Summary

- A differential equation is an equation that involves the derivatives of an unknown function (dependent variable) of one or more variables (independent variables).
- The order of a differential equation is the order of the highest-order derivative which appears in the equation.
- The degree of a differential equation is the power of the highest-order derivative which appears in the equation.
- A solution of an ordinary differential in one dependent variable y on an interval I is a function $y(x)$ which, when substituted for the dependent variable y and its derivatives $y', y'', \ldots$, reduces the differential equation to an identity in the independent variable x over interval I.
- A differential equation is **linear** in a set of one or more of its dependent variables if and only if each term of the equation which contains a variable of the set or any of their derivatives is of the first degree in those variables and their derivatives.
- A function $f(x, y)$ is homogeneous function of degree $\boldsymbol{n}$ in variables x and y if and only if for all values of the variables x, and y and for every positive value of λ, the identity is true:

$$f(\lambda x, \lambda y) = \lambda^n f(x, y), \quad n = 1, 2, 3, \ldots$$

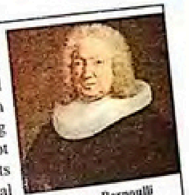
- The differential equation $\dfrac{dy}{dx} = \dfrac{f(x,y)}{g(x,y)}$

is called a homogenous differential equation, if it defines a homogenous function of degree zero.

History

Jacob Bernoulli
(1654)-(1705)

Jacob Bernoulli and Johann Bernoulli we two Swiss mathematicians. They were first interpreters of Leibniz's version of differential equations. Both brothers orgue about Newton's theories and maintained that the Newton's theory of fluxions was plagiarized from the Leibniz's original work. Both brothers worked hard using differential calculus to disprove Newton's principle. The could not accept the theory which Newton had given, that earth and other plants rotate around the sun in elliptical orbits. The first book on differential calculus was written on the bases of Bernoulli brothers ideas by Gabriele Manfred in 1707. Most of the publications were made on differential equations and partial differential equations in 18th century.



Johann Bernoulli
(1667)-(1748)

Unit 11 PARTIAL DIFFERENTIATION

By the end of this unit, the students will be able to:

11.1 Differentiation of function of two variables

i. Define a function of two variables.
ii. Define partial derivative.
iii. Find partial derivatives of a function of two variables.

11.2 Euler's Theorem

i. Define a homogeneous function of degree n.
ii. State and prove Euler's theorem on homogeneous functions.
iii. Verify Euler's theorem for homogeneous functions of different degrees (simple cases).
iv. Use MAPLE command diff to find partial derivatives.

Introduction

The goal of this unit is to extend the methods of single variable differential calculus to functions of two variables. In many practical situations, the value of one quantity may depend on the values of two or more others. For example, the amount of water in a reservoir may depend on the amount of rainfall and on the amount of water consumed by local residents. The current in an electrical circuit may vary with the electromotive force, the capacitance, the resistance, and the impedance in the circuit. The flow of blood from an artery into a small capillary may depend on the diameter of the capillary and the pressure in both the artery and the capillary. The output of a factory may depend on the amount of capital invested in the plant and on the size of the labor force. We will analyze such situations using functions of several variables.

In many problems involving functions of several variables, the goal is to find the derivative of the function with respect to one of its variables when all the others are held constant. In this unit, we need to develop the concept and shall see how it can be used to find slopes and rates of change in case of two variables function.

11.1 Differentiation of the function of two variables

In the real world, physical quantities often depend on two or more variables. For example, we might be concerned with the temperature on a metal plate at various points at time t. The locations of temperature on the plate are given as ordered pairs (x, y), so that the temperature T can be considered as a function of two location variables x and y, as well as a time variable t. The notation of a function of single variable, we might extend this as $T(x, y, t)$. We begin our study of function of two variables by examining this notation and a few other basic concepts.

