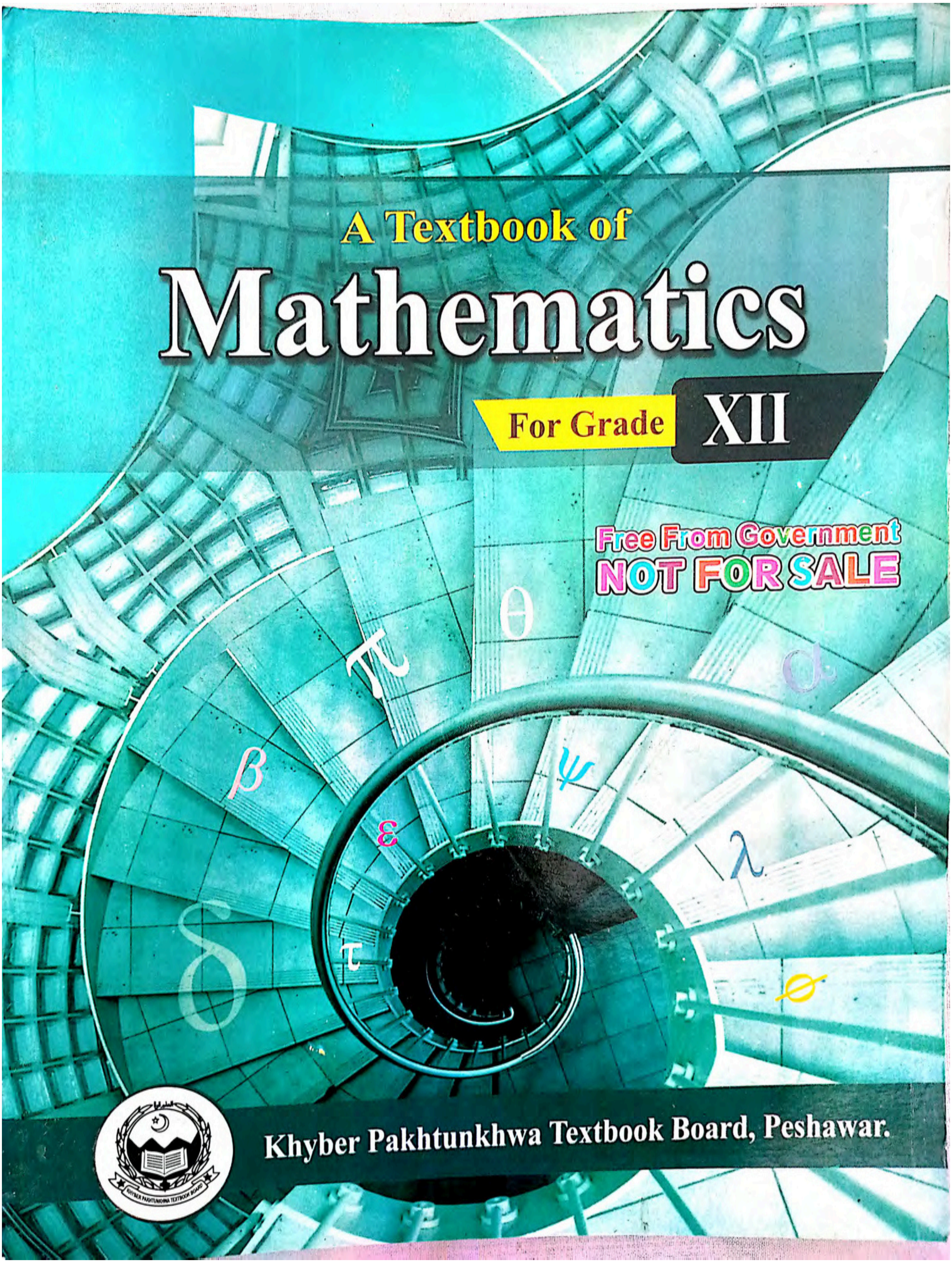

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Mathematics

For Grade XII

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A Textbook of

Mathematics

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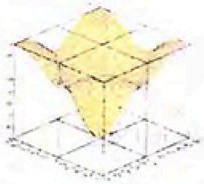

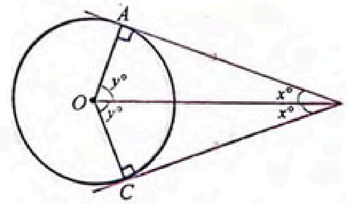

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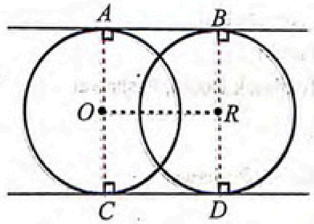

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Unit 1

INTRODUCTION TO SYMBOLIC PACKAGE: MAPLE

By the end of this unit, the students will be able to:

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- i. Recognize MAPLE environment.
- ii. Recognize basic MAPLE commands.
- iii. Use MAPLE as a calculator.
- iv. Use online MAPLE help.

1.2 Polynomials

- i. Use MAPLE commands for,
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  - expanding an expression,
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1.1 Introduction

In the modern age of science and technology, the technical computation has become the heart of problem solving in engineering and mathematics. To help us, MAPLE offers a vast repository of mathematical algorithms converting a wide range of applications. It is a symbolic and numeric computing tools as well as a multi-paradigm programming language maple was conceived at the university of waterloo in 1980. From the first day it continues to be the benchmark software for mathematical and symbolic computation. Maple user interface allows us to harness all the computational power by using context sensitive means, interactive assistant and task templates. In this unit we will learn how to use the basic commands that will allow and lead us into the creative, dynamic and captivating world of MAPLE explorations.

1.1.1 Recognition of MAPLE Environment

Maple software consists of three different parts.

i. User interface

It handles the input of mathematical expressions and different commands. User interface also handles the display of output and the control of the MAPLE worksheet environment.

ii. Kernel

It is a small collection of compiled C code. The entire kernel is loaded when a MAPLE session is started. It contains those essential facilities that required to run maple and perform basic mathematical operations. The components of kernel include the maple programming language interpreter, arithmetics and memory management facilities and fundamental functions. The small size of kernel ensures that the maple system is portable, compact and efficient.

iii. Library

It contains most of the maple routines including functions related to linear algebra, statistics, calculus, graphics and other topics. This library also consists of individual routines and different packages of routines. All the library routines which are implemented in the high level maple programming language that can be viewed and modified. Hence, it is useful to learn the maple programming language so, we can modify the existing code to produce the required routines.

