KINEMATICS

UNIT 2

How fast the bullet move?

Student Learning Outcomes (SLOs)

The students will

- [SLO: P-09-B-01] Differentiate between different types of motion.
- [SLO: P-09-B-02] Differentiate between distance and displacement, speed and velocity.
- [SLO: P-09-B-03] Define and calculate speed.
- [SLO: P-09-B-04] Define and calculate average speed.
- [SLO: P-09-B-05] Differentiate between average and instantaneous speed.
- [SLO: P-09-B-06] Differentiate between uniform velocity and non-uniform velocity.
- [SLO: P-09-B-07] Define and calculate acceleration.
- [SLO: P-09-B-08] Differentiate between uniform acceleration and non-uniform acceleration.
- [SLO: P-09-B-9] Sketch, plot and interpret distance-time and speed-time graphs.
- [SLO: P-09-B-10] Use the approximate value 9.8m/s2 for free fall acceleration near Earth to solve problems.
- [SLO: P-09-B-11] Justify how the gradient of a distance vs time graph gives the speed.
- [SLO: P-09-B-12] Analyse the distance traveled in speed vs time graphs.
- [SLO: P-09-B-13] Derive how the area beneath a speed vs time graph gives the distance traveled.
- [SLO: P-09-B-14] Calculate acceleration from the gradient of a speed-time graph.
- [SLO: P-09-B-15] Justify how the gradient of the speed vs time graph gives the acceleration.
- [SLO: P-09-B-16] State that there is a universal speed limit for any object in the universe that is approximately 3×108 ms-1.

UNIT 2 KINEMATICS

Mechanics is the study of motion. Everywhere we look, objects are moving. We see people moving on roads, some using vehicles. Actually, everything we know is constantly in motion. Celestial objects and our Earth are always moving. Even objects that appear to be still have atoms and molecules that vibrate in continuous motion.

Our formal study of physics starts with kinematics, which is the study of motion without considering its causes. The term "kinematics" comes from Greek and means motion. In this unit, we will only focus on the motion of objects, without concerning ourselves with the forces that cause or change their motion.

2.1 REST AND MOTION

If with passage of time an object does not change its position then it is at rest with respect to an observer and if it is changing its position then it is in motion.

When we look around us, we observe that many objects do not change their position. Thus we consider them at the state of rest. For example a bench in a park fixed under a tree is at rest as there is no change in its position with respect to us while standing near it with the passage of time. On the other hand we also observe that many objects do change their position from one place to another. Hence we consider them to be in the state of motion. For example a car is in motion if there is change in its position with time.

POINT TO PONDER

Interestingly objects can be at rest and in motion at same time. It looks simple to distinguish the rest from motion, for example a car starts, it changes its position with reference to its surrounding, we say that car is moving.

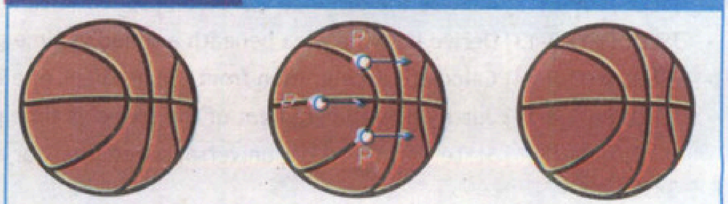
However, we know that Earth is spinning on its axis, so the car along with its road is also in motion. Not only this but Earth is also moving around the sun and the sun along with the rest of the solar system are also moving through our milky way galaxy. Apart from this our galaxy is also traveling through space. How can we say that our car is at rest? This is why when we state an object to be at rest or motion, we specify it reference to some observer.

2.1.1 TYPES OF MOTION

Looking at the motion of object we see that objects move differently. These different types of motion can be broadly categorized in three types translatory motion, rotatory motion and vibratory motion.

A. Translatory motion: If all points of a moving object move uniformly in the same direction, such that there is no change in the object's orientation the object is said to be undergoing translatory motion (also termed as translational motion).

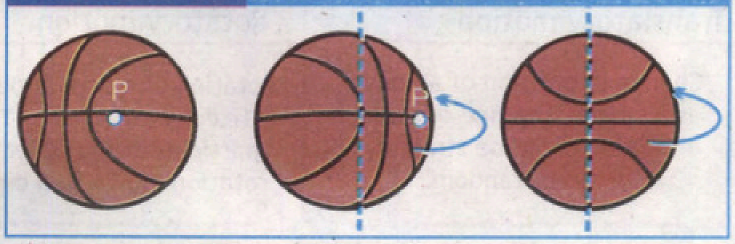
FIGURE 2.1 TRANSLATORY MOTION



A basketball is shown in figure 2.1 as an example of translatory motion. All the three points 'P_1', 'P_2' and 'P_3' moves parallel to each other and there is no change in its orientation relative to a fixed point.

B. Rotatory motion: When an object rotates on its own axis (a line passing through the object), the object is said to be undergoing rotatory motion (also termed as rotational motion). A basketball in figure 2.2 is again shown as an illustration of rotational motion.

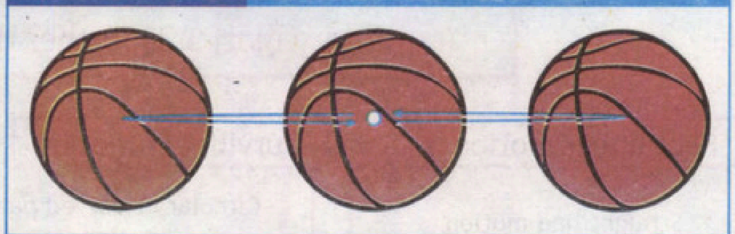
FIGURE 2.2 ROTATORY MOTION



The point '*P*' is rotated around an axis of rotation passing through the center of it.

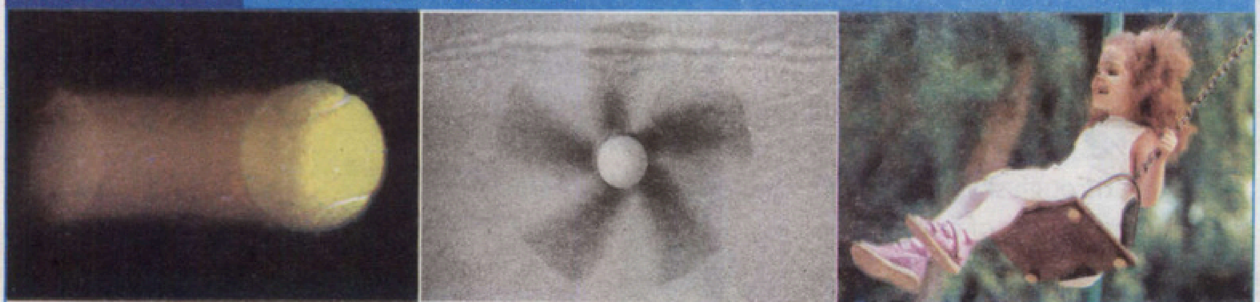
C. Vibratory motion: When an object is moving forward and backward repeatedly about mean position (certain fixed position), the object is said to be undergoing vibratory motion (also termed as vibrational motion) . A basketball in figure 2.3 is shown as an example of vibrational motion.

FIGURE 2.3 VIBRATORY MOTION



The basketball moves back and forth about the mean position. Figure 2.4 shows some daily life examples of types of motion.

FIGURE 2.4 TYPES OF MOTION



Translatory motion | Rotatory motion | Vibratory motion.

Translatory motion is further divided into three types.

- **Rectilinear motion** is the translatory motion of the object in straight line path. For example the motion of train on track, motion of gun shot and motion falling apple.
- **Circular motion** is the translatory motion of an object in which it moves in a curved path. For example the motion of a football when kicked, the motion of roller coaster and the motion of a vehicle in a turn are examples of curvilinear motion. Circular motion is a special case of curvilinear motion in which the radius of rotation remains constant and object moves along a circular path.

Types of Motion

Translatory motion

Change in position of a body as a whole. The line or path of motion could be straight, curved or random.

Examples are motion of ball, car, flying birds, airplane etc.

Rotatory motion

Rotation of a body around a fixed rotation axis. The particles of the object in rotation moves in a circle.

Examples are motion of helicopter rotors, blades of fan etc.

Vibratory motion

The to and fro motion of an object about its mean position. The object in vibration repeats its motion.

Examples are motion of swing, strings of sitar, guitar, etc.

