A TEXTBOOK OF

PHYSICS XII

Balochistan Textbook Board, Quetta.

بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

شروع اللہ کے نام سے جو بڑا مہربان نہایت رحم کرنے والا ہے

A TEXTBOOK OF

PHYSICS XII

Publisher: **New College Publication Quetta**

for

BALOCHISTAN TEXTBOOK BOARD, QUETTA

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Syed Arif Shah Sherazi
Chairman
Balochistan Textbook Board, Quetta.
Ph: 081-2470501
btbb_quetta@yahoo.com

According to the National Curriculum 2006 under the National Textbook and Learning Material Policy 2007. N.O.C No. 1137-41/CB Dated 22/07/2020. Office of the Director Bureau of Curriculum & Extension Centre, Balochistan, Quetta.

Author

Mr. Ghulam Rabbani
Associate Professor,
Govt: Postgraduate Science College Quetta

Ms. Shumaila Fazal
Assistant Professor
Govt: Girls Postgraduate College Quetta Cantt:

Editor

Dr. Hamdullah Khan Tareen (Ph.D Physics)
Professor,
Department of Physics, BUITEMS Quetta.

Dr. Syed Muzamil Bokhari
Assistant Professor, (Department of Physics)
University of Balochistan

Internal Review Committee

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Mr. Wali Muhammad
Lecturer
Govt: Postgraduate Science College, Quetta

Mr. Muhammad Ramzan Barech
Lecturer
Govt: Postgraduate Science College, Quetta

Mr. Abdul Shakoor (Coordinator)
Subject Specialist
Balochistan Text Book Board Quetta

Provincial Review Committee

Dr. Ajab Khan Kasi
Associate Professor
Department of Physics, University of Balochistan

Mr. Ahmed Jan
Associate Professor
Govt: Postgraduate Degree College, Quetta

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Sardar Bahadur Khan Women's University

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Deputy Director (Curriculum)
Bureau of Curriculum and Extension Center

Supervision: **Din Muhammad** (Subject Specialist) BTBB Quetta.

Prepared by: New College Publication, Quetta

Graphics: Farrukh Iqbal / Shoaib Sahil

Printer: New College Publication, Quetta

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Standard abbreviations and symbols used in text

Symbol	Meaning	Symbol	Meaning
A	ampere	Hz	hertz
A°	Angstrom	J	joule
α	temperature co-efficient	K	kelvin
B	magnetic field	K	boltzmann's constant
B	magnetic field density	L	inductance
C	coulomb	m_o	rest mass of electron
C	capacitance	N	north pole
c	speed of light	N	n-type semi conductor
°C	degree celsius	P	dipole moment
Cal	calorie	P	p-type semi conductor
Ci	curie	P	primary coil
D	debye	p	instantaneous power
E	electric field	ρ	resistivity
E	battery (cell)	q	charge
ε	induced e.m.f	Q	total charge
ε_o	permittivity of free space	R	resistance
ε_r	relative permittivity	R_h	rydberg constant
e^-	electron	r	internal resistance
e^+	positron	S	south pole
ev	electron volt	S	secondary coil
F	force	s	second
F	farad	Sv	sievert
F°	degree fahrenheit	T	tesla
f	frequency	T	time period
G	gauss	$T_{1/2}$	half life
G	galvanometer	τ	time constant
G	conductance	τ	torque
σ	conductivity	u	atomic mass unit
σ	charge density	U	potential
I	direct current	V	velocity
i	alternating current	V	potential difference (volt)
H	henry	v	alternating voltage
H	magnetizing force	Wb	weber
h	planck's constant	ω	angular frequency
λ	wavelength	Z	Impedance
λ	decay constant	ϕ	electric flux
η	efficiency	ϕ_B	magnetic flux
γ	photon	μ_o	permeability of free space

Unit 11 ELECTROSTATICS

Major Concepts (21 PERIODS)	Conceptual Linkage
• Force between charges in different media • Electric field • Electric field of various charge configurations • Electric field due to a dipole • Electric flux • Gauss's law and its applications • Electric potential • Capacitors • Energy stored in a capacitor	This chapter is built on Electrostatics Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

- state Coulomb's law and explain that force between two point charges is reduced in a medium other than free space using Coulomb's law.
- derive the expression $E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$ for the magnitude of the electric field at a distance 'r' from a point charge 'q'.
- describe the concept of an electric field as an example of a field of force.
- define electric field strength as force per unit positive charge.
- solve problems and analyze information using $E = F/q$.
- solve problems involving the use of the expression $E = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r^2}$
- calculate the magnitude and direction of the electric field at a point due to two charges with the same or opposite signs.
- sketch the electric field lines for two point charges of equal magnitude with same or opposite signs.
- describe the concept of electric dipole.
- define and explain electric flux.
- describe electric flux through a surface enclosing a charge.

- state and explain Gauss's law.
- describe and draw the electric field due to an infinite size conducting plate of positive or negative charge.
- sketch the electric field produced by a hollow spherical charged conductor.
- sketch the electric field between and near the edges of two infinite size oppositely charged parallel plates.
- define electric potential at a point in terms of the work done in bringing unit positive charge from infinity to that point.
- define the unit of potential.
- solve problems by using the expression $V = \frac{W}{q}$.
- describe that the electric field at a point is given by the negative of potential gradient at that point.
- solve problems by using the expression $E = \frac{V}{d}$.
- derive an expression for electric potential at a point due to a point charge.
- calculate the potential in the field of a point charge using the equation $V = \frac{1}{4\pi\varepsilon_o} \cdot \frac{q}{r}$.
- define and become familiar with the use of electron volt.
- define capacitance and the farad and solve problems by using C=Q/V.
- describe the functions of capacitors in simple circuits.
- solve problems using formula for capacitors in series and in parallel.
- explain polarization of dielectric of a capacitor.
- demonstrate charging and discharging of a capacitor through a resistance.
- prove that energy stored in a capacitor is W=1/2QV and hence $W=1/2CV^2$.

INTRODUCTION

Electrostatics is a branch of physics in which we study about the charges at rest. In this unit, we will not only discuss the behaviour of charged particles at rest but also introduce laws governing it. For example, Coulomb's law which explains the electrostatic force of attraction or repulsion between two stationary point charges separated by some distance. Here a general question arises, how is the electrostatic

> *FOR YOUR INFORMATION*
>
> Sometime a static electricity causes a big problem at gasoline filling station, where the spark can ignite gasoline vapours and make fire. A good safety rule is to touch metal and discharge static charge from your body before you fuel.

force exerted when the charged particles are separated? This "action at a distance" force can be explained by using the electric field theory, i.e. every charge q produces an electric field around it. Now if another charge q_o is brought near to q, then the electric field of q exerts a force on q_o. In the same way, we will also develop the concept of electric field in terms of electric lines of force. These electric lines of force can be studied by using Gauss's law. We will also study these electric lines of force passing through different conducting media called flux. All these are very useful in the understanding of various electrostatic phenomena. Similarly, we will explain the electric potential gained by a charged particle due to work done on it in an electric field between a pair of parallel and opposite charged plates. In the last section of this unit, we will introduce the most important device capacitor and its capacitance in the absence and presence of dielectrics.

11.1 CHARGE AND ITS PROPERTIES

Like mass, electric charge is the intrinsic property of some particles such as electrons etc. The presence of charge causes it to exert or experience a force when it is placed in an electric or magnetic field. Charge is a basic property of matter which is responsible for all electric and magnetic interactions.

A charged comb attracts small pieces of paper.

It is a common experience that when we comb our hair by a plastic comb, then it may attract the nearby small pieces of paper. It is therefore, the comb has gained charges through the rubbing process. The charge which is created by rubbing two objects against each other is called static charge. There are two kinds of charges, which were named as negative and positive by Benjamin Franklin (1706-1790). It is explained by a

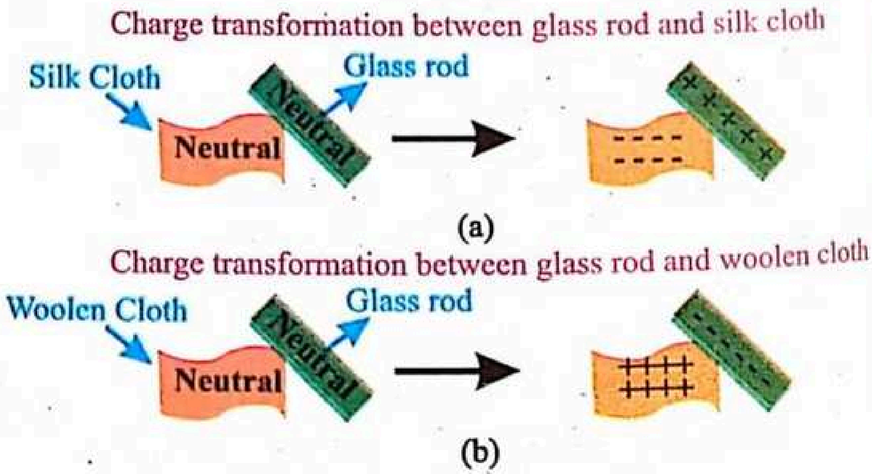


Fig.11.1(a) Transfer of negative charges from glass rod to silk cloth under the process of rubbing (b) transfer of charges from woolen cloth to plastic rod.

simple experiment. When a glass rod is rubbed with silk cloth, electrons are transferred from the glass rod to the silk cloth. The silk cloth now has an excess of electrons and hence becomes negatively charged. Whereas, the glass rod which has deficiency of electrons becomes positively charged as shown in Fig.11.1(a). Similarly, when a hard plastic rod is rubbed with woolen cloth then the wool becomes positively charged whereas, the plastic rod becomes negatively charged. It is shown in Fig. 11.1(b).

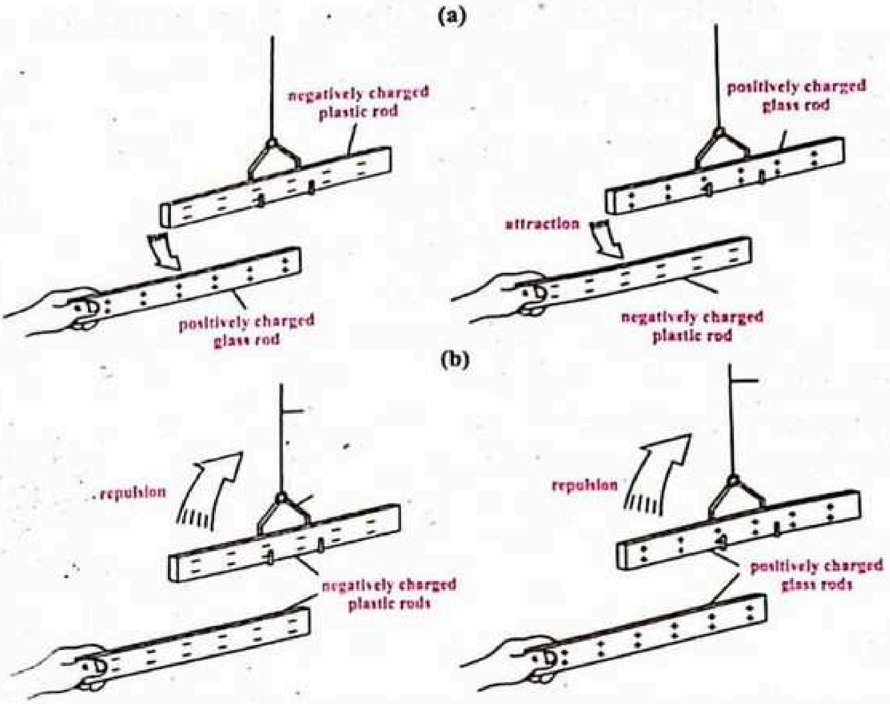


Fig.11.2(a) Unlike charged rods attract to each other. (b) like charged rods repel to each other.

If the positively charged glass rod is brought near to the negatively charged suspended plastic rod then it will attract the plastic rod as shown in Fig. 11.2(a). Similarly, when the negatively charged plastic rod is brought near to the suspended plastic rod which is also negatively charged then they will repel each other as shown in Fig.11.2(b) The same result of repulsion will be observed when two positively charged glass rods are brought near each other. Based on these observations, we have two most important results. First, like charges repel each other and unlike charges attract each other. Secondly, if one material gains some charges, the same magnitude of charges is lost by the other material during the rubbing process. This is known as law of conservation of charges, which states that **"electric charge can neither be created nor be destroyed or the total charges of an isolated system remains constant."**

DO YOU KNOW?

A charge is transferred from one body to another by electrons.

POINT TO PONDER

If an object transfers its charges to another object, will its mass be affected?

POINT TO PONDER

If 2×10^9 electrons are transferred from a glass rod to a piece of silk by rubbing, what will be the net charges on both objects?

FOR YOUR INFORMATION

Like energy and momentum, charge is also a conserved and quantized quantity.

Electric charge is a scalar quantity and its SI unit is coulomb 'C'. As we know that electron is a sub-atomic particle of an atom, it carries negative charge of magnitude 1.6×10^{-19} C. A proton is also a sub-atomic particle of an atom and it carries positive charge of magnitude 1.6×10^{-19} C. This shows that electron and proton both have charges of same magnitude but opposite in nature. One coulomb corresponds to the amount of charge on 6.25×10^{18} electrons. As an atom contains equal number of protons and electrons, so it is electrically neutral at normal condition. It has been observed that the charges are transferred from one body to another body in discrete form. That is, a body may have charges of value $\pm 1e, \pm 2e, \pm 3e \ldots$ respectively but not $\pm 0.5e, \pm 1.5e, \pm 2.1e$ ... etc. where e is charge of an electron or proton.

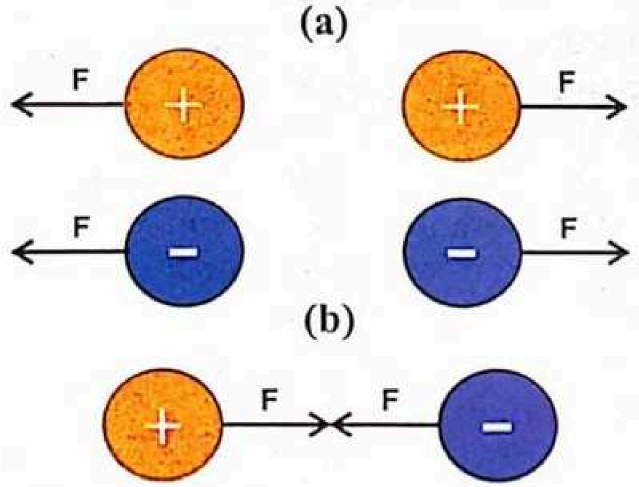


Fig.11.3(a) repulsive force between two like point charges (b) attractive force between two unlike point charges.

11.2 COULOMB'S LAW

As we have discussed that the like charges repel and unlike charges attract each other with a force called electrostatic force of repulsion and attraction respectively. This force was first quantitatively studied by a French military engineer Charles Coulomb under different conditions and formulated a law in 1785 A.D. known as Coulomb's Law, which is stated as:

The magnitude of electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance between them.

> *POINT TO PONDER*
> Does an electrostatic force exist between a charged and an uncharged bodies?

Mathematically, it is expressed as:

Let two point charges q_1 and q_2 which are separated by a distance 'r' as shown in Fig.11.3. According to Coulomb's law the electrostatic force of attraction or repulsion between them is given as:

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

By combining these two relations,

> *DO YOU KNOW?*
> Both electrostatic force between charges and gravitational forces between masses obey inverse square law.

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k\frac{q_1 q_2}{r^2} \quad ......(11.1)$$

where 'k' is constant of proportionality known as Coulomb's or electrostatic constant. Its value depends upon the nature of the medium between the charges. If the medium between the two point charges is free space then the value of 'k' in terms of SI units is $9 \times 10^9\ Nm^2/C^2$ and it can be calculated by the following relation:

$$k = \frac{1}{4\pi\varepsilon_o} \quad ......(11.2)$$

Putting the value of electrostatic constant 'k' from Eq. 11.2 in Eq. 11.1

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} \quad ......(11.3)$$

where 'ε_o' is known as permittivity of free space and its value is $8.85\times10^{-12}\ C^2N^{-1}m^{-2}$.

As force is a vector quantity and the line of action of electrostatic force is along the position vector '$\vec{r}$', so eq. 11.3 can be expressed in term of the unit vector of $\hat{r}$ as;

> **POINT TO PONDER**
> What would be the magnitude of electrostatic force between two point charges, when the separation between them is doubled?

$$\vec{F} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\hat{r} \quad ......(11.4)$$

Eq. (11.4) is the vector form of Coulomb's law.

The electrostatic force between two point charges is a mutual force, i.e. both charges exert the forces on each other, which are same in magnitude but in opposite directions. Let $\vec{F}_{12}$ to be the force which is exerted by q_1 on q_2 along the line joining the two point charges represented by unit vector $\hat{r}_{12}$ as shown in Fig.11.4. Similarly, $\vec{F}_{21}$ be the force which is exerted by q_2 on q_1 along the line joining the two points charges represented by unit vector $(-\hat{r}_{21})$. The negative sign shows that $\hat{r}_{21}$ in opposite direction with respect to $\hat{r}_{12}$. Thus by using Eq. 11.4 the two forces $\vec{F}_{12}$ and $\vec{F}_{21}$ can be expressed as:

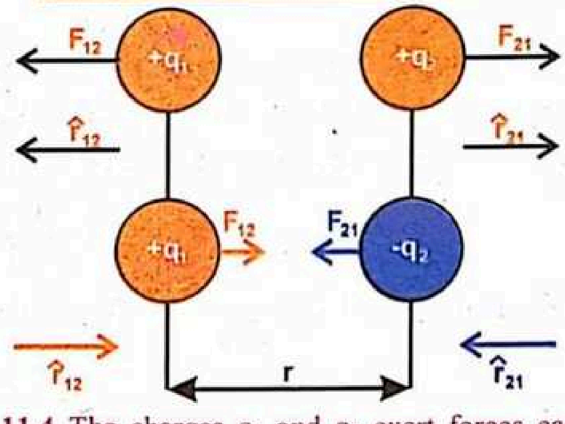


Fig.11.4 The charges q_1 and q_2 exert forces each other which are same in magnitude but in opposite direction.

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{12} \qquad ......(11.5)$$

Similarly,

$$\vec{F}_{21} = \frac{-1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{21} \qquad ......(11.6)$$

Comparing eq. 11.5 and eq. 11.6

$$\vec{F}_{12} = -\vec{F}_{21} \quad ......(11.7)$$

Equation 11.3 shows that the electrostatic force not only depends upon the magnitude of charges and distance between the charges but also depends upon the medium between the charges. If the medium between the charges is an insulating material which is known as dielectric, then we introduce a relative permittivity ε_r which is defined as the ratio between the permittivity of insulating medium ε to the permittivity of free space ε_o, that is:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_o} \quad ......(11.8)$$

Table 11.1
The values of relative permittivity for different materials for static electric field

Material	Relative Permittivity (ε_r)
Vacuum	1.0000
Air	1.0006
PTFE, FEP (Teflon)	2.0
Polypropylene	2.20 to 2.28
Polystyrene	2.4 to 3.2
Wood (Oak)	3.3
Bakelite	3.5 to 6.0
Wood (Maple)	4.4
Glass	4.9 to 7.5
Wood (Birch)	5.2
Glass-Bonded Mica	6.3 to 9.3
Porcelain, Steatite	6.5

Relative permittivity is also known as dielectric constant and its value is always greater than one (as shown in table 11.1). For static electricity, the electrostatic force ' F′ ' between point charges for insulating medium is given as

$$\vec{F'} = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}\hat{r}$$

Using Equation 11.6 $\varepsilon = \varepsilon_o\varepsilon_r$. Therefore

$$\vec{F'} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q_1q_2}{r^2}\hat{r} \quad ...(11.9)$$

By comparing Eq. 11.4 and 11.7 we have

$$\vec{F'} = \frac{1}{\varepsilon_r}\vec{F} \quad ......(11.10)$$

Since, $\frac{1}{\varepsilon_r} < 1$

So, $\vec{F'} < \vec{F}$

This result shows that the electrostatic force of attraction or repulsion between two point charges is greater, if the medium between them is a free space.

Example 11.1

How many electrons constitute a charge of 2C.

Solution:

number of electrons (n) = ?

total charge = Q = 2C

charge on an electron = e = $1.6 \times 10^{-19}C$

charge on 'n' electrons = Q = ne

$$n = \frac{Q}{e}$$

$$n = \frac{2C}{1.6 \times 10^{-19} C}$$

$$n = 1.25 \times 10^{19} \text{ electrons}$$

Example 11.2

The Hydrogen atom consists of a single electron which is orbiting around the nucleus and contains a single proton in its nucleus. Calculate the ratio between electrostatic force and gravitational force between electron and proton in hydrogen atom.

Solution:

Charge on an electron = $q_1 = 1.6 \times 10^{-19}C$

Charge on a proton = $q_2 = 1.6 \times 10^{-19}C$

Mass of electron = $m_1 = 9.11 \times 10^{-31}$ kg

Mass of proton = $m_2 = 1.67 \times 10^{-27}$ kg

Coulomb's constant (k) = 9×10^9 Nm^2 c^{-2}

Gravitational constant (G) = 6.673×10^{-11} Nm^2 kg^{-2}

Distance between electron and proton = r

Now, the electrostatic force between electron & proton is given as

$$F_e = k\frac{q_1 q_2}{r^2} \text{ ......(i)}$$

Similarly, the gravitational force between electron & proton is given as

$$(F_g) = G\frac{m_1 m_2}{r^2} \text{ ......(ii)}$$

Dividing eq. (i) by eq. (ii)

$$\frac{F_e}{F_g} = \frac{k\frac{q_1 q_2}{r^2}}{G\frac{m_1 m_2}{r^2}}$$

$$\frac{F_e}{F_g} = \frac{kq_1 q_2}{Gm_1 m_2}$$

$$\frac{F_e}{F_g} = \frac{(9\times10^9\,\text{Nm}^2\text{c}^{-2})(1.6\times10^{-19}\,\text{C})(1.6\times10^{-19}\,\text{C})}{(6.673\times10^{-11}\,\text{Nm}^2\text{kg}^{-2})(9.11\times10^{-31}\,\text{kg})(1.67\times10^{-27}\,\text{kg})}$$

$$\frac{F_e}{F_g} = \frac{2.304\times10^{-28}}{1.014\times10^{-67}}$$

$$\frac{F_e}{F_g} = 2.27\times10^{39}$$

POINT TO PONDER

Does an electric field exist in empty space?

DO YOU KNOW?

There is no electric field inside the conductor.

The result shows that the electrostatic force between an electron and a proton is extremely large i.e. 2.27×10^{39} times greater than the gravitational force between them for the given same distance.

11.3 ELECTRIC FIELD AND ELECTRIC FIELD INTENSITY

In Coulomb's law, we have studied the electrostatic force of attraction and repulsion between every two point charges. According to field theory, the electrostatic force of attraction and repulsion is transmitted from one charged body to another through a field called electrostatic field, which exists in the region around every electric charge. Electric field is defined as, **the region or space around a charged body in which another charged body can experience force of attraction or repulsion.**

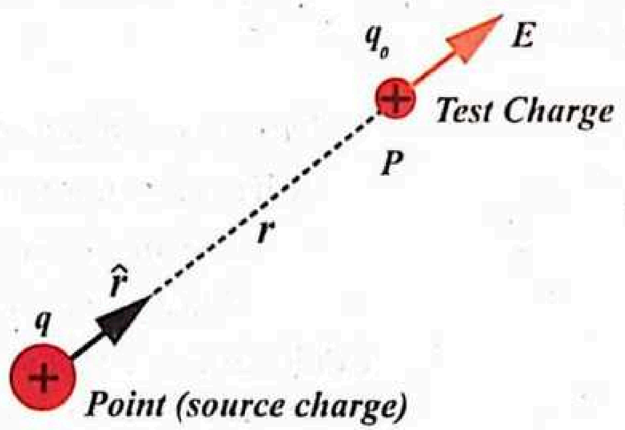


Fig.11.5 Electric field intensity E due to a force by a point charge q on a test charge q_0.

The charge which has a considerable electric field around it, is known as a point charge. It may be positive or negative. Usually, the point charge is being considered as a source charge. Similarly, the charge which has negligible field is known as test charge. It is always taken as a unit positive charge and it is being used to determine the strength of the field of the point charge. The source charge has a very large value of charge as compared with test charge. We assume that test charge moves in the electric field of source charge without causing any appreciable change in its field.

The strength of the electric field at any point is known as electric field intensity or electric intensity. Electric field strength is a vector quantity and it has both magnitude and direction. It is explained as:

Let an electric field is produced by a positive point charge 'q' in the space around it. In order to examine whether, the field exists in space, we introduce a unit positive test charge q_o. As the test charge is very very small in magnitude and it does not affect or disturb the field of the point charge so the electrostatic force is exerted by a point (source) charge on a test charge as shown in Fig.11.5 and it is known as electric intensity. It is represented by $\vec{E}$ and is also defined as a force per unit positive charge q_0 i.e.,

$$\vec{E} = \frac{\vec{F}}{q_o} \quad ......(11.11)$$

Electric intensity is a vector quantity. Its direction is the same as that of the force. It is directed outwards from positive source charge and directed inwards for a negative source charge. The SI unit of electric intensity is NC^{-1}.

11.4 ELECTRIC INTENSITY DUE TO A POINT CHARGE

Consider an electric field due to a point charge q as shown in Fig.11.7. To determine the strength of this field at point 'p' at a distance 'r' from surface of point charge 'q', we place a test charge 'q_o' at that point. Now according to Coulomb's law, the electrostatic force between them is given as

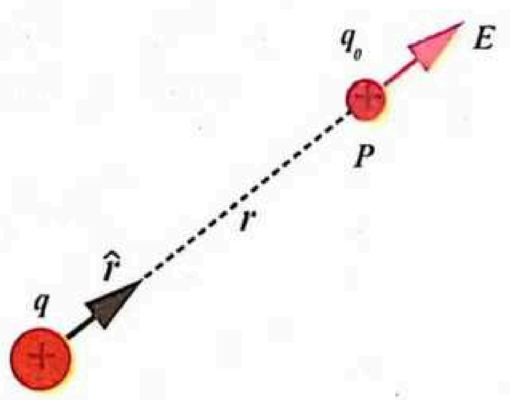


Fig.11.7 Electric intensity due to a point charge q at point 'P' at a distance r from q.

$$F = \frac{1}{4\pi\varepsilon_o}\frac{qq_o}{r^2}$$

$$\frac{F}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

But $\quad E = \dfrac{F}{q_o}$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad ......(11.12)$$

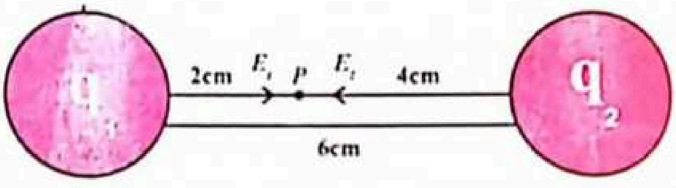
As $\vec{E}$ is a vector quantity, its direction is same as that of the force. So, eq. 11.12 can be expressed in vector form as

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} \quad ...(11.13)$$

This results shows that electric intensity depends upon magnitude of point charge, distance and medium.

Example 11.3

Two charges of the magnitude $15 \times 10^{-9}C$ and $30 \times 10^{-9}C$ are separated by a distance 6cm. Find the intensity of electric field at a point which is at a distance 2cm from the first and 4cm from the second charge respectively.



FOR YOUR INFORMATION
If the charges on a conductor are at rest then the conductor is in static equilibrium.

Solution:

$$q_1 = 15 \times 10^{-9}C$$
$$q_2 = 30 \times 10^{-9}C$$

Distance between q_1 & $q_2 = r = 6cm = 0.06m$

Distance between q_1 & $P = r_1 = 2cm = 0.02m$

Distance between q_2 & $P = r_2 = 4cm = 0.04m$

1. Electric intensity E_1 due to q_1 at point P and at distance $r_1 = 0.02m$ from q_1 is given as

$$E_1 = k\frac{q_1}{r_1^2} = \frac{9\times10^9\times15\times10^{-9}}{(0.02)^2} = \frac{135}{0.0004}$$
$$E_1 = 3.375 \times 10^5\ NC^{-1}$$

2. Electric intensity E_2 due to q_2 at point P and at a distance $r_2 = 0.04m$ from q_2 is given as

$$E_2 = k\frac{q_2}{r_2^2} = \frac{9\times10^9\times30\times10^{-9}}{(0.04)^2} = \frac{270}{0.0016}$$
$$E_2 = 1.69 \times 10^5\ NC^{-1}$$

The resultant intensity 'E' at point P is given by

$$E = E_1 - E_2$$
$$E = 3.375 \times 10^5 - 1.69 \times 10^5$$
$$E = 1.685 \times 10^5 \ NC^{-1}$$ which is directed towards the larger charge i.e., q_2.

11.5 ELECTRIC FIELD LINES:

We have discussed that an electric field is a region around a point charge where an electrostatic force acts on another charged body placed in such region. The electric field is represented by imaginary lines which was introduced by Michael Faraday and these are called electric lines of force. **An electric line of force is the path along which a small positive test charge, placed in an electric field of a point charge can move if free to do so.** The lines of force of an electric field may be straight or curved but its direction is always from positive to negative charged body. It is explained under the following different field patterns.

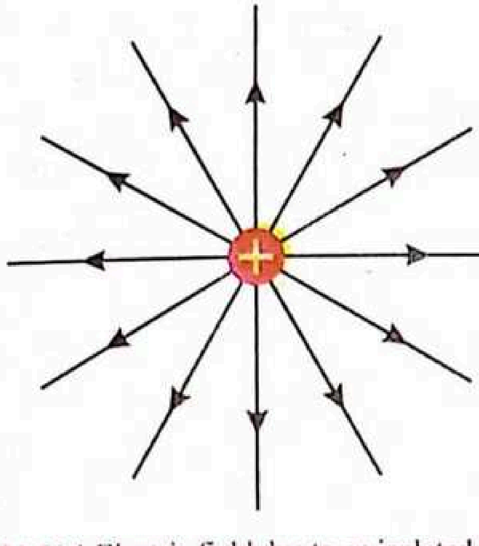


Fig.11.6(a) Electric field due to an isolated positive charge (+q) and the direction of lines are outwards.

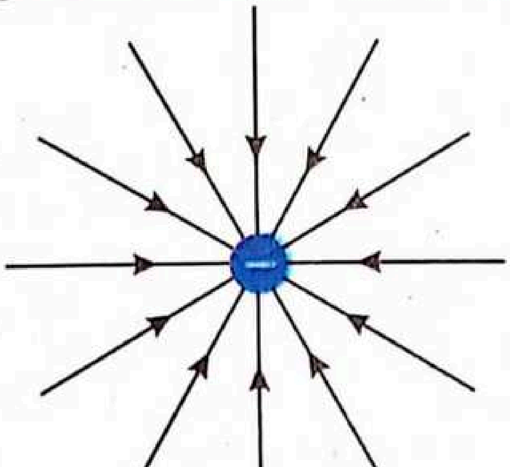


Fig.11.6(b) Electric field due to an isolated negative charge (−q) and the direction of lines are inwards.

When a field is produced by an isolated positive charge (+q) then the lines of force are directed radially outward from the charge in all directions as shown in Fig.11.6(a). It is therefore, when we introduce a positive test charge in this filed, it would be repelled by a point charge in outward direction.

Similarly, when a field is due to an isolated negative charge $(-q)$, then the lines of force of the field are directed radially inward as shown in Fig.11.6(b). If we place a positive test charge in this field, it would be attracted by the point charge in inward direction.

If the field is due to two charges of same magnitude, but one is positive and other is negative then the lines of force start from positive charge surface and terminate at the negative charge surface, as shown in Fig.11.6(c). In this case, we can observe both straight and curved lines. The direction of field at any point along a curved line can be determined by drawing a tangent at that point.

The Fig.11.6(d) shows the field pattern due to two positive charges of same magnitude. In this case the lines of force are repelled to one another. Again, we can observe both straight and curved lines. The strength of the field at the middle point between the two charges is zero. This point is called neutral point of the field.

From the above discussion, we have some important results which are summarized as:

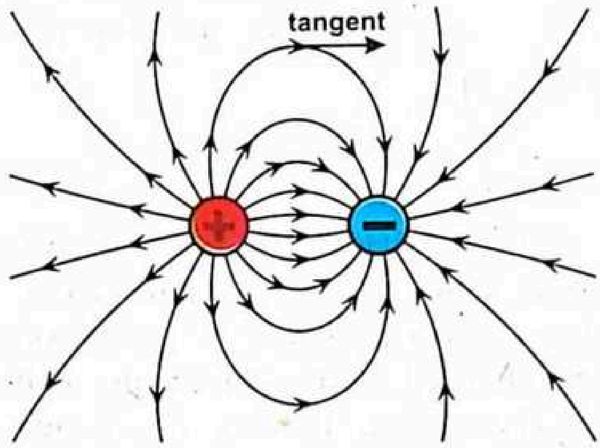


Fig.11.6(c) The electric field pattern due to two charges of same magnitude but having opposite

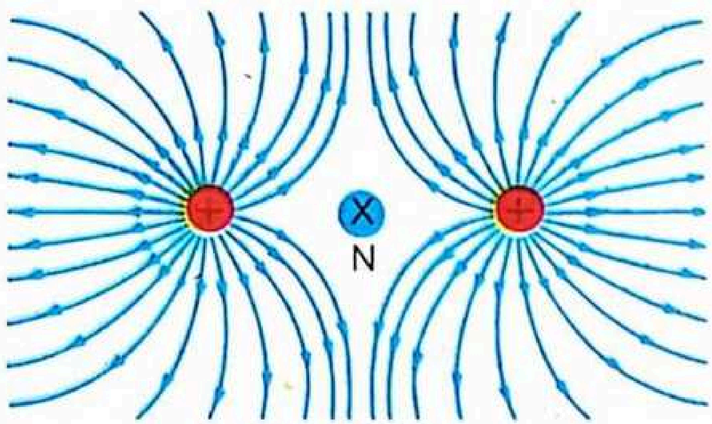


Fig.11.6(d) The electric field pattern due to two charges of same magnitude and having same signs.

1. The electric lines of force initiate from positive charge surface and terminate at negative charge surface.
2. The number of lines drawn is directly proportional to the magnitude of the charge.
3. Curved lines show non-uniform electric field while straight and parallel lines indicate uniform electric field.
4. The tangent at any point on a curved line gives the direction of the field.
5. The lines of force do not exist inside the charged body.
6. The lines of force are closer when the field is stronger, and the lines are farther apart when the field is weaker.
7. Two lines of force never cross each other. If they cross then at the point of intersection, electric intensity would have more than one direction but only one magnitude and it is not possible.

11.6 ELECTRIC DIPOLE

A pair of charges of same magnitude, but one is positive and other is negative separated by a small distance is known as electric dipole. For example, many molecules such as, H_2O, HCl etc. behave as electric dipoles in their normal states. The charges are distributed in these molecules in such a way that the centre of positive charge does not coincide with centre of negative charge, as a result, one end of the molecule is positively charged and the other is negatively charged as shown in Fig. 11.7.

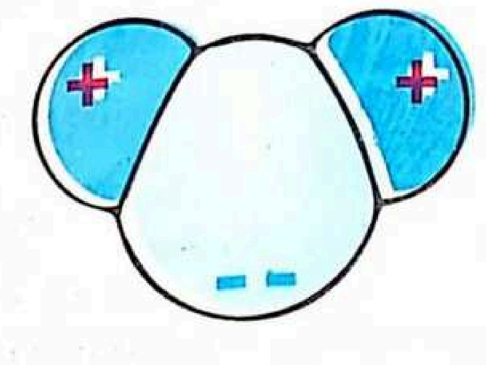


Fig.11.7.A molecule of H_2O behaves as an electric

Let an electric dipole which consists of two charges of equal magnitude but opposite nature or signs (q and −q) separated by a small distance 'd' as shown in Fig.11.8. In electric dipole, the resultant charge is equal to zero, but its corresponding field exists, because the charges are placed at some distance. The line joining the charges is known as dipole axis. It is a vector and its direction is from negative charge to positive charge.

