

- 3 A 2200-kg vehicle traveling at 94 km/h (26 m/s) can be stopped in 21s by gently applying the brakes. It can be stopped in 3.8 s if the driver slams on the brakes. What average force is exerted on the vehicle in both of these stops?
- 4 You want to move a 500-N crate across a level floor. To start the crate moving, you have to pull with a 230-N horizontal force. Once the crate "breaks loose" and starts to move, you can keep it moving at constant velocity with only 200 N. What are the co-efficients of static and kinetic friction?
- 5 Two bodies of masses 3 kg and 5 kg are tied to string which is passed over a pulley. If the pulley has no friction, find the acceleration of the bodies and tension in the string.
- 6 Determine the magnitude of the centripetal force exerted by the rim of a car's wheel on a 45.0-kg tyre. The tyre has a 0.480-m radius and is rotating at a speed of 30.0 m/s.
- 7 A motorcyclist is moving along a circular wooden track of a circus (death well) of radius 5-m at a speed of 10 ms^{-1} . If the total mass of motorcycle and the rider is 150 kg find the magnitude of the centripetal force acting on him?
- 8 A car of mass 1000 kg is running on a circular motor way interchange near Swabi with a velocity of 80 ms^{-1} the radius of the circular motor way interchange is 800 m. How much centripetal force is required?

WEB LINKS

<https://faraday.physics.utoronto.ca/PVB/Harrison/Flash/>
<http://howthingsfly.si.edu/flight-dynamics/newton%E2%80%99s-laws-motion>
<http://www.school-for-champions.com/science/force.htm#.WlhodryWbIU>

Turning helps
in many situations



Unit

4

TURNING EFFECT
OF FORCES

After studying this unit you should be able to:

- ✓ define like and unlike parallel forces.
- ✓ state head to tail rule of vector addition of forces/vectors.
- ✓ describe how a force is resolved into its perpendicular components.
- ✓ determine the magnitude and direction of a force from its perpendicular components.
- ✓ define moment of force or torque as moment = force $\times$ perpendicular distance from pivot to the line of action of force.
- ✓ explain the turning effect of force by relating it to everyday life.
- ✓ state the principle of moments.
- ✓ define the centre of mass and centre of gravity of a body.
- ✓ define couple as a pair of forces tending to produce rotation.
- ✓ prove that the couple has the same moments about all points.
- ✓ define equilibrium and classify its types by quoting examples from everyday life.
- ✓ state the two conditions for equilibrium of a body.
- ✓ solve problems on simple balanced systems when bodies are supported by one pivot only.
- ✓ describe the states of equilibrium and classify them with common examples.
- ✓ explain effect of the position of the centre of mass on the stability of simple objects.

Until now, we have been concerned mainly with translational motion. We discussed the kinematics and dynamics of translational motion (the role of force). In this Chapter we will deal with rotational motion. We will discuss the kinematics and dynamics of rotational motion (involving torque). Our understanding of the world around us will be increased significantly and there may be a few surprises.

We will consider mainly the rotation of rigid objects about a fixed axis. A rigid object is an object with a definite shape that doesn't change, so that the particles composing it stay in fixed positions relative to one another.

2.1 FORCE DIAGRAMS

To study the effects of forces acting on any object, we can apply the skill of drawing force diagrams. Since force is a vector quantity it can be represented by an arrow. Force diagrams are very useful conceptual tools for physics students because they help examine the forces acting on an object. In force diagrams the object on which forces are shown is reduced to a dot at its center and the forces acting on the object are represented by arrows pointing away from it.

For example, if we were to draw a force diagram of this textbook placed at rest on table, we would reduce textbook to a dot, and draw two arrows representing forces acting on it, as shown in Figure 4.1 a. One of the forces is the weight of the book, pulling it downward. The other force is the normal force due to the bench pushing the book upward. The forces in Figure 4.1 a are equal and opposite; that is, the magnitude of the gravitational force is equal to the magnitude of the upward force due to the table. These two forces are an example of balanced forces. When an object is acted on by balanced forces, the forces cancel each other out and the object behaves as though no force is acting on it.

Figure 4.1 b is a force diagram of a textbook in free fall. Assuming negligible air resistance, the only force acting on the book is the force of gravity downward; there is no balancing upward force. As a result, the force due to gravity on the book is unbalanced, and the book accelerates downwards.

Figure 4.1 Force Diagrams

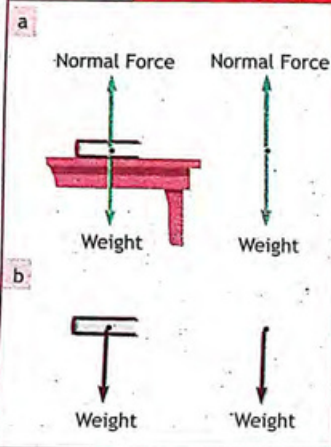


Figure 4.2 Parallel Forces

"If the directions of forces are parallel to each other, even if they are in opposite direction, those forces are called parallel forces. If they are in the same direction they are called 'Like parallel forces' and if they are in the opposite direction they will be known as 'Unlike parallel forces'.

Like parallel forces: For example when we lift a box with double support we are applying like parallel forces from each support, the force from one support may be greater than the other as shown in the figure.



Unlike parallel forces: For example when we apply force with our both hands on steering wheel of a car to turn it the force from one hand may be greater than the other as shown in figure. we are applying unlike parallel forces.



4.2 ADDITION OF FORCES

Addition of forces is a process of obtaining a single force which produces the same effect as produced by a number of forces acting together.

Forces are vectors and may be added geometrically by drawing them to a common scale (Recall chapter 2: Kinematics, representation of vectors) and placing them head to tail. Joining the tail of the first force vector with the head of the last will give another force vector which is the sum of these forces called resultant force (or net force, as you are already familiar with this from last chapter, Newton's Laws). These two terms resultant force and the net force can be used interchangeably.

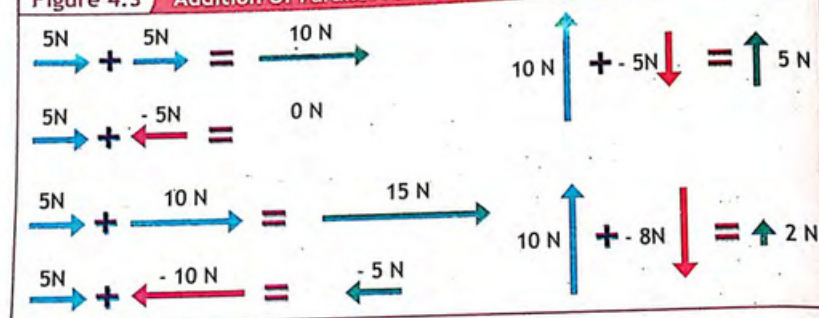
The situation of addition of forces is simple for like and unlike parallel forces i.e.