For illustration, if a company produces x items at a cost of 10 rupees per item, then the total cost $C(x)$ of producing x items is given by: $C(x) = 10x$

The cost is a function of one independent variable, the number of items produced. If the company wants to produce two products, with x of one product at a cost of rupees 10 each, and y of another product at a cost of rupees 15 each, then the total cost to the firm is a function of two independent variables x and y:

$C(x, y) = 10x + 15y$

When $x = 5$ and $y = 12$, the total cost is written with $C(5,12) = 10(5) + 15(12) = 230$ rupees.

11.1.1 Function of two variables

*"A function $z = f(x, y)$ is a **function of two variables x and y**, if for each given pair (x, y), we determine a single value of z." Where, x, y and z are real variables.*

The real numbers x and y are **independent variables**; z is the dependent variable. The set of all ordered pairs of real numbers (x, y) such that $f(x, y)$ is a real number, is **the domain** of f and the set of all values of $f(x, y)$ is the **range**.

Example 1 How to show that $z = f(x,y) = \sqrt{1-x+y}$ is a function of two independent variables x and y? Find also the domain and range of a given function.

Solution For this function, we need to show the transformation of two independent variables (inputs) x and y is just a single dependent variable z. In respect of any two real values of independent variables x and y, say, $x = 2$ and $y = 1$, the function $z = f(2,1) = \sqrt{1-2+1} = 0$ gives response of just one real value of z which is $z = 0$.

The function $z = f(x,y) = \sqrt{1-x+y}$ is therefore declared a function of two independent variables x and y.

> **Remember**
> Functions of more than one independent variable are called **multivariate functions.**

The domain of $f(x, y)$ is the set of all ordered (x, y) for which $\sqrt{1-x+y}$ is defined. We must have $1-x+y \geq 0$ or $y \geq x-1$, in order for the square root to be defined.

In a function $z = f(x,y) = \sqrt{1-x+y}$, we see that $z = f(x, y)$ must be nonnegative and the range of $f(x, y)$ is all $z \geq 0$

11.1.2 Partial derivative

To give clear concept to partial derivative, the problem related to our real-life situations is considered as:

Suppose, a small firm makes only two products, radios and audiocassette recorders. The profit of the firm from these two products is given by: $P(x,y) = 40x^2 - 10xy + 5y^2 - 80$, (i)

Where x is the number of units of radios sold and y is the number of units of recorders sold. How changes in x will (radios) or y (recorders) affects P (profit)?

Suppose that sales of radios have been steady at 10 units; only the sales of recorders vary. The management would like to find the rate (marginal profit/ derivative of the profit function) at which the number of recorders sold.

If x is fixed at 10 units, then this information reduces the profit two variables function to a new **single variable function** that can be found from equation (i) by putting $x = 10$:

$$P(10,y) = 40(10)^2 - 10(10)y + 5y^2 - 80 = 3920 - 100y + 5y^2$$

The function $P(10, y)$ shows the profit from the sales of y recorders, assuming that x is fixed at 10 units. The rate, at which the y number of recorders sold, is the ordinary derivative of $P(10, y)$ with respect to y: $\frac{d}{dy}P(10,y) = -100 + 10y$ (ii)

This represents the per unit profit from y number of audiocassette recorders.

The notation of $\frac{d}{dy}P(10,y)$ is usually stands for **ordinary derivative**, when the function is a single variable function. **In our case, the** profit function (i) is a function of two variables; its rate with respect to y should be a **partial derivative**. For partial derivative with respect to y, the ordinary derivative in equation (ii) is replaced by $\frac{\partial}{\partial y}P(10,y)$ to obtain

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}P(10,y) = P_y = -100 + 10y, \text{ prime notation "/" is not allowed.}$$

Informally,

- the partial derivative $\frac{\partial}{\partial x}f(x,y)$ with respect to x is the derivative of $f(x,y)$ obtained by keeping x as a variable and y as a **constant quantity**.

- the partial derivative $\frac{\partial}{\partial y}f(x,y)$ with respect to y is the derivative of $f(x,y)$ obtained by keeping y as a variable and x as a **constant quantity**.