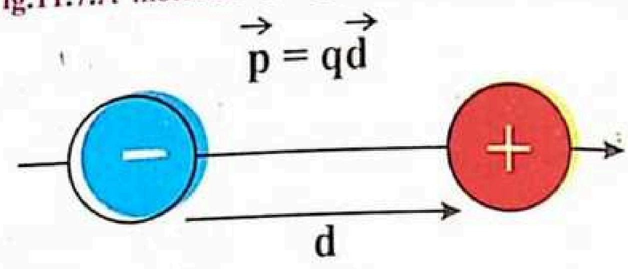


Fig.11.8 Electric dipole due to two point charges of same magnitude but having opposite signs separated by a distance d.

Now the product of magnitude of either of the charge and the distance of separation between the two charges is known as dipole moment. It is represented by $\vec{p}$ and it is expressed as

$$\vec{P} = q\vec{d} \quad ......(11.14)$$

Dipole moment is a vector quantity. Its direction is along the dipole axis, i.e., from negative charge to positive charge. The SI unit of a dipole moment is C m, which is a larger unit. However, a smaller unit used for dipole moment is debye (D).

11.6.1 Electric field due to a dipole

Let an electric field due to a dipole which consists of two charges of equal magnitude, but of opposite nature or signs and are separated by a distance 2d from each other. Let we calculate the strength of this field at point 'C' at a distance 'r' from centre 'O' of the dipole. Such that C is at a distance r − d from the positive charge +q and at a distance r + d from the negative charge −q as shown in Fig.11.9.

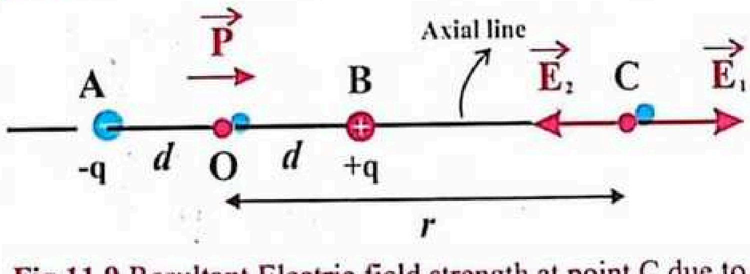


Fig.11.9 Resultant Electric field strength at point C due to a dipole.

The field due to positive charge +q at point 'C' at a distance r − d is given by

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} \quad ......(11.15)$$

Similarly, the field due to negative charge $-q$ at point 'C' and at a distance r + d is given by

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{-q}{(r+d)^2} \quad ......(11.16)$$

The resultant field at point C is given by;

$$E = E_1 + E_2$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{(r-d)^2} - \frac{1}{4\pi\varepsilon_o}\frac{q}{(r+d)^2}$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{(r+d)^2-(r-d)^2}{(r-d)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{4rd}{\left(r^2-d^2\right)^2}\right)$$

$$E = \frac{q}{4\pi\varepsilon_o}\left(\frac{4rd}{\left(r^2\left(1-\frac{d^2}{r^2}\right)\right)^2}\right)$$

As $\frac{d^2}{r^2}$ is very small so it can be neglected.

$$E = \frac{1}{4\pi\varepsilon_o}\frac{2dq(2r)}{r^4(1-0)}$$

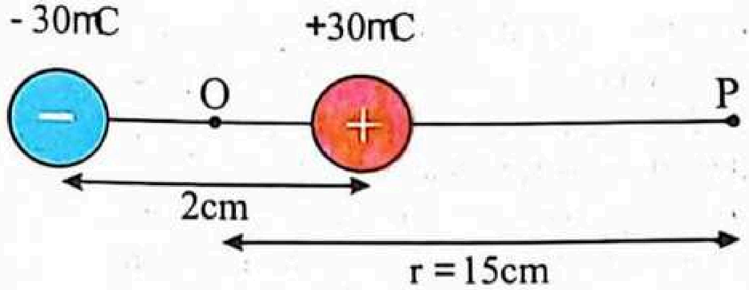
But $\quad (2d)(q) = P$ (Dipole moment)

$$E = \frac{1}{2\pi\varepsilon_o}\frac{P}{r^3} \quad ......(11.17)$$

This is the electric field intensity at a point due to a dipole, which depends upon dipole moment, distance and medium.

Example 11.4

An electric dipole consists of two charges $+30\mu C$ and $-30\mu C$ separated by a distance of 2cm. Calculate the electric field intensity at a point P on the axial line at a distance of 15cm from the mid-point of the dipole.



Solution:

We have,

$$\text{Charge, } q_1 = +30\mu C = 30\times10^{-6}C$$

$$\text{Charge, } q_2 = -30\mu C = -30\times10^{-6}C$$

$$\text{Distance between dipole} = d = 2cm = 0.02m$$

$$\text{Electric field intensity at point } P = E = ?$$

$$\text{Distance between mid-point of dipole and point } P = OP = r = 15cm = 0.15m$$

$$\text{Electrostatic constant} = k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 Nm^2C^{-2}$$

By definition of electric intensity due to a dipole

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3}$$

But $P = qd$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2qd}{r^3}$$

$$E = \frac{9\times10^9 Nm^2C^{-2}\,2\left(30\times10^{-6}C\times0.02m\right)}{(0.15m)^3}$$

$$E = \frac{10.8\times10^3}{0.003375}$$

$$E = 3200\times10^3$$

$$E = 3.2\times10^6 NC^{-1}$$

11.7 ELECTRIC FLUX

We have studied that an electric field is represented by electric lines of force, which originate from a positively charged surface and terminate at a negatively charged surface. Now when an object having an area is placed in an electric field such that the direction of the area is parallel to the direction of electric field as shown in Fig.11.10 then the electric lines of force pass through it and is called electric flux. The word flux comes from a Latin word which means to flow or penetration of field lines through some surface. **Thus, the electric flux is defined as the number of electric lines of force passing through a certain area held perpendicular to the field.**

It is usually represented by a Greek letter ϕ and it depends upon the strength of the field (Electric intensity), the size of the surface area it passes through, and on how the area is oriented with respect to the field. That is, stronger the electric field $\vec{E}$, larger will be the electric flux ϕ. Similarly, the larger the area, the more field lines pass through it, the greater the flux.

Based on these results, the electric flux is defined as the product of electric intensity E and vector area A.

$$\phi = EA \quad ...(11.18)$$

Now if the area is not held perpendicular but it makes an angle 'θ' with the field $\vec{E}$ as shown in Fig.11.11. In this case we take the base component A_x of area $\vec{A}$ which is parallel to the

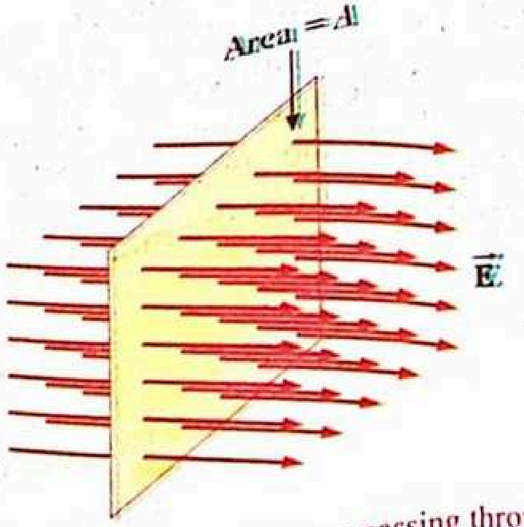


Fig.11.10 Electric lines of forces passing through a surface area perpendicular to the field.

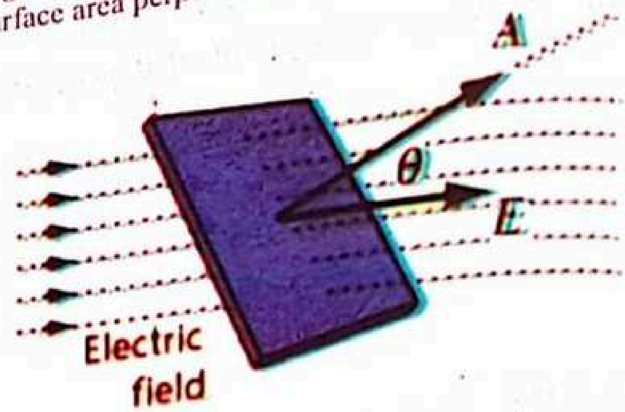


Fig.11.11 Lines of force passing through an area and the direction of area is at an angle 'θ' with the

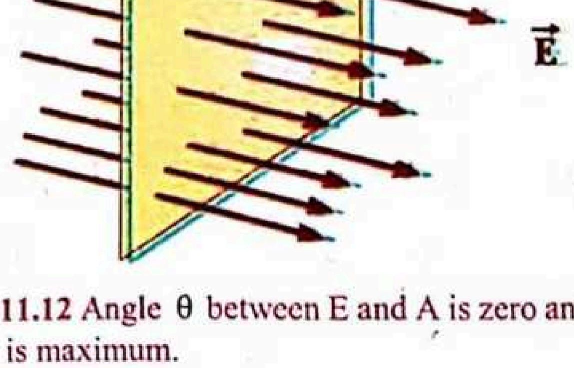


Fig.11.12 Angle θ between E and A is zero and the flux is maximum.

electric lines of force and thus Eq.11.18 becomes

$$\phi = EA_x$$

$$\phi = E\,A\cos\theta \;\ldots\ldots(11.18)$$

$$\phi = \overline{E}\cdot\overline{A} \;\ldots\ldots(11.19)$$

This is an electric flux in terms of the scalar product of $\overline{E}$ and $\overline{A}$. Thus Electric flux is a scalar quantity and its SI unit is Nm^2C^{-1}. The flux can be studied under the following two cases:

Case-I: When area is held perpendicular to $\vec{E}$ such that the vector area $\overline{A}$ becomes parallel to the direction of electric lines of force and angle 'θ' between them is zero as shown in Fig.11.12. Then Eq.11.18 becomes

$$\phi = EA\cos 0^{o}$$

$$\phi = EA$$

This is the maximum electric flux.

Case-II: When surface area held parallel to the electric field such that the vector area becomes perpendicular to the direction of electric lines of force and angle 'θ' between them is 90^0 as shown in Fig.11.13. Then Eq.11.16 becomes

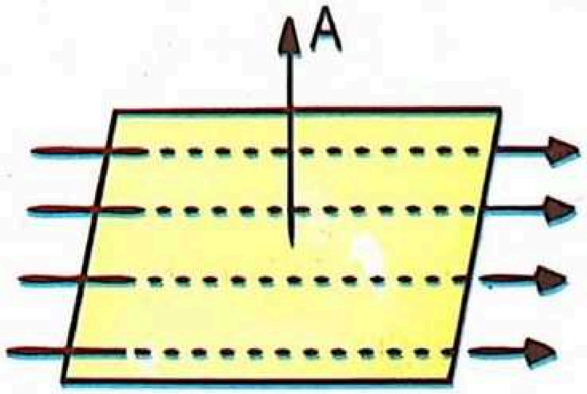


Fig.11.13 Angle θ between E and A is 90^0 and the flux is minimum.

$$\phi = EA\cos 90^{o}$$

$$\phi = 0$$

This is the minimum electric flux or zero flux.

11.8 ELECTRIC FLUX THROUGH A SURFACE ENCLOSING A CHARGE

Consider a point charge q which is enclosed by spherical shape surface of radius 'r' as shown in Fig.11.14. Since the charge is at the centre of the sphere so the electric intensity 'E' at the whole spherical surface remains same (constant). To calculate the total flux due to this point charge, we divide the whole spherical surface of the sphere into 'n' number of small and equal patches of

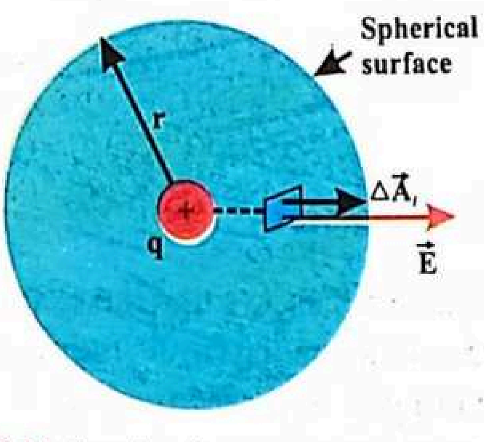


Fig.11.14 A point charge q at the center of the sphere.

area $(\Delta A_1, \Delta A_2, \Delta A_3, \cdots, \Delta A_n)$. It is to be noted that the radius of the sphere is always perpendicular to each small patch. Therefore, the direction of vector area through each patch remains parallel to the direction of intensity $\vec{E}$ and angle 'θ' between them is zero.

Thus, the flux ϕ_1 through small patch of area ΔA_1 is given by

$$\phi_1 = \vec{E} \cdot \Delta\vec{A}_1$$

$$\phi_1 = E\Delta A_1 \cos 0^0$$

$$= E\Delta A_1$$

Similarly, the flux ϕ_2 through ΔA_2 is given by

$$\phi_2 = \vec{E} \cdot \Delta\vec{A}_2 = E\Delta A_2$$

$$\phi_3 = \vec{E} \cdot \Delta\vec{A}_3 = E\Delta A_3$$

.

.

.

$$\phi_n = \vec{E} \cdot \Delta\vec{A}_n = E\Delta A_n$$

> *POINT TO PONDER*
>
> The electric lines of force that can pass through an area, is named as electric flux. Can the same lines of force pass through a volume?

Hence, the total flux

$$\phi = \phi_1 + \phi_2 + \phi_3 \cdots \phi_n$$

$$\phi = E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + \cdots + E\Delta A_n$$

$$\phi = E(\Delta A_1 + \Delta A_2 + \Delta A_3 + \cdots + \Delta A_n)$$

$$\phi = \frac{q}{4\pi\varepsilon_o r^2}\sum_{i=1}^{n}\Delta A_i$$

But surface area of sphere $= \sum \Delta A_i = 4\pi r^2$

$$\phi = \frac{q}{4\pi\varepsilon_o r^2} \cdot 4\pi r^2$$

$$\phi = \frac{q}{\varepsilon_o} \quad \ldots\ldots(11.20)$$

This result shows that the electric flux due to a point charge through the surface of sphere is independent of its radius. It means, the flux through a closed surface does not depend upon a geometrical shape but it depends upon magnitude of charge and medium used.

11.9 GAUSS'S LAW

We have studied in the previous section that the flux due to a point charge at the centre of the sphere is q/ε_o. This shows that the flux is independent of the radius. Therefore, this result can be extended for 'n' number of point charges $q_1, q_2, q_3 \cdots q_n$ which are arbitrarily distributed in a closed surface of non-geometrical or non-uniform shape as shown Fig.11.15. To calculate the total flux due to all the given point charges, we enclose each point charge in a small imaginary sphere within the boundary of the closed surface as shown in Fig.11.16. The surface of this imaginary sphere enclosing the charges is called gaussian surface.

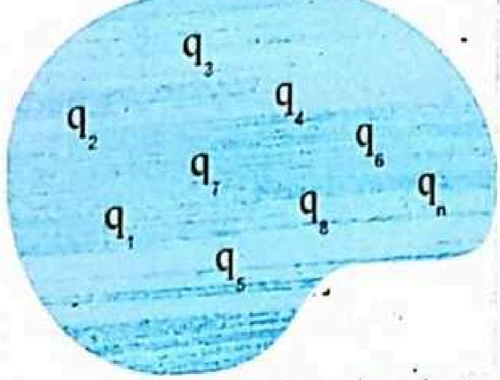


Fig.11.15 There are 'n' number of point charges enclosed by a surface of non-geometrical shape.

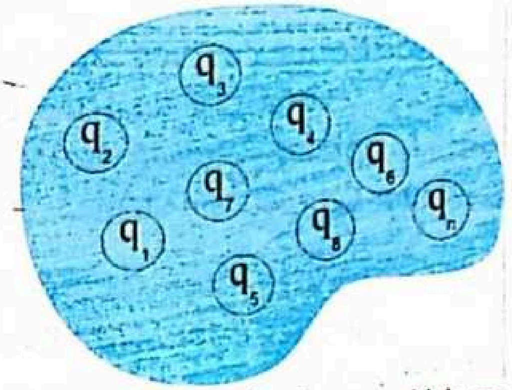


Fig.11.16 The point charges which are enclosed in small spheres.

Thus, the flux ϕ_1 due to a point charge q_1 which is enclosed by a small sphere is given by

$$\phi_1 = \frac{q_1}{\varepsilon_o}$$

Similarly, the flux ϕ_2 due to a point charge q_2 is given by

$$\phi_2 = \frac{q_2}{\varepsilon_o}$$

and $$\phi_3 = \frac{q_3}{\varepsilon_o}$$

$$\vdots$$

$$\phi_n = \frac{q_n}{\varepsilon_o}$$

Total flux can be obtained by adding above equations i.e.,

$$\phi = \phi_1 + \phi_2 + \phi_3 + \cdots + \phi_n$$

$$\phi = \frac{q_1}{\varepsilon_o} + \frac{q_2}{\varepsilon_o} + \frac{q_3}{\varepsilon_o} + \cdots + \frac{q_n}{\varepsilon_o}$$

$$\phi = \frac{1}{\varepsilon_o}(q_1 + q_2 + q_3 + \cdots + q_n)$$

$$\phi = \frac{1}{\varepsilon_o}\text{ (total charge enclosed in the surface)}$$

$$\phi = \frac{1}{\varepsilon_o}(Q) \quad \ldots\ldots(11.21)$$

This is the mathematical form of Gauss's law and it is stated as '**the total flux through a closed surface is equal to $1/\varepsilon_o$ times the total charge enclosed by the surface**'.

11.10 APPLICATIONS OF GAUSS'S LAW

Gauss's law can be applied to calculate the electric intensity due to the different configuration of the charges. Though the charges must be enclosed by a surface called 'Gaussian Surface', but it is not necessary that the surface should have a regular geometrical shape.

(a) Electric field due to an infinite charged sheet

Consider a thin infinite charged sheet of uniform surface charge density σ which is given by

$$\sigma = \frac{\text{charge}}{\text{area}} = \frac{Q}{A} \quad \ldots\ldots (11.22)$$

To calculate the electric field $\overline{E}$ due to this charged sheet at point 'p' near the sheet, we pass a cylinder of cross-section area 'A' through the sheet. This closed cylinder acts as a Gaussian surface as shown in Fig.11.17.

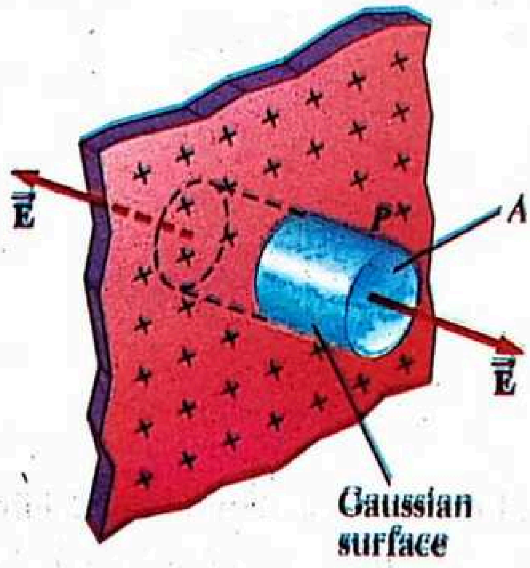


Fig.11.17 A Gaussian-Surface inform of a cylinder passed through a sheet of positive charge.

Since the cylinder is perpendicular to the plane of the sheet, so the direction of electric field is along the cylinder. Now we determine the electric flux through the two flats and one curved surface of the cylinder.

Flux due to the curved surface of the cylinder is

$$\phi_1 = E\,A_1 \cos\theta$$

where the angle 'θ' between electric intensity E and curved area A is 90^0 so

$$\phi_1 = EA_1 \cos 90^0 = 0$$

Similarly, the flux through the flat surfaces or end faces of the cylinder is given by

$$\phi_2 = EA_2 \cos\theta$$

The direction of flat surface of the cylinder is parallel to the electric intensity and angle 'θ' between them is zero

$\therefore$
$$\phi_2 = EA_2 \cos 0^0$$

$$\phi_2 = EA_2$$

The flux for the other flat surface of the cylinder is given by;

$$\phi_3 = EA_3$$

Total flux

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\phi = 0 + EA_2 + EA_2$$

We know that $A_1 = A_2 = A =$ Area of circular surface of cylinder

$$\phi = 2EA \quad ......(11.23)$$

Now, by definition of Gauss's Law

$$\phi = \frac{Q}{\varepsilon_o}$$

But from Eq.11.22; $Q = \sigma A$

$$\phi = \frac{\sigma A}{\varepsilon_o} \quad ......(11.24)$$

Comparing Eq.11.23 and Eq.11.24

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

In vector form

$$\vec{E} = \frac{\sigma}{2\varepsilon_o}\hat{r} \quad ......(11.25)$$

where $\hat{r}$ is a unit vector, perpendicular to the sheet and E is directed away from it. Similarly, if the given sheet is negatively charged then

$$\vec{E} = \frac{-\sigma}{2\varepsilon_o}\hat{r}$$

In this case $\vec{E}$ is directed toward the sheet.

The above results show that the electric field due to a charged sheet depends on its charged density and medium in which the sheet is placed.

(b) Electric field due to two parallel and opposite charged plates

Consider two charged plates of same size with equal and opposite surface charge densities $+\sigma$ and $-\sigma$. The plates are held closed and parallel to each other such that there is a uniform electric field between them.

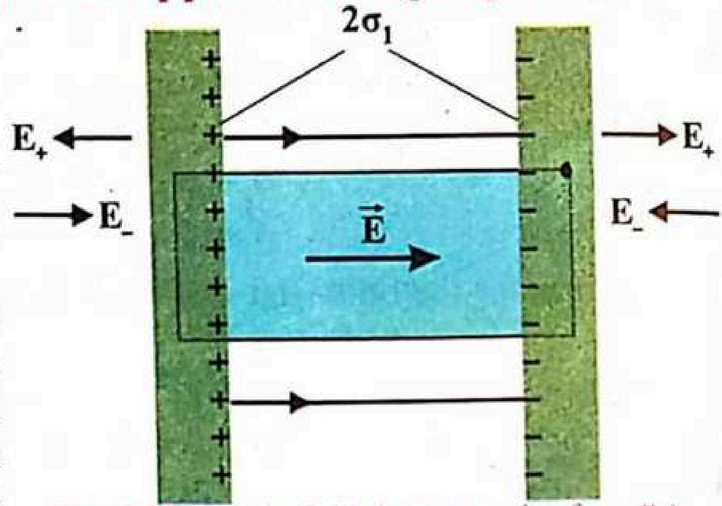


Fig.11.18 Electric field due to a pair of parallel and opposite charged plates.

To calculate the electric field due to these plates by using Gauss's law, we draw a Gaussian surface in the form of a box as shown in Fig.11.18. The electric field outside the plates is zero, because the direction of field (E_+) due to the positive plate is opposite to direction of field (E_-) due to the negative plate, However, there is uniform electric field inside the plates, it is therefore, the magnitude of E_+ and E_- at each point is the same inside the plates. Since each charge plate behaves as a charge sheet, so the field inside the plates due the positive charge plate is given by

$$E_+ = \frac{+\sigma}{2\varepsilon_o} \quad ......(11.26)$$

Similarly, the field inside the plates due to the negative charge plate is given by

$$E_- = -\left(\frac{-\sigma}{2\varepsilon_o}\right) \quad ......(11.27)$$

Negative sign shows that the direction of $\vec{E}_-$ is opposite to the direction of $\vec{E}_+$. Resultant field intensity E between the plates is given by

$$E = E_+ + E_-$$

$$E = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o}$$

$$\vec{E} = \frac{\sigma}{\varepsilon_o}$$

In vector form

$$\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{r} \quad ......(11.28)$$

Where $\hat{r}$ is a unit vector and it shows the direction of $\vec{E}$. i.e., E is directed from positive charged plate to negative charged plate.

(c) Electric field due to the hollow spherical charged conductor

Consider uniformly charged spherical shell of radius 'R'. The electric field due to this charged shell is calculated for two different cases.

I Electric field outside the shell

To calculate the electric field outside the shell, we surrounded it by a closed spherical Gaussian Surface of radius 'r' as shown in Fig.11.19(a) where r > R. Thus, the flux through the Gaussian Surface is given by

$$\phi = \vec{E} \cdot \vec{A}$$

In case of sphere, angle 'θ' between E and A is zero

So $$\phi = E A \cos 0^o$$

$$\phi = EA$$

According to Gauss Law

$$\phi = \frac{q}{\varepsilon_o}$$

$$EA = \frac{q}{\varepsilon_o}$$

Here A = Area of sphere = $4\pi r^2$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad ......(11.29)$$

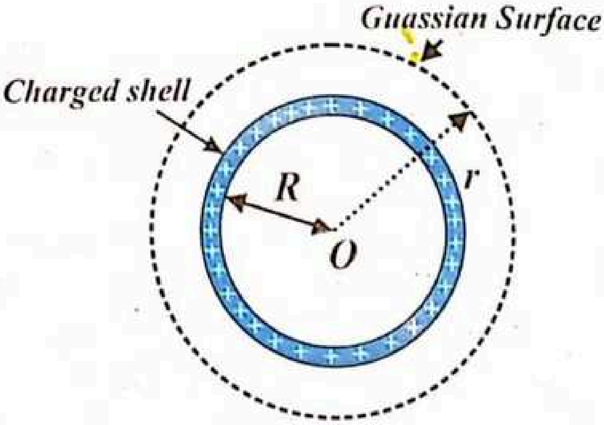


Fig.11.19(a) A uniformly charged spherical shell of radius R surrounded by Gaussian surface of radius r (r > R)

II Electric field inside the shells

Similarly, to calculate the electric field inside the shell, we consider a closed spherical Gaussian surface of radius 'r' and area 'A' inside the shell as shown in Fig.11.19(b) where R > r. Thus, by definition of flux

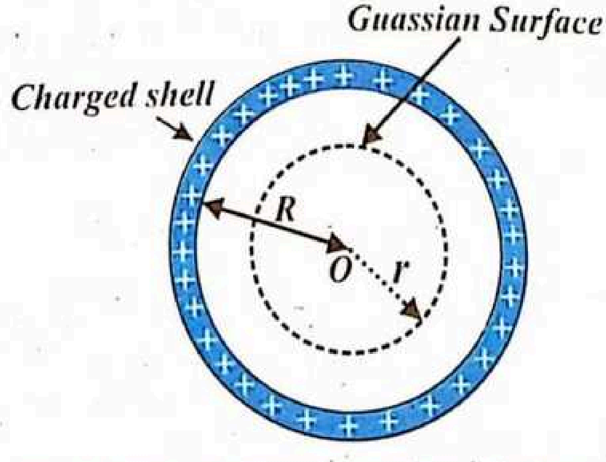


Fig.11.19(b) A Gaussian surface of radius r inside the uniformly charged shell of radius R (R > r)

$$\phi = EA' \quad ......(11.30)$$

But according to Gauss's Law

$$\phi = \frac{q}{\varepsilon_o} = 0 \quad ......(11.31)$$

It may be noted that Gaussian surface does not enclose any charge, therefore electric flux through it is zero. Comparing eq. 11.30 and eq. 11.31

$$E\,A' = 0$$

As $A' \neq 0$

Therefore, $E = 0$

This shows that the electric field inside the charged spherical shell is zero.

11.11 ELECTRIC POTENTIAL

We have studied the force by an electric field on a charged particle in the previous section. Now in this section we study the work done on the charged particle. Consider a uniform electric field due to parallel and opposite charged plates separated by a distance d. The direction of the field is from positive to negative plate. This field exerts a force (qE) on a small positive charge 'q' and the charge moves from point B to point A as shown in Fig.11.20. Now if the charge is to be displaced from point 'A' to point 'B' against the direction of the field, an external force with magnitude (qE) must be applied on it. In this way, the charge gains potential energy called electrical potential energy and it is represented by U.

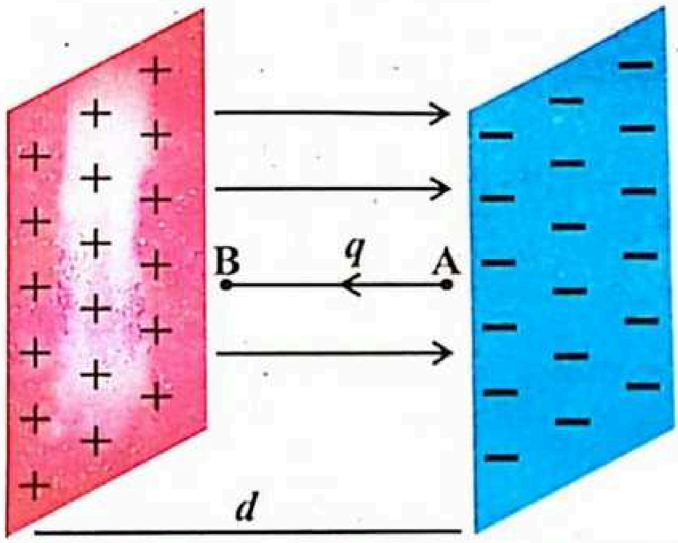


Fig.11.20 Electric field due to parallel and opposite charged plates where a charged particle 'q' is displacing against the direction of electric field E from point A to point B.

Let U_A be potential energy of the charge at point A and U_B be the potential energy of the charge at point B. Indeed, the work done by the applied force against the direction of field causes of change (increase) in electrical potential energy ΔU of the charged particle. Thus, according to work-energy principle,

$$\text{Work} = \Delta U$$

$$W_{AB} = U_B - U_A \quad ......(11.32)$$

Dividing Eq.11.32 by 'q'

$$\frac{W_{AB}}{q} = \frac{U_B}{q} - \frac{U_A}{q} \quad ......(11.33)$$

where $\frac{U_A}{q}$ is the potential energy per unit charge at point 'A' and $\frac{U_B}{q}$ is the potential energy per unit charge at point B. Thus, **the work done per unit charge by the applied force when charge moves against the direction of field is called electrical potential or potential difference.** It is represented by 'V' and hence Eq.11.33 becomes

$$\frac{W_{AB}}{q} = V_B - V_A$$

$$\frac{W_{AB}}{q} = \Delta V_{AB}$$

$$\Delta V = \frac{W}{q} \quad ......(11.34)$$

> ***POINT TO PONDER***
> What will be the value of electric potential when a positive charged particle moves in the direction of electric field?

This is an expression for electric potential difference between two points in an electric field.

Now if the point 'A' is at infinity that is, we bring the unit positive charge from infinity to point 'B' against the field. Then $V_A = 0$ and Eq.11.34 becomes

$$\Delta V = \frac{W}{q}$$

$$V_B - V_A = \frac{W}{q}.$$

$$V_B - 0 = \frac{W}{q}$$

$$V_B = \frac{W}{q}$$

or

$$V = \frac{W}{q} \quad ......(11.35)$$

Electric potential is a scalar quantity and its unit is volt.

Volt

The potential difference between two points is one volt, if one joule of work is done on a moving unit positive charge from one point to the other point against the direction of electric field. It is expressed as

$$1\text{volt} = \frac{1\text{joule}}{1\text{coulomb}}$$

11.12 ELECTRIC FIELD AS POTENTIAL GRADIENT

As the gravitational potential energy of a mass depends on the height, likewise the electrical potential energy of a charged body also depends on its position in the electric field. **Thus the rate of change of electric potential of a charged body with respect to displacement or position in an electric field is called potential gradient.** It is explained as:

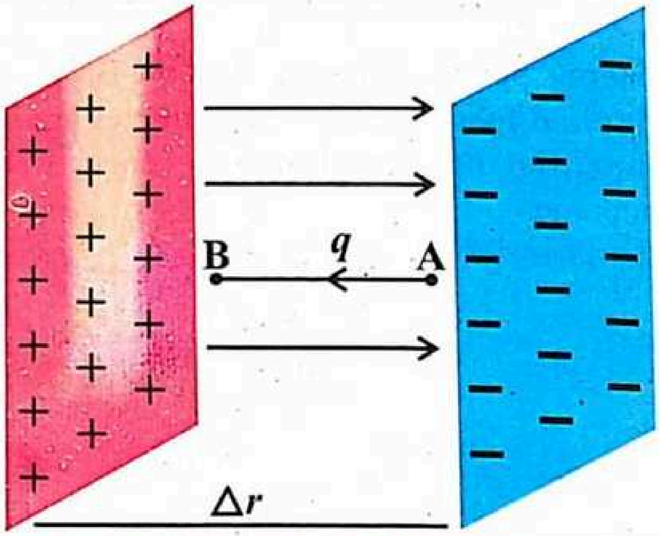


Fig.11.21 Electric potential between two points separated at distance Δr.

Consider a uniform electric field between two parallel and opposite charged plates. When a charged body is displaced against the direction of electric field from point A to point B through a small distance Δr by the applied force (qE) as shown in Fig.11.21, then there is change in electric potential which is given by

$$\Delta V = \frac{W}{q}$$

But $$W = F \cdot \Delta r = qE \cdot \Delta r$$

$$\Delta V = \frac{qE \cdot \Delta r}{q}$$

Here angle 'θ' between E and Δr is 180^0

$$\Delta V = E\Delta r \cos 180^o$$

$$\Delta V = -E\Delta r$$

$$E = -\frac{\Delta V}{\Delta r} \quad ...(11.36)$$

This shows that electric field at any point is equal to the negative of potential gradient. The term $\frac{\Delta V}{\Delta r}$ specifies the change in potential with respect to distance (position in specific direction) is known as potential gradient. The negative sign shows that the direction of E is opposite to the direction in which V increases, where E is directed from higher potential to lower potential. The unit of electric intensity in terms of potential gradient is Vm^{-1} and it is equal to NC^{-1}.

POINT TO PONDER

What will be the work done on the charged particle when it is displaced between two points which have same potential?

11.13 ELECTRIC POTENTIAL AT A POINT DUE TO A POINT CHARGE

Consider an electric field produces by a point positive charge q. The direction of lines of force of this field is radially outward. When a unit positive charge is displaced from infinity inward against the direction of the field then there is change in electric potential and is given by:

$$\Delta V = -E\,\Delta r \quad ......(11.37)$$

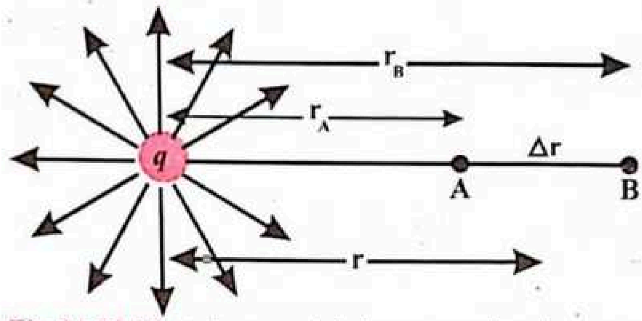


Fig.11.22 Electric potential due to a point charge q at distance r from the center of the point charge.

The negative sign shows that the work is done on a charged particle against the direction of electric field. Eq. 11.37 holds when the electric intensity 'E' remains constant. Since the given field is due to a point charge so its electric intensity E varies inversely with the square of the distance from that point charge. i.e.