Add the magnitudes of vectors in case of like parallel forces.

Subtract the magnitudes of vectors in case of unlike parallel forces.

Few examples are shown in the figure 4.3.

Figure 4.3 Addition Of Parallel Forces



But things get complicated when the forces being added are not an angle to each other (neither parallel nor anti parallel). In such cases we will not make free vectors as above but we would have to draw vectors on a coordinate axis.

As an example consider two persons pulling a cart such that their force vectors are drawn to same scale. Draw F_A making angle θ_A with the x-axis and F_B making angle θ_B with the x-axis, as shown in the figure 4.4.

The following steps must now be followed to add the vectors by head to tail rule.

- Sketch first force vector F_A using same scale according to selected scale in a given direction.
- Now place the tail of the second force vector F_B on the head of the first force vector F_A in the given direction.

Figure 4.4 Addition Of Forces Making An Angle

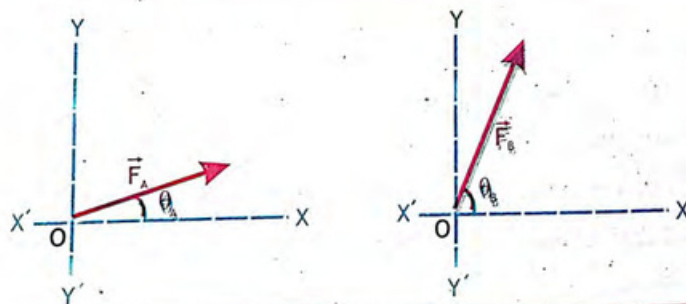
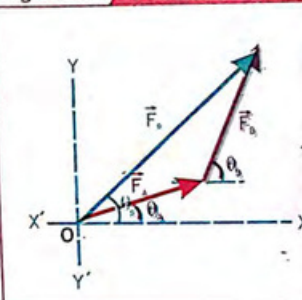


Figure 4.5 Resultant Force



- iii. Now the net force, resultant force F_R can be obtained by joining the tail of the first force vector F_A to the head of the second force vector F_B , putting an arrow on the line pointing away from the origin gives the resultant vector F_R as shown in figure 4.5.

- iv. To determine the magnitude of resultant measure the length of F_R and convert it back according to given scale. To determine the direction of the resultant measure the angle of resultant θ_R with x-axis.

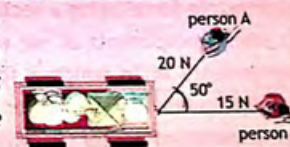
$$\vec{F}_R = \vec{F}_A + \vec{F}_B \quad \text{--- 4.1}$$

Addition of More than two Vectors:

This rule for vector addition can be extended to any number of forces.

Example 4.1 PULLING TRASH IN DIFFERENT DIRECTIONS

Two people, A and B, are pulling a trash cart with two ropes. Person A applies a force 20 N [50° with x-axis] on one rope. Person B applies a force of 15 N [0° with x-axis] on the other rope. Calculate the magnitude of net force on the trash cart.



Scale:

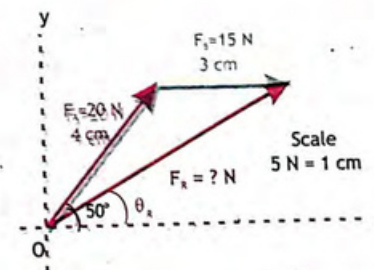
Let 5N = Km

$F_A = 20 \text{ N} = 4 \text{ cm}$

$\theta_A = 50^\circ$ with the x-axis

$F_B = 15 \text{ N} = 3 \text{ cm}$

$\theta_B = 0^\circ$ with the x-axis



Finding the resultant:

We joined the tail of the first to the head of the second vector to get the resultant F_R . We then measured the length of vector F_R which was about 6.2 cm ($6.2 \times 5 = 31$ N), and with the protector we measure the angle $\theta = 30^\circ$ with x-axis.

$F_R = 31$ N, 30° with x-axis — Answer

EXTENSION EXERCISE 4.1

Will it make any difference if you first draw F_B and then F_A to find the resultant?

Assignment 4.1 HEAD TO TAIL RULE

Two forces are applied one force is 25 N [20° with x-axis] and the other force is 10 N [60° with x-axis], find the net resultant force.

4.3 RESOLUTION OF FORCES

The process of splitting a force vector into two or more force vectors is called resolution of forces. The force vectors so obtained are called components.

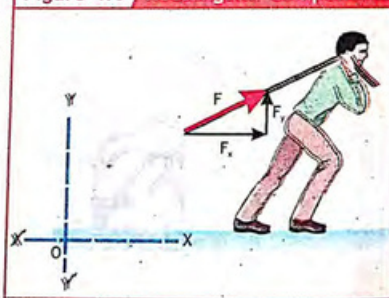
This process is opposite of addition of vectors, in principle force being a vector can be resolved into as many components as we wish. However if these components in to which a force vector is split are perpendicular to each other then such components are called **rectangular components of vector**. Resolving forces into rectangular components help us to analytically add forces rather than needing a ruler or/and protector. Part of the graphical technique is retained, because vectors are still represented by arrows for easy visualization.

Analytical methods are more concise, accurate, and precise than graphical methods, which are limited by the accuracy with which a drawing can be made. Therefore, in physics, very often we

POINT TO PONDER

Is vector addition of more than two forces possible, by head to tail rule?

Figure 4.6 Rectangular Components



need to separate a vector into rectangular components. For example a ice block is being pulled by a boy using rope. We can think of force as tension in the rope. This single force F can be resolved into two components - one directed upwards rightwards along x-axis (F_x) and the other directed upwards along y-axis (F_y) as shown in fig 4.6.

Consider a force F in the Cartesian coordinate system, represented by the line OP , making an angle θ as shown in the figure. 4.7

Draw perpendiculars from point P on x-axis and y-axis which meets the axis at points Q and S respectively. Put arrow head from the direction of O towards Q and S such that they represent vectors as $\vec{F}_x$ (OQ & SP) and $\vec{F}_y$ (OS & QP), becomes the rectangular components of vector $\vec{F}$ (OP).