Translatory motion is further divided into three types

Rectilinear motion

Straight line motion

Example is motion of free falling body

Curvilinear motion

Circular or curved path motion

Example is motion of cars around a turn

Random motion

Irregular motion

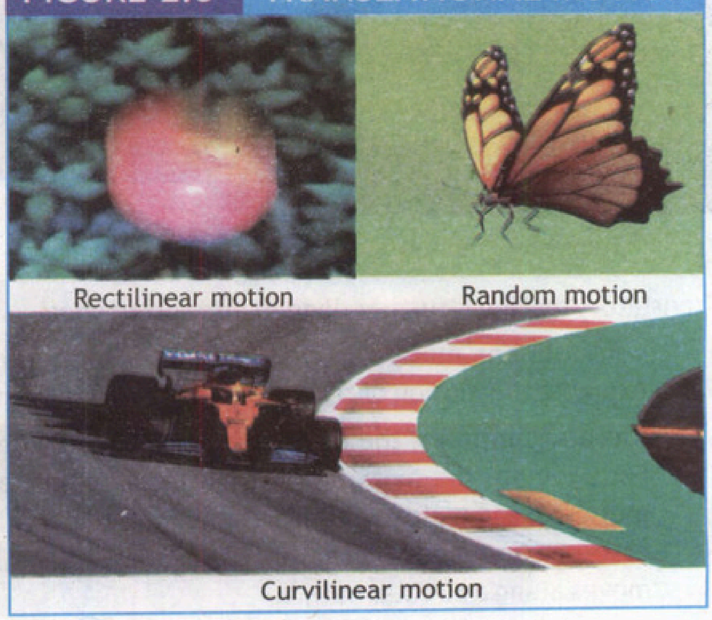
Example is motion of butterfly

An object can have any combination of these types of motion.

- **Random Motion** is the translatory motion of an object with no specific path. For example kites flying through sky, motion of clouds and the motion of butterfly.

Translational motion is seen in various scenarios, covering a wide range of situations. Whether in engineering, physics, or everyday life, objects frequently display this type of motion. It is crucial to comprehend the specific motion type in order to accurately analyze and describe the behavior of moving objects.

FIGURE 2.5 TRANSLATIONAL MOTION



2.2 DISTANCE AND DISPLACEMENT

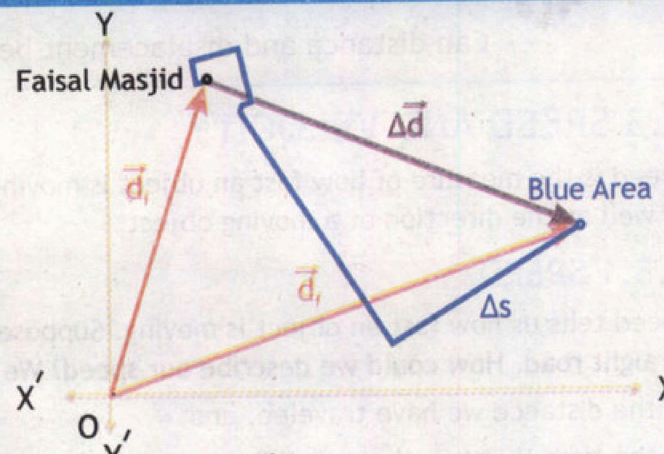
If we are at Faisal Masjid, Islamabad and we want to move to Blue area, Islamabad by searching on google map as shown in figure 2.5 (a), we get a twisted path, showing us the way to reach our destination. However, the straight path as shown in figure 2.5 (b) can be shorter.

'The length of path traveled between two positions is called distance'.

Distance has no direction and therefore it is a scalar quantity. Distance is usually denoted by Δx, Δr, Δs, Δd or Δl, and has SI unit as metre (m).

FIGURE 2.6 DISTANCE AND DISPLACEMENT


(a)



(b)

'The shortest distance from initial position to final position (or straight directed distance) is called displacement'.

Displacement has direction and therefore it is a vector quantity. Displacement has SI unit as metre (same as length).

If an object moves then the object's position changes. This change in position vector '$\Delta\vec{d}$' of an object, from initial position '$\vec{d}_i$' to final position '$\vec{d}_f$' is known as displacement as shown in figure 2.6 (b). Mathematically:

$$\Delta\vec{d} = \vec{d}_f - \vec{d}_i$$

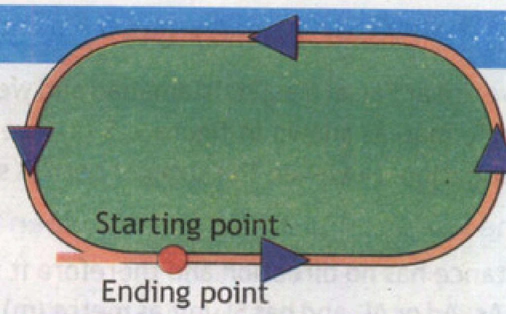
POINT TO PONDER

Here we used symbol Δ (Greek letter delta) for change in position; however, it is used to represent a 'change in' any quantity. For example elapsed time Δt is the change in (or the difference between) the ending time t_f and beginning time t_i.

$$\Delta t = t_f - t_i$$

CAN YOU TELL?

If on a 400 m running track your starting point and ending point is same. How much distance you have covered? What is your displacement?



CAN YOU TELL?

Can displacement be greater than distance?

Can distance and displacement be equal?

2.3 SPEED AND VELOCITY

Speed is the measure of how fast an object is moving, whereas velocity describes the speed as well as the direction of a moving object.

2.3.1 SPEED

Speed tells us how fast an object is moving. **Suppose we are in a car that is moving over a straight road. How could we describe our speed? We need at least two measurements:**

- the distance we have traveled, and
- the time that has elapsed while we covered this distance.

'Measure of the distance covered (Δs) with passage of time (Δt) is called speed (denoted by v)'. Mathematically:

$$speed = \frac{\text{distance}}{\text{elapsed time}} \quad \text{or} \quad v = \frac{\Delta d}{\Delta t} \quad \text{or} \quad v = \frac{s_f - s_i}{t_f - t_i} \qquad (2.1)$$

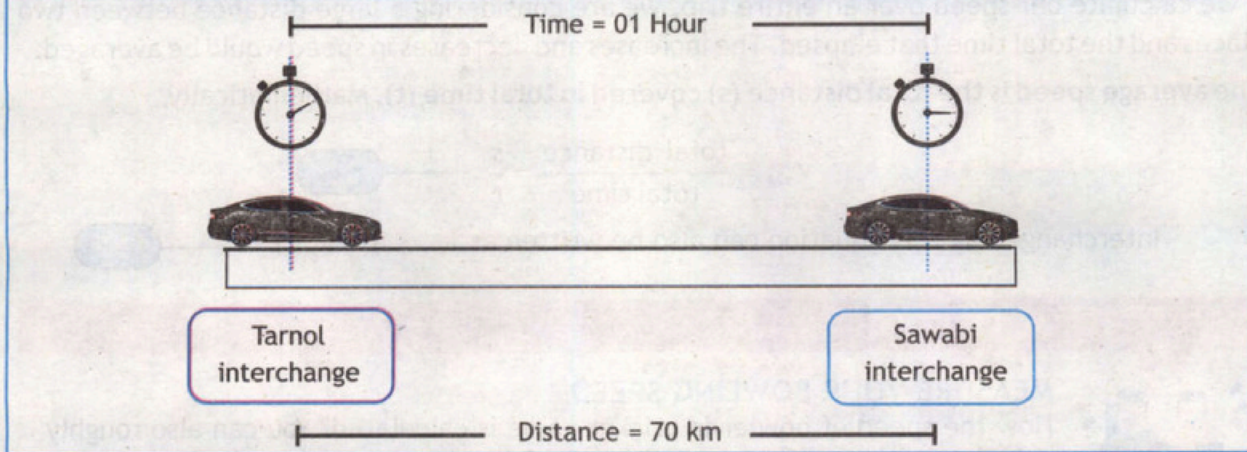
Speed of an object show us the rate at which the object is moving. Speed is a scalar quantity having SI unit of metre per second (m/s or ms^{-1}). The speed will be one 'metre per second' if *an object cover one metre distance in one second.*

Speed tells us how fast an object is moving. An object is fast if it cover large distance in a short time. For example while going from Islamabad to Peshawar through motor-way M1, we leave at Tarnol interchange at 2:00 pm and cross Sawabi interchange at 3:00 pm as shown in figure 2.7. Since Sawabi interchange is about 70 km from Tarnol interchange and it took us one hour therefore our speed can be obtained as:

$$v = \frac{70\,km}{1\,hr} = 70\,{}^{km}\!/_{hr}$$

A fast-moving object covers a relatively large distance in a given amount of time and thus has a high speed. Whereas a slow-moving object covers a relatively small amount of distance in the same amount of time and therefore has a low speed.

FIGURE 2.7 SPEED OF A CAR



POINT TO PONDER

SOME INTERESTING SPEED FACTS

Who is the fastest man on earth? Yes, Usain Bolt. He finished a 100-metre sprint in just 9.58 seconds back in 2009. In that instance, his speed was 10.44 m/s or 37.58 km/h.