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2} \quad ......(11.38)$$

To calculate the electric potential, we consider two points 'A' and 'B' separated by a small distance Δr such that E remains constant between them. Let point 'A' is at distance r_A from the center of point charge and point 'B' is at r_B such that $r_B - r_A = \Delta r$ as shown in Fig.11.22. In order to get the average value of electric intensity, we consider the midpoint between 'A' and 'B' at a distance 'r' from the centre of the point charge. Such that;

$$r = \frac{r_A + r_B}{2}$$

But

$$r_B = r_A + \Delta r$$

$$r = \frac{r_A + r_A + \Delta r}{2}$$

$$r = \frac{2r_A + \Delta r}{2}$$

Squaring on both sides

$$r^2 = \frac{4{r_A}^2 + 4r_A\Delta r + \Delta r^2}{4}$$

As Δr^2 is very very small so we neglect it

$$r^2 = r_A^2 + r_A\Delta r$$

$$r^2 = r_A(r_A + \Delta r) = r_A(r_A + r_B - r_A)$$

$$r^2 = r_A r_B$$

Substitute the value of r^2 in Eq.11.38

$$E = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A r_B}$$

Substitute the value of E in Eq.11.37

$$\Delta V = -E\,\Delta r$$

$$V_A - V_B = -\frac{1}{4\pi\varepsilon_o}\frac{q}{r_A r_B}(r_A - r_B)$$

$$V_A - V_B = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A r_B}(r_B - r_A)$$

$$V_A - V_B = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad \ldots\ldots(11.39)$$

This is the electric potential between two points in an electric field by a point charge. Now if the point 'B' is at infinity ($r_B = \infty$) then $V_B = 0$ and potential at point A is known as absolute potential. So, Eq.11.39, becomes,

$$V_A - 0 = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{\infty}\right) \qquad \because \frac{1}{\infty} = 0$$

$$V_A = \frac{q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - 0\right)$$

$$V_A = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_A}$$

In general, the electric potential 'V' due to a point charge 'q' at distance 'r' from the centre of the point charge is given by

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r} \quad \ldots\ldots(11.40)$$

11.14 ELECTRON VOLT

As we know that the unit of energy is joule, but this is the larger unit and it is difficult to use for atomic physics. To overcome this problem, another smaller unit of energy called electron volt (eV) is used. One electron volt is that amount of electrical potential energy which gained by an electron when it is displaced in an electric field through a potential difference of one volt. Thus, by definition of work

$$W = q\,\Delta V$$

According to work-energy theorem

$$W = \Delta U \quad \text{(gained potential energy)}$$

$$\Delta U = q\,\Delta V$$

Here $q = 1e$

and $\Delta V = 1V$

Therefore $\Delta U = 1eV$ ......(11.41)

As the charge on an electron $e = 1.6 \times 10^{-19}$ C

$$\therefore \quad \Delta U = 1.6 \times 10^{-19}\ \text{CV}$$

$$\Delta U = 1.6 \times 10^{-19}\ \text{J} \quad ......(11.42)$$

Comparing Eq.11.41 and 11.42

$$1eV = 1.6 \times 10^{-19}\ \text{J}$$

Example 11.5

The potential difference between two large parallel and opposite charged metal plates is 160V. If the plates are separated by a distance 4mm then calculate the electric field between them.

Solution:

Potential difference $\Delta V = 160V$

Distance between the plates $\Delta r = 4mm = 4 \times 10^{-3}$ m

Electric field between plates $E = ?$

According to electric field in terms of potential gradient

$$E = \frac{\Delta V}{\Delta r} = \frac{160V}{4 \times 10^{-3} m} = 40 \times 10^{3}\ Vm^{-1}$$

$$E = 40\ kVm^{-1}$$

Example 11.6

When a unit positive charge is displaced at a distance 40cm towards the point charge of magnitude 0.4 μC then calculate the absolute potential.

Solution:

Absolute potential: $V = ?$

Distance: $r = 40cm = 0.4m$

$$q = 0.4\,\mu C = 0.4 \times 10^{-6}\ C = 4 \times 10^{-7} C$$

$$k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^{9}\ Nm^2C^{-2}$$

According to the absolute potential

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

$$V = 9 \times 10^9 \cdot \frac{4\times10^{-7}C}{0.4m}$$

$$V = 9 \times 10^3\ V$$

$$V = 9\ kV$$

11.15 CAPACITOR AND ITS CAPACITANCE

A capacitor is a device that stores electrical energy, or it is a device to store energy in the form of electrical charge producing a potential difference (static voltage) across its plates.

A common capacitor consists of two parallel conducting plates separated by vacuum, air or any other insulator known as dielectric as shown in Fig.11.23. When a capacitor is connected to a battery or other source of voltage, then the potential difference 'V' develops between the plates of the capacitor, such that the plate which is connected to the positive terminal of the source stores positive charges. While the other plate stores the negative charges, which is connected to negative terminal. The experimental results show that these storage of charges (Q) on the plates of the capacitor is directly proportional to the applied potential difference of the source. That is,

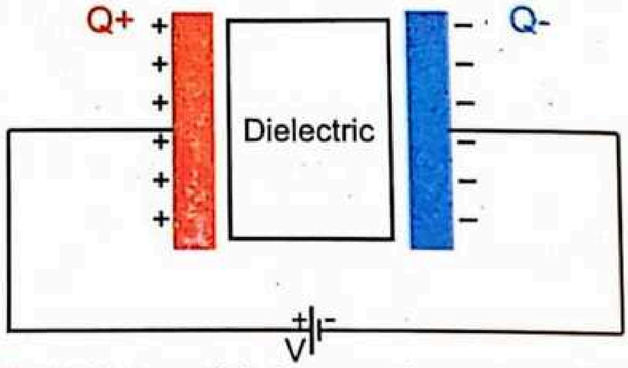


Fig.11.23 A parallel plate capacitor separated by a dielectric material and connected across the source of voltage.

$$Q \propto V$$

$$Q = CV \quad ......(11.43)$$

where 'C' is a constant of proportionality. It is known as the capacitance of the capacitor. It depends upon size of the plates, distance and medium between the plates. The SI unit of the capacitance is Farad (F). It is defined as:

If a charge of one coulomb is stored on one of the plates of the capacitor and it could generate a potential difference of one volt, then the capacitance of a capacitor is said to be one Farad.

The farad is a larger unit of capacitance, for practical purposes, the smaller units such as millifarad (mF=10^{-3} F), microfarad ($\mu F = 10^{-6} F$), nano-farad ($nF = 10^{-9} F$) and picofarad (pF = 10^{-12} F) are used.

11.16 CAPACITANCE OF PARALLEL PLATE CAPACITOR

A parallel plate capacitor consists of two parallel metal plates of same size separated by a distance 'd' and connected with a source of a voltage as shown Fig.11.24. Let 'A' be the area of each plate and σ be its surface charge density. If the medium between the plates of capacitor is vacuum or air then its capacitance is given by;

$$C = \frac{Q}{V} \quad ......(11.44)$$

When voltage 'V' is applied across the plates of capacitor then with the storage of charges, an electric field 'E' is setup between the plates and is given by;

$$E = \frac{\sigma}{\varepsilon_o}$$

but $\sigma = \frac{Q}{A}$

Therefore $E = \frac{Q}{A\varepsilon_o} \quad ......(11.45)$

Similarly, by definition of electric field in terms of potential gradient,

$$E = \frac{V}{d} \quad ......(11.46)$$

Comparing Eq.11.45 and Eq.11.46

$$\frac{Q}{A\varepsilon_o} = \frac{V}{d}$$

$$\frac{CV}{A\varepsilon_o} = \frac{V}{d}$$

$$C = \frac{A\varepsilon_0}{d} \quad ......(11.47)$$

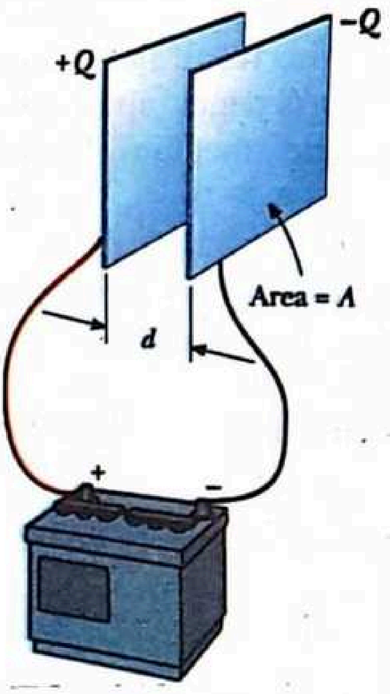


Fig.11.24 A parallel plate capacitor consist of a pair of plates separated by a distance d and the area of each plate is A.

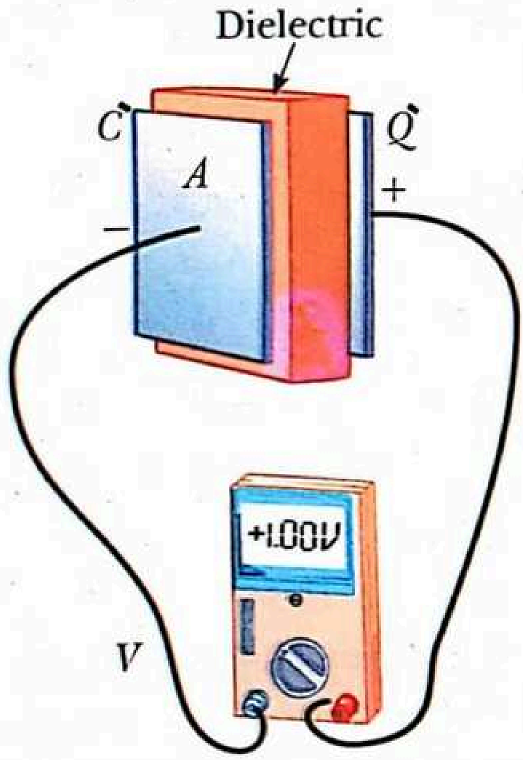


Fig.11.25 The dielectric material between the plates of the capacitor.

This is the capacitance of a parallel plate capacitor in the presence of air or vacuum between its plates. This result shows that the capacitance of a capacitor depends upon the area of the plates, distance and medium between the plates of capacitor.

> ***FOR YOUR INFORMATION***
> A capacitor is a passive component that can be used in both electric and electronic circuits.

Faraday found experimentally that when the free space between the two plates of the capacitor is occupied by insulator called dielectric then for the same applied voltage, more charges store on the plates of the capacitor. To explain this, we consider two parallel plate capacitors, each of Area 'A' separated by a distance d. Let the space between the plates occupied by some dielectric material of permittivity $(\varepsilon = \varepsilon_0 \varepsilon_r)$ as shown in Fig.11.25. When the voltage 'V' is applied the induced electric field E' between the plates is given by

$$E' = \frac{Q'}{A\varepsilon_o \varepsilon_r}$$

But $$E' = \frac{V}{d}$$

$$\frac{V}{d} = \frac{Q'}{A\varepsilon_o \varepsilon_r}$$

$$\frac{V}{d} = \frac{C'V}{A\varepsilon_o \varepsilon_r}$$

$$C' = \frac{A\varepsilon_o \varepsilon_r}{d}$$

But $$C = \frac{A\varepsilon_o}{d}$$

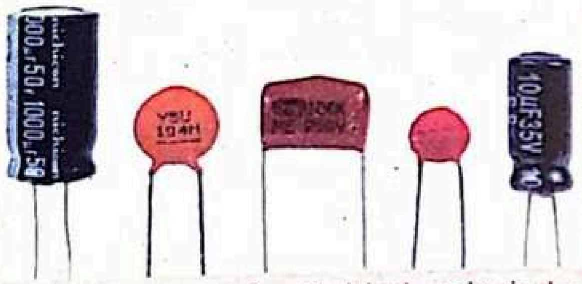
$$C' = \varepsilon_r C \quad ......(11.48)$$

As $\varepsilon_r > 1$

So $C' > C$

The relative permittivity ε_r is also called dielectric constant. The values of dielectric constant for different materials are shown in table 11.2. The dielectric constant is defined as; **the ratio of the capacitance of a capacitor with dielectric substance as a medium**

FOR YOUR INFORMATION



External structure of cylindrical, spherical and parallel plate capacitors.

Table 11.2

Dielectric Constant of Materials	
Air	1.00
Aisimag 196	5.70
Bakelite	4.90
Cellulose	3.70
Fiber	6.00
Formica	1.75
Glass	7.75
Mica	5.10
Mycalex	7.10
Paper	3.00
Plexiglass	2.80
Polyethlene	2.30
Polystyrene	2.60
Porcelain	5.57
Pyrex	4.00
Quartz	3.80
Steatite	5.80
Teflon	2.10

between the plates to the capacitance with air or vacuum as medium between the plates. That is,

$$\varepsilon_r = \frac{C'}{C} \quad ......(11.49)$$

11.16.1 Combination of capacitors

Capacitors are manufactured with different values of capacitance and voltage. Sometimes these standard values do not match with the circuit requirement. To overcome this problem, we can obtain these required values by combining the capacitors either in series or in parallel, such combination of capacitor are explained as:

I Capacitors in series

A series circuit is one when all the capacitors are connected in the form of a chain (one after the other) and the flow of charges are along the single path. Hence, when the capacitors are connected in series the storage of charges on each capacitor is the same. It is explained as:

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in series as shown in Fig.11.26. When the potential difference V is applied across them, the right plate of C_3 is negatively charged $(-Q)$, it induces an equal and opposite charge (+Q) on the left plate of C_3. Where, the total charge $(-Q)$ has moved from the left plate of C_3 to the right plate of C_2. Again, the charge (+Q) induces on the left plate of C_2. Finally, the negative charges are induced on the right plate of 'C_1' from the left plate of C_2 which also repels the negative charges from the left plate of C_1 toward the battery and the charges (+Q) left on it. Thus, it is obvious that when capacitors are connected in series, the charges on each capacitor is the same, but the voltage drop across each capacitor is different. i.e.,

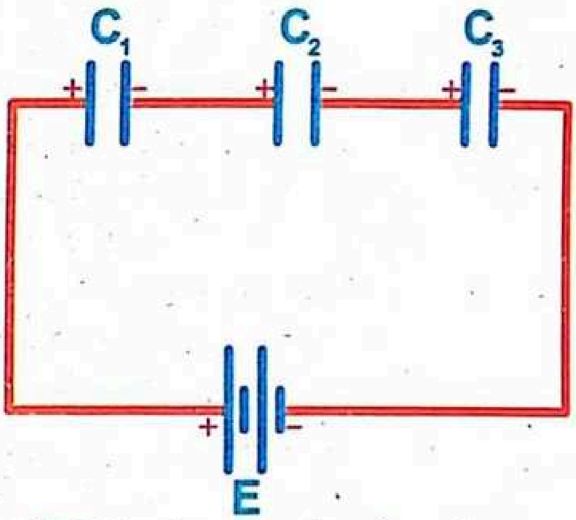


Fig.11.26 A series connection of capacitors across the potential difference.

$$V = V_1 + V_2 + V_3$$

As $\therefore Q = CV$

$$\therefore V = \frac{Q}{C}$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots\ldots(14.50)$$

FOR YOUR INFORMATION

When capacitors are connected in series, their equivalent capacitance is always less than each individual capacitance.

This result shows that the equivalent capacitance of a series combination is always less than any individual capacitance connected in series, or the reciprocal of equivalent capacitance in series combination is equal to sum of reciprocal of the individual capacitance.

II Capacitors in Parallel

In parallel circuit, the capacitors are connected across one another and flow of charges is divided into all the given paths. Consider three capacitors of capacitances C_1, C_2 and C_3 which are connected in parallel as shown in Fig.11.27. When the potential difference 'V' is applied across them, then the same potential is drawn across each capacitor, but the charges stored on the three capacitors are Q_1, Q_2 and Q_3. So,

$$Q = Q_1 + Q_2 + Q_3$$

$$CV = C_1V + C_2V + C_3V$$

$$C_{eq} = C_1 + C_2 + C_3 \ldots\ldots(14.51)$$

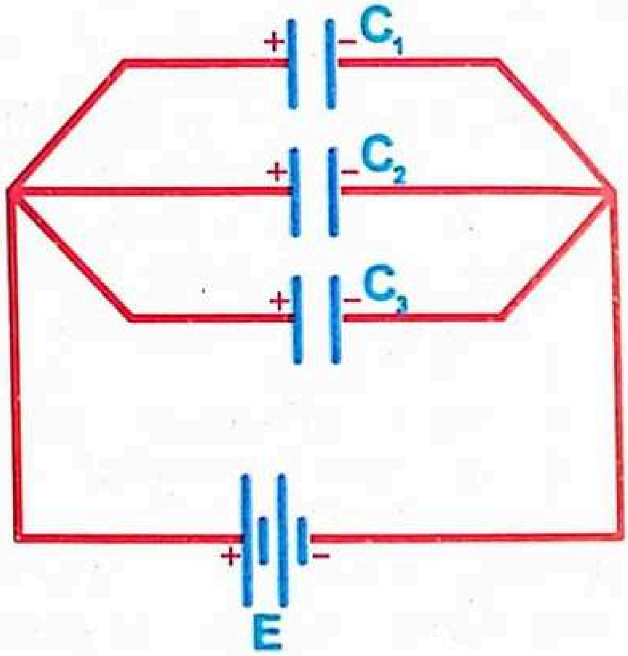


Fig.11.27 A parallel connection of capacitors across the potential difference

This result shows that the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitance, or the equivalent capacitance in parallel combination is equal to sum of the individual capacitance.

11.17 ELECTRIC POLARIZATION OF DIELECTRIC

The insulating materials e.g., glass, mica, paper etc. are called dielectrics. They transmit electric effect without conduction. As we know that each dielectric material consists of a number of atoms or molecules and in each atom, there is positively charged nucleus which is surrounded by negatively charged electronic cloud as shown in Fig.11.28.

When the dielectric material is placed between the plates of charged capacitor then their atoms are subjected to external electric field E_e. The negatively charged electrons are attracted towards the positively charged plate of the capacitor, and the positively charged nucleus is attracted towards the negatively charged plate of the capacitor. As a result, the charges of the atoms are displaced from their original position and form dipoles as shown in Fig.11.29. This formation of electric dipole in the presence of electric field is known as electric polarization of the dielectric.

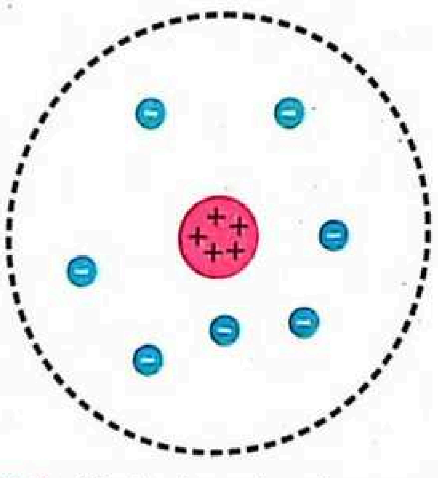


Fig.11.28 Positively charged nucleus surrounded by negatively charged electrons.

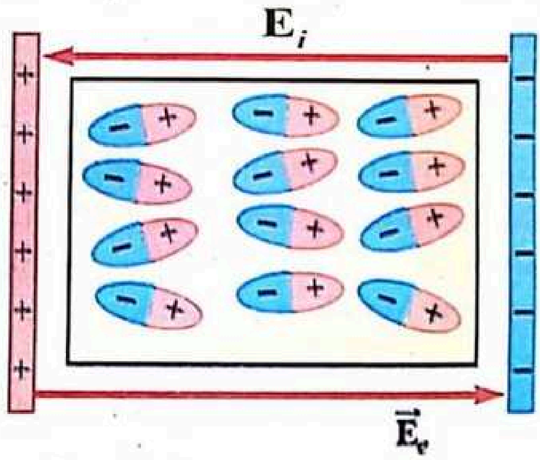


Fig.11.29 Polarization of atoms of dielectric material between the plates of the capacitor.

In the process of polarization of the dielectric between the plates of the capacitor, we have observed that the negative charges are induced on one side near the positive plate and an equal positive charges on the opposite side near the negative charged plate. These induced charges produce an internal electric field E_i and it opposes the applied external field E_e as shown in Fig.11.29. Thus, their resultant field is given by

$$E = E_e - E_i \quad \text{......(11.51)}$$

This shows that in the presence of dielectric, the electric field is reduced, and it causes smaller potential drop across the plates of the capacitor. As the relation $C = \frac{Q}{V}$ shows that at constant Q, the capacitance 'C' is inversely proportional to the applied potential difference 'V'. Thus, it is clear that in the presence of dielectric the capacitance of a capacitor has been increased.

> ***FOR YOUR INFORMATION***
> When capacitors are connected in parallel, their equivalent capacitance is always greater than each individual capacitance.

> ***DO YOU KNOW?***
> The capacitance of a capacitor is increased by inserting a dielectric material between its plates.

11.18 CHARGING AND DISCHARGING OF A CAPACITOR

When a resistor R and a capacitor C with a source of voltage V are connected in series as shown in Fig.11.30 then it is called RC-series circuit. It is being used for charging of a capacitor. Now when the switch 'S_1' in RC-series circuit is closed then the process of charging of the capacitor is started. This process of charging is not an instantaneous one, but it takes some time. The rate of charging or discharging of a capacitor depends on the product of resistance R and capacitance C. As the unit of product of RC is that of time, so this product is termed as time constant and it is represented by τ. Mathematically the charging of a capacitor can also be studied under the following expression

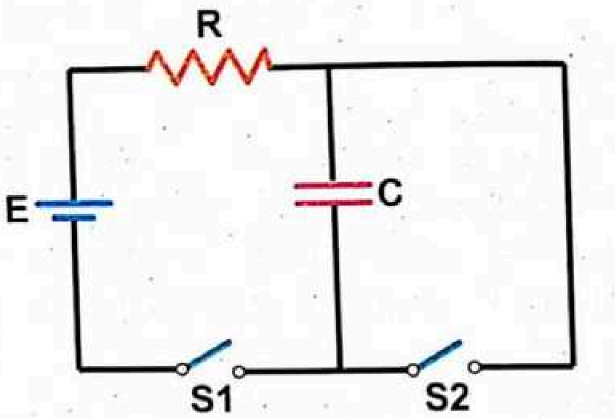


Fig.11.30 Charging of a capacitor in RC-series circuit.

$$q = q_0(1 - e^{-\frac{t}{RC}}) \quad ......(11.52)$$

This relation shows that the nature of charging of a capacitor is exponential, where q_0 represents the maximum charge on the capacitor that stores after an infinite length of time. It means, the rate of charging of a capacitor is different at its different stages, graphically it is explained as:

If time 't' = 1τ (1RC),

then eq. 11.52 becomes

$$q = q_0\left(1 - e^{\frac{-\tau}{\tau}}\right)$$

$$q = q_0(1 - e^{-1})$$

$$q = q_0\left(1 - \frac{1}{e}\right)$$

$$q = q_0\left(1 - \frac{1}{2.718}\right)$$

$$q = 0.63\ q_0 \quad ......(11.53)$$

This gives us that after one time constant, the capacitor will be charged about 63%.

If time t= 2τ (2RC)

then $$q = q_0\left(1 - e^{\frac{-2\tau}{\tau}}\right)$$

$$q = q_0(1 - e^{-2})$$

$$q = 0.86\ q_0 \quad ......(11.54)$$

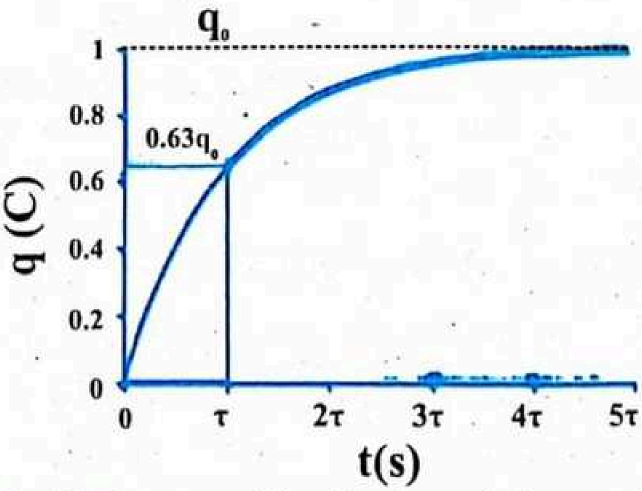


Fig.11.31 A curved line in charge – time graph shows charging of a capacitor.

The shows that the capacitor is charged about 86% after 2 time constant.

Similarly, after five time constant a capacitor acquires a charge very close to its maximum value q_0. Graphically, a curved line in charge-time (q-t) graph shows charging of a capacitor as shown in Fig.11.31.

After maximum charging of a capacitor, when switch S_1 is opened and switch S_2 is closed in RC-circuit as shown in Fig.11.30, then there is flow of charges from the capacitor and hence the process of discharging of the capacitor is started.

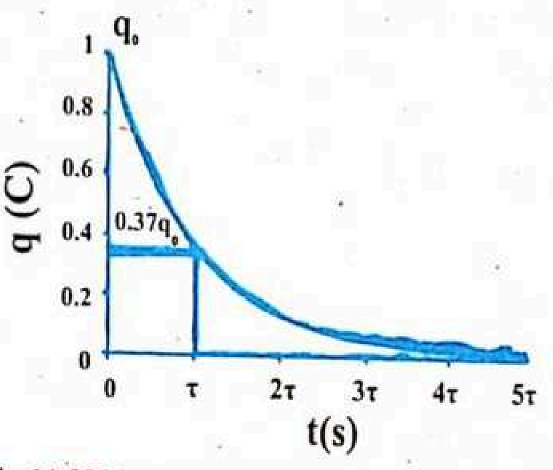


Fig.11.32 A curved line in charge – time graph shows a discharging of a capacitor

The rate of discharging also depends upon the product of R and C. Mathematically, the relation for discharging of a capacitor is given by

$$q = q_o e^{-\frac{t}{RC}} \quad ......(11.55)$$

This shows that the charge stored on the capacitor also decreases exponentially and graphically the discharging of a capacitor is explained as;

If time 't' = 1τ (1RC)

Then $q = q_0 e^{-1} = 0.37 q_0$ ......(11.56)

It means a capacitor is discharged about 63% or there are 37% charges left on the capacitance after one time constant.

Similarly, if time 't'=2τ (2RC)

Then $q = q_0 e^{-2} = 0.14 q_0$ ......(11.57)

This shows that a capacitor is discharged about 86% after two time constants.

After five time constants, the capacitor will be almost discharged. Graphically, a curved line in charge - time graph as shown in Fig.11.32 gives us discharging of a capacitor.

11.19 ENERGY STORED IN A CAPACITOR

As we know that a capacitor is a device which stores the charges. A charged capacitor has an electric field between its plates. This field has ability to store the energy in the form of electrical potential energy, it is explained as: Consider a parallel plate capacitor which is connected with source of potential difference 'V'.The charges from the source begin to flow towards the plates of the capacitor. The potential difference across the plates of the capacitor increases until it attains the same value as that of the source. It means there is work has to done in carrying the charges from the source onto the plates of the capacitor. This work done is stored in the form of electrical potential energy in the electric field between the plates of the capacitor. Its value can be calculated as: By the definition of work;

$$\text{Work} = F.\Delta r$$
$$= qE\cdot\Delta r$$
$$= q\frac{\Delta V}{\Delta r}\cdot\Delta r$$
$$\text{Work} = q\Delta V \quad ....(11.58)$$

> ***FOR YOUR INFORMATION***
> A capacitor has potential in terms of electrical energy.
> A battery has potential in terms of chemical energy.

As the potential difference across the capacitor rises from 0 to V so its average value is given by;

$$\Delta V = \frac{0+V}{2} = \frac{V}{2}$$

Also, $q = CV$

Thus, substituting the values of ΔV and q in Eq.11.58

$$\text{Work} = CV\cdot\frac{V}{2}$$

$$\text{Work} = \frac{1}{2}CV^2$$

This work is stored in a capacitor in terms of electrical potential energy. It is represented by ΔU. According to work-energy theorem:

$$\text{Work} = \Delta U$$

$$\Delta U = \frac{1}{2}CV^2 \quad ......(11.59)$$

This is the electrical P.E. stored in a capacitor, when the medium between the plates is air or vacuum. If the medium between the plates is dielectric material, then,

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

and $V = Ed$

Thus Eq 11.59 becomes

$$\Delta U = \frac{1}{2}\cdot\frac{A\varepsilon_0\varepsilon_r}{d}\cdot E^2d^2$$

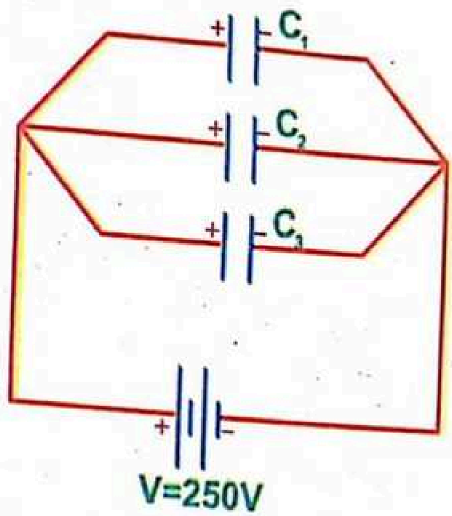
$$\Delta U = \frac{1}{2}\cdot\varepsilon_0\varepsilon_r E^2(\text{volume}) \quad \therefore Ad = \text{volume of dielectric medium between the plates}$$

Energy per unit volume i.e., energy density is given by;

$$U = \frac{\Delta U}{\text{volume}}$$

$$U = \frac{\frac{1}{2}\varepsilon_o\varepsilon_r E^2(\text{volume})}{\text{volume}}$$

$$U = \frac{1}{2}\varepsilon_o\varepsilon_r E^2 \quad ......(11.60)$$

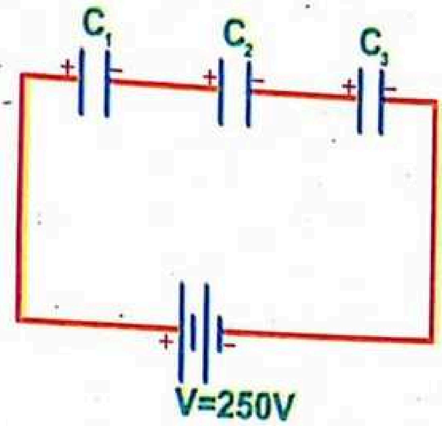


Example 11.7

Three capacitors of capacitances $10\,\mu F$, $50\,\mu F$ and $25\,\mu F$ respectively are given. Calculate:

i. Its total capacitance and charges on each capacitor when these are connected in parallel to a 250V supply.

ii. Total capacitance and potential difference across each capacitor when these are connected in series.



Solution:

Let

$$C_1 = 10\mu F = 10\times10^{-6}F$$
$$C_2 = 50\mu F = 50\times10^{-6}F$$
$$C_3 = 25\mu F = 25\times10^{-6}F$$

i. Total capacitance when capacitors are connected in parallel $C = ?$
Charges on each capacitor, Q_1, Q_2 and $Q_3 = ?$
Applied voltage $= V = 250v$

ii. Total capacitance when capacitors are connected in series $= C = ?$
V_1, V_2 and $V_3 = ?$ (when capacitors in series)

i. Let the three capacitors connected in parallel as shown in figure then their equivalent capacitor 'C' is given as:

$$C_{eq} = C_1 + C_2 + C_3$$
$$C_{eq} = 10 + 50 + 25$$
$$C_{eq} = 85\,\mu F$$

As the capacitors are connected in parallel so each capacitor has a same potential difference of 250V across it. To find the charge stored on each capacitor we have:

$$Q_1 = C_1V = 10\mu F\times250V = 2500\,\mu C$$
$$Q_2 = C_2V = 50\mu F\times250V = 12500\,\mu C$$
$$Q_3 = C_3V = 25\mu F\times250V = 6750\,\mu C$$

ii. Now when the three capacitors are connected in series, as shown in figure, then their equivalent capacitance is given by

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\frac{1}{C_{eq}}=\frac{1}{10\mu F}+\frac{1}{50\mu F}+\frac{1}{25\mu F}$$

$$\frac{1}{C_{eq}}=\frac{5+1+2}{50}$$

$$\frac{1}{C_{eq}}=\frac{8}{50}=\frac{4}{25}$$

$$C_{eq}=\frac{25}{4}\mu F=6.25\mu F$$

As $$Q=C_{eq}V=6.25\mu F\times 250V$$

$$Q=1562.5\,\mu C$$

As the capacitors are connected in series so each capacitor has same charge equal to $1562.5\,\mu C$. To find the potential difference across each capacitor we have

$$V_1=\frac{Q}{C_1}=\frac{1562.5\mu C}{10\mu F}=156.25V$$

$$V_2=\frac{Q}{C_2}=\frac{1562.5\mu C}{50\mu F}=31.25V$$

$$V_3=\frac{Q}{C_3}=\frac{1562.5\mu C}{25\mu F}=62.5V$$

Example 11.8

A capacitor is charged with 9nC and has 120V potential difference between its terminals. Compute its capacitance and the energy stored in it.

Solution:

We have

$$q=9nC=9\times 10^{-9}C$$
$$V=120V$$
$$C=?$$
$$\text{energy }(\Delta U)=?$$

As $$q=CV$$

$$C = \frac{q}{V} = \frac{9\times10^{-9}C}{120V} = 0.075\times10^{-9}F = 75\times10^{-12}F$$

$$C = 75pF \qquad \therefore 1pF = 1\times10^{-12}F$$

$$\Delta U = \frac{1}{2}CV^2$$

$$\Delta U = \frac{1}{2}\left(75\times10^{-12}F\right)\left(120V\right)^2$$

$$\Delta U = \frac{1}{2}\times75\times10^{-12}\times14400$$

$$\Delta U = 540000\times10^{-12}J$$

or $$\Delta U = 0.54\times10^{-6}J$$

$$\Delta U = 0.54\mu J$$

SUMMARY

- **Electrostatics:** It is the branch of physics in which we study the charged particles at rest.
- **Charge:** Like mass, electric charge is the intrinsic property of matter which causes it to exert or experience a force when placed in an electric or magnetic field.
- **Coulomb's Law:** This law is stated that the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between them.
- **Electric Field:** The region around a point charge in which another charge can experience force of attraction or repulsion is known as electric field. The electric field is represented by lines known as electric lines of force.
- **Electric Intensity:** The electric intensity is defined as a force experienced by a unit positive charge placed at any point in an electric field.
- **Electric dipole:** A pair of charges of equal magnitude but opposite sign separated by a small distance.
- **Dipole moment:** The product of magnitude of charge (either positive or negative) and the distance between the two charges is known as dipole moment.
- **Electric Flux:** The number of electric field lines passing through a certain element of area is known as electric flux.
- **Gauss's Law:** This law is stated as "the flux through any closed surface is equal to $\frac{1}{\varepsilon_o}$ times the total charges enclosed in it."

- **Potential Difference:** The difference in electric potential between two points in an electric field or an electric circuit. It is measured in terms of voltage.
- **Electric Potential:** The amount of work on a unit positive charge in moving it from infinity to a point in the electric field against the direction of electric field is known as electric potential at that point. Its unit is volt.
- **Electron Volt:** Electron volt is the unit of energy where 1eV is equal to 1.6×10^{-19}J. It is the amount of energy gained or lost by an electron when it is moved across an electric potential difference of one volt.
- **Capacitor:** A capacitor is a device that stores electric charge.
- **Dielectric:** The insulator or insulating material placed between the plates of the capacitor is known as dielectric.
- **Polarization:** The process in which the molecules of the dielectric materials between the plates of the capacitor forms dipoles is known as polarization.
- **Time Constant:** The product of resistance and capacitance (RC) in the given circuit is termed as time constant OR It is a time in which a capacitor is charged or discharged about 63% of its fully charged value.

EXERCISE

Choose the most appropriate option.

