive charge 'q' and the charge moves from point B to point A as shown in Fig.11.20. Now if the charge is to be displaced from point 'A' to point 'B' against the direction of the field, an external force with magnitude (qE) must be applied on it. In this way, the charge gains potential energy called electrical potential energy and it is represented by U.

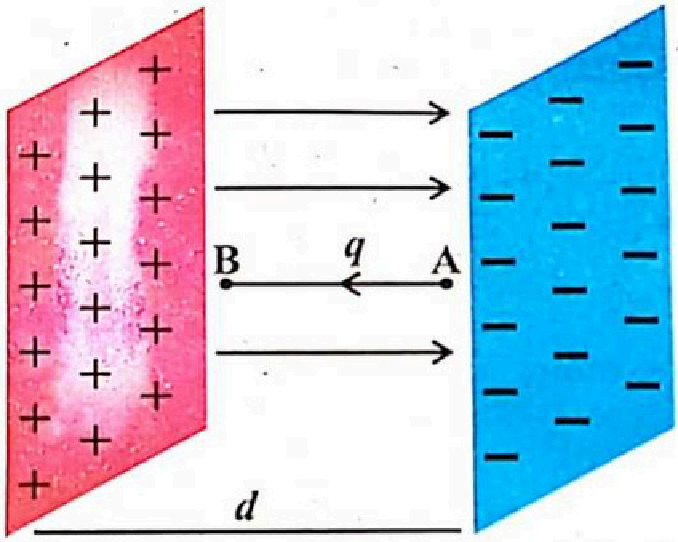


Fig.11.20 Electric field due to parallel and opposite charged plates where a charged particle 'q' is displacing against the direction of electric field E from point A to point B.

Let U_A be potential energy of the charge at point A and U_B be the potential energy of the charge at point B. Indeed, the work done by the applied force against the direction of field causes of change (increase) in electrical potential energy ΔU of the charged particle. Thus, according to work-energy principle,

$$\text{Work} = \Delta U$$

$$W_{AB} = U_B - U_A \quad ......(11.32)$$

Dividing Eq.11.32 by 'q'

$$\frac{W_{AB}}{q} = \frac{U_B}{q} - \frac{U_A}{q} \quad ......(11.33)$$

where $\frac{U_A}{q}$ is the potential energy per unit charge at point 'A' and $\frac{U_B}{q}$ is the potential energy per unit charge at point B. Thus, **the work done per unit charge by the applied force when charge moves against the direction of field is called electrical potential or potential difference.** It is represented by 'V' and hence Eq.11.33 becomes

$$\frac{W_{AB}}{q} = V_B - V_A$$

$$\frac{W_{AB}}{q} = \Delta V_{AB}$$

$$\Delta V = \frac{W}{q} \quad ......(11.34)$$

> ***POINT TO PONDER***
> What will be the value of electric potential when a positive charged particle moves in the direction of electric field?

This is an expression for electric potential difference between two points in an electric field.

Now if the point 'A' is at infinity that is, we bring the unit positive charge from infinity to point 'B' against the field. Then $V_A = 0$ and Eq.11.34 becomes

$$\Delta V = \frac{W}{q}$$

$$V_B - V_A = \frac{W}{q}.$$

$$V_B - 0 = \frac{W}{q}$$

$$V_B = \frac{W}{q}$$

or

$$V = \frac{W}{q} \quad ......(11.35)$$

Electric potential is a scalar quantity and its unit is volt.

Volt

The potential difference between two points is one volt, if one joule of work is done on a moving unit positive charge from one point to the other point against the direction of electric field. It is expressed as

$$1\text{volt} = \frac{1\text{joule}}{1\text{coulomb}}$$

11.12 ELECTRIC FIELD AS POTENTIAL GRADIENT

As the gravitational potential energy of a mass depends on the height, likewise the electrical potential energy of a charged body also depends on its position in the electric field. **Thus the rate of change of electric potential of a charged body with respect to displacement or position in an electric field is called potential gradient.** It is explained as:

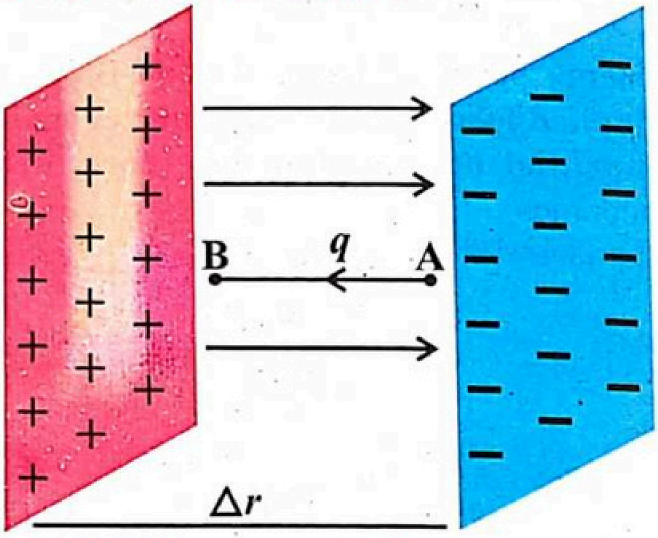


Fig.11.21 Electric potential between two points separated at distance Δr.

Consider a uniform electric field between two parallel and opposite charged plates. When a charged body is displaced against the direction of electric field from point A to point B through a small distance Δr by the applied force (qE) as shown in Fig.11.21, then there is change in electric potential which is given by

$$\Delta V = \frac{W}{q}$$

But $$W = F \cdot \Delta r = qE \cdot \Delta r$$

$$\Delta V = \frac{qE \cdot \Delta r}{q}$$

Here angle 'θ' between E and Δr is 180^0

$$\Delta V = E\Delta r \cos 180^o$$

$$\Delta V = -E\Delta r$$

$$E = -\frac{\Delta V}{\Delta r} \quad \ldots(11.36)$$

This shows that electric field at any point is equal to the negative of potential gradient. The term $\frac{\Delta V}{\Delta r}$ specifies the change in potential with respect to distance (position in specific direction) is known as potential gradient. The negative sign shows that the direction of E is opposite to the direction in which V increases, where E is directed from higher potential to lower potential. The unit of electric intensity in terms of potential gradient is Vm^{-1} and it is equal to NC^{-1}.

> ***POINT TO PONDER***
> What will be the work done on the charged particle when it is displaced between two points which have same potential?

