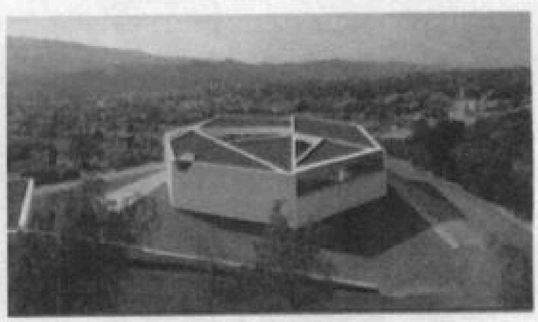
UNIT
09

GEOMETRY AND POLYGONS

In this unit the students will be able to:

- Differentiate between mathematical statement and its proof.
- Differentiate between an axiom, conjecture and theorem.
- Formulate simple deductive proofs (algebraic proofs that require showing LHS equal to RHS e.g. $(x-3)^2+5=x^2-6x+14$
- Identify similarity of polygons and area and volume of similar figures.
- Solve problems using relationship between areas of similar figures and volumes of similar solids.
- Solve real life problems that involve the properties of regular polygons, triangles and parallelograms.
- Solve real life problems using the following loci and the method of intersecting loci for sets of points in two dimensions which are:
  - at a given distance from a given point.
  - at a given distance from a given straight line.
  - equidistant from two given points.
  - equidistant from two given intersecting straight lines.



Geometry has many uses. It is used whenever we ask questions about the size, shape, volume, or position of an object. As a school subject, it helps to develop logical reasoning. Architects and engineers use geometry in planning buildings, bridges, and roads.
Geometry is used by navigators to guide boats, planes, and even space ships. Military personnel use geometry to guide vessels and aim guns and missiles. Almost everything you do in your daily life involves geometry in some way. We observe many geometrical shapes used in the construction of buildings like masjids, forts and in other buildings.

Demonstrative Geometry

It is a branch of geometry in which geometrical statements are proved through logical reasoning. The reasons can be taken from given information, basic assumptions or already proved results and their corollaries etc.

Reasoning

Reasoning is a way of proving results. The statements without logical reasoning are not acceptable. Different forms of reasoning are accepted in different cases. There are two types of reasoning.

i. Inductive reasoning **ii.** Deductive reasoning

i. Inductive Reasoning

In inductive reasoning we work from some specific observations to a general result.

For example, if your school starts at 8:00 am daily and you left your home at 7:15 am for school today, you arrived at school on time. So, to arrive at school on time you should leave your home 45 minutes before the school time daily.

Inductive reasoning is commonly used in science. It is not always valid logically because it is not always accurate to assume a general principle to be correct. In above example, perhaps 'today' there was less traffic, and if you leave the house at 7:15 am. On any other day, it might take longer and you might be late for school due to heavy traffic.

ii. Deductive Reasoning

In deductive reasoning we work from some general result to a specific conclusion.

For example, if your school starts at 8:00 am and you leave your home at 7:15 am daily for school, you arrive at school on time. So, to arrive at school on time today you should leave your home 45 minutes before the school time.

Example 1:

Prove that $3x + 9 = 3(x + 3)$

Solution:

Let $x = 1$

LHS $= 3(1) + 9 = 12$, RHS $= 3(1 + 3) = 3(4) = 12$

Similarly, the statement is true for all real numbers.

Hence proved.

Example 2:

Find $(102)^2 = ?$

Solution:

General: $(a + b)^2 = a^2 + b^2 + 2ab$

Check Point

Tell whether the following statement follows inductive or deductive approach?

Statement: If a line segment AB passes through point O, then the line AB (or ray AB) will also pass through point O.

Particular: $(100+2)^2 = 100^2 + 2^2 + 2 \times 100 \times 2$

$= 10000 + 4 + 400 = 10404$

Applicability of Deductive Approach

Deductive approach is suitable for giving practice to the student in applying the formula or principles or generalization which has been already arrived at. This method is very useful for fixation and retention of facts and rules as it provides adequate drill and practice.

Mathematical Statement

When we solve any problem in mathematics, our solution is either right or wrong. There is no midway to solve the problems.

> A mathematical statement is a meaningful composition of words that can be either true or false. It may contain words and symbols.

Examples:

1. Look at the following sentences:
   (i) Sum of 2 and 5 is 7.
   (ii) Square root of 36 is 6.
   (iii) No line can pass through a point.

   Here the first two sentences are true and third one is false.

   ∴ Above are mathematical statements.

2. $2x + 4 = 2(x + 2)$

   This is a mathematical statement since both sides of the sentence are equal when any real number is substituted for x.

3. Now observe the following sentence.
   (i) Product of numbers x and y is 10.
   We are not sure the statement is true or false as the values of x and y are not known to us.
   ∴ The above statement is an open sentence.
   (ii) If we write the sentence as follows:
   'Product of numbers x and y when $x = 5$ and $y = 2$, is 10'.
   Then the sentence is a mathematical statement (being true).
   (iii) Again, if we write the sentence as follows:
   'Product of numbers x and y when $x = 4$ and $y = 3$, is 10'.
   Then the sentence is again a mathematical statement (being false).

Check Point

Which of the mathematical statements are true?
(a) Square of 12 is 144.
(b) Product of 4, 5 and 8 is 165.
(c) Log 1 = 0
(d) $2^6 = 12$

Proposition

A statement which may or may not be true is called a proposition.

There are three parts of proposition.

(i) **The premise**

It is an assumption that something is true.

(ii) **The argument**

The logical chain of reasoning that leads from the premises to the conclusion is known as an argument.

(iii) **The conclusion**

The result obtained after giving argument to the premises is called conclusion.

The conclusion must be true if the premises are true and the argument is valid.

Key Fact

A mathematical statement consists of two parts. First is the hypothesis or assumptions and the second is the conclusion.

Axiom, Conjecture and Theorem

Fundamental Assumptions

Fundamental assumptions are statements which are regarded true without any proof. These assumptions play an important role in geometry.

1. Axiom

The word 'Axiom' is derived from the Greek word 'Axioma' which mean 'true without any proof'. Thus, an axiom is a mathematical statement which is assumed to be true without any proof. Axioms are truths that have been derived on the basis of everyday experience and form the basis for all other derivations. Axioms deal with numbers and their relations.

For example:

(i) A number is always equal to itself (reflexive property).

i.e. if x is any real number then $x = x$.

In geometry we can say that if $\overline{AB}$ is any line segment then AB = AB.

(ii) If $a = b$ and $b = c$ then $a = c$ (Transitive property)

Or in geometry, if $\angle A = \angle B$ and $\angle B = \angle C$, then $\angle A = \angle C$.

Key Fact

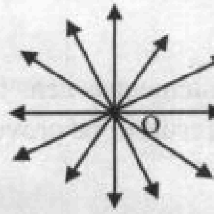
Axioms are also known as properties and first seven axioms are called properties of equations.

2. Postulate

Axioms related to geometry are called postulates.

Some postulates of geometry are given below.

(i) Infinite number of lines can pass through a point.



(ii) Through two different points one and only one line can pass.

(iii) If two points of a line lie in a plane, then whole line lies in that plane.

3. Conjecture

A statement that is believed to be true but its truth has not been proved is called a conjecture. In other words, it is a true statement that needs proof.

For example, observe the following pattern of numbers:

4, 8, 12, 16, ___.

If we are asked, 'what is next number in the pattern?'. We observe that each next number is 4 more than previous one. So, the answer is 20.

$\therefore$ One of the conjectures is that, **'the next number is $16 + 4 = 20$'**.

Another conjecture could be **'the next number is $16 + 5 - 1 = 20$'**.

None of the two has a proof but both conjectures follow simple mathematical rules and axioms.

Key Fact

Conjectures play a very important role in problem-solving in all branches of mathematics including Geometry, where the solution is not always apparent and is generated by following a series of steps. Each of these steps is a 'conjecture' over the previous step'.

Proof

A proof is a series of conjectures and axioms (postulates) and proved theorems that combine together to give a true result.

No assumptions can be made in a mathematical proof. Every step must be proved in the logical sequence. Mathematical proofs use deductive reasoning where a conclusion is drawn from multiple premises. The premises in the proof are called statements.

Theorem

The word theorem is derived from Greek word **'theorien'** which means "behold, contemplate or consider."

A mathematical statement which can be proved or supposed provable through logical reasons is called a theorem.

Important Steps of Proof of a Theorem

i. Statement

A description of a geometrical theorem in words should be written first. It is called statement of the theorem.

ii. Figure

After writing the statement a neat figure should be drawn to explain or understand the given information and the result to be proved.

iii. Given

In this step only given information should be written symbolically so that it becomes easy to use in the proof.

iv. Required to Prove

After given information it is necessary to write the result symbolically which is to be proved so that our attention does not divert from the main objective at any stage.

v. Construction

The necessary addition in the figure can be done to prove the theorem easily. This addition in the figure is called construction.

vi. Proof

It is the most important step. It consists of statements and facts along with their reasons through which we obtain the required results.

Corollary

Some results which can be deduced directly from theorems are called corollaries.

Converse of a Theorem

If given and to prove of a theorem are interchanged, the new statement is called converse of a theorem. It is not necessary that converse of a theorem is also a theorem.

EXERCISE 9.1

1. What is the difference between axiom and conjecture?
2. Which of the following are mathematical statements?
   - (i) Difference of 19 and 12 is 7. (ii) $-2+7-3=2$
   - (iii) $34+16 \neq 50$ (iv) $a+b=9$
   - (v) $(a+b)^2=a^2+2ab+b^2$ (vi) $2+2\times 2=6$
   - (vii) The product of x and y is smaller than 5.
   - (viii) If x is real then either $x<0$ or $x>0$ or $x=0$.
   - (ix) If $a>b$ and $b>c$ then $a<c$
   - (x) $xy+z=12$ (xi) $s-t=4$ if $s=4$ and $t=0$
3. The sum of a and b is equal to 0.
   Is this sentence a mathematical statement? If no! How can we make it a mathematical statement?
4. Prove $(x+1)^2+5=x^2+2x+6$ by taking $x=2$, 5 and 10.
5. Find the next number in the pattern using conjecture.
   1, 3, 7, 15, 31, ____
   State the conjecture used.
6. Which of the following are axioms? How many of the axioms are postulates?
   - (i) If $a=b$ then $b=a$ (ii) 2 plus 2 make 4.
   - (iii) One and only one line can pass through two points.
   - (iv) If two sides of a triangle are equal then opposite angles are also equal.

(v)	Product of two negative real numbers is always greater than zero.
(vi)	All right angles are equal to one another.
(vii)	The whole is greater than its part.
(viii)	If $a > b$ and $c > d$ then $a + c > b + d$
(ix)	It is possible to extend a line segment continuously in both directions.
(x)	When we add three consecutive even numbers, their sum is even.

7. Explain all the steps of geometrical proof.

Similarity of Polygons

Similar Figures

Two or more figures that have the same shape but not the same size are called similar figures. Figures (i) and (ii) below represent the pairs of similar figures.

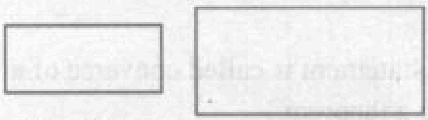


Fig. (i)

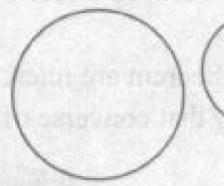


Fig. (ii)

The symbol for similarity is '~'. Thus, if two figures A and B are similar, then we write A ~ B.

Example 3: Are the following pairs of figures similar? Explain.

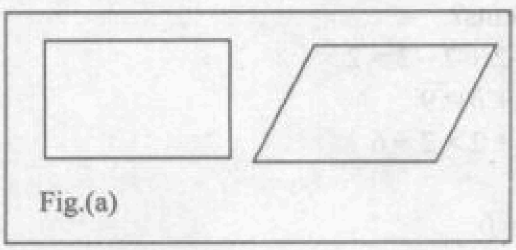


Fig.(a)

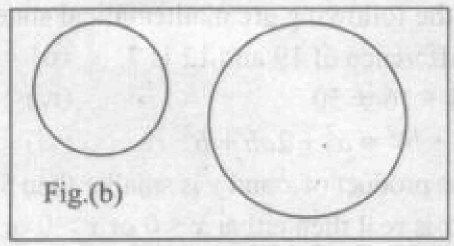


Fig.(b)

Solution:

In Fig.(a), two quadrilaterals do not have the same shape. Hence, they are not similar.

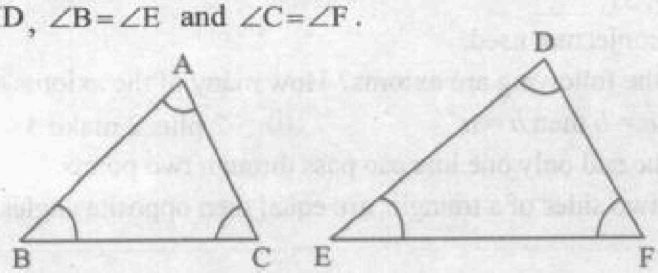
In Fig.(b), two circles have the same shape but not the same size, therefore they are similar.