1. The electrostatic force between two point charges is independent of one of the following quantities
   (a) Magnitude of charges (b) Temperature of the charges
   (c) Distance between charges (d) Medium between charges
2. If the distance between two equal charges is reduced to half and the magnitude of charges is also decreased to half, then the force between them will be
   (a) Remain same (b) Decreased to half
   (c) Increased to double (d) Becomes four time
3. The concept of electric field was introduced by
   (a) Coulomb (b) Faraday (c) Gauss (d) Ampère
4. The number of electrons or protons which constitutes a charge of one coulomb is
   (a) 6.25×10^{-18} (b) 6.25×10^{18} (c) 1.6×10^{-19} (d) 1.6×10^{19}
5. For static electricity the value of relative permittivity is always
   (a) Zero (b) Less than one (c) Equal to one (d) Greater than one
6. A metallic charged sphere is placed in uniform electric field E, the electric field inside the sphere will be
   (a) E (b) Less than E (c) Greater than E (d) Zero

7. The electric intensity E at a point in a field due to a dipole depends upon distance 'r' the relationship between them is

(a) $E \propto r$ (b) $E \propto \frac{1}{r}$ (c) $E \propto \frac{1}{r^2}$ (d) $E \propto \frac{1}{r^3}$

8. The dipole moment is defined as the product of

(a) Charge and distance (b) Charge and displacement
(c) Charge and force (d) Charge and electric field

9. Debye is the unit of

(a) Electric field (b) Electric charge
(c) Lines of force (d) Dipole moment

10. The electric flux is maximum when angle 'θ' between area vector and electric intensity is

(a) 0° (b) 45° (c) 60° (d) 90°

11. Electric flux is independent of one of the following quantities

(a) Charge (b) Medium (c) Field (d) Distance

12. The dimension of an electric potential is same as that of

(a) Work (b) Work per unit charge
(c) Electric field per unit charge (d) Electric force per unit charge

13. Electron volt is the unit of

(a) Energy (b) Charge
(c) Current (d) Electric Potential

14. The energy of an electron which accelerates through a potential difference of 1000V is

(a) 1.6×10^{-22}J (b) 1.6×10^{-20}J (c) 1.6×10^{-19}J (d) 1.6×10^{-16}J

15. The unit of surface charge density is

(a) cm (b) $C\ m^{-1}$ (c) $C\ m^{2}$ (d) $C\ m^{-2}$

16. Capacitance of a capacitor does not depend upon

(a) Area of plate (b) Distance between plates
(c) Medium between plates (d) Material of the plates

17. In the presence of dielectric material, the electric field between the plates of the capacitor will be

(a) Zero (b) Remain same (c) Decreased (d) Increased

18. When the potential difference across the capacitor is decreased by dielectric then the capacitance of the capacitor will be

(a) Zero (b) Remain same (c) Decrease (d) Increase

19. The unit of product of resistance and capacitance is equal to unit of

(a) Time (b) Work
(c) Potential difference (d) Current

20. A capacitor is approximately full charged after
(a) Two time constant (b) Three time constant
(c) Four time constant (d) Five time constant

SHORT QUESTIONS

1. What is the effect of the medium on the electrostatic force between two point charges?
2. What do you know about dielectric constant or relative permittivity?
3. How does the electrostatic force exert on one charge by another charge?
4. Why the two lines of force do not cross each other?
5. How can you determine the direction of electric intensity?
6. Differentiate between point charge and test charge.
7. What do you know about the dipole axis and its direction?
8. Is dipole moment vector or scalar? If vector, then where is its direction in an electric field?
9. When the electric flux will be minimum and maximum?
10. What do you know about the Gaussian Surface?
11. What is the electric field inside the hollow charged spherical shell?
12. What do you know about the electric potential gradient?
13. What is the relation between work and electrical potential energy?
14. Define one electron volt and give its numerical value.
15. What is the absolute electric potential?
16. What is the capacitance of a capacitor?
17. What is the effect of dielectric on the capacitance of a capacitor?
18. What is the polarization of dielectric?
19. How does dipole moment produce in dielectric material?
20. How does the electric field between the plate of the capacitor reduce by dielectric?
21. What is RC-circuit and its function?
22. What is meant by time constant of the RC-circuit?
23. What is the unit of product of resistance and capacitance?

COMPREHENSIVE QUESTIONS

1. Define and explain charges, their kinds and properties.
2. State and explain Coulomb's law, also discuss the magnitude of electrostatic force of attraction or repulsion between two point charges in the absence and presence of dielectric.
3. What do you know about the electric field and electric field intensity? Explain the magnitude and direction of electric field intensity?

4. What are the characteristics of the electric lines of force? Explain the electric field in terms of lines of force due to a positive charge, a negative charge and two identical positive or negative charges.
5. What is electric dipole? Calculate electric field due to a dipole.
6. Define electric flux and discuss how does the electric flux become maximum and minimum?
7. State and explain Gauss's law. Also discuss the various applications of Gauss's law.
8. What do you know about the electric potential? Define potential gradient and calculate potential due to a point charge.
9. Define capacitor and its capacitance. Also calculate the capacitance of a parallel plate capacitor.
10. State and explain series and parallel combinations of capacitors.
11. What is electric polarization? Discuss the effect of dielectric on the capacitance of a capacitor.
12. State and explain the process of charging and discharging of a capacitor.

NUMERICAL PROBLEMS

1. How many electrons are contained in 1C and 3C of charges?
(6.25×10^{18} electrons, 1.9×10^{19}eletrons)
2. Determine the force of repulsion between two free electrons spaced 0.5 Angstrom ($1A^0 = 10^{-10}$m). **(92nN)**
3. Two coins lie 2m apart on a table, and carry identical charge. How large is the charge on each if a coin experiences a force of 3N? **(3.65×10^{-5}C)**
4. Two positive charges of magnitudes $q_1 = 5\mu C$ and $q_2 = 2\mu C$ are separated by 50cm from each other. At what point the electric intensity due to these charges will be zero? **(0.3m from q_1)**
5. Find the force that an electron experience in an electric field of $1000NC^{-1}$. If the electron is free to move. Find the distance covered by it in 10ns. The mass of electron is 9.11×10^{-31}kg. **(1.6×10^{-16}N)(0.88mm)**
6. The diameter of a hollow metallic sphere is 10cm and the sphere carries charge of $80\mu C$. Find the electric intensity (i) at a distance 60cm from the centre of the sphere and (ii) at the surface of the sphere.
(2×10^6N/C, 2.88×10^8N/C)
7. An electric dipole consists of two charges $+10\mu C$ and $-10\mu C$ separated by distance 0.5cm. Calculate the electric field intensity at a point on the axial line at a distance of 5cm from the midpoint of the dipole. **(7.2×10^6 N/C)**
8. Find the electric flux through each face of a hollow cube of side 5cm. If a charge of $6\mu C$ is placed at its centre. **(1.133×10^5 Nm^2C^{-1})**

9. The potential difference between two metal plates is 120V. The plate separation is 3mm. Find the electric field between the plates. **(40KV/m)**

10. A charged particle remains stationary between the two horizontal opposite charged plates due to its weight and the electric force by the field. Find the potential difference between the plates. Where the distance between the plates is 2cm, mass of particle 4×10^{-13}kg and charge on particle 2.4×10^{-18}C. **(32.6KV)**

11. A capacitor of 5μF is charged by a 12V battery. Find the charge and energy stored on it. **(60μC, 3.6×10^{-4}J)**

12. Three capacitor have capacitance 2μF, 5μF and 7μF. Calculate their equivalent capacitance when they are connected in (i) Series (ii) Parallel **(1.19μF,14μF)**

Unit 12

CURRENT ELECTRICITY

Major Concepts (36 PERIODS)

- Steady current
- Electric potential difference
- Resistivity and its dependence upon temperature.
- Internal resistance
- Power dissipation in resistance
- Thermoelectricity
- Kirchhoff's Laws
- The potential divider
- Balanced potentials (Wheatstone bridge and potentiometer)

Conceptual Linkage

This chapter is built on
Current Electricity Physics
X

Students Learning Outcomes

After studying this unit, the students will be able to:

- described the concept of steady current.
- state Ohm's law.
- define resistivity and explain its dependence upon temperature.
- define conductance and conductivity of conductor.
- state the characteristics of a thermistor and its use to measure low temperatures.
- distinguish between e.m.f. and p.d. using the energy considerations.
- explain the internal resistance of sources and its consequences for external circuits.
- describe some sources of e.m.f.
- describe the conditions for maximum power transfer.
- describe the thermocouple and its function.
- explain variation of thermoelectric e.m.f. with temperature.
- apply Kirchhoff's first law as conservation of charge to solve problem.
- apply Kirchhoff's second law as conservation of energy to solve problem.
- describe the working of rheostat in the potential divider circuit.

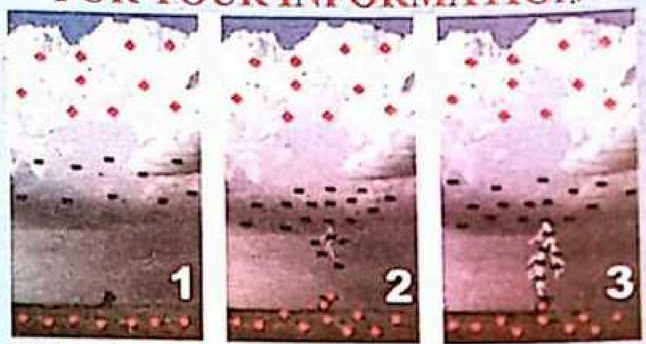
- describe what is a Wheatstone bridge and how it is used to find unknown resistance.
- describe the function of potentiometer to measure and compare potentials without drawing any current from the circuit.

INTRODUCTION

In the previous unit, we have studied the electrostatics i.e., the charges at rest. In this unit, we will study the rate of flow of electric charges passing through a point which is termed as electric current. Such motion of charges take place within a conducting closed path, the path is called an electric circuit. The main function of the electric circuit is to transfer the electrical energy (current) from electrical source to the loads. The loads may be bulb, heating element, fan, motor, etc., which convert the electrical energy into the other forms of energy, such as: sound, light, heat, mechanical energy, chemical energy etc. For example, a stereo system converts electrical energy into sound, a toaster or bulb into heat and light, a motor into a mechanical energy etc. Moreover, the electrical energy or current powers our computers, televisions, CD players, air conditioners, refrigerators etc.

In this unit, we will explain not only the sources of electric current but also the factors which are related with the current, such as resistance and resistivity, conductance and conductivity, potential difference and electromotive force etc. We will discuss power generation in a source and its dissipation in an electric circuit. Similarly, we will also introduce the circuits analysis by using ohm's law as well as Kirchhoff's two laws. i.e., node analysis and loop analysis. In the same way, we will explain the potential dividers (Wheatstone bridge and potentiometer) in the last section of this unit.

FOR YOUR INFORMATION



Lightening occurs mainly in warm climates. As warm water vapors rise in air, they bushes against ice crystals high in the air above, producing charges. The ice crystals gain a slight positive charges and the undraft carries them to the top of a cloud is usually positively charged with the bottom negatively charged. Lightening is the bolt that arcs between these regions and between the cloud and the ground below.

DO YOU KNOW

An electric shock is a violent disturbance of the nervous system caused by an electrical discharge or current through the body.

12.1 STEADY CURRENT

The study of flow of either positive or negative charges through a conducting medium with time is known as electric current. Usually, it is represented by 'I' and it is defined as **the rate of flow of charges.** If the amount of charge ΔQ flow through a conductor in time Δt, then the electric current can be expressed as:

$$I = \frac{\Delta Q}{\Delta t} \quad ......(12.1)$$

The SI unit of current is Ampere. **When one coulomb charge flows in a conductor in one second, then the current is said to be one ampere.** In other words, the current of one ampere is equal to the flow of combined charge of 6.25×10^{18} electrons through a conductor in one second. Now if the magnitude of rate of flow of charges remain constant then it is called a steady current.

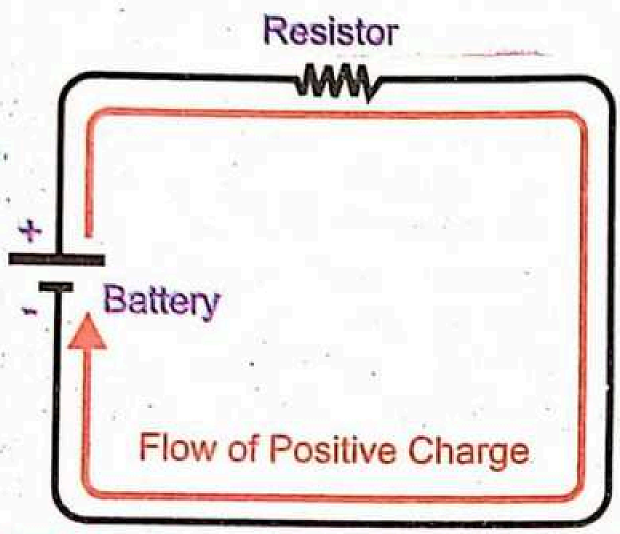


Fig.12.1(a) Conventional current due to flow of charges from positive terminal to negative terminal of the battery.

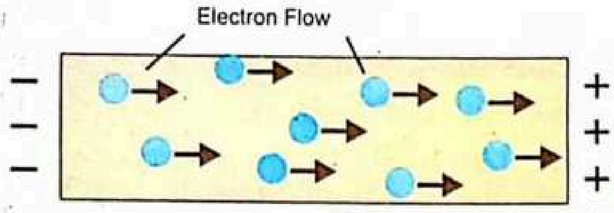


Fig.12.1(b) Electronic current due to flow of electrons.

The flow of charges may be positive or negative charges. The current due to flow of positive charges from positive terminal of the battery towards the negative terminal as shown in Fig. 12.1(a) is called conventional current. The current in metals is due to flow of free electrons as shown in Fig. 12.1(b), is known as electronic current. In semiconductor materials such as germanium and silicon, the current is due to the flow of electrons and holes, this is also known as electronic current. Similarly, in case of conducting liquid called electrolyte or ionized gas called plasma, the current in these medium is due to both negatively and positively charged ions as shown in Fig. 12.1(c) and its resultant current (I) is given by

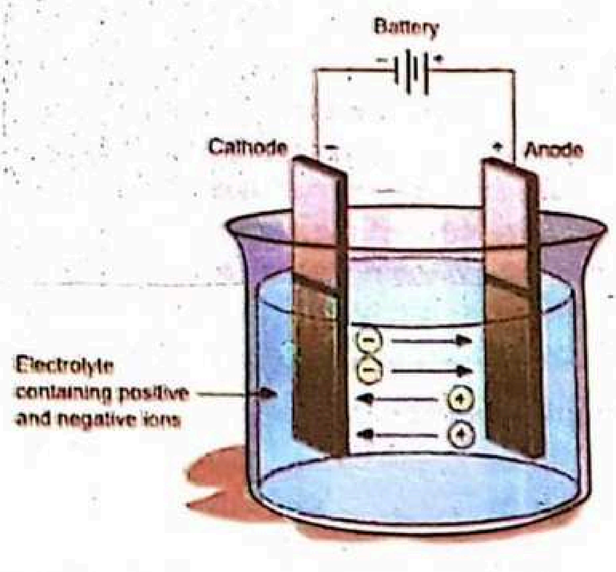


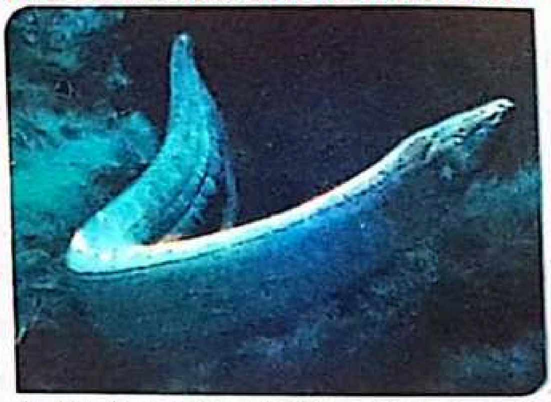
Fig.12.1(c) Current due to flow of positive and negative ions in electrolyte.

$$I = I_+ + I_- \quad ......(12.2)$$

12.1.1 The direction of current through a metallic conductor

In the beginning, the scientists believed that the current is due to the flow of positive charges through a conductor when it is connected with the terminals of the battery. It is therefore, the positive terminal of the battery has high potential while its negative terminal has low potential. Later on, the experimental observations proved that the current is indeed due to the flow of electrons through a conductor when it is connected with the terminals of the battery. For example, metals such as silver, copper, aluminum etc. have free electrons which are moving randomly in the absence of any electric field and their speed depends upon temperature. Now when one of the metallic conductor say copper is connected across the source (cell) than an electric field is produced across the copper and it is directed from positive towards the negative plate of the cell as shown in Fig.12.2. The free electrons of the copper experience attractive force ($F = qE$) due to the anode and repulsive force due to the cathode. As a result, the free electrons of the copper start drifting in one direction from −ve terminal towards the +ve terminal of the cell as shown in Fig.12.2. Hence, this result has confirmed that the current is actually due to the flow of negative charges in a metallic conductor but its direction is taken as in its opposite direction.

INTERESTED INFORMATION



A 20 foot long electric eel is a south American electric fish has a number of cells called electro plaques that produces emf of 600V. The current due to this emf is used to stun its enemies and to kill its prey.

POINT TO PONDER

Why is it advised to wear rubber soled shoes while handling electric appliances?

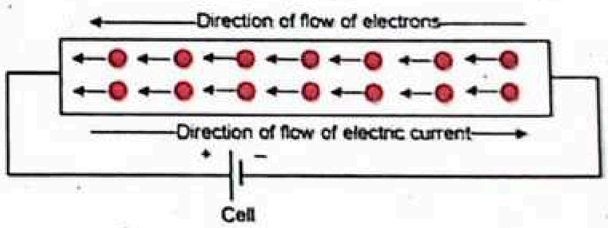


Fig.12.2 The flow of electrons toward the positive terminal of the battery while the direction of current from positive to negative terminal.

Example 12.1

If 1×10^{13} electrons flow through a conductor in 1ms, calculate the current in Ampere through the conductor.

Solution:

Number of electrons = $n = 1 \times 10^{13}$ electrons

Charge on an electron = $e = 1.6 \times 10^{-19}$ C

Time taken = $t = 1\text{ms} = 1 \times 10^{-3}$ s

So, Charge on 1×10^{13} electrons $= Q = ne = 1.6\times10^{-19}\,C\times 1\times10^{13}$

$$Q = 1.6\times10^{-6}\,C$$

Thus, the flow of current through the conductor is given by

$$I = \frac{Q}{t} = \frac{1.6\times10^{-6}\,C}{1\times10^{-3}\,s}$$

$$I = 1.6\times10^{-3}\,A$$

$$I = 1.6\,mA$$

12.2 SOURCES OF STEADY CURRENT

As the flow of fluid between two points requires a difference in pressure between the given points, similarly flow of charges between two points through a conducting medium takes place only when a potential difference exists across these two points. For example, let two conductors at different potential are connected by a metallic wire. Then there will be flow of current through the wire but such current is for very short period of time because the potential of both conductors become same due to decreasing the flow of charges. Like a steady flow of river, we require a steady current in our circuit. It is possible only when we have suitable source. Such source was discovered by Italian scientist Volta in 1800 and is called voltaic cell. A simple source of current as shown in Fig.12.3 consists of two metallic plates which are immersed in a dilute sulfuric acid (H_2SO_4) known as electrolyte. One plate is made of copper called anode and the other plate is made of zinc called cathode. Now due to the chemical action within the source, the electrons are released by the copper plate and collected by the zinc plate. Thus, under this process the copper plate becomes positively charged while the zinc plate becomes negatively charged. In this way, Volta was

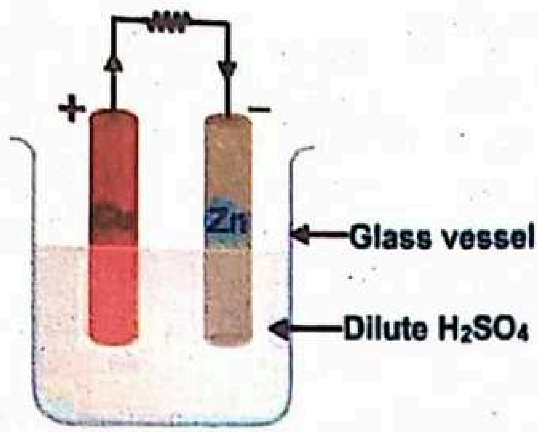
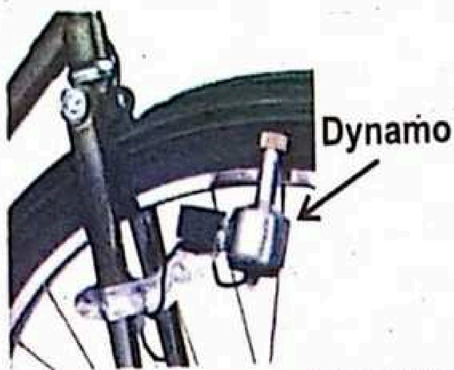
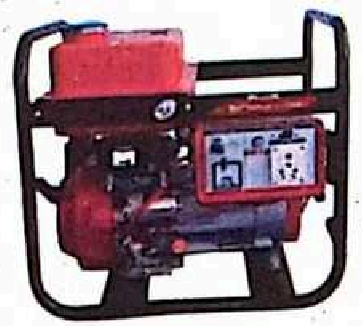


Fig.12.3 A schematic diagram for source of a current.



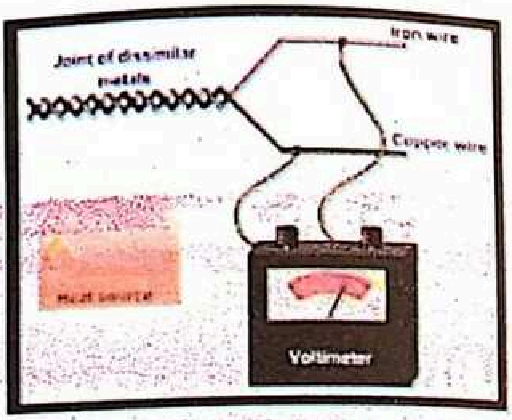
A dynamo as a source of electricity



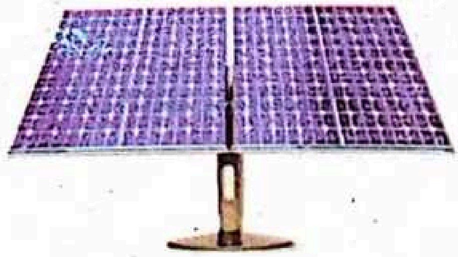
An electric generator

succeeded in the development of potential difference. When these two plates (electrodes) are connected externally by a wire, then there will be flow of electrons from the cathode towards the anode and its uniform flow depends upon the continuous chemical reaction.

After voltaic cells, other cells were also discovered by using different materials and different electrolytes; such as Daniell cell, Laclanche cell etc. All these cells are not rechargeable and called primary cells. Later on, the rechargeable cells, such as nickel cadmium cell, lead acid cell have also been constructed. These are called a secondary cells. There are number of other sources of current which are summarized as;



A thermocouple


A solar cell

i. **Dynamo:** It converts mechanical energy into electrical energy.

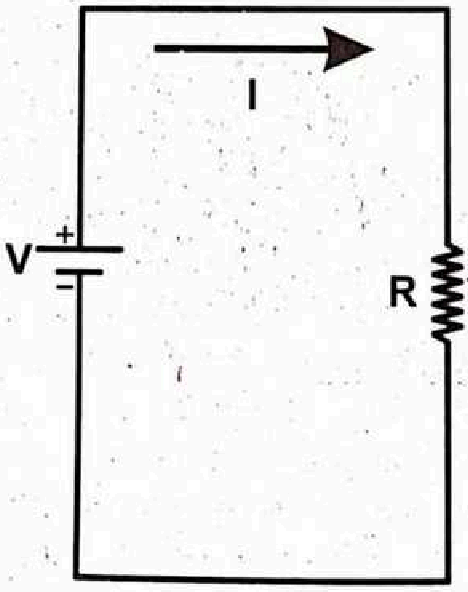
ii. **Generators:** It also converts mechanical energy into electrical energy.

iii. **Thermo-couple:** It converts the heat energy into electrical energy.

iv. **Solar or photo cell:** It converts solar energy into electrical energy.

12.3 OHM'S LAW

As we have discussed that when the potential difference is applied across the conductor, its free electrons start drifting in one direction. As a result, there are collision between the free electrons with the atoms or ions of the conductor. These collisions cause of opposition to the flow of current and such opposite to the flow of current is called resistance of the conductor.



At constant resistance, the applied voltage across the conductor is directly proportional to the current

A German scientist G.S Ohm studied the relation among the following three parameters, i.e., the voltage applied across the conductor, the current through the conductor and the resistance of the conductor. Thus, by summing his experimental results, he formulated a law in 1826, known as Ohm's Law which relates the above stated parameters and it is stated as; **the applied voltage across the conductor is directly proportional to the steady current, if the temperature and other physical conditions of the conductor remain constant.**

Mathematically, Ohm's law can be expressed as

$$V \propto I$$

$$V = I\,R \quad \ldots\ldots(12.3)$$

where 'R' is the constant of proportionality and it is known as resistance of the conductor. The value of resistance depends upon temperature and other physical states of the conductor. Its SI unit is ohm and it is denoted by 'Ω'. **A conductor has a resistance of one ohm, if a current of one ampere flows through it by applying a potential difference of 1volt across its ends.**

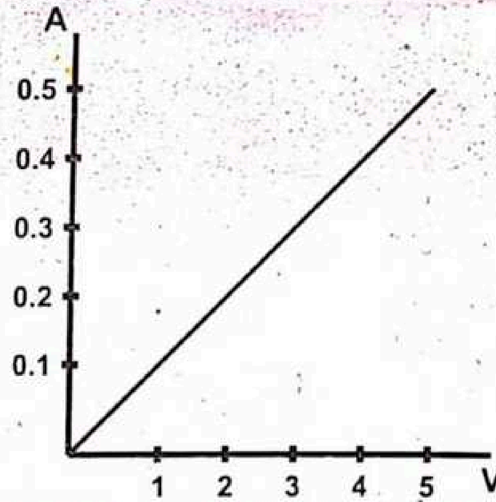


Fig.12.4(a) A straight line between V & I due to Ohmic device.

The materials which obey Ohm's law and have constant resistance over a large range of voltage are known an Ohmic materials. For example, a carbon resistor, nichrome or eureka wire, and metallic conductors etc. are known as Ohmic materials.

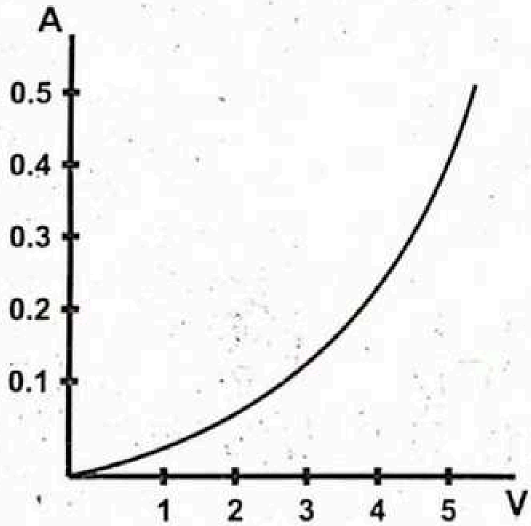


Fig.12.4(b) A curved line between V & I due to a non-ohmic device

For ohmic materials, the graph of voltage versus current is a straight line as shown in Fig. 12.4(a). Such straight line graph is known as ohmic characteristics.

On the other hand, those materials having resistance that varies with current or voltage non-linearly are known as non-ohmic materials. For example, filament of a bulb, semiconductor diode etc. are non-ohmic devices and the graph of voltage versus current is a curved line i.e. not linear as shown in Fig.12.4(b). Such curved line is known as non-ohmic characteristics of a non-ohmic device.

DO YOU KNOW?

- Flow of current is directly proportional to the potential difference.
- Flow of heat is directly proportional to the temperature difference.
- Flow of fluid is directly proportional to the pressure difference.

Example 12.2

What is the current through a resistor of 16Ω when the potential difference of 240 volts is applied across it.

Solution:

Current = I = ?

Resistance of the resistor = R = 16Ω

Applied voltage = V = 240v

According to Ohm's law

$$V = I R$$

$$I = \frac{V}{R}$$

$$I = \frac{240V}{16\Omega}$$

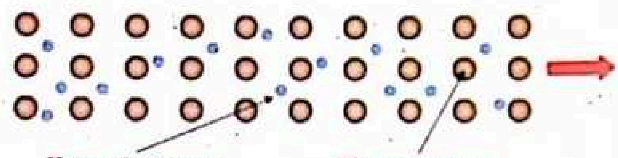
$$I = 15A$$

> ***FOR YOUR INFORMATION***
> Current is a flow of charge, pressured into motion by voltage and hampered by resistance.

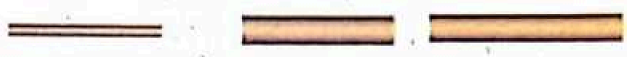
> ***FOR YOUR INFORMATION***
> An ohmic device is one for which, under constant physical condition such as temperature, the resistance is constant for all current that pass through it.
> A non-ohmic device is one for which the resistance is different for different currents passing through it.

12.4 RESISTIVITY

Nearly, all the materials have some resistance to flow of current. e.g., silver, copper, aluminum etc. have small resistance and they are called conductors. While, some materials such as glass, rubber, wood etc. offer high resistance to the flow of current and they are called insulators. **Therefore, the resistance of a material or conductor in its one cubic metre is known as its specific resistance or resistivity. In other words, the resistance of a wire of 1m length and has cross section area of 1m² is called its resistivity.** Its value can be calculated as, since the resistance of a conductor is due to the collision between the free moving electrons with the atoms or ions of the conductor when a potential difference is applied across it. So one can say that the resistance of the conductor depends upon its length and cross-sectional area, because for a long conductor, the electrons undergo greater number of collision and hence there will be more resistance of the conductor. Also resistance increases by decreasing cross-sectional area of the conductor. Thus the experimental results show



A resistance due to the collision between the moving charges and the atoms of the conductor.



A resistance is directly proportional to the length and inversely proportional to the cross sectional area of conductor that the resistance of a conductor is directly proportional to its length ℓ and inversely proportional to its cross-sectional area A. i.e.,

$$R \propto \ell$$

$$R \propto \frac{1}{A}$$

> *DO YOU KNOW*
> A single cell supplies 1.5V. Inside a 12V battery, there are eight cells of 1.5V each.

Combing these two relations

$$R \propto \frac{\ell}{A}$$

$$R = \rho\frac{\ell}{A} \quad ......(12.4)$$

where 'ρ' is a constant of proportionality known as specific resistance or resistivity of the material and it depends upon the nature of the material. That is, conductors have low resistivity and insulators have high resistivity. The value of the resistivity of different materials are shown in table 12.1. Mathematically the value of 'ρ' can be calculated as

$$\rho = \frac{RA}{\ell} \quad ......(12.5)$$

The unit of 'ρ' is Ωm.

Table 12.1

Resistivity (Ω m) of some materials

Material	Resistivity
Carbon (graphene)	1×10^{-8}
Silver	1.59×10^{-8}
Copper	1.68×10^{-8}
Gold	2.44×10^{-8}
Calcium	3.36×10^{-8}
Zinc	5.90×10^{-8}
Latium	9.28×10^{-8}
Iron	1.0×10^{-8}
Platinum	1.06×10^{-7}
Tin	1.09×10^{-7}
Lead	2.2×10^{-7}
Carbon (graphite)	2.5×10^{-6} to 5.0×10^{-6}
Sea water	2×10^{-1}
Drinking water	2×10^{1} to 2×10^{3}
Silicon	6.40×10^{2}
Deionized water	1.8×10^{5}
Glass	1×10^{10} to 1×10^{14}
Carbon (diamond)	1×10^{12}
Hard rubber	1×10^{13}
Sulfur	1×10^{15}
Air	1.3×10^{16} to 3.3×10^{16}
Teflon	10×10^{22} to 10×10^{24}

12.4.1 Conductance

Conductance is the reciprocal of resistance. It is represented by G and is expressed as;

$$\text{Conductance} = \frac{1}{\text{Resistance}}$$

$$G = \frac{1}{R} \quad ......(12.6)$$

The SI unit of conductance is per ohm or 'mho' (Ω^{-1}) also known as Siemen.

It is defined as the ability to which an object conducts electricity. It may be expressed in terms of ratio of the current to the potential difference and mathematically, $G = \frac{I}{V} = \frac{1}{R}$, i.e., the lower the resistance, the higher the conductance.

12.4.2 Conductivity

Conductivity is also an electrical property of materials which is reciprocal of the resistivity of the conductor. It is denoted by 'σ' (sigma) and it is expressed in terms of the reciprocal of resistivity i.e.,

$$\text{Conductivity} = \frac{1}{\text{Resistivity}}$$

$$\sigma = \frac{1}{\rho} \quad ......(12.7)$$

> ***FOR YOUR INFORMATION***
>
> Since R and G are reciprocal to each other, their value can be calculated as:
>
> $$R = \frac{V}{I} \text{ and } G = \frac{I}{V}$$

By definition of resistance in terms of resistivity

$$R = \rho\frac{\ell}{A} \quad \text{(from Eq.12.4)}$$

But $\quad R = \dfrac{1}{G}$ and $\rho = \dfrac{1}{\sigma}$

$$\frac{1}{G} = \frac{1}{\sigma}\frac{\ell}{A}$$

$$\Rightarrow \quad G = \sigma\frac{A}{\ell}$$

$$\sigma = \frac{G\ell}{A}$$

The SI unit of conductivity is $\Omega^{-1}m^{-1}$ or mho.m^{-1}. The values of conductivity of different materials are listed in table 12.2.

Table 12.2

Electrical Conductivity of Some material

Material	Conductivity $(\Omega^{-1}m^{-1})$
Silver	66.7×10^6
Copper	64.1×10^6
Gold	49.0×10^6
Aluminum	40.8×10^6
Rhodium	23.3×10^6
Zinc	28.2×10^6
Nickel	16.4×10^6
Cadmium	14.7×10^6
Iron	11.2×10^6
Platinum	10.2×10^6
Palladium	9.3×10^6
Tin	8.7×10^6
Chromium	7.9×10^6
Lead	5.3×10^6
Titanium	2.3×10^6
Mercury	1.0×10^6
Carbon-graphite	$(1.5-20) \times 10^4$

12.4.3 Variation of resistivity with temperature

It is an experimental fact that the resistance of a conductor depends upon the temperature i.e., the resistance of a material increases with increase in its temperature. For example, the atoms in a solid are always vibrating about their mean position at room temperature. Now when the temperature increases, the amplitude of vibration of atoms also increases and the free electrons undergo greater number of collisions. Hence the resistance of the conductor increases with increase in temperature.

Let a metallic conductor having resistivity R_o at temperature 0°C, when its temperature is increased to t°C then its resistance becomes R_t.