11.13 ELECTRIC POTENTIAL AT A POINT DUE TO A POINT CHARGE

Consider an electric field produces by a point positive charge q. The direction of lines of force of this field is radially outward. When a unit positive charge is displaced from infinity inward against the direction of the field then there is change in electric potential and is given by:

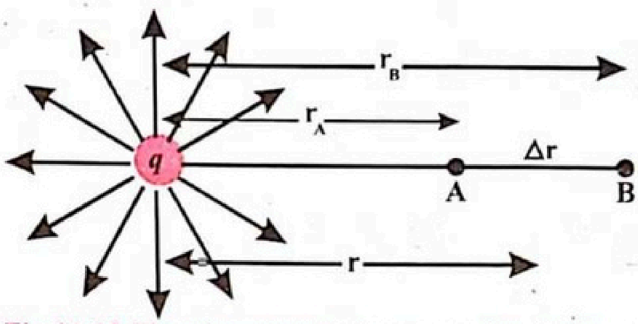


Fig.11.22 Electric potential due to a point charge q at distance r from the center of the point charge.

$$\Delta V = -E\,\Delta r \quad \text{......(11.37)}$$

The negative sign shows that the work is done on a charged particle against the direction of electric field. Eq. 11.37 holds when the electric intensity 'E' remains constant. Since the given field is due to a point charge so its electric intensity E varies inversely with the square of the distance from that point charge. i.e.

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad \text{......(11.38)}$$

To calculate the electric potential, we consider two points 'A' and 'B' separated by a small distance Δr such that E remains constant between them. Let point 'A' is at distance r_A from the center of point charge and point 'B' is at r_B such that $r_B - r_A = \Delta r$ as shown in Fig.11.22. In order to get the average value of electric intensity, we consider the midpoint between 'A' and 'B' at a distance 'r' from the centre of the point charge. Such that;

$$r = \frac{r_A + r_B}{2}$$

But

$$r_B = r_A + \Delta r$$

$$r = \frac{r_A + r_A + \Delta r}{2}$$

$$r = \frac{2r_A + \Delta r}{2}$$

Squaring on both sides

$$r^2 = \frac{4r_A{}^2 + 4r_A\Delta r + \Delta r^2}{4}$$

As Δr^2 is very very small so we neglect it

$$r^2 = r_A^2 + r_A\Delta r$$

$$r^2 = r_A(r_A + \Delta r) = r_A(r_A + r_B - r_A)$$

$$r^2 = r_A r_B$$

Substitute the value of r^2 in Eq.11.38

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A r_B}$$

Substitute the value of E in Eq.11.37

$$\Delta V = -E\,\Delta r$$

$$V_A - V_B = -\frac{1}{4\pi\varepsilon_o}\frac{q}{r_A r_B}(r_A - r_B)$$

$$V_A - V_B = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A r_B}(r_B - r_A)$$

$$V_A - V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad ......(11.39)$$

This is the electric potential between two points in an electric field by a point charge. Now if the point 'B' is at infinity ($r_B = \infty$) then $V_B = 0$ and potential at point A is known as absolute potential. So, Eq.11.39, becomes,

$$V_A - 0 = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{\infty}\right) \qquad \because \frac{1}{\infty} = 0$$

$$V_A = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - 0\right)$$

$$V_A = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A}$$

In general, the electric potential 'V' due to a point charge 'q' at distance 'r' from the centre of the point charge is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \quad ......(11.40)$$

11.14 ELECTRON VOLT

As we know that the unit of energy is joule, but this is the larger unit and it is difficult to use for atomic physics. To overcome this problem, another smaller unit of energy called electron volt (eV) is used. One electron volt is that amount of electrical potential energy which gained by an electron when it is displaced in an electric field through a potential difference of one volt. Thus, by definition of work

$$W = q\,\Delta V$$

According to work-energy theorem

$$W = \Delta U \quad \text{(gained potential energy)}$$

$$\Delta U = q\Delta V$$

Here $q = 1e$

and $\Delta V = 1V$

Therefore $\Delta U = 1eV$ ......(11.41)

As the charge on an electron $e = 1.6 \times 10^{-19}$ C

$$\therefore \quad \Delta U = 1.6 \times 10^{-19}\ CV$$

$$\Delta U = 1.6 \times 10^{-19}\ J \quad ......(11.42)$$

Comparing Eq.11.41 and 11.42

$$1eV = 1.6 \times 10^{-19}\ J$$

Example 11.5

The potential difference between two large parallel and opposite charged metal plates is 160V. If the plates are separated by a distance 4mm then calculate the electric field between them.

Solution:

Potential difference $\Delta V = 160V$

Distance between the plates $\Delta r = 4mm = 4 \times 10^{-3}$ m

Electric field between plates $E = ?$

According to electric field in terms of potential gradient

$$E = \frac{\Delta V}{\Delta r} = \frac{160V}{4\times10^{-3}m} = 40 \times 10^{3}\ Vm^{-1}$$

$$E = 40\ kVm^{-1}$$

Example 11.6

When a unit positive charge is displaced at a distance 40cm towards the point charge of magnitude $0.4\ \mu C$ then calculate the absolute potential.

Solution:

Absolute potential: $V = ?$

Distance: $r = 40cm = 0.4m$

$$q = 0.4\ \mu C = 0.4 \times 10^{-6}\ C = 4\times10^{-7} C$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^{9}\ Nm^2C^{-2}$$

According to the absolute potential

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

$$V = 9 \times 10^9 . \frac{4\times10^{-7}C}{0.4m}$$

$$V = 9 \times 10^3 V$$

$$V = 9 kV$$

11.15 CAPACITOR AND ITS CAPACITANCE

A capacitor is a device that stores electrical energy, or it is a device to store energy in the form of electrical charge producing a potential difference (static voltage) across its plates.

