6. Define equilibrium. Explain its types and state the two conditions of equilibrium.
7. State and explain principle of moments with examples.
8. What is center of mass or center of garvity. Explain how CM/CG can be determined? Is there any difference between CM and CG?
9. Explain the stability of the objects with reference to position of center of mass.

NUMERICAL QUESTIONS

1. To open a door force of 15 N is applied at 30° to the horizontal, find the horizontal and vertical components of force.
2. A bolt on a car engine needs to be tightened with a torque of 40 Nm. You use a 25-cm-long wrench and pull on the end of the wrench perpendicularly. How much force do you have to exert?
3. Sana, whose mass is 43 kg, sits 1.8 m from the center of a seesaw. Faiz, whose mass is 52 kg, wants to balance Sana. How far from the center of the seesaw should Faiz sit?
4. Two kids of weighing 300 N and 350 N are sitting at the ends of 6 m long see-saw. The seesaw is pivoted at its centre. Where would a third kid sit so that the see-saw is in equilibrium in the horizontal position? The weight of third kid is 250 N. (Ignore the weight of see-saw)
5. Two children push on opposite sides of a door during play. Both push horizontally and perpendicular to the door. One child pushes with a force of 20 N at a distance of 0.60 m from the hinges, and the second child pushes at a distance of 0.50 m. What force must the second child exert to keep the door from moving? Assume friction is negligible.
6. A construction crane lifts building material of mass 1500 kg by moving its crane arm, calculate moment of force when moment arm is 20 m. After lifting the crane arm, which reduces moment arm to 12 m calculate moment.

WEB LINKS

http://www.learneasy.info/MDME/MEMmods/MEM30005A/add_forces/add_forces.html
https://courses.lumenlearning.com/boundless-physics/chapter/center-of-mass/
http://www.cyberphysics.co.uk/topics/forces/stability.htm

Unit 5 GRAVITATION

CHECK LIST

After studying this unit you should be able to:

- ✓ state Newton's law of gravitation.
- ✓ explain that the gravitational forces are consistent with Newton's third law.
- ✓ explain gravitational field as an example of field of force.
- ✓ define weight (as the force on an object due to a gravitational field.)
- ✓ calculate the mass of earth by using law of gravitation.
- ✓ solve problems using Newton's law of gravitation.
- ✓ explain that value of 'g' decreases with altitude from the surface of earth.
- ✓ discuss the importance of Newton's law of gravitation in understanding the
- ✓ motion of satellites.

The study of gravity has always been a central theme in physics, from Galileo's early experiments on free fall in the seventeenth century to Stephen Hawking's work on black holes in recent years. Perhaps the grandest milestone in this endeavor, was the discovery by Newton of the law for gravitation.

Our Milky Way galaxy is a disk-shaped collection of gas, dust, and billions of stars, including our Sun and solar system. Image shows how our galaxy would look if we could view it from outside. Earth is near the edge of the disk of the galaxy, about 26,000 light-years (2.5×10^{20} m) from its central bulge. Our galaxy is a member of the Local Group of galaxies, which includes the Andromeda galaxy at a distance of 2.5×10^6 light-years.

The force that binds together these progressively larger structures, from moon all the way to galaxy clusters is known as the gravitational force. Gravity is action-at-a-distance force which acts even when the interacting objects are not in physical contact. This force not only holds us on Earth but also reaches out across intergalactic space and acts between galaxies. This force is our focus in this unit.

5.1 LAW OF UNIVERSAL GRAVITATION

Newton expressed the force of attraction between different bodies in the universe in the form of a law known as the law of universal gravitation; which is stated as follows

'Every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres'.

Consider two spherical bodies of masses 'm_1' and 'm_2' separated by distance 'r' as shown in figure 5.1.

By definition of Newton's law of universal gravitation, the force of gravity F_g is

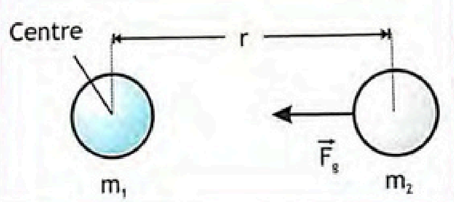
$$F_g \propto m_1 \times m_2 \quad \text{——— (1)}$$