Thus, change in resistivity $= \Delta R = R_t - R_o$

change in temperature $= \Delta T = t - 0 = t$

The analysis of experimental data on resistance as a function of temperature shows that change in resistance of the conductor is directly proportional to both initial resistance of the conductor and raise in temperature. That is,

$$\Delta R \propto R_0$$

$$\Delta R \propto \Delta T$$

By combining these two relations

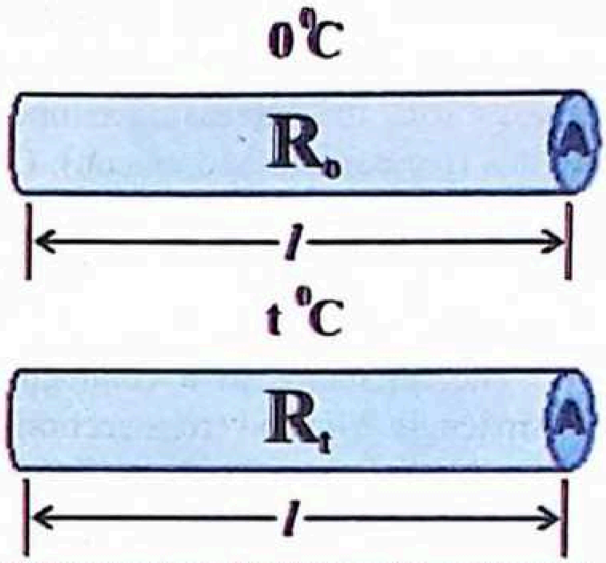
$$\Delta R \propto R_0 \Delta T$$

$$\Delta R = \alpha R_0 \Delta T \quad ......(12.9)$$

where ' α ' is constant of proportionality and it is known the temperature co-efficient of resistivity. Its value can be calculated as,

$$\alpha = \frac{\Delta R}{R_o \Delta T}$$

$$\alpha = \frac{R_t - R_o}{R_o t} \quad ......(12.10)$$



Different values of resistance of a conductor at different temperatures.

DO YOU KNOW

A fuse is a safety device that serves to protect the circuit components and wiring in the event of short circuit. Excessive current melt the fuse conductor which blows the fuse.

The SI unit of 'α' is K^{-1}. Let 'ℓ' be the length of the uniform conductor and 'A' be its cross-sectional area. Similarly, ρ_0 be its resistivity at 0°C and ρ_t be its resistivity at t°C then Eq.12.10 can be expressed in terms of resistivity as;

$$\alpha = \frac{\frac{\rho_t \ell}{A} - \frac{\rho_o \ell}{A}}{\frac{\rho_o \ell}{A} \cdot t}$$

$$\alpha = \frac{\rho_t - \rho_o}{\rho_o t}$$

$$\rho_t - \rho_o = \alpha \rho_o t$$

$$\rho_t = \rho_o + \alpha \rho_o t$$

$$\rho_t = \rho_o (1 + \alpha t) \quad ......(12.11)$$

Table 12.3

Color	Temperature Coefficient $[(°C)^{-1}]$
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminium	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.92×10^{-3}
Lead	3.9×10^{-3}
Nichrome	0.4×10^{-3}
Carbon	-0.5×10^{-3}
Germanium	-48×10^{-3}
Silicon	-75×10^{-3}

The values of temperature co-efficient for various materials are given in Table 12.3. The results show that the value of ' α ' is positive when resistance of the material increases with the increasing temperature such as copper, silver tungsten etc. Similarly, the value of 'α' is taken as negative when the resistance of the material decreases with the increasing temperature such as carbon and all semiconductor materials (germanium and silicon). On the other hand, the value of 'α' is taken as zero, when the resistance of the material remains constant with changes in temperature.

Example 12.3

The resistance of a conductor of uniform length 20m and cross-section area $1mm^2$ is 0.4Ω. Determine the resistivity of the conducting material.

Solution:

Resistance of the conductor $R = 0.4\Omega$

Length of the conductor $\ell = 20m$

Cross-section area of the conductor $A = 1mm^2$

Or $A = 1 \times 10^{-6} m^2$

Resistivity of the conductor $\rho = ?$

Resistivity of the conductor can be calculated by using Eq.12.4, that is,

$$R = \rho \frac{\ell}{A}$$

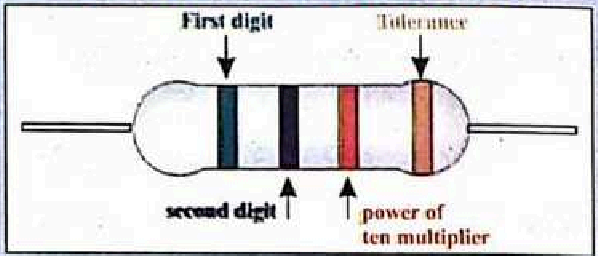
$$\rho = \frac{RA}{\ell}$$

$$\rho = \frac{(0.4\Omega)\times(1\times10^{-3}m^2)}{20m}$$

$$\rho = 0.02 \times 10^{-3}\ \Omega m$$

$$\rho = 2\times10^{-5}\Omega m$$

FOR YOUR INFORMATION



The resistor colour code

Colour	Digit	Multiplier	Tolerance
Black	0	10^0 or 1	
Brown	1	10^1	
Red	2	10^2	±2%
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold		10^{-1}	±5%
Silver		10^{-2}	±10%
No colour			±20%

The numerical value of resistance of a carbon resistor can be determined by its color code, which consist of four bands, three at one end, fourth on the other. Where the first two bands specify the first two digits and the third band gives the number of zeroes. The fourth band at the other end gives the value of tolerance in percentage.

Example 12.4

A tungsten filament of a bulb has a resistance of 50Ω at $20°C$ and 467Ω at $2000°C$. Determine the value of temperature co-efficient of resistance of the tungsten.

Solution:

$$R_1 = 50\Omega$$

$$R_2 = 467\Omega$$

$$t_1 = 20°C$$

$$t_2 = 2000°C$$

Change in temperature $\Delta T = 2000 - 20 = 1980°C$

Now, by using Eq.12.11,

$$R_2 = R_1(1+\alpha\Delta T)$$

$$\alpha = \frac{R_2 - R_1}{R_1\Delta T}$$

$$\alpha = \frac{467\Omega - 50\Omega}{(50\Omega)(1980^\circ C)}$$

$$\alpha = 0.0042^\circ C^{-1}$$

12.5 THERMISTOR

Thermistor is a temperature dependent resistor, its resistance changes very fast even with small change of temperature. The term thermistor is combination of thermal and resistor. Majority of the thermistors are working under the negative temperature co-efficient of resistance, however the positive temperature coefficient of a thermistors are also available. Thermistors are made from the semiconductor oxides of nickel, cobalt, copper, iron etc. They are constructed in different shapes such as beads, discs or rods etc. under different conditions as shown in Fig.12.5.

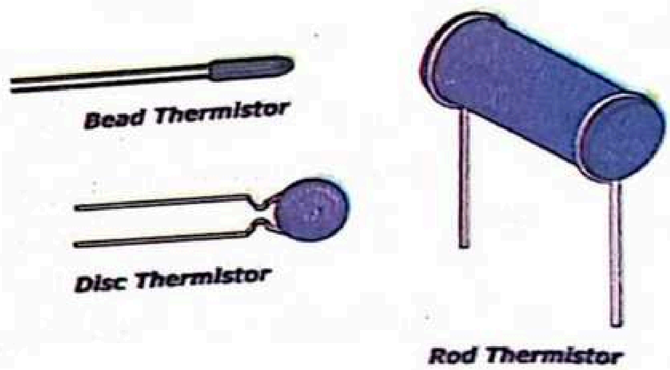


Fig.12.5 Different types of thermistors.

Thermistors have several applications which are summarized as:

i. A negative temperature coefficient thermistor is being used to safeguard against current surge in a circuit.
ii. Thermistors are being used for voltage stabilization.
iii. It is being used as a temperature sensor.
iv. Thermistors are used to measure very low temperatures of the order of 10K.

12.6 ELECTROMOTIVE FORCE (e.m.f.) AND POTENTIAL DIFFERENCE

To maintain a steady current in a closed electric circuit, a source of electrical energy is required. The source may be battery, generator, solar cell, thermocouple etc. All these are known as sources of electromotive force (e.m.f.). When any one of them is connected to a circuit which consists of a resistor 'R', then an electric field is produced in the circuit directed from the high potential to the low potential as shown in Fig.12.6. The electrons from the negative terminal are forced by the source of e.m.f.

to move against the direction of the field. Thus, **the energy supplied, by the source per unit charge to move the charge in circuit from the low potential to the high potential is called its electromotive force.** It always raises the potential of the charges and it is expressed as

$$\text{e.m.f. } (E) = \frac{W}{q}$$

$$W = qE \text{ ......(12.12)}$$

The SI unit of e.m.f. is volt.

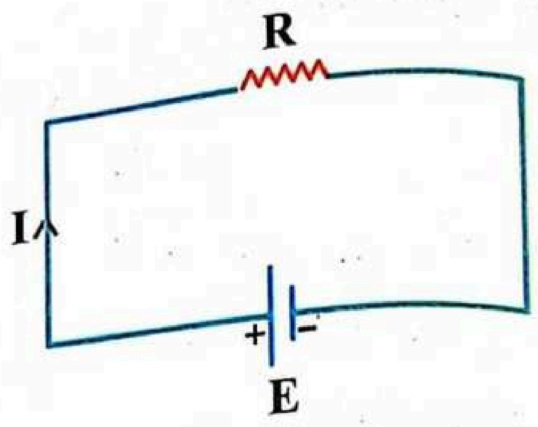


Fig.12.6 An electric circuit consists of a resistor and a source of e.m.f. (E).

On the other hand, when the source delivers the electrical energy into the circuit, then **work done per unit charge between two points (A and B) as shown in Fig.12.7, is known as the potential difference.** It is represented by V and it is expressed as;

$$W = qV \text{ ......(12.13)}$$

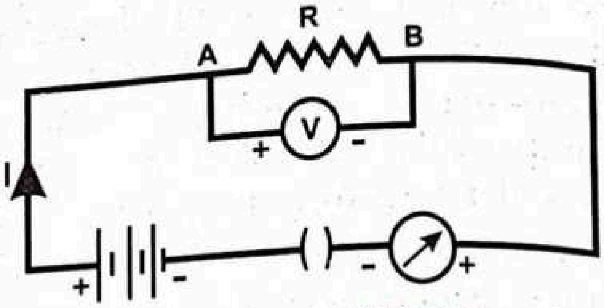


Fig.12.7 The electric potential between point A and point B in an electric circuit.

An ideal source of e.m.f. is one which maintains a constant potential difference between the two points in the circuit. Thus, by comparing Eq.12.12 and Eq.12.13 we get;

$$qE = qV$$

$$E = V \quad (\text{As } V = IR)$$

$$E = IR$$

$$I = \frac{E}{R} \text{ ......(12.14)}$$

DO YOU KNOW

The word 'open' means no flow of charges (current) in the given circuit and the word 'closed' is applied for the flow of charges (current) in the circuit.

Each real source of e.m.f. always has some internal resistance 'r' as show in Fig.12.8, and this resistance is due to the electrolyte present inside the source. Thus, in the presence of internal resistance, the potential difference across a real source in a circuit is not equal to the e.m.f. because some energy is dissipated through 'r'. Thus, when a current I is

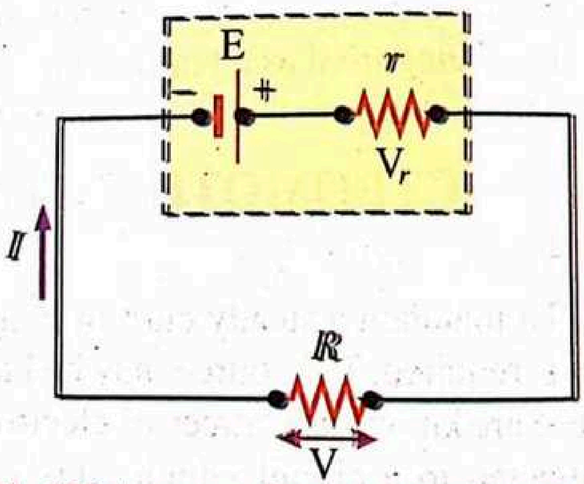


Fig.12.8 A source of e.m.f. which has an internal resistance r.

flowing from the negative terminal of the source to the positive terminal through the external resistance 'R' and internal resistance 'r', then the potential difference 'V' between the terminals of the source is given by

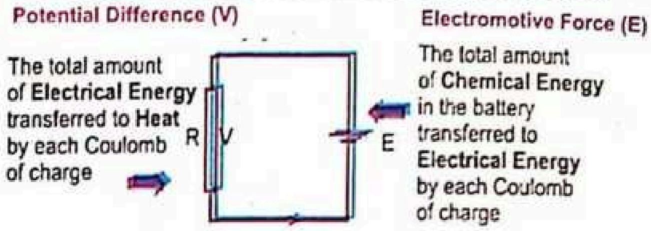
$$V = E - V_r$$

$$IR = E - Ir$$

$$E = I(R + r)$$

$$I = \frac{E}{R + r} \quad \text{......(12.15)}$$

Potential difference vs Electromotive force



This equation gives us few important results about the e.m.f. and the potential difference. That is,

1. If $r \neq 0$ and the source delivers the current into the circuit then the e.m.f. of the source will be greater than the potential difference $(E > V)$.
2. If the switch is opened and there is no flow of current in the given circuit i.e. there is no voltage drop across the internal resistance. Then the e.m.f. will be equal to the potential difference $(E = V)$
3. The e.m.f. also equals to the potential difference if the internal resistance is zero $(r = 0)$. But it is an ideal case.

12.6.1 Maximum power transfer

We have discussed that a source of e.m.f. generates electrical energy and transfers it in an electrical circuit. The rate of transfer of this electrical energy is termed as electrical power which is dissipated in resistor R in terms of voltage 'V' and current 'I'. Now if we neglect the internal resistance of the source, then according to law of conservation of energy, the power delivered by source is equal to power dissipation in load resistor R i.e.,

Power delivered to R = Power generation

$$P_{out} = \frac{\text{energy transferred}}{\text{time}}$$

$$P_{out} = \frac{V\Delta Q}{\Delta t}$$

as $\quad \frac{\Delta Q}{\Delta t} = I$

$$P_{out} = VI$$

and $\quad V = IR$

$$P_{out} = I^2R \quad \text{......(12.16)}$$

> **POINT TO PONDER**
> Since power is directly proportional to I^2 or V^2, if current through resistor or voltage across resistor is doubled then power will be?

A real source has some internal resistance. Substitute the value of current in terms of resistance 'r' from Eq.12.15 into Eq.12.16.

$$P_{out} = \frac{E^2}{(R+r)^2}R$$

$$= \frac{E^2R}{(R-r)^2+4rR}$$

$$= \frac{E^2}{\frac{(R-r)^2+4rR}{R}}$$

$$P_{out} = \frac{E^2}{\frac{(R-r)^2}{R}+4r} \quad \ldots\ldots(12.17)$$

If R = r then the denominator of Eq.12.17 is minimum and then the power P_{out} will be maximum. Thus, it is concluded that a load resistor 'R' will receive maximum power only if its resistance is equal to the internal resistance 'r' of the source of e.m.f. which may be a cell, battery or power supply etc. Hence, we get the value of maximum power by putting R = r in Eq.12.17.

$$P_{max} = \frac{E^2}{4r} \quad \ldots\ldots(12.18)$$

FOR YOUR INFORMATION

Table for power consumption of various household appliances.

Appliances	Power Consumption
Phone Charger	5w
LED	8w
Tube Light	22w
Computer Monitor	25w
LED TV (32")	30w
DVD Player	26w
Freezer	40w
Wall Fan	50w
Ceiling Fan	60w
Deep Freezer	70w
Laptop Computer	75w
Incandescent Bulb (100w)	100w
Refrigerator	150w
Television (25")	150w
Washing Machine	500w
Laser Printer	700w
Air Conditioner	2000w
Oven	2150w

Example 12.5

The potential difference of a cell on open circuit is 6V which falls to 4V when current of 2A is drawn from the cell. Find the internal resistance of the cell.

Solution:

Electromotive force of a cell = E = 6V

Potential difference = V = 4V

Current drawn from source = I = 2A

Internal resistance = r = ?

As we know

$$V = E - Ir$$

$$r = \frac{E-V}{I}$$

POINT TO PONDER

A same voltage of 220V is applied across the two bulbs but the resistance of one bulb is higher than the other. Explain which of the bulb glows more brightly?

$$r = \frac{6V - 4V}{2A}$$
$$r = 1\Omega$$

FOR YOUR INFORMATION

A node is a point in an electric circuit which joins two or more branches. The total current at the nod is zero.

12.7 THERMOCOUPLE

A thermocouple produces a temperature dependent voltage as a result of the thermoelectric effect, and this voltage can be used to measure temperature. The thermocouple was discovered by the German physicist Thomas Johann Seeback in 1821. It consists of two wires of different metallic conductors forming an electrical junction as shown in Fig.12.9, such that one junction is the cold and the other junction is hot. If the temperature difference between these two junctions exists, then an e.m.f. of a few millivolt can be obtained. It is measured by a device attached with the thermocouple.

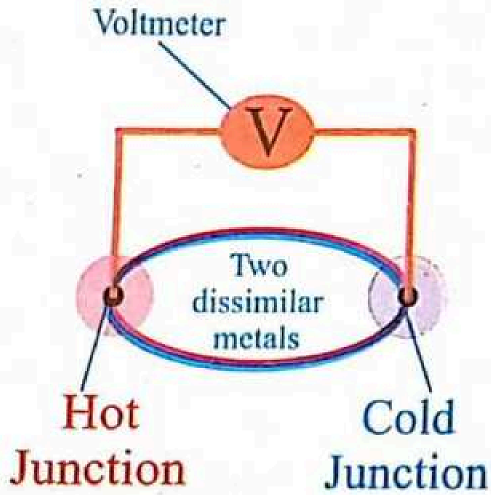


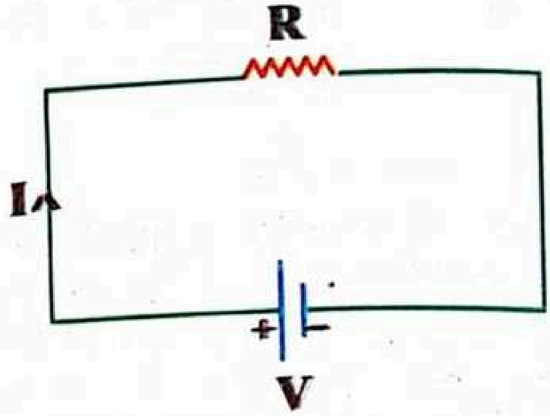
Fig.12.9 A schematic diagram of a thermocouple.

For example, let one wire is of bismuth and the other one is the antimony. If the one junction is placed in melting ice of temperature 0°C and the other one is at temperature 100°C then an e.m.f. of about 10mV is produced for a temperature difference of 100°C. If the temperature of both the junction is same, equal and opposite e.m.f. will be produced at both junctions and the resultant current flowing through the junction is zero. Thus, the total e.m.f. of a thermocouple does not only depend upon the nature of the metal of the wires but also on the temperature difference between two junctions.

Thermocouple are widely used in science and industry. Their applications include temperature measurement for furnaces, gas turbine exhaust, diesel engines, and other industrial processes. They are also used in homes, offices and markets as the temperature sensors in thermostats and as flame sensors in safety devices.

12.8 KIRCHHOFF'S LAW

An electric circuit consists of a source of e.m.f. and a number of resistors connected in either series or parallel. When potential difference 'V' is applied then there is flow of current 'I' through a resistor. The specification of the three parameters, i.e., the applied voltage, current and resistance for a given circuit is termed as circuit analysis. When the circuit is simple, which consists of a single source and a single resistor, then we can apply Ohm's law for its analysis. However, when the circuit is complex, which contains more than one source and a number of resistors then we cannot use Ohm's law directly for its analysis. For this, Kirchhoff has introduced two rules known as Kirchhoff's rules or Kirchhoff's laws. In the first rule, we can determine the flow of current in different branches of the circuit with the help of node in the given circuit. While, in the second rule, we can determine the voltage drop by using loop.



A simple circuit contains a single source of e.m.f. and a single resistor.

Node: A point in an electric circuit where two or more branches meet is called node or junction. The current at such point (node) has its minimum value.

Loop: A closed path in an electric circuit is called loop.

12.8.1 Kirchhoff's current law

The Kirchhoff's first law is also known as Kirchhoff's current law (KCL). This law is based upon law of conservation of charges and it is stated as **"the algebraic sum of current at node point in an electric circuit is zero"**, that is,

$$\sum I = 0 \;......(12.19)$$

Consider five resistors which meet at a node point 'O' as shown in Fig.12.10. The currents I_1, I_4 and I_5 are flowing towards the node and the current I_2 and I_3 are flowing away from the node. If we take the current flowing towards the node as positive and the current flowing away from the node as negative, then by applying Kirchhoff's current law at the node point 'O' we have;

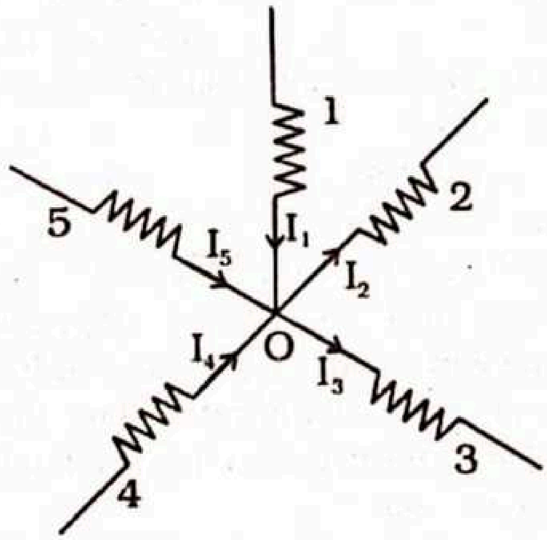


Fig.12.10 The sum of flow of current towards the node equals to the sum of flow of current away from the node.

$$I_1 + I_4 + I_5 + (-I_2) + (-I_3) = 0$$

$$I_1 + I_4 + I_5 = I_2 + I_3 \;...(12.20)$$

This is a mathematical form of KCL and it shows that the sum of current entering any node in circuit must be equal to the sum of the

DO YOU KNOW?
The node which potential is taken as zero called datum node.

currents leaving that node. In other words, the amount of charge arriving at the node is equal to the amount of charge leaving the node. This is named as conservation of charges.

Node analysis

Node analysis is based upon KCL and a number of steps are employed for the analysis of a given circuit. These steps are summarized as

1. Label all the nodes. i.e., the points in a circuit where two or more elements are met.
2. Mark the potential of the nodes such as: V_1, V_2, V_3 etc.
3. Identify the node at which potential is zero. Such node is called reference or datum node.
4. Apply KCL at each node except the datum node where incoming current is taken as positive and the outgoing as negative.
5. Determine the number of equations which are equal to the number of nodes excluding the datum node.
6. Solve all the obtained equations simultaneously and calculate the unknown required quantities.

Example 12.6

Calculate the currents I_1, I_2 and I in the given electric circuit as shown in Fig.12.11.

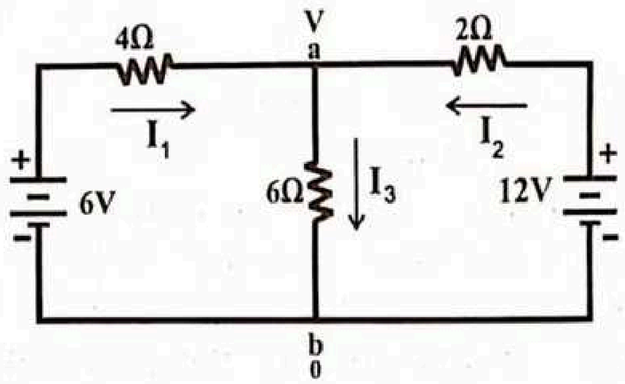


Fig.12.11

Solution:

We have

$$R_1 = 4\Omega$$
$$R_2 = 2\Omega$$
$$R_3 = 6\Omega$$
$$E_1 = 6V$$
$$E_2 = 12V$$
$$I_1 = ?$$
$$I_2 = ?$$
$$I_3 = ?$$

> ***FOR YOUR INFORMATION***
> A loop is a closed path in an electric circuit. The total voltage drop in the loop is taken as zero.

Applying KCL on node 'a' which is at potential 'V'. The potential of node 'b' is taken 0.

$$I_1 + I_2 - I_3 = 0 \qquad \text{......(12.21)}$$

But according to Ohm's law

$$I = \frac{V}{R}$$

Eq. 12.21 becomes

$$\frac{E_1 - V}{R_1} + \frac{E_2 - V}{R_2} - \frac{V - 0}{R_3} = 0$$

$$\frac{6 - V}{4} + \frac{12 - V}{2} - \frac{V}{6} = 0$$

$$18 - 3V + 72 - 6V - 2V = 0$$

$$11V = 90$$

$$V = 8.18V$$

Thus

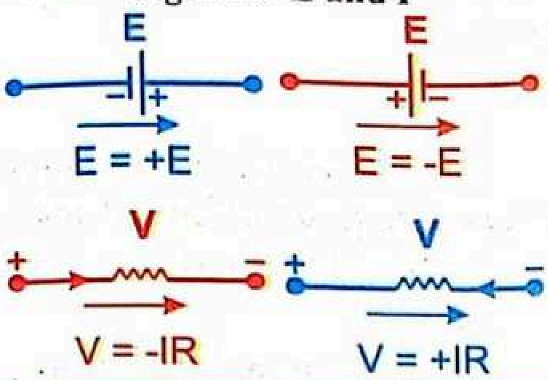
$$I_1 = \frac{E_1 - V}{R_1} = \frac{6 - 8.18}{4} = -0.4545\,A$$

$$I_2 = \frac{E_2 - V}{R_2} = \frac{12 - 8.18}{2} = 1.91A$$

$$I_3 = \frac{V}{R_3} = \frac{8.18}{6} = 1.36A$$

FOR YOUR INFORMATION

Signs for E and I



12.8.2 Kirchhoff's voltage law

The Kirchhoff's second law is also known as Kirchhoff's Voltage Law (KVL). This law is based upon the law of conservation of energy and it is stated as **"the algebraic sum of the potential difference in a closed loop is equal to zero"**, that is;

$$\sum V = 0 \;......(12.22)$$

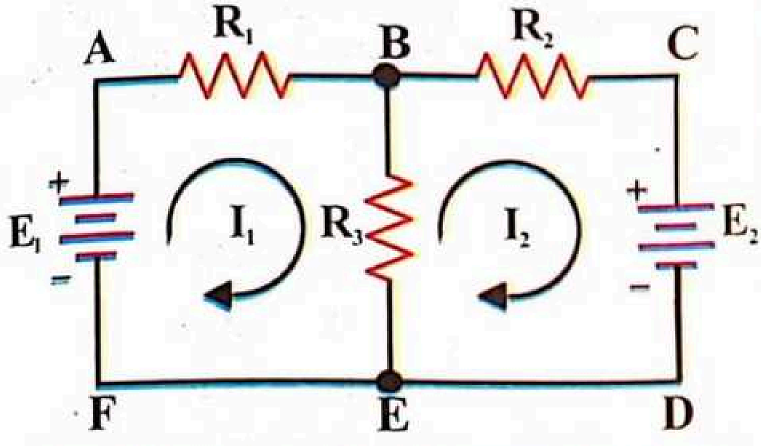


Fig.12.12 An electrical circuit consist of two loops with two sources of e.m.f. and three resistors.

In order to explain Kirchhoff's voltage law, we consider an electrical circuit which consist of two loops ABEF and BCDE with two sources of e.m.f. E_1 and E_2 as shown in Fig. 12.12. The voltage drop across resistors R_1, R_2 and R_3 are taken as V_1, V_2 and V_3 respectively. While applying KVL, it is very important to assign proper signs to voltage drop and e.m.f. in the given closed circuit. In this regard, the flow of the current in each loop is taken as clockwise. That is, when current passes through a resistor 'R' in clockwise direction than there is fall in potential because current flows a higher to a lower potential. So the term IR is taken as negative. For anticlockwise direction, there is rise in potential because current flows a lower to a higher potential. Hence the term IR is taken as positive. Similarly, if the direction of the current is from negative terminal to positive terminal of the battery then there is rise in potential and E is taken as positive. When the direction of current is from positive terminal to negative terminal of the battery then there is fall in potential and E is taken as negative. Hence under these conditions, we apply KVL on the given two loops:

Loop ABEF

$$E_1 - V_1 - V_3 = 0$$

$$E_1 - I_1R_1 - (I_1 - I_2)R_3 = 0 \qquad ......(12.23)$$

Loop BCDE

$$-E_2 - V_2 - V_3 = 0$$

$$-E_2 - (I_2 - I_1)R_2 - I_2R_3 = 0 \qquad ......(12.24)$$

Loop analysis

Loop analysis is based upon KVL and a number of steps are employed for the analysis of the given circuit. These steps are summarized as

1. Identify the number of loops in the given circuit.
2. The current must be taken as clockwise in each loop.
3. Label the current in each loop such as I_1, I_2, I_3 etc.
4. Apply KVL for each loop.
5. Determine the number of equations. Note that the number of equations is equal to the number of loops.
6. Solve the equations and calculate the unknown quantities.

Example 12.7

Determine the current in each loop of the circuit which consists of three resistors of resistance 1Ω, 2Ω and 3Ω respectively and two sources of e.m.f. as shown in Fig.12.13. The potential differences of the two sources are 5V and 10V.

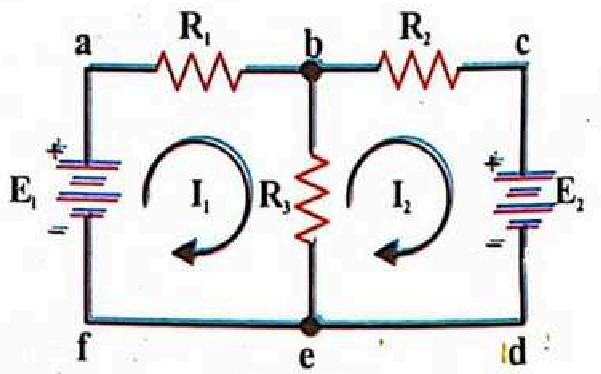


Fig.12.13

Solution:

We have

$$R_1 = 1\Omega$$
$$R_2 = 2\Omega$$
$$R_3 = 3\Omega$$
$$E_1 = 5V$$
$$E_2 = 10V$$
$$I_1 = ?$$

$$I_2 = ?$$

By applying KVL to the loop abef

$$I_1R_1 + (I_1 - I_2)R_3 - E_1 = 0$$

$$I_1(1) + (I_1 - I_2)(3) - 5 = 0$$

$$-4I_1 - 3I_2 = 5 \text{ ......(12.25)}$$

Again by applying KVL to the loop bcde

$$(I_2 - I_1)R_3 + I_2R_2 + E_2 = 0$$

$$(I_2 - I_1)(3) + I_2(2) + 10 = 0$$

$$-3I_1 + 5I_2 = -10 \text{ ......(12.26)}$$

Multiplying eq. 12.25 by 5 and eq. 12.26 by 3 then adding them

$$20I_1 - 15I_2 = 25$$

$$-9I_1 + 15I_2 = -30$$

$$11I_1 + 0(I_2) = -5$$

Or $$I_1 = -\frac{5}{11}\text{A }]$$

From eq. 12.26

$$5I_2 = -10 + 3I_1 = -10 + 3\left(\frac{-5}{11}\right) = -10 - \frac{15}{11}$$

$$5I_2 = \frac{-125}{11}$$

Or $$I_2 = -\frac{25}{11}\text{A}$$

The negative signs of I_1 and I_2 indicate that these currents are in the anticlockwise direction i.e. opposite to assumed clockwise direction.

Example 12.8

An electrical network consists of five resistors R_1, R_2, R_3, R_4 and R_5 of resistance $3\Omega, 5\Omega, 2\Omega, 6\Omega$ and 4Ω respectively are connected with a source of e.m.f. (E) about 2V as shown in Fg.12.14. Calculate the current in each resistance.

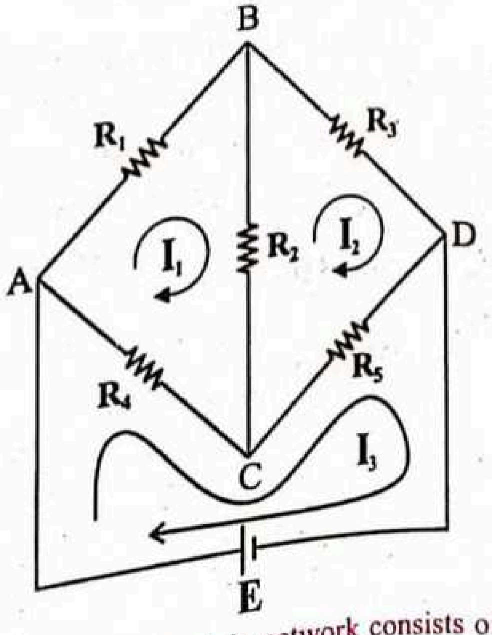


Fig.12.14 An electric network consists of five resistance and a source of e.m.f.

Solution:

We have

$$R_1 = 3\Omega$$

$$R_2 = 5\Omega$$
$$R_3 = 2\Omega$$
$$R_4 = 6\Omega$$
$$R_5 = 4\Omega$$
$$E = 2V$$
$$I_1 = ?$$
$$I_2 = ?$$
$$I_3 = ?$$

Applying KVL to the three loop of the given circuit.

Loop ABCA

$$-I_1R_1 - (I_1 - I_2)R_2 - (I_1 - I_3)R_4 = 0$$
$$-3I_1 - (I_1 - I_2)5 - (I_1 - I_3)6 = 0$$
$$-14I_1 + 5I_2 + 6I_3 = 0$$

or $$14I_1 - 5I_2 - 6I_3 = 0 \quad ......(12.27)$$

Loop BCDB

$$-I_2R_3 - (I_2 - I_1)R_2 - (I_2 - I_3)R_5 = 0$$
$$-2I_2 - 5(I_2 - I_1) - 4(I_2 - I_3) = 0$$
$$5I_1 - 11I_2 + 4I_3 = 0 \quad ......(12.28)$$

Loop ACDA

$$-(I_3 - I_1)R_4 - (I_3 - I_2)R_5 + E = 0$$
$$-6(I_3 + I_1) - 4(I_3 - I_2) + 2 = 0$$
$$6I_1 + 4I_2 - 10I_3 = -2 \quad ......(12.29)$$

Multiplying Eq.12.27 by 4 and Eq.12.28 by 6 and adding them

$$56I_1 - 20I_2 - 24I_3 = 0$$
$$30I_1 - 66I_2 + 24I_3 = 0$$
$$86I_1 = 86I_2$$
$$I_1 = I_2 \quad ......(12.30)$$

Now Eq.12.27 becomes

$$14I_1 - 5I_2 - 6I_3 = 0$$
$$9I_1 - 6I_3 = 0$$
$$3I_1 = 2I_3$$
$$I_3 = \frac{3}{2}I_1 \quad ......(12.31)$$

Putting the values of I_2 and I_3 from Eq.12.30 and Eq.12.31 in Eq.12.29 we get

$$6I_1 + 4I_1 - 10 \cdot \left(\frac{3}{2} I_1\right) = -2$$

$$-5I_1 = -2$$

$$I_1 = \frac{2}{5} = 0.4A$$

$$I_2 = \frac{2}{5} = 0.4A$$

$$I_3 = \frac{3}{2} I_1 = 0.6A$$

> ***POINT TO PONDER***
> Why is a three pin plug used in some electrical appliances?

