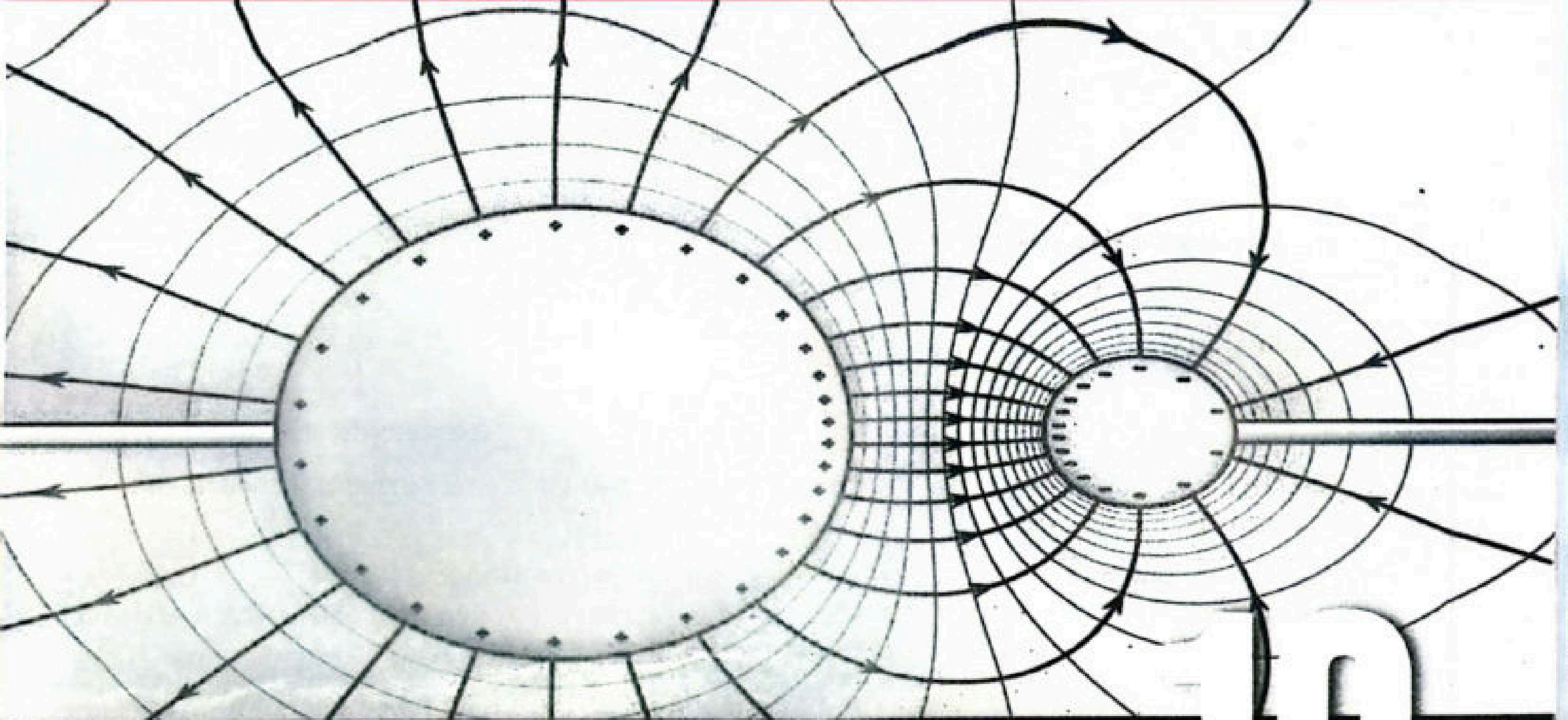


ELECTRIC POTENTIAL AND CAPACITOR

19

Student Learning Outcomes (SLOs)

The student will

- define and calculate electric potential [At a point as the work done per unit positive charge in bringing a small test charge from infinity to the point. Use $V = \frac{q}{4\pi\epsilon_o r}$ for the electric potential in the field due to a point charge].
- use the fact that the electric field at a point is equal to the negative of potential gradient at that point.
- state how the concept of electric potential leads to the electric potential energy of two point charges and use $E_p = \frac{qQ}{4\pi\epsilon_o r}$
- define and calculate capacitance [as applied to both isolated spherical conductors and to parallel plate capacitors].
- Derive and apply formulae for the combined capacitance of capacitors in series and in parallel.
- use the capacitance formula for capacitors in series and in parallel.
- determine the electric potential energy stored in a capacitor from the area under the potential charge graph [Use $W = 1/2QV = 1/2CV^2$ to solve physics related problems].
- analyze graphs of the variation with time of potential difference, charge and current for a capacitor discharging through a resistor [use $\tau = RC$ for the time constant for a capacitor discharging through a resistor]
- Use equations of the form $x = x_o \exp(-t/RC)$ [where x could represent current, charge or potential difference for a capacitor discharging through a resistor]
- List the use of capacitors in various household appliances [such as in flash guns, refrigerators, electric fans, rectification circuits, etc.]
- Illustrate how bioelectricity is generated in animals. [- cells control the flow of specific charged elements across the membrane with proteins that sit on the cell surface and create an opening for certain ions to pass through. These proteins are called ion channels.
  - When a cell is stimulated; it allows positive charges to enter the cell through open ion channels. The inside of the cell then becomes more positively charged, which triggers further electrical currents that can turn into electrical pulses, called action potentials.
  - The bodies of many organisms use certain patterns of action potentials to initiate the correct movements, thoughts and behaviors.]
- State that there are several species of aquatic life such as Electrophorus Electricus, that can naturally generate external electric shocks through internal biological mechanisms that act as batteries.
- Explain, with examples of animals with this ability, that electroreception is the ability to detect weak naturally occurring electrostatic fields in the environment.

Imagine a world where energy is rare and devices can't function efficiently. Understanding how energy is stored and released is crucial for harnessing its power. In this chapter, we'll explore the fundamental concepts of energy storage, starting with electric potential energy and its relationship to energy storage. We'll then explore the working of capacitors, devices that store energy and regulate its flow.

Through interactive examples, diagrams, and real-life applications, you'll gain a deeper understanding of: electric potential energy, capacitors and its combination, energy stored in capacitors, charging and discharging of capacitors. Presence of electric potential in living organism (i.e., bioelectricity) will also be discussed at the end of this unit.