and $$F_g \propto \frac{1}{r^2} \quad \text{——— (2)}$$

combining equation 1 and equation 2 we get

$$F_g \propto \frac{m_1 m_2}{r^2}$$

Figure 5.1 UNIVERSAL GRAVITATION

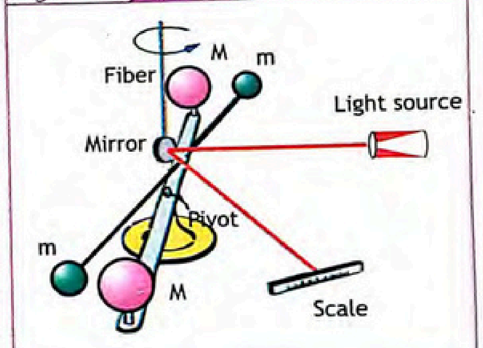


To change sign of proportionality into equality a constant is included

$$F_g = G\frac{m_1 m_2}{r^2} \quad \text{——— 5.1}$$

Where G is constant of proportionality and is known as gravitational constant. The value of gravitational constant (as determine by experiment) G = 6.67×10^{-11} $Nm^2\,kg^{-2}$. It does not depend on the medium (vacuum, air, water, concrete etc.) between the two bodies. The above formula is a mathematical expression of Newton's law of universal gravitation.

Figure 5.2 TORSION BALANCE



Cavendish used an apparatus like this to measure the gravitational attraction between the two suspended spheres (m) and the two on the stand (M) by observing the amount of torsion (twisting) created in the fiber. Distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

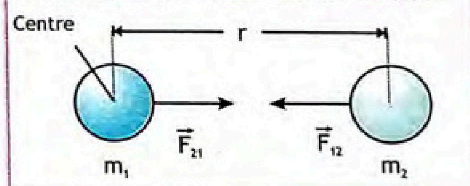
Newton's Third Law of Motion and Universal Gravitation:

We can see that the force acting on mass m_2 due to mass m_1 is F_{12}. Also the force acting on mass m_1 due to mass m_2 is same force F_{21} both these forces are equal but opposite in direction. Therefore, we can say that the forces acting on two bodies due to gravitation force is the example of action and reaction forces.

$$\vec{F}_{12} = -\vec{F}_{12} \quad \text{——— 5.2}$$

For example, Earth pulls on the Moon and the Moon pulls on Earth with a force of equal magnitude. At Earth's surface, Earth pulls down on a 1.0 kg mass with a force of magnitude 9.8 N, and the

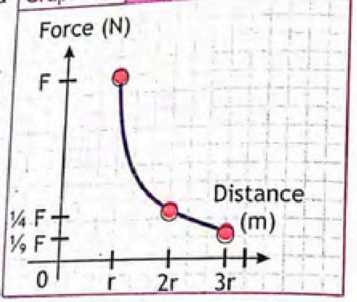
Figure 5.3 UNIVERSAL GRAVITATION AND NEWTON'S THIRD LAW OF MOTION



1.0 kg mass pulls upward on Earth with a force of magnitude 9.8 N.

The gravitational force is an inverse-square force: it decreases by a factor of 4 when the distance increases by a factor of 2, it decreases by a factor of 9 when the distance increases by a factor of 3, and so on. Graph 5.1 is a plot of the magnitude of the gravitational force as a function of the distance.

Graph 5.1



Example 5.1 CAN YOU ATTRACT ANOTHER PERSON GRAVITATIONALLY

Suppose you have mass of 40 kg and a 45 kg person is sitting on a bench close to you, such that the distance between centers of you and the person is 0.5 m. Estimate the magnitude of the gravitational force you exert on that person.

SOLUTION

GIVEN:

mass m_1 = 40 kg

mass m_2 = 45 kg

distance r = 0.5 m

Gravitational Constant $G = 6.673 \times 10^{-11}\ Nm^2kg^{-2}$

REQUIRED:

Gravitational force F_g = ?

By Newton's law of universal gravitation

$$F_g = G\frac{m_1 m_2}{r^2}$$

putting values $$F_g = 6.67 \times 10^{-11}\ Nm^2kg^{-2}\ \frac{40\,kg \times 45\,kg}{(0.5m)^2}$$

or $$F_g = 6.67 \times 10^{-11}\ \frac{N\cancel{m^2}}{\cancel{kg^2}}\ \frac{1800\,\cancel{kg^2}}{0.25\,\cancel{m^2}}$$

therefore $$F_g = 4.8 \times 10^{-7}\ N$$ — Answer

This means we attract each other but a force of about 10^{-7} N is unnoticeably small unless extremely sensitive instruments are used. Even very large scale objects like ships and building have very small gravitational attraction.

Assignment 5.1 GRAVITATIONAL FORCE ON MOON

The mass of earth is 6×10^{24} kg and that of the moon is 7.4×10^{22} kg. If the distance between the earth and the moon is 3.84×10^5 km, calculate the force exerted by the earth on the moon.

INFORMATION

Light from distant stars and galaxies takes so long to reach us that we are actually seeing these objects as they appeared in the past. As we look up at the sky, we are really looking back in time. For example, the Sun's light takes almost 8.5 minutes to travel to Earth, so we see the Sun as it looked 8.5 minutes ago. The nearest star to us, Proxima Centauri, is 4.2 light-years away, so it appears as it was 4.2 years ago. The nearest galaxy is 2.5 million light-years away, and it looks as it did when our australopithecus hominid ancestors walked the planet. The farther away something is, the further back in time it appears.

