Student Learning Outcomes (SLOs)

The student will

- state that electromagnetic radiation has a particulate nature.
- Explain and apply the photonic model of light to solve problems [use E = hf to solve problems, and use the electronvolt (eV) as a unit of energy]
- Explain that a photon has momentum [including that the momentum is given by $p=E/c$ (connect with the idea that light can exert a force)]
- describe that photoelectrons may be emitted from a metal surface when it is illuminated by electromagnetic radiation.
- describe and use the terms threshold frequency and threshold wavelength.
- explain photoelectric emission in terms of photon energy and work function energy.
- state and apply $hf=\emptyset + 1/2mv_{max}^2$
- explain why the maximum kinetic energy of photoelectrons is independent of intensity, whereas the photoelectric current is proportional to intensity.
- Juxtapose the evidence for light as a wave and as a particle [Explain that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature].
- Analyze qualitatively the evidence provided by electron diffraction for the wave nature of particles.
- Explain and apply the de Broglie wavelength to solve problems [use $\lambda= h/p$ to solve problems].
- State that there are discrete electron energy levels in isolated atoms (e.g. atomic hydrogen).
- explain the appearance and formation of emission and absorption line spectra.
- use $hf = \Delta E$ to solve problems.
- Describe the Compton effect qualitatively.
- Explain the phenomena of pair production and pair annihilation.
- Explain how electron microscopes achieve very high resolution.
- State and explain Heisenberg's uncertainty principle qualitatively.
- Use the uncertainty principle to explain why empirical measurements must necessarily have uncertainty in them.

Quantum theory is the foundation of Modern Physics. At the smallest scales, the classical laws for real cases no longer apply, so quantum physics explains the nature and behaviour of matter and energy at the atomic and subatomic levels. Quantum physics is the branch of physics that studies the behavior of matter and energy at the atomic and subatomic level. In this realm, the rules of classical physics are replaced by strange and counterintuitive phenomena. Quantum physics is not just a curiosity, but has led to countless technological innovations, from transistors and lasers to computer chips and medical imaging.

In this chapter, we'll study the fundamental principles and fascinating concepts of quantum physics that help us in exploring our universe.

21.1 ELECTROMAGNETIC WAVE

In 1864, the British physicist James Clerk Maxwell made the remarkable ideas that accelerated electric charges generate linked electric and magnetic disturbances that can travel through space, as shown in Fig. 21.1. These electric field (E) and magnetic field (B) can sustain each other, forming an electromagnetic wave that propagates through space with speed about 3×10^8 m s^{-1}. The direction of propagation is the direction of the vector product $E \times B$. Electric and magnetic components of an electromagnetic wave are perpendicular to each other and to the direction of motion.
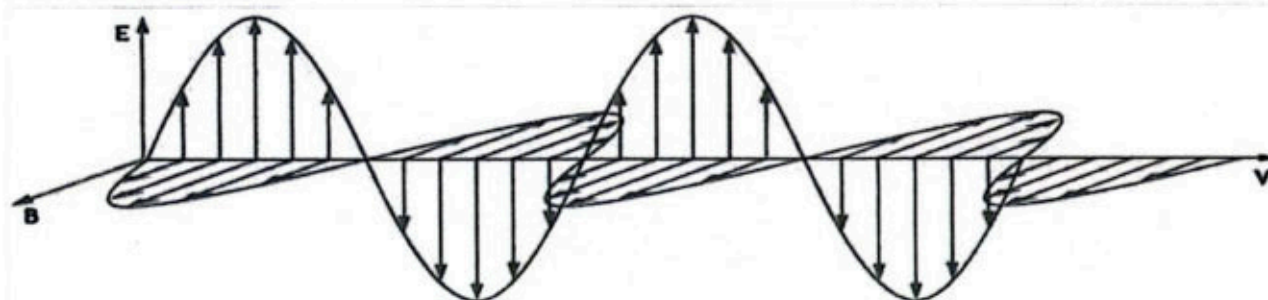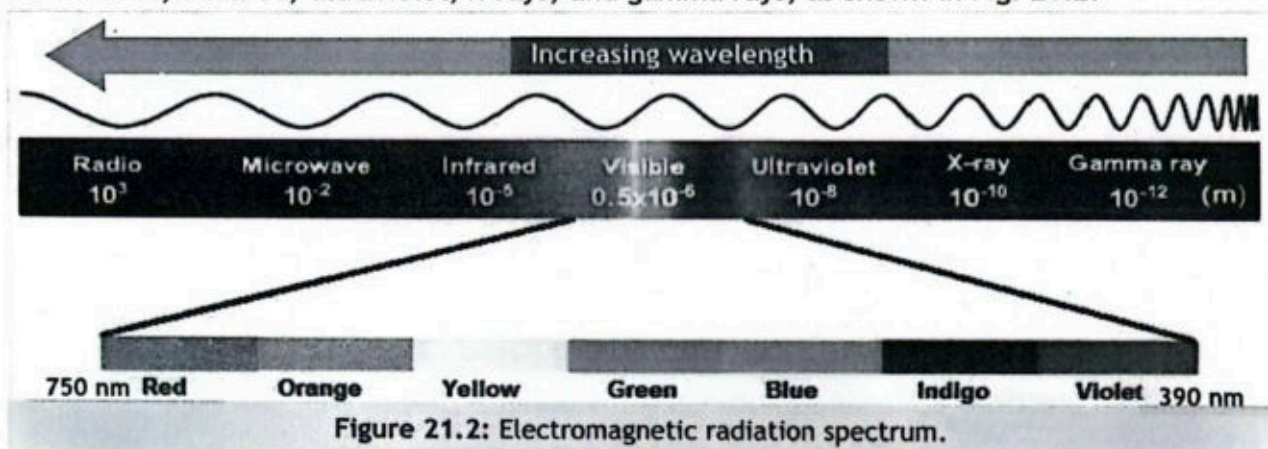

Figure 21.1: A pair of electric (red) and magnetic (blue) fields, propagating together as an electromagnetic wave in the direction indicated by the arrow at the speed of light.

Visible light emitted by the glowing filament of light bulb is an example of electromagnetic wave. There are many other waves which are electromagnetic in nature, e.g. radio waves, microwaves, infrared, ultraviolet, x-rays, and gamma rays, as shown in Fig. 21.2.