By mastering these concepts, you'll unlock the secrets of energy storage and its role in shaping our modern world. Get ready to explore the exciting world of energy storage and its applications!

19.1 ELECTRIC POTENTIAL ENERGY AND ELECTRIC POTENTIAL

Electricity is a form of energy that powers our devices and gadgets. Electric circuits usually use electric energy and transfer it to other forms like heat, light, or motion. The stored energy of an electric circuit is called electric potential energy. For example, we know that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines. Batteries are typically a few volts, the outlets in your home produce 220 volts, and power lines can be as high as hundreds of thousands of volts. Energy and voltage are two different quantities. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. Here, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

Consider a point charge q_o is placed in between two oppositely charged parallel plates, as shown in Fig. 19.1. Force F experienced by the charge q_o in the electric field E is:

$$F = q_o E$$

If the charge q_o is allowed to move freely in an electric field, it will move from positive plate to negative plate and acquire kinetic energy. If the charge is moved against the electric field, energy is required.

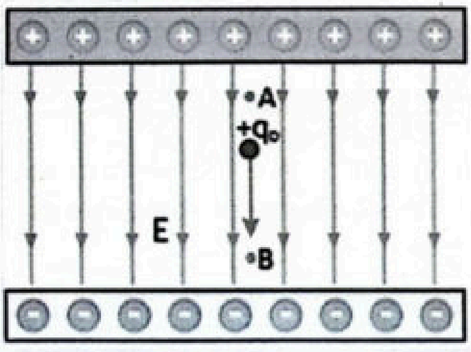


Figure 19.1: A charge is placed between two oppositely charged parallel plates.

Electric potential energy is the energy that is needed to move a charge against an electric field.

One needs more energy to move a charge further in the electric field, but also more energy to move it through a stronger electric field. So, there are two factors on which the electric potential energy of an object depends: Its own electric charge and its relative position with other electrically charged objects.

Thus, the electric potential energy can also be defined as:

Electric potential energy is the energy possessed by a unit charge if located at any point in the space, around a field charge.

The change in potential energy ΔU of charge q_o is equal to the "work done by the force in carrying the charge q_o from point B to the point A against the electrical field" i.e.,

$$\Delta U = W_{BA}$$

or $$U_A - U_B = W_{BA} \qquad \text{\_\_\_\_\_\_ (19.1)}$$

Where U_A and U_B represent the potential energies at point A and B respectively. W_{BA} is the work done by the force in carrying a positive charge q_o from point B to point A without disturbing the equilibrium state of the charge. Unit of electric potential energy is joule (J).

The concept of electric potential is used to express the effect of an electric field of a source in terms of the location within the electric field. A test charge with twice the quantity of charge would possess twice the potential energy at a given location; yet its electric potential at that location would be the same as any other test charge. A positive test charge would be at a high electric potential when held close to a positive source charge and at a lower electric potential when held further away. In this sense, electric potential becomes simply a property of the location within an electric field. Suppose that the electric potential at a given location is 12 Joules per coulomb, then that is the electric potential of a 1 coulomb or a 2 coulomb charged object. Stating that the electric potential at a given location is 12 joules per coulomb, would mean that a 2 coulomb object would possess 24 joules of potential energy at that location and a 0.5 coulomb object would experience 6 joules of potential energy at the location. Thus, Electric potential is purely location dependent. Electric potential is the potential energy per unit charge.

The potential difference between two points is defined as:

Potential difference is the work done in moving a unit positive charge from one point to another keeping the charge in electrostatic equilibrium.

If V_A and V_B are the electric potentials at points A and B, respectively, then the potential difference between these two points is

$$\Delta V = V_A - V_B = \frac{W_{AB}}{q_o} = \frac{\Delta U}{q_o} \qquad \text{\_\_\_\_\_\_ (19.2)}$$

It can also be written as:

$$\Delta U = q_o \Delta V \qquad \text{\_\_\_\_\_\_ (19.3)}$$

Unit of Electric Potential

The unit of electric potential is joule per coulomb (J/C) or volt (V).

The potential difference between two points in an electric field is one volt if one joule of work is done in moving one coulomb of charge from one point to the other.

So, $$1 \text{ volt} = \frac{\text{joule}}{\text{coulomb}}$$

The following multiples and sub-multiples of volt are commonly used;

$$1\ mV = 10^{-3}\ V,\ 1\ \mu V = 10^{-6}\ V,\ 1\ kV = 10^{3}\ V,\quad 1\ MV = 10^{6}\ V,\ 1\ GV = 10^{9}\ V$$

Example 19.1: You have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver?

Given: For the motorcycle battery, $q_m = 5000\ C$

For the car battery, $q_c = 60{,}000\ C$

$\Delta V = 12.0\ V$

To Find: a) $\Delta U_m = ?$ **b)** $\Delta U_c = ?$

Solution: a) The total energy delivered by the motorcycle battery is

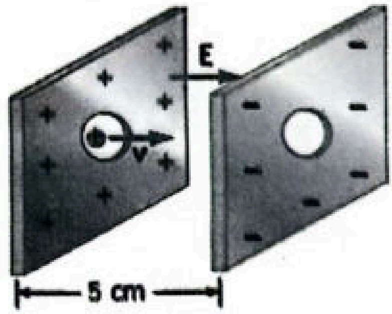
$$\Delta U_m = q\,\Delta V = (5000\ C)\ (12.0\ V) = 6.00 \times 10^4\ J.$$

b) The total energy delivered by the car battery is

$$\Delta U_c = q\,\Delta V = (60{,}000\ C)(12.0\ V) = 7.20 \times 10^5\ J$$

Assignment 19.1

A proton is injected at a speed of $1.0\times10^6\ m\ s^{-1}$ between two plates 5.0 cm apart, as shown in Figure. The proton accelerates across the gap and exits through the opening. (a) What must the electric potential difference be if the exit speed is to be $3.0\times10^6\ m\ s^{-1}$? (b) What is the magnitude of the electric field intensity between the plates, if it is assumed constant?



19.2 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

The equation of electric potential ΔV in constant electric field E is given by:

$$\Delta V = -E\,\Delta r$$

This equation is true only for constant electric field E. For most of the cases electric field is not uniform. Here we derive an expression of electric potential for such non uniform field. Let us suppose an isolated charge +Q is placed at origin of coordinate system, as shown in Fig. 19.2.