A. Getting started with Maple

The maple software runs on different systems and platforms. It depends on the platform and system. It is convenient to use if you have windows based operating system (installed maple software package 14, 18 or any latest package).

When a maple session is started, the maple prompt command (>) is displayed like [>.

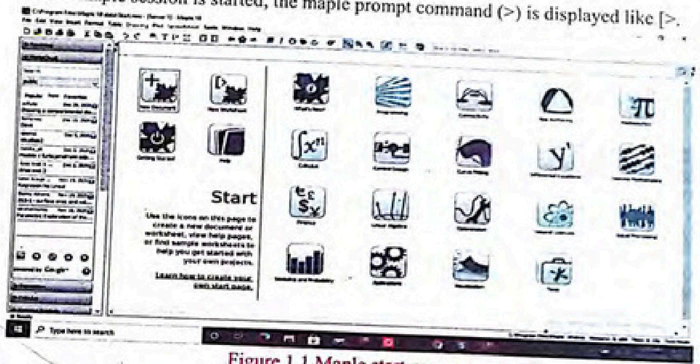


Figure 1.1 Maple start menu

This prompt character will show at the upper left of the worksheet indicates that maple is waiting to receive input in the form of maple statement.

When you are finished with the maple session, you will leave the program by selecting "Exit" under the file menu (upper left of the maple tools bar) as shown in Figure 1.2.

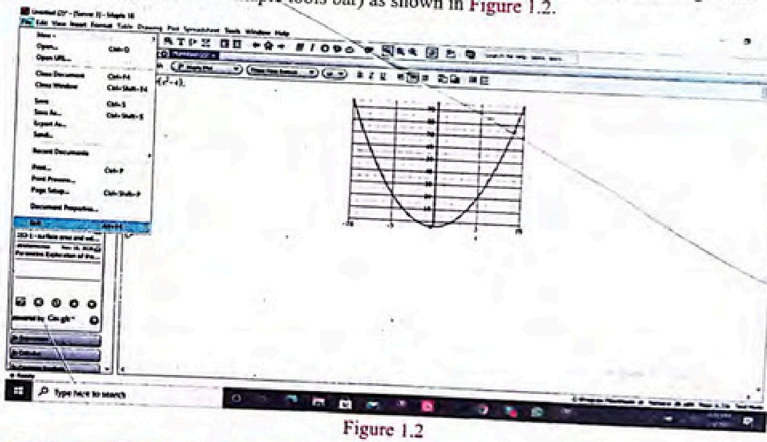


Figure 1.2

1.1.2 Recognition of basic MAPLE commands

A MAPLE command is a statement of calculation followed by a semicolon or a colon. Following are some commands followed by the displayed results.

Enter the commands on your worksheet and verify the given results. When you get to "Save the file", select "Save" under the "File" menu or use CTRL-S to save your working. **For example,**

```
> 7 + 17;                         24            (1)
> 12^2 + 15*2 - 3;                171           (2)
> 12 + 4*(6 + 1 - 10);            0             (3)
>
```

Remember

If you do not include a semicolon or colon at the end of a command, MAPLE will interpret the next command line as a continuation of the previous command.
The symbol +, −, *, / and ^ (or **) denote addition, subtraction, multiplication, division and exponential (4^2 = 4**2 = 4^2 = 16).
When a string of operations are specified in a command, MAPLE first does exponentiations then multiplication and divisions, then additions and subtractions. To change the order, we use parentheses.

i. Save command: To save a variable in our current MAPLE session to a file, type the following at the command prompt.

Save variablename, "Filename.m";

Replace the variable name with your variable name and replace the file name with your file name, but keep **.m** extension.

The Save command saves the variable as a MAPLE assignment statement. If the value of your variable depends on other variables, you must save them as well.

You can save more than one variable by giving all variable names to the save command.

Save variablename1, variablename2, variablename3, "filename.m";

ii. Editing commands: If you make a mistake in a command or want to change a command, you can go back and edit the command.

iii. Exact arithmetic and floating-point (evalf) commands: MAPLE calculates fractions (exact arithmetic) unless you specify that you want decimals (floating-point arithmetic) with the evalf function ("evalf" stands for "evaluate using floating-point arithmaetic").

$> \frac{27}{29} + \frac{14}{29} - \frac{4}{29};$ $\qquad \frac{37}{29}$ (4)

$> evalf\left(\frac{27}{29} + \frac{14}{29} - \frac{4}{29}, 4\right);$ $\qquad 1.276$ (5)

The argument 4 in the evalf command specifies the number of significant figures you want in the result. If you omit this command, you will get ten significant figures:

$> evalf\left(\frac{27}{29} + \frac{14}{29} - \frac{4}{29}\right);$ $\qquad 1.275862069$ (6)

\>

iv. Maple internal memory clearing command: To clear the internal memory during a maple session. We use 'restart' command or click the restart icon on the tool bar of the worksheet. e.g.

[> restart

when you enter this command, the maple session returns to its startup state. All the values reset to their initial values.

v. Enlistment of variables: Use the colon-equal symbol (:=) to define variables that is, to assign values to them. Once you have defined a variable, simply typing its name will show its value, and using the name in a formula will cause the value to be substituted. **For example,**

$> A := 25;\ B := 145;$ $\qquad A := 25$, $B := 145$ (7)

\>

If you want to string commands together on the same line, then:

$> A := 5;\ B := 10;\ C := 12;$ $\qquad A := 5$, $B := 10$, $C := 12$ (8)

$> 2 \cdot A + 4 \cdot B + 5 \cdot C;$ $\qquad 110$ (9)

$> \frac{4 \cdot A}{B};$ $\qquad 2$ (10)

\>

1.1.3 Use of MAPLE as a calculator

You click-start, then select-program <Maple 18 < click-"Maple Calculator" to obtain;

(a) Maplesoft (TM) Graphing Calculator Overview

This graphically scientific calculator is available for use as part of your Maple(TM) installation or via a Web Server running MapleNet(TM). The calculator use Maple for calculations.