Thus, current through $R_1 = I_1 = 0.4A$

current through $R_2 = I_1 - I_2 = 0$

current through $R_3 = I_2 = 0.4A$

current through $R_4 = I_3 - I_1 = 0.2A$

current through $R_5 = I_3 - I_2 = 0.2A$

12.9 RHEOSTAT

A *rheostat* is a variable resistance used to control the current or as a voltage divider to control voltage in an electric circuit. It is connected in series with a load resistor to adjust the current by increasing or decreasing its resistance. A rheostat is made by winding the nichrome (resistance) wire around an insulating ceramic core in a cylindrical form as shown in Fig.12.15(a). It has three terminals A, B and C such that the terminal A and terminal B are fixed at the two ends of the resistance coil while the terminal C is the adjustable terminal connected with a slider contact. The slider contact is connected with the resistance coil and it can move along the coil from

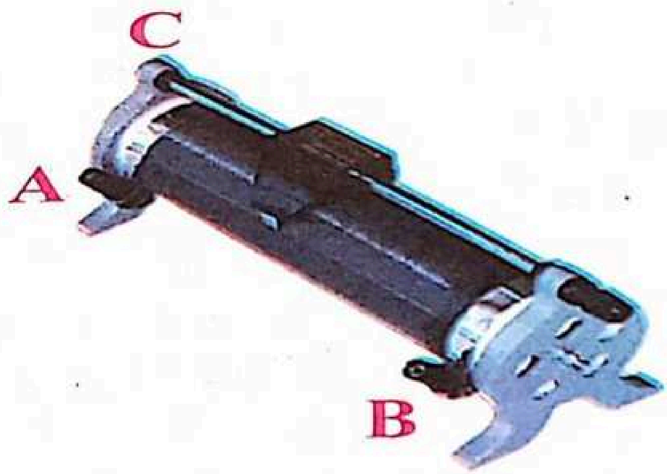


Fig.12.15(a) A Rheostat

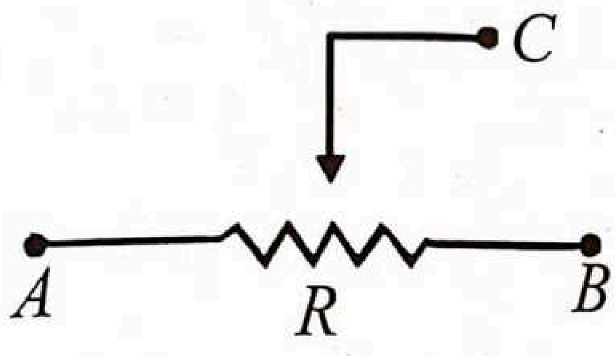


Fig.12.15(b) Circuit diagram of a Rheostat

its one end to the other end. The resistance of the rheostat is varied when the slider is moved over its resistive path. The equivalent circuit of rheostat is shown in Fig.12.15(b) In order to get a variable resistance from rheostat, the set of connections A and C or B and C are used in the circuit.

Rheostat as a variable resistance

The resistance of rheostat depends upon its resistive path. For example, let we use the terminals A and C. Now if the slider is moved towards the terminal 'A' and when they are close to each other than the rheostat offers minimum resistance and allows a large amount of current. Similarly, if the slider is moved towards the terminal 'B' and they are close to each other, then the rheostat offers a maximum resistance and allows a small amount of current. This is the function of a rheostat as a variable resistance.

Rheostat as a potential divider

A rheostat can also be used as a potential divider. For instance, supply voltage E is applied across the fixed terminals 'A' and 'B' as shown in Fig.12.16. The applied potential E is divided by the variable terminal 'C' connected with the slider in the ratio of the resistance between AC and BC. Let 'R' be the total resistance of the rheostat between the fixed terminal A and B then the flow of current between them can be calculated by using Ohm's law

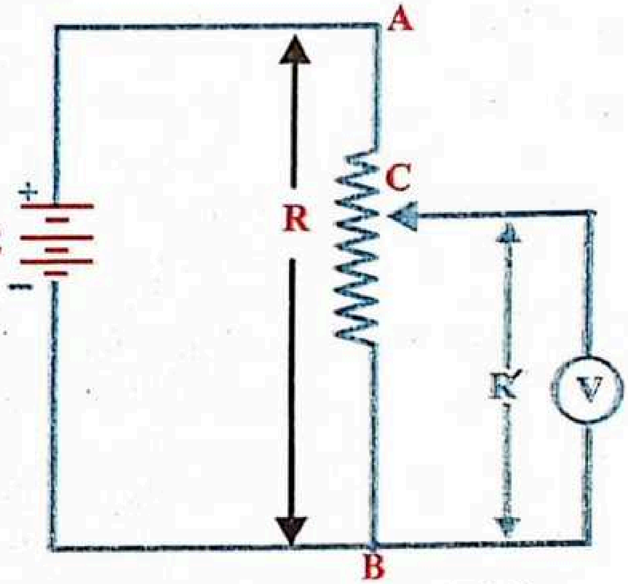


Fig.12.16 A rheostat as a potential divider.

$$E = IR$$

$$I = \frac{E}{R} \quad \ldots(12.32)$$

Similarly, let R' be the resistance between the fixed terminal 'B' and the variable sliding terminal 'C' then the potential difference V across BC is given by

$$V = IR' \quad \ldots\ldots(12.33)$$

Putting the value of I from eq. 12.32 in eq. 12.33

$$V = \frac{E}{R}R'$$

$$V = \left(\frac{R'}{R}\right)E \quad \ldots\ldots(12.34)$$

This is the basic principle of rheostat as a potential divider, that is the potential difference 'V' varies by varying the resistance R'.

12.10 WHEATSTONE BRIDGE

A Wheatstone bridge is an electrical network which is used to measure the unknown resistance. It was introduced by Charles Wheatstone. It consists of four resistances 'P', 'Q', 'R' and 'X', such that the value of resistances P and Q are known. The value of the resistance R is also known but it is variable, and the value of the resistance X is unknown, which is to be determined. All the resistances are connected in the form of a closed loop ABCDA as shown in Fig.12.17. The source of e.m.f. is connected across the junctions A and C and the galvanometer is connected across the junctions B and D. This electrical network is called a bridge, because, the galvanometer bridges the two circuit branches by a third branch B and D. When the key (K) is closed then there is flow of current through galvanometer. Now by varying the value of variable resistance 'R' until galvanometer shows no deflection that is, there is no flow of current through the galvanometer then the bridge is said to be balanced and this is the working principle of the Wheatstone bridge.

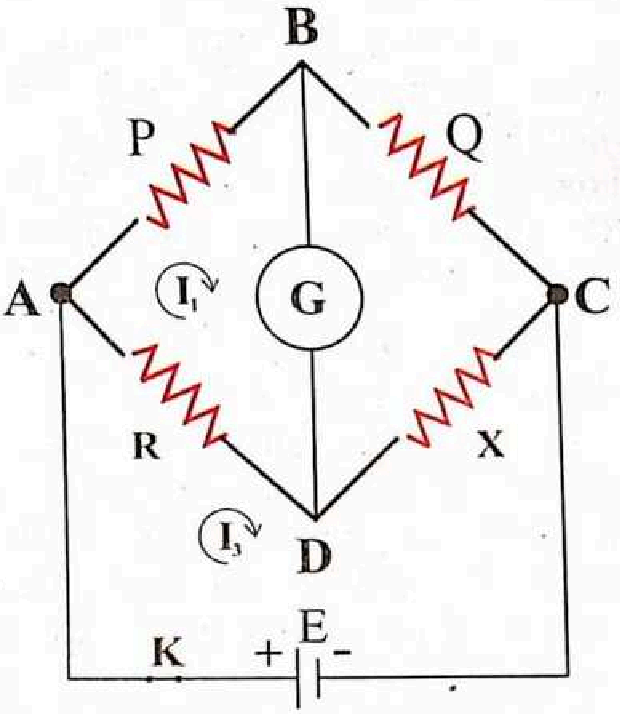


Fig.12.17 A Wheatstone bridge circuit consists of four resistors with a source of e.m.f.

Fig.12.16 shows that there are three loops ABDA, BCDB and ADCA having current I_1, I_2 & I_3 respectively. If we take all the current clockwise then by applying KVL to we get first two loops.

For loop ABDA

$$I_1P + (I_1 - I_2)G + (I_1 - I_3)R = 0 \;......(12.35)$$

For loop BCDB

$$I_2Q + (I_2 - I_3)X + (I_2 - I_1)G = 0 \;......(12.36)$$

As the flow of current through galvanometer is zero, so

$$I_1 - I_2 = 0 \quad \text{i.e.,} \quad I_1 = I_2$$

Thus Eq.12.35 and Eq.12.36 become

$$I_1P = -(I_1 - I_3)R \quad ......(12.37)$$

$$I_2Q = -(I_2 - I_3)X$$

But $\quad I_1 = I_2$

$$I_1Q = -(I_1 - I_3)X \quad ......(12.38)$$

Dividing Eq.12.38 by Eq.12.37

$$\frac{Q}{P} = \frac{X}{R}$$

$$X = R\frac{Q}{P} \quad ......(12.39)$$

Thus, to determine the value of the unknown resistance we can use Eq.12.39. It is true only, under the balanced condition of the bridge i.e., when the flow of current through the galvanometer is zero.

12.11 POTENTIOMETER

A potentiometer is a device used to measure the unknown potential difference or to compare the e.m.f. of sources. Although, we have other devices for the measurement of potential difference between two points, such as, voltmeter, cathode ray oscilloscope etc., but the potentiometer has more advantages over the others. For example, a potentiometer is a simple, inexpensive, has high degree of accuracy and does not draw any current.

A potentiometer consists of a long conducting wire with uniform area of cross-section. Usually its length is 4m to 5m which is stretched on a wooden board as shown in Fig.12.17. A potentiometer contains three terminals A, B and C. The terminal 'A' and the terminal 'B' are at the two opposite ends of the potentiometer. The terminal 'C' through galvanometer is connected with the Jockey and it can slide over the wire AB. A cell which is known as the divider cell is used across the ends of the wire. With the help of rheostat, a constant potential difference can

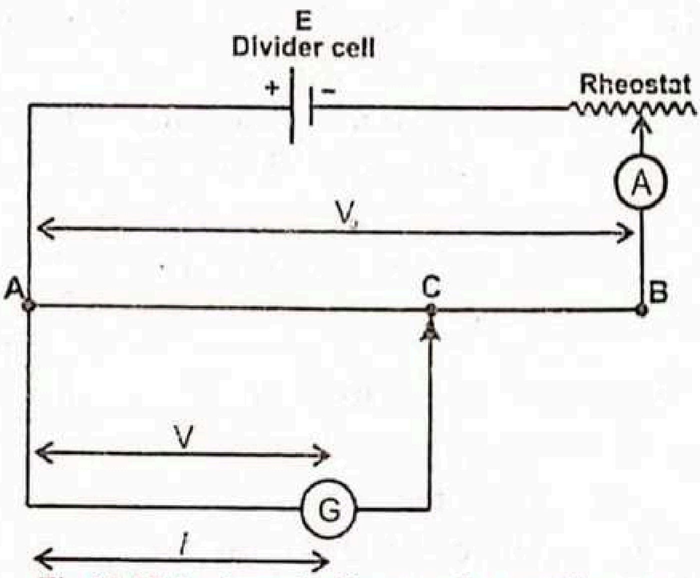


Fig.12.17 A schematic diagram of a potentiometer.

be maintained across the ends of the wire. An ammeter is also used in order to observe the current in the given circuit.

As the wire is of uniform cross-section therefore the potential difference 'V_0' across the two fixed terminals 'A' and 'B' is uniformly distributed over the entire length AB of the wire. However, the potential difference 'V' between the terminal 'A' and the sliding contact point 'C' is proportional to the length of wire between them. If ℓ be the length between terminals 'A' and 'C'. Then

$$V \propto \ell$$

$$V = k\ell$$

$$k = \frac{V}{\ell} \quad \text{......(12.40)}$$

Where 'k' is constant known as potential gradient, i.e., fall of potential per unit length.

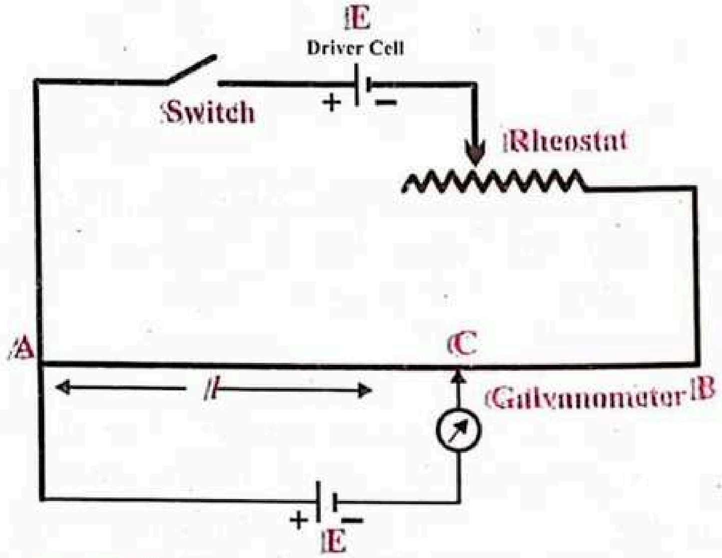
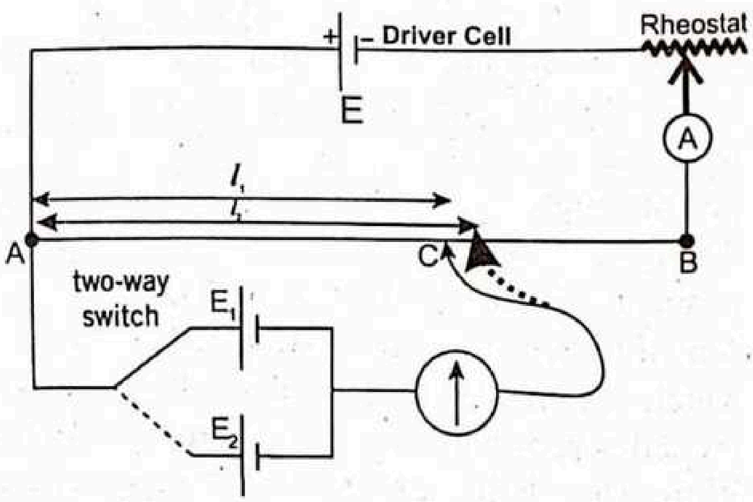


Fig.12.18 Measurement of e.m.f. of a cell by using potentiometer.

Determination of potential difference by potentiometer

A potentiometer can be used to measure the potential difference between two points in an electric circuit. Fig.12.18 shows how the potentiometer can be used to determine the e.m.f. (E_1) of a cell. The positive terminal of the given cell is connected with the positive terminal of the driver cell and the negative terminal of E is connected with the jockey through the galvanometer. Now the sliding contact jockey is tapped along the slide wire AB until the galvanometer shows no deflection, i.e. the current through the galvanometer is zero. Then the e.m.f. of the given cell (E) is equal to the potential difference across the points A and C of length ℓ.



Comparison of e.m.f. of two cells by potentiometer

A potentiometer can also be used to compare the e.m.f. of two cells. The arrangement for comparing the e.m.f. of two cells as shown in

Fig.12.19 which consists of two cells E_1, and E_2. The e.m.f. of cell E_1 is known and it is called standard cell, while, the e.m.f. of cell E_2 is unknown. It is called a test cell and its e.m.f. is to be compared with the standard cell. The positive terminals of the cells are connected with the positive terminal of the driver cell. The negative terminals of the cells are connected to the two terminals of two way key. The third common terminals of the two way key is connected to jockey through the galvanometer. An ammeter is also used for the measurement of current flowing in the circuit.

Let ℓ_1 be the balanced length from the terminal A to the terminal C, for standard cell (E_1), then

$$\text{e.m.f. of cell } (E_1) = \text{P.D across the length } \ell_1.$$

$$E_1 = k\ell_1 \quad ......(12.41)$$

Similarly, let 'ℓ_2' be the balanced length between A and 'C' for test cell (E_2), then

$$E_2 = k\ell_2 \quad ......(12.42)$$

Dividing Eq.12.41 by Eq.12.42

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2} \quad ......(12.43)$$

This result shows that the ratio of the e.m.f.s is equal to the ratio of balancing lengths and this is the working principles of comparing of e.m.f. of two cells.

SUMMARY

- **Electric Current:** The rate of flow of charges through a conductor is called electric current.
- **Ampere:** If one coulomb charge is moving through a point in the given circuit in one second then its corresponding current is one ampere.
- **Steady Current:** A continuous flow of current whose magnitude remains constant is known as steady current.
- **Ohm's Law:** This law states that at constant temperature, the applied voltage across the conductor is directly proportional to the current. Mathematically,

  $$V = IR$$
- **Resistance:** Resistance is the measurement of opposition to the flow of electric current, its SI unit is ohm.
- **Conductance:** The reciprocal of the resistance is called conductance.
- **Resistivity:** The resistance in one metre cube of a conductor is called its resistivity or specific resistance.
- **Conductivity**: The reciprocal of resistivity is called conductivity.

- **Thermistor:** Thermistors is a temperature dependent resistor, its resistance varies even with small change of temperature. Most thermistor have negative temperature coefficient of resistance.
- **Electromotive Force:** The work done by the source per unit charge to move the charge in a circuit from the low potential to the high potential is called electromotive force.
- **Thermocouple:** A thermocouple is an electric device to produce an e.m.f. by heat and it is being used to measure the temperature.
- **Kirchhoff's First Law:** The algebraic sum of current at a junction in an electric circuit is equal to zero.
- **Kirchhoff's Second Law:** The algebraic sum of potential difference in a closed loop or closed electric circuit is equal to zero.
- **Rheostat:** A rheostat is a variable resistance used to control the electric current.
- **Wheatstone Bridge:** A Wheatstone bridge is an electric circuit used to measure the unknown resistance.
- **Potentiometer:** A potentiometer is a device used to measure potential difference and e.m.f. of a source.

EXERCISE

Choose the best option.

1. A flow of 10^7 electrons per second through a conductor produces a current of
(a) 1.6×10^{-12}A (b) 1.6×10^{12}A (c) 1.6×10^{-26}A (d) 1.6×10^{26}A
2. The first source of e.m.f. was discovered by
(a) Ampere (b) Volta (c) Ohm (d) Coulomb
3. Which one of the given cells is not rechargeable?
(a) Laclanche (b) Nickel (c) Cadmium (d) Lead acid
4. Which one of the given sources converts light energy into electrical energy?
(a) Dynamo (b) Generator (c) Photocell (d) Thermocouple
5. Which one of the following material is non-ohmic:
(a) Gold (b) Germanium (c) Copper (d) Silver
6. What will be resistance of the wire when its length is doubled
(a) Remain Same (b) Half
(c) Double (d) Triple
7. If the resistivity of the conductor is $2\times10^{-6}\Omega$m then its conductivity is
(a) $2\times10^{6}\Omega^{-1}m^{-1}$ (b) $5\times10^{6}\Omega^{-1}m^{-1}$
(c) $5\times10^{-5}\Omega^{-1}m^{-1}$ (d) $5\times10^{5}\Omega^{-1}m^{-1}$
8. Siemen is a unit of conductance and it is equal to

(a) ohm (b) mho (c) mho . m (d) mho $.m^{-1}$

9. When resistance of the material increases by increasing the temperature then the value of co-efficient of resistivity will be
(a) Constant (b) Zero (c) Positive (d) Negative

10. The unit of temperature co-efficient is
(a) ^{o}C (b) $^{o}C^{-1}$ (c) ^{o}C . m (d) $^{o}C^{-1}$. m

11. In the presence of internal resistance of the source, which one of the following relations between potential difference (V) and e.m.f. (E) is correct
(a) $E = 0$ (b) $E = V$ (c) $E > V$ (d) $E < V$

12. The unit of electromotive force is
(a) Newton (b) Joule (c) Watt (d) Volt

13. The source of e.m.f. transfers its maximum power to the external circuit when (r = internal resistance, R = load resistance)
(a) $r = 0$ (b) $r = R$ (c) $r < R$ (d) $r > R$

14. Kirchhoff's 1st law is based upon law of conservation of
(a) Current (b) Charge (c) Voltage (d) Energy

15. The node in an electric circuit which has zero potential is known as
(a) Node (b) Antinode (c) Negative node (d) Datum node

16. When the direction of current is from negative to positive terminal of the battery then 'E' is taken as
(a) Zero (b) Normal (c) Positive (d) Negative

17. If current passes through a resistor in anticlockwise direction. Then the electric potential will be
(a) Zero (b) Remain same (c) Raised (d) Fall down

18. An electrical device which controls the current is known as
(a) Thermocouple (b) Rheostat
(c) Thermistor (d) Wheatstone bridge

19. The electrical device which is being used to compare the e.m.f. of two cells is known as
(a) Rheostat (b) Wheatstone Bridge
(c) Potentiometer (d) Galvanometer

SHORT QUESTIONS

1. How does the conventional current differ from the electronic current?
2. How can you point out the direction of electric current?
3. What do you know about the drift of the free electrons in a conductor?
4. Which kind of current you observe in electrolyte plasma?
5. What are the functions of copper and zinc plates in the source of e.m.f.?
6. What do you know about the ohmic characteristics of a ohmic device?

7. Which type of graph will be obtained between V and I for non-ohmic conductor?
8. How does resistivity of a conductor depend upon temperature?
9. Distinguish between positive and negative temperature coefficients of resistance.
10. Under what condition, the potential difference and electromotive force give the same value.
11. When the source delivers its maximum power?
12. What is the working principle of a thermocouple?
13. What do you know about the circuit analysis?
14. What do you know about node and loop in an electric circuit?
15. What are the sign of currents when it flows toward and away from the node?
16. What are the basic principles of Kirchhoff's two laws?
17. Under what condition you use Kirchhoff's Laws for circuit analysis?
18. Which electrical device is being used as a potential divider?
19. What is the difference between Rheostat and Potentiometer?
20. What is the working principle of Wheatstone Bridge?

COMPREHENSIVE QUESTIONS

1. Define and explain steady current, direction of current and sources of current.
2. State and explain Ohm's law. Also discuss its applications for ohmic and non-ohmic devices.
3. What do you know about resistivity and conductivity of a conductor? Describe how the resistivity of a conductor depends upon the variation of its temperature.
4. Describe thermistor and its application in daily life.
5. What do you know about electromotive force and potential difference? Under what conditions, their values are same in an electric circuit.
6. What is electrical power? How does the maximum power transfer take place in an electric circuit?
7. What do you know about a thermocouple and its function?
8. State and explain Kirchhoff's two rules for circuit analysis.
9. State and explain node analysis and loop analysis by using Kirchhoff's two rules.
10. Describe a rheostat and its working principle. Explain how a rheostat can be used as a variable resistance as well as a potential divider.
11. What do you know about Wheatstone bridge? Draw its circuit diagram and explain its working principle.
12. State and explain potentiometer, its function and applications.

NUMERICAL PROBLEMS

1. How many electrons flow through a light bulb each second, if the current through the bulb is 0.8A. **(5×10^{18})**
2. A flow of charges of 150C through a wire in 2 hours what is the amount of current in the wire. **(21mA)**
3. A metal rod is 4m long and 6mm in diametre. Compute its resistance if the resistivity of the metal is $1.76 \times 10^{-8} \Omega m$. **($2.5 \times 10^{-3} \Omega$)**
4. Calculate the potential difference across the two ends of the wire of resistance 5Ω. If the charges of 720C pass through it per minute. **(60V)**
5. A 10m long wire of diametre 0.2cm has a resistance 3Ω. Find the resistivity of the material of the wire also calculate its conductivity. **($9.4 \times 10^{-7} \Omega m$, $1.06 \times 10^{6} \Omega^{-1} m^{-1}$)**
6. The resistance of a coil is 140Ω at 20°C. If current is passed through it, its temperature rises, and its resistance becomes 160Ω at 40^0C. Calculate the temperature co-efficient of resistance of the coil. **(0.0071^0C^{-1})**
7. A copper wire has resistance of 3Ω at 0^0C. What is its resistance at 60^0C. The temperature coefficient of resistance of copper is 0.0039^0C^{-1}. **(3.77Ω)**
8. A battery has an e.m.f. of 12.5v and an internal resistance of $2.4 \times 10^{-3}\Omega$. If the load current is 20A. Find the terminal voltage. **(12.45V)**
9. Determine the current through each resistor in the electric circuit as shown in Fig.12.20, where R_1 is 10Ω, R_2 is 6Ω, R_3 is 10Ω, E_1 is 6V and E_2 is 3V. **($I_1 = 0.3A, I_2 = 0, I_3 = 0.3A$)**
10. Find the current through each resistor in the electrical network as shown in Fig.12.21. **($I_1 = 0.21A, I_2 = 0.14A, I_3 = 0.36A$)**
11. If the e.m.f. of a battery is balanced by a length of 80cm on a potentiometer wires while, the e.m.f. of 1.03v of a standard cell is balanced by a length of 55cm. Then determine the e.m.f. of the battery? **(1.5V)**

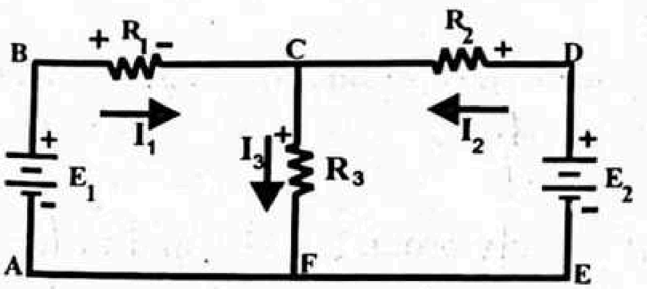


Fig.12.20

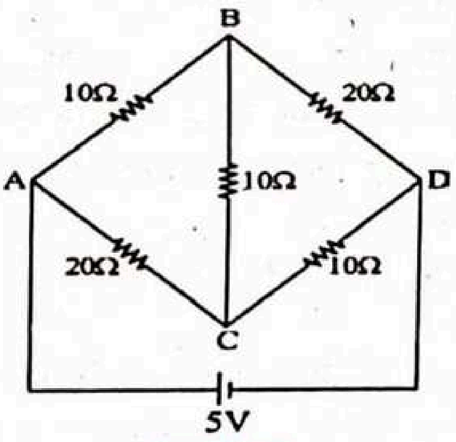


Fig.12.21

Unit 13

ELECTROMAGNETISM

Major Concepts (18 PERIODS)

- Magnetic field of current – carrying conductor
- Magnetic force on a current-carrying conductor
- Magnetic flux density
- Ampere's law and its application in solenoid
- Force on a moving charged particle in a magnetic field
- e/m of an electron
- Torque on a current carrying coil in a magnetic field
- Electro-mechanical instruments

Conceptual Linkage

This chapter is built on Electromagnetism Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

- explain that magnetic field is an example of field of force produced either by current-carrying conductors or by permanent magnets.
- describe magnetic effect of current.
- describe and sketch field lines pattern due to a long straight wire.
- explain that a force might act on a current-carrying conductor placed in a magnetic field.
- investigate the factors affecting the force on a current carrying conductor in a magnetic field.
- solve problems involving the use of $F = BIL \sin\theta$.
- define magnetic flux density and its units.
- describe the concept of magnetic flux (ϕ_B) as scalar product of magnetic field (B) and area (A) using the relation $\phi_B = B_\perp A = B \cdot A$.
- state Ampere's law.
- apply Ampere's law to find magnetic flux density around a wire and inside a solenoid.
- describe quantitatively the path followed by a charged particle shot into a magnetic field in a direction perpendicular to the field.
- explain that a force may act on a charged particle in a uniform magnetic field.

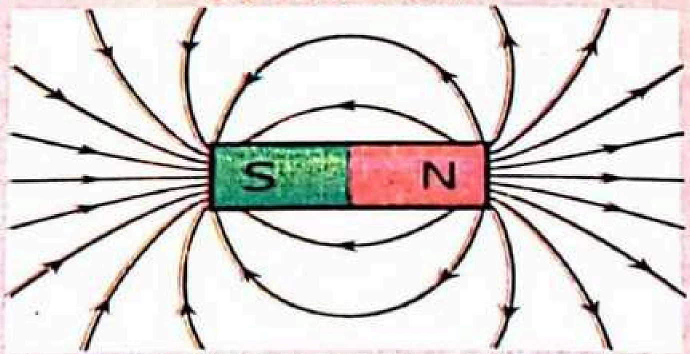
- describe a method to measure the e/m of an electron by applying magnetic field and electric field on a beam of electrons.
- predict the turning effect on a current carrying coil in a magnetic field and use this principle to understand the construction and working of a galvanometer.
- explain how a given galvanometer can be converted into a voltmeter or ammeter of a specified range.
- describe the use of avometer / multimeter (analogue and digital)

INTRODUCTION

A naturally occurring Lodestone was first mined at magnesia, Anatolia in Turkey. It was named as magnetite, Fe_3O_4. It has a property to attract iron pieces. If a bar-shaped of this permanent magnet is suspended from its midpoint by a piece of string so that it can swing freely, it will rotate until its one end points to the earth's geographic north pole. This end is called north (N) pole of bar magnet. The other end points to the earth's geographic south Pole called south (S) pole. The same idea is being used to construct a simple compass. Like electric charges, the like or similar poles repel, and the unlike or opposite poles attract each other with a force called magnetic force.

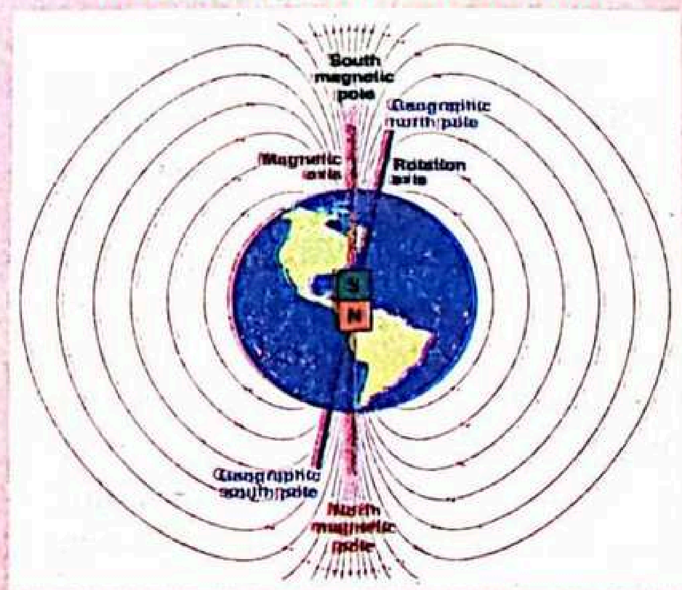
In 1820, Oersted discovered the relationship between magnetism and electricity. He observed that a compass needle was deflected by a current carrying wire. A few years later, Michelson Faraday discovered that an electric current can be produced in a circuit by moving a magnet near the circuit. These

DO YOU KNOW?



Like electric dipole, a magnetic dipole consist of a south pole and a north pole.

INTERESTED INFORMATION



The Earth's magnetic field is approximately that of a dipole, like that of the fictitious bar magnetic. Where the south magnetic pole is towards the geographic north pole and the north magnetic pole is towards the geographic south pole.

observations show that an electric field creates a magnetic field. Such interaction, or production of magnetism due to electricity implies to electromagnetism. Magnetism and electromagnetism are being used in several fields of daily life, such as in electric motors, loud speaker, TV picture tubes, microwaves oven, tapes, disk drives, computer printer, MRI (magnetic resonance imaging) etc.

In this unit we will determine the magnetic field, magnetic flux and magnetic force that acts on a moving charge as well as on a current carrying wire which is placed in the applied magnetic field. Similarly, we also explain the construction, principle and working of some electromagnetic instruments, such as: galvanometer, voltmeter, ammeter and Avometer.

INTERESTING INFORMATION

When you insert your credit, debit or ATM card into the Automated Teller Machine (ATM). A magnetic strip on a card contains millions of tiny magnetic domains held together by a resin binder. The machine reads the information encoded on the magnetic strip and it makes your access to your account.

POINT TO PONDER

Does every magnetic material have a north and south pole?

13.1 MAGNETIC FIELD

An iron ore called lodestone is a naturally occurring magnetic rock found near the ancient city of 'Magnesia' (in western Turkey). This is a reason that why this lodestone is named as a 'Magnet'. If a suspended bar-shaped magnet is free to rotate its one end points north, called north pole, and its other end points towards south and is called south pole. The like poles repel and unlike poles attract each other with a force, as shown in Fig.13.1. Such force of attraction or repulsion between unlike or like poles is known as magnetic force. Since each pole produces a magnetic field around it, so the field of one pole exerts a force on the nearby other pole and vice versa. Like electric field, magnetic field is represented by

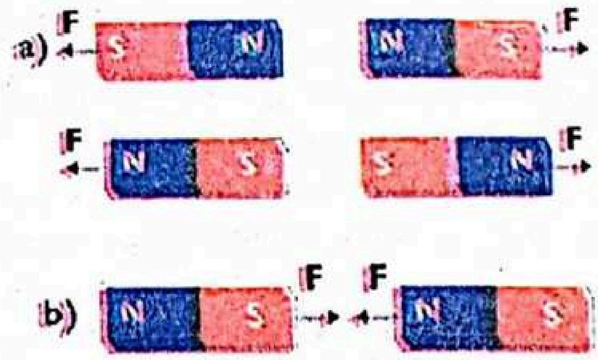


Fig.13.1(a) Like poles repel each other (b) Unlike poles attract to each other.

DO YOU KNOW

Like electric field lines, magnetic field lines also never cross each other but instead push apart of each other.

magnetic lines of force $\vec{B}$ which provides both magnitude and direction of the field. The magnetic field vector $\vec{B}$ at any point is tangent to the field line as shown in Fig.13.2.

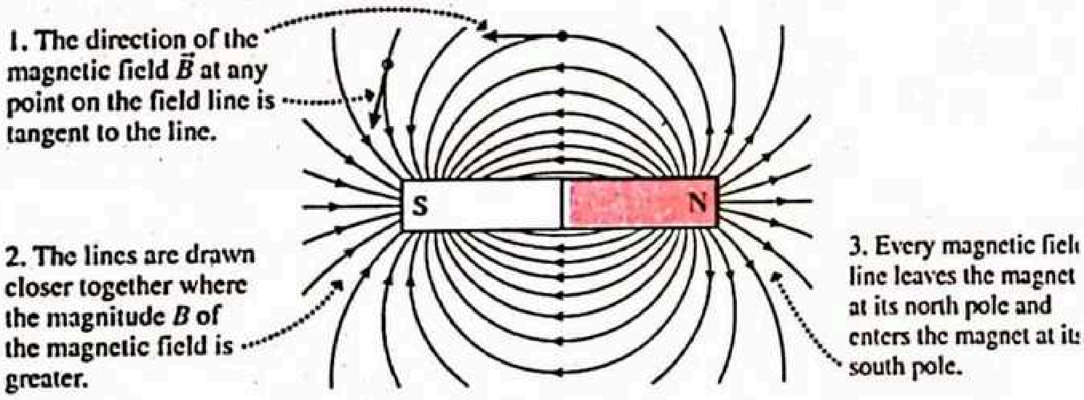


Fig.13.2 Magnetic field vector 'B' is tangent to the field line.

H.C. Orested was the first person who discovered the magnetic effect of electric current in 1820. According to him, a current carrying-conductor can also produces magnetic effect. He verified his notion by performing a simple experiment. The experimental setup consists of a wire AB which is connected across the source of e.m.f. and a compass needle placed parallel to wire as shown Fig.13.3(a). If the switch S is opened and there is no current in the wire, the needle remains parallel to the wire i.e., the needle does not deflect. However, when the switch is closed and the current starts to flow in the wire, the needle deflects as shown in Fig.13.3(b). Thus, this deflection shows that a current carrying wire produces a magnetic field around it. The strength of the magnetic field depends upon the magnitude of current.

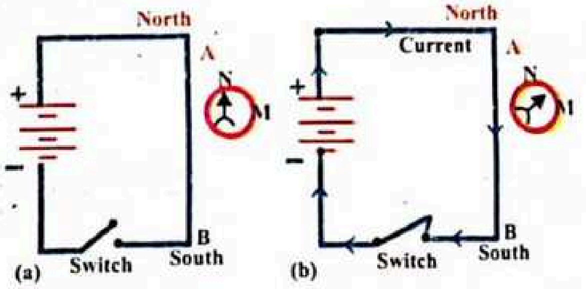


Fig.13.3(a) No deflection, when the wire carries no current (b) There is deflection, when the wire carries a current.