11.1.3 Partial derivatives of a function of two variables

"If $z = f(x,y)$ is a function of two variables, then the first partial derivatives of $z = f(x, y)$ with respect to x and y are the functions f_x and f_y respectively, defined by,

$$\frac{\partial f}{\partial x} = f_x(x,y) = f_x = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x} \quad \text{(i)}$$

$$\frac{\partial f}{\partial y} = f_y(x,y) = f_y = \lim_{\Delta y \to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y} \quad \text{(ii)}$$

Provided that limits exist.

Example 2 If function is $f(x,y) = x^3y + xy^2$. Find partial derivatives f_x and f_y.

Solution The partial derivatives of $f(x, y)$ w.r.t. x and y are the following:

$$\frac{\partial f}{\partial x} = f_x = \frac{\partial}{\partial x}\left(x^3y + xy^2\right) = 3x^2y + y^2, \quad y \text{ is constant}$$

$$\frac{\partial f}{\partial y} = f_y = \frac{\partial}{\partial y}\left(x^3y + xy^2\right) = x^3 + 2xy, \quad x \text{ is constant}$$

Example 3 The function is $z = x^2\sin(3x + y^3)$. Find z_x and z_y at a point $\left(\frac{\pi}{3}, 0\right)$.

Solution The partial derivative of $z(x, y)$ w.r.t. x is:

$$z_x = \frac{\partial}{\partial x}\left[x^2\sin(3x+y^3)\right], \quad y \text{ is constant}$$

$$= 2x\sin(3x+y^3) + x^2\cos(3x+y^3)\frac{\partial}{\partial x}\left(3x+y^3\right) = 2x\sin(3x+y^3) + x^2\cos(3x+y^3)(3+0)$$

$$= 2x\sin(3x+y^3) + 3x^2\cos(3x+y^3)$$

$$\left[z_x\right]_{\left(\frac{\pi}{3},0\right)} = 2\frac{\pi}{3}\sin\left(\frac{3\pi}{3}\right) + 3\frac{\pi^2}{9}\cos\left(3\frac{\pi}{3}\right) = \frac{2\pi}{3}\sin\pi + \frac{3\pi^2}{9}\cos\pi = -\frac{\pi^2}{3}$$

The partial derivative of $z(x, y)$ w.r.t. y is:

$$z_y = \frac{\partial}{\partial y}\left[x^2\sin(3x+y^3)\right], \quad x \text{ is constant}$$

$$= x^2\frac{\partial}{\partial y}\left[\sin(3x+y^3)\right] = x^2\cos(3x+y^3)\frac{\partial}{\partial y}\left(3x+y^3\right) = x^2\cos(3x+y^3)(0+3y^2)$$

$$= 3x^2y^2\cos(3x+y^3)$$

$$\left[z_y\right]_{\left(\frac{\pi}{3},0\right)} = 3\left(\frac{\pi^2}{9}\right)(0)^2\cos\left(\frac{3\pi}{3}\right) = 0$$

Example 4 Suppose that the temperature of the water at the point on a river where a nuclear power plant discharges its hot waste water is approximated by $T(x,y) = 2x + 5y + xy - 40$ (i)

Where x represents the temperature of the river water in degree Celsius before it reaches the power plant and y is the number of megawatts (in hundreds) of electricity being produced by the plant.

(a). Find and interpret $T_x(9,5)$. *(b).* Find and interpret $T_y(9,5)$.

Solution

a. The partial derivative of (i) w.r.t. x is the rate of change in T with respect to x: $T_x = 2 + y$, y is constant

This rate with $x = 9$ and $y = 5$ is $[T_x]_{(9,5)} = 2+y = 2+5 = 7$ the approximate change in temperature resulting from a one degree increase in input water, if the input electricity y remains constant at 500 megawatts.