A common capacitor consists of two parallel conducting plates separated by vacuum, air or any other insulator known as dielectric as shown in Fig.11.23. When a capacitor is connected to a battery or other source of voltage, then the potential difference 'V' develops between the plates of the capacitor, such that the plate which is connected to the positive terminal of the source stores positive charges. While the other plate stores the negative charges, which is connected to negative terminal. The experimental results show that these storage of charges (Q) on the plates of the capacitor is directly proportional to the applied potential difference of the source. That is,

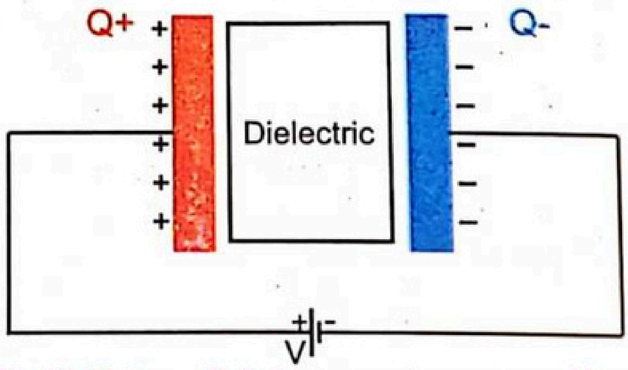


Fig.11.23 A parallel plate capacitor separated by a dielectric material and connected across the source of voltage.

$$Q \propto V$$

$$Q = CV \text{ ......(11.43)}$$

where 'C' is a constant of proportionality. It is known as the capacitance of the capacitor. It depends upon size of the plates, distance and medium between the plates. The SI unit of the capacitance is Farad (F). It is defined as:

If a charge of one coulomb is stored on one of the plates of the capacitor and it could generate a potential difference of one volt, then the capacitance of a capacitor is said to be one Farad.

The farad is a larger unit of capacitance, for practical purposes, the smaller units such as millifarad ($mF=10^{-3}$ F), microfarad ($\mu F = 10^{-6} F$), nano-farad ($nF = 10^{-9} F$) and picofarad ($pF = 10^{-12}$ F) are used.

11.16 CAPACITANCE OF PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two parallel metal plates of same size separated by a distance 'd' and connected with a source of a voltage as shown Fig.11.24. Let 'A' be the area of each plate and σ be its surface charge density. If the medium between the plates of capacitor is vacuum or air then its capacitance is given by;

$$C = \frac{Q}{V} \quad ......(11.44)$$

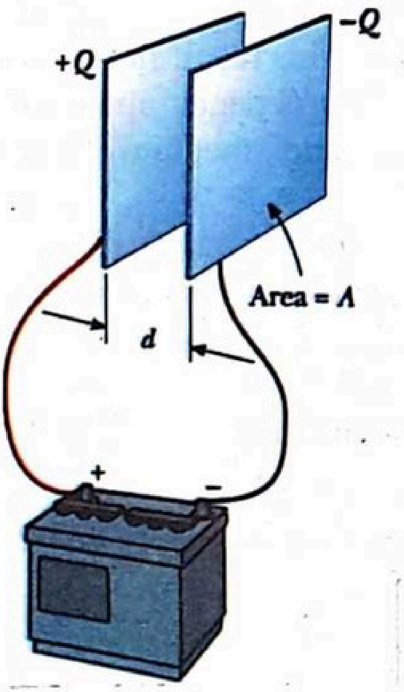


Fig.11.24 A parallel plate capacitor consist of a pair of plates separated by a distance d and the area of each plate is A.

When voltage 'V' is applied across the plates of capacitor then with the storage of charges, an electric field 'E' is setup between the plates and is given by;

$$E = \frac{\sigma}{\varepsilon_o}$$

but $\sigma = \frac{Q}{A}$

Therefore $E = \frac{Q}{A\varepsilon_o} \quad ......(11.45)$

Similarly, by definition of electric field in terms of potential gradient,

$$E = \frac{V}{d} \quad ......(11.46)$$

Comparing Eq.11.45 and Eq.11.46

$$\frac{Q}{A\varepsilon_o} = \frac{V}{d}$$

$$\frac{CV}{A\varepsilon_o} = \frac{V}{d}$$

$$C = \frac{A\varepsilon_0}{d} \quad ......(11.47)$$

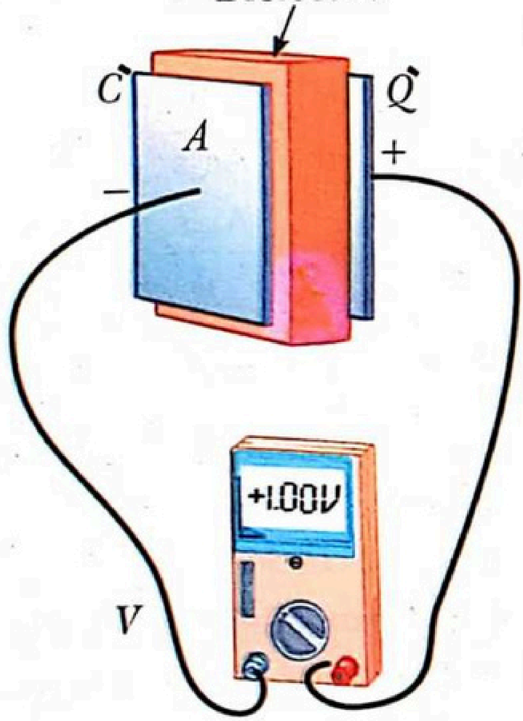


Fig.11.25 The dielectric material between the plates of the capacitor.

This is the capacitance of a parallel plate capacitor in the presence of air or vacuum between its plates. This result shows that the capacitance of a capacitor depends upon the area of the plates, distance and medium between the plates of capacitor.

> ***FOR YOUR INFORMATION***
> A capacitor is a passive component that can be used in both electric and electronic circuits.

Faraday found experimentally that when the free space between the two plates of the capacitor is occupied by insulator called dielectric then for the same applied voltage, more charges store on the plates of the capacitor. To explain this, we consider two parallel plate capacitors, each of Area 'A' separated by a distance d. Let the space between the plates occupied by some dielectric material of permittivity $(\varepsilon = \varepsilon_0\varepsilon_r)$ as shown in Fig.11.25. When the voltage 'V' is applied the induced electric field E' between the plates is given by

$$E' = \frac{Q'}{A\varepsilon_o\varepsilon_r}$$

But

$$E' = \frac{V}{d}$$

$$\frac{V}{d} = \frac{Q'}{A\varepsilon_o\varepsilon_r}$$

$$\frac{V}{d} = \frac{C'V}{A\varepsilon_o\varepsilon_r}$$

$$C' = \frac{A\varepsilon_o\varepsilon_r}{d}$$

But

$$C = \frac{A\varepsilon_o}{d}$$

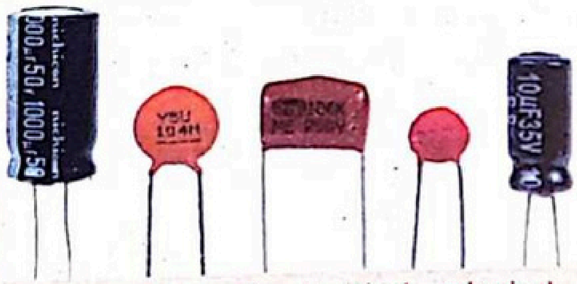
$$C' = \varepsilon_r C \quad ......(11.48)$$

As $\varepsilon_r > 1$

So $C' > C$

The relative permittivity ε_r is also called dielectric constant. The values of dielectric constant for different materials are shown in table 11.2. The dielectric constant is defined as; **the ratio of the capacitance of a capacitor with dielectric substance as a medium**

FOR YOUR INFORMATION



External structure of cylindrical, spherical and parallel plate capacitors.