On toolbar,

- use the "setting tab" to control the basic computation settings for the calculator.
- use the "Math tab" to select functions to apply, from basic functions to linear algebra to statistics.
- use the "Graph tab" to control over how graphs are displayed and what they display.
- use the "Data tab" to control the data used to produce a graph or the data you have tabulated directly.
- use the "variable tab" to control the variables you have assigned and their values.

To involve the graphing calculator, use your mouse to press the "Math tab" and select functions to apply. This will build up your expression for your in the input area, which is just below the session history area on the left side of the calculator. When you are ready to evaluate your expression, press ENTER key on your keyboard. Alternatively, you can press the "Graph button", to graph the expression, or the "data button", to tabulate values for the expression.

Example 1 Differentiate $f(x) = x^2 + 4x + 4$ with respect to x at a point $x = 2$. The steps required for obtaining the graphing-calculator result are:

Solution Click-Math tab < Click-Calculus < Click-Differentiate to obtain:

Diff(l)

Cursor "1" requires A: the expression x^2+4x+4, X: the differentiation of $f(x)$ with respect to x and P: the differentiation of $f(x)$ at a point x =2:

Diff(A,X,P)

Diff(x^2 + 4x + 4, x,2)

Click-ENTER 8

Example 2 Integrate x^2+4x+4, with respect to x over the interval [0,1]. The graphing – calculator result is as under:

Solution

Int(1)

Cursor "1" requires A: the function x^2+4x+4, X: the integration of $f(x)$ with respect to x, P: the lower limit $x = 0$ and Q: the upper limit $x = 1$ of the integral.

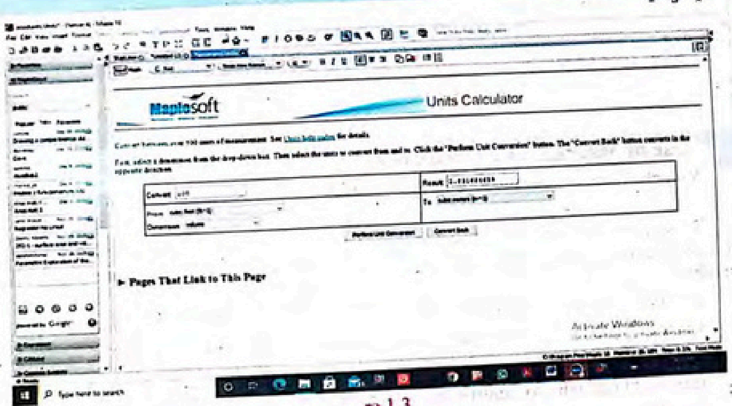
Int(A,X,P,Q)

Int(x^2 + 4x + 4,x,0,1) 6.333333

(b) Unit calculator

To use maple as a calculator simply type in your worksheet command prompt e.g.

[>? Assistant calculator



re 1.3

Then you will see the unit calculator as shown in the Figure 1.3.

1.1.4 Online MAPLE help

You can get help with MAPLE syntax by using the HELP menu, as described previously. If you have a question about a particular command, you can quickly get help by typing a question mark followed by the name (no semicolon). For example,

[> ? Addition

will open a window containing information about what the "addition" does and how to use it as shown in Figure 1.4. Click on the little "Cross" box at the upper left of the window to close down the little window.

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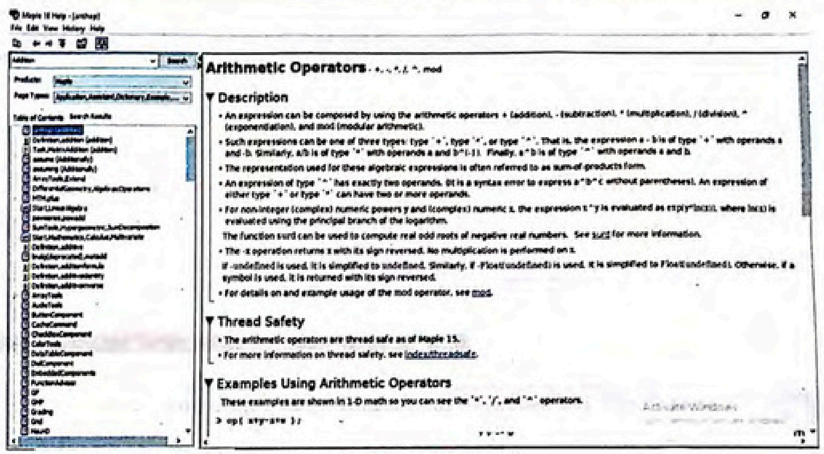

Figure 1.4

1.2 Polynomials

We are familiar about polynomials since our previous grades. The factorization of a polynomial, expansion of an expression can be solve through direct MAPLE commands and context menus.

1.2.1 Use of MAPLE commands for factoring a polynomial

(a) Commands

```
> factor(x^2 + 7·x + 12);
                                   (x+4)(x+3)                    (1)
>
```

(b) Context menu result

You can use Maple's context menus to perform a wide variety of mathematical and other operations. Enter the polynomial and place your cursor on the last end of the polynomial or expressions and right-click. The command will show you full information about factorization on line by typing. Then choose the factor option from the open window as shown in Figure 1.5. The context menu offers several operation to choose form according to the expression that you are using. The above result through context menu is as under:

```
> . - 7·x + 12
>       ^2+7*x+12 );
                                   (x+4)(x+3)                    (2)
>
```

The result is obtained through right-click on the last end of the expression by selecting "Factor" on the context menus. As shown in Figure 1.5.

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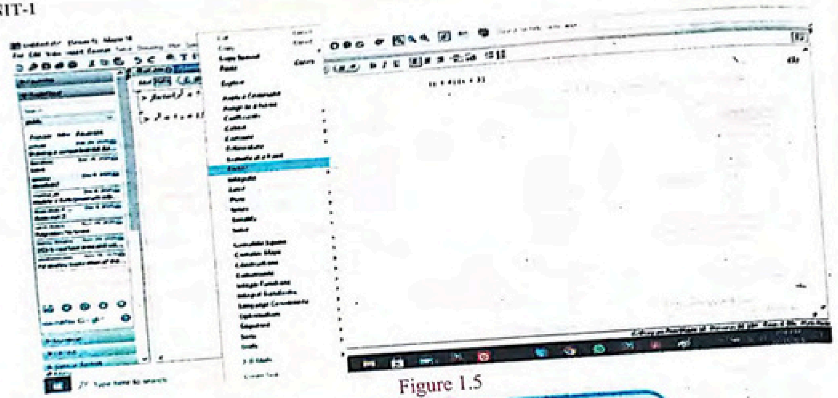

Figure 1.5

1.2.2 Use of MAPLE commands for expanding an expression

(a) Command

Use MAPLE command **"expand"** before parenthesis to expand the given expression.