The slowest animal in the world, the crown goes to the 3-toed sloth. And, the average speed of them is about 0.00134112 m/s or 0.0048 km/h. You would have seen garden snails or turtles moves which is faster than this rate.

The fastest animal in the world is Peregrine Falcon, it can attain a maximum speed of up to 108.333 m/s or 390 km/h. Cheetah is the fastest animal in the land can reach a fastest speed of 33.33 m/s or 120 km/h.

This means that our car is moving at 70 km/hr neither speeding up nor slowing down. However, it is usually difficult to maintain a same speed. Other cars and distractions can cause us to reduce speed or at times we have to increase speed of our car.

A. AVERAGE SPEED

If we calculate our speed over an entire trip, we are considering a large distance between two places and the total time that elapsed. The increases and decreases in speed would be averaged.

The average speed is the total distance (s) covered in total time (t). Mathematically,

$$v_{ave} = \frac{\text{total distance}}{\text{total time}} = \frac{s}{t} \qquad (2.2)$$

Interchangeably this equation can also be written as $s = v_{ave}\,t$ (2.3)

ACTIVITY

MEASURE YOUR BOWLING SPEED

How the speed of bowler in cricket game is calculated? You can also roughly calculate your bowling speed. First carefully measure the length of the cricket pitch in metres from bowlers delivery stride mark to where the batter is standing. Now give a stop watch to your friend and ask him to start the stopwatch as you release the ball and stop it once it reaches the batter. To get the speed in m/s divide the length of the pitch by the time in the stop watch. For comparison with speed of international bowlers, we would require to convert this speed to kph or pmh.

B. INSTANTANEOUS SPEED

We see sign boards on road reading, sharp turn ahead reduce speed 'speed limit 70 km/hr'.Certainly this sign board does not refer to our average speed, but the speed at which we are moving at that particular instant of time. The speed at any specific instant of time is called the instantaneous speed.

If we are not looking at the speedometer of car we only have a rough idea of how fast we are moving, and how much we should reduce speed. However, looking at the speedometer, on the other hand, we will know exactly how fast we are going at that instant of time.

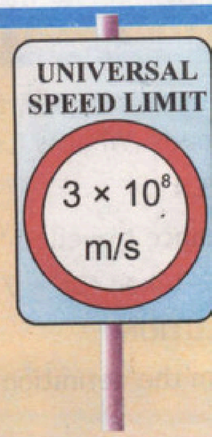
FIGURE 2.8 SPEEDOMETER OF A CAR

C. UNIFORM AND VARIABLE SPEED

'If an object covers equal distances in equal intervals of time we say that the object is moving with uniform speed'. In uniform speed object does not get slower or faster and maintains the same speed.

DO YOU KNOW?

When it comes to fastest measured speed, the limit is set by the laws of physics themselves as the 'speed of light'. Albert Einstein realized that, a light ray appears to move at-the same speed, regardless of whether it's moving towards us or away from us. No matter how fast you travel or in what direction, all light always moves at the same speed. Moreover, anything that's made of matter can only approach, but never reach, the speed of light. If you don't have mass, you must move at the speed of light; if you do have mass, you can never reach it.



The speed of light is in a vacuum is about 299,792,458 m/s or 299,792 km/s (which is approximately 3×10^8 ms^{-1}). At this speed, you can revolve around the Earth 7.5 times in a second. In comparison the speed of sound in the air is roughly 343 m/s or 767 mph or 1235 km/h. That means the speed of light is so much faster than the speed of sound.

EXAMPLE 2.1: REACTION TIME OF BATSMAN

Shoaib Akhtar made a record in Word cup 2003 against England by bowling at a speed of 161.3 km/h. If the batsman is at a displacement of 17.5 m from the bowler, what should be the reaction time for the batsman to play such a delivery?

GIVEN

Speed of ball $v = 161.3\ km/h = \dfrac{161.3 \times 1000\ m}{3600\ s} = 44.8\ m/s$

Distance covered by ball $s = 17.5\ m$

REQUIRED

time $t = ?$

SOLUTION

From the definition of average speed, equation 2.2 we have $v_{ave} = \dfrac{\text{total distance}}{\text{total time}} = \dfrac{s}{t}$

$$t = \frac{s}{v_{ave}} \quad \text{Putting values} \quad t = \frac{17.5m}{44.8\,m/s}$$

Hence $\boxed{t = 0.39\ s}$ Answer

The batsman should react in just 0.39 seconds to play this delivery. These are typical reaction times player deal in game of cricket.

DO YOU KNOW?

Pakistani Cricketer Shoaib Akhtar bowled the fastest recorded ball in the history of cricket in the World Cup match at Newlands South Africa. This match was played between Pakistan and England and the ball was faced by Nick Knight (former England opener).

EXAMPLE 2.2: FASTEST TRAIN IN THE WORLD

Shanghai's Maglev, the fastest train, travelled a distance of 30 kilometres in 7 minutes and 30 seconds. What is its speed? Convert the speed to km/h.

GIVEN

Distance travelled 'Ds' = 30 km = 30 × 1000 m = 30,000 m

Time taken 'Dt' = 7 min 30 s = (7 × 60) s + 30 s = 420 s + 30 s = 450 s

REQUIRED

speed v = ?

SOLUTION

From the definition of speed, equation 2.1 we have: $v = \frac{\Delta s}{\Delta t}$

Putting values $v = \frac{30{,}000\,m}{450\,s}$

Hence $\boxed{v = 66.67\,m/s}$ Answer

Maglev is a system of train transportation that uses two sets of electromagnets: one set to repel and push the train up off the track, and another set to move the elevated train ahead, taking advantage of the lack of friction.

Conversion in km/h

Converting m to km and s to h

$$v = \left[66.67\frac{m}{s}\right] \times \left[\frac{3600}{1}\frac{s}{h}\right] \times \left[\frac{1}{1000}\frac{km}{m}\right] = 240.01\,km/h$$

$\boxed{v = 240.01\,km/h}$ Answer

This is a much greater speed as compared to the speed limits on motor ways (120 km/h)

2.3.2 VELOCITY

Velocity is similar to speed, but a direction is needed for the description of velocity. **'Measure of displacement ($\Delta\vec{d}$) with passage of time (Δt) is called velocity (denoted by $\vec{v}$)'. Mathematically**

$$velocity = \frac{\text{displacement}}{\text{elapsed time}} \quad \text{or} \quad \vec{v} = \frac{\Delta\vec{d}}{\Delta t} \quad \text{or} \quad \vec{v} = \frac{\vec{d}_f - \vec{d}_i}{t_f - t_i} \qquad (2.4)$$

Velocity is a vector quantity having same direction as displacement vector. The SI unit of velocity is metre per second (m/s). When we know both the speed and the direction of an object, we simply call it as velocity.

For straight-line motion in one direction, speed and velocity have same magnitudes because the lengths of the distance and the displacement are the same. The distinction between them in this case is that a displacement direction must be specified for the velocity.

A. AVERAGE VELOCITY

The average velocity is the total displacement ($\vec{d}$) covered in total time (t). Mathematically,

$$\vec{v} = \frac{\vec{d}}{t} \qquad (2.5)$$

B. INSTANTANEOUS VELOCITY

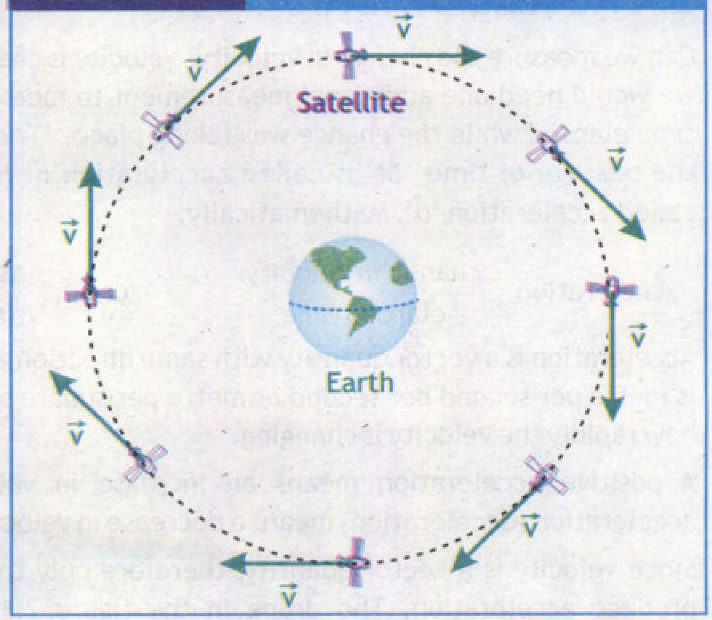
If velocity is measured by keeping the time interval small, such velocity is termed as instantaneous velocity. To calculate velocity both the speed and direction for that moment of time need to be specified.