Table 11.2

Dielectric Constant of Materials

Air	1.00
Aisimag 196	5.70
Bakelite	4.90
Cellulose	3.70
Fiber	6.00
Formica	1.75
Glass	7.75
Mica	5.10
Mycalex	7.10
Paper	3.00
Plexiglass	2.80
Polyethlene	2.30
Polystyrene	2.60
Porcelain	5.57
Pyrex	4.00
Quartz	3.80
Steatite	5.80
Teflon	2.10

between the plates to the capacitance with air or vacuum as medium between the plates. That is,

$$\varepsilon_r = \frac{C'}{C} \quad ......(11.49)$$

11.16.1 Combination of capacitors

Capacitors are manufactured with different values of capacitance and voltage. Sometimes these standard values do not match with the circuit requirement. To overcome this problem, we can obtain these required values by combining the capacitors either in series or in parallel, such combination of capacitor are explained as:

I Capacitors in series

A series circuit is one when all the capacitors are connected in the form of a chain (one after the other) and the flow of charges are along the single path. Hence, when the capacitors are connected in series the storage of charges on each capacitor is the same. It is explained as:

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in series as shown in Fig.11.26. When the potential difference V is applied across them, the right plate of C_3 is negatively charged $(-Q)$, it induces an equal and opposite charge (+Q) on the left plate of C_3. Where, the total charge $(-Q)$ has moved from the left plate of C_3 to the right plate of C_2. Again, the charge (+Q) induces on the left plate of C_2. Finally, the negative charges are induced on the right plate of 'C_1' from the left plate of C_2 which also repels the negative charges from the left plate of C_1 toward the battery and the charges (+Q) left on it. Thus, it is obvious that when capacitors are connected in series, the charges on each capacitor is the same, but the voltage drop across each capacitor is different. i.e.,

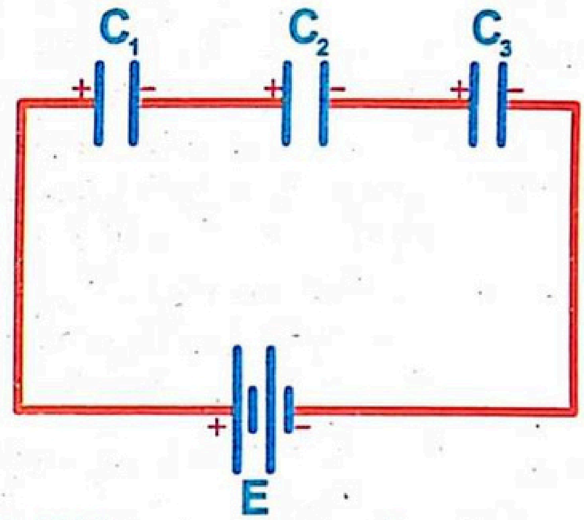


Fig.11.26 A series connection of capacitors across the potential difference.

$$V = V_1 + V_2 + V_3$$

As $\quad \therefore Q = CV$

$$\therefore V = \frac{Q}{C}$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad ......(14.50)$$

> ***FOR YOUR INFORMATION***
> When capacitors are connected in series, their equivalent capacitance is always less than each individual capacitance.

This result shows that the equivalent capacitance of a series combination is always less than any individual capacitance connected in series, or the reciprocal of equivalent capacitance in series combination is equal to sum of reciprocal of the individual capacitance.

II Capacitors in Parallel

In parallel circuit, the capacitors are connected across one another and flow of charges is divided into all the given paths. Consider three capacitors of capacitances C_1, C_2 and C_3 which are connected in parallel as shown in Fig.11.27. When the potential difference 'V' is applied across them, then the same potential is drawn across each capacitor, but the charges stored on the three capacitors are Q_1, Q_2 and Q_3. So,

$$Q = Q_1 + Q_2 + Q_3$$

$$CV = C_1V + C_2V + C_3V$$

$$C_{eq} = C_1 + C_2 + C_3 \quad ......(14.51)$$

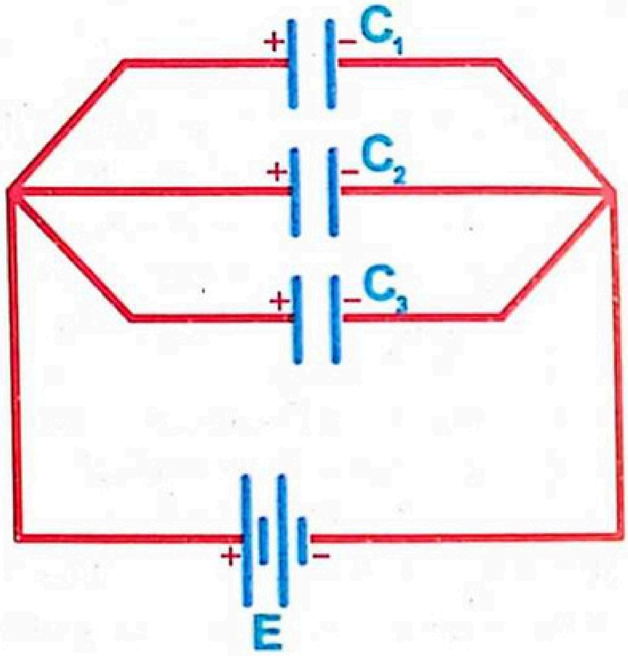


Fig.11.27 A parallel connection of capacitors across the potential difference

This result shows that the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitance, or the equivalent capacitance in parallel combination is equal to sum of the individual capacitance.

11.17 ELECTRIC POLARIZATION OF DIELECTRIC

The insulating materials e.g., glass, mica, paper etc. are called dielectrics. They transmit electric effect without conduction. As we know that each dielectric material consists of a number of atoms or molecules and in each atom, there is positively charged nucleus which is surrounded by negatively charged electronic cloud as shown in Fig.11.28.