5.2 MASS OF EARTH

The determination of gravitational constant helped scientists to calculate the mass of earth, sun and other celestial objects.

Let an object of mass 'm_o' be placed on the surface of the Earth. The distance between the center of the body and the earth is (nearly) equal to the radius of earth 'r_E'. If the mass of earth is 'm_E' then the force F_g with which the earth attracts the body towards its centre is given by law of gravitation.

$$F_g = G\frac{m_o m_E}{r_E^2} \quad \text{--- (1)}$$

We know that the force of gravity is equal to the weight of the body.

$$F_g = W = m_o g \quad \text{--- (2)}$$

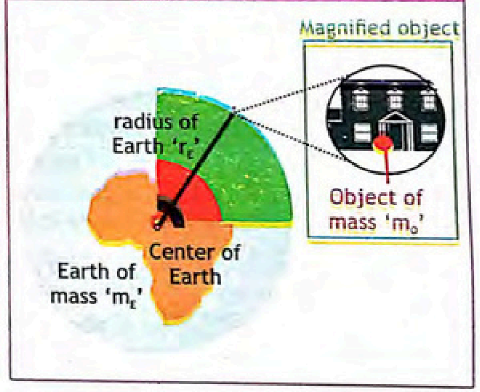
comparing equation 1 and equation 2 we get

$$m_o g = G\frac{m_o m_E}{r_E^2}$$

or $$g = G\frac{m_E}{r_E^2}$$

re-arranging $$m_E = \frac{g r_E^2}{G} \quad \text{--- (5.3)}$$

Figure 5.4 MASS OF EARTH



Since Gravitational Constant $G = 6.673 \times 10^{-11}\ Nm^2kg^{-2}$

for earth we have acceleration due to gravity $g = 9.8\ ms^{-2}$ ($9.8\ Nkg^{-1}$) and radius of Earth $r_E = 6.4 \times 10^6$ m. Putting these values in equation 5.3 we get

$$m_E = \frac{9.8\,Nkg^{-1}(6.4\times10^6\ m)^2}{6.67\times10^{-11}\ Nm^2\ kg^{-2}}$$

or
$$m_E = \frac{9.8\,\cancel{N}\cancel{kg}^{-1}\times 4.1\times10^{13}\ \cancel{m^2}}{6.67\times10^{-11}\ \cancel{N}\cancel{m^2}\ kg^{-2}}$$

therefore $m_E = 6\times10^{24}\ kg$

TIP $Nkg^{-1} = ms^{-2}$

Since $N = kg\ ms^{-2}$

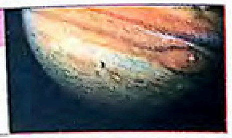
Dividing both sides by kg

$N/kg = ms^{-2}$

or $N\ kg^{-1} = ms^{-2}$

Thus the mass of earth is approximately 6×10^{24} kg. This means that if we know acceleration due to gravity 'g' and radius 'r' of any planet or star we can calculate its mass.

Example 5.2 MASS OF JUPITER



Jupiter is the largest planet in the solar system. Radius of Jupiter is $r_J = 7.15 \times 10^7$ m, the acceleration due to gravity on Jupiter is 24.7 N/kg. Calculate the mass of Jupiter.

SOLUTION

GIVEN:

Radius of Jupiter $r_J = 7.15 \times 10^7$ m

Acceleration due to gravity at Jupiter $g_J = 24.7\ N/kg$

Gravitational Constant $G = 6.673 \times 10^{-11}\ Nm^2kg^{-2}$

REQUIRED:

Mass of Jupiter m_J = ?

The mass of a planet or star can be calculated by using equation 4.4

$$m_J = \frac{g_J r_J^2}{G}$$

putting values
$$m_J = \frac{24.7\ N/kg\times(7.15\times10^7\ m)^2}{6.673\times10^{-11}\ Nm^2kg^{-2}}$$

or
$$m_J = \frac{24.7\ \cancel{N}\cancel{kg}^{-1}\times5.11\times10^{15}\ \cancel{m^2}}{6.673\times10^{-11}\ \cancel{N}\cancel{m^2}kg^{-2}}$$

or
$$m_J = 1.891\times10^{16+11}kg$$

therefore
$$m_J = 1.89\times10^{27}\,kg$$ Answer

Thus the mass of Jupiter is about 300 times the mass of the earth.

Assignment 5.2 MASS OF MOON

If the radius of the moon is 1.74×10^6 m and have acceleration due to gravity on its surface as $1.6\ ms^{-2}$. Calculate the mass of moon.