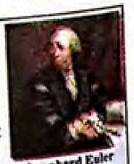
b. The partial derivative of (i) w.r.t. y is the rate of change in T with respect to y: $T_y = 5+x$, x is constant. This rate with $x = 9$ and $y = 5$ is $[T_y]_{(9,5)} = 5+x = 5+9 = 14$ the approximate change in temperature resulting from a one megawatt increase in production of electricity if the input water temperature x remains constant at 9°C.

Exercise 11.1

1. If $f(x,y) = x^2y + xy^2$ and t is any real number, then find out the following:
   a. $f(0,0)$ b. $f(-1,0)$ c. $f(0,-1)$
   d. $f(t,t)$ e. $f(t,t^2)$ f. $f(1-t,t)$
2. The function is $f(x,y,z) = x^2ye^{2x} + (x+y-z)^2$. Find the function value at the following points:
   a. $f(0,0,0)$ b. $f(1,-1,1)$ c. $f(-1,1,-1)$
   d. $\frac{\partial}{\partial x}f(x,x,x)$ e. $\frac{\partial}{\partial y}f(1,y,1)$ f. $\frac{\partial}{\partial z}f(1,1,z^2)$
3. Find the partial derivatives f_x and f_y of each of the following functions:
   a. $f(x,y) = \sin(x^2)\cos y$ b. $f(x,y) = \sqrt{3x^2+y^4}$
   c. $f(x,y) = xy^3\tan^{-1}y$ d. $f(x,y) = x^3 + x^2y + xy^2 + y^3$
   e. $f(x,y) = \sin^{-1}xy$ f. $f(x,y) = x^2e^{x+y}\cos y$
4. The production function z for the United States was once estimated as:
   $z = f(x,y) = x^{0.7}y^{0.3}$
   Where x stands for the amount of labor and y stands for the amount of capital. Find the marginal productivity of labor $\left(\frac{\partial z}{\partial x}\right)$ and of capital $\left(\frac{\partial z}{\partial y}\right)$.
5. A similar production function for Canada is:
   $z = f(x,y) = x^{0.4}y^{0.6}$
   Where x stands for the amount of labor and y stands for the amount of capital. Find the marginal productivity of labor $\left(\frac{\partial z}{\partial x}\right)$ and of capital $\left(\frac{\partial z}{\partial y}\right)$.
6. If $f(x,y) = x^2y + xy^2$, then find f_x and f_y by using definition of partial derivatives.

History

Leonhard Euler was a Swiss Mathematician, Physicist, Astronomer and Engineer. He made the important and influential contributions in many branches of Mathematics, such as calculus, graph theory, topology and analytic number theory. He also made significant contribution in mechanics, fluid dynamics, optics and music theory, He was the first person who introduced $f(x)$ to denoted the function f applied to the argument 'X'. In 1735 he introduced a theorem known by his name Euler theorem.



Leonhard Euler (1707-1783)

11.2 Euler's Theorem

The specialty of Euler's theorem is to verify the degree of a homogeneous function. The homogeneous function is a function $z = f(x,y)$ not altered if the real numbers x and y of a function $z = f(x,y)$ are stretched or squeezed by any real scalar quantity λ.

11.2.1 Homogeneous function of degree n

"A function $f(x,y)$ is said to be a homogeneous function of degree n if, for all values of λ and some constant values of n, we have $f(\lambda x, \lambda y) = \lambda^n f(x,y)$ " (i)

For example,

a. $f(x,y) = 3x+4y$ is a homogeneous function of degree 1.
   Since, $f(\lambda x,\lambda y) = 3(\lambda x) + 4(\lambda y) = \lambda(3x) + \lambda(4y) = \lambda(3x,4y) \Rightarrow f(\lambda x,\lambda y) = \lambda f(x,y)$

b. $f(x,y) = 3x^2 + 4y^2$ is a homogeneous function of degree 2.
   Since, $f(\lambda x,\lambda y) = 3(\lambda x)^2 + 4(\lambda y)^2$
   $= \lambda^2(3x^2, 4y^2)$
   $\Rightarrow f(\lambda x,\lambda y) = \lambda^2 f(x,y)$

In general, A function $f(x_1,x_2,x_3,\dots,x_n)$ is said to be homogeneous of degree n if, for all values of λ and some constant values of n, we have $f(\lambda x_1,\lambda x_2,\lambda x_3,\dots,\lambda x_n) = \lambda^n f(x_1,x_2,x_3,\dots,x_n)$.