Increasing wavelength							
Radio 10^3	Microwave 10^{-2}	Infrared 10^{-5}	Visible 0.5×10^{-6}	Ultraviolet 10^{-8}	X-ray 10^{-10}	Gamma ray 10^{-12}	(m)

750 nm Red   Orange   Yellow   Green   Blue   Indigo   Violet 390 nm

Figure 21.2: Electromagnetic radiation spectrum.

The visible wavelength spectrum of electromagnetic wave lies in between 750 nm (Red) and 390 nm (Violet). The electromagnetic wave travels through vacuum or through any specific medium from one point to another point depending on energy.

Plank's Quantum Theory

Planck's quantum theory is the fundamental theory of quantum mechanics. In 1900, Plank studied the electromagnetic radiation emitted from different atoms and molecules, and proposed a theory which was in complete agreement with experiments at all wavelengths. According to this theory:

1) Different atoms and molecules can emit or absorb energy (E) in discrete amount only, which is given by:

$$E = n\,hf \qquad\qquad \text{_____ (21.1)}$$

Here n = 1, 2, 3, ... is called quantum number, $h = 6.626 \times 10^{-34}$ J s is the plank's constant and f is the frequency of the radiation.

The energies of the molecules are said to be quantized, and the allowed energy state are called quantum states. Atoms or molecules emit or absorb energy only by jumping from one quantum state to another.

2) The smallest amount of energy that can be emitted or absorbed in the form of electromagnetic radiation is known as quantum (plural quanta).

The development of Plank's theory gave the birth to the Quantum Physics.

Example 21.1: What is the frequency of a photon whose energy is 66.3 eV?
Given: Energy of Photon = E = 66.3 eV = $66.3 \times 1.6 \times 10^{-19}$ J
To Find: Frequency = f = ?
Solution: Using the equation: $\quad E = hf$

or $\qquad\qquad f = \dfrac{E}{h} = \dfrac{66.3 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.6 \times 10^{16}$ Hz

Assignment 21.1
What will be the photon energy for a wavelength of 5000 angstroms, if the energy of a photon corresponding to a wavelength of 7000 angstroms is 4.23×10^{-19} J?

21.2   PHOTOELECTRIC EFFECT

The word photoelectric is the combination of two words Photo; means light and electric; means electron, it means that this process defines the interaction between photon and electron.

Photoelectric effect is the process of emitting the electrons from the metal surface when the metal surface is exposed to an electromagnetic radiation of sufficiently high frequency.

The emitted electrons are called **photoelectrons** because they are liberated by means of light.

For Your Information
The first discovery of the photoelectric effect was made by Hertz, who was also the first to produce the electromagnetic waves predicted by Maxwell.

The setup to observe the photoelectric effect is shown in Fig. 21.3. An evacuated glass tube contains a metal plate connected to the negative terminal of a battery. Another metal plate is maintained at a positive potential by the battery. When the tube is kept in dark, the ammeter (A) shows zero reading, indicating that there is no current in the circuit. However, when light of the appropriate frequency falls on the metal surface, a current is detected by the ammeter, indicating a flow of charges across the gap between the metal surface and the detector. The current associated with this process arises from electrons emitted from the negative plate and collected at the positive plate. Ultraviolet light is required in the case of emission of electrons from an alkali metal.
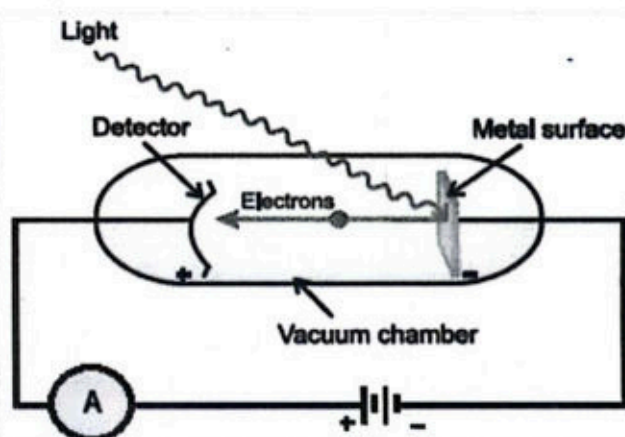

Figure 21.3: Schematic diagram of photoelectric effect setup.

Effect of Applied Voltage on Photoelectric Current

A graph of the photoelectric current versus the potential difference (V) between the metal plate and the detector for different light intensities is shown in Fig. 21.4. It can be noted from the graph that:

- The current increases as the incident light intensity increases.
- For large values of V, the current reaches a maximum value, corresponding to the case where all photoelectrons are collected at anode (detector).
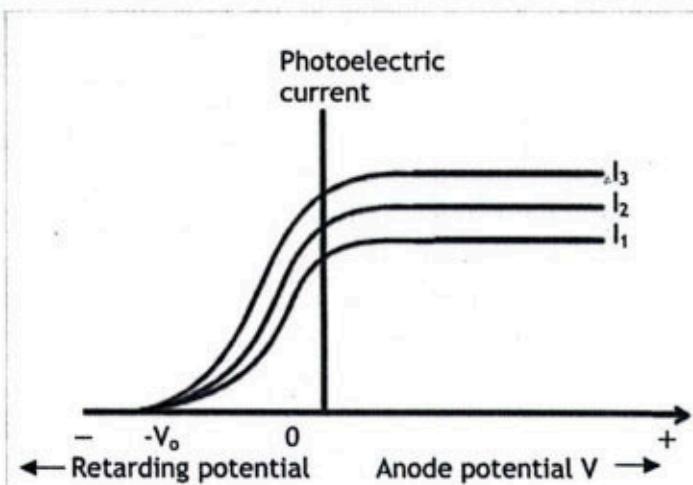

Figure 21.4: Graph of the photoelectric current versus the potential difference.

Thus, brighter light of constant frequency causes an increase in current (more electrons ejected) but does not cause the individual electrons to gain higher energies. It means that the maximum K.E of the electrons is independent of the intensity of the light. Classical physics says that more intense light has larger amplitude and thus delivers more energy. That should not only enable a larger number of electrons to escape from the metal; it should also enable the electrons emitted to have more K.E.