$> expand((x^2+2)\cdot(x^3+4\cdot x-3));$

$$x^5+6x^3-3x^2+8x-6 \quad (3)$$

(b) Context menu result

Entre the given expression and place your cursor on the last end of the expression and right click. Then choose expand option from the opened window as shown in the Figure 1.6.

$> (x^2+2)\cdot(x^3+4\cdot x-3)$

```
> expand ( (x^2+2) * (x^3+4*x-3) ) ;
```

$$x^5+6x^3-3x^2+8x-6 \quad (4)$$

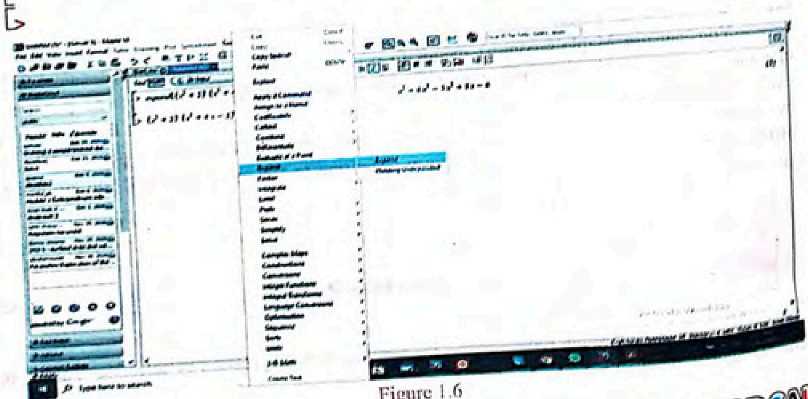

Figure 1.6

1.2.3 Use of MAPLE commands for simplifying an expression

(a) Command

$> simplify\left((25)^{\frac{1}{2}}+\frac{9}{6}-\frac{2}{3}\right);$

$$\frac{35}{6} \quad (5)$$

(b) Context menu result

Enter the given expression and place your cursor on the last end of the expression and right click. Then choose "simplify" from the opened window as shown in the Figure 1.7.

$> (25)^{\frac{1}{2}}+\frac{9}{6}-\frac{2}{3}$

```
> simplify ( 25^ (1/2) +9* (1/6) -2/3 ) ;
```

$$\frac{35}{6} \quad (6)$$

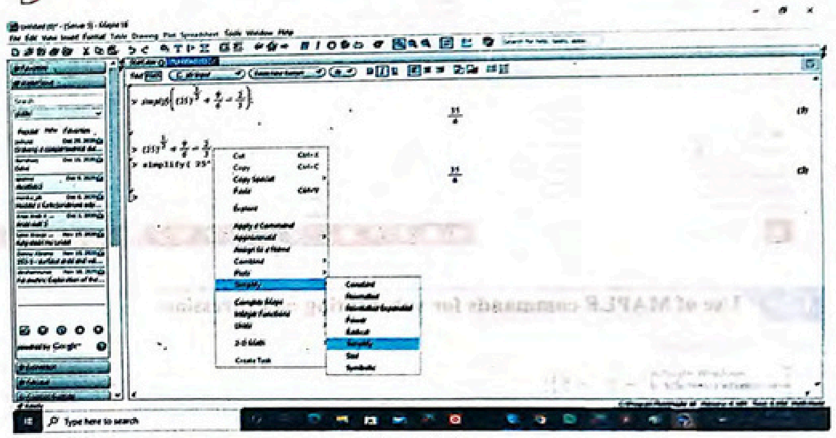

Figure 1.7

1.2.4 Use of MAPLE commands for simplifying a rational expression

(a) Command

$> simplify\left(\frac{t^2-9}{t^2+7\cdot t+12}\right);$

$$\frac{t-3}{t+4} \quad (7)$$

(b) Context menu result

Enter the given expression and place your cursor on the last end of the expression and right click. Then select "simplify" option as shown in the Figure 1.8.

> $\frac{t^2-9}{t^2+7\cdot t+12}$

> simplify ((t^2–9) / (t^2+7* t+12)) ;

$$\frac{t-3}{t+4} \quad (8)$$

>

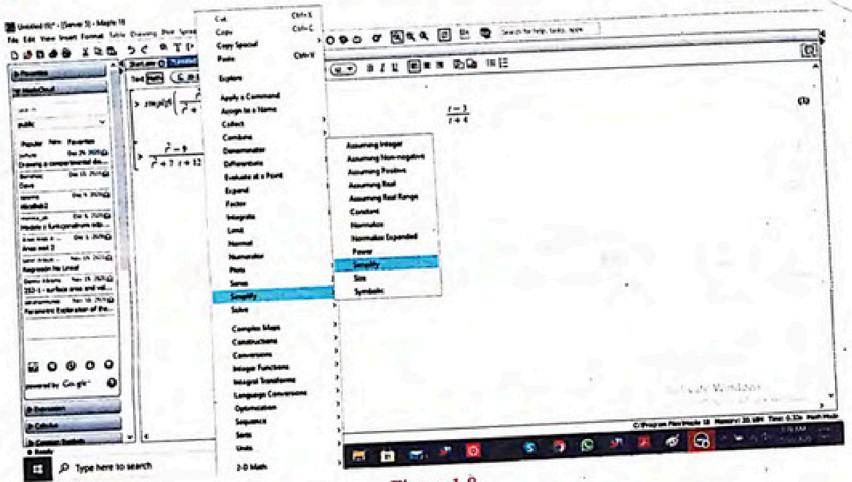

Figure 1.8

1.2.5 Use of MAPLE commands for substituting on expression

(a) Command

> $subs(t=20, t^2-2\cdot t+5)$;

$$365 \quad (9)$$

>

(b) Context menu result

Enter the given expression and place your cursor on last end of the expression and right click. Then select "evaluate at a point" option as shown in the Figure 1.9 and Figure 1.10 respectively.