Class Activity 1:

Look at the following triangles ABC and DEF. They have the same shape but not the same size. Therefore ΔABC and ΔDEF are similar.

Now measure corresponding angles of both the triangles.

You will see that: $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.



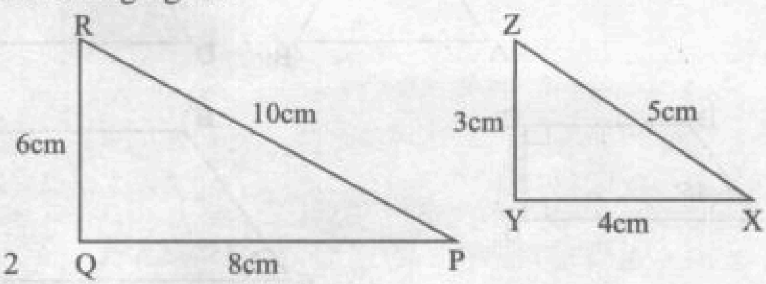
> If two or more figures are similar then their corresponding angles are congruent and vice versa.

Key Fact

(i) If triangles ABC and DEF are similar then, we write $\Delta ABC \sim \Delta DEF$ and read it as "ΔABC is similar to ΔDEF".

(ii) In the above figure if $\angle A = \angle D$, $\angle B = \angle E$ then $\angle C = \angle F$ as the sum of measures of angles of triangle is 180°.

Class Activity 2: Draw the following figures.



In ΔPQR and ΔYYZ

$$\frac{PQ}{XY} = 2, \frac{QR}{YZ} = 2, \frac{PR}{XZ} = 2$$

$$\therefore \frac{PQ}{XY} = \frac{QR}{YZ} = \frac{PR}{XZ}$$

i.e. corresponding sides are proportional.

Hence $\Delta PQR \sim \Delta XYZ$

> If two or more figures are similar then ratios of corresponding sides are equal and vice versa.

Class Activity 3:

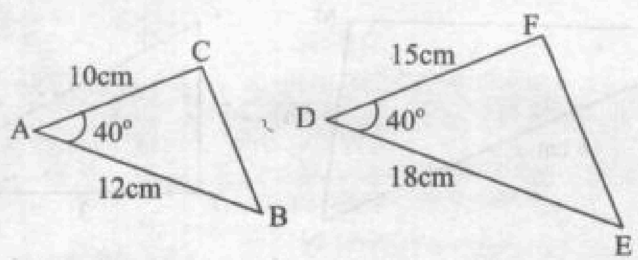
Look at the following figures.

In ΔABC and ΔDEF

$$\angle A = \angle D = 40^\circ$$

And $\frac{AC}{DF} = \frac{10}{15} = \frac{2}{3}$

Also $\frac{AB}{DE} = \frac{12}{18} = \frac{2}{3}$



As $\angle A \cong \angle D$ and $\frac{AC}{DF} = \frac{AB}{DE} = \frac{2}{3}$

$\therefore \Delta ABC \sim \Delta DEF$

> If in a correspondence of two triangles ratios of two pairs of corresponding sides and one pair of corresponding angles are equal then triangles are similar.

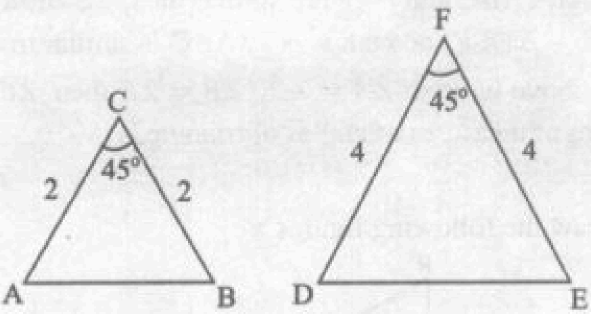
Example 4:

Check whether the following figures are similar.

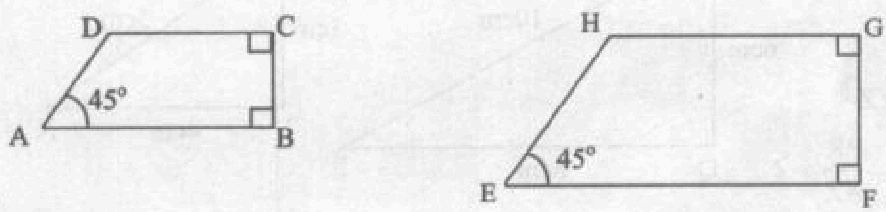
All measurements are in centimetres.

> **Check Point**
>
> In above figure, measure ($\angle C$, $\angle F$), ($\angle F$, $\angle B$) and $\frac{BC}{EF}$. What do you notice?

(i)



(ii)



Solution:

(i) $\frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{2}$ and $\angle C \cong \angle F$

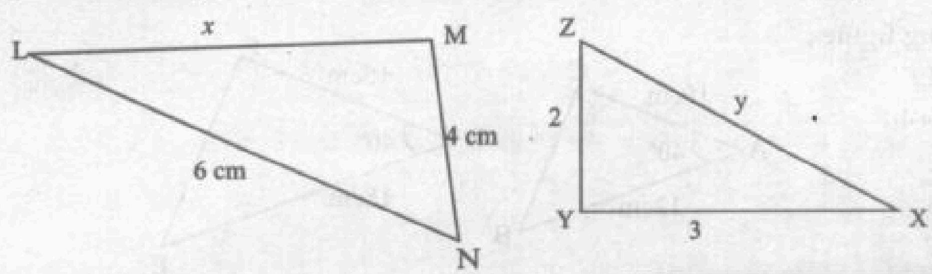
So $\Delta ABC \sim \Delta DEF$

(ii) In quadrilaterals ABCD and EFGH

$\angle A = \angle E$, $\angle B = \angle F$, $\angle C = \angle G$ and thus $\angle D = \angle H$

Therefore, both quadrilaterals are similar.

Example 5: In the figure below $\Delta LMN \sim \Delta XYZ$. Find values of x and y.



Solutions: Since the two triangles are similar.

$$\frac{x}{3} = \frac{6}{y} = \frac{4}{2}$$

$$\frac{x}{3} = \frac{4}{2} \quad \text{and} \quad \frac{6}{y} = \frac{4}{2}$$

$$x = 2 \times 3 \quad \text{and} \quad \frac{6}{y} = 2$$

> **Key Fact**
>
> Two triangles are similar if two sides of one triangle are proportional to the corresponding two sides of the second triangle and angles between the proportional sides are congruent.

$x = 6$ and $6 = 2y$

$\therefore$ $x = 6$ and $y = 3$

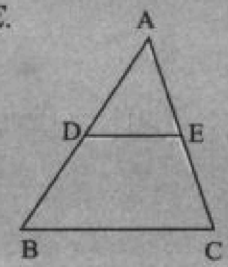
Key Fact

(i) In ΔABC, if $\overline{DE} \,//\, \overline{BC}$, then $\angle B = \angle D$ and $\angle C = \angle E$.

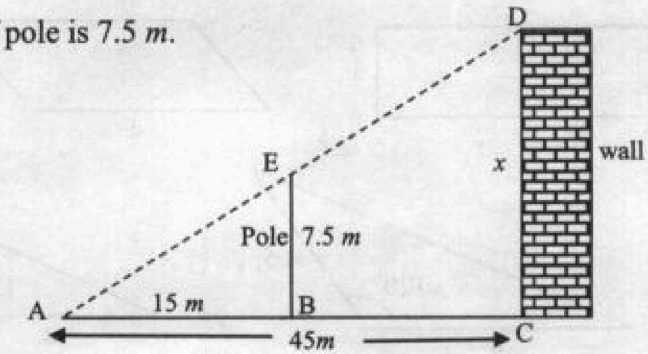
(ii) Corresponding angles in ΔABC and ΔADE are equal. So, the triangles are similar.

(iii) If ΔABC is isosceles or equilateral so is ΔADE

(iv) Also $\frac{AD}{DB} = \frac{AE}{EC}$ and $\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$



Example 6: In the figure height of pole is 7.5 *m*. Find the height of wall.



Solution:

Let height of wall = $CD = x$

Join A to E and D.

Here $\overline{BE} \parallel \overline{CD}$ in the ΔACD

$$\therefore \frac{AB}{AC} = \frac{BE}{CD}$$

$$\frac{15}{45} = \frac{7.5}{x}$$

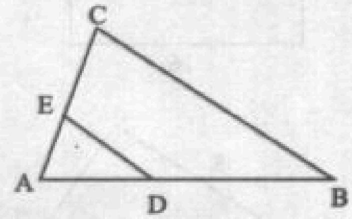
$$15x = 45 \times 7.5$$

$$x = \frac{45 \times 7.5}{15} = 22.5$$

So, height of wall = 22.5*m*

Example 7:

In ΔABC, $AB = 10\ cm$, $AC = 5\ cm$. Take D and E on $\overline{AB}$ and $\overline{AC}$ respectively such that $AD = 4\ cm$ and $AE = 2\ cm$. Is $\overline{DE} \,//\, \overline{BC}$?



Solution: As $\frac{AD}{DB} = \frac{4}{6} = \frac{2}{3}$ [As $BD = AB - AD$] .... (i)

And $\dfrac{AE}{EC} = \dfrac{2}{3}$ [As $EC = AC - AE$] .....(ii)

From (i) and (ii)

$$\frac{AD}{DB} = \frac{AE}{EC},$$

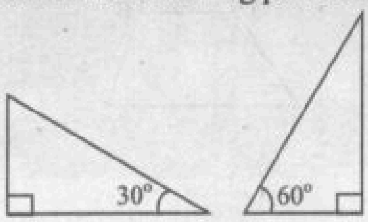
which shows that DE intersects AB and AC in the same ratio.

$\therefore \quad \overline{DE} \parallel \overline{BC}$

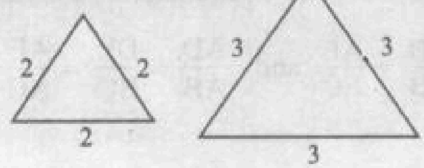
EXERCISE 9.2

1. Which of the following pairs of figures are similar?

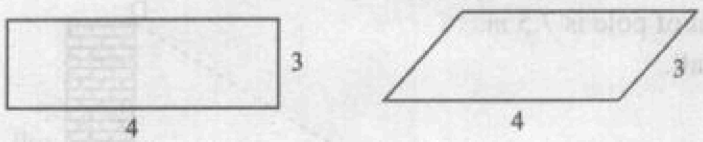
(i)



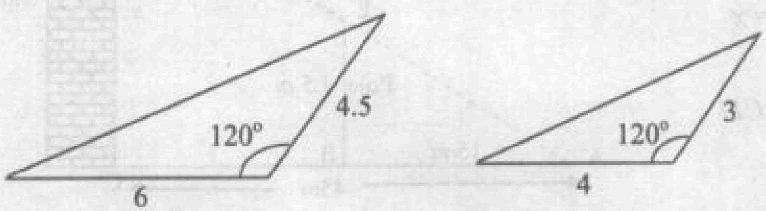
(ii)



(iii)



(iv)



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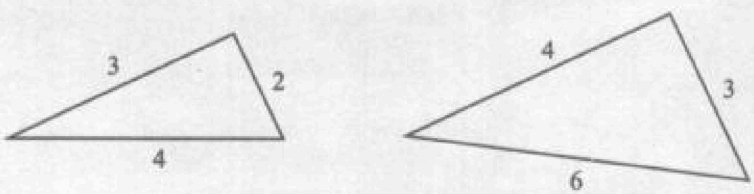

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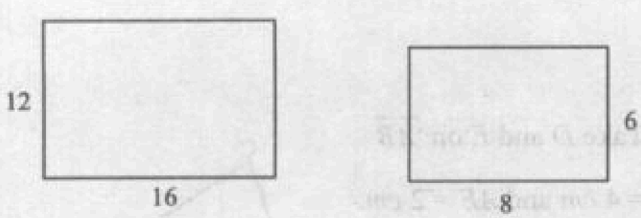