By head to tail rule of vector addition, we know that force $\vec{F}$ is the vector sum of $\vec{F}_x$ and $\vec{F}_y$, mathematically

$$\vec{F} = \vec{F}_x + \vec{F}_y \quad \text{--- 4.2}$$

Components represented in terms of force: Consider a triangle ΔOPQ , without taking the sides as vectors. This forms a right angle triangle for which we have.

$$\cos \theta = \frac{\text{Base}}{\text{hyp}} = \frac{OQ}{OP}$$

$$\text{or} \quad \cos \theta = \frac{F_x}{F}$$

$$\text{therefore} \quad F_x = F \cos \theta \quad \text{--- 4.3}$$

$$\text{similarly} \quad \sin \theta = \frac{\text{Perp}}{\text{hyp}} = \frac{QP}{OP}$$

$$\text{or} \quad \sin \theta = \frac{F_y}{F}$$

Figure 4.7 Rectangular Components

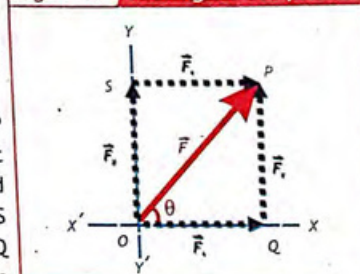


TABLE 4.1: ANGLE VALUES

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0.000	1.000	0.000
30°	0.866	0.500	0.577
45°	0.707	0.707	1.000
60°	0.866	0.500	1.732
90°	1.000	0.000	infinite

therefore $F_y = F \sin \theta$ — 4.4

This means that we can calculate the components analytically just by knowing the value of force and the angle.

Force represented in terms of its Components:
Since triangle ΔOPQ , from a right angle triangle, therefore we can use Pythagorus theorem, which states that

$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

taking square root on both sides

$$\sqrt{(\text{hyp})^2} = \sqrt{(\text{base})^2 + (\text{perp})^2}$$

such that

$$\text{hyp} = \sqrt{(\text{base})^2 + (\text{perp})^2}$$

or

$$OP = \sqrt{(OQ)^2 + (QP)^2}$$

therefore

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{— 4.5}$$

The magnitude of vector can now be determined if the values of the magnitudes of components are known. To determine the direction in right angle triangle ΔOPQ , we have $\tan \theta$ as

$$\tan \theta = \frac{\text{perp}}{\text{base}} = \frac{QP}{OQ}$$

or

$$\tan \theta = \frac{F_y}{F_x}$$

therefore

$$\theta = \tan^{-1} \frac{F_y}{F_x} \quad \text{— 4.6}$$

Example 4.2 PULLING A BOX ON FLOOR

Divia is pulling a box on the floor with a force of 20 N making an angle of 60° with the horizontal. Find the horizontal and vertical components of this force.

GIVEN

Force $F = 20.0 \text{ N}$

Angle $\theta = 60^\circ$

REQUIRED:

(a) Horizontal component of force $F_x = ?$

(b) Vertical component of force $F_y = ?$

(a) Horizontal component of force F_x is the force that is involved in moving the box along the floor. Using equation 4.3

$$F_x = F \cos \theta$$

putting values $F_x = 20 \text{ N} \cos 60^\circ$

therefore $F_x = 10 \text{ N}$ — Answer

(a) For vertical component of force F_y , we will use equation 4.4

$$F_y = F \sin \theta$$

putting values $F_y = 20 \text{ N} \sin 60^\circ$

therefore $F_y = 17.3 \text{ N}$ — Answer

To check the answer, we can convert the components back into magnitude and direction.

$$F = \sqrt{F_x^2 + F_y^2}$$

putting values $F = \sqrt{(10)^2 + (17.3)^2}$

or $F = \sqrt{100 + 299.3}$

therefore $F = 19.9 \text{ N} = 20 \text{ N}$

The magnitude is nearly equal to 20 N, therefore our calculations are correct, now to check the angle

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

putting values $\theta = \tan^{-1} \frac{17.3}{10}$

or $\theta = \tan^{-1} 1.73$

therefore $\theta = 60^\circ$

As the angle is equal to 60° , therefore our calculations are correct.

Assignment 4.2 TILLING GARDEN

While tilling your garden, you exert a force on the handles of the tiller that has components $F_x = 85 \text{ N}$ and $F_y = 13 \text{ N}$. The x-axis is horizontal and the y-axis points up. What are the magnitude and direction of this force?

ROTATIONAL MOTION:

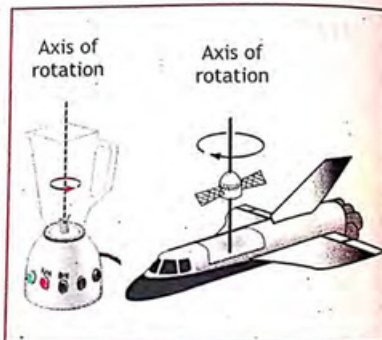
Motion where all points of an object moves about single fixed axis (which can be external is called rotational motion). In force diagrams we have seen that extended object (an object that occupies non zero space) forces can be shown by reducing object to a dot at its center. But there are some situations for which we need to consider an extended object—a system of particles for which the size and shape do make a difference and cannot be neglected. The following terms will be used in our study of rotational dynamics in this chapter.

Rigid objects:

Rigid objects are objects of fixed form that do not distort or deform (change shape) as they move. The study of rotational dynamics becomes easier if we consider the objects to be rigid.

Axis of Rotation:

For rotational motion we have to consider an axis of rotation. Axis of rotation is a line about which rotation takes place. This line remain stationary during rotational motion of the extended object, while the other points of the body move in circles about this line, it may be a pivot, hinges or any other support.

**4.4 CENTER OF MASS**

An extended rigid body is made of large number of small interconnected particles. The masses of all particles together make the mass of the body. The center of mass of the body is the point about which mass is equally distributed in all directions.

In force diagrams we have seen that extended object (that is, an object that has size) forces can be shown by reducing object to a dot at its center. The most suitable location of this point is center of mass.

Observations indicate that even if an object rotates, or several parts of a system of objects move relative to one another, there is one point that moves in the same path that a particle would move if subjected to the same net force. This point is called the center of mass (abbreviated CM). The general motion of an extended object (or system of objects) can be considered as the sum of the

translational motion of the CM, plus rotational, vibrational, or other types of motion about the CM.

Figure 4.8 MOTION OF BASEBALL BAT

As an example, consider the motion of the center of mass of the base ball bat thrown in Fig. 4.8 a; the CM follows a parabolic path even when the base ball bat rotates, as shown in Fig. 4.8 b. Other points in the rotating base ball bat follow more complicated paths.

The center of mass is the point at which we can imagine all the mass of an object to be concentrated. Thus, the center of mass is also the point at which we can imagine the force of gravity acting on the entire object to be acting. If we can imagine all of the mass to be concentrated at this point when calculating the force due to gravity, it is legitimate to call this point the center of gravity, a term that can often be used interchangeably with center of mass.

The point where whole weight of the body appear to act is called center of gravity. In most physics problems, one of the forces acting on the body is its weight. The weight doesn't act at a single point; it is distributed over the entire body. But we can always assume that the entire force of gravity (weight) is concentrated at a point called the center of gravity (abbreviated 'CG').