TIP REMEMBER THESE VALUES

Gravitational constant $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$

Radius of earth $r_E = 6.4 \times 10^6$ m

TABLE 5.1: SELECTED DATA FOR THE SOLAR SYSTEM

Planet	Radius (km)	Mass (10^{24} kg)	g (m/s^2)	Orbital Period (yr)
Mercury	2,400	0.330	3.7	0.241
Venus	6,050	4.87	8.9	0.615
Earth	6,370	5.97	9.8	1
Mars	3,400	0.642	3.7	1.88
Jupiter	71,500	1890	24.7	11.9
Saturn	60,300	568	9.0	29.4
Uranus	25,600	86.8	8.7	83.8
Neptune	24,800	102	11.0	164
Sun	696,000	1,990,000	274	--

5.3 GRAVITATIONAL FIELD

Gravity is a field force:

Newton was not able to explain how objects can exert forces on one another without coming into contact. He developed a mathematical theory to describe gravity, but he did not had a physical explanation for how gravity works. Scientists later developed a theory of fields to explain how gravity and other forces operate.

According to this theory, masses create a gravitational field in space. (Similarly, charged objects generate an electric field and magnets develop a magnetic field.) A gravitational force is

INFORMATION

We have eight planets in our Solar System. However, outside of our Solar System there are thousands of other planets. The extra-solar planets or exo-planets are in orbit around another star. So far we have almost 3500 confirmed new worlds, with another 2000 awaiting confirmation.

an interaction between a mass and the gravitational field created by other masses.

At any point, Earth's gravitational field can be described by the gravitational field strength, abbreviated as g. The value of 'g' is equal to the magnitude of the gravitational force exerted on a unit mass at that point,

or $g = F_g/m$.

The gravitational field (g) is a vector with a magnitude of 'g' that points in the direction of the gravitational force.

Gravitational field strength (free-fall acceleration) 'g':

Consider an object that is free to accelerate and is acted on only by gravitational force. According to Newton's second law, a = F/m. As seen earlier, g is defined as F_g/m, where F_g is gravitational force. Thus, the value of 'g' at any given point is equal to the acceleration due to gravity. For this reason, g = 9.8 m/s² on Earth's surface. Although gravitational field strength and free-fall acceleration are equivalent, they are not the same thing. For instance, when you hang an object from a spring scale, you are measuring gravitational field strength. Because the mass is at rest (in a frame of reference fixed to Earth's surface), there is no measurable acceleration.

Figure 5.5 Gravitational Field

Figure 5.5 shows gravitational field vectors at different points around Earth. As shown in the figure, gravitational field strength rapidly decreases as the distance from Earth increases, as you would expect from the inverse-square nature of Newton's law of universal gravitation.

Weight changes with location:

In chapter 3 Dynamics, we learned that weight is the magnitude of the force due to gravity, which equals mass times free-fall acceleration. We can now refine our definition of weight as mass times gravitational field strength.

The two definitions are mathematically equivalent, but our new definition helps to explain why your weight changes with our location in the universe.

Newton's law of universal gravitation shows that the value of g depends on mass of all reacting bodies and distance to it.

For example, consider an object (e.g. tennis ball) of mass m_o placed on surface of earth. Let m_E be the mass of the earth and r_E is the distance between their centers. The gravitational force between the tennis ball and Earth is as follows:

$$F_g = G\frac{m_o m_E}{r_E^2} \quad \text{——} (1)$$

The gravitational field strength Fg by Newton's second law is

$$F_g = W = m_o g \quad \text{——} (2)$$

comparing equation 1 and equation 2 we get

$$\cancel{m_o} g = G\frac{\cancel{m_o} m_E}{r_E^2}$$

therefore

$$g = G\frac{m_E}{r_E^2} \quad \text{——} \boxed{5.4}$$

Inserting the known values of

$m_E = 6 \times 10^{24}$ kg

and $r_E = 6.4 \times 10^6$ m

along with $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$,

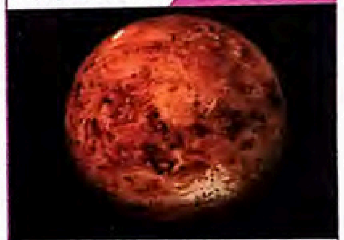
we find

$$g = 6.67 \times 10^{-11}\ Nm^2kg^{-2}\ \frac{6 \times 10^{24}\ kg}{(6.4 \times 10^6\ m)^2}$$

therefore $g = 9.77\ ms^{-2}$

or $g = 9.8\ ms^{-2}$

TID BITS



The hottest planet is not the closest planet to the Sun.
Even though Mercury is the closest planet to the Sun, it is not actually the hottest. Mercury does not have any atmosphere meaning that this planet is only hot in the daytime when it is directly facing the Sun. At this stage temperatures can rise to 425°C but at night the planet's temperature can drop down to a freezing -180°C. Venus (shown in picture) is the hottest planet. Its thick clouds trap the Sun's heat causing Venus to be a sizzling 500°C all of the time!