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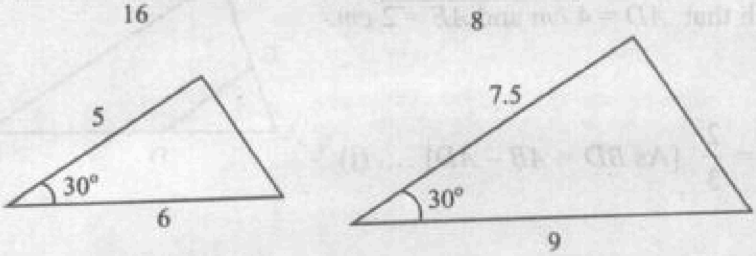
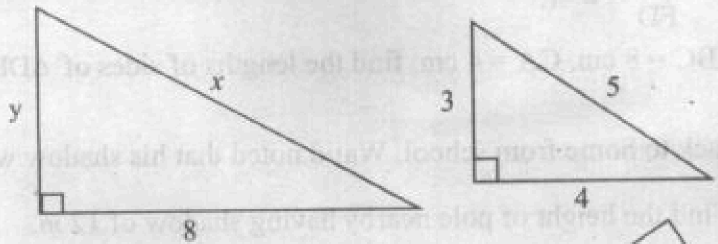


2. **Find the unknown quantities in the following similar figures. All measurements are in centimetres.**

(i)



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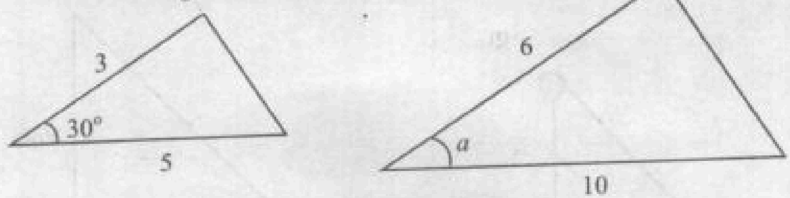
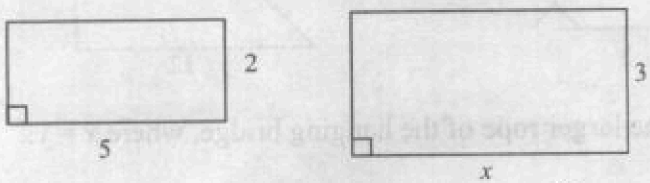
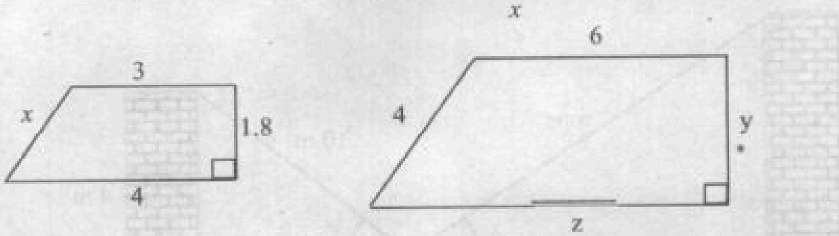


(iii)



(iv)



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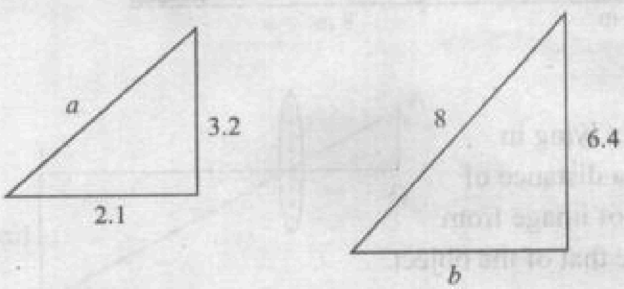

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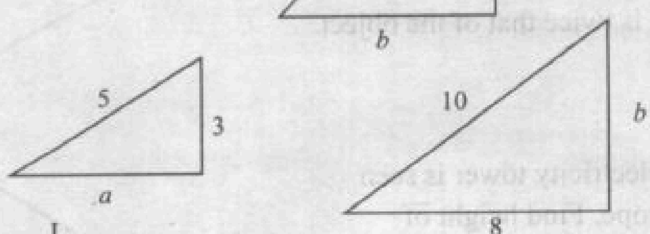

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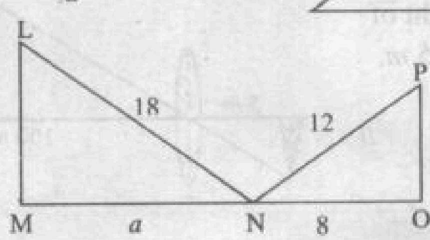
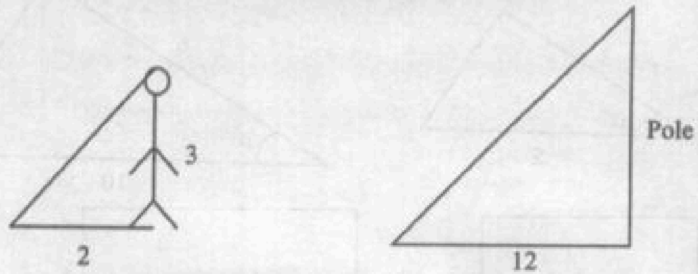


3. Two Triangles ABC and DEF are similar.

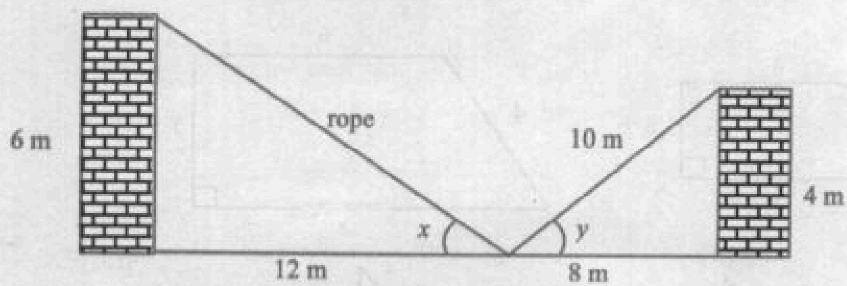
and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = 2$

If AB = 6 cm, BC = 8 cm, CA = 4 cm, find the lengths of sides of ΔDEF.

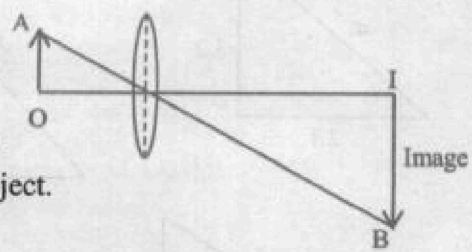
4. While going back to home from school, Wajid noted that his shadow was $\frac{2}{3}$ of his height. Find the height of pole nearby having shadow of 12 *m*.



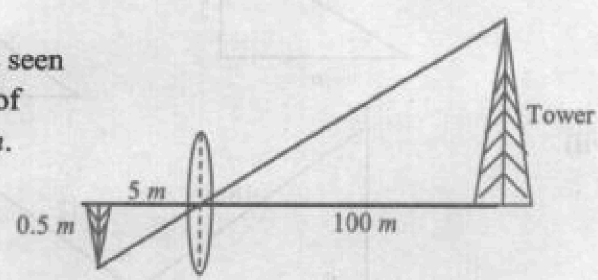
5. Find the length of the larger rope of the hanging bridge, where $x = y$.



6. In the figure, OA is object lying in front of a convex lens at a distance of 10 *cm*. Find the distance of image from the lens if its size is twice that of the object.

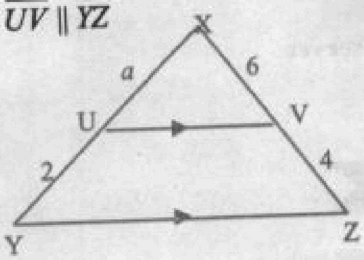


7. In the figure, an electricity tower is seen through the telescope. Find height of tower if height of its image is 0.5 *m*.

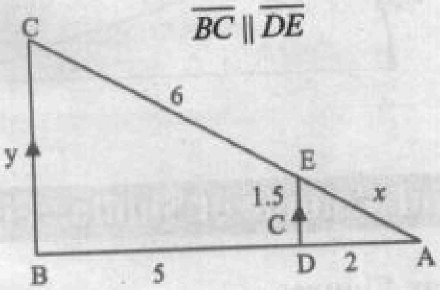


8. Find the values of unknown quantities in the following figures. All the measurements are in centimeters.

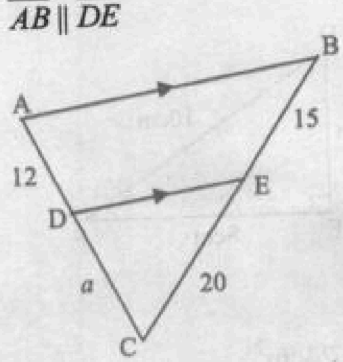
(a) $\overline{UV} \parallel \overline{YZ}$



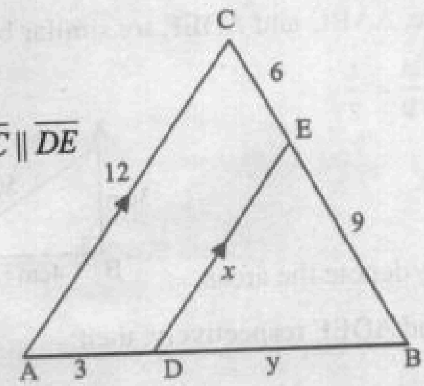
(b) $\overline{BC} \parallel \overline{DE}$



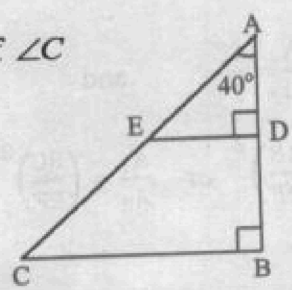
(c) $\overline{AB} \parallel \overline{DE}$



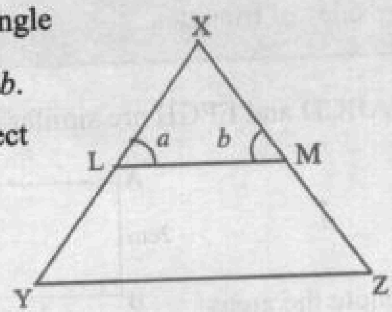
(d) $\overline{AC} \parallel \overline{DE}$



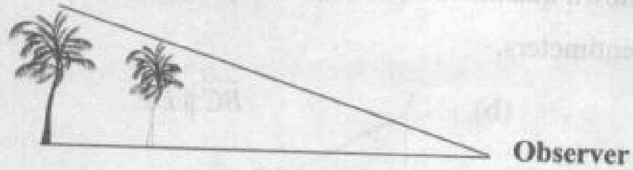
9. In the following figure, find the measure of $\angle C$ and $\angle AED$. Is $\overline{ED} \parallel \overline{CB}$?



10. In the figure, ΔXYZ is an equilateral triangle and $\overline{LM} \parallel \overline{YZ}$. Find measure of a and b. What type of triangle is XLM with respect to sides and angles?



11. Find the height of shorter tree if longer one is 12 m high from the ground level, where observer is at a distance of 25 m from the shorter tree and distance between both trees is 5 m.

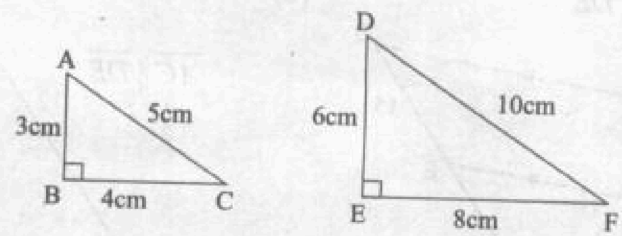


Area and Volume of Similar Figures

Area of Similar Figures

1. In the figure, ΔABC and ΔDEF are similar because:

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}=\frac{1}{2}$$



If A_1 and A_2 denote the areas of ΔABC and ΔDEF respectively, then:

$$A_1=\frac{1}{2}\times 4\times 3=6\text{cm}^2 \quad \text{and} \quad A_2=\frac{1}{2}\times 8\times 6=24\text{cm}^2$$

Now $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}=\frac{1}{2}$ and $\frac{A_1}{A_2}=\frac{6}{24}=\frac{1}{4}=\left(\frac{1}{2}\right)^2$

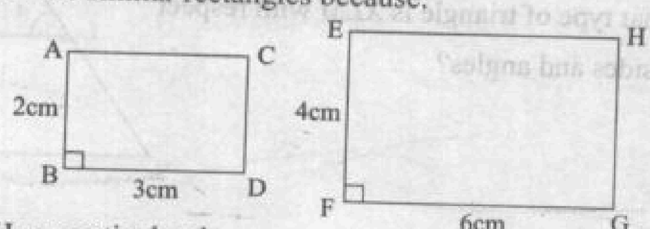
Therefore: $\frac{A_1}{A_2}=\left(\frac{AB}{DE}\right)^2$ or $\frac{A_1}{A_2}=\left(\frac{BC}{EF}\right)^2$ or $\frac{A_1}{A_2}=\left(\frac{CA}{FD}\right)^2$

We notice that:

> Ratio of areas of two triangles is equal to the square of the ratios of any two corresponding sides of triangles.