- When V is negative (i.e. when the battery in the circuit is reversed) the photoelectrons are repelled by the negative plate. Only those electrons having a K.E greater than 'eV' will reach the detector, where 'e' is the charge on electron.
- If V is greater than or equal to V_o, called the stopping potential, no electrons will reach the detector and the current will be zero. The stopping potential is independent of the radiation

intensity. The maximum K.E of the photoelectrons is related to the stopping potential through the relation.

$$K.E_{max} = eV_0 \underline{\hspace{3cm}} (21.2)$$

Effect of Intensity of Incident Radiation on Photoelectric Current

If the frequency of the incident radiation and the potential difference (V) between the cathode and the anode is kept constant and the intensity of incident radiation is varied, then it is found that the photoelectric current increases linearly with the intensity of incident radiation, as shown in Fig. 21.5. Since the photoelectric current is directly proportional to the number of photoelectrons emitted per second, so it implies that:

The number of photoelectrons emitted per second is proportional to the intensity of incident radiation.
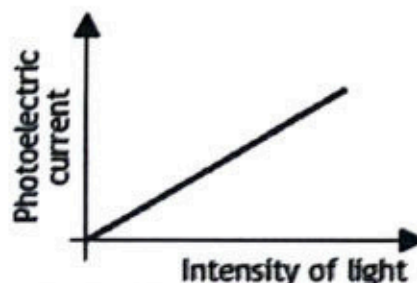

Figure 21.5: Graph of the photoelectric current versus intensity of light.

Effect of Frequency of Incident Radiation on K.E of Photoelectrons

The maximum K.E of photoelectrons depends on the frequency of the incident radiation, as shown in Fig. 21.6. If the incident light is very dim (low intensity) but high frequency, electrons with large K.E are released. Classical physics gives no explanation for the frequency dependence.

For a given metal, there is a threshold frequency 'f_0' below which no electrons are emitted, how intense the incident light may be. Classical physics has no explanation for the frequency dependence.

Figure 21.6: Plot between K.E of photoelectron and frequency of incident light.

Work Function (Φ)

The minimum amount of energy required to escape the electron from metal surface is known as the work function (Φ) of the substance.
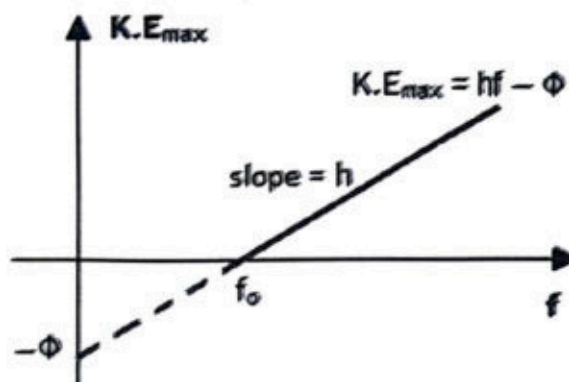
Photon (Quantum) Theory of Photoelectric Effect

In the year 1905, Albert Einstein reinterpreted Planck's theory to further explain the photoelectric effect. Basically, Planck's work led Einstein in determining that light exists in discrete quanta of energy, called photons.

According to Einstein, the emission of photoelectron is the result of the interaction between a single photon of the incident radiation and an electron in the metal. When the photon's energy (E=hf) is transferred to an electron in a metal, a part of its energy (Φ) is used by the electron to break away from the metal and the rest appears as the maximum K.E of the electron. i.e.,

$$E = \Phi + K.E_{max} \underline{\hspace{2cm}} (21.3)$$

This is called **Einstein's photoelectric equation**. Here, $\Phi = hf_o$ is the work function.

When K.E of the photoelectron is zero, the frequency 'f' is equal to threshold frequency f_o, hence the Einstein's photoelectric equation becomes:

$$hf_o = \Phi$$

Hence, we can also write Einstein's photoelectric equation as:

$$K.E_{max} = E - hf_o$$

- If $E < \Phi$, there will be no photoelectric effect.
- If $E = \Phi$, the photoelectric effect occurs, but the kinetic energy of the expelled photoelectron is 0.
- If $E > \Phi$, the photoelectric effect will occur and the expelled electron possesses kinetic energy.

There are so many practical utilizations of photoelectric effect; for example, photocells, photoconductive devices, and solar cell, etc.

Example 21.2: A metal whose work function is 4.2 eV is irradiated by radiation whose wavelength is 2000 Å. Find the maximum kinetic energy of emitted electron.

Given: Work function = Φ = 4.2 eV = $4.2 \times 1.6 \times 10^{-19}$ J = 6.72×10^{-19} J

Wavelength of radiation = λ = 2000 Å = 2000×10^{-10} m = 2×10^{-7} m

To Find: Maximum kinetic energy = $K.E_{max}$ = ?

Solution: By Einstein's photoelectric equation: $K.E_{max} = hf - \Phi$

or $$K.E_{max} = \frac{hc}{\lambda} - \Phi$$

Putting values, we get: $$K.E_{max} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7}} - 6.72 \times 10^{-19} = 2.1015 \text{ eV}$$

Thus, maximum kinetic energy of emitted electron is 2.015 eV.

Assignment 21.2

Radiation of wavelength 3000 Å falls on a photoelectric surface for which work function is 1.6 eV. What is the stopping potential for emitted electron?

21.3   COMPTON'S EFFECT

Arthur H. Compton in 1923, conducted an experiment in which he made incident a beam of X-rays of wavelength λ, on a block of graphite. He found that the scattered X-rays had a slightly longer wavelength λ' than the incident X-rays, as shown in Fig. 21.7. The amount of energy reduction depended on the angle at which the X-rays were scattered. This phenomenon is known as Compton's effect.

Compton's effect is the phenomenon where X-rays rays scatter off electrons, transferring energy and momentum, and increasing the wavelength of the scattered radiation.

The change in wavelength $\Delta\lambda$, between a scattered X-ray and an incident X-ray is called the **Compton shift**.