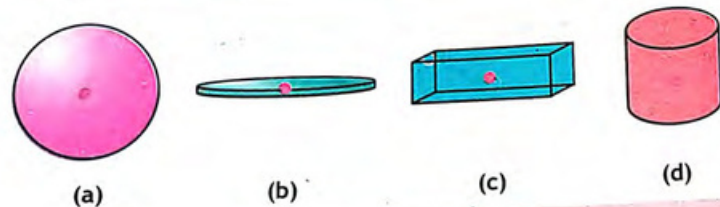
Difference between Center of Gravity (CG) and Center of Mass (CM):

The acceleration due to gravity decreases with altitude; but for small objects this variation is negligible, the body's center of gravity is for most of the time same as its center of mass.

Center of Gravity (CG) and Center of Mass (CM) for symmetric objects with uniform density:

The center of gravity of a homogeneous (having same density throughout) sphere, cube, circular disk, or rectangular plate is at its geometric center. The center of gravity of a right circular cylinder or cone is on the axis of symmetry, and so on.

Figure 4.9 Center Of Mass For Symmetric Object

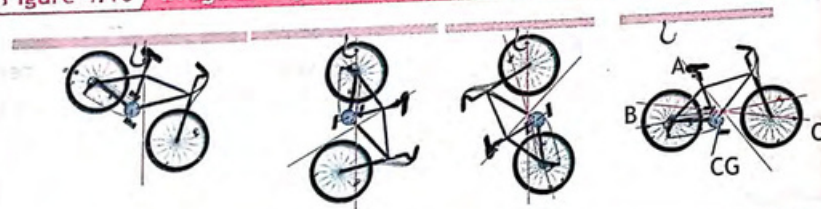


Center of mass/gravity (CM/CG) is obvious by inspection and located at the center of symmetric object

Center of Gravity (CG) and Center of Mass (CM) for irregular objects:

As the centre of mass of a uniform wooden rod is at its mid-point, but when massive metal is attached to light wooden rod, the CM/CG is now non-uniform and we can find CM/CG by hanging this object from various locations.

Figure 4.10 Irregular Object Center Of Mass

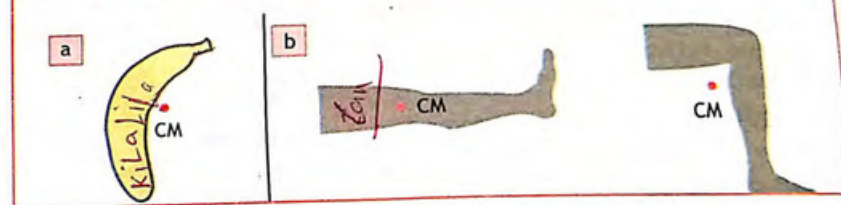


One way to determine the balance point of a three-dimensional object (such as bicycle) is to hang it randomly from at least three different points, as shown in Figure. The point of intersection of all three plumb lines is the bicycle's centre of gravity. This point is also the object's centre of mass.

The CM/CG may or may not lie inside the mass, some times the CM/CG is

outside the distribution of mass. For example Banana in the figure 4.11 a has center of gravity outside the mass distribution. Also the center of mass may change its location depending upon the orientation of the object. If parts of an object change position relative to each other the location of CM/CG will change. For example, leg shown in figure 4.11(b). When leg is stretched the mass is inside the body, but when the leg is bent the change in mass distribution changes CM/CG and is shifted outside the body.

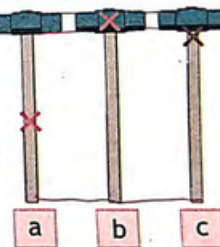
Figure 4.11 CENTER OF MASS OUTSIDE MASS DISTRIBUTION



SELF ASSESSMENT

massive head

lighter handle



A sledgehammer has a massive metal head attached to a much lighter wooden handle. Which location is most likely to be the sledgehammer's center of mass?

POINT TO PONDER



The figures show a high jumper using different techniques to get over the crossbar. Which technique would allow the jumper to clear the highest setting of the bar? ☐

LAB WORK

- To find the tension in the strings by balancing a metre rod on the stands.
- To find the weight of an unknown object by using vector addition of forces.

4.5 TORQUE OR MOMENT OF FORCE

Force can be used to produce rotation in an object, for example in opening a door or tightening a nut with spanner or wrench. **Turning effect produced in a body about a fixed point due to applied force is called torque or moment of force.** Physicists usually use the term "torque," while engineers usually use "moment".

Torque is the cause of changes in rotational motion and is similar to force, which causes changes in translational motion. This means that torque play the same role in rotation as force in translation.

Quantitatively Torque = force applied $\times$ perpendicular distance from the axis of rotation. This perpendicular distance from the axis of rotation to the line of action of force is called moment arm (or lever arm) 'd'.

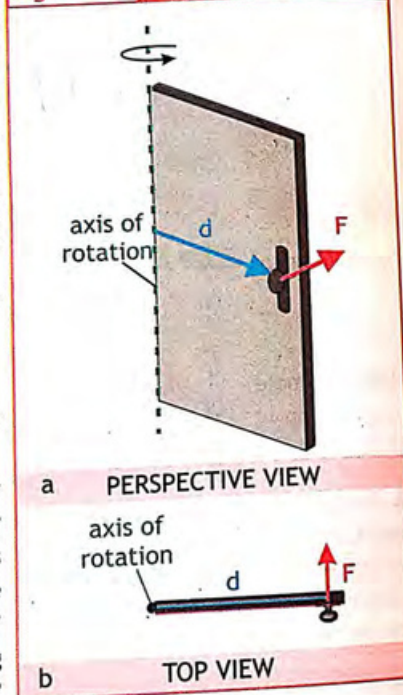
Thus torque ' τ ' will be the product of applied force ' F ' and moment arm 'd'.

$$\tau = F \times d \quad \text{4.7}$$

The SI unit of torque is "newton metre" (Nm).