Equation 5.4 shows that the value of 'g' does not depend upon the mass of the body. This means that light and heavy bodies should fall towards the centre of the earth with the same acceleration. This equation also shows that gravitational field strength depends only on mass of earth 'm_E' and radius of earth 'r_E'. As earth is not a perfect sphere, and there are mountains and trenches so our distance from Earth's center varies as we change location, so our weight also varies. On the surface of any planet, the value of g, as well as our weight, will depend on the planet's mass and radius.

TECHNOLOGY

The variation in the value of 'g' on Earth is used to detect the presence of minerals and oil. Geophysicists use sensitive instruments, called gravimeters, to detect small variations in 'g' when they search for new deposits of ore or oil. Gold and silver deposits increase the value of 'g', while deposits of oil and natural gas decrease 'g'. Picture is an example of a map that shows different measured values of g as lines, where each line represents a specific value of g.

Example 5.3 VALUE OF 'g' AT SUN'S SURFACE

What is the free-fall acceleration at the surface of the sun? As mass of Sun is 1.99×10^{30} kg and having radius of 6.96×10^{8} m.

SOLUTION

GIVEN:

Radius of Sun $r_s = 6.96 \times 10^8$ m

Mass of Sun $m_s = 1.99 \times 10^{30}$ kg

Gravitational Constant $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}

REQUIRED:

free-fall acceleration at Sun $g_s = ?$

The value of g at sun's surface can be calculated by equation 5.4 as

$$g = G\frac{m_S}{r_S^2}$$

$$g = 6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2}\,\frac{1.99 \times 10^{30}\,\text{kg}}{(6.96 \times 10^{8}\,\text{m})^2}$$

therefore $g = 274\,ms^{-2}$ — Answer

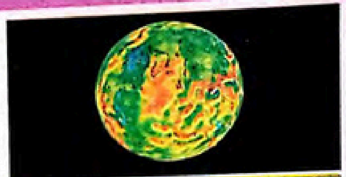
Notice that the mass of the Sun is almost a million times more than the mass of the Earth, and having gravitational acceleration of about 30 times that of earth.

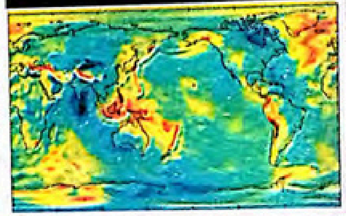
Assignment 5.3 FREE FALL AT MOON

An astronaut of mass 65.0 kg (weighing 637 N on earth) is walking on the surface of the Moon, which has a mean radius of 1.74×10^{6} m and a mass of 7.35×10^{22} kg. What is the weight of the astronaut on moon? What is the free-fall acceleration at the surface of the moon?

INFORMATION CHART

This global model of the Earth's gravitational strength was constructed from a combination of surface gravity measurements and satellite tracking data. It shows how the acceleration of gravity varies from the value at an idealized "sea level" that takes into account the Earth's non-spherical shape. (The Earth is somewhat flattened at the poles—its radius is greatest at the equator.) Gravity is strongest in the red areas and weakest in the dark blue areas.

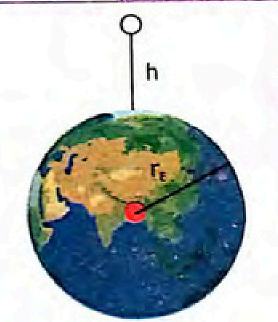




5.4 VARIATION of 'g' WITH ALTITUDE

The value of 'g' at a given place depends upon the distance from the centre of earth. Greater the distance from the centre of the earth, smaller will be the value of 'g'. Equation 5.3 shows that the value of 'g' varies inversely as the square of distance. Let 'g_h' be the value of acceleration due to garvity at a height 'h' from the surface of the earth we can modify Equation 5.4 as .

Figure 5.6



$$g_h = G\frac{m_E}{(r_E + h)^2} \quad \text{—— (1)}$$

equation 5.3 can also be written as

$$g = G\frac{m_E}{r_E^2}$$

or $$Gm_E = g r_E^2 \quad \text{—— (2)}$$

putting equation 2 in equation 1, we get $$g_h = \frac{g r_E^2}{(r_E + h)^2} \quad \text{—— 5.5}$$

Equation 5.6 shows that as we move away from the center of earth the value of 'g' decreases. The change in the value of g is significant only at very large distances.