> $t^2-2\cdot t+5$

> eval (t^2–2*t+5, t = 20) ;

$$365 \quad (10)$$

>

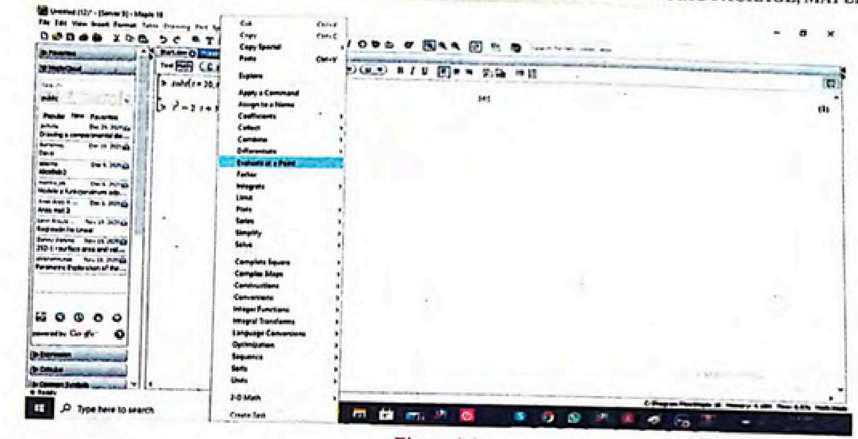

Figure 1.9

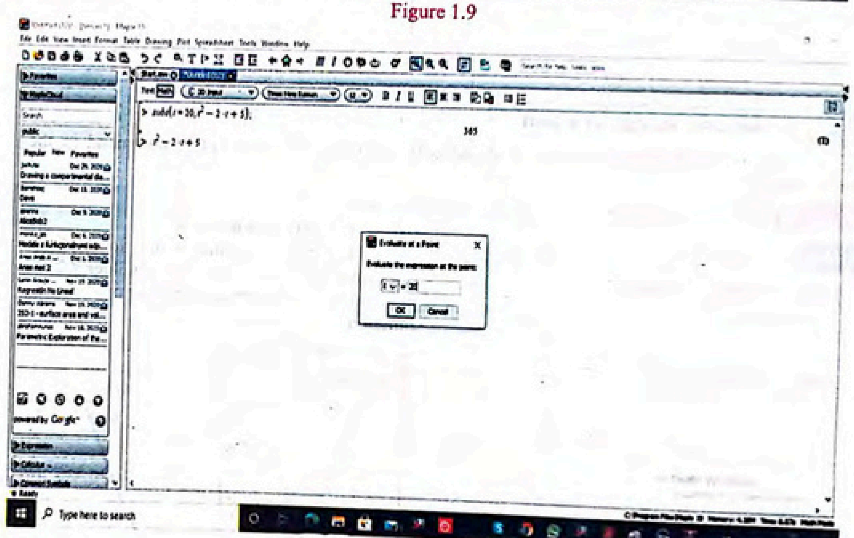

Figure 1.10

1.3 Graphics

1.3.1 Plot a two dimensional graph

To plot any two dimensional graph in MAPLE we use "plot" command in the command prompt.

Example 3 Plot the graph of x^2+3x+8.

Solution The graph of x^2+3x+8 is shown in the Figure 1.11 and 1.12 respectively.

(a) Command

> $plot(x^2+3\cdot x+8)$;

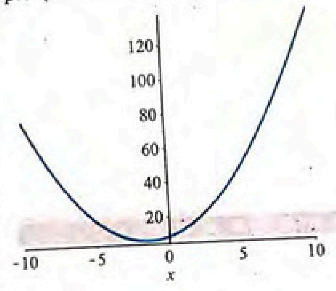

Figure 1.11

(b) Context menu result

This result is obtained through right click on the last end of the expression by selection "plots< 2-D plot" on the context menu.

> $x^2+3\cdot x+8$
> smartplot (x^2+3*x+8);

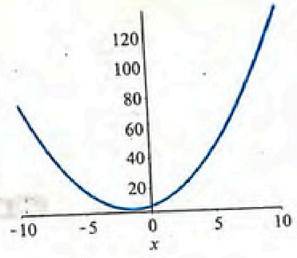

Figure 1.12

1.3.2 Domain and Range of a plot

To plot a two dimensional graph with domain and range. We use "plot" command in the command prompt.

Example 4 Plot the graph of x^2+2 having domain −3 to 3.

Solution The graph of x^2+2 is shown in the Figure 1.13 and 1.14 respectively.

(a) Command

> $plot(x^2+2, x=-3..3)$;

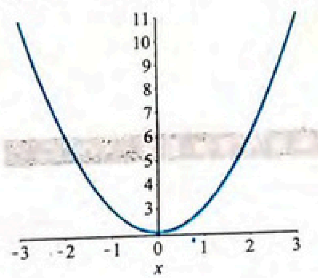

Figure 1.13

(b) Context menu result

Enter an expression or function, right click on it and select plots < plot builder < 2-D plot and enter the domain for the expression or function.

> $x^2+2, x=-3..3)$;
> plot (x^2+2, x = -3 .. 3);

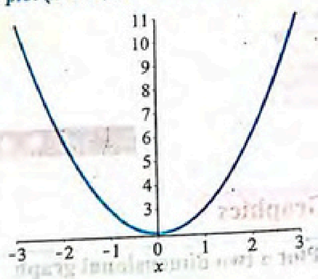

Figure 1.14

1.3.3 Sketch parametric equation

To sketch the parametric equation in MAPLE. We use MAPLE "plot" as discussed in the following example.

Example 5 Sketch the graph of parametric equation $x=\cos(t)$ and $x=\sin(t)$.

Where $-4<t\le 4$, $x=-2$ to 2 and $y=-2$ to 2

Solution

(a) Command

> $plot([\cos(t), \sin(t), t=-4..4], -2..2, -2..2)$;

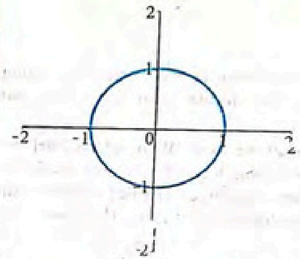

Figure 1.15

(b) Context menu result

Enter an expression, or function, right click on it and select plots < plot builder < 2-D parametric plot and domain for the parametric t.