2. In the figure, ABCD and EFGH are similar rectangles because:

$$\frac{AB}{EF}=\frac{BD}{FG}=\frac{1}{2}$$



If A_1 and A_2 denote the areas of rectangles ABCD and EFGH respectively, then:

$A_1 = 2 \times 3 = 6\text{cm}^2$ and $A_2 = 4 \times 6 = 24\text{cm}^2$

Now $\frac{AB}{EF} = \frac{BD}{FG} = \frac{1}{2}$ and $\frac{A_1}{A_2} = \frac{6}{24} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$

Therefore: $\frac{A_1}{A_2} = \left(\frac{AB}{EF}\right)^2$ or $\frac{A_1}{A_2} = \left(\frac{BD}{FG}\right)^2$

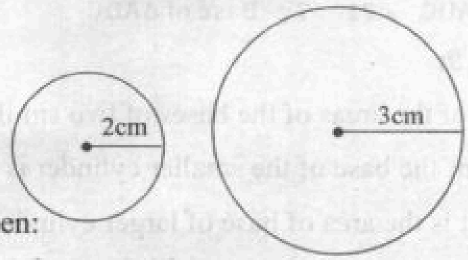
The same can be proved for other types of quadrilaterals. Thus:

Ratio of areas of two quadrilaterals is equal to the square of the ratios of any two corresponding sides of quadrilaterals.

3. Figure shows two similar circles with radii 2cm and 3cm respectively.

If $r_1 = 2\text{cm}$ and $r_2 = 3\text{cm}$, then:

$\frac{r_1}{r_2} = \frac{2}{3}$



If A_1 and A_2 denote the areas of circles with radii r_1 and r_2 respectively, then:

$A_1 = \pi \times 2^2 = 4\pi\ \text{cm}^2$ and $A_2 = \pi \times 3^2 = 9\pi\ \text{cm}^2$

Now $\frac{r_1}{r_2} = \frac{2}{3}$ and $\frac{A_1}{A_2} = \frac{4\pi}{9\pi} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$

Therefore: $\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$

Thus:

Ratio of areas of two circles is equal to the square of the ratios of radii of both circles.

Key Fact

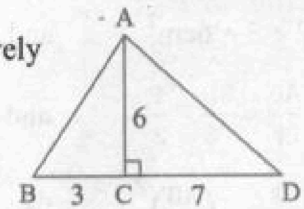
The ratio of areas of two similar figures is equal to the square of ratios of any two corresponding lengths of the figures.

If A_1 and A_2 denote the areas of two similar figures and, *a* and *b* denote the corresponding lengths of the figures, then:

$$\frac{A_1}{A_2} = \left(\frac{a}{b}\right)^2 = k^2$$

Example 8:

Triangles ABC and ADC with bases 3cm and 7cm respectively have common height 6 cm.
Prove that the ratio of their areas is equal to ratio of bases of both triangles.



Solution:

Area of $\Delta ABC = \frac{1}{2} \times 3 \times 6 = 9\text{cm}^2$

Area of $\Delta ADC = \frac{1}{2} \times 7 \times 6 = 21\text{cm}^2$

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADC} = \frac{9}{21} = \frac{3}{7} = \frac{\text{Base of } \Delta ABC}{\text{Base of } \Delta ADC}$$

Key Fact

Ratio of areas of two triangles with common height is equal to the ratio of bases of the two triangles.

$$\frac{A_1}{A_2} = \frac{b_1}{b_2} = k$$

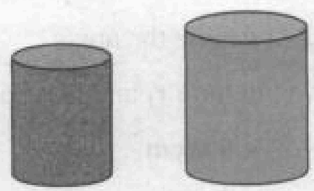
Example 9:

The ratio of the areas of the bases of two similar cylinders is 9 : 4.

The area of the base of the smaller cylinder is 240cm^2.

(i) What is the area of base of larger cylinder?

(ii) Write the ratio of heights of both cylinders.



Solution:

(i) If A_1 and A_2 are areas of bases of larger and smaller cylinders respectively, then:

$$\frac{A_1}{A_2} = \frac{9}{4} \quad \Rightarrow \quad \frac{A_1}{240} = \frac{9}{4}$$

$$\Rightarrow \quad A_1 = \frac{9}{4} \times 240 = 540\text{cm}^2$$

(ii) Again, if h_1 and h_2 are heights of larger and smaller cylinders respectively, then:

$$\left(\frac{h_1}{h_2}\right)^2 = \frac{A_1}{A_2} \quad \Rightarrow \quad \left(\frac{h_1}{h_2}\right)^2 = \frac{540}{240} = \frac{9}{4}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{3}{2}$$

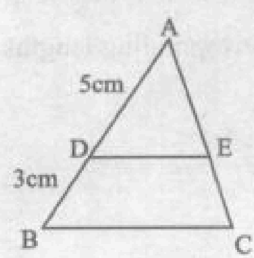
Example 10:

In the figure, $\overline{BC} \parallel \overline{DE}$. Find:

(i) $\frac{DE}{BC}$ (ii) $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC}$

(iii) find the area of ΔADE if area of ΔABC is 256cm^2.

(iv) area of trapezium DBCE.



Solution:

(i) $\frac{DE}{AB} = \frac{AD}{AB} = \frac{5}{8}$

(ii) $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$

(iii) $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{25}{64} \Rightarrow \text{Area of } \Delta ADE = \frac{25}{64} \times \text{Area of } \Delta ABC$

$\Rightarrow \text{Area of } \Delta ADE = \frac{25}{64} \times 256 \text{ cm}^2 = 100 \text{ cm}^2$

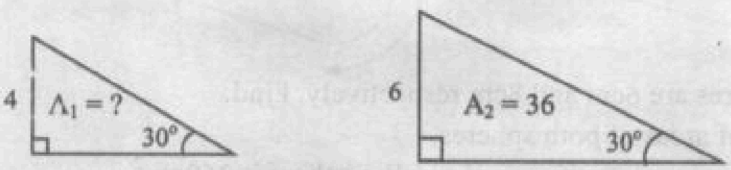
(iv) Area of trapezium DBCE = Area of ΔABC – Area of ΔABC

$= 256 - 100 = 156 \text{ cm}^2$

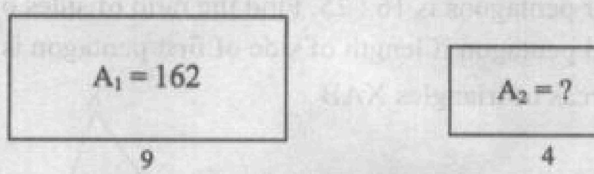
EXERCISE 9.3

1. Following pairs of shapes are similar. Find unknown area in each case. All measurements are in cm and cm^2.

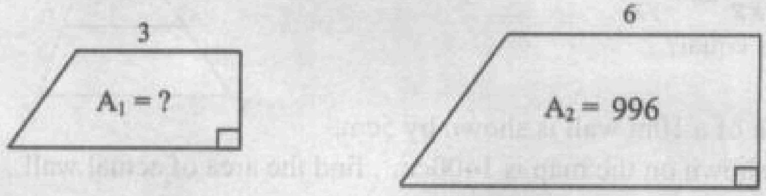
(i)



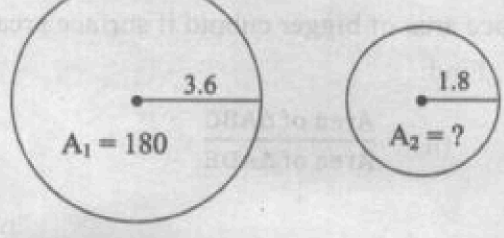
(ii)



(iii)



(iv)

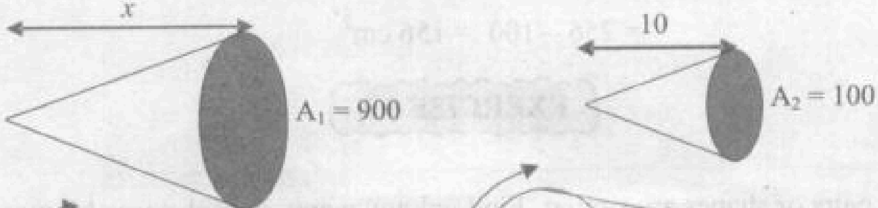


2. Following pairs of shapes are similar. Find unknown length x in each case. All measurements are in cm and cm^2.

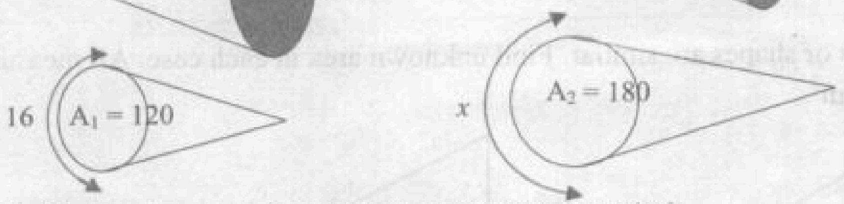
(i)

(ii)

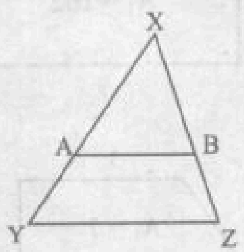
(iii)



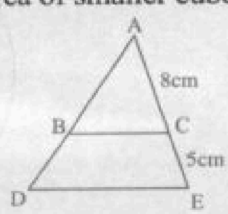
(iv)



3. Radii of two spheres are 6cm and 8cm respectively. Find:
   (i) the ratio of areas of both spheres.
   (ii) area of larger sphere if area of smaller sphere is 360cm^2.
   (iii) area of smaller sphere if area of larger sphere is 1600cm^2.
4. Ratio of areas of two regular pentagons is 16 : 25. Find the ratio of sides of pentagons. Also find length of side of second pentagon if length of side of first pentagon is 8cm.
5. In the figure, $\overline{AB} \parallel \overline{YZ}$. If areas of triangles XAB and XYZ are in the ratio 25 : 36, find $\frac{XB}{XZ}$ and $\frac{AB}{YZ}$.
   Are the ratios equal?



6. In a map, length of a 10m wall is shown by 5cm.
   If area of wall shown on the map is 1400cm^2, find the area of actual wall.
7. Two cuboids are similar. Height of smaller cuboid is one-third of bigger one.
   (i) Find the ratio of surface area of larger cuboid to that of smaller one.
   (ii) Find the surface area of bigger cuboid if surface area of smaller cuboid is 350cm^2.
8. In the figure, $\overline{BC} \parallel \overline{DE}$. Find:
   (i) $\frac{BC}{DE}$ and $\frac{AB}{AD}$ (ii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE}$



(iii) the area of ΔABC if area of ΔADE is $507 cm^2$.

(iv) area of quadrilateral BDEC.

What type of the quadrilateral is?

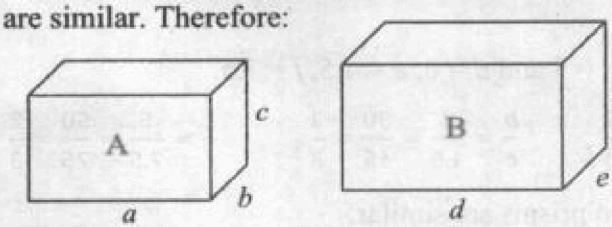
Volume of Similar Solids

Two solids are said to be similar if they have same shape. The ratio of corresponding lengths (sides etc.) of two similar solids is constant called the scale factor of the solids.

1. In the figure two cuboids are similar. Therefore:

$$\frac{a}{d}=\frac{b}{e}=\frac{c}{f}=k$$

$$\Rightarrow a=dk, b=ek, c=fk$$



Now if V_1 is volume of cuboid A and V_2 is the volume of cuboid B, then

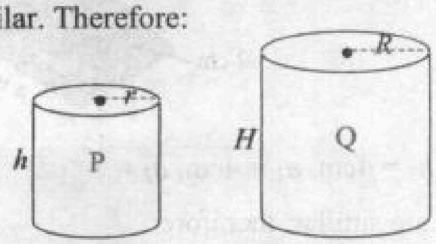
$$\frac{V_1}{V_2}=\frac{abc}{def}=\frac{dkekfk}{def}=k^3 \quad \Rightarrow \quad V_1=k^3 V_2$$

If two cuboids are similar then volume of one cuboid is k^3 times the volume of the other cuboid.

2. In the figure two cylinders P and Q are similar. Therefore:

$$\frac{r}{R}=\frac{h}{H}=k$$

$$\Rightarrow \quad r=Rk \quad \text{and} \quad h=Hk$$



Now if V_1 is volume of cylinder P and V_2 is the volume of cylinder Q, then

$$\frac{V_1}{V_2}=\frac{\pi r^2 h}{\pi R^2 H}=\frac{r^2 h}{R^2 H}=\frac{R^2k^2Hk}{R^2H}=k^3 \quad \Rightarrow \quad V_1=k^3 V_2$$

If two cylinders are similar then volume of one cylinder is k^3 times the volume of the other cylinder.