Example 5.4 METEOROID IN EARTH'S FIELD

A meteoroid (a chunk of rock) is at 4.4×10^7 m from the earth, what is the value of free fall acceleration 'g' at this point due to earth.

SOLUTION

GIVEN:

Radius of earth $r_E = 6.4 \times 10^6$ m

Acceleration due to gravity $g = 9.8\ ms^{-2}$

height $h = 4.4 \times 10^7$ m $= 44 \times 10^6$ m

REQUIRED:

free-fall acceleration at height g_h = ?

The value of 'g_h' at height h, can be calculated by using equation 5.5

$$g_h = \frac{g r_E^2}{(r_E + h)^2}$$

putting values $$g_h = \frac{9.8\,ms^{-2} \times (6.4 \times 10^6\ m)^2}{(6.4 \times 10^6\ m + 44 \times 10^6\ m)^2}$$

or $$g_h = \frac{9.8\,ms^{-2} \times (6.4 \times 10^6\ m)^2}{(50.4 \times 10^6\ m)^2}$$

or $$g_h = \frac{9.8\,ms^{-2} \times 4.1 \times 10^{13}\ \cancel{m^2}}{2.54 \times 10^{15}\ \cancel{m^2}}$$

therefore $g_h = 0.16\,ms^{-2}$ — Answer

EXTENSION EXERCISE 5.1

Confirm the value of 'g' for Jupiter given in Example 5.2, Using equation 5.5

Assignment 5.4 Value Of g At Specific Height

Calculate the value of g at 1000 km and 35900 km above the earth surface.

TABLE 5.1: CHANGE IN GRAVITATIONAL ACCELERATION WITH ALTITUDE

Altitude (km)	a (m/s²)	Altitude Example
0.0	9.8	Mean Earth radius
8.8	9.8	Mt. Everest
36.6	9.7	Highest manned balloon
400	8.7	Space shuttle orbit
35 700	0.2	Communications satellite

POINT TO PONDER

The magnitude of g on the moon's surface is about 1/6 of the value of g on Earth's surface. Can you infer from this relationship that the moon's mass is 1/6 of Earth's mass? Why or why not?

5.5 MOTION OF ARTIFICIAL SATELLITES

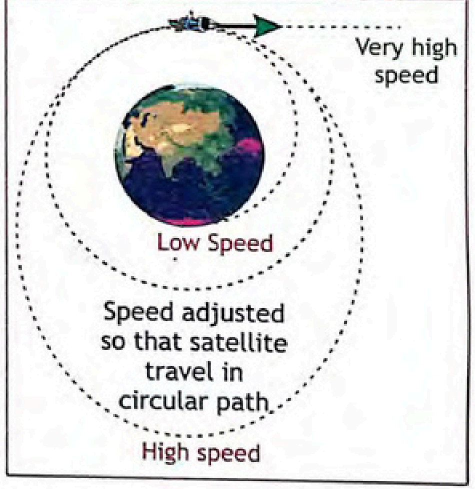
A **Satellite** is a naturally occurring or man-made object that orbits another object larger than itself by force of gravity.

A natural body orbiting another body so large that center of mass is well within the larger body is called natural satellites of that body. Six of the major planets possess natural satellites often termed as moons.

Any object purposely placed into orbit of earth or other planets, stars or sun are termed as artificial satellites. First artificial satellites was launched in 1957, since then thousands have been sent for communication industry (internet, mobile phones, TV communication etc.), weather, military purposes and scientific study.

A satellite is put into orbit by moving it to high altitude and then accelerating it to a sufficiently high tangential speed with the use of rockets, as shown in Fig. 5.7. If the speed is too high, the spacecraft either move in elliptical orbit or will not be confined by the Earth's gravity and will escape, never to return. If the speed is too low, it will fall back to Earth. Satellites are typically put into circular (or nearly circular) orbits, because such orbits require the least take off speed.

Figure 5.7 SATELLITE MOTION



You may be wondering "What keeps a satellite up?" The answer is: its high speed. If a satellite in orbit stopped moving, it would fall directly to Earth. But at sufficient speed a satellite would orbit in space around a larger body.