When the dielectric material is placed between the plates of charged capacitor then their atoms are subjected to external electric field E_e. The negatively charged electrons are attracted towards the positively charged plate of the capacitor, and the positively charged nucleus is attracted towards the negatively charged plate of the capacitor. As a result, the charges of the atoms are displaced from their original position and form dipoles as shown in Fig.11.29. This formation of electric dipole in the presence of electric field is known as electric polarization of the dielectric.

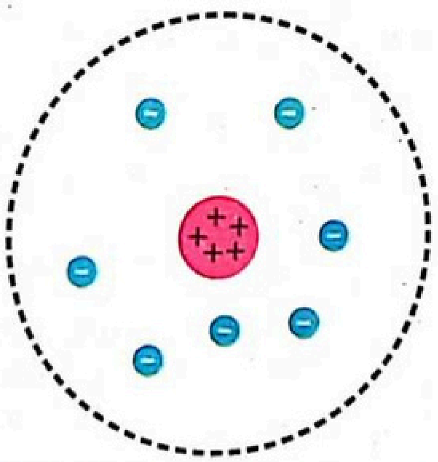


Fig.11.28 Positively charged nucleus surrounded by negatively charged electrons.

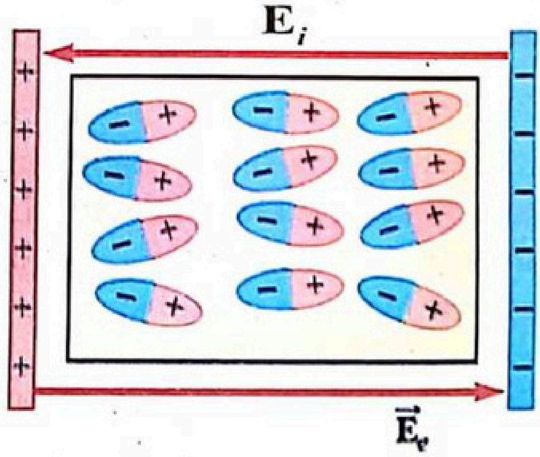


Fig.11.29 Polarization of atoms of dielectric material between the plates of the capacitor.

In the process of polarization of the dielectric between the plates of the capacitor, we have observed that the negative charges are induced on one side near the positive plate and an equal positive charges on the opposite side near the negative charged plate. These induced charges produce an internal electric field E_i and it opposes the applied external field E_e as shown in Fig.11.29. Thus, their resultant field is given by

$$E = E_e - E_i \quad \ldots\ldots(11.51)$$

This shows that in the presence of dielectric, the electric field is reduced, and it causes smaller potential drop across the plates of the capacitor. As the relation $C = \frac{Q}{V}$ shows that at constant Q, the capacitance 'C' is inversely proportional to the applied potential difference 'V'. Thus, it is clear that in the presence of dielectric the capacitance of a capacitor has been increased.

FOR YOUR INFORMATION

When capacitors are connected in parallel, their equivalent capacitance is always greater than each individual capacitance.

DO YOU KNOW?

The capacitance of a capacitor is increased by inserting a dielectric material between its plates.

11.18 CHARGING AND DISCHARGING OF A CAPACITOR

When a resistor R and a capacitor C with a source of voltage V are connected in series as shown in Fig.11.30 then it is called RC-series circuit. It is being used for charging of a capacitor. Now when the switch 'S_1' in RC-series circuit is closed then the process of charging of the capacitor is started. This process of charging is not an instantaneous one, but it takes some time. The rate of charging or discharging of a capacitor depends on the product of resistance R and capacitance C. As the unit of product of RC is that of time, so this product is termed as time constant and it is represented by τ. Mathematically the charging of a capacitor can also be studied under the following expression

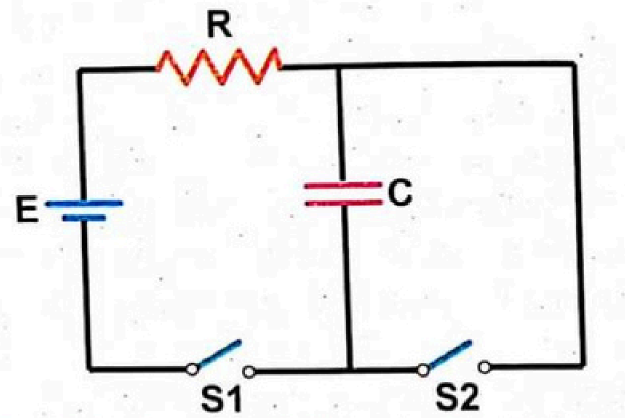


Fig.11.30 Charging of a capacitor in RC-series circuit.

$$q = q_0(1 - e^{-\frac{t}{RC}}) \quad ......(11.52)$$

This relation shows that the nature of charging of a capacitor is exponential, where q_0 represents the maximum charge on the capacitor that stores after an infinite length of time. It means, the rate of charging of a capacitor is different at its different stages, graphically it is explained as:

If time 't' = 1τ (1RC),

then eq. 11.52 becomes

$$q = q_0\left(1 - e^{\frac{-\tau}{\tau}}\right)$$

$$q = q_0(1 - e^{-1})$$

$$q = q_0\left(1 - \frac{1}{e}\right)$$

$$q = q_0\left(1 - \frac{1}{2.718}\right)$$

$$q = 0.63\, q_0 \quad ......(11.53)$$

This gives us that after one time constant, the capacitor will be charged about 63%.

If time t= 2τ (2RC)

then $$q = q_0\left(1 - e^{\frac{-2\tau}{\tau}}\right)$$

$$q = q_0\,(1 - e^{-2})$$

$$q = 0.86\, q_0 \quad ......(11.54)$$

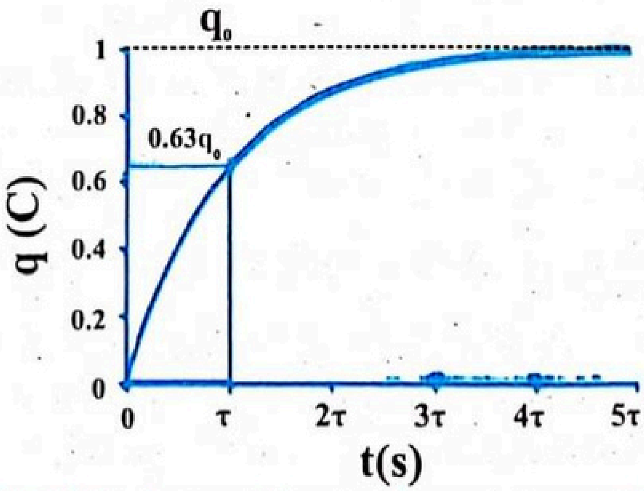


Fig.11.31 A curved line in charge – time graph shows charging of a capacitor.