C. UNIFORM AND VARIABLE VELOCITY

'If an object covers equal displacements in equal intervals of time we say that the object is moving with uniform velocity. **Uniform velocity is the velocity that does not change otherwise it is called variable velocity.**

To produce variable velocity (a change in velocity), either the speed or the direction is changed (or both are changed). A satellite moving with a constant speed in a circular orbit around Earth does not have a constant velocity since its direction of movement is constantly changing as shown in figure 2.9.

FIGURE 2.9 MOTION OF A SATELLITE



EXAMPLE 2.3: VELOCITY OF A CAR

A car travels a curvy track of length 800 metres in 40 seconds. The straight path is about 600 metres between starting point and ending point, which the same car travels in 36 seconds. What is the car's (a) average speed and (b) average velocity?

GIVEN

Length of curvy track = Distance Δd = 800 m

Time taken 'Δt' = 40 s

Length of straight path = Displacement Δd = 600 m

Time taken 'Δt' = 40 s

REQUIRED

(a). Average speed vave = ?

(b). Average Velocity $\vec{vave}$ = ?

SOLUTION

From the definition of speed and velocity, we have

(a). Average Speed= $v_{ave} = \frac{\text{Total distance}}{\text{Total time}} \Rightarrow v_{ave} = \frac{s}{t}$

Putting vales: $v_{ave} = \frac{800\text{ m}}{40\text{ s}} \Rightarrow \boxed{v_{ave} = 20\text{ m/s}}$ Answer

(b). Average Velocity= $\vec{v}_{ave} = \frac{\text{Total displacement}}{\text{Total time}} \Rightarrow \vec{v}_{ave} = \frac{\vec{d}}{t}$

Putting values: $\vec{v}_{ave} = \frac{600\text{m}}{36\text{s}} \Rightarrow \boxed{\vec{v}_{ave} = 16.67\text{ m/s}}$ Answer

2.4 ACCELERATION

Can we measure the change in velocity? Velocity is changed by changing speed, direction or both, we would need one additional measurement to measure change in velocity, which is how much time elapsed while the change was taking place. **'The measure of change in velocity '$\Delta\vec{v}$' with the passage of time 'Δt' is called acceleration $\vec{a}$.** (or) Time rate of change in velocity '$\Delta\vec{v}$' is called acceleration '$\vec{a}$'. Mathematically:

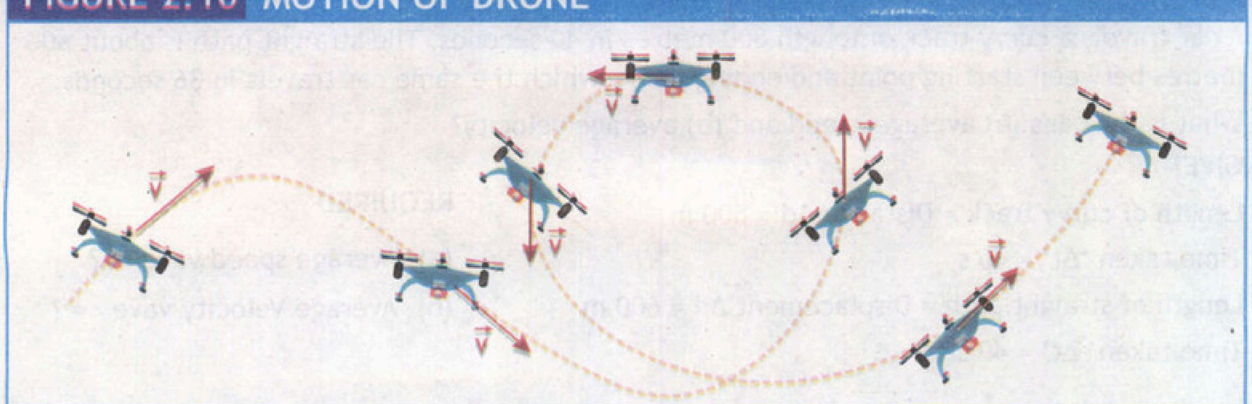
$$\text{acceleration} = \frac{\text{change in velocity}}{\text{elapsed time}} \quad \text{or} \quad \vec{a} = \frac{\Delta\vec{v}}{\Delta t} \quad \text{or} \quad \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \qquad (2.6)$$

Acceleration is a vector quantity with same direction as change in velocity. SI Unit of acceleration is metre per second per second or metre per square second (m/s^2). Acceleration is a measure of how rapidly the velocity is changing.

A positive acceleration means an increase in velocity with time, whereas the negative acceleration (deceleration) means a decrease in velocity with time.

Since velocity is a vector quantity, therefore only the change in direction of velocity can also produce acceleration. The drone in the figure 2.10 is accelerating because it is changing directions.

FIGURE 2.10 MOTION OF DRONE

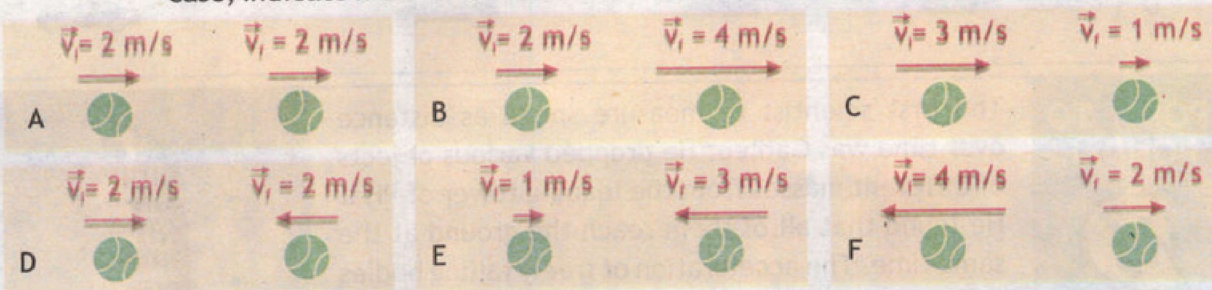


Uniform and Non-uniform Acceleration: **When an object is changing its velocity at the same rate each second we call it uniform acceleration.** A body has uniform acceleration if it has equal changes in velocity in equal intervals of time.

Non-uniform acceleration occurs when an object's velocity changes, but this change is not steady over time. In simple terms, the rate at which the object's velocity changes is not the same throughout its movement. Acceleration, which is the measure of velocity change, is therefore not constant in non-uniform acceleration. Understanding non-uniform acceleration is important in physics to explain the movement of objects affected by changing forces. This is a common and practical situation since many real-life scenarios involve forces that vary over time, resulting in non-uniform acceleration.

CAN YOU TELL?

The initial velocity v_i and final velocity v_f of a tennis ball at two different points in time is shown below. The direction of the ball is indicated by the arrow. For each case, indicate if there is an acceleration and show the direction of acceleration.



EXAMPLE 2.4: ACCELERATION OF CHEETAH

Cheetah (fastest land animal) can accelerate its speed from zero to 26.8 m/s in just three seconds. Suppose the Cheetah has started running towards East, find its acceleration.

GIVEN

Initial velocity v_i = 0 m/s (East)

Final velocity v_f = 26.8 m/s (East)

Time taken Δt = 3 s

REQUIRED

acceleration $\vec{a}$ = ?

SOLUTION

From the definition of acceleration, equation 2.6 we have $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$

Putting values $\vec{a} = \dfrac{26.8\ m/s - 0\ m/s}{3\ s}$

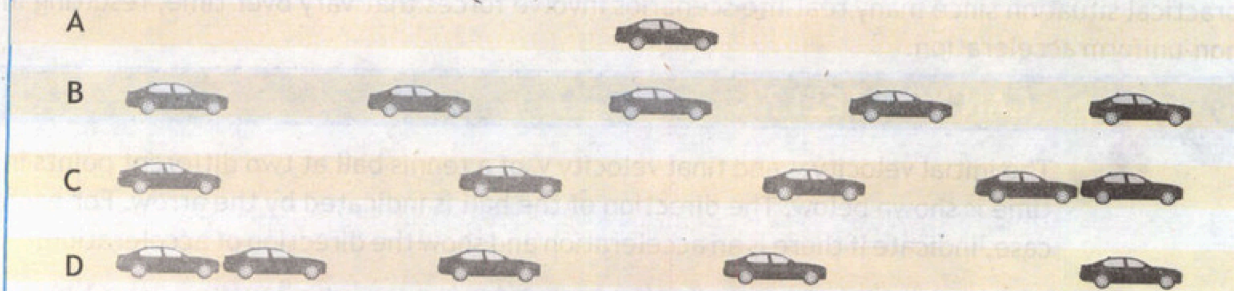
$$\boxed{\vec{a} = 9.93\ m/s^2}$$ Answer

That is a big value, as typical cars have accelerations of only 3 to 4 m/s^2

CAN YOU TELL?