Key Fact

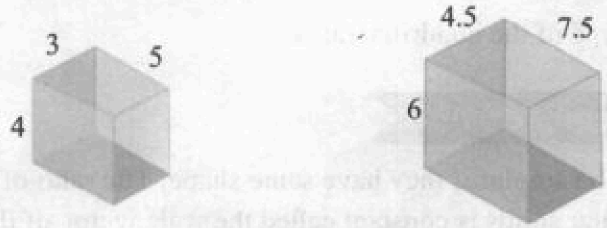
If l_1 and l_2 are any two corresponding lengths of two similar figures then the ratio of corresponding volumes is:

$$\frac{V_1}{V_2}=k^3=\left(\frac{l_1}{l_2}\right)^3 \quad \text{where } k=\frac{l_1}{l_2}$$

If both solids are made from the same material or have the same density, the ratio of their masses is given by:

$$\frac{m_1}{m_2}=k^3=\left(\frac{l_1}{l_2}\right)^3$$

Example 11: Check whether the prisms are similar or not?



Solution:

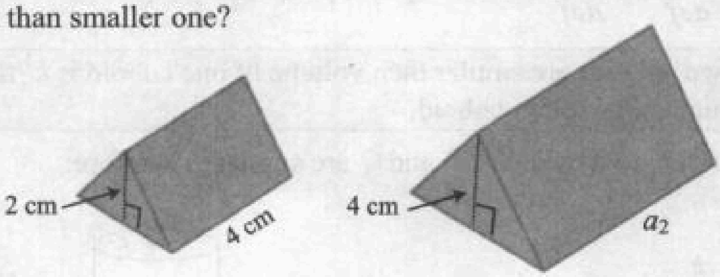
Let $a = 4$, $b = 3$, $c = 5$ and $d = 6$, $e = 4.5$, $f = 7.5$.

Then: $\frac{a}{d} = \frac{4}{6} = \frac{2}{3}$, $\quad \frac{b}{e} = \frac{3}{4.5} = \frac{30}{45} = \frac{2}{3}$, $\quad \frac{c}{f} = \frac{5}{7.5} = \frac{50}{75} = \frac{2}{3}$

Therefore, the two prisms are similar.

Example 12:

Find a_2 it the following solids are similar. Also find the ratio of volumes of both. How many times is the volume of larger solid than smaller one?



Solution:

Let $h_1 = 2\text{cm}$, $h_2 = 4\text{cm}$, $a_1 = 4\text{cm}$, $a_2 = ?$

As the figures are similar, therefore:

$$\frac{h_1}{h_2} = \frac{a_1}{a_2} \Rightarrow a_2 = \frac{a_1 h_2}{h_1} = \frac{4 \times 4}{2} = 8\text{cm}$$

Now, $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3 = \left(\frac{2}{4}\right)^3 = \frac{1}{8}$

$\Rightarrow \quad V_2 = 8V_1$

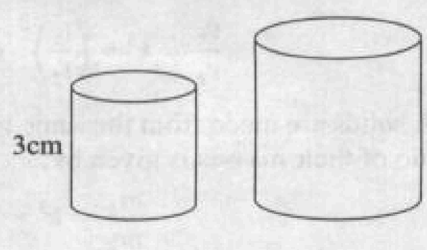
Hence, volume of larger solid is 8 times the volume of smaller one.

> **Check Point**
>
> If two solids are similar, what is the ratio of their surface areas, and what is the ratio of their volumes?

Example 13:

In the figure two geometrically similar cylinders are shown. Find:

(i) the ratio of volume of smaller cylinder to larger cylinder.



(ii) curved surface area of smaller cylinder if that of larger one is 250cm^2.

(iii) volume of larger cylinder if volume of smaller one is 162cm^3.

Solution:

(i) If V_1 and V_2 are volumes of smaller and larger cylinders respectively, then:

$$\frac{V_1}{V_2} = \left(\frac{3}{5}\right)^3 \quad \Rightarrow \quad \frac{V_1}{V_2} = \frac{27}{125}$$

(ii) If A_1 and A_2 are areas of smaller and larger cylinders respectively, then:

$$\frac{A_1}{A_2} = \left(\frac{3}{5}\right)^2 \Rightarrow \frac{A_1}{250} = \frac{9}{25}$$

$$\Rightarrow A_1 = \frac{9}{25} \times 250 = 90\text{cm}^2$$

(iii) $$\frac{162}{V_2} = \left(\frac{3}{5}\right)^3 \quad \Rightarrow \quad \frac{162}{V_2} = \frac{27}{125}$$

$$\Rightarrow 27 \times V_2 = 162 \times 125 \quad \Rightarrow V_2 = \frac{162 \times 125}{27} = 750\text{cm}^3$$

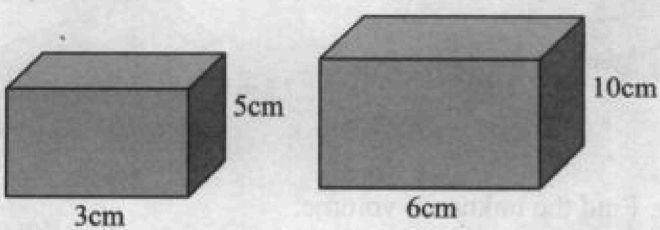
Key Fact

The ratio of volumes of two similar solids is equal to the cube of the ratio of any two corresponding lengths of the two solids.

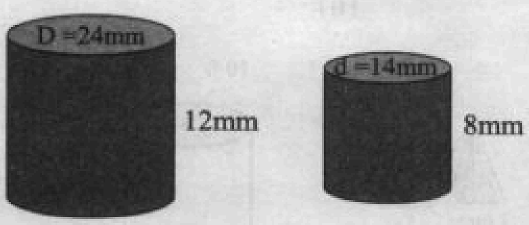
EXERCISE 9.4

1. Determine whether the solids are similar or not.

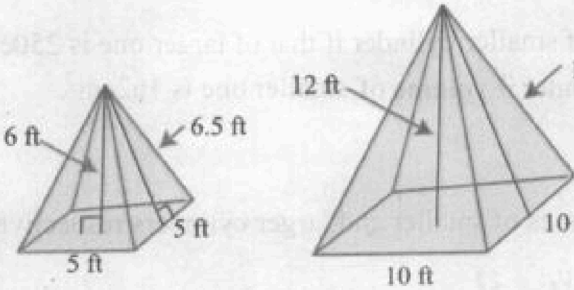
(i)



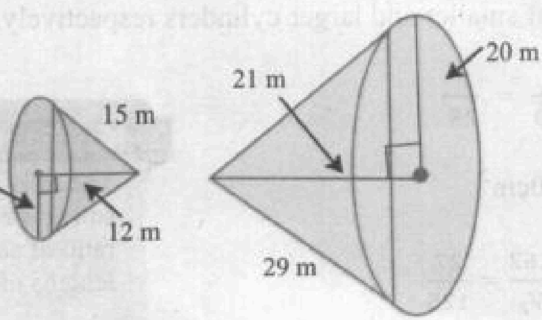
(ii)



(iii)

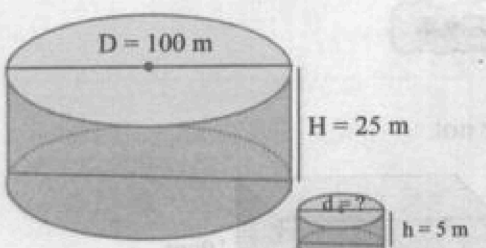


(iv)

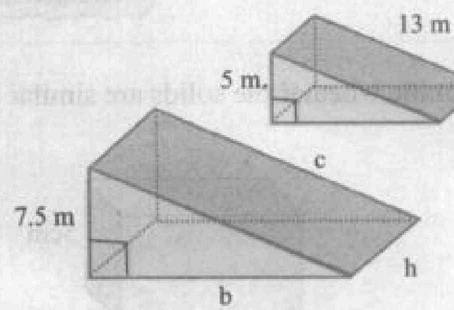


2. **Solids are similar. Find the values of unknowns. Also find the ratios of volume of solids.**

(i)

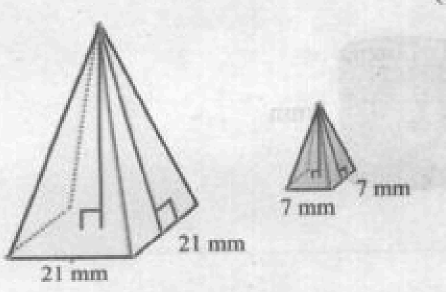


(ii)



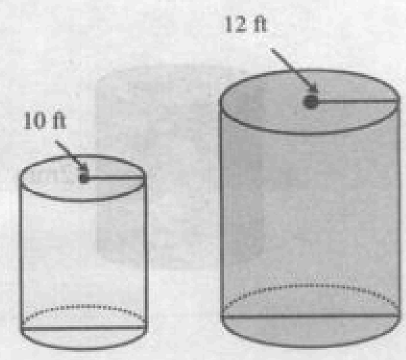
3. **Solids are similar. Find the unknown volume.**

(i)



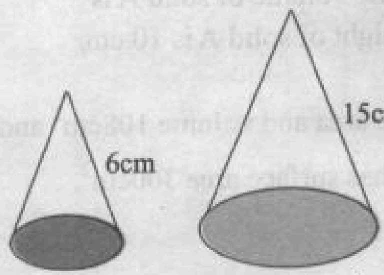
$V_1 = 9000\text{mm}^3$ $\quad V_2 = ?$

(ii)



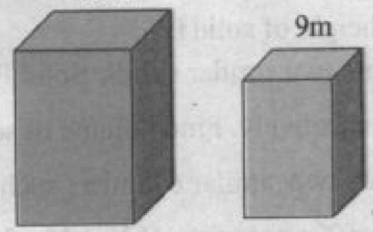
$V_1 = ?$ $\quad V_2 = 360\text{ft}^3$

(iii)



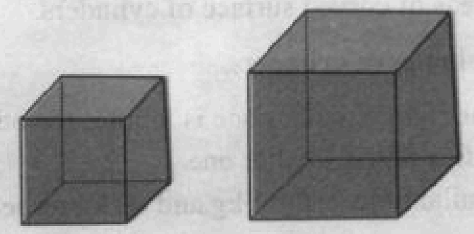
$V_1 = ?$ $V_2 = 460\text{cm}^3$

(iv)



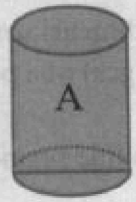
$V_1 = 288\text{m}^3$ $V_2 = ?$

4. Find the ratio of scale factors of the following pairs of similar solids.

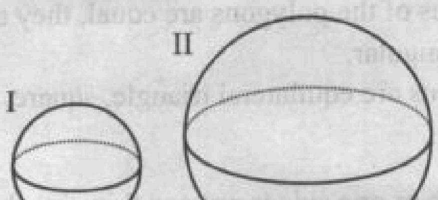


$V = 512\ \text{m}^3$ $V = 1728\ \text{m}^3$

5. Two swimming pools are similar with a scale factor of 4 : 5. The amount of chlorine mixture to be added is proportional to the volume of water in the pool.
   If three cups of chlorine mixture are needed for the smaller pool, how much of the chlorine mixture is needed for the larger pool?
6. A model bus is built with a scale of 1 : 10. The model bus has a volume of 30 m^3.
   What is the volume of the actual bus?
7. Solid A shown below is similar to solid B (not shown) with a scale factor of 2 : 3. Find the surface area and volume of solid B if surface area and volume of solid A are $130\pi\ \text{cm}^2$ and $280\pi\ \text{cm}^3$ respectively.



8. Solid I is similar to Solid II. Find the scale factor of solid I to solid II.



$V = 8\pi \text{ft}^3$ $V = 125\pi \text{ft}^3$

9. Solid A and solid B are mathematically similar. The volume of solid A is $32\ cm^3$. The volume of solid B is $108\ cm^3$. The height of solid A is 10 cm. Find the height of solid B.
10. P and Q are two similar solids. Solid P has surface area and volume $108 cm^2$ and $135 cm^3$ respectively. Find volume of solid Q if Q has surface area $300 cm^2$.
11. X and Y are two similar cylinders such that:
    base area of X : base area of Y = 16 : 25. Find:
    (i) ratio of heights of both cylinders.
    (ii) Ratio of areas of curved surface of cylinders.
    (iii) Ratio of volumes of cylinders.
12. The volume of one right circular cone is 8 times the other one. If the radius of larger cone is 12cm, find the radius of the smaller one.
13. Masses of two similar objects are 8 kg and 27 kg respectively. If the height of first object is 2m, what is the height of second object?