> $\cos(t), \sin(t)$
> plot ([cos (t), sin (t), t = -4 .. 4]);

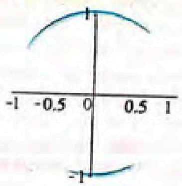

Figure 1.16

1.3.4 Know plotting options

The plotting options listed below can provide the commands that create 2-D plots. These options can be used with the "plot" command and are generally available to all Maple commands that generate two dimensional plots. The help page of Maple for a particular command provides more detail about the plotting options that accepts.

Options must be added at the end of the given sequence.

We can use an interactive method of exploring the options in the "plot" command by using the Interactive Plot Builder.

- **Adaptive**

adaptive = n or truefalse

When we are plotting a function over an interval. The interval is sampled at a number of points, controlled by **"sample"** and **"numpoints"**. Adaptive plotting, where necessary, subdivides these intervals to attempt to get a better representation of the given function. This subsampling can be turned off by setting the **"adaptive"** option to **false**. By default, this option is set to **true**, and intervals are subdivided at most six times in trying to improve the plot. By setting this option to a non-negative integer, we can control the maximum number of six times that subintervals are divided.

- **Annotation**

annotation = t

The annotation option allows us to add descriptive text to a 2-D curve or point plot. The term "point plot" in this context means a collection of points that are treated as a single plot element, such that created by the "dataplot" command. The text which is included mathematical expressions, appears when the pointer hovers over the plot element to which it is associated. An annotation for a curve or a point, which appears whenever the pointer is placed over the element.

- **Axes**

axes=f

Specifies the type of axes, one of: **boxed, frame, none, or normal.**

- **Axes font**

axesfont = l

Font for the labels on the tick marks of the axes, specified in the same manner as **font**. This option overrides values specified for the **font**.

- **Axis**

axis=t or axis[dir]=t

The **axis** and **axis[dir]** options allows us to provide information about one or more axes to plotting commands such as "plot" and " plot3d". For example, the axis color, locations of tick marks and gridlines, and use of logarithmic scales.

With the **axis** = **t** option, the information provided in **t** is applied to all the axes. With the **axis[dir]** = **t** option, **t** is used for only the direction(s) specified in **dir** (direction), which can be the single value at (x-axis), (y-axis) or (z-axis), or a sequence of two of these values. Multiple **axis[dir]** options, with different values of **dir**, can be used to specify different information for different axes. The commands for 2-D plotting do not accept the **axis[3]** option.

The axis information is given by **t**. The list **t** may contain one or more of the sub options.

- **Axis coordinates**

axiscoordinates = t

Normally a coordinate system is used to display of the axes. The value **t** can be either cartesian of polar Cartesian axes are displayed by default. If **t** is **polar**, then radial and angular axes are generated. This option is used together with the **cords** = **polar** option.

> **Note**
> This option is only available in the Standard interface. In the Classic interface, the coordinates are always Cartesian.

- **Background**

background = t

The value of **t** can be the name of an image file, as a string, a name, a **datatype** = **float**, Array as used with the Image Tools package, or a color. A plot can have a single background image or color.

If the size option is omitted then the dimensions of the plot can be determined by the dimensions of the image. If the **size** option is provided then the image is displayed with the dimensions of the plot.

A color may be given as a color tools [color] object or as a color string. If **t** is a string, then it is first interpreted as a filename. If the file **t** does not exist, it is then assumed to be a color.

- **Caption**

caption = c

The value **c** can be an arbitrary expression. It can also be a list consisting of the caption followed by the **font** option. The default is no caption.

- **Caption font**

captionfont = l

This option defines the font for the plot caption, specified in the same manner as **font**. This option over-rides values specified for **font**.

- **Color**

color = n or colour = n

This allows us to specify the color of the curves to be plotted.

- **Color scheme**

colorscheme = t or colourscheme = t

This allows us to apply a color scheme to a surface or set of points.

- **Coordinate view**

coordinateview = $[r_1...r_2, a_1...a_2]$

This option is used when the **axis coordinates** option has the value **polar**. When that is the case, then $r_1...r_2$ specifies the radial range that is to be displayed and $a_1...a_2$ specifies the angular range.

- **Cords**

cords = cname

The value **cname** is one of the choices listed on the coords help page. The cartesian axes are displayed by default. To generate polar axes with polar plots, we use **axiscoordinates** = **polar** option along with the **coords** = **polar** option.

- **Discount**

discount = t

This allows us for detection of discontinuities.

- **Filled**

filled = truefalse or list

If the **filled** option is set to **true**, the area between the curve and the x-axis is given by a solid color. The value of the **filled** option can also be a list containing one or more sub options (color, style or transparency). These options are applied only to the filled area, and not to the original curve itself. This option does not work with non-cartesian coordinate systems.

- **Filled regions**

filledregions = truefalse

If the **filledregions** option is set to **true**, the regions defined by the curves are filled with different colors. This option is valid only by using the following commands:

"contourplot", "implicitplot" and "listcontplot". This option does not work with non-Cartesian coordinate systems.

- **Font**

font = l

This option defines the font for the plot title, caption, axis tick mark labels, and axis labels if no values have been specified for the **axes font, captionfont, label font, or title font** options. The value **l** is a list of the form **[family, style, size]**.

The value of **family** can be one of **Times, Courier, Helvetica,** or **Symbol**. It can also be any font name supported by your system, for example, **Times New Roman** and **Calibri** in Windows. The first letter of the **family** name must be capitalized.

The value of **style** can be omitted or one of **roman, bold, italic, bold italic, oblique,** or **bold oblique**. The **Symbol** family does not accept a style option. The final value, **size**, is the point size to be used.

Gridline

gridlines=truefalse

When **gridlines = true** or **gridlines** is provided, default gridlines are drawn. The default is **gridlines = false**.If the **axis** option is also provided and contains a **gridlines** sub option, then that option over rides this **gridlines** option.

Labels

labels = [x, y]

This option specifies labels for the axes. By default labels are the names of the variables in the original function to be plotted, if these are available otherwise, no labels will be used.

Label directions

labeldirections = [x,y]

This option specifies the direction in which labels are printed along the axes. The values of x and y should be **horizontal** or **vertical**. The default direction of any labels is **horizontal**.