For example, to open the door force ' F ' is applied at perpendicular distance 'd' from the axis of rotation as shown in the figure 4.12. Increasing the applied force ' F ' or the moment arm 'd' increases the torque ' τ '. Reducing applied force ' F ' or moment arm 'd' decreases torque ' τ '. For example if we apply more force we will forcefully produce rotation in the door and small force at the same distance from the axis of rotation will produce less torque. Similarly as we move away from the hinges (where there is axis of rotation) moment arm 'd'

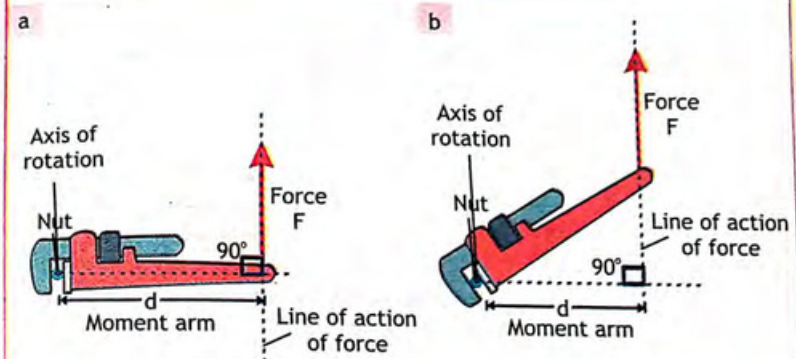
Figure 4.12 DOOR PHYSICS



increases and torque also increases for same applied force. Furthermore, if there is no force no torque will be produced, similarly a force applied at the pivot point will cause no torque since the moment arm would be zero ($d = 0$).

However force is not always applied perpendicularly, in this situation we have to extend the line of action of force applied ' F ' and take the moment arm 'd' as perpendicular distance from the axis of rotation to the line of action of force.

Figure 4.13 MOMENT ARM



For example consider a wrench which is used to open a nut as shown in figure 4.13 (a) the torque is produced. However if we continue to apply the same force in the same direction, after the nut has rotated as in figure 4.13 (b), our definition of moment arm will reduce it in magnitude and torque will decrease.

Moment arm is key to the operation of the lever, pulley, gear, and many other simple machines.

There are two senses of rotation. If the force is capable of rotating the body in clockwise direction, the torque is known as **clockwise torque**.

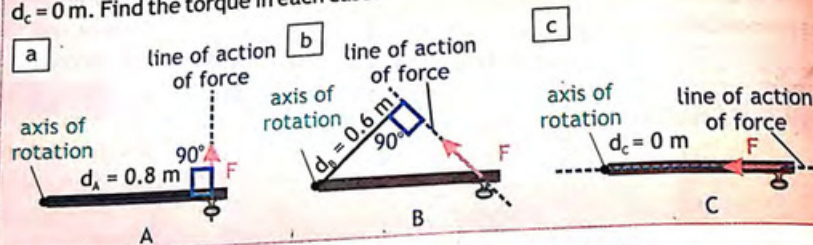
Similarly, the force is capable of producing rotation in the anti-clockwise direction, the torque is known as **anti-clockwise torque**.

Conventionally, clockwise torque is taken as negative, whereas anticlockwise torque is taken as positive.



Example 4.3 OPENING A DOOR

In Figure a force (magnitude 55 N) is applied to a door. However, the arms are different in the three parts of the drawing: (a) $d_A = 0.80$ m, (b) $d_B = 0.60$ m, and (c) $d_C = 0$ m. Find the torque in each case.

**GIVEN:**

Force $F = 55$ N
Moment arm $d_A = 0.80$ m
Moment arm $d_B = 0.60$ m
Moment arm $d_C = 0.00$ m

REQUIRED:

- (a) torque $\tau_A = ?$
(b) torque $\tau_B = ?$
(c) torque $\tau_C = ?$

In each case the lever arm is the perpendicular distance between the axis of rotation and the line of action of the force.

- (a) Using the definition of torque, the equation 4.7 can be written as

$$\tau_A = F \times d_A$$

putting values $\tau_A = 55 \text{ N} \times 0.80 \text{ m}$

therefore $\tau_A = 44 \text{ Nm}$ — **Answer**

- (b) Using the definition of torque, the equation 4.7 can be written as

$$\tau_B = F \times d_B$$

putting values $\tau_B = 55 \text{ N} \times 0.60 \text{ m}$

therefore $\tau_B = 33 \text{ Nm}$ — **Answer**

- (c) Using the definition of torque, the equation 4.7 can be written as

$$\tau_C = F \times d_C$$

putting values $\tau_C = 55 \text{ N} \times 0.00 \text{ m}$

therefore $\tau_C = 0 \text{ Nm}$ — **Answer**

Because the lever arm is different in each case, the torque is different, even

though the magnitude of the applied force is the same. In parts a and b the torques are positive, since the forces tend to produce a counter clockwise rotation of the door. In part c the line of action of passes through the axis of rotation (the hinge). Hence, the lever arm is zero, and the torque is zero.

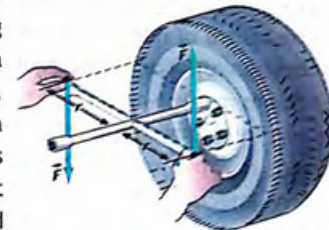
Assignment 4.3 FORCE ON BOTTLE OPENER

20 Nm torque is required to open a soda bottle. A boy with a bottle opener apply a force perpendicularly at 0.1 m, what is the magnitude of force required.

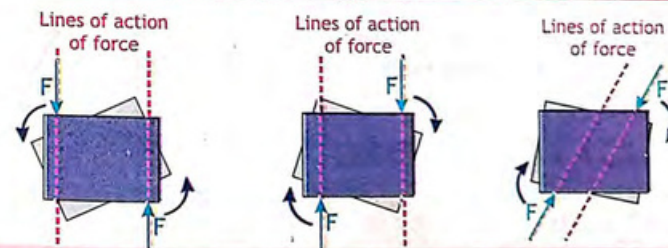
**4.6 COUPLE**

A special case of moments is a couple.

Two equal and opposite parallel forces acting along different lines on a body constitute a couple. It does not produce any translation, but only rotation. The resultant force of a couple is zero. but, the resultant of a couple is not zero; it is a pure moment. The shortest distance between two couple forces is called couple arm.



Example of couple is shown in the figure, also the forces that two hands apply to turn a steering wheel are often a couple. Each hand grips the wheel at points on opposite sides of the shaft. When they apply a force that is equal in magnitude yet opposite in direction the wheel rotates.

Figure 4.14 Couple Acting On Book

For example apply forces on this physics textbook with your fingers such that the line of action is different. No matter how you apply these forces the book will turn.

Similarly in our day-to-day life, we come across many objects which work on the principle of couple. For example Exerting force on bicycle pedals, winding up the spring of a toy car, opening and closing the cap of a bottle and turning of a water tap etc.



4.7 EQUILIBRIUM

We know that forces tend to cause either translational or rotational motion, depending on the direction and position of the force applied with respect to the centre of mass. Now we turn our attention to one specific effect of an application of force: static equilibrium. To achieve true static equilibrium, two conditions must be met. First, to avoid translation (moving from place to place), the net force directed through the centre of mass of the object must be zero. Second, to avoid rotation, the net torque on the object must also be zero. The study of objects in equilibrium is called STATICS. Equilibrium is defined as

"The state of a body in which under the action of several forces acting together there is no change in translational motion as well as rotational motion is called equilibrium".