The shows that the capacitor is charged about 86% after 2 time constant.

Similarly, after five time constant a capacitor acquires a charge very close to its maximum value q_0. Graphically, a curved line in charge-time (q-t) graph shows charging of a capacitor as shown in Fig.11.31.

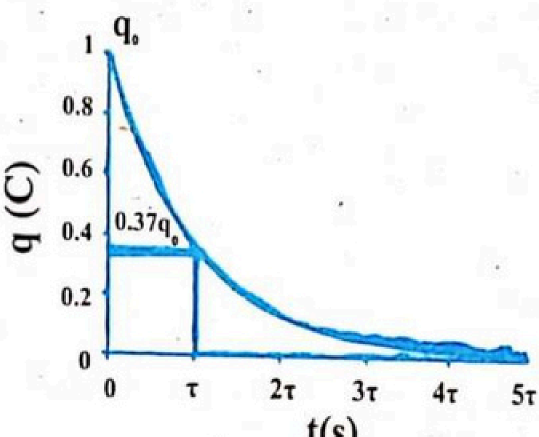


Fig.11.32 A curved line in charge – time graph shows a discharging of a capacitor

After maximum charging of a capacitor, when switch S_1 is opened and switch S_2 is closed in RC-circuit as shown in Fig.11.30, then there is flow of charges from the capacitor and hence the process of discharging of the capacitor is started. The rate of discharging also depends upon the product of R and C. Mathematically, the relation for discharging of a capacitor is given by

$$q = q_o e^{-\frac{t}{RC}} \quad ......(11.55)$$

This shows that the charge stored on the capacitor also decreases exponentially and graphically the discharging of a capacitor is explained as;

If time 't' = 1τ (1RC)

Then $\quad q = q_0e^{-1} = 0.37q_0$ ......(11.56)

It means a capacitor is discharged about 63% or there are 37% charges left on the capacitance after one time constant.

Similarly, if time 't'= 2τ (2RC)

Then $\quad q = q_0e^{-2} = 0.14q_0$ ......(11.57)

This shows that a capacitor is discharged about 86% after two time constants.

After five time constants, the capacitor will be almost discharged. Graphically, a curved line in charge - time graph as shown in Fig.11.32 gives us discharging of a capacitor.

11.19 ENERGY STORED IN A CAPACITOR

As we know that a capacitor is a device which stores the charges. A charged capacitor has an electric field between its plates. This field has ability to store the energy in the form of electrical potential energy, it is explained as: Consider a parallel plate capacitor which is connected with source of potential difference 'V'.The charges from the source begin to flow towards the plates of the capacitor. The potential difference across the plates of the capacitor increases until it attains the same value as that of the source. It means there is work has to done in carrying the charges from the source onto the plates of the capacitor. This work done is stored in the form of electrical potential energy in the electric field between the plates of the capacitor. Its value can be calculated as: By the definition of work;

$$\text{Work} = F.\Delta r$$
$$= qE \cdot \Delta r$$
$$= q\frac{\Delta V}{\Delta r} \cdot \Delta r$$
$$\text{Work} = q\Delta V \quad ....(11.58)$$

> ***FOR YOUR INFORMATION***
> A capacitor has potential in terms of electrical energy.
> A battery has potential in terms of chemical energy.

As the potential difference across the capacitor rises from 0 to V so its average value is given by;

$$\Delta V = \frac{0+V}{2} = \frac{V}{2}$$

Also, $q = CV$

Thus, substituting the values of ΔV and q in Eq.11.58

$$\text{Work} = CV \cdot \frac{V}{2}$$
$$\text{Work} = \frac{1}{2}CV^2$$

This work is stored in a capacitor in terms of electrical potential energy. It is represented by ΔU. According to work-energy theorem:

$$\text{Work} = \Delta U$$
$$\Delta U = \frac{1}{2}CV^2 \quad ......(11.59)$$

This is the electrical P.E. stored in a capacitor, when the medium between the plates is air or vacuum. If the medium between the plates is dielectric material, then,

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

and $V = Ed$

Thus Eq 11.59 becomes

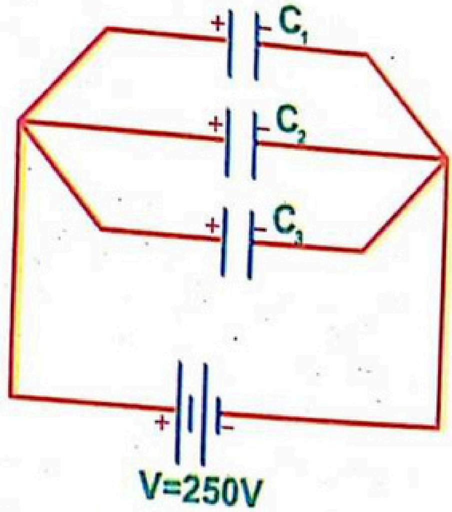
$$\Delta U = \frac{1}{2} \cdot \frac{A\varepsilon_0\varepsilon_r}{d} \cdot E^2 d^2$$
$$\Delta U = \frac{1}{2} \cdot \varepsilon_0\varepsilon_r E^2 (\text{volume}) \quad \therefore Ad = \text{volume of dielectric medium between the plates}$$

Energy per unit volume i.e., energy density is given by;

$$U = \frac{\Delta U}{\text{volume}}$$

$$U = \frac{\frac{1}{2}\varepsilon_o \varepsilon_r E^2 (\text{volume})}{\text{volume}}$$

$$U = \frac{1}{2}\varepsilon_o \varepsilon_r E^2 \quad ......(11.60)$$

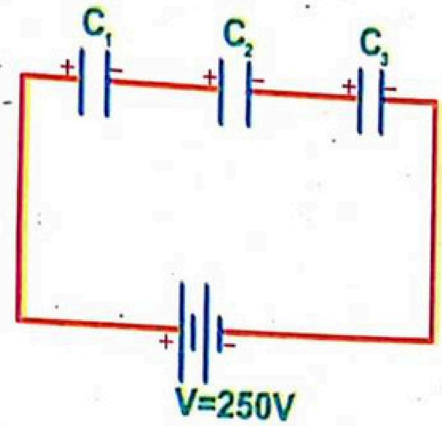


Example 11.7

Three capacitors of capacitances $10\,\mu F$, $50\,\mu F$ and $25\,\mu F$ respectively are given. Calculate:

i. Its total capacitance and charges on each capacitor when these are connected in parallel to a 250V supply.

ii. Total capacitance and potential difference across each capacitor when these are connected in series.