Label font

labelfont = l

The font of the labels on the axes of the plot, specified in the same manner as **font**. This option overrides values specified for **font**.

Legend

legend = s

If the plot command is being used to plot multiple curves, then **s** can be a list containing a legend entry for each curve.

Legend style

legendstyle = s

Since the value **s** is a list consisting of one or more sub options. The sub options are available for the **legendstyle** option include **font = f** and **location = loc**. The **location = loc** sub option allows us **top, bottom, right** and **left** for **loc**.

Line style

linestyle = t

It controls the line style of curves. The **line style** value **t** can be one of the following names: **solid, dot, dash, dash dot, long dash, space dash,** or **space dot**. The default value of **t** is **solid**. The value **t** can also be an integer from 1 to 7, where each integer represents a line style, as given in the order above.

Num points

numpoints = n

Specifies the minimum number of points to be generated. The default number of points is 200.

Remember

Plot employs an adaptive plotting scheme which automatically does more work where the function values do not lie close to a straight line.

Resolution

resolution = n

This sets the horizontal display resolution of the device in pixels. The default resolution is n=800.

The value of **n** is used to determine when the adaptive plotting scheme terminates.

Sample

sample = [l]

A list of numerical values which is to be used for the initial sampling of the function. Normally, the function is sampled at additional points. To restrict sampling to only these values we include the **adaptive = false** option.

Scaling

scaling = s

It controls scaling of the graph. The value of **s** is **unconstrained** by default, which means the plot is scaled to fit the plot window. The **constrained** value causes all axes to use the same scale.

Size

size = [w, h]

We use this to specify the size of the plot window. We can set the size of the plot window by specifying the number of pixels, a proportion of worksheet width, or a ratio, such as a square, the golden ratio, or a custom ratio.

Smart view

smartview = truefalse

This is used to determine an appropriate view of the plot data. The plot command generates data based on the range provided by us or on a default range if this is not provided. When the **smartview = true** option is provided, a view that tries to present the important regions of the data is computed. To show all data computed, use the **smartview = false** option. The **default** setting of **smartview** is **true**. This option is available for the plot command and only applies to curves not points, polygons or text.

Style

style=s

The plot style should be one of **line, point, point line, polygon,** or **polygon outline**. The names in parentheses are aliases for the option values. The styles **line**, **polygon**, and **polygon outline** all draw curves by interpolating between the sample points. The **point** style results in a plot of the points only. The default style, **polygon outline**, draws any polygons as filled with an outline. The **polygon** style shows the polygons with no outline, whereas **line** draws the polygons as outlines only. The **point line** style is a combination of the **point** and **line** styles.

Symbol

symbol=s

Symbol for points in the plot, where the value **s** is one of **asterisk, box, circle, cross, diagonalcross, diamond, point, solidbox, solidcircle, soliddiamond**

- **Symbol size**

symbolsize = n

The size of a symbol used in plotting can be given by a natural number. This does not affect the symbol **POINT**. The symbol size is **10** by default.

- **Thickness**

thickness = n

This option specifies the thickness of lines in the plot. The **thickness** = **n** must be a non-negative number. A value with 0 produces the thinnest line. By default the value is 1.

- **Tick marks**

Tickmarks = [m, n]

The values **m** and **n** specify the tick mark placement for the *x-axis* and *y-axis* respectively and can take an integer specifying the number of tick marks, a list of values specifying locations, a list of equations each having the form **location = label**, a name, or a **spacing** structure.

- **Title**

title=t

We can give a title to the plot. The value t can be an arbitrary expression. The value **t** can also be a list consisting of the title followed by the font option. There is no title by default for a plot.

- **Title font**

titlefont = l

It specified in the same manner as font. This option overrides values specified for **font**.

- **Transparency**

transparency = t

This option specifies the transparency of the plot surface. The **transparency** = **t** must evaluate to a floating-point number in the range [0, 1]. 0 means "not transparent" but 1 means "fully transparent."

- **Use units**

useunits = t

This option, with **t** set to **true**, indicates that units are part of the function and should be included in the axes labels. The value **t** can also be a list of units.

- **View**

view = [xmin...xmax, ymin...ymax]

View option indicates the minimum and maximum coordinates of the curve to be displayed on the screen. By default it is determined by the **smartview** option: if **smartview** = **false** is given, then all the plot data will be displayed; if **smartview** = **true** or the **smartview** option is not given, then the plot structure will be analyzed to determine a reasonable view of the data which allows you to see the significant features of the data.

Remember

i. If the same option is provided more than once, with different values, then the final value specified is generally the one used.

ii. All above options are available for the Standard Worksheet interface.

Example 6 Plot the graph of sec(x) and give the title as;

title="Graph\n of Secant Function "

titlefont = ["Times New Roman", 20]

Solution

```
> plot(sec(x), x=-2 π.. 2 π, title=Graph of Secant Function", titlefont= ["Times New Roman", 20]);
```

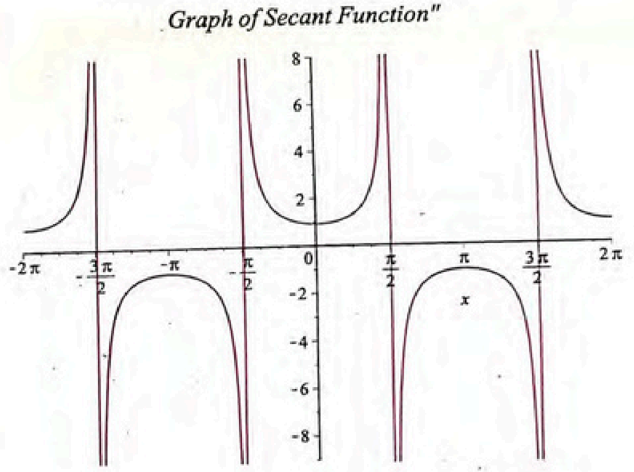

Figure 1.17

This example uses the following axes and graph properties:

title="Graph\n of Secant function"

axes = framed

style = point

symbol = asterisk

symbolsize = 20

tickmarks = [spacing (Pi), default]

1.4 Matrices

The command will show you full information about matrices on line by typing: [> ?*matrices*

1.4.1 Recognition of matrix and vector entry arrangement

(a) Command

```
> with(linalg);
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian] (1)