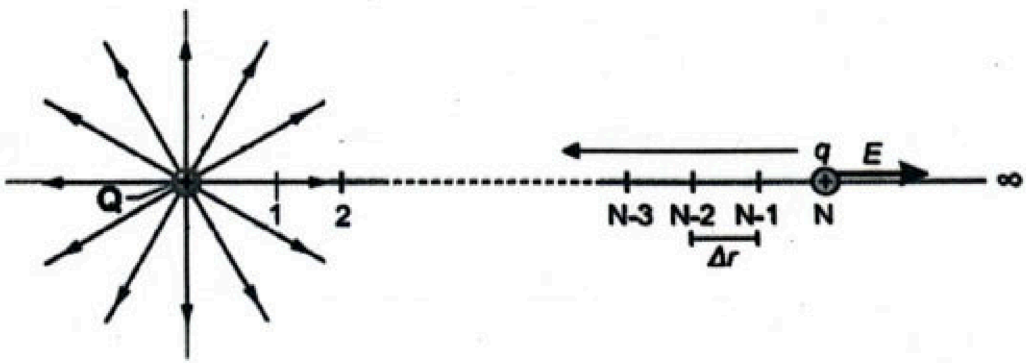


Figure 19.2: A test charge q is moved from a far point N to a point 1 against electric field E.

Let a test charge q is moved from a far point N to a point 1 against electric field E by applying force F. As the electric field does not remain constant but varies with the square of the distance from the charge +Q. So, we divide this large distance into small steps of length Δr such that E remains constant over each step. Here $r_1, r_2, r_3, \ldots, r_N$ are the distances of point 1, 2, 3, ... , N from charge Q, respectively. Let us find the electric potential from N to N−1.

The work done during the 1st step from N to N−1 is;

$$\Delta W_{N\to N-1} = F\,\Delta r\cos\theta$$

As $F = qE$, so we get;

$$\Delta W_{N\to N-1} = q\,E\,\Delta r\cos\theta$$

As E and Δr are in opposite direction, so we putting $\theta = 180^\circ$, hence;

$$\Delta W_{N\to N-1} = q\,E\,\Delta r\cos 180^\circ$$

$$= q\,E\,\Delta r\,(-1)$$

$$= -q\,E\,\Delta r$$

Putting $E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$ and $\Delta r = r_{N-1} - r_N$, we get;

So,
$$\Delta W_{N\to N-1} = -q\left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)(r_{N-1} - r_N) = q\left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)(r_N - r_{N-1})$$

As we know that r_N and r_{N-1} are very close to each other. So, for this step $r^2 = r_{N-1} r_N$, hence

$$\Delta W_{N\to N-1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{r_N - r_{N-1}}{r_{N-1} r_N}\right)$$

$$\Delta W_{N\to N-1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right) \quad \text{—— (i)}$$

Similarly,

$$\Delta W_{N-1\to N-2} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right) \quad \text{—— (ii)}$$

$$\vdots \qquad \vdots$$

$$\Delta W_{2\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \text{—— (iii)}$$

So total work done in bringing the charge from N to 1 is calculated by adding the work in each step, i.e.,

$$\Delta W_{N\to 1} = \Delta W_{N\to N-1} + \Delta W_{N-1\to N-2} + \ldots + \Delta W_{2\to 1}$$

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right) + \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right) + \ldots + \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left[\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right) + \left(\frac{1}{r_{N-2}} - \frac{1}{r_{N-1}}\right) + \ldots + \left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]$$

$$\Delta W_{N\to 1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_N}\right) \quad \text{—— (iv)}$$

If we bring the test charge q from infinity, then;

$$r_N = \propto \text{ and } \frac{1}{r_N} = \frac{1}{\propto} = 0$$

So Eq. (iv) becomes

$$\Delta W_{N\to1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - 0\right)$$

or

$$\Delta W_{N\to1} = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r_1}\right)$$

We replace r_1 by r, to obtain a general expression for the electric potential energy 'U' at distance r from Q, i.e.,

$$U = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r}\right) \quad \text{\_\_\_\_\_\_} \quad (19.4)$$

The electric potential at any point in an electric field is equal to work done in bringing a unit positive charge from infinity to that point keeping it in equilibrium. The electric potential at distance r from Q is calculated by using formula:

$$V = \frac{U}{q}$$

Using Eq. (19.4), we get:

$$V = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r}\right) \div q$$

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{Q}{r}\right) \quad \text{\_\_\_\_\_\_} \quad (19.5)$$

The Eq. (19.5) gives the potential at distance r from the charge Q. This equation shows that the potential is same at all the points that are equidistant from an isolated source charge Q.

19.3 CAPACITORS

Capacitors are a common component in most electronic devices. These are used to store energy. Typically, commercial capacitors have two identical, parallel conducting plates separated by a distance d, as shown in Fig. 19.3, is called a parallel plate capacitor. The medium between the plates is air or a sheet of some insulator. This medium is known as dielectric. When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, +Q and -Q, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance. When a charge is

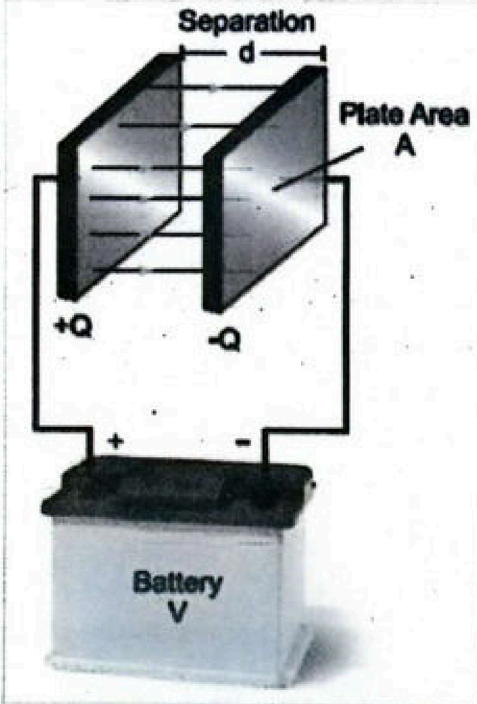


Figure 19.3: Parallel plate capacitor attached to a battery.

transferred to one of the plates say (A) due to electrostatic induction it would induce charge Q on the inner surface of the other plate (B). Mutual attraction between the charges keeps them bound on the inner surface of two plates and thus the charge remains stored in the capacitor even after removal of the battery.