Properties of Regular Polygons

Polygon

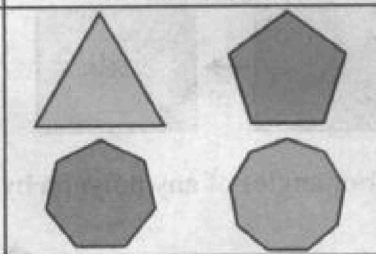
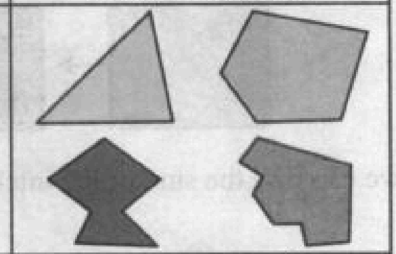
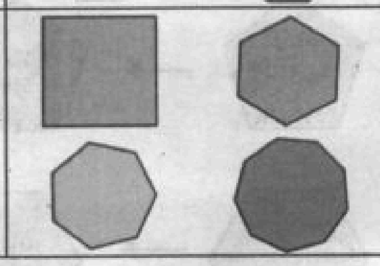
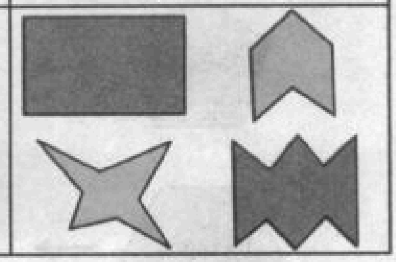
'Polygon' is the Greek word where 'poly' means 'many' and 'gon' means 'angles'.
A polygon is a two-dimensional convex figure that has a finite number of sides (at least three sides). The sides (edges) of a polygon are made of straight line segments connected end to end to form a closed shape.
The point where two line segments meet is called vertex of polygon.

Regular Polygon

If all the sides and interior angles of the polygons are equal, they are known as regular polygons. Regular polygons are also equiangular.
The examples of regular polygons are equilateral triangle, square, pentagon, hexagon etc.

Irregular Polygon

If in a polygon at least one angle or one side is not equal in size then the polygon is called irregular.

	Regular Polygons	Irregular Polygons
With odd number of sides		
With even number of sides		

Key Fact

- A polygon with 5 sides is called pentagon.
- A polygon with 6 sides is called hexagon.
- A polygon with 7 sides is called heptagon.
- A polygon with 8 sides is called octagon.
- A polygon with 9 sides is called nonagon.
- A polygon with 10 sides is called decagon.

Check Point

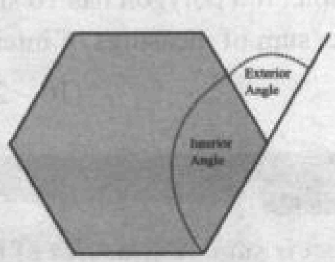
can you search for the name of 15-sided polygon?

Interior Angle of Polygon

Interior angles are the angles that are inside the polygon formed by two adjacent sides.

Exterior Angle of Polygon

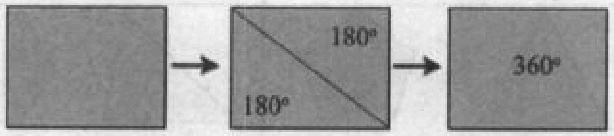
Exterior angle is the angle formed by any side of the polygon and the extension of its adjacent side.



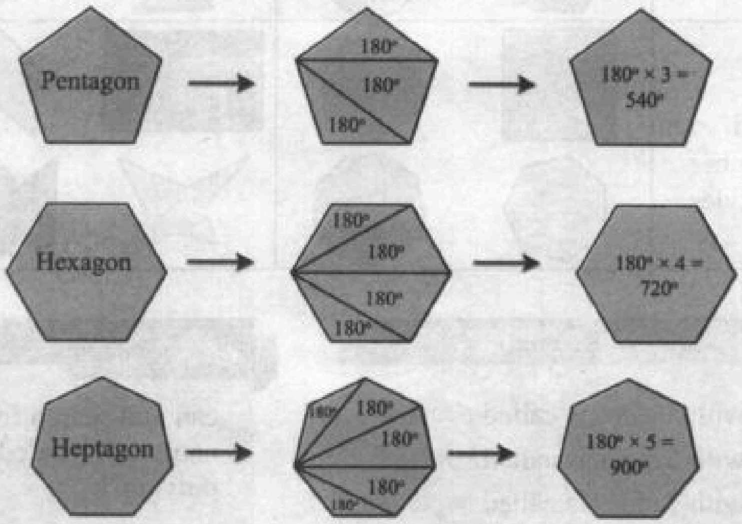
Sum of Measures of Interior Angles of a Polygon

(i) Sum of measures of interior angles in a triangle is 180°.

(ii) Sum of measures of interior angles in a quadrilateral is 360°.



In this way, we can find the sum of the interior angles of any polygon by splitting it into triangles.



From the above discussion, it can be concluded that:
The sum of measures of all the interior angles of a polygon having n sides is:

$$(n-2) \times 180°$$

For example, if a polygon has 10 sides, then $n = 10$.
Therefore, sum of measures of interior angles is:

$$(10-2) \times 180° = 1440°$$

Check Point

What is sum of measures of interior angles of polygons having?
(i) 9 sides (ii) 17 sides

Interior Angle of a Regular Polygon

The measure of each interior angle of n-sided regular polygon is given by the formula:

$$\frac{(n-2) \times 180°}{n}$$

Example 14:

Find interior angle of a regular nonagon.

Solution:

A nonagon has 9 sides, therefore $n = 9$

If θ is an interior angle of the nonagon then:

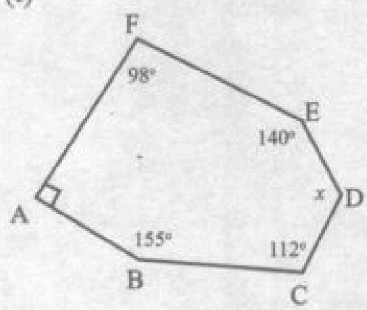
$$\theta = \frac{(9-2)\times 180^\circ}{9} = 140^\circ$$

> **Check Point**
>
> Find the measure of interior angle of a regular polygon having 16 sides.

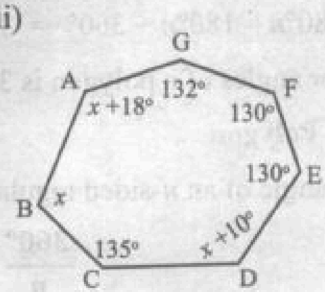
Example 15:

Find the value of x in the following polygons.

(i)



(ii)



Solution:

(i) The given polygon is hexagon and sum of measures of its interior angles is 720°.

$\therefore \quad 90^\circ + 155^\circ + 112^\circ + x + 140^\circ + 98^\circ = 720^\circ$

$\Rightarrow \quad x + 595 = 720$

$\Rightarrow \quad x = 720 - 595 = 125$

(ii) The given polygon is heptagon and sum of measures of its interior angles is 900°.

$\therefore \quad (x + 18^\circ) + x + 138^\circ + (x + 10^\circ) + 130^\circ + 130^\circ + 132^\circ = 900^\circ$

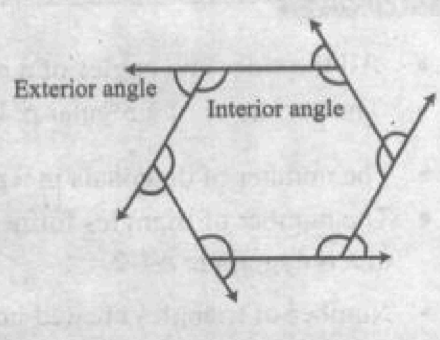
$\Rightarrow \quad 3x + 558^\circ = 900^\circ$

$\Rightarrow \quad 3x = 900^\circ - 558^\circ = 342^\circ$

$\Rightarrow \quad x = \frac{342^\circ}{3} = 114^\circ$

Sum of Measures of Exterior Angles of a Polygon

Figure shows a regular hexagon.



As interior angle of a regular hexagon is 120° and sum of measures of an interior and exterior angles is 180°, therefore:

Interior angle + Exterior angle = 180°

120° + Exterior angle = 180°

$\Rightarrow$ Exterior angle = $180^\circ - 120^\circ = 60^\circ$

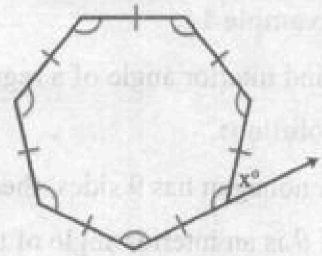
Now, hexagon has 6 sides, therefore:

Sum of measures of exterior angles of regular hexagon $= 6 \times 60^\circ$

$= 360^\circ$

For a regular polygon having n sides:

exterior angle + interior adjacent angle = 180°



Sum of all exterior angles + Sum of all interior angles

$$= n \times 180°$$

So, sum of all exterior angles

$$= n \times 180° - \text{Sum of all interior angles}$$

Sum of all exterior angles $= n \times 180° - (n-2) \times 180°$

$$= n \times 180° - n \times 180° + 2 \times 180°$$

$$= 180°n - 180°n + 360° = 360°$$

$\therefore$ Sum of measures of exterior angles of a polygon is 360°.

Exterior Angle of a Regular Polygon

The measure of each exterior angle of an n-sided regular polygon is:

$$\frac{360°}{n}$$

Example 16:

Exterior angle of a regular polygon is 120°. Identify the name of polygon.

Solution:

Measure of exterior angles = 360°

Measure of one exterior angle = 120°

Number of exterior angles $= \frac{360°}{120°} = 3$

Since the polygon has 3 exterior angles therefore, it has 3 sides.

Hence it is an equilateral triangle.

Key Fact

- All the sides and angles of a regular polygon are equal.
- The perimeter of a regular polygon with n sides is equal to the n times of a side measure.
- The number of diagonals in a polygon with n sides are $n(n-3)/2$.
- The number of triangles formed by joining the diagonals from one corner of a polygon are $n-2$.
- Number of triangles created inside a polygon is 2 less than number of sides of polygon.
- Interior and exterior angles add up to 180°.

Example 17:

In a certain polygon, the sum of the measures of all the interior angles is equal to twice that of the exterior angles. What is the name of that polygon?

Solution:

Given that

Sum of interior angles = 2 × the sum of exterior angles

If n is number of sides of polygon, then:

$$(n-2) \times 180^\circ = 2 \times 360^\circ$$

$$\Rightarrow\ n-2 = \frac{2 \times 360^\circ}{180^\circ} \ \Rightarrow\ n-2 = 4 \ \Rightarrow\ n = 6$$

Hence the polygon is hexagon.

Example 18:

The measure of the exterior angles of a polygon are $(x+4)^\circ$, $(3x-4)^\circ$, $(7x-3)^\circ$, $(2x+3)^\circ$, $(8x-1)^\circ$ and $(9x+1)^\circ$.

(i) Identify polygon. (ii) Find x. (iii) Find the measure of each angle.

Solution:

(i) Since there are 6 exterior angles in the polygon, therefore polygon is hexagon.

(ii) As the sum f measures of exterior angles in a polygon is 360°, therefore:

$$(x+4)^\circ + (3x-4)^\circ + (7x-3)^\circ + (2x+3)^\circ + (8x-1)^\circ + (9x+1)^\circ = 360^\circ$$

$$\Rightarrow\ x + 4^\circ + 3x - 4^\circ + 7x - 3^\circ + 2x + 3^\circ + 8x - 1^\circ + 9x + 1^\circ = 360^\circ$$

$$\Rightarrow\ 30x = 360^\circ \quad \Rightarrow\ x = 12^\circ$$

(iii) Substituting the value of x in above expressions of angles, we get:

$(12+4)^\circ$, $(3 \times 12 - 4)^\circ$, $(7 \times 12 - 3)^\circ$, $(2 \times 12 + 3)^\circ$, $(8 \times 12 - 1)^\circ$, $(9 \times 12 + 1)^\circ$

Or $16^\circ, 32^\circ, 81^\circ, 27^\circ, 95^\circ, 109^\circ$

EXERCISE 9.5

1. Find the number of sides of a regular polygon if each of its exterior angle is:

   (a) 45° (b) 60° (c) 120° (d) 40°

2. Draw a regular pentagon whose exterior angles are p, q, r, s, t and each interior angle is k. What is the measure of:

   (a) each interior angle (b) each exterior angle

3. Each interior angle of a polygon is five times the exterior angle of the polygon. Find the number of sides.
4. Find the minimum interior angles and maximum exterior angles possible in a regular polygon. Give reasons to support your answer.
5. Find the exterior angle of a regular polygon of:

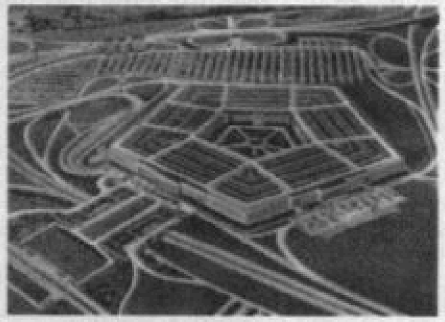
   (a) 5 sides (b) 9 sides (c) 15 sides (d) 20sides

6. The ratio between an exterior angle and the interior angle of a regular polygon is 1 : 2. Find:

   (a) the measure of each exterior angle.
   (b) the measure of each interior angle.
   (c) the number of sides in the polygon.