The car is depicted after equal time intervals, can you determine that in which picture A, B, C and D, the car is

at rest	☐	speeding up	☐
moving at a constant speed	☐	slowing down	☐



POINT TO PONDER

The first scientist to measure speed as distance over time was Galileo. He dropped various objects of different masses from the leaning tower of Pisa. He found that all of them reach the ground at the same time. The acceleration of freely falling bodies is called gravitational acceleration or acceleration due to gravity denoted by 'g'.

2.5 MOTION DUE TO GRAVITY

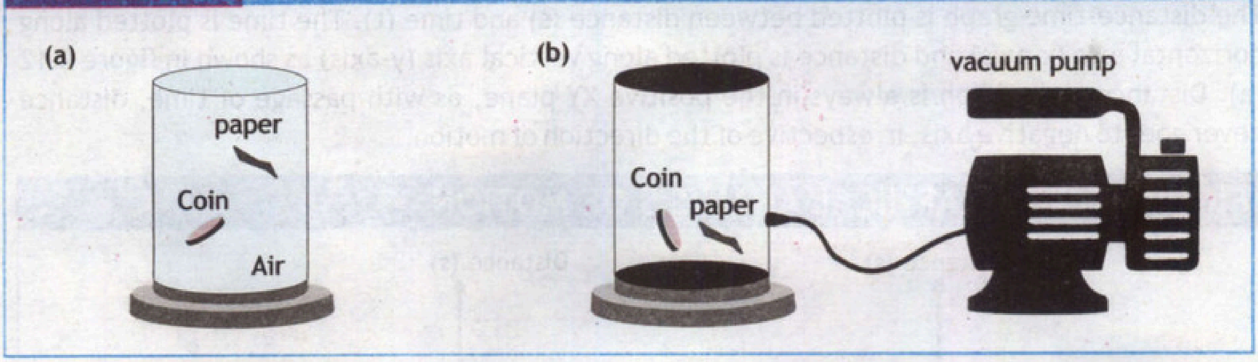
If you drop a ball and large stone from the roof of your school building, which of them will reach the bottom first? All the freely falling objects have the same acceleration called the acceleration due to gravity (g) and is independent of their masses.

The acceleration due to gravity is directed downward, toward the center of the earth. Near the earth's surface, g is approximately

$$g = 9.80\ m/s^2 \text{ or } 32.2\ ft/s^2$$

For large object the presence of air resistance is neglected, however if we drop a small piece of paper and coin. The coin will fall faster than a sheet of paper due to air resistance as in Figure 2.11 (a). However, when air is removed, as in Figure 2.11 (b), the coin and the paper will experience the same acceleration due to gravity, and both the coin and the paper will fall at same rate.

FIGURE 2.11 FREE FALL EXPERIMENT



When an object moves with the gravity acceleration due to gravity is taken as positive (+g) and when object moves against gravity (like an object thrown up), acceleration due to gravity is taken as negative (-g).

EXAMPLE 2.5 ACCELERATION DUE TO GRAVITY

A block of mass 2 kg is left from the top of a building. How much time will the block take to reach the ground if it strikes the ground with a speed of 78.5 m/s? (Ignore air resistance).

GIVEN

Mass of the block 'm' = 2 kg

Initial speed 'v_i' = 0 m/s

Final speed 'v_f' = 78.5 m/s

Acceleration due to gravity 'g' = 9.8 m/s²

REQUIRED

Time to reach the ground 'Δt' = ?

SOLUTION

From the definition of acceleration, acceleration due to gravity can also be written as

$$g = \frac{v_f - v_i}{\Delta t} \quad \text{rearranging for time} \quad \Delta t = \frac{v_f - v_i}{g}$$

Putting values $$\Delta t = \frac{78.5 m/s - 0 m/s}{9.8 m/s^2}$$

Therefore, $\boxed{\Delta t = 8 s}$ Answer

So, the block will reach the ground in 8 seconds.

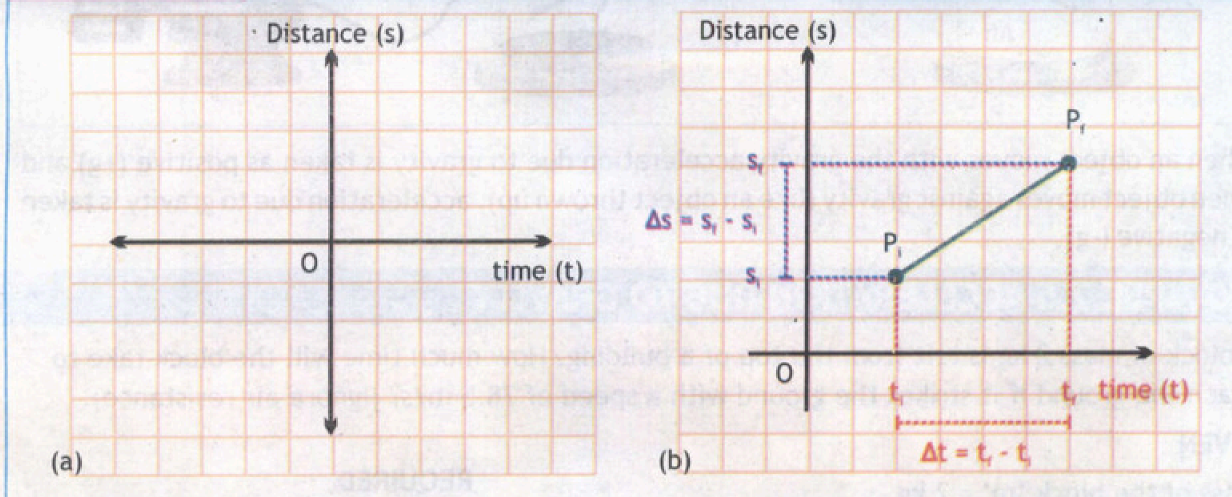
2.6 GRAPHICAL ANALYSIS OF MOTION

Graph (horizontal and vertical lines at equal distances) is an efficient method to show relationship between physical quantities. Graph use coordinate systems to show relationship in various quantities.

2.6.1 DISTANCE-TIME GRAPH

The distance-time graph is plotted between distance (s) and time (t). The time is plotted along horizontal axis (x-axis) and distance is plotted along vertical axis (y-axis) as shown in figure 2.12 (a). Distance time graph is always in the positive XY plane, as with passage of time, distance never goes to negative axis, irrespective of the direction of motion.

FIGURE 2.12 GRAPHICAL ANALYSIS OF MOTION



The gradient (or slope) of distance time curve gives speed. The gradient of the graph means vertical coordinate difference divided by horizontal coordinate difference. The gradient in distance-time graph can be calculated as

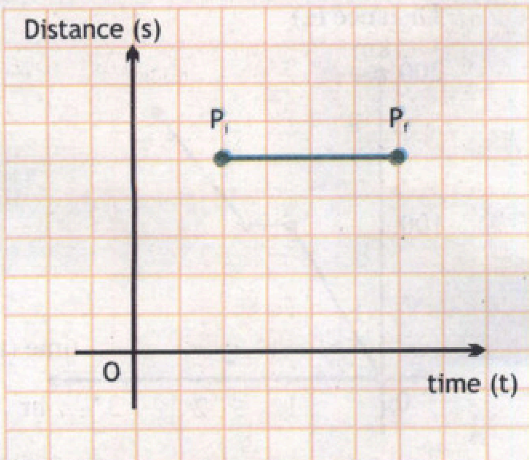
1. Choose two points P_i and P_f for which the gradient is to be determined.
2. Determine the coordinates P_i (t_i, s_i) and P_f (t_f, s_f), by drawing perpendicular on each axis from both points as shown in figure 2.11 (b).
3. Determine the difference between horizontal-cordinates ($\Delta t = t_f - t_i$) and vertical-coordinates ($\Delta s = s_f - s_i$).
4. Dividing the difference in vertical-coordinates ($\Delta s = s_f - s_i$) by difference in horizontal-cordinates ($\Delta t = t_f - t_i$) gives gradient. Mathematically

$$gradient = \frac{\Delta s}{\Delta t} = \frac{s_f - s_i}{t_f - t_i} = v \qquad (2.7)$$

from equation 2.1 it is definition of speed, therefore $\quad gradient = v$

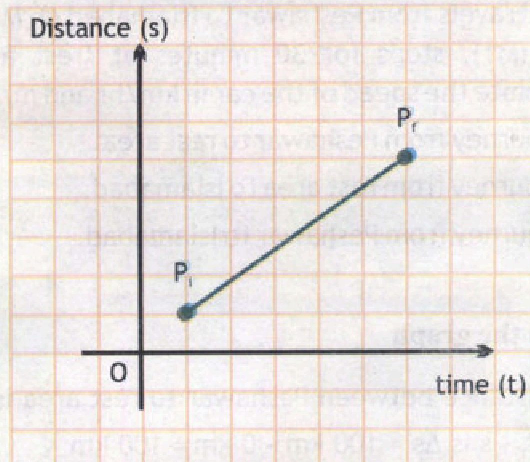
Thus by looking at the graph we get the idea about the speed of a body, shown in figure 2.13.