```
> X=matrix(3, 3, [1, 4, 2, 2, 3, 4, 5, 4, 6]);
```

$$X = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 4 \\ 5 & 4 & 6 \end{bmatrix} \quad (2)$$

```
> Y=matrix(3, 3, [5, 3, 4, 1, 2, 0, 9, 3, 7]);
```

$$Y = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 2 & 0 \\ 9 & 3 & 7 \end{bmatrix} \quad (3)$$

```
> Z=matrix(3, 3, [1, 0, 1, 2, 3, 1, 0, 5, 1]);
```

$$Z = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 1 \\ 0 & 5 & 1 \end{bmatrix} \quad (4)$$

```
> c := ⟨1,2,3⟩;
```

$$c := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad (5)$$

```
> r := ⟨1|2|3⟩;
```

$$r := \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad (6)$$

```
>
```


(b) Using Palettes

Use cursor button to select matrix palette. Click- "matrix" <click-choose(for the umber of rows and columns of a required matrix) <click-data type (to select integers entries of the rows and columns of a required matrix), then finally click-"insert matrix" and press ENTER key to obtain a required matrix.

$$> \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 4 \\ 5 & 4 & 6 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 4 \\ 5 & 4 & 6 \end{bmatrix} \quad (7)$$

1.4.2 Applying matrix operations

(a) Matrix addition

```
> with(linalg);
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian] (1)

```
> X := Matrix([[1, 2, 3], [1, 3, 0], [1, 4, 3]]);
```

$$X := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (2)$$

```
> Y := Matrix([[1, 2, 5], [6, 3, 0], [1, 4, 3]]);
```

$$Y := \begin{bmatrix} 1 & 2 & 5 \\ 6 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (3)$$

```
> matadd(X, Y);
```

$$\begin{bmatrix} 2 & 4 & 8 \\ 7 & 6 & 0 \\ 2 & 8 & 6 \end{bmatrix} \quad (4)$$

```
>
```

(b) Matrix multiplication

```
> with(linalg);
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian] (1)

```
> X := Matrix([[1, 2, 3], [1, 3, 0], [1, 4, 3]]);
```

$$X := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (2)$$

```
> Y := Matrix([[1, 2, 5], [6, 3, 0], [1, 4, 3]]);
```

$$Y := \begin{bmatrix} 1 & 2 & 5 \\ 6 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (3)$$

```
> multiply(X, Y);
```

$$\begin{bmatrix} 16 & 20 & 14 \\ 19 & 11 & 5 \\ 28 & 26 & 14 \end{bmatrix} \quad (4)$$

```
>
```

(c) Using Palettes

To obtain the result simply right click on the last "matrix" then click simplify or press ENTER.

$$> \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 5 \\ 6 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 8 \\ 7 & 6 & 0 \\ 2 & 8 & 6 \end{bmatrix} \quad (1)$$

```
> simplify ( Matrix (%id = 18446744074594159542) +Matrix (%id = 18446744074594159662) );
```

$$\begin{bmatrix} 2 & 4 & 8 \\ 7 & 6 & 0 \\ 2 & 8 & 6 \end{bmatrix} \quad (2)$$

```
>
```


1.4.3 Inverse and transpose of a matrix

(a) Inverse of a matrix

```
> with(linalg);
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian] (1)

```
> X := Matrix([[1, 2, 3], [1, 3, 0], [1, 4, 3]]);
```

$$X := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (2)$$

```
> Y := Matrix([[1, 2, 5], [6, 3, 0], [1, 4, 3]]);
```

$$Y := \begin{bmatrix} 1 & 2 & 5 \\ 6 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (3)$$

```
> inv := inverse(X);
```

$$inv := \begin{bmatrix} \frac{3}{2} & 1 & -\frac{3}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \end{bmatrix} \quad (4)$$

```
> inv := inverse(Y);
```

$$inv := \begin{bmatrix} \frac{3}{26} & \frac{7}{39} & -\frac{5}{26} \\ -\frac{3}{13} & -\frac{1}{39} & \frac{5}{13} \\ \frac{7}{26} & -\frac{1}{39} & -\frac{3}{26} \end{bmatrix} \quad (5)$$

```
>
```

(b) Transport of a matrix

```
> with(linalg);
```

[BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian] (1)

```
> X := Matrix([[1, 2, 3], [1, 3, 0], [1, 4, 3]]);
```

$$X := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (2)$$

```
> Y := Matrix([[1, 2, 5], [6, 3, 0], [1, 4, 3]]);
```

$$Y := \begin{bmatrix} 1 & 2 & 5 \\ 6 & 3 & 0 \\ 1 & 4 & 3 \end{bmatrix} \quad (3)$$

```
> trans := transpose(X);
```

$$trans := \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 0 & 3 \end{bmatrix} \quad (4)$$

```
> trans := transpose(Y);
```

$$trans := \begin{bmatrix} 1 & 6 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 3 \end{bmatrix} \quad (5)$$

```
>
```


(c) Using Palettes

Inverse of a matrix

To obtain the result through context menu. Right-click on the last end of the matrix by selecting "Standard operations" and then right-click on the "inverse" to obtained the inverse of a matrix.

$$> \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

```
> LinearAlgebra:-MatrixInverse( Matrix(%id = 18446744074459087686) );
```

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \quad (1)$$

Similarly,

Transpose of a matrix

To obtain the result through context menu. Right-click on the last end of the matrix by selecting "Standard operations" and then right-click on the "transpose" to obtained the inverse of a matrix.

$$> \begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 6 \\ 3 & 5 & 1 \end{bmatrix}$$

```
> LinearAlgebra:-Transpose( Matrix(%id = 18446744073883260558) );
```

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 2 & 5 \\ 3 & 6 & 1 \end{bmatrix} \quad (2)$$

Determinant of a matrix

To obtain the result through context menu. Right-click on the last end of the matrix by selecting "Standard operations" and then right-click on the "determinant" to obtained the inverse of a matrix.

$$> \begin{bmatrix} 3 & 2 & 5 \\ 1 & 5 & 7 \\ 2 & 9 & 4 \end{bmatrix}$$

```
> LinearAlgebra:-Determinant( Matrix(%id = 18446744073883262118) );
```

$$-114 \quad (3)$$

```
>
```