7. Is it possible to have a regular polygon each of whose exterior angle is 50°? Give reason to support your answer.
8. Name the polygon whose sum of interior angles is equal to the sum of its exterior angles.
9. The sum of all the interior angles of a regular polygon is four times the sum of its exterior angles. Identify the polygon.
10. An exterior angle of a regular polygon is 12°. What is the sum of all the interior angles?
11. Prove that each interior angle and its corresponding exterior angle in any polygon are supplementary.
12. Find the number of sides in a regular polygon when the measure of each exterior angle is 72°.
13. The exteriors angles of a pentagon are $(y + 5)°$, $(2y + 3)°$, $(3y + 2)°$, $(4y + 1)°$ and $(5y + 4)°$ respectively. Find the measure of each angle.
14. A convex polygon has 14 diagonals. Find the number of sides of the polygon.
15. Find the sum of all the interior angles of a polygon having 13 sides.
16. The sum of all the interior angles of a polygon is 2880°. How many sides does the polygon have?

Real Life Problems Involving Regular Polygons, Triangles and Parallelograms

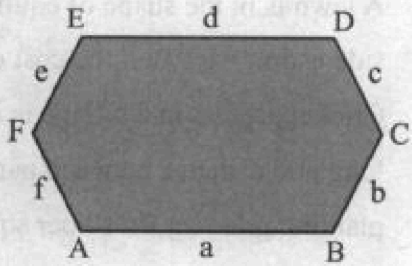


The variety of polygons are commonly used in the modern constructions. Because of its reasonably strong design, the triangle is commonly used in construction. The usage of the polygons minimizes the number of resources used to construct a structure like lowering costs and increasing profits in a corporate setting. The rectangle is another polygon which is used in a variety of applications. For example, most of televisions are rectangular to make watching easier and more enjoyable. Photo frames and phone screens are in the same boat.

Perimeter of Polygon

Perimeter of any polygon is sum of measures of all sides.

Perimeter of pentagon shown in adjoining figure, is:



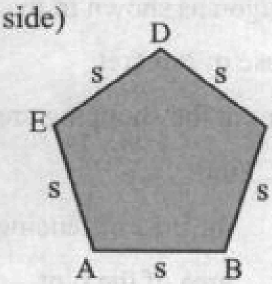
$$P = a + b + c + d + e + f$$

Perimeter of Regular Polygon

Perimeter of regular polygon = (number of sides) × (length of one side)

$$P = n \times s$$

Perimeter of a regular pentagon shown in adjoining figure, is:



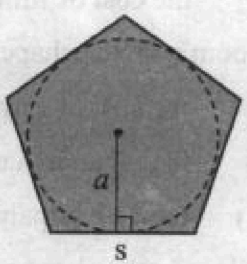
$$P = s + s + s + s + s = 5\,s$$

Area of Regular Polygon

Area of a regular polygon is defined as:

$$A = \frac{1}{2}(P)(a)$$

Where 'P' is perimeter of regular polygon and '*a*' is called apothem. Substituting the value of 'P', we have:



$$A = \frac{1}{2}(n\,s)(a) = \frac{n}{2} \times (sa)$$

Where 's' denotes the length of side.

$$\boxed{\therefore\ A = \frac{1}{2}(P)(a) = \frac{n}{2} \times (sa)}$$

Example 19:

Basement of a water tank is in the shape of regular pentagon having the side length 8 feet. Its apothem is 7 feet. Find the perimeter and area of basement of tank.

Solution:

Number of sides of basement = 5

Side length of basement = s = 8 ft

Apothem of basement = 7 ft

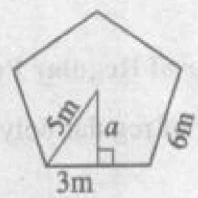
Perimeter of basement = 5 × 8 = 40 ft

Area of basement = $\frac{1}{2}(40)(7) = 140$ ft^2

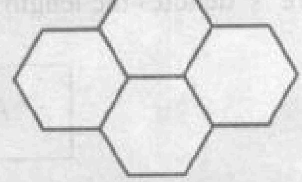
> **Key Fact**
>
> Apothem is the length of perpendicular drawn from centre of a regular polygon to any side. It is also the radius of inscribed circle of a regular polygon.

EXERCISE 9.6

1. A lawn is in the shape of equilateral triangle. Find the perimeter of lawn if length of one side is 5m. Also find the cost of boundary wall of lawn @ Rs. 220 per metre.
2. Cricket ground in a village is in the shape of parallelogram. One side of ground is 65m long and distance between parallel sides having length 65m is 42m. Find the cost of planting grass @ Rs.10 per square metre.
3. Base of a minaret of a Masjid is built in the shape of regular pentagon as shown in the figure. Find the perimeter and area of base of minaret.



4. A plot in the shopping area is in the shape of regular hexagon. One side of the plot is 10m long. Find:
   (i) the cost of fencing the plot @ Rs.160 per metre.
   (ii) area of the plot.
   (iii) the cost of filling the plot @ Rs. 500 per m^2.
5. A room is in the shape of square having perimeter of 48 feet. Find:
   (i) the cost of carpeting the floor @ Rs. 350 per m^2.
   (ii) the inner area of each wall if room is 10 feet high.
   (iii) the cost of painting inner sides of the room @ Rs.100 per m^2.
6. A tile is in the shape of regular hexagon. Each side of the tile is one foot long. Find the perimeter of four tiles joined together as shown in the figure.



Locus

A locus is the set of points that satisfy a given condition. It is a path traced by a moving point which moves according to some given geometrical condition.

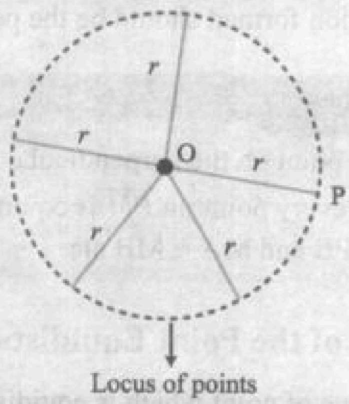
Key Fact

- Every point which satisfies the given geometrical conditions will lie on the locus.
- Every point lying on the locus will satisfy the given geometrical condition.
- The plural of 'locus' is 'loci'. The word locus is derived from the word location.

Locus of a Point from a Fixed Point

A circle with centre 0 and radius r cm is the locus of a point P moving in a plane in such a way that its distance from a fixed-point 0 is always equal to r cm.

This theorem helps to determine the region formed by all the points which are located at the same distance from a single point.

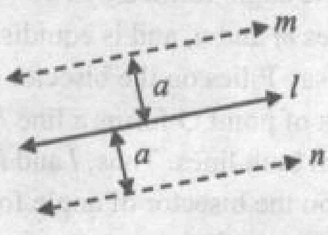


Key Fact

The locus of points in the plane equidistant from a given point is a circle, and the set of points in three-space equidistant from a given point is a sphere.

Locus of a Point from a Given Straight Line

Locus of a point that moves in such a way that its distance from a fixed line l is always equal to a cm. As clear from the figure, the locus of moving point is a pair of straight lines m and n each parallel to l and are located on either side of l at a distance a cm from the line l.

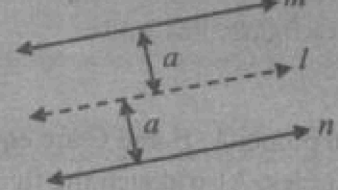


This theorem helps to find the region formed by all the points which are located at the same distance from a single line.

Key Fact

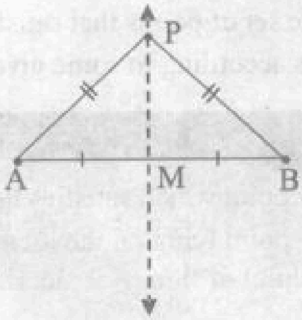
The locus of the point which is equidistant from the two parallel lines say m and n, is considered to be a line l parallel to both the lines m and n and it should be halfway between them.

This theorem helps to find the region formed by all the points which are at the same distance from the two parallel lines.



Locus of a Point Equidistant from Two Given Points

The locus which is equidistant from the two given points say A and B, is the perpendicular bisectors of the line segment that joins the two points.



Here, in the figure $\overleftrightarrow{PM}$ is the perpendicular bisector of $\overline{AB}$.

This theorem helps to determine the region formed by all the points which are located at the same distance from points A and B.

The region formed should be the perpendicular bisector of the line segment AB.

Key Fact

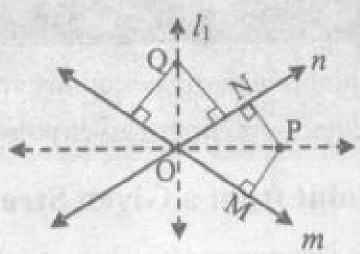
Every point on the perpendicular bisector of AB is equidistant from A and B.

Thus, every point on $\overleftrightarrow{PM}$ is equidistant from fixed points A and B.

PA = PB and MA = MB etc.

Locus of the Point Equidistant from Two Given Intersecting Lines

The locus of point which is equidistant from the two intersecting lines say m and n, is considered to be a pair of lines that bisects the angle formed by the two lines m and n.



In the figure, locus of point P forms a line l which bisects the angle formed by two intersecting lines m and n, and is equidistant from both lines. We say P lies on the bisector of angle formed by m and n.

Similarly, locus of point Q forms a line l_1 which bisects the angle formed by lines m and n, and is equidistant from both lines. Thus, l and l_1 are the pairs of lines that bisect the angle formed at O. We say Q lies on the bisector of angle formed by m and n.

This theorem helps to find the region formed by all the points which are located at the same distance from the two intersecting lines.

Key Fact

The locus of every point on the angle bisector of two intersecting lines, is equidistant from the lines.

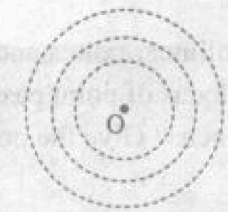
In the above figure, PM = PN

Example 20:

Loci of three points A, B and C are equidistant from a fixed point O. Prove that they form concentric circles. Also sketch the figure.

Solution:

Since the loci of the three points A, B and C are equidistant from a fixed point O, therefore, point O is the centre of circles formed by the movement of three points.



Hence the three circles formed are concentric circles.

Example 21:

A point P is moving parallel to a straight line l at a distance of 5m.

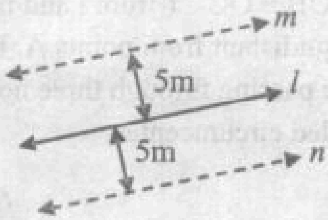
(i) Draw the locus of the path.

(ii) Prove that the point is moving in straight line.

(iii) How many paths are possible for the moving point P?

(iv) Another point Q is moving 8m away from the line l. Explain why the point Q is not the part of the locus?

Solution:



(i) Locus of the path of point P is possible along the lines m and n that are 5m apart from l.

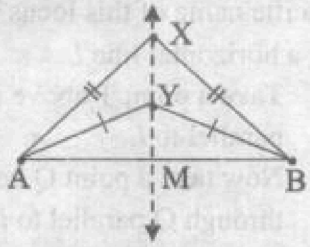
(ii) As the point P is moving parallel to straight line l, therefore it follows straight path too.

(iii) Two paths which are along m and n.

(iv) As the point Q is moving along straight line 8m away from l, therefore it is not part of locus of point P which is 5m away from l.

Example 22:

Figure shows two isosceles triangles XAB and YAB on the same base AB. Show the produced line XY bisects AB and is perpendicular to AB.



Solution:

Draw line through X and Y intersecting AB at M.

As ΔXAB is isosceles, therefore:

$XA = XB$ (i)

Which shows that X is equidistant from A and B and hence lies on the perpendicular bisector of AB. Similarly, ΔYAB is isosceles, therefore:

$YA = YB$ (ii)

Which shows that Y is equidistant from A and B and hence lies on the perpendicular bisector of AB.

This shows that XY is perpendicular bisector of AB.

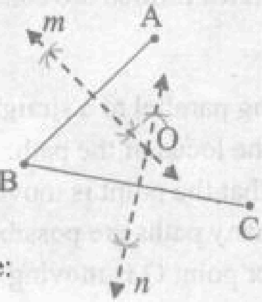
Also, (i) and (ii) show that every point on XY is equidistant from A and B.

Since M lies on XY, therefore M is mid-point of AB and consequently XYM is perpendicular bisector of AB.

Example 23:

Take three non-collinear points and find a point O which is equidistant from these non-collinear points. Draw the locus of point passing through these three non-collinear points. What could be the name of this locus? Give the point O a specific name.