Therefore for complete equilibrium two conditions must be met.

A) First Condition of Equilibrium:

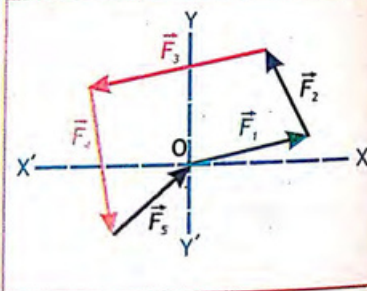
When the vector sum of all the forces acting on the body is ZERO then the first condition of equilibrium is satisfied. Mathematically if $\vec{F}_{net}$ is the sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$ then

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0$$

$$\text{or } \vec{F}_{net} = \sum_{i=1}^{i=n} \vec{F}_i = 0 \quad \text{--- 4.8}$$

For an object to satisfy the first condition of equilibrium the condition is

Figure 4.15 First Condition



that force polygon must close. Mathematically if $\vec{F}_{net}$ is the sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4, \vec{F}_5$ by head to tail rule it must be ZERO as shown in figure 4.15.

B) Second Condition of equilibrium:

When the vector sum of all the Torques acting on the body is ZERO then the second condition of equilibrium is satisfied. If τ_{net} is the sum of torques $\tau_1, \tau_2, \tau_3, \dots, \tau_n$ then mathematically

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3 + \dots + \tau_n = 0$$

$$\text{or } \tau_{net} = \sum_{i=1}^{i=n} \tau_i = 0 \quad \text{--- 4.9}$$

For complete equilibrium both the first and second conditions of equilibrium must be satisfied. Consider the following example in which a wheel is acted upon by forces.

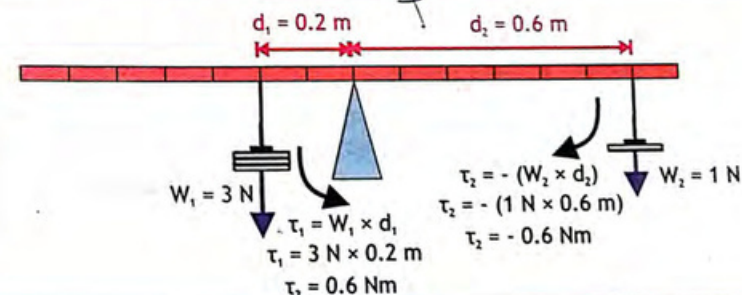
PRINCIPLE OF MOMENTS

Second condition of equilibrium can also be stated as the principle of moments, which states that

'For an object in equilibrium, the sum of the clockwise moments taken about the pivot must be equal to the sum of anti-clockwise moments taken about the same pivot'. To balance torques or moment of force, apart from force the perpendicular distance from the axis of rotation also play the key role.

For example, if we suspend weight of 3 N at 0.2 m from the pivot, it exerts the same torque as 1 N weight at 0.6 m from the fulcrum. A uniform meter stick will balance on pivot under

Figure 4.16 Principal Of Moments



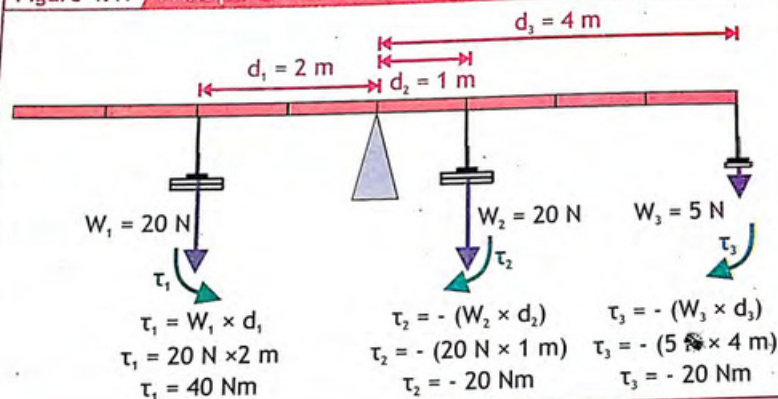
these conditions as shown in the figure 4.16. In the figure 4.16 anticlockwise torques are taken as negative, which leads to second condition of equilibrium that the sum of both these torques must be zero.

$$\tau_2 + \tau_2 = 0.6 \text{ Nm} - 0.6 \text{ Nm}$$

$$\tau_2 + \tau_2 = 0 \text{ Nm}$$

Similarly three or more torques around a pivot (as axis of rotation) can also balance each other. For example in figure 4.17, there is only one anticlockwise moment about the turning point, but two clockwise moments add up to balance it.

Figure 4.17 Principle Of Moments



Anticlockwise moment

$$\tau_1 = 40 \text{ Nm}$$

Clockwise moment

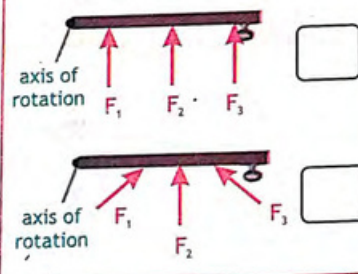
$$\tau_2 + \tau_3 = -20 \text{ Nm} - 20 \text{ Nm} = -40 \text{ Nm}$$

For second condition of equilibrium to be satisfied the sum of all these torques must be zero.

$$\tau_1 + \tau_2 + \tau_3 = 40 \text{ Nm} - 20 \text{ Nm} - 20 \text{ Nm} = 0 \text{ Nm}$$

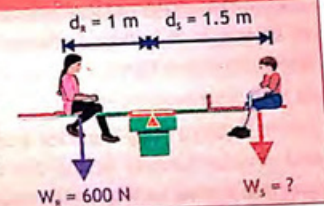
CONCEPT TEST

Which force of equal magnitude will produce larger torque?



Example 4.4 SEESAW BALANCE

Two Children Romiasa and Sanaan are sitting on a seesaw. The see-saw is balanced on a pivot as Romiasa is at 1 m and Sanaan at 1.5 m from the pivot (as shown in figure). If Romiasa weight is 600 N, what is the weight of Sanaan?