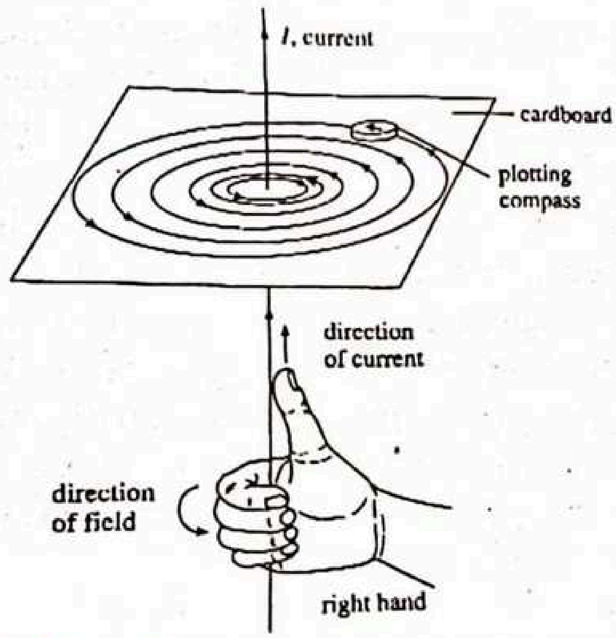


Fig.13.4(a) Magnetic lines of force in form of concentric circles due to a current carrying a wire, which is detected by plotting compass.

Magnetic field due to current carrying a long straight wire

Consider a current carrying long straight wire, according to Orested a magnetic field is produced around it. The pattern of such magnetic field is in form of closed concentric circles in a plane which is perpendicular to the wire. It can be detected by using a small plotting compass placed near the wire as shown in Fig.13.4(a). The experiments show that the strength of such field depends upon current, medium and distance from the wire. Similarly, the direction of magnetic lines of force can be determined by "right hand rule".

Right hand rule

The direction of magnetic lines of force due to a current carrying long wire can be determined by "right hand rule". According to this rule, the wire is grasped in the right hand such that if the thumb is pointing in the direction of current, then the curved fingers of the hand will give the direction of magnetic field as shown in Fig. 13.4(b).

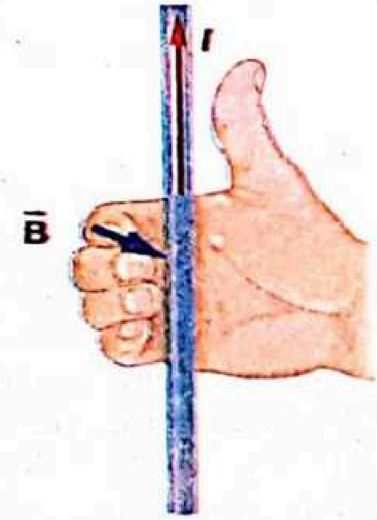


Fig.13.4(b) Right hand rule shows the direction of magnetic lines of force.

13.2 FORCE ON A CURRENT CARRYING CONDUCTOR IN UNIFORM MAGNETIC FIELD

We have studied that a current carrying conductor produces a magnetic field around it. Now if such conductor is placed in an applied magnetic field, as shown in Fig.13.5, then these two magnetic fields i.e., the magnetic field due to a current carrying conductor and the applied magnetic field interact with each other, as a result, a force is exerted on the conductor. The observations show that the magnitude of such magnetic force depends upon the following four factors.

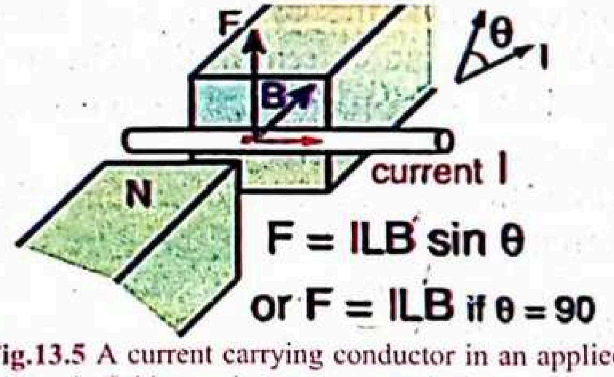


Fig.13.5 A current carrying conductor in an applied magnetic field experiences a magnetic force.

i. Current: The magnitude of force exerts on the conductor is directly proportional to the magnitude of current flowing through the conductor.

$$F_m \propto I$$

ii. Length of the conductor: The magnitude of magnetic force is also directly proportional to the length of the conductor within the applied external magnetic field

$$F_m \propto \ell$$

iii. **Strength of the field:** The magnitude of the magnetic force is directly proportional to the magnitude of the applied magnetic field B.

$$F_m \propto B$$

iv. **Directions:** If the length of the conductor is perpendicular to the direction of the applied field then a maximum force acts on the conductor. However, if the length is parallel to the direction of field, the conductor experiences no force. It means, the magnitude of force is also depends upon the factor of $\sin\theta$. That is,

$$F_m \propto \sin\theta$$

Combining all the above results we may write;

$$F_m \propto I\ell B\sin\theta$$
$$F_m = kI\ell B\sin\theta$$

where 'k' is constant of proportionality. It is dimensionless and if its value is 1 in SI units then

$$F_m = I\ell B\sin\theta \text{......(13.1)}$$

In vectors form

$$\vec{F}_m = I\left(\vec{\ell}\times\vec{B}\right) \text{......(13.2)}$$

If the length of the conductor is perpendicular to direction of magnetic field and angle 'θ' between them is 90^0 then magnetic force on the conductor is maximum. It is given by

$$F_m = I\ell B \qquad \therefore \sin 90^\circ = 1$$

$$B = \frac{F_m}{I\ell} \text{......(13.3)}$$

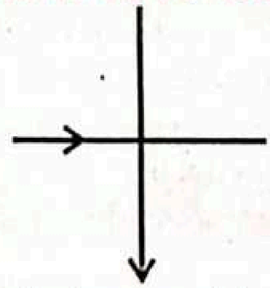
Magnetic field is a vector quantity and its SI unit is Tesla (T) i.e., **the strength of the magnetic field is said to be one tesla if it exerts a force of one newton on a conductor of length one metre through which current of one ampere is passing through it.** Mathematically, it is expressed as;

$$1T = \frac{1N}{1A - 1m}$$

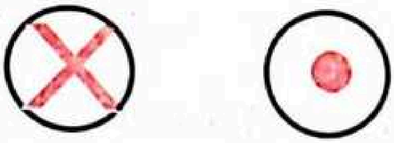
POINT TO PONDER

Why does a picture become distorted when a magnetic bar is brought near to the screen of TV, Computer Monitor or Oscilloscope?

POINT TO PONDER



Two conductors are at right angle in form of a plane carry equal currents. At what point in the plane that their magnetic field is zero?



Current into the page   Current coming out ofthe page

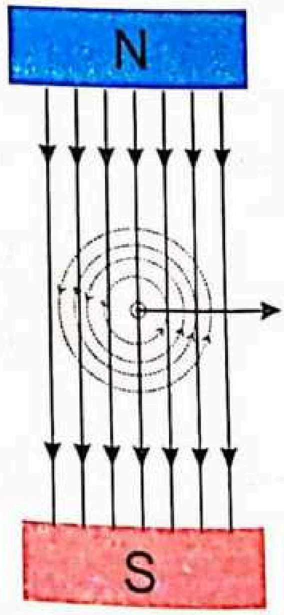


Fig.13.6(a) A magnetic force due to a current carrying a' conductor in uniform magnetic field.

The magnetic field $\vec{B}$ can also be measured in terms of Gauss (G) where,

$$1 \text{ Tesla(T)} = 10^4 \text{ Gauss(G)}$$

In order to determine the direction of force, we have a current carrying conductor perpendicularly in uniform magnetic field, if the current flow is out of the page then it is represented by symbol dot (•), as shown in Fig.13.6(a). Similarly, if the flow of current is into the page then it is represented by symbol cross (X).

Due to the interaction these two fields reinforce each other on the left side of the conductor and give a strong magnetic field while cancel each other on the right side of the conductor and give a weak magnetic field. Thus, the direction of the force on the conductor will be directed from stronger to weaker side at right angle to both the length of conductor and magnetic field.

The direction of force on the conductor can further be explained by Fleming's left hand rule. **According to this rule, stretched the thumb, forefinger and middle finger of the left hand perpendicularly in such a way that the forefinger is along the magnetic field, middle finger is along the current flow then thumb indicates the direction of force as shown in Fig.13.6(b).**

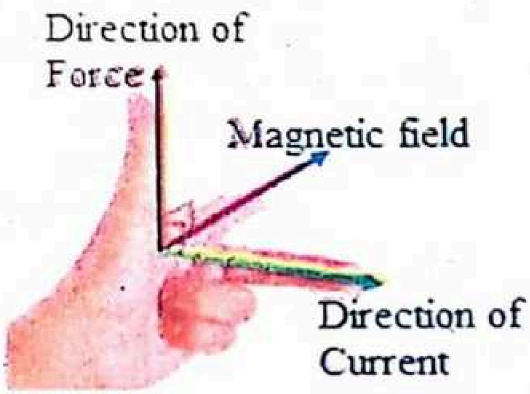


Fig.13.6(b) Flaming left hand rule explaining the directions of force, magnetic field and current.

Example 13.1

A straight conductor of length 20cm carrying a current of 10A in a uniform magnetic field of strength 0.4T. What is the force on the wire when it is (a) at right angles to the field and (b) at 45^0 to the field.

Solution:

Length of the conductor = ℓ = 20cm = 0.2m

Current = I = 10A

Strength of the field = B = 0.4T

(a) Force = F_1 = ?

$\theta_1 = 90^o$

(b) Force = F_2 = ?

$\theta_2 = 45^o$

By definition of force on a current carrying a conductor,

(a)

$$F_1 = I\ell B \sin\theta_1$$

$$F_1 = (10A)(0.2m)(0.4T)\sin 90^o$$

$$F_1 = 0.8N$$

(b) $F_2 = I\ell B \sin\theta_2$

$F_2 = (10A)(0.2m)(0.4T) \sin 45^\circ$

$F_2 = (0.8)(0.707)$

$F_2 = 0.57N$

13.3 MAGNETIC FLUX AND MAGNETIC FLUX DENSITY

A magnetic field can be represented by imaginary lines of force called magnetic field lines. Like electric flux, the magnetic flux is also defined as, **the number of magnetic field lines passing through a certain area held perpendicular to the direction of field** as shown in Fig.13.7. Magnetic flux is represented by ϕ_B. Quantitatively, it is equal to the scalar product of magnetic field strength '$\vec{B}$' and vector area $\vec{A}$. i.e.,

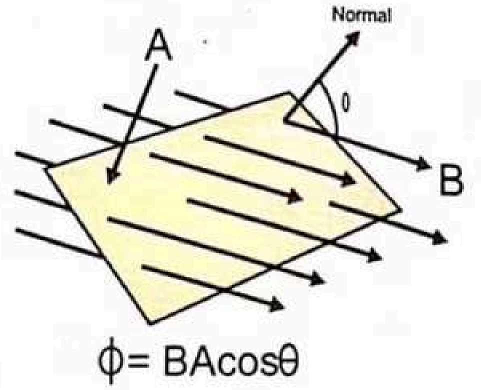


Fig.13.7 The magnetic field lines passing through area.

$$\phi_B = \vec{B} \cdot \vec{A}$$

$$\phi_B = BA\cos\theta \quad ......(13.4)$$

Magnetic flux is a scalar quantity and it can be studied under the following two cases:

Case I: If area is held perpendicular to the direction of field B then the direction of vector area $\vec{A}$ is parallel to the direction of $\vec{B}$ and angle 'θ' between them is zero as shown in Fig.13.8. Thus Eq.13.4 becomes

$$\phi_B = BA\cos 0^\circ$$

$$\phi_B = BA \quad ......(13.5)$$

This is the maximum flux.

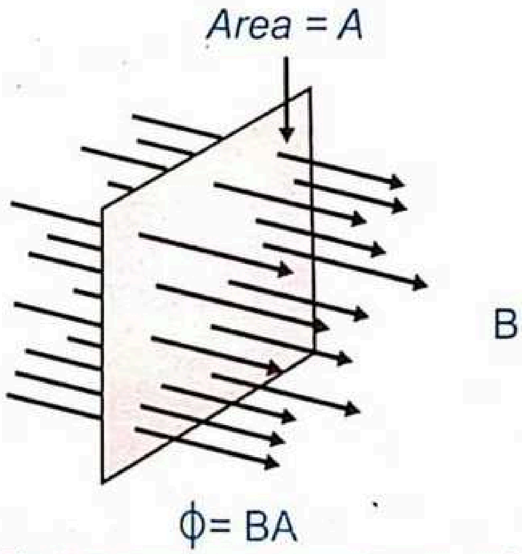


Fig.13.8 Area held perpendicular and angle θ between B and A is 0^0.

Case II: Similarly, if the area A is placed parallel to the direction of field B then the direction of vector area $\vec{A}$ is perpendicular to the direction of $\vec{B}$ and angle 'θ' between them is 90° as shown in Fig.13.9, then Eq.13.4 becomes

$$\phi_B = BA\cos 90^\circ$$

$$\phi_B = 0$$

This is the minimum flux.

The SI unit of magnetic flux is Weber (Wb), which can be derived by using Eq.13.5.

$$\phi_B = BA$$

$$1\ Wb = 1T \cdot 1m^2$$

or $$1\ Wb = \frac{1N}{A \cdot m} \cdot m^2$$

$$1\ Wb = 1N \cdot m \cdot A^{-1}$$

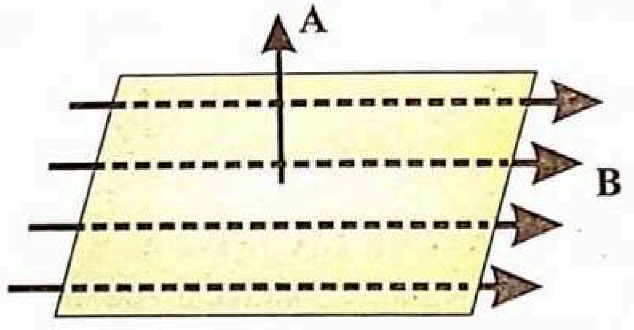


Fig.13.9 Area held parallel and angle θ between B and A is 90^0.

Magnetic flux density

Magnetic flux density is defined as **the magnetic flux per unit area held perpendicular to the direction of field strength 'B'.** It is measured in terms of ratio between magnetic flux ϕ_B and unit area 'A' by using Eq.13.5

$$B = \frac{\phi_B}{A} \quad ......(13.6)$$

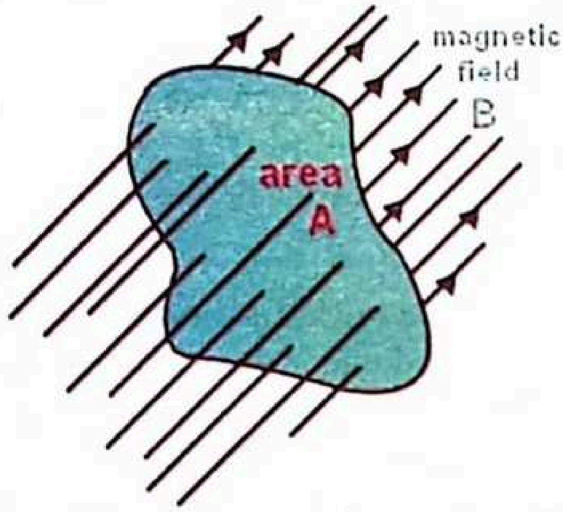
The unit of magnetic flux density is Wbm^{-2} and it is equal to tesla. i.e.,

$$Wbm^{-2} = \left(\frac{N \cdot m}{A}\right) m^{-2}$$

$$Wbm^{-2} = \frac{N \cdot m^{-1}}{A}$$

$$Wbm^{-2} = \frac{N}{A \cdot m}$$

$$Wbm^{-2} = T$$

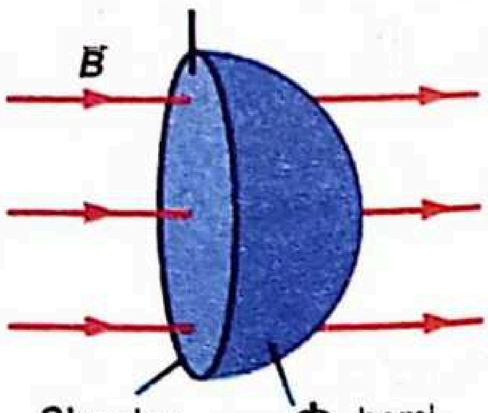


Magnetic lines of force passing through a unit area.

This shows that magnetic field strength and magnetic flux density both have same unit.

Example 13.2

A hemispherical surface of radius 5cm is placed in a magnetic field of strength 0.6T. If the direction of the surface is along the direction of the field then calculate the flux through the hemispherical surface.



Solution:

Radius of hemisphere = R = 5cm = 0.05m

Magnetic field strength = B = 0.6T

Angle between B and A = θ = 0°

Magnetic flux = ϕ_B = ?

By definition of magnetic flux

$$\phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$
$$\phi_B = BA\cos 0$$
$$\phi_B = BA$$

But area of hemispherical surface $= \pi R^2$

$$\phi_B = B\pi R^2$$
$$\phi_B = (0.6T)(3.14)(0.05m)^2$$
$$\phi_B = 4.7 \times 10^{-3}\ Wb$$

13.4 AMPERE'S LAW

We have studied that a current carrying conductor produces a magnetic field around it in the form of a closed circular loop of radius 'r' as shown in Fig.13.10. The experiment shows that the direction of the magnetic field is tangent at each point of the circular loop and its strength is directly proportional to the current flowing through the wire and inversely proportional to the distance from the wire. Mathematically, these two results can be expressed as,

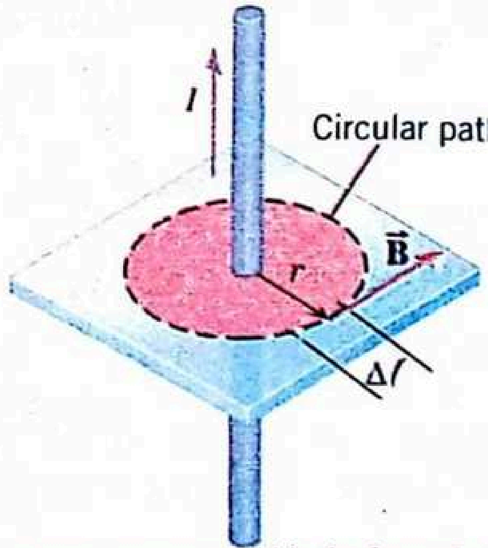


Fig.13.10 A magnetic field in the form of circular loop of constant radius r around steady current carrying a conductor.

$$B \propto \frac{I}{r}$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r} \quad ......(13.7)$$

Here $\frac{\mu_0}{2\pi}$ is the constant of proportionality. The parameter 'μ_0' is called the permeability of free space and its value is $4\pi\times10^{-7}\ TmA^{-1}$.

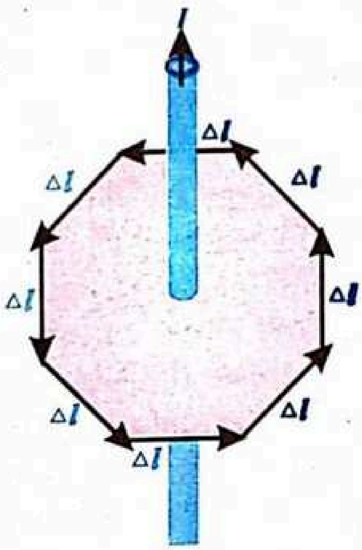


Fig.13.11 An arbitrary closed path around a current carrying a conductor

Equation 13.7 was derived for the magnetic field in form of a close circular loop around a steady current-carrying conductor. However, when the magnetic field is along an arbitrary closed path called 'amperean path' around the current-carrying conductor as show in Fig.13.11, then we cannot use Eq. 13.7 directly. For this Ampere has expressed a general relation between current and magnetic field along an arbitrary closed path.

Consider the arbitrary path around the current-carrying conductor which consists of 'n' number of small segments, such that each segment has same length equals to $\Delta\ell$ and is parallel to the magnetic field B, where

$$\vec{B}\cdot\Delta\vec{\ell} = B\Delta\ell\cos\theta = B\Delta\ell \qquad \therefore \theta = 0^\circ$$

According to Ampere, **the sum of all such product, $(B\cdot\Delta\ell)$ over the closed path is equal to μ_0 times of the total current that passes through the surface bounded by the closed path.** This statement is called Ampere's circuital law and it can be expressed as,

$$(B\cdot\Delta\ell)_1 + (B\cdot\Delta\ell)_2 + \cdots + (B\cdot\Delta\ell)_n = \mu_0 I$$

$$\sum_{i=1}^{n}(B\cdot\Delta\ell)_i = \mu_0 I \qquad \ldots\ldots(13.8)$$

This is the mathematical form of Ampere's circuital law.

13.4.1 Magnetic field due to a current carrying solenoid

A solenoid is a long coil of conducting wire with many turns. When current is passed through a solenoid then a uniform magnetic field is produced in it as shown in Fig.13.12. The solenoid acts as a bar magnet when it carries current i.e., it becomes a strong electromagnet. The magnetic field lines are entering at its one end and emerging from the other end. The lines emerging end of the solenoid acts as a north pole and the line entering end acts as a south pole as shown in Fig.13.13. The magnetic field inside the solenoid is not only uniform but also stronger. Whereas, the field outside the solenoid is weaker and negligible.

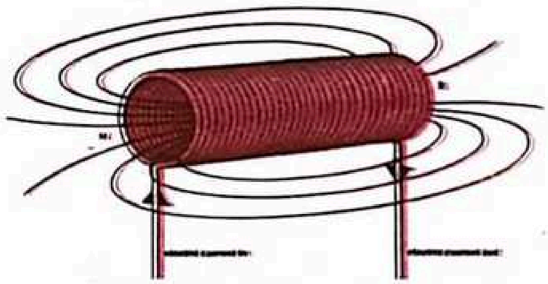


Fig.13.12 A solenoid in a cylindrical form.

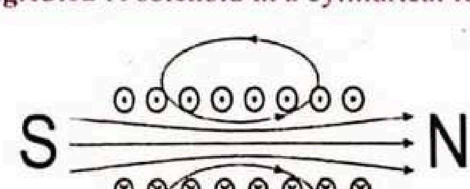


Fig.13.13 Polarities of uniform and strong magnetic field that produced inside the solenoid.

Consider a current-carrying solenoid of length 'ℓ'. Here we assume that magnetic field outside the solenoid is zero. To calculate the value of magnetic field B inside the solenoid using Ampere's law, we consider a rectangular closed path abcd such

> ***DO YOU KNOW***
> **The magnetic polarities of a solenoid can be verified with a compass.**

that ab=l_1, bc=l_2, cd=l_3 and da=l_4, as shown in Fig.13.14. The sum of the products of magnetic field B and lengths of rectangular closed path is expressed as,

$$\sum \vec{B}\cdot\Delta\vec{\ell} = \vec{B}\cdot\vec{\ell}_1 + \vec{B}\cdot\vec{\ell}_2 + \vec{B}\cdot\vec{\ell}_3 + \vec{B}\cdot\vec{\ell}_4$$

$$\sum \vec{B}\cdot\Delta\vec{\ell} = B\ell_1\cos 0^\circ + B\ell_2\cos 90^\circ$$

$$+(0)(\ell_3)\cos 180^\circ + B\ell_4\cos 270^\circ$$

$$\sum \vec{B}\cdot\Delta\vec{\ell} = B\ell_1 + 0 + 0 + 0$$

$$\sum \vec{B}\cdot\Delta\vec{\ell} = B\ell_1 \quad \ldots\ldots(13.9)$$

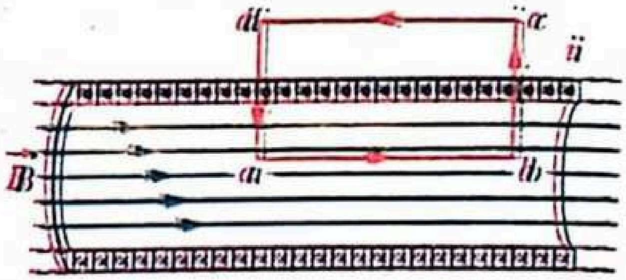


Fig.13.14 To calculate magnetic field B considering a closed rectangular path abcd on a solenoid.

If the number of turns per unit length is 'n' then total number of turns of solenoid of length 'ℓ' is $N = n\ell$ and total number of turns in length ℓ_1 is $n\ell_1$.

Thus, total current in closed path abcda = $n\ell_1 I$

According to Ampere's Law.

$\Sigma B\cdot\Delta\ell = \mu_0$ (total current enclosed)

POINT TO PONDER

By what factor will the magnetic field inside a solenoid increase if both the number of turns and the current are doubled?

$$\sum \vec{B}\cdot\Delta\vec{\ell} = \mu_0 n\ell_1 I \quad \ldots\ldots(13.10)$$

Comparing the left-hand sides of Eq.13.9 and Eq.13.10

$$B\ell_1 = \mu_0 n\ell_1 I$$

$$\Rightarrow \quad B = \mu_0 nI \quad \ldots\ldots(13.11)$$

The direction of field B is along the axis of solenoid. The result of Eq. 13.11 shows that magnetic field B is independent of the position within the solenoid. It means the field is uniform within a long solenoid. Eq. 13.11 is also valid to the solenoid which has more than one layer of windings because B does not depend on the radius of the solenoid. A much stronger magnetic field can be produced if windings of solenoid are made on magnetic material that is use of an iron core.

Example 13.3

A 10cm long solenoid has 400 turns of wire and carries a current of 2A. Calculate the magnetic field inside the solenoid.

Solution:

Length of the solenoid = ℓ = 10cm = 0.1m

Total number of turns of solenoid = N = 400

Current through the solenoid = I = 2A

Magnetic field inside the solenoid = B = ?

Number of turns in length per unit length $= n = \frac{N}{l}$

$$n = \frac{400}{0.1} = 4000 \text{ turns /m}$$

The magnetic field inside the solenoid is given by

$$B = \mu_o nI$$

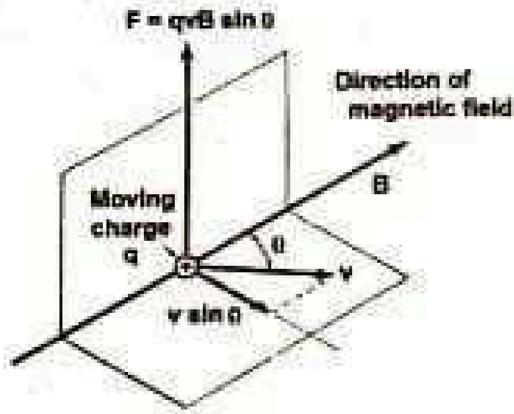
$$B = 4\pi \times 10^{-7} \times 4000 \times 2$$

$$B = 0.01 \text{ T}$$

$$B = 10^{-2} \text{ T}$$

13.5 FORCE ON A MOVING CHARGED PARTICLE IN A MAGNETIC FIELD

As we know the rate of flow of charges through a conductor is known as current. We have already studied that when current-carrying conductor is placed in a uniform magnetic field, it experiences a magnetic force. Indeed, the magnetic force acts on the moving charges because a beam of charged particles which are moving with uniform velocity equals to a steady current in the direction of their motion. The magnetic force on the charge is due to interaction between the applied magnetic field and the induced magnetic field around the moving charges.



The magnetic force F acting on a charge q moving with velocity v

$\vec{F} = q\vec{v} \times \vec{B}$

Consider a charge particle **q** is moving with velocity **v** in a uniform magnetic field **B**, it experiences a magnetic force. The magnitude of this force depends upon the following factors;

i. **Charge:** The magnitude of magnetic force is directly proportional to the magnitude of charge.

$$F_m \propto q$$

ii. **Magnetic Field:** The force is directly proportional to the strength of the magnetic field.

$$F_m \propto B$$

iii. **Velocity:** The magnetic force also directly proportional to the velocity of the charge particle.

$$\mathbf{F_m \propto v}$$

iv. **Direction:** It is found experimentally that when a charge particle is moving perpendicular to the direction of magnetic field, a maximum force exerts on it. However, when its motion is along the direction of the field, it experiences no force. This shows that the magnitude of the magnetic force is directly proportional to $\sin\theta$

$$\mathbf{F_m \propto \sin\theta}$$

Combining all the above results, we get a relation,

$$F_m \propto qBv\sin\theta$$

$$F_m = kqBv\sin\theta$$

where 'k' is a constant of proportionality. In SI units its value is one and is dimensionless,

$$F_m = qvB\sin\theta$$

POINT TO PONDER

A force exerts on a moving charged particle in a magnetic field, but in what direction it moves that the force does not exert on it?

In vector form

$$\vec{F}_m = q(\vec{v}\times\vec{B}) \quad ......(13.12)$$

This result is applied for both types of charges i.e. positive and negative charges. The direction of the magnetic force can be determined by using right hand rule as; place or draw the vectors $\vec{v}$ and $\vec{B}$ with their tail together. Imagine rotating $\vec{v}$ towards $\vec{B}$ through the smaller angle between them by curl of fingers of right hand in the direction of rotation. The erected thumb points in the direction of force on moving charge as shown in Fig.13.15. In this case we have considered a positively charged particle (proton). i.e., when the proton enters into a magnetic field, it experiences a force in the upward direction as given by the vector $\vec{v}\times\vec{B}$. Hence, the proton is deflected in the upward direction as shown in Fig.13.16(a). If the moving particle is negatively charged such as an electron then the direction of force will be opposite to that of positive charge. When the electron enters into a magnetic field, a magnetic force acts on it in the

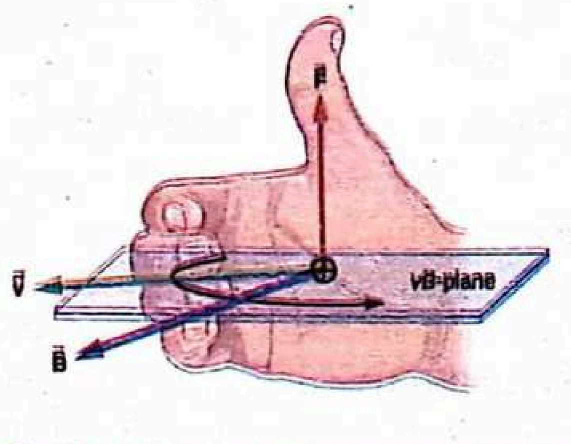


Fig.13.15 Right hand rule showing a direction of force.

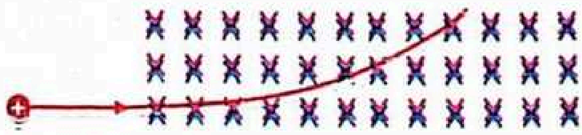


Fig.13.16(a) The proton is deflected upward under the action of magnetic force.

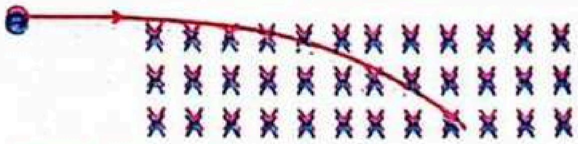


Fig.13.16(b) The electron is deflected downward under the action of magnetic force.

downward direction. As a result, the electron is deflected in the downward direction, as shown in Fig.13.16(b).

Example 13.4

An ion (q = +2e) enters into a magnetic field of strength 1.2T with a velocity 2.5×10^5 m s^{-1} perpendicular to the field. Determine the force that acts on the ion.

Solution:

Charge on an ion = q = +2e

$$q = 2.(1.6 \times 10^{-19})C$$

$$q = 3.2 \times 10^{-19}\ C$$

Magnetic field strength = B = 1.2T

Velocity of ions = v = 2.5×10^5 m s^{-1}

Angle between field and velocity = $\theta = 90^\circ$

The force on a the given ion is given by

$$F = qvB\sin\theta$$

$$F = (3.2 \times 10^{-19}C)(2.5 \times 10^5 m\ s^{-1})(1.2T)\sin 90^\circ$$

$$F = 9.6 \times 10^{-14}\ N$$

13.6 DETERMINATION OF e/m OF AN ELECTRON

Consider an electron which is moving with a constant velocity $\vec{v}$ and enters in a uniform magnetic field $\vec{B}$ such that the direction of its motion is perpendicular to direction of the field $\vec{B}$. It is to be noted that the direction of the magnetic field is into the page as shown in Fig.13.17. Thus, the force acting on the moving electron through $\vec{B}$ is given by:

$$\vec{F}_m = -e\left(\vec{v} \times \vec{B}\right) \quad ......(13.13)$$

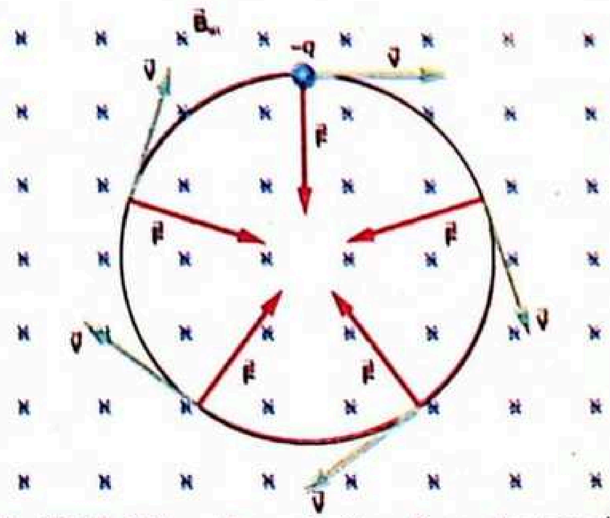


Fig.13.17 When the velocity of an electron is perpendicular to the magnetic field then the magnetic force acting on it equals to the centripetal force.

This magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$. So, it changes the direction of velocity of electron, but the magnitude of the velocity remains same. Thus, under the action of this constant force the electron will move along a circular path as shown in Fig.13.17. The magnitude of magnetic force on electron is given by

$$F_m = evB\sin\theta = evB \qquad \therefore \theta = 90^\circ$$

According to right hand rule, the magnetic force is always directed towards the centre of the circle. Therefore, it provides the necessary centripetal force to the electron of mass m to move it along a circular path of radius r. Thus, we have;

$$F_m = F_c$$

$$evB = \frac{mv^2}{r}$$

$$\frac{e}{m} = \frac{v}{Br} \quad \ldots\ldots(13.14)$$

If the values of v, B and r are known then we could determine the e/m i.e., charge to mass ratio. In this regard, the radius 'r' can be measured by making circular path of electron visible. It is taken place by using gas (hydrogen or helium) filled tube at a low pressure placed in the magnetic field. The molecules of the gas are excited by the elastic collision of electrons with them. Now during deexcitation, the molecules emit light and hence the circular path of electron becomes visible as shown in Fig.13.18. In this way, the radius of the ring can be determined easily.

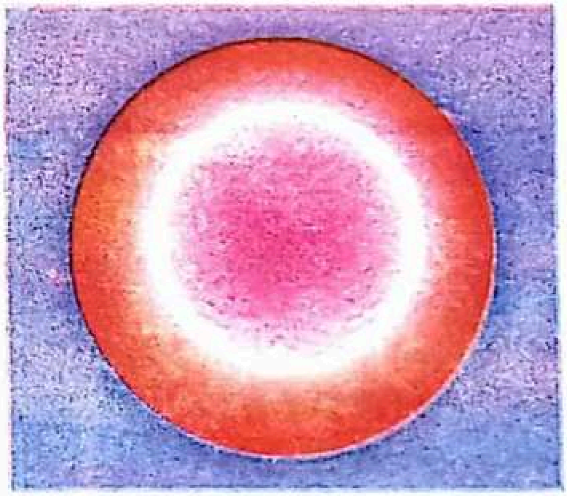


Fig.13.18 A visible circular path of electrons.