FIGURE 2.13 DISTANCE - TIME GRAPH



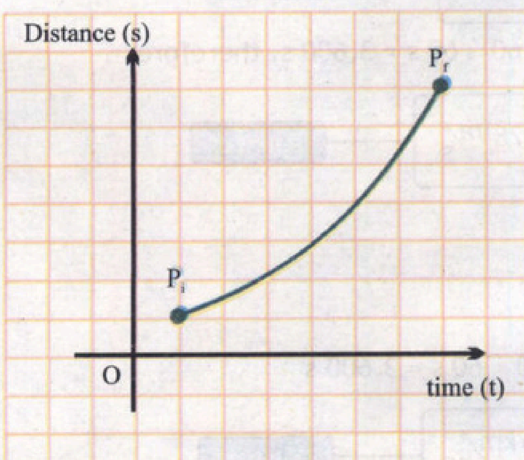
(a) BODY AT REST (ZERO SPEED)

Time is passing and no change in distance is seen. It means the body is at rest. Since there is no slope so the speed is zero.



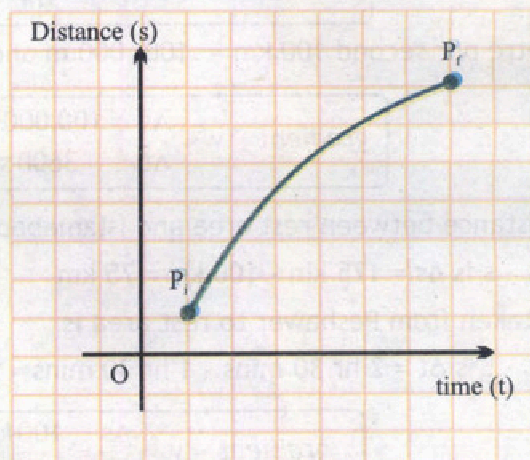
(b) BODY MOVING WITH CONSTANT SPEED

The distance is increasing linearly with time. The slope is constant therefore object is moving with uniform speed.



(c) BODY MOVING WITH VARIABLE SPEED

Increasing speed (accelerating): The distance is changing non-linearly with time (curving up). The slope is increasing therefore object is increasing its speed.



(d) BODY MOVING WITH VARIABLE SPEED

Decreasing Speed (decelerating): The distance is changing non-linearly with time (curving down). The slope is changing therefore object is decreasing its speed.

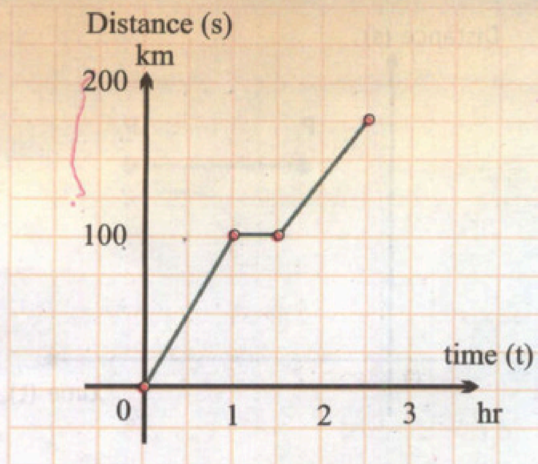
EXAMPLE 2.5: PESHAWAR TO ISLAMABAD THROUGH M1

A car travels from Peshawar to Islamabad on Motorway (M1), stops for 30 minutes at 'rest area'. Calculate the speed of the car in km/hr and m/s for

(a) Journey from Peshawar to rest area.

(b) Journey from rest area to Islamabad.

(c) Journey from Peshawar to Islamabad.



From the graph

(a) Distance between Peshawar to rest area is

$\Delta s = s_f - s_i$ is Δs = 100 km - 0 km = 100 km

time taken from Peshawar to rest area is

$\Delta t = t_f - t_i$ is Δt = 1 hr - 0 hr = 1 hr

$$gradient = v = \frac{\Delta s}{\Delta t} = \frac{100\,km}{1hr} = 100\,{}^{km}/_{hr}$$ Answer

In metre per second 100 km = 100, 000 m and 1 hr = 60 ´ 60 s = 3,600 s, therefore

$$gradient = v = \frac{\Delta s}{\Delta t} = \frac{100{,}000\,m}{3600\,s} = 27.78\,{}^{m}/_{s}$$ Answer

(b) Distance between rest area and Islamabad is

$\Delta s = s_f - s_i$ is Δs = 175 km - 100 km = 75 km

time taken from Peshawar to rest area is

$\Delta t = t_f - t_i$ is Δt = 2 hr 30 mins - 1 hr 30 mins = 1 hr = 60 ´ 60 s = 3,600 s

$$gradient = v = \frac{\Delta s}{\Delta t} = \frac{100\,km}{1hr} = 100\,{}^{km}/_{hr}$$ Answer

In metre per second 75 km = 75, 000 m and 1 hr = 60 ´ 60 s = 3,600 s, Therefore

$$gradient = v = \frac{\Delta s}{\Delta t} = \frac{75{,}000\,m}{3600\,s} = 20.83\,{}^{m}/_{s}$$ Answer

(c) Distance from Peshawar to Islamabad is:

$\Delta s = s_f - s_i$ is Δs = 175 km - 0 km = 175 km

time taken from Peshawar to Islamabad is

$\Delta t = t_f - t_i$ is Δt = 2 hr and 30 mins - 0 hr = 2.5 hr

$$gradient = v = \frac{\Delta s}{\Delta t} = \frac{175\,km}{2.5\,hr} = 70\,{}^{km}\!/\!_{hr}$$ Answer

In metre per second

175 km = 175, 000 m

and 2 hr and 30 mins = 2 (60 ´ 60) s + (60 ´ 30) s = 7,200 s + 1,800 s = 9,000 s

$$gradient = v = \frac{\Delta s}{\Delta t} = \frac{175{,}000\,m}{9{,}000\,s} = 19.44\,{}^{m}\!/\!_{s}$$ Answer

2.6.2 SPEED - TIME GRAPH

Speed-time graph is the graph plotted between speed (v) and time (t). In this graphical analysis the speed is plotted along vertical axis (y-axis) and time along horizontal axis (x-axis). Speed time graph serve two purposes

- Slope of the graph gives magnitude of acceleration
- Area under the graph gives distance traveled.

The slope of speed time curve gives magnitude of acceleration. As discussed in the graph for the distance graph, the slope of velocity time graph gives by definition the magnitude (value) of acceleration

The gradient (or slope) of speed time curve gives magnitude of acceleration. The gradient of the graph means vertical coordinate difference divided by horizontal coordinate difference. The gradient in distance-time graph can be calculated as:

1. Choose two points 'P_i' and 'P_f' for which the gradient is to be determined.

2. Determine the coordinates '$P_i\,(t_i, v_i)$' and '$P_f\,(t_f, v_f)$', by drawing perpendicular on each axis from both points as shown in figure 2.9 (b).

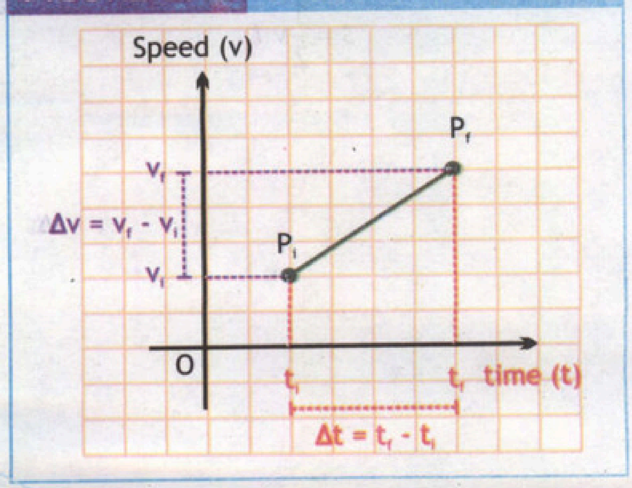
3. Determine the difference between horizontal-cordinates ($\Delta t = t_f - t_i$) and vertical-coordinates ($\Delta v = v_f - v_i$).

4. Dividing the difference in vertical-coordinates ($\Delta v = v_f - v_i$) by difference in horizontal-cordinates ($\Delta t = t_f - t_i$) gives gradient. Mathematically

$$gradient = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = |a| \qquad (2.8)$$

From equation 2.8 we can conclude that the gradient of velocity - time graph gives the magnitude of acceleration.

FIGURE 2.13 SPEED TIME GRAPH



B. Area under speed time graphs represent the distance traveled: If the motion of a body represented by the speed time graph is symmetric shape then the area can be calculated using appropriate formula for geometrical shapes.