SOLUTION

GIVEN: Romiasa's Weight $W_R = 600 \text{ N}$

Romiasa's Moment arm $= d_R = 1 \text{ m}$

Sanaan's Moment arm $= d_S = 1.5 \text{ m}$

Here Romiasa is producing anticlockwise torque, while Sanaan is producing clockwise torque, Therefore by second condition of equilibrium

$$\tau_R - \tau_S = 0$$

As the see-saw is balanced, therefore we can write second condition of equilibrium as principle of moments 'The Sum of the clockwise moments = sum of the anticlockwise moments

$$\tau_R = \tau_S$$

or

$$W_R d_R = W_S d_S$$

dividing both sides by d_S

$$W_S = \frac{W_R d_R}{d_S}$$

putting values

$$W_S = \frac{600 \text{ N} \times 1 \text{ m}}{1.5}$$

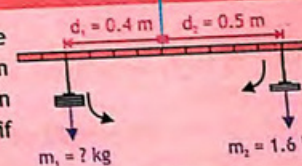
therefore

$$W_S = 400 \text{ N} \quad \text{Answer}$$

Hence the weight of Sanaan is 400 N.

Assignment 4.4 CALCULATING MASSES

With a beam two masses m_1 and m_2 are suspended at distance 0.4 m and 0.5 m respectively from suspension point as shown in the figure. Ignoring the weight of the balance, if $m_2 = 1.6 \text{ kg}$, what is the mass m_1 ?



Types of Equilibrium:

The effect of force is to produce change in translational motion and effect of torque is to produce change in rotational motion. Thus the equilibrium is divided into two types.

A) Static equilibrium:

When a body is at rest under the action of several forces acting together and several torques acting the body is said to be in static equilibrium. For example a book resting on the table is in static equilibrium, the weight mg of the book is balanced by a normal reaction force from the table surface.

B) Dynamic equilibrium:

When a body is moving at uniform velocity under the action of several forces acting together the body is said to be in dynamic equilibrium. It is further divided in to two types as shown in figure 4.18.

Figure 4.18 Types Of Dynamic Equilibrium**I) Dynamic Translational Equilibrium:**

When a body is moving with uniform linear velocity the body is said to be in dynamic translational equilibrium. For example a paratrooper falling down with constant velocity is in dynamic translational equilibrium.

**II) Dynamic Rotational Equilibrium:**

When a body is moving with uniform rotation the body is said to be in dynamic rotational equilibrium. For example a compact disk (CD) rotating in CD Player with constant angular velocity is in dynamic rotational equilibrium.

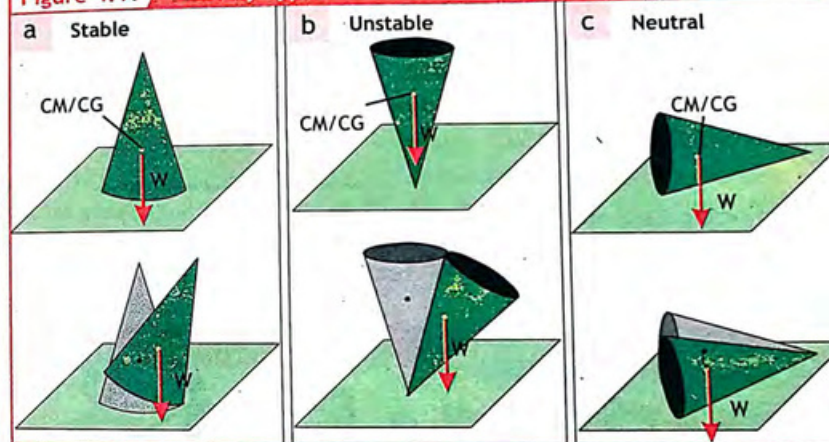
LAB WORK

- To verify the principle of moments by using a metre rod balanced on a wedge.
- To find the weight of an unknown object by using principle of moments.

4.8 STABILITY

Stability is a measure of how hard it is to displace an object or system from **EQUILIBRIUM**. The conditions of equilibrium does not specify whether an object is stable or not. The stable object does not topple easily. The position of the centre of mass of a body affects whether or not it topples over easily. This is important in the design of such things as tall vehicles (which tend to overturn when rounding a corner), racing cars, reading lamps and even drinking glasses.

Three cases are studied in statics. They differ in the effect on the center of mass of a small displacement.

Figure 4.19 Stability Types**a) Stable equilibrium:**

A body is in stable equilibrium if when slightly displaced and then release it returns to its previous position. As an example the cone in position a is stable as shown in figure 4.19 a. Its centre of mass rises when it is displaced. It regain its position back because its weight has a moment about the point of contact that acts to reduce the displacement.

b) Unstable equilibrium:

A body is in unstable equilibrium if it moves further away from its previous position when slightly displaced and released. As an example the cone in position b is unstable as shown in figure 4.19 b.

Its centre of mass falls when it is displaced, because there is a moment which increases the displacement.

c) **Neutral equilibrium:**

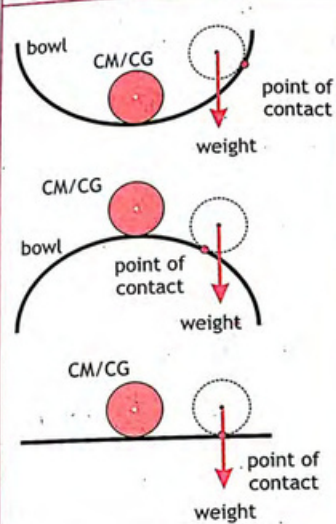
A body is in neutral equilibrium if it stays in its new position when displaced. As an example the cone in position B is neutral as shown in figure 4.19 c. Its centre of mass does not rise or fall because there is no moment to increase or decrease the displacement.

Similar demonstration for three cases of stability can be done by displacing a ball in a round bowl, as shown in figure 4.20. In each case think about what will happen when the ball is displaced slightly?

An object's stability is improved by:

- lowering the center of mass; or
- increasing the area of support; or
- by both.

Figure 4.20 Stability



INTERESTING INFORMATION



The CG/CM of race cars is kept low and tyres wide to decrease the chances for overturning of the car.

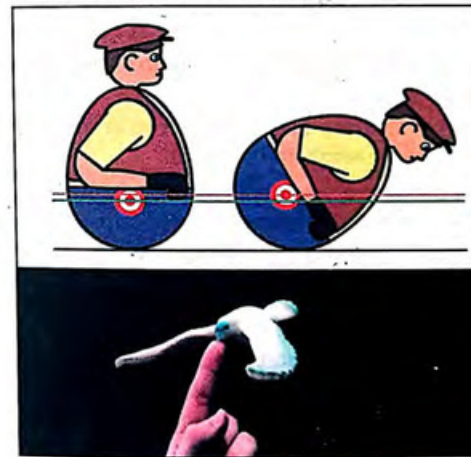
INTERESTING INFORMATION



The base of long glass and reading lamps is made heavy to lower CG/CM and large to increase area of support and thus to increase stability.

Equilibrium is stable as long as the vertical line from the centre of mass remains inside the area of the base of the body. If a disruption moves the vertical line from the centre of mass outside of the base, then the equilibrium is unstable. Neutral equilibrium exists when any disruptive force acts horizontally but the vertical height of the centre of mass remains unchanged. Taller people are generally less stable than shorter people because their higher centre of mass can be more easily pushed outside base of the structure, sometimes called the footprint. Some toys are more stable, then they look as they right by themselves if they are disturbed.