Remember

A homogeneous function can also define another way.
"A function (x, y) is said to be homogeneous function of degree n if it is expressed in the form $f(x,y) = x^n.f\left(\frac{y}{x}\right)$"

Example 5 Show that the function $f(x,y) = 2xy + y^2$ is a homogeneous function of degree 2.

Solution The function $f(x,y) = 2xy + y^2$ is a homogeneous function of degree 2, if the identity (i) is true:

$$f(\lambda x,\lambda y) = 2(\lambda x)(\lambda y) + (\lambda y)^2 = 2\lambda^2xy + \lambda^2y^2 = \lambda^2(2xy + y^2) = \lambda^2 f(x,y),\ n = 2$$

Thus, the given function is a homogeneous function of degree 2.

11.2.2 Verification of Euler's theorem for homogeneous functions of different degrees

Statement: If $z = f(x,y)$ is continuously differentiable and defines a homogeneous function of degree n, then

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz \qquad \text{(i)}$$

Proof: If $z = x^n f\left(\frac{y}{x}\right)$, then, its partial derivatives with respect x and y are the following:

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}x^n f\left(\frac{y}{x}\right)$$

$$\frac{\partial z}{\partial x} = nx^{n-1}f\left(\frac{y}{x}\right) + x^n f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right) = nx^{n-1}f\left(\frac{y}{x}\right) - yx^{n-2}f'\left(\frac{y}{x}\right) \qquad \text{(ii)}$$

$$\frac{\partial z}{\partial y} = x^n f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) = x^{n-1}f'\left(\frac{y}{x}\right) \qquad \text{(iii)}$$

The addition of the products of (ii) by x and y (iii) by y to obtain the Euler's method of order n:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=x\left[nx^{n-1}f\left(\frac{y}{x}\right)-yx^{n-2}f'\left(\frac{y}{x}\right)\right]+y\left[x^{n-1}f'\left(\frac{y}{x}\right)\right]=nx^{n}f\left(\frac{y}{x}\right)-yx^{n-1}f'\left(\frac{y}{x}\right)+yx^{n-1}f'\left(\frac{y}{x}\right)$$

$$=nx^{n}f\left(\frac{y}{x}\right)=nz \qquad \text{(iv)}$$

Example 6 Use Euler's theorem to verify that the function $z=f(x,y)=ax^2+2bxy+cy^2$ is a homogeneous function of degree 2.

Solution The homogeneous function and its derivatives

$$z=f(x,y)=ax^2+2bxy+cy^2,\quad \frac{\partial z}{\partial x}=2ax+2by,\quad \frac{\partial z}{\partial y}=2bx+2cy$$ are used in Euler's result (iv) to confirm the degree of homogeneous function:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=x(2ax+2by)+y(2bx+2cy)\ =2ax^2+2bxy+2bxy+2cy^2=2(ax^2+2bxy+cy^2)=2z,\ n=2$$

The Euler's procedure confirmed the second degree of homogeneous function.