Speed in circular orbit calculation:

In circular orbit a satellite has a constant tangential speed called orbital velocity. Consider a satellite of mass m_s revolving in a circular orbit with velocity v from earth of mass m_E. Let r be the distance between the center of earth and the center of satellite as shown in figure 5.8. Then the gravitational force by Newton's law of universal gravitation is

$$F_g = \frac{Gm_E m_s}{r^2} \quad \text{—— (1)}$$

Then centripetal force to keep satellite of mass m, revolving in a circular orbit of radius r with velocity 'v' is

$$F_C = \frac{m_s v^2}{r} \quad \text{—— (2)}$$

Then centripetal force is provided by gravitational force therefore

$$F_g = F_C \quad \text{—— (3)}$$

Putting values from equation 1 and equation 2 in equation 3, we get

$$\frac{\cancel{m_s}v^2}{\cancel{r}} = \frac{G\cancel{m_s}m_E}{r^{\cancel{2}}}$$

or $$v^2 = \frac{Gm_E}{r}$$

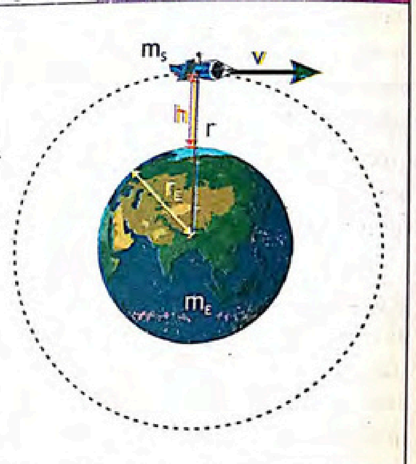
taking square root on both sides

$$\sqrt{v^2} = \sqrt{\frac{Gm_E}{r}}$$

therefore $$v = \sqrt{\frac{Gm_E}{r}}$$

or $$v = \sqrt{\frac{Gm_E}{r_E + h}} \quad \text{—— 5.6}$$

Figure 5.8 ORBITAL SPEED



Where h is the height of satellite from surface of earth and r_E is the radius of earth. The equation 5.6 shows that orbital speed depends upon the mass of Earth and the distance from the center of the Earth to the center of mass of the satellite and does not depend upon the mass of satellite.

Which means that for particular distance from the center of earth, all the satellite should have the same orbital speed irrespective of the size of satellite.

Geostationary communications satellites are placed in a circular orbit that is 3.59 $\times 10^7$ m = 35.9 $\times 10^6$ m above the surface of the earth. Their orbital speed can be calculated by equation 5.6 as

$$v = \sqrt{\frac{Gm_E}{r_E + h}}$$

putting values $$v = \sqrt{6.673 \times 10^{-11}\, Nm^2kg^{-2}\, \frac{6 \times 10^{24}\, kg}{(35.9 + 6.4) \times 10^6\, m}}$$

therefore $$v = 3.07 \times 10^3\, ms^{-1}$$

or $$v = 3.07\, kms^{-1}$$

This equation can also be used to measure the velocity of any planet or celestial object moving in a circular path.

Example 5.5 SPEED OF EARTH AROUND SUN

The mass of the Sun is 1.99 $\times 10^{30}$ kg, and the radius of the Earth's orbit around the Sun is 1.5×10^{11} m. From this, calculate the orbital speed of the Earth.

SOLUTION

GIVEN:

Radius r = 1.5×10^{11} m

Mass of Sun m_s = 1.99 $\times 10^{30}$ kg

Gravitational Constant G = 6.673 $\times 10^{-11}$ Nm^2kg^{-2}

REQUIRED:

Orbital Speed v = ?

the orbital speed is $$v = \sqrt{G\frac{m_s}{r}}$$

putting values $$v = \sqrt{6.673 \times 10^{-11}\, Nm^2kg^{-2}\, \frac{1.99 \times 10^{30}\, kg}{1.5 \times 10^{11}\, m}}$$

therefore $$v = 3.0 \times 10^4\, m/s = 30\, km/s$$ —— Answer

Assignment 5.5 ORBITAL SPEED OF A SATELLITE

If a satellite orbits the earth at 2,000 km above sea level, how fast must the orbiting satellite travel to maintain a circular orbit?

KEY POINTS

Law of Gravitation: Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.

Gravitational acceleration: The acceleration of bodies falling freely towards the earth is called gravitational acceleration 'g'. The value of 'g' decreases as altitude increases.

Artificial Satellite: The objects which are used for communication and space research revolve round the Earth under the force of gravity, which provides the necessary centripetal force.

Orbital Velocity: It is the velocity of a satellite which moves around the Earth at a specific height.

PROJECTS

GROUP - A

SUPARCO: What is SUPARCO? Research Pakistan Space Program 2040 and Develop a presentation on space prospects in Pakistan?

GROUP - B

Dream Space Visit: Research how do space scientists and engineers know what kinds of science instruments (cameras, spectrometers, etc.) to put on space craft that are destined for other planets, moons, asteroids, or comets? How do they decide what they will want to measure once they get to, say, Saturn's moon Titan or Jupiter's moon Europa? Select a place for your visit, then design a two day trip to that world, and describe the things you will carry with you. Write an essay and place it school library for reference.

GROUP - C

Pluto: In 2006 a new definition of planet was set and Pluto was demoted to dwarf planet, research the reason and present it in chart.