In order to explain this effect, Compton assumed that if a photon behaves like particle, its collision with other particles is similar to that between two billiard balls. Hence, both energy and momentum must be conserved. If the incident photon collides with an electron initially at rest, the photon transfers some of its energy and momentum to the electron. Consequently, the energy and frequency of the scattered photon are lowered and its wavelength increases.

Figure 21.7: Compton's Effect.

By applying conservation of energy, we have

$$hf = K.E + hf' \qquad \text{(21.4)}$$

Here, "hf" and "hf' " represent the energy of the incident and scattered photon, respectively, K.E is the kinetic energy given to the recoiling electron. By applying conservation of momentum, we have:

$$p = p_o + p' \qquad \text{(21.5)}$$

Where p_o and p' represents the momentum of scattered photon and recoiling electron.

According to classical electromagnetic theory, **EM waves carry momentum** of magnitude E/c, where E is the energy of waves and c is the speed of light. Thus, the momentum of a photon is

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \qquad \text{(21.6)}$$

Take the incident photon's direction along the x-axis, we can write the Eq. (21.6) in components as:

Along x-axis
$$\frac{h}{\lambda} = p_e \cos\Phi + \frac{h}{\lambda'}\cos\theta \qquad \text{(21.7)}$$

Along y-axis
$$0 = -p_e \sin\Phi + \frac{h}{\lambda}\sin\theta \qquad \text{(21.8)}$$

From Eq. (21.6), (21.7) and (21.8), Compton derived the following relationship

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta) \qquad \text{or} \qquad \Delta\lambda = \frac{h}{m_o c}(1 - \cos\theta) \qquad \text{(21.9)}$$

The quantity $\frac{h}{m_o c}$ = 0.00243 nm is constant and is known as the Compton wavelength because it has the dimension of a wavelength. It is cleared from this expression that Compton's shift depends only on scattering angle. Since $(1 - \cos\theta)$ is always positive thus $\lambda' > \lambda$. Compton's shift does not depend upon; wavelength of incident photon and nature of scattering material.
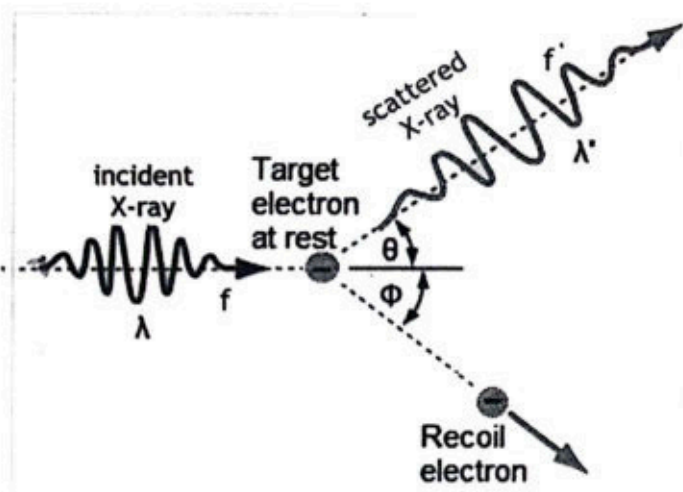
Example 21.3: An X-ray of 0.20 nm is incident on a target. What is the Compton shift that can be expected at a 45°?

Given: $\lambda = 0.20$ nm $\qquad \theta = 45°$

To Find: $\lambda' - \lambda = ?$

Solution: The Compton shift can be calculated from the relation:

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

Putting values, we get: $\qquad \lambda' - \lambda = \dfrac{6.63\times10^{-34}}{(9.11\times10^{-31})(3\times10^8)}(1 - \cos 45°) = 7.11\times10^{-13}$ m

Assignment 21.3

An X-ray of 71 pm is incident on a target. What is the smallest shift that can be expected at 60°? Find the wavelength of the scattered X-ray.

21.4 PAIR PRODUCTION

Pair production is the process of creation of a subatomic particle and its antiparticle. When a photon passes near a heavy nucleus, an electron-positron pair is created, as shown in Fig. 21.8. The photon is totally absorbed in this process.

When a high energy photon interacts with matter, photon energy is changed into an electron-positron pair. This process is called pair production.

Figure 21.8: Pair production.

Phenomenon of pair production is evidence of conversion of energy into mass. Three parameters are conserved during the pair production; Charge, Momentum, and Energy.

Conservation of Charge: Charges must be conserved in this process. As electron and positron having opposite charge means that the sum of net charge of pairs is zero, which is actually equal to that of photon before the collision. Therefore, the conservation of electric charge will be observed in this process. It is impossible for a photon to produce a single electron, a single positron, two electrons, or two positrons, because the photon has zero charge and then charge will not be conserved.

Conservation of Momentum: Momentum must also be conserved in this process. If $\dfrac{hf}{c}$ is the momentum of photon, mv_{e^+} is the momentum of positron and mv_{e^-} is the momentum of electron, then apply conservation of momentum, we have:

$$\frac{hf}{c} = mv_{e^+} + mv_{e^-} \qquad\qquad \text{_____ (21.11)}$$

Conservation of Energy: Energy must be conserved in the process of pair production to occur, i.e.,

$$E_{Photon} = E_{electron} + E_{positron} \qquad\qquad \text{_____ (21.12 a)}$$

As the total energy of a particle with mass 'm' is the sum of its kinetic energy (K.E) and its rest mass energy ($m_o c^2$), so Eq. (21.12) can be written as:

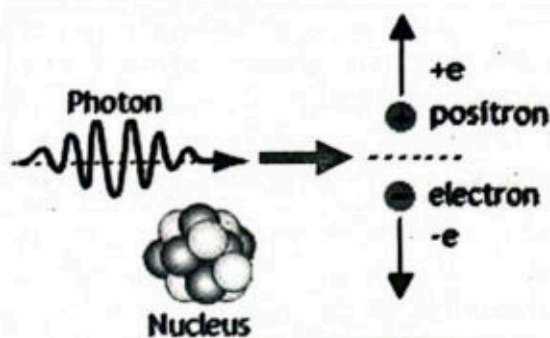$$hf = (K.E_{electron} + m_o c^2) + (K.E_{positron} + m_o c^2) \qquad \text{_____ (21.12 b)}$$

Where, $E = m_0c^2 = 0.511$MeV. Thus, a photon must have energy of at least $2m_0c^2$ in order to create an electron-positron pair. If the photon energy is greater than $2m_0c^2$ (= 1.02 MeV) the excess energy appears as K.E of the electron and positron.