For example consider the figure 2.14 in which the speed time graph of the object in motion is represented by a rectangle. The area of rectangle is

$$Area = width \times length$$

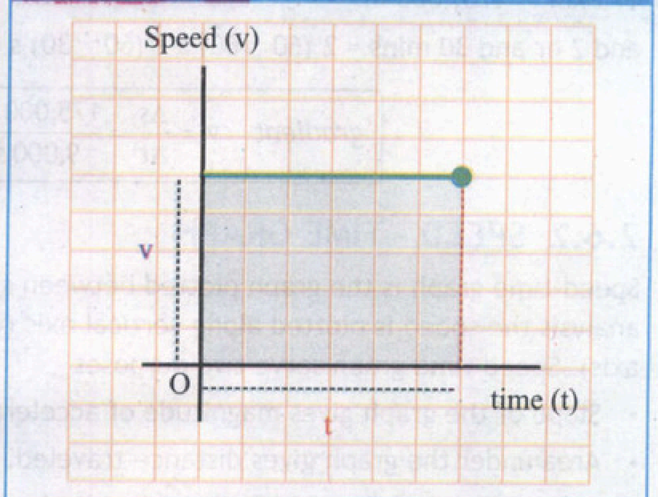
$$Area = v \times t$$

The distance by average speed is given by equation 2.3, is also

$$s = v\,t$$

Thus the area under speed time graphs represent the distance traveled.

FIGURE 2.14 RECTANGULAR AREA



Similarly consider the figure 2.15 in which the speed time graph of the object in motion is represented by a triangle. The area of triangle is

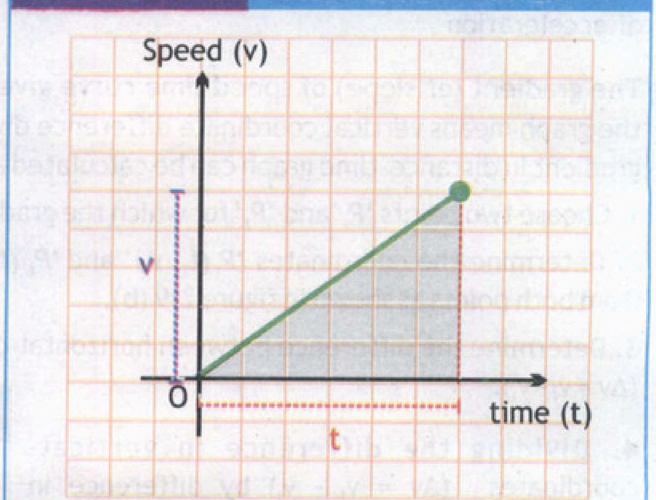
$$Area = \frac{1}{2} width \times length$$

$$Area = \frac{1}{2} v \times t$$

The distance by average speed is given by equation 2.4, is again

$$s = \frac{1}{2} v\,t$$

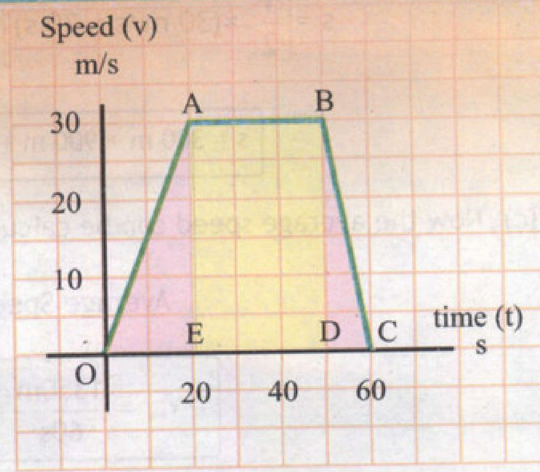
FIGURE 2.15 TRIANGULAR AREA



EXAMPLE 2.6: GRAPHICAL REPRESENTATION OF SPEED OF CAR

A car increases its speed from zero to a 30 m/s in 20 s. Then it moves with uniform speed for the next 30 seconds and then the driver applies brakes and the speed of the car decreases uniformly to zero in 10 s. The graph is plotted for the journey, use this graph to calculate:

(a). magnitude of acceleration (i) in first 20 s (ii) from 20 s to 50 s and (iii) in last 10 s (b). total distance covered (c). average speed



(a) The slope of the graph will give magnitude of acceleration.

$$\text{Slope} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \text{Magnitude of acceleration}$$

(i) For the first 20 seconds, OA line represents the slope

$$\text{Magnitude of acceleration} = |\vec{a}| = \frac{(30 - 0)\ \text{m/s}}{(20 - 0)\ \text{s}} = \frac{30\ \text{m/s}}{20\ \text{s}}$$

$$\boxed{|\vec{a}| = 1.5\ \text{m/s}^2}$$ Answer

(ii). From 20 s to 50 s, the slope is represented by line AB

$$\text{Magnitude of acceleration} = |\vec{a}| = \frac{(30 - 30)\ \text{m/s}}{(50 - 20)\ \text{s}} = \frac{0\ \text{m/s}}{30\ \text{s}}$$

$$\boxed{|\vec{a}| = 0\ \text{m/s}^2}$$ Answer

(iii). In the last 10 seconds, the slope is represented by BC

$$\text{Magnitude of acceleration} = |\vec{a}| = \frac{(0 - 30)\ \text{m/s}}{(60 - 50)\ \text{s}} = \frac{-30\ \text{m/s}}{10\ \text{s}}$$

$$\boxed{|\vec{a}| = -3\ \text{m/s}^2}$$ Answer

The negative sign shows that the car is slowing down.

(b). Now the total distance covered is equal to the area under the speed-time graph.

Total distance covered = Area of triangle OAE + Area of rectangle ABDE + Area of triangle CBD

$$s = \left[\frac{1}{2} \times (30\ m/s \times 20\ s)\right] + \left[30 m/s \times 30\ s\right] + \left[\frac{1}{2} \times (30\ m/s \times 10\ s)\right]$$

$$\boxed{s = 300\ m + 900\ m + 150\ m = 1350\ m}$$ Answer

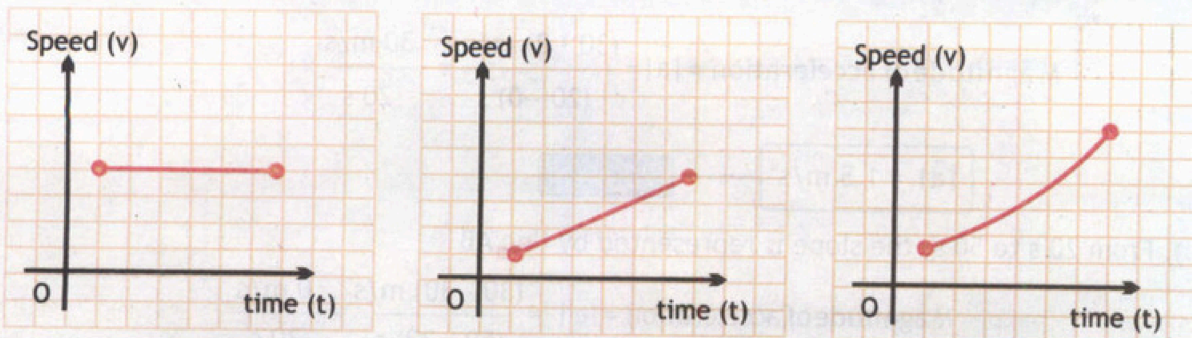
(c). Now the average speed can be calculated when distance s is divided by total time t

$$\text{Average Speed} = \frac{\text{Total distance covered}}{\text{Total time}}$$

$$\boxed{v_{ave} = \frac{1350m}{60s} = 22.5\, m/s}$$ Answer

CAN YOU TELL?

What does the slope of these graphs tells about acceleration?



SUMMARY

Position is the distance and direction of a body from a fixed reference point.

Distance is the length of a path traveled by an object.

Displacement is the shortest distance from the initial and final position of a body.

Speed is time rate of change of distance and is a scalar quantity.

Velocity is the time rate of change of displacement and is a vector quantity.

Acceleration is the time rate of change of velocity and is a vector quantity.

Gradient of distance-time graph gives speed of the body.

Gradient of speed-time graph gives acceleration of the body.

Area under the speed-time graph gives distance travelled by the body.

EXERCISE

MULTIPLE CHOICE QUESTIONS

QI. Choose best possible option:

1. Change in position of a body from initial to final point is called:

A. Distance B. Displacement C. Speed D. Velocity

2. Motion of a screw of rotating fan is:

A. Circular motion B. Vibratory motion C. Random motion D. Rotatory motion

3. A cyclist is travelling in a westward direction and produces a deceleration of 8 m/s^2 to stop. The direction of its acceleration is towards

A. North B. East C. South D. West

4. A girl walks 3 km towards west and 4 km towards south. What is the magnitude of her total distance and displacement respectively?