Capacitance of a Capacitor

Experimentally it is seen that the charge stored on a capacitor is directly proportional to the voltage applied across it. If a charge 'Q' is transferred to one of the plates of a capacitor when the potential difference applied between the plates is 'V', then

$$Q \propto V$$

or $$Q = CV$$

or $$C = \frac{Q}{V} \quad \_\_\_\_\_\_ (19.6)$$

Where 'C' is a constant of proportionality, called the capacitance of capacitor. Capacitance can be defined as:

The capability of a capacitor to store charges is called its capacitance.

The unit of capacitance is the farad (F), named for Michael Faraday (1791-1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt.

If a capacitor stores a charge of 1 coulomb (a very large amount of charge) having the potential difference of 1 volt between the plates, then the capacitance is called 1farad.

One farad is, thus, a very large capacitance. Typical capacitors range from picofarad (1 pF = 10^{-12} F) to millifarad (1 mF = 10^{-3} F).

Capacitance of a Parallel Plate Capacitor

Consider a parallel plate capacitor connected to a voltage source, as shown in Fig. 19.4. Let the area of each plate is A and the separation between plates is d. It built +Q and −Q charges on the plates. If the positive plate is at potential V_1 and negative plate is at potential V_2, then the electric field strength E between the plates is:

$$E = -\frac{\Delta V}{\Delta r} = -\frac{(V_2 - V_1)}{d} = \frac{V_1 - V_2}{d}$$

$$E = \frac{V}{d} \quad \_\_\_\_\_\_ (i) \qquad (\because V_1 - V_2 = V)$$

Where, d is separation between the plates. The strength of the electric field also depends on the number of charges on the plates. As, Q is the charge on either of the plate of area A, so the surface charge density on the plate is:

$$\sigma = \frac{Q}{A}$$

Also, as the electric field intensity E between the plates of capacitor is given by applying Gauss's law:

$$E = \frac{\sigma}{\varepsilon_o} \quad \text{\_\_\_\_\_\_ (ii)}$$

substituting the value of σ in Eq. (ii), we get:

$$E = \frac{Q}{A\varepsilon_o} \quad \text{\_\_\_\_\_\_ (iii)}$$

Combining Eqs. (i) and (iii), we get:

$$\frac{Q}{A\varepsilon_o} = \frac{V}{d}$$

or

$$\frac{Q}{V} = \frac{A\varepsilon_o}{d}$$

Material Science and Technology

Capacitors are mainly made of ceramic, glass, or plastic, depending upon purpose and size. A computer chip contains number of capacitors.

As the medium between the plate is air, or vacuum, here we put $Q/V = C_{vac}$, hence we get:

$$C_{vac} = \frac{A\varepsilon_o}{d} \quad \text{\_\_\_\_\_\_ (19.7 a)}$$

When an insulating material called dielectric is inserted between the plates of a capacitor, then capacitance of a capacitor is enhanced by a factor of ε_r, and is given as:

$$C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d} \quad \text{\_\_\_\_\_\_ (19.7 b)}$$

By dividing Eq. (19.7) by Eq. (19.6), we get:

$$\varepsilon_r = \frac{C_{med}}{C_{vac}} \quad \text{\_\_\_\_\_\_ (19.8)}$$

The ratio of the capacitance of a capacitor with the dielectric medium between the plates to that of the capacitance of the same capacitor when the space is evacuated is called the relative permittivity ε_r of the material.

The equation $C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d}$ shows that value of capacitance C depends upon:

- The area of the plates.
- The distance between the plates.
- The medium (dielectric) between the plates.

Example 19.2: What is the capacitance of a parallel plate capacitor with metal plates, each of area 1.00 m^2, separated by 1.00 mm? What charge is stored in this capacitor if a voltage of 3.00×10^3 V is applied to it?

Given: Area of the plate = A = 1.00 m^2

Distance between the plates = d = 1.00 mm $\Delta V = 3.00 \times 10^3$ V

To Find: a) C = ? b) Q = ?

Solution: a) Here we use: $C = A\epsilon_0/d$

Entering the given values into the equation, we get:

$$C = (1.00)(8.85\times10^{-12}) \div 1.00\times10^{-3}$$

or $$C = 8.85\times10^{-9}\ F = 8.85\ nF$$

b) Now the charge stored in the capacitor can be found by the equation: $Q = C\,V$

Entering the known values into this equation gives:

$$Q = (8.85\times10^{-9})(3.00\times10^{3}) = 26.6\ \mu C$$

Assignment 19.2

What is the capacitance of a large Van de Graff generator's terminal, given that it stores 8.00 mC of charge at a voltage of 12.0 mV?

19.4 COMBINATIONS OF CAPACITORS

Several capacitors can be connected together to be used in a variety of applications. Multiple connections of capacitors behave as a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. Capacitors can be arranged in two simple and common types of connections, known as series and parallel combinations.

19.4.1 Series Combination of Capacitors

When the capacitors are connected plate to plate i.e., right plate of one capacitor is connected to left plate of the next capacitor, then it is called series combination. Fig. 19.4 shows series combination of three capacitors connected between points A and B.

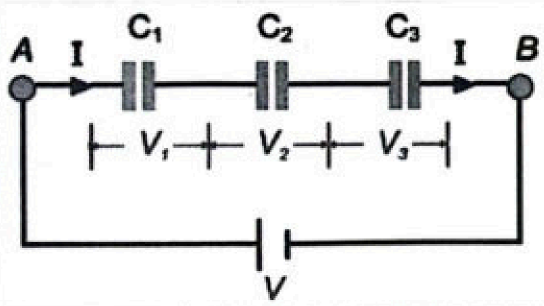


Figure 19.4: Series combination of capacitors.

If V is potential of the battery and V_1, V_2 and V_3 are potential of capacitors C_1, C_2 and C_3 respectively. Then;

$$V = V_1 + V_2 + V_3 \quad \text{\_\_\_\_\_\_ (19.9)}$$

As the battery will supply same amount of charge Q to each capacitor, so putting values of V_1, V_2 and V_3 in Eq. (19.9), we get:

$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} \quad \text{\_\_\_\_\_\_ (i)}$$

Here, we use the formula Q = CV or C = Q/V. In series combination, due to *electrostatic induction* charge on each capacitor is same and equal to charge supplied by battery i.e.,

$$Q = Q_1 = Q_2 = Q_3$$

So, Eq. (i) becomes:

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \quad \text{or} \quad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Put $\frac{V}{Q} = \frac{1}{C_e}$, so above equation becomes:

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Similarly, for n number of capacitors connected in series, we can write as:

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \quad \text{\_\_\_\_\_ (19.9)}$$

The Eq. (19.9) show that the equivalent capacitance of series capacitors is always less than any individual capacitance in the combination.