21.5   PAIR ANNIHILATION

Pair annihilation is the reverse process of pair production. Pair production is the process in which energy (photon) is converted into mass (electron-positron) and annihilation is the process in which mass (electron-positron) is converted into energy (photon).

Pair annihilation is a process in which an electron-positron pair combines to produce two photons.

It means during collision of electron and positron, these particles are disappeared and radiation is produced in term of gamma ray photons. Pair annihilation cannot create just one photon, because it is required to conserve both energy and momentum. The movement of photons in opposite direction justify the conservation of momentum.

The total energy of the two photons must be equal to the total energy of the electron-positron pair. Annihilation of the pair then produces two photons, each with energy $E = hf = m_0c^2 = 0.511$ MeV, traveling in opposite direction, as shown in Fig. 21.9. In general;

Figure 21.9: Pair Annihilation.

$$2m_0c^2 + K.E_{electron} + K.E_{positron} = \text{Energy of two gamma photons} \qquad _____ (21.13)$$

Some other examples of pair annihilation are proton-antiproton annihilation and Higgs production. Besides confirming the photon model of EM radiation, pair annihilation and pair production clearly illustrate Einstein's idea about mass and rest energy.

21.6   THE WAVE-PARTICLE DUALITY

Interference, diffraction and polarization are the basic properties of all electromagnetic waves. As light shows all the three effects, therefore, light can be considered as a wave. But Photoelectric effect can be explained only when we consider light as made by small energy packets (particles) called photon. It means photoelectric effect shows the particle nature of light. The light displays the dual nature, i.e., particle *characteristics as well as wave* characteristics.

In 1924, the French physicist Louis De Broglie predicted that if light (which characteristically demonstrated as pure wave properties) can have particle-like properties, then particle can also behave as a wave. So, particles of matter obey a wave equation just as photon does. Compton's investigation showed that the momentum p of a photon having wavelength λ is described by the equation;
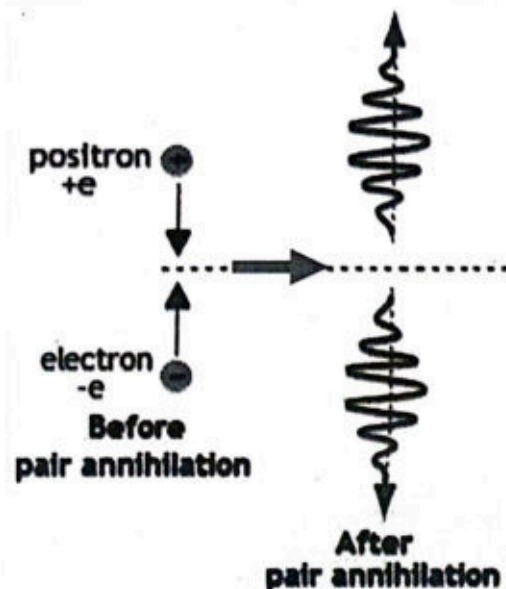
$$p = \frac{h}{\lambda}$$

or
$$\lambda = \frac{h}{p} \quad _____ (21.14)$$

Eq. (21.14) is called De Broglie relation and is a general formula that applies to material particles as well as to photons. This relation shows that a wavelength is associated with a moving particle, greater the particle's momentum the shorter its wavelength and vice versa. For macroscopic objects of heavy masses, their corresponding wavelengths are very small to be measured. For example, De Broglie wavelength of an object having mass 0.5 kg moving with a speed of 10 m s^{-1} is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(0.5)(10)} = 1.3 \times 10^{-34} \text{ m}$$

This wavelength is too small to be observed. De Broglie wavelength of an electron moving with a typical speed of 10^6 m s^{-1} is

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{(9.1 \times 10^{-31})(10^6)} = 7.3 \times 10^{-10} \text{ m}$$

This wavelength is comparable with the spacing between the atoms in a crystal, which is suitable for diffraction and interference. Thus, the wavelengths of very small particle of matter are observable.

There are so many practical applications of De-Broglie equations, for example; the theory behind the construction of electron microscope is the De-Broglie theory, which is commonly used now a day to study the deep observation of microscopic organisms like viruses, bacteria etc.

Example 21.4: A boy through a stone having mass 5.0 mg with the help of a catapult. If the stone moves with speed of 8.0 m s^{-1}, calculate De Broglie wavelength associated with the stone.

Given: m = 50 mg = 5.0×10^{-6} kg   v = 8 m s^{-1}
To Find: λ = ?

Solution: **The de-Broglie wavelength can be calculated by using:** $\lambda = \dfrac{h}{mv}$

Putting values, we get: $\lambda = \dfrac{6.63 \times 10^{-34}}{(5 \times 10^{-6})(8)} = 1.66 \times 10^{-29} \text{ m}$

Assignment 21.4

The speed of rifle bullet is 1500 m s^{-1}. If its mass is 20 g then find the De Broglie wavelength associated with it. Is this wavelength observable? Give reason.

21.7 ELECTRON MICROSCOPE

The electron microscope is a device that is based on the wave nature of electron. In microscope, accelerated electrons are used as illumination source. This microscope helps us to observe objects in detail even up to nano-scale with remarkably high resolution. Tungsten metal (material) and high potential source are used for excitation of electrons to form an electrons stream like a beam of light. Magnetic coils are used for focusing the electrons- beam on the

target substance. The theoretical aspect of this set is; by increasing the applied potential, current is increased significantly and resultantly the strength of magnetic lens (magnetic coils) is increased.

Electron-microscope is basically practical usage and demonstration of wave nature of electrons, which is thousand times shorter than visible light which enables the electron microscope distinguish details not visible with optical microscope i.e. visible light wavelength is 4000 A° to 7000 A°, while electrons accelerated to 10,000 KeV and acquire wavelength about 0.12 A°. A beam of extremely fast-moving electrons is used instead of the light source of a conventional light microscope. The sample must be carefully prepared and placed in a vacuum chamber.