A. 7 km, 7 km B. 1 km, 7 km C. 7 km, 1 km D. 7 km, 5 km

5. A rider is training a horse. Horse moves 60 metres towards right in 3 seconds. Then it turns back and travels 30 metres in 2 seconds. Find its average velocity?

A. 6 m/s B. 18 m/s C. 35 m/s D. zero

6. If a cyclist has acceleration of 2m/s^2 for 5 seconds, the change in velocity of the cyclist is

A. 2 m/s B. 10 m/s C. 20 m/s D. 15 m/s

7. A car is moving with velocity of 10 m/s. If it has acceleration of 2 m/s2 for 10 seconds. What is final velocity of the car?

A. 30 m/s B. 20 m/s C. 10 m/s D. 15 m/s

8. When the slope of a body's displacement-time graph increases, the body is moving with:

A. increasing velocity B. decreasing velocity

C. constant velocity D. all of these

9. A ball is thrown straight up, what is the magnitude of acceleration at the top of its path?

A. zero B. 9.8 m/s^2 C. 4.9 m/s^2 D. 19.6 m/s^2

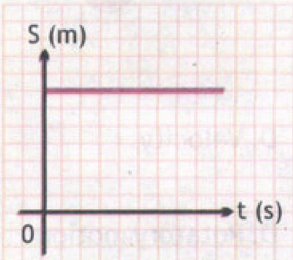
10. Slope of distance-time graph is:

A. velocity B. acceleration C. speed D. displacement

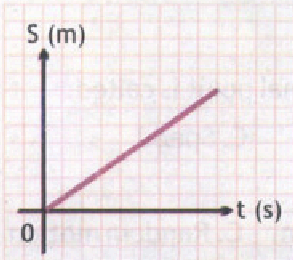
11. Area under speed-time graph is equal to ____________ of moving body:

A. distance B. change in velocity C. uniform velocity D. acceleration

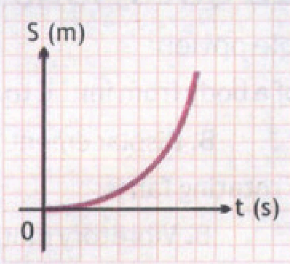
12. In 5 s a car accelerates so that its velocity increases by 20m/s. The acceleration is
    A. $0.25\ m/s^2$ B. $4\ m/s^2$ C. $25\ m/s^2$ D. $100\ m/s^2$
13. Ball dropped freely from a tower reaches ground in 4s, the speed of impact of ball is:
    A. 0 m/s   B. 2.45 m/s   C. 19.6 m/s   D. 39.2 m/s
14. Which of following distance time graphs represents increasing speed of a car?



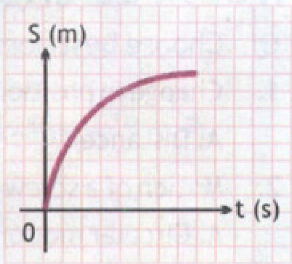
A.



B.



C.



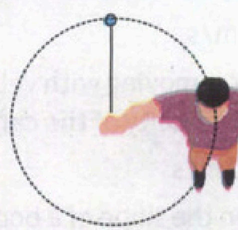
D.

SHORT RESPONSE QUESTIONS

QII. Give a short response to the following questions

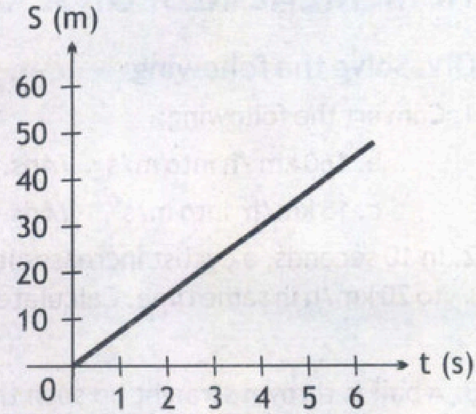
1. In a park, children are enjoying a ride on Ferris wheel as shown. What kind of motion the big wheel has and what kind of motion the riders have?
2. A boy moves for some time, give two situations in which his displacement is zero but covered distance is not zero?
3. A stone tied to string is whirling in circle, what is direction of its velocity at any instant?


Side view


Top view

4. Is it possible to accelerate an object without speeding it up or slowing it down?
5. Can a car moving towards right have direction of acceleration towards left?
6. With the help of daily life examples, describe the situations in which:
    a. acceleration is in the direction of motion.
    b. acceleration is against the direction of motion.
    c. acceleration is zero and body is in motion.

7. Examine distance-time graph of a motorcyclist (as shown), what does this graph tell us about the speed of motorcyclist? Also plot its velocity-time graph.



DISTANCE - TIME GRAPH

8. Which controls in the car can produce acceleration or deceleration in it?
9. If two stones of 10 kg and 1 kg are dropped from a 1 km high tower. Which will hit the ground with greater velocity? Which will hit the ground first? (Neglect the air resistance)
10. A 100 g ball is just released (from rest) and another is thrown downward with velocity of 10 m/s, which will have greater acceleration? (Neglect the air resistance)

LONG RESPONSE QUESTIONS

QIII. Give a detailed response to questions below.

1. Differentiate between rest and motion. With the help of example, show that rest and motion are relative to observer?
2. What are different types of motion? Define each type of motion with examples from daily life.
3. What are scalars and vectors? Give examples. How are vectors represented symbolically and graphically?
4. Define the term position. Differentiate between distance and displacement.
5. Differentiate between speed and velocity. Also define average speed, uniform and variable speeds, average velocity, uniform and variable velocities.
6. What are freely falling bodies? What is gravitational acceleration? Write down sign conventions for gravitational acceleration? Write three equations of motion of a freely falling body?
7. Draw distance-time graphs for rest, uniform speed, increasing speed and decreasing speed.
8. Draw speed-time graphs for zero acceleration, uniform acceleration, uniform deceleration. Also show that area under speed time graph represents distance covered by the body.

NUMERICAL RESPONSE QUESTIONS

QIV. Solve the following

1. Convert the following:

   a. 160 km /h into m/s *(Ans. 44.44 m/s)* b. 36 m/s into km/h *(Ans. 129.6 km/h)*

   c. 15 km/h^2 into m/s^2 *(Ans. 0.001 m/s^2)* d. 1 m/s^2 into km/h^2 *(Ans. 12,960 km/hs^2)*

2. In 10 seconds, a cyclist increases its speed from 5 km/h to 7 km/h, while a car moves from rest to 20 km/h in same time. Calculate and compare acceleration in each case.

   (Ans. 0.055 m/s^2 and 0.55 m/s^2)

3. A ball is thrown straight up such that it took 2 seconds to reach the top after which it started falling back. What was the speed with which the ball was thrown up? *(Ans. 19.6 m/s)*

4. A car is moving with uniform velocity of 20 m/s for 20 seconds. Then brakes are applied and it comes to rest with uniform deceleration in 30 s. Plot the graph to calculate this distance using speed time graph? *(Ans. 1 km)*

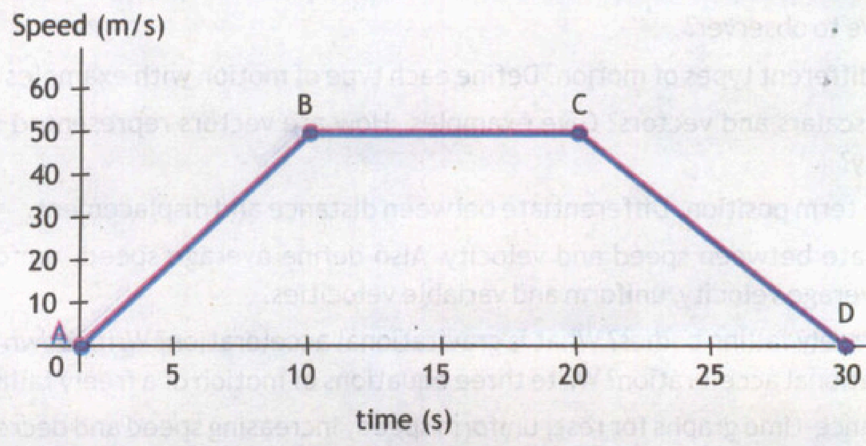
5. A girl starts her motion by a racing bicycle in a straight line at a speed of 50 km/h. Her speed is changing at a constant rate. If she stops after 60 s, what is her acceleration? *(Ans. 0.23 m/s^2)*

6. Consider the following speed time graph. Tell:

   a. Which part of the graph is showing acceleration, deceleration and zero acceleration?

   b. Calculate covered distance from 10 seconds to 20 seconds from the graph.

   (Ans. (a) OA, BC, AB, (b) 500 m)



SPEED - TIME GRAPH