Solution:



Let A, B and C be three non-collinear points in the plane.
Join AB and BC.
Draw perpendicular bisector m of AB and n of BC.
Both bisectors meet at O.
As O lies on perpendicular bisector of AB, therefore:

$OA = OB \quad$ (i)

Again, as O lies on perpendicular bisector of BC, therefore:

$OB = OC \quad$ (ii)

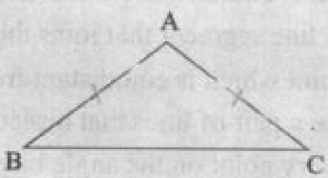
$\Rightarrow \quad OA = OB = OC \quad$ (from i and ii)

Hence O is equidistant from points A, B and C, and if we draw a circle with centre O, we get a locus of circle passing through three non-collinear points A, B and C called circumcircle. Point O is called circumcenter.

EXERCISE 9.7

1. Draw AB = 6cm. Bisect AB at O and draw a locus of point P equidistant from O and above AB. What could be the name of locus?
2. Draw coordinate axes. Take two points A and B on x-axis and y-axis respectively at a distance of 4.5cm each. Draw a locus of points from A to B equidistant from origin. What is specific name of this locus? How many such loci can be drawn around the origin?
3. Draw a horizontal line l.
   - (i) Take a point T above l at a distance of 3cm and draw a locus of points through T parallel to l.
   - (ii) Now take a point Q below l at a distance of 3.2cm and draw a locus of points through Q parallel to l.
   - (iii) What is distance between both loci?
4. Diagram shows a circle with centre P. X and Y are two points on the circumference of the circle.
   - (i) Draw locus of points which are equidistant from X and Y through P.
   - (ii) Take another point Z on the locus outside the circle and draw another circle of radius PZ.
   - (iii) What is the relation of this circle with given circle?

5. Draw a line AB. Let P and Q be two points not on AB but coplanar with AB. Draw the locus of points from P to Q.
   (i) What is the name of that locus?
   (ii) Is there any point on the locus PQ if extended which lies on line AB?
   (iii) How can we take points P and Q such that no point of the above locus lies in line AB?
   (iv) How can we take points P and Q such that every point of line AB may lie on locus?
6. Draw an equilateral triangle PQR of suitable measurement.
   (i) Draw right bisectors of any two sides and locate a point A where both bisectors meet.
   (ii) Draw angle bisectors of any two vertices and locate a point B where both bisectors meet.
   (iii) What is the relation between locus of A and B?
7. Figure shows an isosceles triangle ABC. Prove that the locus of bisector of angle A is right bisector of side BC.



8. Draw three non-collinear points in the plane. Find the locus of the points which are equidistant form these three points. How many such points exist?
9. Take two lines AB and CD inclined at 60° intersecting at O.
   (i) Draw a locus of points which are equidistant from both lines.
   (ii) Draw bisector of 60°.
   (iii) What is relation between locus of points equidistant from lines and angle bisector?
   (iv) Draw bisector of adjacent angle at O find the relation between both angle bisectors.

KEY POINTS

- A mathematical statement is a meaningful composition of words that can be either true or false.
- A proof is a series of conjectures and axioms (postulates) and proved theorems that combine together to give a true result.
- An axiom is a mathematical statement which is assumed to be true without any proof.
- A statement that is believed to be true but its truth has not been proved is called a conjecture.

- A mathematical statement which can be proved through logical reasons is called a theorem.
- Two or more figures that have the same shape but not the same size are called similar figures.
- The ratio of areas of two similar figures is equal to the square of ratios of any two corresponding lengths of the figures.
- The ratio of volumes of two similar solids is equal to the cube of the ratio of any two corresponding lengths of the two solids.
- If all the sides and interior angles of the polygons are equal, they are known as regular polygons.
- If in a polygon at least one angle or one side is not equal in size then the polygon is called irregular.
- The sum of measures of all the interior angles of a polygon having n sides is:

$$(n-2) \times 180^\circ$$

- Sum of measures of exterior angles of a polygon is 360°.
- The locus of points in the plane equidistant from a given point is a circle.
- The locus which is equidistant from the two given points say A and B, is the perpendicular bisectors of the line segment that joins the two points.
- The locus of point which is equidistant from the two intersecting lines say m and n, is considered to be a pair of lines that bisects the angle formed by the two lines m and n.
- The locus of every point on the angle bisector of two intersecting lines, is equidistant from the lines.

MISCELLANEOUS EXERCISE 9

1. Encircle the correct option in the following.

(i) Which of the following is polygon?
(a) circle (b) pyramid (c) quadrilateral (d) sphere

(ii) Which of the following is regular polygon?
(a) kite (b) rhombus (c) rectangle (d) square

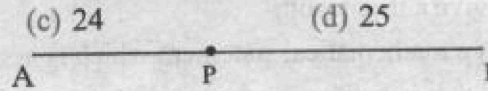
(iii) If two triangles are similar, their corresponding sides are.
(a) proportional (b) equal (c) congruent (d) parallel

(iv) What is the sum of interior angles for an irregular hexagon?
(a) 120° (b) 720° (c) 135° (d) 360°

(v) What is the sum of interior angles for a regular 12 sided polygon?
(a) 1800° (b) 2160° (c) 1980° (d) 360°

(vi) How many sides a regular polygon has if its exterior angle is 15?
(a) 20 (b) 21 (c) 24 (d) 25

(vii) In the figure, $AB = 21$ cm and

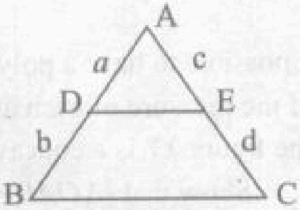


P divides AB in the ratio 3 : 4. What is length of $\overline{AP}$?
(a) 8 cm (b) 10 cm (c) 12 cm (d) 9 cm

(viii) In the figure, $\frac{a}{b} = \frac{c}{d}$. Which one is true?

(a) $DE = BC$ (b) $DE > BC$

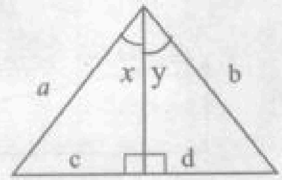
(c) $\overline{DE} \cong \overline{BC}$ (d) $\overline{DE} \parallel \overline{BC}$



(ix) In the figure if $x = y$. Then the value of b is:

(a) $\frac{ac}{d}$ (b) $\frac{ad}{c}$

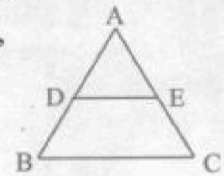
(c) $\frac{cd}{a}$ (d) $\frac{c}{cd}$



(x) In the figure, ΔABC is equilateral and $\overline{DE} \parallel \overline{BC}$, then ΔADE is

(a) right angled (b) scalene

(c) isosceles (d) equilateral

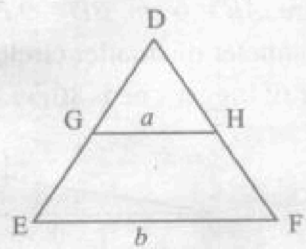


(xi) In the figure, $\overline{GH} \parallel \overline{EF}$.

Then $a : b = ?$

(a) $DG : DE$ (b) $DG : DH$

(c) $DG : GE$ (d) $DE : EF$



(xii) What is the sum of all the exterior angles of a 13-sided polygon whose one interior angle is equal to x^{o}?

(a) $90^{o} + x$ (b) 360^{o} (c) $360^{o} + x$ (d) $180^{o} + x$

(xiii) Which polygon has both its interior and exterior angles the same?

(a) pentagon (b) triangle (c) square (d) hexagon

(xiv) The formation or expression of an opinion or theory without sufficient evidence for proof is known as:

(a) axiom (b) conjecture (c) corollary (d) theorem

(xv) A mathematical statement that is proved true based on already accepted statements is called:

(a) axiom (b) conjecture (c) postulate (d) theorem

(xvi) A mathematical statement that is assumed to be true without proof is called:

(a) axiom (b) conjecture (c) postulate (d) theorem

(xvii) Two solids with equal ratios of corresponding linear measures:

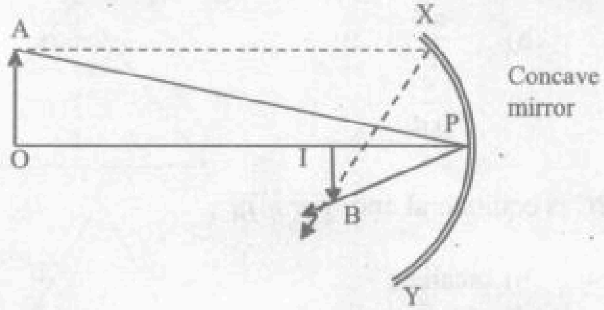
(a) are similar (b) are congruent

(c) have different area (d) have same volume

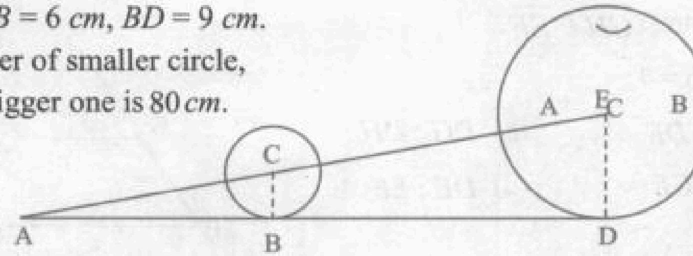
2. Find the exterior angle of a polygon with 6 sides.

3. Is it possible to have a polygon, in which sum of interior angles is 9 right angles?

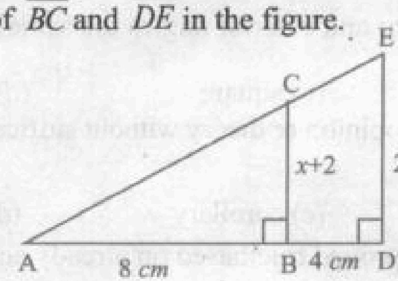
4. Is it possible to have a polygon whose sum of interior angles is 7200°?
5. Find the measure of each angle of a regular nonagon.
6. In the figure XY is a concave mirror OA is object and IB is its image.
   (i) Show that $\Delta OAP \sim \Delta IBP$
   (ii) Find height of object if $IB = 2\ cm$, $OP = 10\ cm$, $IP = 4\ cm$.



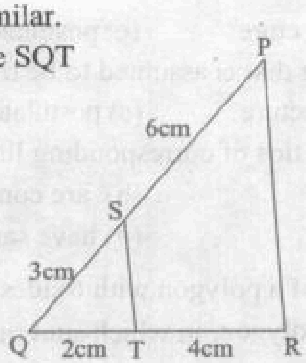
7. In the figure, $AB = 6\ cm$, $BD = 9\ cm$.
   Find the diameter of smaller circle,
   if diameter of bigger one is $80\ cm$.



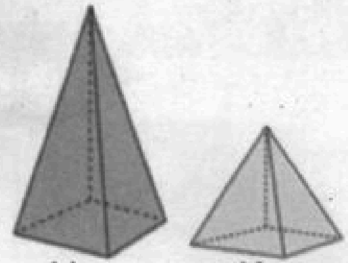
8. Prove that if vertex angles of two isosceles triangles are equal then the two triangles are similar.
9. Find the length of $\overline{BC}$ and $\overline{DE}$ in the figure.



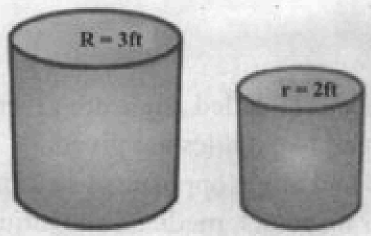
10. Triangles SQT and PQR are similar.
    Find the ratio of area of triangle SQT
    to that of triangle PQR.



11. The following pyramids are similar and larger pyramid has a surface area of 392 cm^2. What is the surface area of smaller pyramid?

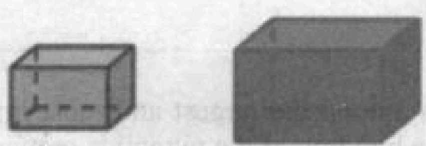


12. The two cylinders are similar. What is the volume of the larger cylinder if the volume of smaller cylinder is 40 ft^3.



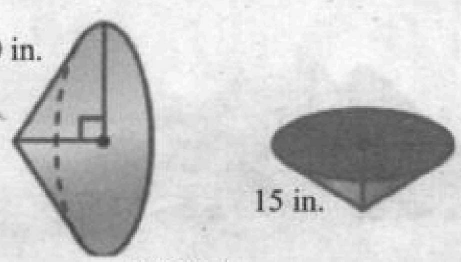
13. The following pairs of solids are similar. Find the surface area of red solid.

(i)



Surface area = 336 m^2

(ii)



Surface area = 1800 in^2.