Ray diagram of electron microscope is shown in Fig. 21.10. The electron beam is passed through a series of coil-shaped electromagnets that replace conventional optical lenses. There are several components of electron microscope; electron gun, condenser lens, sample stage (magnetic object), objective lens,

Figure 21.10: Electron microscope.

intermediate lens, projector lens, detector or screen and vacuum system. The electron gun and condenser lens are typically located in the upper part of the microscope. The sample stage and objective lens are situated in the middle section, the intermediate and projector lenses are located in lower part of microscope.

The detector or screen is usually located at the very bottom. The image may take the form of photograph, often referred to as an electron micrograph.

Any microscope is capable of detecting details that are comparable in size to the wavelength of radiation used to illuminate the object. Electron can be accelerated to very high kinetic energies, giving them a very short wavelength about 100 times shorter than those of the visible light (used in optical microscopes). As a result, electron microscope is able to provide details about 100 times smaller. The study of metals and crystals became easy with the introduction of an electron microscope.

21.8 HEISENBERG'S UNCERTAINTY PRINCIPLE

On microscopic level, we cannot measure any property of a particle without interacting with it in some way; this introduces unavoidable uncertainty into the result. One can never measure all the properties exactly. According to Planck's point of view; $E = hc/\lambda$, means a photon with
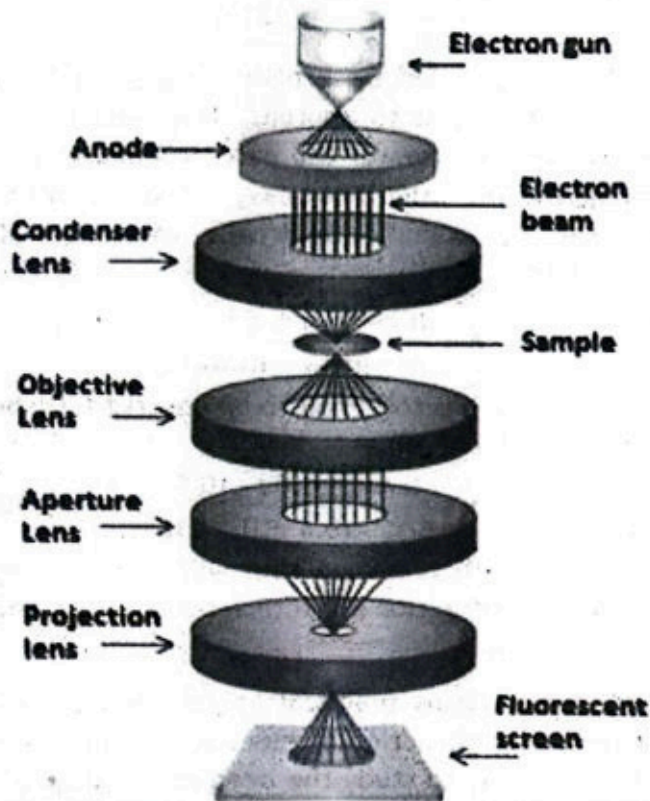
a short wavelength has a large energy. Formulated by the German physicist and Nobel laureate Werner Heisenberg in 1927, the uncertainty principle states that we cannot find both the position and speed of a particle, such as a photon or electron, with perfect accuracy. It is a consequence of the dual nature of matter. According to Heisenberg uncertainty;

It is impossible to accurately find both, the position and momentum of an object simultaneously.

According to this principle; The product of uncertainties in determining the position and momentum of a particle at the same instant is approximately equal to $\hbar$ (read as 'h-cut');

whereas $\hbar = \dfrac{h}{2\pi} = 1.054 \times 10^{-34}$ J s.

If position of a particle has an uncertainty Δx and that of the corresponding momentum is Δp, then uncertainties are found to be related in general by the inequality as:

$$\Delta x . \Delta p \geq \hbar$$

It is impossible to know both the position and momentum exactly; i.e., if $\Delta x = 0$ and $\Delta p = \infty$. In quantum mechanics a particle is described by a wave, the term position refers; point where the wave is concentrated. The uncertainty in position means; the position is uncertain to the degree that the wave is spread out. While the uncertainty in momentum means; the momentum is uncertain to the degree that the wavelength is unclear. These uncertainties are inherent in the physical world and have nothing to do with the skill of observer; due to the very small value of angular Planck's constant, these uncertainties are not observable in normal everyday situations.

If we want accuracy in position, we must use photons of shorter wavelength because the best resolution we can get is about the wavelength of the radiation used. Short wavelength radiation implies high frequency, high energy photons. When these collide with the electrons, they transfer more momentum to the target. If we use longer wavelength i.e., less energetic photons, we compromise resolution and position.

If the energy is in the form of electromagnetic waves, the limited time available restricts the accuracy with which we can determine the frequency of the electromagnetic waves. Let's consider that the minimum uncertainty in the number of waves under observation is equal to number of them that we counted divided by the time interval, the uncertainty in the frequency can be measured as;

$$\Delta f \geq \frac{1}{\Delta t} \quad \text{i.e., } \Delta E = \hbar \Delta f.$$

Another kind of uncertainty principle concerns uncertainties in simultaneous measurements of the energy of a quantum state and its lifetime, if ΔE is the uncertainty in determining the energy of the system and Δt is the uncertainty in determining the time to which this determination refers, then the uncertainty principle for energy and time can be expressed as;

$$\Delta E . \Delta t \geq \hbar$$

A system that remains in metastable state for a very long time can have a well-defined energy, but if it remains in a state for only a short time the uncertainty in energy must be

correspondingly greater. The uncertainty ΔE depends on the time interval Δt during which the system remains in the given state.

Example 21.5: Calculate the uncertainty in the momentum of an electron if uncertainty in its position is 1 Å (10^{-10} m).

Given: $\Delta x = 10^{-10}$ m

To Find: $\Delta p = ?$

Solution: According to the uncertainty principle:

$$\Delta x.\ \Delta p \geq \frac{h}{2\pi}$$

or

$$\Delta p \geq \frac{h}{2\pi \Delta x}$$

By putting values, we get:

$$\Delta p = \frac{6.63 \times 10^{-34}}{2(3.14)(10^{-10})} = 5.28 \times 10^{-25} \text{ kg m s}^{-1}$$

Assignment 21.5

Let's consider a ball with a mass of 0.1 kg moving at a speed of 30 m s^{-1}. The uncertainty in its momentum is 0.000001 of its actual momentum. Determine the uncertainty in its position?