GROUP - D

Extra Solar Planet: Explore the discovery of planets around other stars. What methods did the astronomers use? What measurements did they take? Write a research article.

GROUP - E

Sky Observation: Consult your instructor or other sources to find out what

planets are observable in the evening during the current month. Venus, Jupiter, or Mars are usually the best candidates.

a. Locate the planet visually and observe it with binoculars if possible. How does the planet differ in appearance from that of nearby stars?

b. Sketch the position of the planet relative to nearby stars for several nights. How does this position change?

EXERCISE

MULTIPLE CHOICE QUESTIONS

1. Two masses are separated by a distance r. If both masses are doubled, the force of interaction between the two masses changes by a factor
   A. 2 B. 4 C. 1/2 D. 1/4
2. The radius of earth r_E is
   A. 9.8 m B. 6.67×10^{-11} m C. 6×10^{24} m D. 6.4×10^{6} m
3. The S.I. unit for gravitational constant "G" is
   A. $N\,kg^2$ B. $N\,m^2\,kg^{-2}$ C. $N\,m^2\,kg^2$ D. $N\,m^{-2}\,kg^2$
4. What is the mass on planet Mercury of an object that weighs 784 N on the earth's surface?
   A. 80.0 kg B. 118 C. 784 kg D. More information needed
5. An object is orbiting a planet with orbital speed v, if the radius is same and mass is increased 4 times, by what factor orbital speed will change
   A. 2 B. 4 C. 1/2 D. 1/4
6. The value of 'g' at the surface of the moon is;
   A. $9.8\,m\,s^{-2}$ B. $1.63\,m\,s^{-2}$ C. $4.9\,m\,s^{-2}$ D. $8.9\,m\,s^{-2}$
7. The value of 'g' ________ with altitude
   A. increases B. decreases C. gets ZERO D. remains the same
8. When a body is moved from sea level to the top of a mountain, There is change in the body's
   A. mass B. weight C. none D. both mass and weight
9. The value of 'g' at equator is
   A. Same at poles B. Larger at poles
   C. Smaller at poles D. none

CONCEPTUAL QUESTIONS

Give a brief response to the following questions.

1. If there is an attractive force between all objects, why don't we feel ourselves gravitating toward nearby massive buildings?
2. Does the sun exert a larger force on the Earth than that exerted on the sun by the Earth? Explain.
3. What is the importance of gravitational constant 'G'? Why is it difficult to calculate?
4. If Earth somehow expanded to a larger radius, with no change in mass, how would your weight be affected? How would it be affected if Earth instead shrunk?
5. What would happen to your weight on earth if the mass of the earth doubled, but its radius stayed the same?
6. Why lighter and heavier objects fall at the same rate toward the earth?
7. The value of 'g' changes with location on earth, however we take same value of 'g' as $9.8\ ms^{-2}$ for ordinary calculations. Why?
8. Moon is attracted by the earth, why it does not fall on earth?
9. Why for same height larger and smaller satellites must have same orbital speeds?

COMPREHENSIVE QUESTIONS

Give an extended response to the following questions.

1. State and explain the law of Universal Gravitation. Also show that the law obeys Newton's third law of motion.
2. Determine the mass of Earth by applying law of gravitation.
3. What is gravitational field and gravitational field strength? Show that the weight of an object changes with location.
4. How is the value of 'g' changing by going to higher altitude? Write the relevant formula.
5. Derive the formula for the orbital speed of an artificial satellite.

NUMERICAL QUESTIONS

1. Pluto's moon Charon is unusually large considering Pluto's size, giving them the character of a double planet. Their masses are 1.25×10^{22} kg and 1.9×10^{21} kg, and their average distance from one another is 1.96×10^{4} km. What is the gravitational force between them?
2. The mass of Mars is 6.4×10^{23} kg and having radius of 3.4×10^{6} m. Calculate the gravitational field strength (g) on Mars surface.
3. Titan is the largest moon of Saturn and the only moon in the solar system known to have a substantial atmosphere. Find the acceleration due to gravity on Titan's surface, given that its mass is 1.35×10^{18} and its radius is 2570 km.
4. At which altitude above Earth's surface would the gravitational acceleration be $4.9\ m/s^2$?
5. Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 6×10^{24} kg and the radius of Earth is 6.4×10^{6} m, what is the satellite's orbital speed?
6. The distance from center of earth to center of moon is 3.8×10^{8} m. Mass of earth is 6×10^{24} kg. What is the orbital speed of moon?
7. The Hubble space telescope orbits Earth ($m_E = 6 \times 10^{24}$ kg) with an orbital speed of 7.6×10^{3} m/s. Calculate its altitude above Earth's surface.

WEB LINKS

http://suparco.gov.pk/webroot/index.asp

https://www.nasa.gov/

http://www.ist.edu.pk/

https://www.space.com/