Solution:

Let

$$C_1 = 10\mu F = 10\times10^{-6} F$$
$$C_2 = 50\mu F = 50\times10^{-6} F$$
$$C_3 = 25\mu F = 25\times10^{-6} F$$

i. Total capacitance when capacitors are connected in parallel C = ?
Charges on each capacitor, Q_1, Q_2 and Q_3 = ?
Applied voltage = V = 250v

ii. Total capacitance when capacitors are connected in series = C = ?
V_1, V_2 and V_3 = ? (when capacitors in series)

i. Let the three capacitors connected in parallel as shown in figure then their equivalent capacitor 'C' is given as:

$$C_{eq} = C_1 + C_2 + C_3$$
$$C_{eq} = 10 + 50 + 25$$
$$C_{eq} = 85\,\mu F$$

As the capacitors are connected in parallel so each capacitor has a same potential difference of 250V across it. To find the charge stored on each capacitor we have:

$$Q_1 = C_1 V = 10\mu F \times 250V = 2500\,\mu C$$
$$Q_2 = C_2 V = 50\mu F \times 250V = 12500\,\mu C$$
$$Q_3 = C_3 V = 25\mu F \times 250V = 6750\,\mu C$$

ii. Now when the three capacitors are connected in series, as shown in figure, then their equivalent capacitance is given by

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\frac{1}{C_{eq}}=\frac{1}{10\mu F}+\frac{1}{50\mu F}+\frac{1}{25\mu F}$$

$$\frac{1}{C_{eq}}=\frac{5+1+2}{50}$$

$$\frac{1}{C_{eq}}=\frac{8}{50}=\frac{4}{25}$$

$$C_{eq}=\frac{25}{4}\mu F=6.25\mu F$$

As $$Q=C_{eq}V=6.25\mu F\times 250V$$

$$Q=1562.5\,\mu C$$

As the capacitors are connected in series so each capacitor has same charge equal to $1562.5\,\mu C$. To find the potential difference across each capacitor we have

$$V_1=\frac{Q}{C_1}=\frac{1562.5\mu C}{10\mu F}=156.25V$$

$$V_2=\frac{Q}{C_2}=\frac{1562.5\mu C}{50\mu F}=31.25V$$

$$V_3=\frac{Q}{C_3}=\frac{1562.5\mu C}{25\mu F}=62.5V$$

Example 11.8

A capacitor is charged with 9nC and has 120V potential difference between its terminals. Compute its capacitance and the energy stored in it.

Solution:

We have

$$q=9nC=9\times 10^{-9}C$$
$$V=120V$$
$$C=?$$
$$\text{energy }(\Delta U)=?$$

As $$q=CV$$

$$C = \frac{q}{V} = \frac{9\times10^{-9}C}{120V} = 0.075\times10^{-9}F = 75\times10^{-12}F$$

$$C = 75pF \qquad \therefore 1pF = 1\times10^{-12}F$$

$$\Delta U = \frac{1}{2}CV^2$$

$$\Delta U = \frac{1}{2}(75\times10^{-12}F)(120V)^2$$

$$\Delta U = \frac{1}{2}\times75\times10^{-12}\times14400$$

$$\Delta U = 540000\times10^{-12}J$$

or

$$\Delta U = 0.54\times10^{-6}J$$

$$\Delta U = 0.54\mu J$$

SUMMARY

- **Electrostatics:** It is the branch of physics in which we study the charged particles at rest.
- **Charge:** Like mass, electric charge is the intrinsic property of matter which causes it to exert or experience a force when placed in an electric or magnetic field.
- **Coulomb's Law:** This law is stated that the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between them.
- **Electric Field:** The region around a point charge in which another charge can experience force of attraction or repulsion is known as electric field. The electric field is represented by lines known as electric lines of force.
- **Electric Intensity:** The electric intensity is defined as a force experienced by a unit positive charge placed at any point in an electric field.
- **Electric dipole:** A pair of charges of equal magnitude but opposite sign separated by a small distance.
- **Dipole moment:** The product of magnitude of charge (either positive or negative) and the distance between the two charges is known as dipole moment.
- **Electric Flux:** The number of electric field lines passing through a certain element of area is known as electric flux.
- **Gauss's Law:** This law is stated as "the flux through any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the total charges enclosed in it."

- **Potential Difference:** The difference in electric potential between two points in an electric field or an electric circuit. It is measured in terms of voltage.
- **Electric Potential:** The amount of work on a unit positive charge in moving it from infinity to a point in the electric field against the direction of electric field is known as electric potential at that point. Its unit is volt.
- **Electron Volt:** Electron volt is the unit of energy where 1eV is equal to 1.6×10^{-19}J. It is the amount of energy gained or lost by an electron when it is moved across an electric potential difference of one volt.
- **Capacitor:** A capacitor is a device that stores electric charge.
- **Dielectric:** The insulator or insulating material placed between the plates of the capacitor is known as dielectric.
- **Polarization:** The process in which the molecules of the dielectric materials between the plates of the capacitor forms dipoles is known as polarization.
- **Time Constant:** The product of resistance and capacitance (RC) in the given circuit is termed as time constant OR It is a time in which a capacitor is charged or discharged about 63% of its fully charged value.

EXERCISE

Choose the most appropriate option.

1. The electrostatic force between two point charges is independent of one of the following quantities
   (a) Magnitude of charges (b) Temperature of the charges
   (c) Distance between charges (d) Medium between charges
2. If the distance between two equal charges is reduced to half and the magnitude of charges is also decreased to half, then the force between them will be
   (a) Remain same (b) Decreased to half
   (c) Increased to double (d) Becomes four time
3. The concept of electric field was introduced by
   (a) Coulomb (b) Faraday (c) Gauss (d) Ampère
4. The number of electrons or protons which constitutes a charge of one coulomb is
   (a) 6.25×10^{-18} (b) 6.25×10^{18} (c) 1.6×10^{-19} (d) 1.6×10^{19}
5. For static electricity the value of relative permittivity is always
   (a) Zero (b) Less than one (c) Equal to one (d) Greater than one
6. A metallic charged sphere is placed in uniform electric field E, the electric field inside the sphere will be
   (a) E (b) Less than E (c) Greater than E (d) Zero

7. The electric intensity E at a point in a field due to a dipole depends upon distance 'r' the relationship between them is