19.4.2 Parallel Combination of Capacitors

When two or more capacitors are connected between the same two points, then the combination is called parallel combination. Figure 19.5 shows parallel combination of three capacitors connected between points A and B.

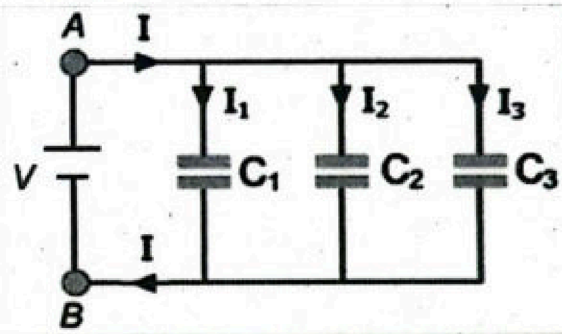


Figure 19.5: Parallel combination of capacitors.

If Q is charge supplied by the battery and Q_1, Q_2 and Q_3 are charges stored on each capacitor C_1, C_2 and C_3 respectively. Then

$$Q = Q_1 + Q_2 + Q_3 \quad \text{\_\_\_\_\_ (i)}$$

As the battery will supply same amount of voltage V to each capacitor, so putting values of Q_1, Q_2 and Q_3 in Eq. (i), we get:

$$Q = C_1 V + C_2 V + C_3 V$$

Or

$$Q = V(C_1 + C_2 + C_3)$$

$$\frac{Q}{V} = C_1 + C_2 + C_3 \quad \text{\_\_\_\_\_ (ii)}$$

Put $\frac{Q}{V} = C_e$ in Eq. (ii), we get:

$$C_e = C_1 + C_2 + C_3$$

Similarly, for 'n' number of capacitors connected in parallel, we can write as:

$$C_e = C_1 + C_2 + C_3 + \ldots \quad \text{\_\_\_\_\_ (19.10)}$$

The Eq. (19.10) shows that the equivalent capacitance of parallel capacitors is larger than any of the individual capacitances.

For Your Information

If N identical capacitors of capacitance C are connected in series, then effective capacitance is C/N.
If N identical capacitors of capacitance C are connected in parallel, then effective capacitance is CN.

Example 19.3: Find the equivalent capacitance of two capacitors of capacitance 4 μF and 8 μF. (a) When connected in series (b) When connected in parallel.

Given: $C_1 = 4\ \mu F$ $\quad$ $C_2 = 8\ \mu F$

To Find: a) For series combination, C_e = ? $\quad$ b) For parallel combination, C_e = ?

Solution: a) **For series combination**

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad \frac{1}{C_e} = \frac{C_1 + C_2}{C_1 C_2}$$

or $$C_e = \frac{C_1 C_2}{C_1 + C_2}$$

Putting values, we get:

$$C_e = \frac{(4\mu)(8\mu)}{4\mu + 8\mu} = \frac{32 \times 10^{-12}}{12 \times 10^{-6}} = 2.66 \times 10^{-6}\ F = 2.66\ \mu F$$

b) For parallel combination

$$C_e = C_1 + C_2$$

or $$C_e = 4\ \mu F + 8\ \mu F = 12\ \mu F$$

Assignment 19.3

Find the equivalent capacitance of three capacitors of capacitance 4 μF, 6 μF and 8 μF, connected in series.

19.5 ENERGY STORED IN A CAPACITOR

Capacitor stores energy in the form of electric field. Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge 'Q' and voltage 'V' on the capacitor. The capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage 'V' on it. The charge 'Q' on the capacitor is directly proportional to its potential difference 'V'. The graph of charge against potential difference is therefore a straight-line graph through the origin, as shown in the Fig. 19.6.

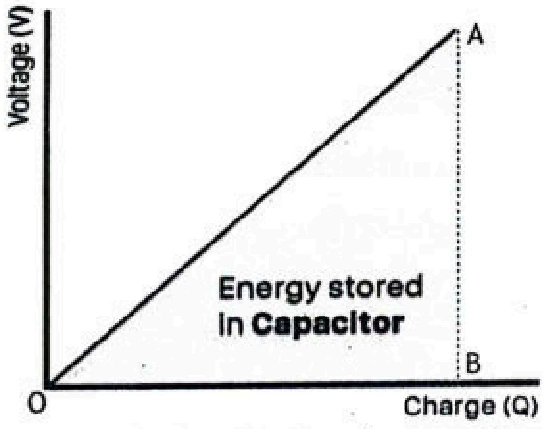


Figure 19.6: Graph of Potential-Difference verses Charge.

The electric potential energy (E) stored in a capacitor can be determined from the area under the potential-charge graph which is equal to the area of a right-angled triangle:

$$E = \text{area of a right-angled triangle OAB}$$

or $$E = \frac{1}{2} \times Base \times Height$$

Where the base is Q, and the height is V, i.e.,

$$E = \frac{1}{2} Q V \quad \text{\_\_\_\_\_ (9.11 a)}$$

This energy is directly related to the amount of charge moved onto the plates and the voltage across them. Substituting Q = CV, the energy can also be defined as:

$$E = \frac{1}{2}(CV) \times V$$

or $$E = \frac{1}{2} CV^2 \quad \text{\_\_\_\_\_ (9.11 b)}$$

By substituting the potential V=Q/C, the work done can also be defined in terms of just the charge and the capacitance:

or $$E = \frac{Q^2}{2C} \quad \text{\_\_\_\_\_ (9.11 c)}$$

The Eqs. (19.11 a, b, c) are valid for calculating energy stores in a capacitor.

For Your Information

A battery stores electrical energy and releases it through chemical reactions. This means that it can be quickly charged but the discharge is slow. Unlike the battery, a capacitor temporarily stores electrical energy through distributing charged particles on plates to create a potential difference. A capacitor can take a shorter time than a battery to charge up and it can release all the energy very quickly.