By making centre of mass vertically below the point of support makes state of the object a stable equilibrium. This mechanism is employed in various toys and equipments that restore themselves up when disturbed, few examples as pictures are shown.



TID-BITS



To stay balanced while walking the tightrope, this acrobat must keep her center of mass directly above the rope. If the center of mass shifts to one side, gravity will exert a torque on the acrobat, tending to cause a rotation about the rope and she will fall.

To stay balanced, the acrobat must create a counter-torque having equal magnitude and opposite sign. That is why tightrope walkers carry a long pole. Because the torque due to a force is the product of the magnitude of the force and the moment arm, the gravitational forces on the ends of the pole exert significant torques. Thus, the acrobat controls the net torque acting on her center of gravity by manipulating the pole. Also, the acrobat holds the pole low, moving her center of gravity closer to the rope.

ACTIVITY 4.1 RIDE A BUS

This is an experiment to perform while standing in a bus or a train. Stand facing sideways. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Now stand facing forward. How do you move your body to readjust the distribution of your mass as the bus accelerates and decelerates? Why is it easier and safer to stand facing sideways rather than forward? Note: For your safety (and those around you), make sure you are holding onto something while you carry out this activity!

POINT TO PONDER



Two workers are carrying a long, heavy steel beam (Fig). Which one is exerting a larger force on the object? How can you tell?

KEY POINTS

Parallel Forces: The directions of forces are parallel to each other if they are in opposite direction, those forces are called parallel forces. If they are of same direction then they are called "like parallel forces" and if they are in the opposite direction they will be known as "Unlike parallel forces."

Addition of Vectors: Addition of vectors means the combination of two or more than two vectors to get a single vector.

Resolution of Vector: The process of splitting one vector by two or more than two vector components are called the resolution of vector.

Moment of a force or Torque: The measure of an object tendency to rotate about some point O is called torque or moment of force.

Moment of a force = force $\times$ perpendicular distance of the force to the pivot point.

The SI unit of the moment of a force is Nm.

Principle of moments: The principle of moments states that for an object in equilibrium, the sum of the clockwise moments taken about the pivot must be equal to the sum of the anti-clockwise moments taken about the same pivot.

KEY POINTS

Center of Mass: The center of mass of the body is the point about which mass is equally distributed in all directions.

Centre of Gravity: The centre of gravity is defined as a single point where the whole weight of an object appears to act.

Couple: A couple consists of two forces that are equal in magnitude, but opposite in direction.

Equilibrium: When a number of forces are simultaneously acting on a body and the body does not change its state of rest or of uniform motion, then the body is said to be in a state of equilibrium.

Stability: The stability of an object refers to the ability of the object to return to its original position when the force that changed its orientation is removed.

PROJECTS

GROUP - A

AUTOMOBILE ENGINES: Automobile engines are rated by the torque that they produce. Research and explain why torque is an important quantity to measure for automobile.

GROUP - B

BICYCLES TORQUES: Research different types of bicycles, describe exactly which measurements you would need to make in order to identify the torques at work during a ride on a specific bicycle. Suggest a considerations torque efficient design of bicycle ride. If possible design and construct a torque efficient bicycle model.

GROUP - C

SIMPLE MACHINES: Write a publication essay for school library on simple machines and explain how these machines use the principle of moments. Make a chart for display in the school laboratory.

GROUP - D

SELF RESTORE MECHANISM: Research different self restore mechanisms to design and build a toy from (e.g. trash toys in your home) that restore itself after it is displaced

GROUP - E

SEE SAW AS WEIGHING MACHINE: Make a see saw as weighing device from plank (wooden or any easily available thing) and discuss its limitations and donate it to the physics lab.

EXERCISE

- MULTIPLE CHOICE QUESTIONS**
- Conventionally anti clock wise torque is taken as _____.
A. Negative B. positive C. parallel D. zero
 - A door requires a minimum torque of 80 Nm in order to open it. What is the minimum distance of the handle from the hinges, if the door is to be pulled with a force at the handle not greater than 100N?
A. 0.6 m B. 1 m C. 0.4 m D. 0.8 m
 - Two children are balanced on opposite sides of a seesaw. If one child leans inward toward the pivot point, her side will
A. rise B. fall
C. insufficient data D. neither rise nor fall
 - A body in equilibrium must not be
A. at rest B. moving C. rotating D. accelerating
 - The torque in uniformly rotating fan having blade of length 0.5 m is
A. 0.5 Nm B. 2 Nm C. -0.5 Nm D. 0 Nm
 - A force of 100 N is applied perpendicularly at 0.5 m, to turn nut of wheel of a bus. The torque acting on nut is
A. 500 Nm B. 50 Nm C. 5 Nm D. 0.005 Nm
 - The shortest distance between two couple forces is
A. moment arm B. couple arm
C. radius D. double moment
 - A girl pushes to open a door perpendicularly with a force of 25 N at 0.6 m from the hinge, the torque is
A. 41.6 Nm B. 25.6 Nm C. 15 Nm D. 0 Nm
 - The angle at which x and y components of force are equal is
A. 0° B. 30° C. 45° D. 60°
 - CM is different from CG, When we have nonuniform
A. shape of object B. mass of object
C. gravitational force D. none of these

CONCEPTUAL QUESTIONS

Give a brief response to the following questions.

- Can the rectangular component of the vector be greater than the vector itself? Explain.
- Explain why door handles are not put near hinges?
- Can a small force ever exert a greater torque than a larger force? Explain.
- Why it is better to use a long spanner rather than a short one to loosen a rusty nut?
- The gravitational force acting on a satellite is always directed towards the centre of the earth. Does this force exert torque on satellite?
- Can we have situations in which an object is not in equilibrium, even though the net force on it is zero? Give two examples.
- Why do tightrope walkers carry a long, narrow rod?
- Why does wearing high-heeled shoes sometimes cause lower back pain?
- Why is it more difficult to lean backwards. Explain?
- Can a single force applied to a body change both its translational and rotational motion? Explain.
- Two forces produce the same torque. Does it follow that they have the same magnitude? Explain. e. Describe the path of the brick after you suddenly let go of the rope.

COMPREHENSIVE QUESTIONS

Give an extended response to the following questions.

- What are force diagrams? Define like and unlike parallel forces with example.
- Explain the addition of forces, in connection with head to tail rule.
- Define moment of a force. Give its mathematical description and elaborate the factors on which it depends?
- What is resolution of forces? Explain with an example how forces can be resolved into rectangular components.
- What is couple? explain with examples.