(a) $E \propto r$ (b) $E \propto \frac{1}{r}$ (c) $E \propto \frac{1}{r^2}$ (d) $E \propto \frac{1}{r^3}$

8. The dipole moment is defined as the product of

(a) Charge and distance (b) Charge and displacement

(c) Charge and force (d) Charge and electric field

9. Debye is the unit of

(a) Electric field (b) Electric charge

(c) Lines of force (d) Dipole moment

10. The electric flux is maximum when angle 'θ' between area vector and electric intensity is

(a) 0° (b) 45° (c) 60° (d) 90°

11. Electric flux is independent of one of the following quantities

(a) Charge (b) Medium (c) Field (d) Distance

12. The dimension of an electric potential is same as that of

(a) Work (b) Work per unit charge

(c) Electric field per unit charge (d) Electric force per unit charge

13. Electron volt is the unit of

(a) Energy (b) Charge

(c) Current (d) Electric Potential

14. The energy of an electron which accelerates through a potential difference of 1000V is

(a) 1.6×10^{-22}J (b) 1.6×10^{-20}J (c) 1.6×10^{-19}J (d) 1.6×10^{-16}J

15. The unit of surface charge density is

(a) cm (b) $C\ m^{-1}$ (c) $C\ m^{2}$ (d) $C\ m^{-2}$

16. Capacitance of a capacitor does not depend upon

(a) Area of plate (b) Distance between plates

(c) Medium between plates (d) Material of the plates

17. In the presence of dielectric material, the electric field between the plates of the capacitor will be

(a) Zero (b) Remain same (c) Decreased (d) Increased

18. When the potential difference across the capacitor is decreased by dielectric then the capacitance of the capacitor will be

(a) Zero (b) Remain same (c) Decrease (d) Increase

19. The unit of product of resistance and capacitance is equal to unit of

(a) Time (b) Work

(c) Potential difference (d) Current

20. A capacitor is approximately full charged after
(a) Two time constant
(b) Three time constant
(c) Four time constant
(d) Five time constant

SHORT QUESTIONS

1. What is the effect of the medium on the electrostatic force between two point charges?
2. What do you know about dielectric constant or relative permittivity?
3. How does the electrostatic force exert on one charge by another charge?
4. Why the two lines of force do not cross each other?
5. How can you determine the direction of electric intensity?
6. Differentiate between point charge and test charge.
7. What do you know about the dipole axis and its direction?
8. Is dipole moment vector or scalar? If vector, then where is its direction in an electric field?
9. When the electric flux will be minimum and maximum?
10. What do you know about the Gaussian Surface?
11. What is the electric field inside the hollow charged spherical shell?
12. What do you know about the electric potential gradient?
13. What is the relation between work and electrical potential energy?
14. Define one electron volt and give its numerical value.
15. What is the absolute electric potential?
16. What is the capacitance of a capacitor?
17. What is the effect of dielectric on the capacitance of a capacitor?
18. What is the polarization of dielectric?
19. How does dipole moment produce in dielectric material?
20. How does the electric field between the plate of the capacitor reduce by dielectric?
21. What is RC-circuit and its function?
22. What is meant by time constant of the RC-circuit?
23. What is the unit of product of resistance and capacitance?

COMPREHENSIVE QUESTIONS

1. Define and explain charges, their kinds and properties.
2. State and explain Coulomb's law, also discuss the magnitude of electrostatic force of attraction or repulsion between two point charges in the absence and presence of dielectric.
3. What do you know about the electric field and electric field intensity? Explain the magnitude and direction of electric field intensity?

4. What are the characteristics of the electric lines of force? Explain the electric field in terms of lines of force due to a positive charge, a negative charge and two identical positive or negative charges.
5. What is electric dipole? Calculate electric field due to a dipole.
6. Define electric flux and discuss how does the electric flux become maximum and minimum?
7. State and explain Gauss's law. Also discuss the various applications of Gauss's law.
8. What do you know about the electric potential? Define potential gradient and calculate potential due to a point charge.
9. Define capacitor and its capacitance. Also calculate the capacitance of a parallel plate capacitor.
10. State and explain series and parallel combinations of capacitors.
11. What is electric polarization? Discuss the effect of dielectric on the capacitance of a capacitor.
12. State and explain the process of charging and discharging of a capacitor.

NUMERICAL PROBLEMS

1. How many electrons are contained in 1C and 3C of charges?
(6.25×10^{18} electrons, 1.9×10^{19}eletrons)
2. Determine the force of repulsion between two free electrons spaced 0.5 Angstrom ($1A^0 = 10^{-10}$m). **(92nN)**
3. Two coins lie 2m apart on a table, and carry identical charge. How large is the charge on each if a coin experiences a force of 3N? **(3.65×10^{-5}C)**
4. Two positive charges of magnitudes $q_1 = 5\mu C$ and $q_2 = 2\mu C$ are separated by 50cm from each other. At what point the electric intensity due to these charges will be zero? **(0.3m from q_1)**
5. Find the force that an electron experience in an electric field of $1000NC^{-1}$. If the electron is free to move. Find the distance covered by it in 10ns.The mass of electron is 9.11×10^{-31}kg. **(1.6×10^{-16}N)(0.88mm)**
6. The diameter of a hollow metallic sphere is 10cm and the sphere carries charge of 80μC. Find the electric intensity (i) at a distance 60cm from the centre of the sphere and (ii) at the surface of the sphere.
(2×10^6N/C, 2.88×10^8N/C)
7. An electric dipole consists of two charges +10μC and −10μC separated by distance 0.5cm. Calculate the electric field intensity at a point on the axial line at a distance of 5cm from the midpoint of the dipole. **(7.2×10^6 N/C)**
8. Find the electric flux through each face of a hollow cube of side 5cm. If a charge of 6μC is placed at its centre. **(1.133×10^5 Nm^2C^{-1})**

9. The potential difference between two metal plates is 120V. The plate separation is 3mm. Find the electric field between the plates. **(40KV/m)**

10. A charged particle remains stationary between the two horizontal opposite charged plates due to its weight and the electric force by the field. Find the potential difference between the plates. Where the distance between the plates is 2cm, mass of particle 4×10^{-13}kg and charge on particle 2.4×10^{-18}C. **(32.6KV)**

11. A capacitor of 5μF is charged by a 12V battery. Find the charge and energy stored on it. **(60μC, 3.6×10^{-4}J)**

12. Three capacitor have capacitance 2μF, 5μF and 7μF. Calculate their equivalent capacitance when they are connected in (i) Series (ii) Parallel **(1.19μF,14μF)**