Example 7 If $u=\tan^{-1}\dfrac{x^3+y^3}{x-y}$, then, show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$$

Solution The function $u=(x,y)$ is not a homogeneous function, however, it can be reduced to homogeneous form by introducing a new variable z:

$$z=\tan u=\frac{x^3+y^3}{x-y}=\frac{x^3\left(1+\left(\frac{y}{x}\right)^3\right)}{x\left(1-\left(\frac{y}{x}\right)\right)}=x^2f\left(\frac{y}{x}\right)$$

The given function is a homogeneous of degree 2. The Euler's theorem in this situation is:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2z,\quad n=2 \qquad \text{(i)}$$

The partial derivatives of $z=\tan u$ w.r.t. x and y

$$\frac{\partial z}{\partial x}=\sec^2u\frac{\partial u}{\partial x},\quad \frac{\partial z}{\partial y}=\sec^2u\frac{\partial u}{\partial y}$$

are substituting in (i) to obtain the required result:

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2z$$

$$\sec^2u\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)=2z,\ z=\tan u$$

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2z\cos^2u=2\frac{\sin u}{\cos u}\cos^2u=2\sin u\cos u=\sin 2u$$

11.2.3 MAPLE command "*diff*" to find partial derivatives

Look at the following example ·he use of MAPLE command "diff" is illustrated in this examples.

Example 8 Use MAPLE command "diff" to find the partial derivation of

(*a*). $f(x,y)=x^3+y^3+3xy^2+4x^2y$ w.r.t, variables x and y.

(*b*). $f(x,y)=y\sin x+x\cos y+x^2$ w.r.t, variables x and y.

Solution

a. **Command:**

```
> diff(x^3 + y^3 + 3·x·y^2 + 4·x^2y, x);
                                        3x^2 + 3y^2 + 8xy
> diff(x^3 + y^3 + 3·x·y^2 + 4·x^2y, y);                     (2)
                                        3y^2 + 6xy + 4x^2
```

b.

```
> diff(y sin(x) + x·cos(y) + x^2, x);                         (1)
                                        y cos(x) + cos(y) + 2x
> diff(y sin(x) + x·cos(y) + x^2, y);                         (2)
                                        sin(x) − x sin(y)
```

Using Palettes: Use cursor button to select expression in which you are interested. In this problem, the expression is partial derivative palette. Click-partial derivative palette, insert the given function, then press "ENTER" key to obtain the partial derivatives of a given function:

$> \frac{\partial}{\partial x}(x^3+y^3+3\cdot x\cdot y^2+4x^2\cdot y)$

$$3x^2+3y^2+8xy \qquad (1)$$

$> \frac{\partial}{\partial y}(x^3+y^3+3\cdot x\cdot y^2+4x^2\cdot y)$

$$3y^2+6xy+4x^2 \qquad (2)$$

Exercise 11.2

1. Are the following functions homogeneous?

   a. $u=f(x,y)=\sin^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$?   b. $z=f(x,y)=\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$?

   c. $z=f(x,y)=x^3e^{\frac{y}{x}}-3y^2\sqrt{x^2+y^2}$ d. $z=f(x,y)=(x^2+3y^2)^{\frac{1}{3}}$

2. Verify Euler's theorem for the following homogeneous functions:

   a. $z=f(x,y)=ax^2+2hxy+by^2$ b. $z=f(x,y)=(x^2+xy+y^2)^{-1}$

3. If $u=f\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$.

4. If $z=xyf\left(\frac{x}{y}\right)$, then show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2z$.

5. a. If $u=\tan^{-1}\dfrac{x^2+y^2}{x+y}$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin(u).\cos(u)$

b. If $u=\sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0.$

6. Use MAPLE command "*diff*" to find the partial derivation of

a. $f(x,y)=\frac{x^3+y^3}{x^2+y^2}, w\cdot r\cdot t' y'$

b. $-\frac{\sin^2 x}{y}+x\cos(y) w\cdot r\cdot t' y'$

c. $\frac{1}{y}\tan(x)-x\cot^2(y) w\cdot r\cdot t' x'$

d. $\left(\frac{4t^3+3s^2}{s-t}\right) w\cdot r\cdot t' s'$

Review Exercise 11

1. **Choose the correct option.**

i. In the function $z=f(x,y)$, x and y are:
(a). alphanumeric variables (b). dependent variables
(c). independent variables (d). dependent constants