21.9 DISCRETE ENERGY LEVELS IN AN ATOM AND ITS SPECTRA

The electrons orbiting an atom can only occupy certain allowed orbits (also called shells). Each orbit has a discrete energy state of the atom e.g. E_1, E_2, E_3 and so on, as shown in the Fig. 21.11. Electrons can move from one allowed energy level to another only by gain or loss of certain amount of energy.

- Energy is required for an electron to move from a lower to a higher energy level. This transition is called an excitation.

- Energy is released if the electron moves from a higher to a lower energy level. This transition is called a de-excitation.

Electrons can move to a higher or lower energy level by absorbing or emitting electromagnetic

Figure 21.11: Excitation and de-excitation of an isolated H-atom.

radiation with a frequency f. This frequency depends on the difference of energy between the specific energy levels involved in the transition. The frequency of the radiation obeys the condition:
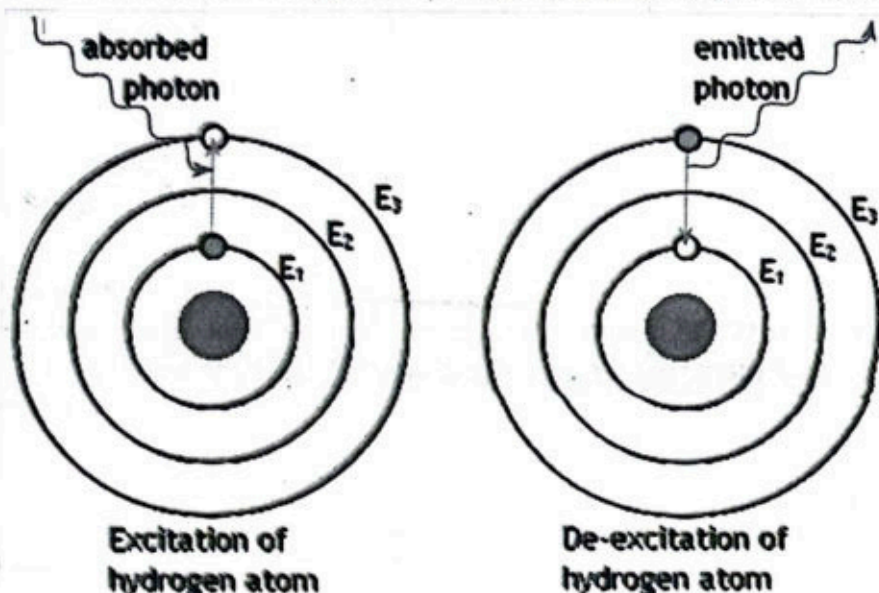
$$hf = E_n - E_p$$

Where E_n and E_p are higher and lower energy states respectively.

Photons emitted from such transitions, when allowed to pass through a narrow slit and a prism (or diffraction grating), will give rise to a set of line spectrum, unique to the element, as shown in Fig. 21.12. A spectrum is produced by using a prism or diffraction grating to separate the various wavelengths in a beam of light. A line spectrum contains a discrete set of wavelengths. It is a characteristic feature of an element.
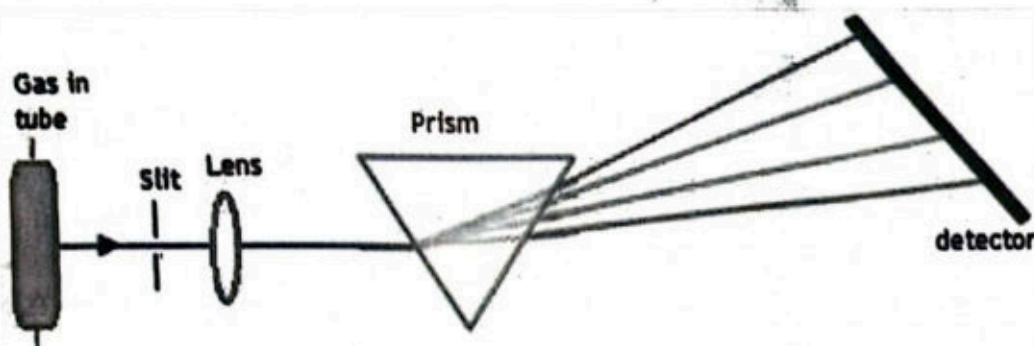

Figure 21.12: Setup for obtaining spectrum.

An **emission spectrum** of an element consists of a series of separate bright lines of definite frequencies (or wavelengths) on a dark background. It is produced when a stream of photons of different frequencies is passed through a narrow slit and normally through a diffraction grating. These photons are emitted randomly from transitions (from higher to lower excited states or the ground state) in the excited atoms of the element in a vapour or gas at low pressure. Emission spectra of Hydrogen is shown in Fig. 21.13 (a). Since each element has a unique set of orbital electrons, the emission line spectrum of an element is also unique, enabling it to be used as a means of identification of the element.
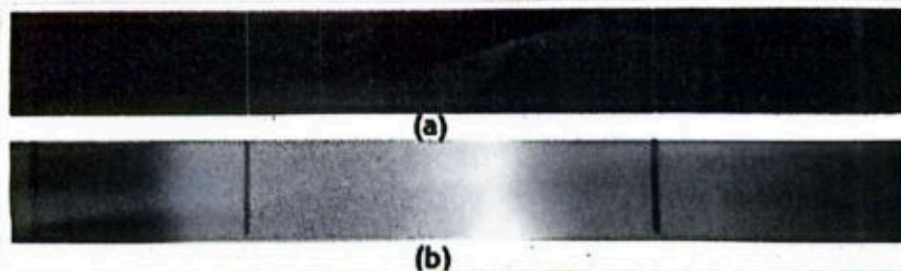
Figure 21.13: (a) H-emission spectrum (b) H-absorption spectrum.