Uses of Capacitor

Capacitors are very important circuit element in Electronics. Following are a few applications of capacitor in daily life:

- Capacitors are used for filtering unwanted frequencies in radio and TV set.
- Capacitors are used in camera flashes to store and quickly release a large amount of electrical energy.
- Refrigerator capacitors are usually used to keep the compressor running.
- Capacitor is used to increase speed of the fan.
- Capacitor is used in rectification circuit to act as a filter to reduce ripple voltage.

Example 19.4: Capacitance of a capacitor is 50 F. It is charged to a potential of 100 V. Calculate the energy stored in it.

Given: C = 50 F V = 100 V
To Find: E = ?

Solution: Here we use the formula:

$$E = \frac{1}{2} CV^2$$

Putting values, we get:

$$E = \frac{1}{2}(50)(100)^2$$

$$E = 250 \times 10^3\ J \qquad \text{or} \qquad E = 250\ kJ$$

Assignment 19.4

Calculate the energy stored in a capacitor having 5 C of charge and a potential difference of 15 V.

19.6 CHARGING AND DISCHARGING OF A CAPACITOR

Consider an RC (resistor-capacitor) circuit for charging of a capacitor, as shown in Fig. 19.7 (a). When the switch is set at point 1, a battery of e.m.f 'ε' starts charging the capacitor through the resistor R. The charge builds up gradually on the plates to the maximum value of Q_o.

When a capacitor charges, electrons flow onto one plate and move off the other plate. This process will be continued until the potential difference across the capacitor is equal to the e.m.f of the battery.

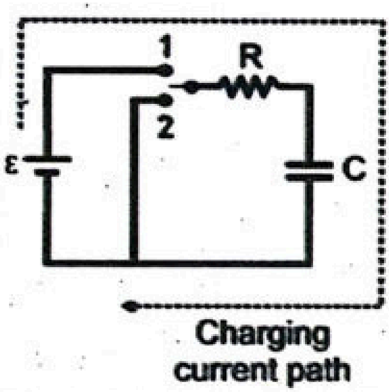


Figure 19.7 (a): Charging circuit for a capacitor.

The rate of flow of charge will not be linear. At the start, the current will be at its highest value but will gradually decrease to zero. Voltage on the capacitor is initially zero and rises rapidly at first, since the initial current is a maximum. Suppose at t = 0, charge on a capacitor is zero i.e. Q = 0. It can be shown experimentally that after time t, as charge builds up on the plates, it repels more charge that is arriving so the current drops as the charge on the plates increases. Charging will stop when the potential difference between the capacitor plates is equal to the e.m.f. of the battery.

Maximum charge on capacitor = Capacitance × e.m.f. of battery

Experiments show that the charging process of a capacitor is not linear but shows the exponential behavior. The exponential behavior for charging of capacitor is written in equation form as:

$$Q = Q_o\left(1 - e^{-t/RC}\right) \qquad \text{———— (19.12)}$$

Where, e = 2.7182 is a constant. The graph of charging process, between time t and charge Q, is shows in Fig. 19.7 (b). According to this graph, Q = 0 at t = 0 and increases gradually to its maximum value Q_o.

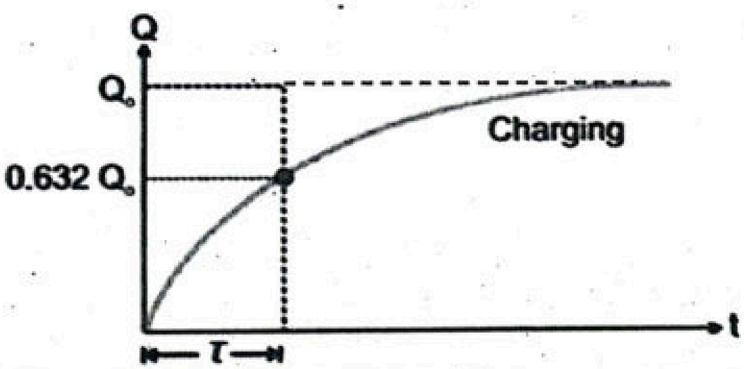


Figure 19.7 (b): Charging graph for a capacitor.

The time during which 63.2 % of its maximum value charge is deposited on the plates of the capacitor is called time constant (τ) of an RC circuit. This can be seen by putting t=RC in Eq. (19.12), i.e.,

$$Q = Q_o\left(1-e^{-1}\right) = Q_o\left(1-\frac{1}{2.718}\right) = Q_o(0.632)$$

or $\frac{Q}{Q_o} = 0.632$ or $\frac{Q}{Q_o} = 63.2\ \%$

Smaller the resistance or the capacitance, the smaller the time constant, the faster the charging and the discharging rate of the capacitor, and vice versa.

When a capacitor is discharging, the current will be highest at the start. This will gradually decrease until reaching 0, when the current reaches zero, the capacitor is fully discharged as there is no charge stored across it. The circuit for discharging of a charged capacitor is shown in Fig. 19.8 (a).

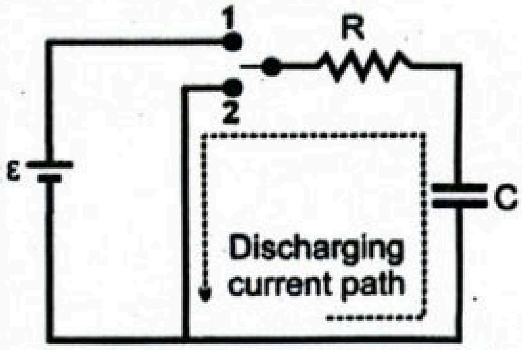


Figure 19.8 (a): Discharging circuit for a capacitor.

When the switch is set at terminal 2, the charge $-Q$ on the negative plate can now flow through the resistance and neutralize the charge $+Q$ on the positive plate of the capacitor. Assume that a fully charged capacitor begins discharging at time t = 0. It can be shown that charge left on either plate at time t is:

$$Q = Q_o e^{-t/RC} \qquad \text{\_\_\_\_\_\_ (19.13}$$

The graph of discharging in Fig. 19.8 (b) shows that discharging begins at t = 0 when Q = Q_o, and decreases gradually. When t = RC, the magnitude of charge remaining on each plate is:

$$Q = Q_o\ (0.367)$$

or $\frac{Q}{Q_o} = 0.367$

or $\frac{Q}{Q_o} = 36.7\ \%$

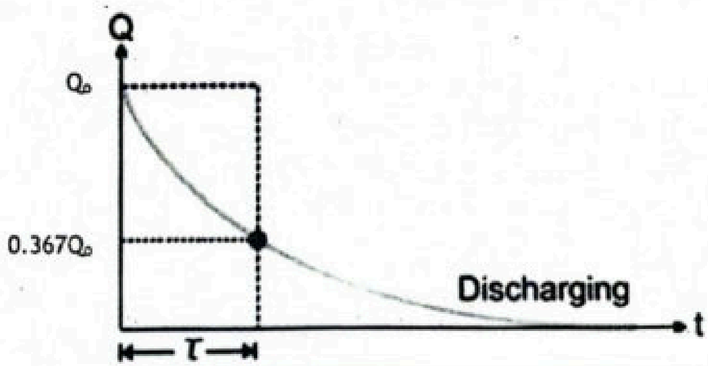


Figure 19.8 (b): Discharging graph for a capacitor.