ii. If $f(x,y)=\sqrt{x^2+y^2-1}$ then $f(1,5)$ is:
(a). 25 (b). 5 (c). 125 (d). $3\sqrt{3}$

iii. If $f(x,y)=x^2+y^2$ then f_x or $\frac{\partial f}{\partial x}$ is:
(a). $2y$ (b). $2x$ (c). $2x+2y$ (d). $2x+y^2$

iv. $diff\left(\frac{1}{x+y},y\right)$; is:
(a). $\frac{1}{x^2}$ (b). $-\frac{1}{y^2}$ (c). $-\frac{1}{(x+y)^2}$ (d). $\frac{1}{(x+y)^2}$

v. The Euler's Theorem stated that:
(a). $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=nz$ (b). $x\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=nz$ (c). $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$ (d). $-x\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=nz$

vi. The functions of more than one independent variables are called:
(a). monomial functions (b). polynomial functions
(c). multivariate function (d). none of these

vii. If $f(x,y,z)=\sqrt{x^3+y^3-z}$, then $f(1,0,1)$ is:
(a). 1 (b). -1 (c). 0 (d). 2

viii. If $f(x,y)=\cos y\cdot e^{3x}$ then f_x is:
(a). $3\cos y\cdot e^{3x}-\sin y$ (b). $3\cos ye^{3x}$
(c). $3e^{3x}\cos y$ (d). $8\cos ye^{3x}$

ix. If $f(x,y)=3xe^y+x^3y^2$ then f_y is:
(a). $3xe^y+2x^3y$ (b). $3x^ye+2y3x^2$
(c). $3xe^y-2x^3y$ (d). $3x^2y^2+3e^y$

Summary

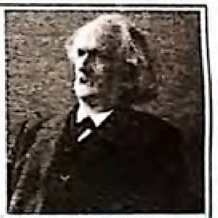
- A function $z=f(x,y)$ is a **function of two variables x and y**, if a unique of z is obtained from each ordered pair of real numbers (x,y). The real numbers x and y are **independent variables**; z is the **dependent variable.** The set of all ordered pairs of real numbers (x,y) such that $f(x,y)$ is a real number, is **the domain** of f and the set of all values of $f(x,y)$ is the **range.**
- A polynomial function in x and y is the sum of functions of the form
$f(x,y)=Cx^m y^n$
- If $z=f(x,y)$ is a function of two variables, then the first partial derivatives of $z=f(x,y)$ with respect to x and y are the functions f_x and f_y respectively, defined by,
$$f_x(x,y)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x},\quad f_y(x,y)=\lim_{\Delta x\to 0}\frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$$
- A function $f(x,y)$ is a **homogeneous** function of degree n in variables x and y, if for all values of the variables and for every positive value of λ, for which the identity is true:
$f(\lambda x,\lambda y)=\lambda^n f(x,y)$
- The specialty of **Euler's theorem** is to verify the degree of a homogeneous function. The homogeneous function is a function $z=f(x,y)$ not altered if the real numbers x and y of a function $z=f(x,y)$ are stretched or squeezed by any real scalar quantity λ.

History

Carl Neumann was German mathematician. He studied physics from his father and later became a mathematician. His father was professor at Konigsberg university. In 1875 be introduce the new standard notation (d-hat) during a lecture on the mechanical theory of heat. The symbol was popularized by his name as Neumann notation or δ (Greek delta). He worked on the drichlet principle and can be considered one of the initiators of the theory of integral equations. The Neumann series. That is analogous to the geometric series

$$\frac{1}{1-x}=a+x+x^2+x^3+\ldots$$

is named after him.



Carl Neumann
(1832)-(1925)

Carl Neumann also founded a mathematical research journal mathematische amalen. The Neumann boundary condition for different types of ordinary differential equations and partial differential equation is also named after him. He also developed an interest in thermodynamics via the overlap of heat and electricity.