An **absorption line spectrum** of an element consists of a series of separate dark lines of definite frequencies (or wavelengths) on a coloured background. The coloured background is produced when a stream of photons of different frequencies from a white light source (e.g. tungsten filament lamp) is passed through a narrow slit and a diffraction grating. The cool vapour of the element concerned is placed between the white light source and the narrow slit, such that its atoms may absorb excitation energy from photons incident from the white light source. The unabsorbed photons from the white light source will be incident on the screen with the original

intensity. After the absorption, the excited atoms will eventually go back to the ground state by emitting the same photons absorbed earlier. Absorption spectra of Hydrogen are shown in Fig. 21.13 (b).

SUMMARY

❖ When electromagnetic radiation is absorbed by a material, electronically-charged particles are released from the material (or even within itself). This phenomenon is called the photoelectric effect.

❖ Conservation of momentum implies that if two or more bodies interact with each other in an isolated system (that is, no external force is acting upon them), then the total momentum of all the bodies remains constant.

❖ The electrons which are emitted from a metal surface upon the influence of light are called photoelectrons.

❖ Threshold energy is the minimum amount of energy needed by the electron to break free from the metal and eject from it.

❖ Threshold frequency is the lowest frequency of electromagnetic radiation that will produce a photoelectric effect in a material.

❖ The threshold wavelength is the largest possible wavelength of the incident radiation which allows photoemission to take place. No photoemission occurs if the wavelength is higher than the threshold.

Formula Sheet

$E = hf$ 　　　　　 $K.E_{max} = eV_0$ 　　　　　 $E = \Phi + K.E_{max}$

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) \qquad \Delta x. \Delta p \geq \hbar \qquad \Delta E. \Delta t \geq \hbar$$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) Which of the following spectral regions has the highest energy?
   A. Infrared 　　　　 B. Violet 　　　　 C. Red 　　　　 D. Blue

2) Which of the following statements is true about a photon?
   A. A photon has zero mass and zero momentum.
   B. A photon has finite mass and a finite value of momentum.
   C. A photon has zero mass but finite value of momentum.
   D. A photon has finite mass but zero momentum.

3) What happens to the kinetic energy of the emitted electrons when the light is incident on a metal surface?
   A. It varies with the frequency of light. 　　　 B. It varies with the light intensity.
   C. It varies with the speed of light. 　　　 D. It varies irregularly.

4) A photoelectric cell is a device which
   A. Converts light energy into electricity.
   B. Converts electricity into light energy.
   C. Stores light energy.
   D. Stores electricity.

5) Which property does the Compton Effect describe about photons?
   A. Mass
   B. Momentum
   C. Wave properties
   D. Speed rates

6) What happens to a high energy photon after it strikes an electron?
   A. decreases frequency
   B. decreases wavelength
   C. increases energy
   D. increases momentum

7) Energy of gamma photon having a wavelength of 1A° is:

   A. 12.4×10^4 eV
   B. 12.4×10^4 eV
   C. 12.4×10^4 eV
   D. 12.4×10^4 eV

8) If uncertainty in the position of an electron is zero, the uncertainty in its momentum will be

   A. less than $h/4\pi$
   B. greater than $h/4\pi$
   C. zero
   D. infinite

9) What is the process by which a particle and its antiparticle annihilate each other?

   A. Pair production
   B. Pair annihilation
   C. Nuclear fission
   D. Nuclear fusion

10) Which of the following particles is produced during pair annihilation?

   A. Photon
   B. Electron
   C. Proton
   D. Neutron

Short Questions

1) Prove that energy and momentum of Photon are directly proportional to each other.
2) Convert 800 MeV into joules.
3) Write the two phenomena to describe photon-electron interaction.
4) In the interpretation of the photoelectric effect, how is it known that an electron does not absorb more than one photon?
5) How you can determine the work function from a plot of the stopping potential versus the frequency of the incident radiation in a photoelectric effect experiment? Can you determine the value of Planck's constant from this plot?
6) Speculate how increasing the temperature of a photo electrode affects the outcomes of the photoelectric effect experiment.
7) Which aspects of the photoelectric effect cannot be explained by classical physics?
8) What is the physical significance of Compton's effect?
9) Differentiate between Compton's shift and Compton's wavelength.
10) How can you say that scattering angle of photon plays effective role in measurement of Compton's shift?
11) What are the conditions required for pair production to occur?
12) What is meant by de Broglie wavelength?
13) Which phenomenon provides evidence for particulate nature of electromagnetic radiation?

Comprehensive Questions

1) What is photoelectric effect? How quantum physics explain its results.
2) What is Compton's effect? Discuss in detail.
3) State and discuss the Heisenberg Uncertainty Principle.

4) Discuss Pair production of electron as practical implementation of conversion of energy into mass correspondences to of Eisenstein energy-mass equation.
5) Explain Annihilation process as practical implementation of conversion of mass into energy correspondences to of Eisenstein energy-mass equation.
6) What do you understand by the term 'Dual nature of light'? Discuss.
7) Explain that electromagnetic radiation has a particulate nature.
8) Explain how electron microscopes achieve very high resolution.
9) There are discrete electron energy levels in isolated atoms. Explain this statement by the appearance and formation of emission and absorption line spectra.

Numerical Problems

1) In an experiment, the work function of potassium surface is found to be 2.1 eV. What should be the wavelength of incident radiation if the stopping potential for the electron is 0.43 V?

(Ans: 5.91×10^{-7} m)

2) Light of wavelength 5000 Å falls on a metal surface having work function of 1.9 eV. Find (a) Energy of photon in electron-volt. (b) Kinetic energy of photo-electrons emitted. (c) Stopping potential. (Ans: 2.48 eV, 0.58 eV, 0.58 V)

3) Calculate the energies of the photons associated with the (i) violet light of 413 nm. (ii) X-rays of 0.1 nm. (iii) radio waves of 10 m. (Ans: 3eV; 12424eV; 1.24×10^{-7} eV)

4) Calculate the minimum energy required by a photon to transfer half of its energy to an electron at rest. (Ans: 0.25 MeV)

5) An electron is placed in a box about the size of an atom that is about 1 Å. What is the velocity of the electron? (Ans: 7.29×10^6 m s^{-1})

6) A 50 keV X-ray is scattered through an angle of 90°. What is the energy of the X-ray after Compton scattering? (Ans: 45.5 keV)