A capacitor with smaller values of time constant (RC), lead to a more rapid discharge. The Eq. (19.12) and Eq. (19.13) can also be used for current and potential difference, if written as:

For Charging: $x = x_o\left(1-e^{-t/RC}\right)$ ______ (19.14)

For Discharging: $x = x_o\, e^{-t/RC}$ ______ (19.15)

Here x could represent current, charge or potential difference for a capacitor discharging through a resistor.

19.7 BIOELECTRICITY

Bioelectricity refers to the generation or action of electric currents or voltages in biological processes. Electric activity in living tissue is a cellular phenomenon that depends on the cell membrane. The cell membrane acts like a capacitor that stores energy as electrically charged ions on opposite sides of the cell membrane.

In most solutions, ions of a given electric charge are accompanied by ions of opposite charge, so that the solution itself has no net charge. If two solutions of different concentrations are separated by a membrane that allows one kind of ion to pass but block the other, producing equal and opposite net charges in the two solutions. This concentration imbalance gives rise to an electric-potential difference between the solutions.

In living cells, there are two types of solutions: those found inside and outside the cell. The cell membrane (as shown in Fig. 19.9), separating inside from outside, is semipermeable. Cell membrane allows certain ions to pass through while blocking others. In particular, nerve-cell and muscle-cell membranes are slightly permeable to positive potassium ions (K+), which diffuse outward, leaving a net negative charge in the cell. Ion channels allow specific inorganic ions (such as: Na^+, K^+, Ca^{2+}, or Cl^-) to diffuse rapidly down their electrochemical gradients across the lipid bilayer.

Cells control the flow of specific charged elements across the membrane with proteins that sit on the cell surface and create an opening for certain ions to pass through. These proteins are called ion channels. Ion channel spans the membrane and make hydrophilic tunnels across it, allowing their target molecules to pass through by diffusion. Channels are very selective and will accept only one type of molecule (or a few closely related molecules) for transport.

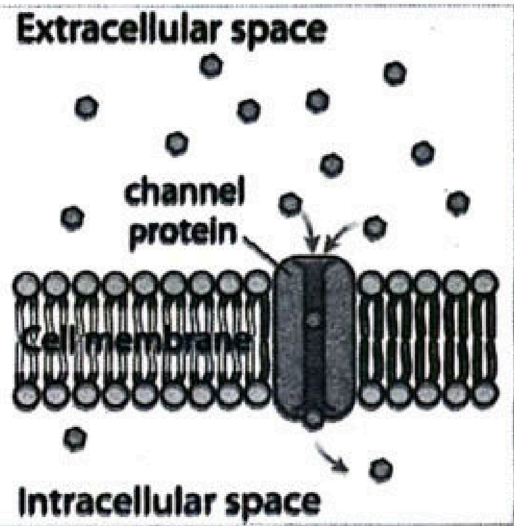


Figure 19.9: A channel protein, which forms a tunnel allowing a specific molecule to cross the membrane (down its concentration gradient).

When a cell is stimulated; it allows positive charges to enter the cell through open ion channels. The inside of the cell then becomes more positively charged, which triggers further electrical currents that can turn into electrical pulses, called action potentials. The bodies of many organisms use certain patterns of action potentials to initiate the correct movements, thoughts and behaviors.

The bioelectric potential across a cell membrane is typically about 50 millivolts; this potential is known as the resting potential. Bioelectric phenomena include fast signaling in nerves and the triggering of physical processes in muscles or glands.

There are several species of aquatic life, such as Electrophorus Electricus, that can naturally generate external electric shocks through internal biological mechanisms that act as batteries.

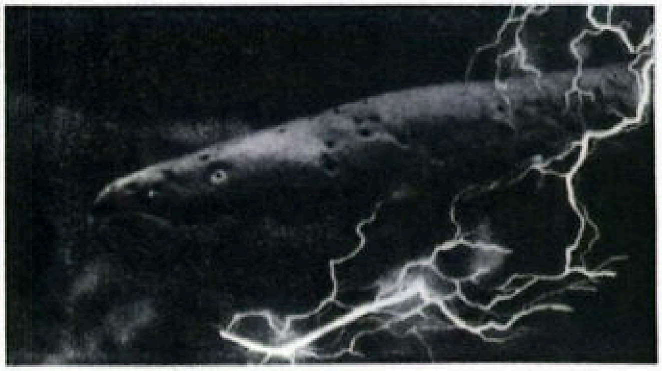
Such species are called electrogenic animals. Electric eels are probably the best known electrogenic animals. Some types of bacteria, yeast and fish are also electrogenic.

There are also animals that can detect electricity. They're called electroreceptive. Some electroreceptive animals are echidnas, platypuses, bees, spiders, dolphins, sharks and rays.

Most electrogenic animals are also electroreceptive. But there are many electroreceptive animals that are not electrogenic.

For Your Information

Electric eels are known for their ability to stun their prey by generating electricity, delivering shocks up to 860 volts.



SUMMARY

- The potential difference between two points is defined as "the work done in moving a unit positive charge from one point to another keeping the charge in electrostatic equilibrium".
- The potential difference between two points in an electric field is one volt if one joule of work is done in moving one coulomb of charge from one point to the other.
- The potential gradient represents the rate of change of potential with displacement.
- Capacitors are electronic devices used to store electric energy.
- The capability of a capacitor to store charges is called its capacitance.
- If a capacitor stores a charge of 1 coulomb (a very large amount of charge) having the potential difference of 1 volt between the plates, then the capacitance is called 1 farad.
- Equivalent capacitance of series capacitors is always less than any individual capacitance in the combination.
- The equivalent capacitance of parallel capacitors is larger than any of the individual capacitances.
- Bioelectricity refers to the generation or action of electric currents or voltages in biological processes.
- Electric eels are known for their ability to stun their prey by generating electricity, delivering shocks up to 860 volts.
- Electroreception is the ability of some animals to detect weak naturally occurring electrostatic fields in the environment.

Formula Sheet

$U_A - U_B = W_{BA}$

$\Delta U = q_o \Delta V U = \frac{Qq}{4\pi\varepsilon_o}\left(\frac{1}{r}\right)$

$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{Q}{r}\right)$

$Q = CV$

$\sigma = \frac{Q}{A}$

$C_{vac} = \frac{A\varepsilon_o}{d}$

$C_{med} = \frac{A\varepsilon_o\varepsilon_r}{d}$

$\varepsilon_r = \frac{C_{med}}{C_{vac}}$

$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$

$C_e = C_1 + C_2 + C_3 + \ldots$

$E = \frac{1}{2}CV^2$

$E = \frac{Q^2}{2C}$

$Q = Q_o\left(1 - e^{-t/RC}\right)$

$Q = Q_o\, e^{-t/RC}$

EXERCISE

Multiple Choice Questions

Encircle the correct option.

1) Which of the following is equivalent to 1.0 volt.

   A. newton/second B. joule/second

   C. joule/coulomb D. coulomb/joule

2) The quantity electric potential is defined as the amount of _____.

   A. electric potential energy B. force acting upon a charge

   C. potential energy per unit charge D. force per unit charge

3) A negatively charged particle is placed in a uniform electric field directed from South to North. In which direction will the particle move after it is released?

   A. East B. South C. North D. North-West

4) What is the charge that appears on the plates of the 10 pF capacitor if it is connected to a battery of 9 V.

   A. 90 pC B. 90 mC C. 90 kC D. 90 μC

5) What is the voltage that should be applied to the 5 μF capacitor to accumulate 1μC charge on its plates?

   A. 0.2 V B. 0.2 mV C. 0.2 kV D. 0.2 μV

6) Capacitor is a device used to_________

   A. store electrical energy B. vary the resistance

C. store magnetic energy D. dissipate energy

7) What is the total capacitance when three capacitors, C_1, C_2 and C_3 are connected in parallel?

A. $\frac{C_1}{C_2+C_3}$ B. $C_1+C_2+C_3$ C. $\frac{C_2}{C_1+C_3}$ D. $\frac{1}{C_1+C_2+C_3}$

8) If three capacitors of capacitances 1 F, 2 F and 10 F are connected in parallel then their equivalent capacitance will be:

A. 10 F B. 15 F C. 13 F D. 20 F

9) When capacitors are connected in parallel, the total capacitance is always ________ the individual capacitance values.

A. Greater than B. Less than C. Equal to D. Cannot be determined

10) What is the capacitance of a capacitor?

A. The ratio of charge to electric potential difference

B. The ratio of electric potential difference to charge

C. The product of charge and electric potential difference

D. The ratio of electric field strength to charge density

11) What is the electric potential difference between two points?

A. The work done in moving a unit charge between the two points

B. The force exerted on a unit charge at one of the points

C. The electric field strength between the two points

D. The distance between the two points

12) A 220 Ω resistor is in series with a 2.2 μF capacitor. The time constant is

A. 48 μs B. 480 μs C. 2.42 μs D. 24 μs

13) The formula for electrostatic potential is ___________.

A. Electrostatic potential = Work done × charge

B. Electrostatic potential = Work done/charge

C. Electrostatic potential = Work done + charge

D. Electrostatic potential = Charge Work/done

Short Questions

1) What is the relationship between electric potential and electric potential energy?

2) If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

3) What are the units of (a) electric potential difference (b) electric potential energy (c) Capacitance?

4) What is the net amount of charge on a charged capacitor?

5) Write some applications of capacitors in real life.

6) Would you place the plates of a parallel-plate capacitor closer together or farther apart to increase their capacitance?

7) What is meant by electroreception?

8) If you were asked to design a capacitor in which small size and large capacitance were required, what would be the two most important factors in your design?

9) If a capacitor is fully charged and then left for discharging. How much charge will be left on the plate of the capacitor after time equal to one time constant?

Comprehensive Questions

1) Define and explain the term electric potential. Derive an expression for electric potential at a field point due to a source charge.

2) What is meant by potential gradient? Explain.

3) By using graph, derive an expression for the energy stored in a capacitor.

4) Derive the expression for the equivalent capacitance for parallel combination of capacitors.

5) Derive the formula for the combined capacitance of capacitors in series.

6) Explain the process of charging and discharging of a capacitor.

7) How bioelectricity is generated in animals? Explain.

Numerical Problems

1) A heart defibrillator delivers 4×10^2 J of energy by discharging a capacitor initially at 1×10^4 V. What is its capacitance?

(Ans: 8 μF)

2) Two capacitors of capacitance $C_1 = 6\ \mu$ F and $C_2 = 3\ \mu$ F are connected in series. Calculate the equivalent capacitance.

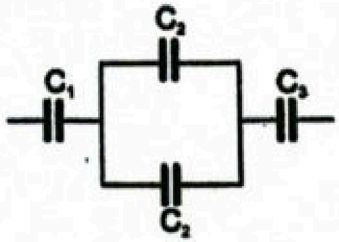
(Ans: 2μF)

3) A $2200\mu F$ capacitor is charged up with a 1.5 V cell. Calculate the charge and energy stored in the capacitor.

(Ans: 3.3 mC, 0.00248 J)

4) Find the equivalent capacitance for the following circuit if $C_1 = 1$ pF, $C_2 = 0.5$ pF, $C_3 = 1$ pF.

(Ans: 1/3 pF)



5) Suppose electrons in a TV tube are accelerated through a potential difference of 2.0×10^4 V from the heated cathode (where they are produced) toward the screen (which also serves as the anode), 25.0 cm away.

(a) At what speed would the electrons impact the phosphors on the screen? Assume they accelerate from rest and ignore relativistic effects.

(b) What's the magnitude of the electric field, if it is assumed constant?

(Ans: 8.38×10^7 m s^{-1} (b) 8.0×10^4 V m^{-1})

6) A network of five capacitors of capacitance C is connected to a 100 V supply, as shown in below figure. Determine the equivalent capacitance of the network. **(Ans: 5C/6)**