Similarly, the velocity of electrons can be determines when they are accelerated before entering into the magnetic field by applying potential difference 'V_o'. These accelerated electrons gain kinetic energy $\left(\frac{1}{2}mv^2\right)$ which equals to eV_o.

That is,

$$\frac{1}{2}mv^2 = eV_o$$

$$v = \sqrt{\frac{eV_0}{m}}$$

Substituting the value of e in eq. 13.14.

$$\frac{e}{m} = \frac{1}{Br}\sqrt{\frac{2eV_2}{m}}$$

$$\frac{e^2}{m^2} = \frac{1}{B^2r^2}\frac{2eV_o}{m}$$

$$\frac{e}{m} = \frac{2V_o}{B^2r^2} \quad ......(13.15)$$

The velocity of electron can also be determined by velocity selector method. The arrangement of such method consists of the applied electric and magnetic fields at right angle between the two plates as shown in Fig.13.19.

When an electron of mass 'm' charge 'e' enters perpendicularly with velocity 'v' in the region occupied by mutually perpendicular electric and magnetic fields, it experiences both electric force $(+eE)$ and magnetic force $(-evB)$. Negative sign shows that the magnetic force has same magnitude as that of electric force but in opposite direction. Hence, under the action of these two equal forces, the electron is in the state of equilibrium. i.e.,

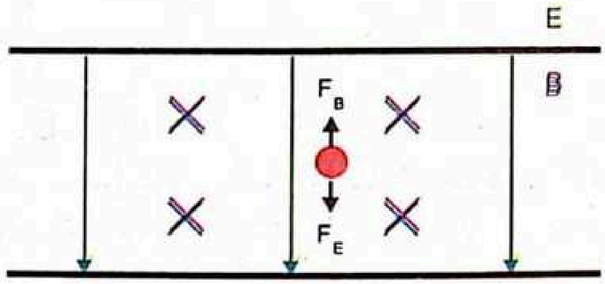


Fig.13.19 When an electron enters in the region occupied by electric and magnetic fields at right angle. It experiences electric and magnetic forces which are same in magnitude but in opposite direction.

$$F_e + F_B = 0$$

$$eE - evB = 0$$

$$vB = E$$

$$v = \frac{E}{B}$$

Substituting the value of v in Eq.13.14. We get

$$\frac{e}{m} = \frac{E}{B^2r} \quad ......(13.16)$$

Example 13.5

A proton of mass 1.67×10^{-27} kg and charge 1.6×10^{-19}C is moving in a circle of radius 0.4m in a magnetic field of strength 1.2T. Find (i) speed and (ii) kinetic energy of proton.

Solution:

Mass of proton = m = 1.67×10^{-27}kg

Charge on a proton = e = 1.6×10^{-19}c

Radius of the circular path of proton = r = 0.4m

Magnetic field strength = B = 1.2T

(i) Speed of proton = v = ?

(ii) K.E. of proton = ?

i. To find the speed of proton in uniform magnetic field along a circular path, we use the following equation

$$v = \frac{eBr}{m}$$

$$v = \frac{(1.6\times10^{-19}\,C)(1.2T)(0.4m)}{1.67\times10^{-27}\,kg}$$

$$v = 4.6 \times 10^{7}\ m\ s^{-1}$$

ii $$K.E. = \frac{1}{2}mv^2$$

$$K.E. = \frac{1}{2}(1.67\times10^{-27}\,kg)(4.6\times10^{7}\,m s^{-1})^2$$

$$K.E. = 17.7\times10^{-13}\,J = \frac{17.7\times10^{-13}}{1.6\times10^{-19}}eV = 1.1\times10^{6}\,eV$$

$$K.E. = 11MeV$$

13.7 TORQUE ON A CURRENT CARRYING COIL IN A MAGNETIC FIELD

Consider a current-carrying rectangular coil 'abcd' of length L and width x which is placed in a uniform magnetic field such that the direction of field is parallel to the direction of the plane of the rectangular coil as shown in Fig.13.20. Also, the coil is capable to rotate about its axis. We have studied in the previous section that when a current carrying conductor is placed in a magnetic field, such that the length of the conductor is perpendicular to the direction of field then it experiences a magnetic force, $\vec{F} = I\left(\vec{L}\times\vec{B}\right) = ILB\sin\theta\,\hat{n}$. Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{L}$ and $\vec{B}$ and it indicates the direction of force. In case of rectangular coil, no forces act on its side 'ab' and 'cd' because these two lengths are parallel to the field and angle between B and these two sides is

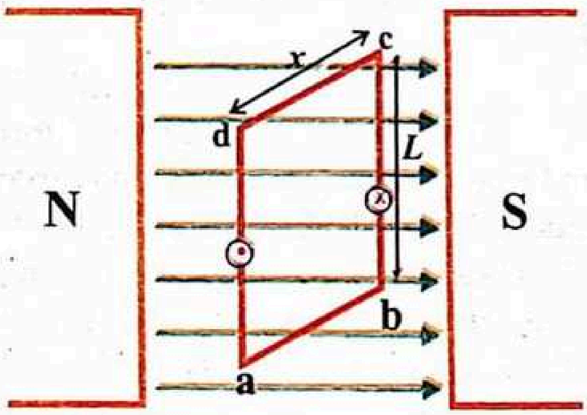


Fig.13.20 Torque on a current carrying a rectangular coil in a uniform magnetic field.

zero. Therefore, $\vec{F}_2 = \vec{F}_4 = I(\vec{L} \times \vec{B}) = 0$. On the other hand, the magnetic forces act on the sides bc and ad because these two sides are perpendicular to the field. Thus, the magnitude of these forces is given by

$$F_1 = F_2 = ILB \qquad \therefore \theta = 90^\circ$$

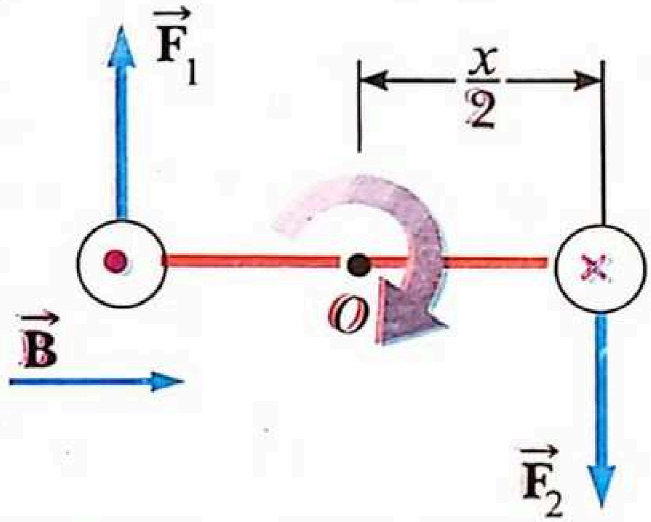


Fig.13.21(a) A magnetic field parallel to the plane of the loop, where F_1 and F_2 act perpendicularly on moment arm x/2 and produce maximum torque.

Where the force F_1 is directed out of the plane (page), while the force F_2 is directed into the plane. These two forces are same in magnitude but in opposite directions, Therefore, they produce the rotation in the coil about an axis which is known as torque, as $\frac{x}{2}$ is the moment arm of each force, the magnitude of the net torque is given by

$$\tau = F_1 \frac{x}{2} + F_2 \frac{x}{2}$$

$$\tau = ILB\frac{x}{2} + ILB\frac{x}{2}$$

$$\tau = xLIB$$

where xL is the vector area of the rectangular coil and its magnitude is A. Therefore,

$$\tau = IAB \qquad ...(13.16)$$

This is the maximum torque produced by a current-carrying rectangular coil. Equation 13.16 hold only when the field is parallel to the plane of the coil as shown in Fig. 13.21(a). Now if there is some angle 'θ' between the field and area 'A' of the coil as shown in Fig.13.21(b) then the torque will be due to the vertical component of moment arm $\frac{x}{2}\sin$ as shown in Fig.13.21(b). In this case, the magnitude of the resultant torque is given as

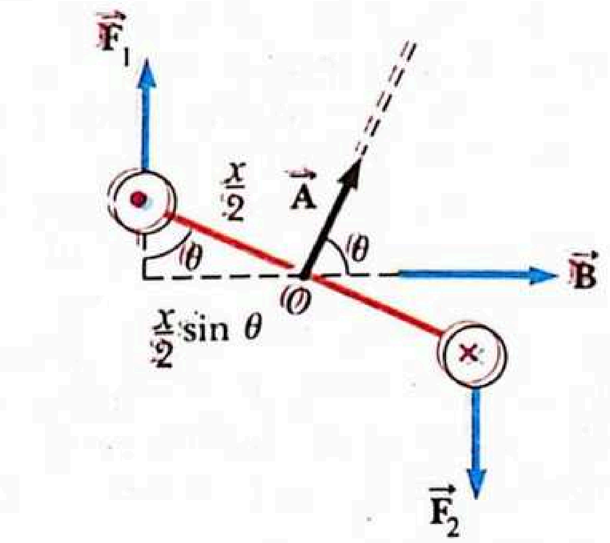


Fig.13.21(b) There is some angle between field $\vec{B}$ and area $\vec{A}$ of the loop and torque is due to the forces F_1 and F_2 and vertical components of moment arm (x/2 sinθ)

$$\tau = F_1 \frac{x}{2} \sin\theta + F_2 \frac{x}{2} \sin\theta$$

$$\tau = IAB\ \sin\theta$$

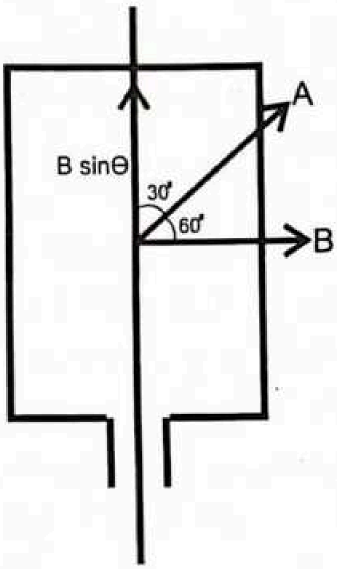
If the rectangular coil has 'N' number of turns. Then

$$\tau = INAB\ \sin\theta \quad ......(13.17)$$

This result shows that the rotation (torque) due to current carrying coil in magnetic field depends upon the magnitude of current, number of turns of the coil, magnetic field strength, area 'A' of the coil and the angle 'θ' between the field B and plane area 'A' of the coil.

Example 13.6

The plane of a rectangular coil makes an angle of 60° with the direction of a uniform magnetic field of strength 0.9T. The coil has 50 turns and the magnitude of its plane area is $0.12m^2$. If it carries a current of 10A then calculate the torque acting on the coil.



Solution:

Current through the coil = I = 10A

Strength of field = B = 0.9T

Number of turns of coil = N = 50

Angle between area and field = $\theta = 90^\circ - 60^\circ$

$= 30^\circ$

Area of the plane of the coil = $0.12m^2$

Torque = τ = ?

The torque on a current carrying a coil is given by

$$\tau = INAB\ \sin\theta$$

$$\tau = (10A)(50)(0.12m^2)(0.9T)\sin 30^\circ$$

$$\tau = 27Nm$$

13.8 GALVANOMETER

A galvanometer is a sensitive electrical instrument used to detect or measure a small electric current. The most commonly used type of galvanometer is the moving-coil galvanometer. The working principle of galvanometer is based upon the fact that when a current-carrying coil is placed in a uniform magnetic field, it is acted upon by forces on its both end sides which produces a deflecting torque given as:

$$\tau = INAB\sin\theta$$

where I is the current in the rectangular coil, N is the number of turns of the coil, 'A' area of coil, 'B' is the uniform applied magnetic field, θ is the angle between the field and direction of plane of the coil.

Construction

A moving coil galvanometer consists of a rectangular coil 'abcd' which contains 'N' number of turns of insulated copper wire wounded on a non-magnetic light frame. The coil is suspended with the help of a phosphor bronze wire *x* between the curved N and S poles of a powerful U-shaped permanent magnet, such that it is free to rotate as shown in Fig.13.22(a). Inside the coil, a soft iron cylinder is fixed between the curved faces of the poles. Its function is to make the field radial, uniform and stronger. The suspension phosphor-bronze wire *x* is also serves as one current lead of the coil. The other terminal of the coil is connected to a spring 'y' of phosphor-bonze having a few turns. It is used as the second current lead. A small plane mirror is also attached to the top suspension wire. It helps to measure the deflection of the coil by lamp and scale arrangement.

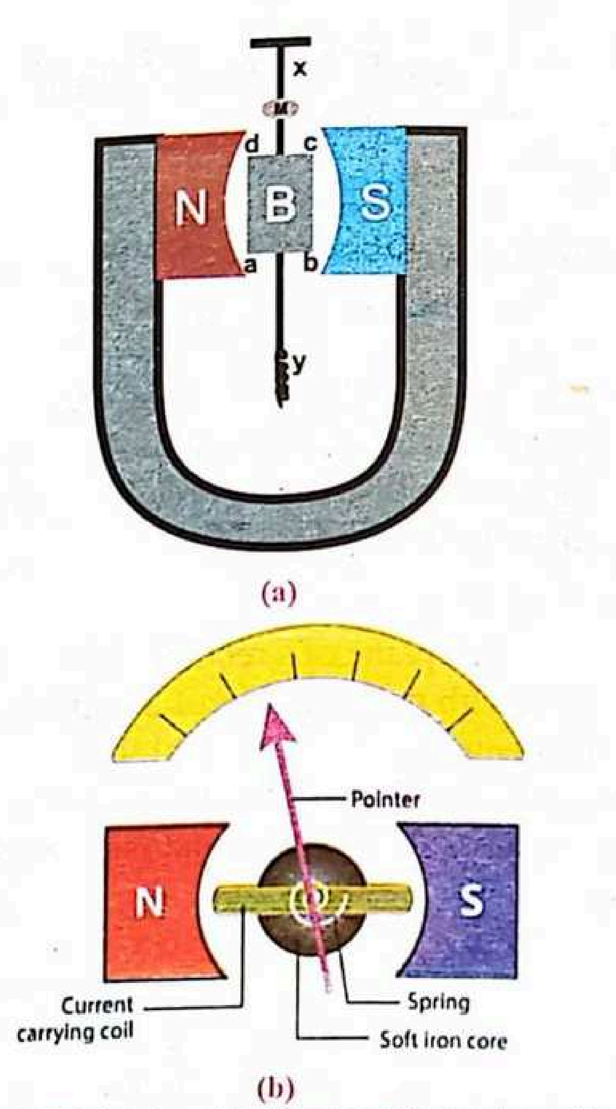


Fig.13.22 A schematic diagram of a moving coil galvanometer.

Working

When a current is passed through the coil, two forces of the same magnitude but in opposite direction act on the two sides of the coil called a couple. This couple produces a deflecting torque τ_d which is given by;

$$\tau_d = INAB\sin\theta$$

Since the coil is placed in a radial magnetic field, where angle 'θ' between the field and the direction of the plane of the coil is 90°. Therefore, the coil experiences a maximum torque. i.e.,

$$\tau_d = INAB \ldots\ldots(13.18)$$

Due to this maximum deflection torque, the coil rotates, and this rotation of coil produces a twist in the phosphor-bronze wire which causes the restoring torque. This restoring torque is directly proportional to the angle 'θ' through which the wire is twisted. i.e.,

$$\tau_r \propto \theta$$

$$\tau_r = c\theta \ldots\ldots(13.19)$$

where 'c' is a constant of proportionality called torsion constant of the suspension wire. Its unit is Nm per degree. The coil rotates until the restoring torque becomes equal to the deflection torque. i.e., when the coil is at the state of equilibrium, i.e.,

$$\tau_d = \tau_r$$

$$INAB = c\theta$$

$$I = \frac{c}{NAB}\theta \ldots\ldots(13.20)$$

As all the terms of $\frac{c}{NAB}$ are constant so,

$$I \propto \theta$$

This shows that the deflection of the coil is directly proportional to the current passing through it. Also, this result leads to the development of a linear scale of a galvanometer. Usually, there are two methods of observing the angle of deflection of the coil, which are explained as under:

In a sensitive galvanometer, if a small current passes through a coil, it produces a large deflection. Such deflection is observed by means of a small plane mirror

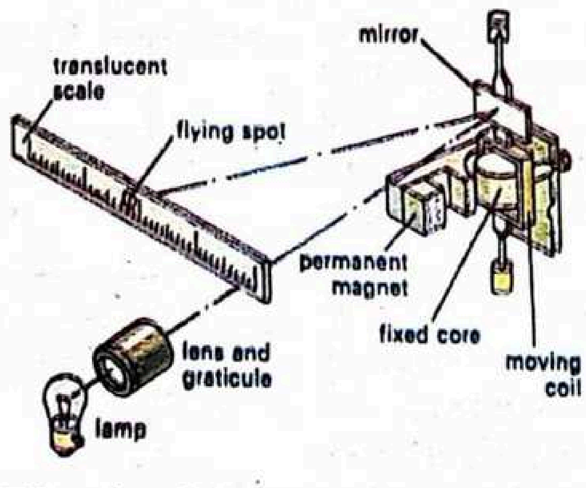


Fig.13.23 Observing of deflection by lamp and scale arrangement.

attached to the suspension wire along with a lamp and scale arrangement as shown in Fig.13.23. A beam of light from the lamp is focused on the mirror of the galvanometer. It is reflected from mirror and produces a spot on a scale placed at a distance of one metre from the galvanometer. Now when the coil rotates, the mirror also rotates with the coil and the spot of light moves along the scale. The displacement of the spot of light on the scale is proportional to the angle of deflection.

The second type of galvanometer is a pivoted coil galvanometer which is being used in the laboratories of school, colleges and other educational institutions. Such type of galvanometer is a less sensitive, where the coil is pivoted between two jeweled bearings. The restoring torque is provided by two hair springs which also work as current leads. A light aluminum pointer is attached to the coil which moves over the calibrated scale as shown in Fig.13.24. This moment of the pointer provides the measurement of the angle of deflection of the coil.

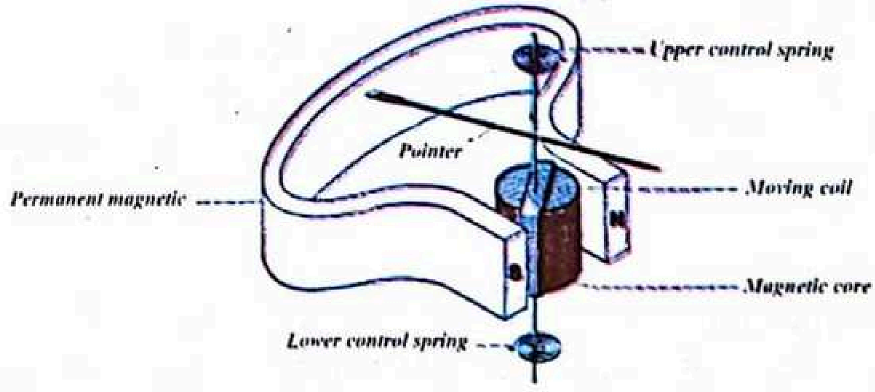


Fig.13.24 A schematic diagram of a pivoted type galvanometer.

13.8.1 Ammeter

An ammeter is an electrical device used to measure the electric current passing through a circuit. A galvanometer is a sensitive instrument and its pointer shows full scale deflection for a very small current even for current of milli ampere. Thus, in practice, an ordinary galvanometer cannot be used for large current and its measurement. To overcome this problem, we convert a galvanometer into an ammeter by connecting a suitable low resistance in parallel with it as shown in Fig.13.25. Such low resistance diverts (by passes) the

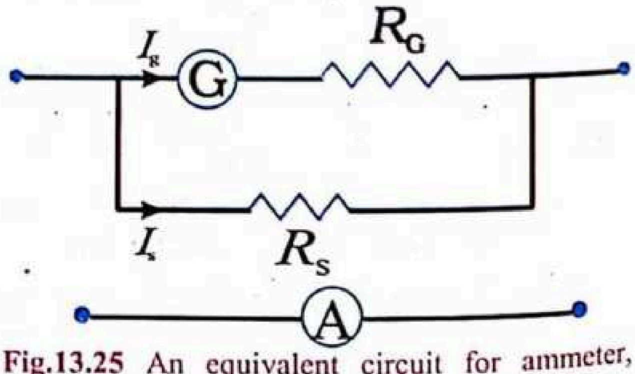


Fig.13.25 An equivalent circuit for ammeter, where a shunt resistance is connected parallel with a galvanometer.

extra current and hence it is named as shunt resistance R_S. This shunted galvanometer is called an ammeter.

To find the required value of shunt resistance R_S for a given range of ammeter, we allow flow of current I across the circuit of ammeter. A fraction of this current of value I_g passes through a galvanometer of resistance R_g, while the remaining large amount of current of value $(I - I_g)$ passes through the shunt resistance. As shunt resistance is parallel to the galvanometer so there is same potential difference across both shunt resistance and galvanometer thus, we have;

$$V_s = V_g$$

$$(I - I_g)R_s = I_g R_g$$

$$R_S = \left(\frac{I_g}{I - I_g}\right) R_g \quad \ldots\ldots(13.21)$$

The above equation can be used to calculate the required shunt resistance for given galvanometer in order to convert it for any range of ammeter.

Since ammeter is being used to measure the current, so it should always be connected in series and the current flow through the component of a circuit can be calculated by using the following relation:

$$I = I_g\left(\frac{R_g}{R_S} + 1\right) \quad \ldots\ldots(13.22)$$

13.8.2 Voltmeter

A voltmeter is an electrical device used to measure the potential difference between two points in an electric circuit. As we have discussed that a galvanometer can measure a current only in milliampere due its very low resistance. It means that the potential difference can also be applied across the galvanometer in millivolts. But in practice, our requirement is to measure the high potential difference. To overcome this problem, we convert a galvanometer into voltmeter by connecting a suitable high resistance R_x in series with it as shown in Fig.13.26.

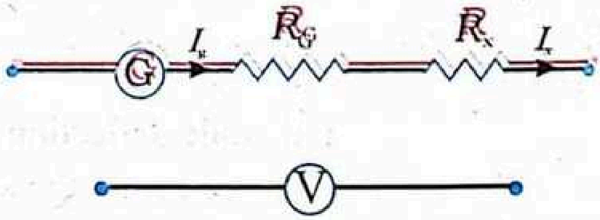


Fig.13.26 An equivalent circuit for voltmeter, where a high resistance is connected in series with a galvanometer

Now when a potential difference 'V' is applied across the voltmeter, then only a fraction of volts V_g drops across the galvanometer and the remaining high voltage $(V - V_g)$ drops across the high resistance R_x. Since the deflection of galvanometer is proportional to the current I_g flowing through it and I_g is proportional to the potential difference. Therefore, the scale is calibrated to indicate the voltage across the voltmeter.

Suppose we want to measure the applied potential difference in volts by using a galvanometer of resistance R_g having full scale deflection current I_g, we connect a high resistance R_x in series with the galvanometer. It may be noted that the same amount of current is passed through both high resistance and galvanometer but the value of the potential difference across each is different. Thus, their resultant potential difference 'V' is given as;

$$V = V_g + V_x$$
$$V = I_gR_g + I_gR_x$$
$$V = I_g(R_g + R_x)$$
$$\frac{V}{I_g} = R_g + R_x$$
$$R_x = \frac{V}{I_g} - R_g \quad ......(13.23)$$

The high resistance R_x obtained by Eq. 13.23 has to be connected in series with galvanometer to provide the potential difference of desired range. Since voltmeter is being used for measurement of potential difference between two points of the given circuit, it should always be connected in parallel.

Example 13.7

A galvanometer gives full-scale reading of 25mA when potential difference across its terminals is 75mV. How it can be used (i) as an ammeter of range 100A and (ii) as a voltmeter of range 750V?

Solution:

Full scale deflection current $= I_g = 25\text{mA} = 25 \times 10^{-3}\text{A}$

Potential difference across the terminal $= V_g = 75\text{mV} = 75 \times 10^{-3}\text{V}$

Shunt resistance $= R_s = ?$

(i) If $I = 100\text{A}$

High resistance $= R_x = ?$

(ii) If $V = 750\text{V}$

$$\text{Resistance of galvanometer} = R_g = \frac{V_g}{I_g} = \frac{75\times10^{-3}}{25\times10^{-3}} = 3\Omega$$

(i)
$$\text{Shunt resistance} = R_s = \frac{I_gR_g}{I - I_g} = \frac{25\times10^{-3}\times3}{100-0.025}$$

$$R_s = \frac{75\times10^{-3}}{99.975}$$
$$R_s = 0.00075\Omega$$

(ii) High resistance $= R_h = \frac{V}{I_g} - R_g$

$$R_x = \frac{750}{25\times10^{-3}} - 3$$
$$R_x = 30000 - 3$$
$$R_x = 29997\Omega$$

13.9 AVOMETER (MULTIMETER)

An Avometer is a multipurpose electrical device for measuring alternating/direct current, voltages and resistance. The name Avometer comes from AVO and meter which means ampere meter, voltmeter and ohmmeter.

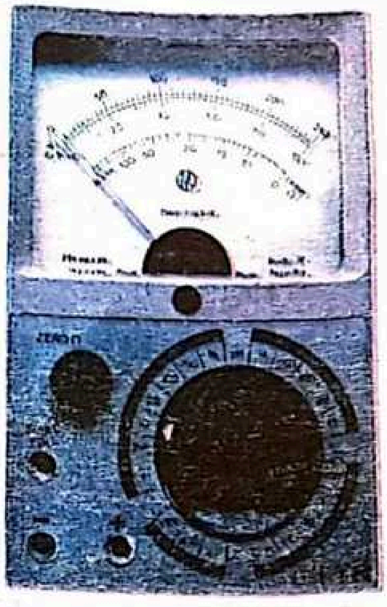


Fig.13.27 An Analogue Avometer

An Avometer may be of analogue or digital type. The analog type has the pointer and scale system as shown in Fig.13.27. However, digital multimeter has a numerical display screen as shown in Fig.13.28, where the digital values in terms of amperes, volts and ohms are displayed automatically with decimal point on it. Such meter also eliminates the human error.

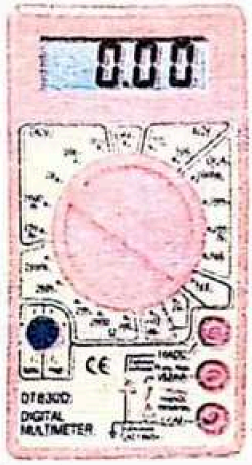


Fig.13.28 A digital Avometer

An Avometer is basically a sensitive moving coil galvanometer which is arranged with a necessary network of resistances, a battery and switching system as shown in Fig.13.29. It has four terminals. When one terminal of X and the other of terminal of Y is selected, then the required quantity can be measured. Now all the three parts of Avometer circuit are explained as;

I The current measurement part

As we know, when a low shunt resistance is connected in parallel to a galvanometer then such arrangement converts a galvanometer into ammeter. The range of such meter can be extended when series combination of low resistance R_1, R_2 and R_3 are connected in parallel with a galvanometer as shown in Fig.13.30. Such arrangement provides the measurement of current in the range from milli amperes to amperes. Alternating current (A.C.) can also be measured by Avometer when a diode is connected with it. Here the diode is used as a rectifier, i.e. a diode converts A.C. into D.C.

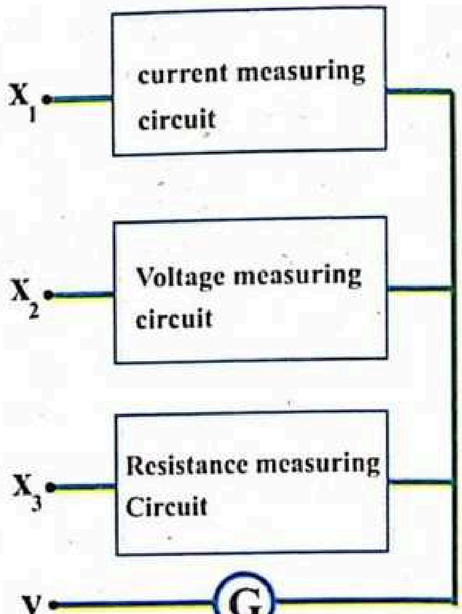


Fig.13.29 A network of an avometer.

II The voltage measurement part

We have already explained that when a high resistance is connected in series with a galvanometer, then such arrangement converts a galvanometer into voltmeter. The range of this instrument can further be increased, when a number of high resistances R_1, R_2, and R_3, are connected in series with a galvanometer as shown in Fig.13.31. This network gives the measurement of potential difference in different range such as 10V, 50V and 250V etc.

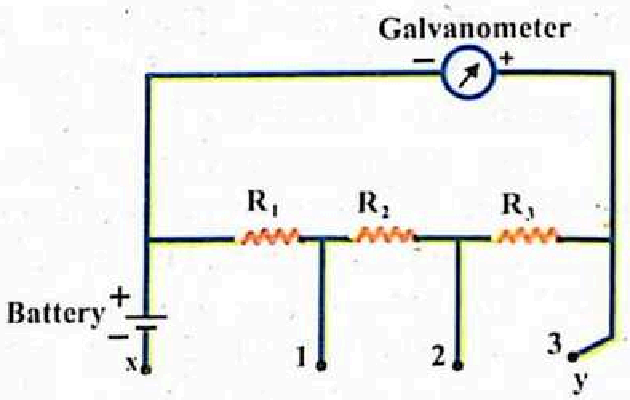


Fig.13.30 A circuit diagram for current measuring of an Avometer.

Avometer can also be used to measure the A.C. voltage when a diode is connected with it. Where A.C. voltage is first converted into DC voltage by the diode then the measurement of voltage is taken place.

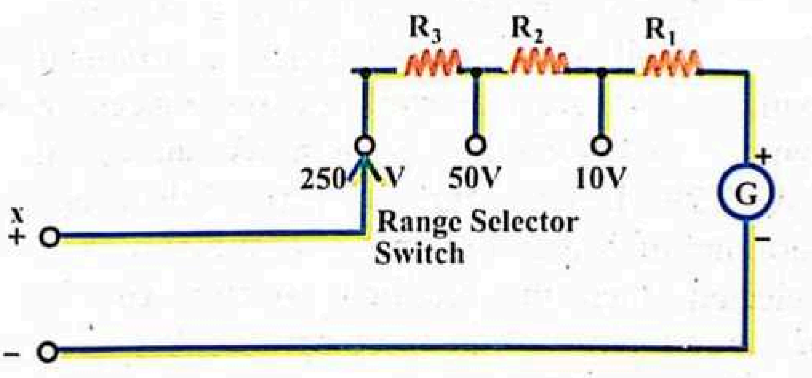


Fig.13.31 A circuit diagram for voltage measuring of an avometer.

III The resistance measurement part

A circuit for an ohmmeter in a Avometer consists of a galvanometer, a variable resistance R_v and a source of e.m.f. E connected in series as shown in Fig.13.32. The resistance 'R' to be measured is connected between terminals x and y.

First, the variable resistance R_v is adjusted so that when the terminals x and y are short circuited, that is, when R = 0, the galvanometer deflects full scale. Then the circuit is opened, i.e., nothing is connected between the terminals x and y, so resistance is infinity and current is zero. Thus, the deflection of the meter is also zero. Finally, when a resistance 'R_n' (n = 1, 2, 3, …) is connected between the terminals x and y, the galvanometer deflects to some intermediate point. This point is calibrated as a resistance. The range of ohmmeter can further be extended by introducing different resistances of different values such R_1, R_2, R_3 and so on.

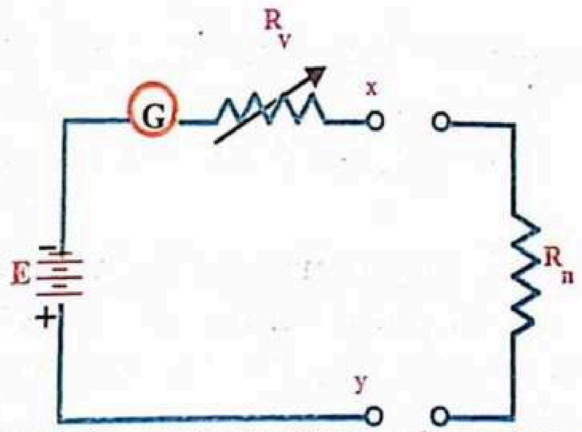


Fig.13.32 A circuit diagram for resistance measuring of an avometer.

SUMMARY

- **Magnetic Field:** The region around the magnetic or the current carrying conductor in which a magnetic effect or force can be experienced is called magnetic field.
- **Magnetic Force on Current Carrying Conductor:** When a current carrying conductor of length 'ℓ' is placed perpendicularly in uniform magnetic field B, it is acted upon by a magnetic force given as;

$$\vec{F} = I(\vec{\ell} \times \vec{B})$$

- **Magnetic Flux:** The number of magnetic lines of force passing through certain element of an area is called magnetic flux. It is equal to the dot product of field strength 'B' and vector area A.

$$\phi_m = \vec{B} \cdot \vec{A}$$

- **Magnetic Flux Density:** The magnetic flux per unit area held perpendicular to field lines is called magnetic flux density.
- **Tesla:** Tesla is the unit of magnetic field strength or magnetic flux density. The strength of the magnetic field is said to be 1T, if it exerts a force of 1N on a conductor when flow of current through it is 1A, also

$$1T = 10^4 G$$

- **Ampere's Law:** The sum of the dot product of B and $\Delta\ell$, i.e., $(\vec{B} \cdot \Delta\vec{\ell})$ over the closed loop around the current-carrying conductor is equal to μ_0 times of the total current surrounded by the closed loop. i.e.,

$$\Sigma \vec{B} \cdot \Delta\vec{\ell} = \mu_0 I$$

- **Solenoid** A solenoid is a long coil of conducting wire wounded in many turns which produces a uniform magnetic field inside it when current is passed through it. The magnetic field due to current-carrying solenoid is given by

$$\vec{B} = \mu_0 n I$$

- **Force on a charge particle moving through magnetic field:** When a charged particle is moving with velocity 'v' in a magnetic field experiences a force. The force on a charged particle is perpendicular to both the direction of the field and direction of the motion of charged particle. The magnetic force is given as;

$$F = q(\vec{v} \times \vec{B})$$

- **Torque on a current carrying coil:** When a current carrying rectangular coil is placed in a uniform magnetic field, the coil experiences a torque given by;

$$\tau = INAB\ \sin\theta$$

- **Galvanometer:** It is an electrical device used to detect a small electric current.
- **Ammeter:** It is a device used to measure the large current passing through a circuit.
- **Voltmeter:** It is an electrical instrument used to measure the potential difference between two point in an electric circuit.
- **Avometer:** It is a multimeter used to measure current, voltage and resistance in an electric circuit.

EXERCISE

Multiple choice questions.

1. A maximum force that acts on a current carrying conductor placed in a magnetic field, when angle between the field and length of the conductor is
(a) 0^0 (b) 45^0 (c) 90^0 (d) 180^0
2. Tesla in terms of base units is equal to
(a) kg $m^{-1}A^{-1}$ (b) kg mA (c) kg $s^{-1}A^{-1}$ (d) kg $s^{-2}A^{-1}$
3. One Gauss is equal to
(a) 1T (b) 10^2T (c) 10^{-4}T (d) 10^4T
4. The unit of magnetic flux is
(a) Tesla (b) Weber (c) Gauss (d) Henry
5. The magnetic flux is maximum when angle between magnetic field and vector area is
(a) 0^0 (b) 30^0 (c) 60^0 (d) 90^0
6. Magnetic flux density is defined in terms of
(a) Tesla (b) wb m^{-2} (c) $NA^{-1}m^{-1}$ (d) All of them
7. A charged particle is moving along X-axis in a magnetic field along the Y-axis. The direction of magnetic force acting on it is

(a) along x-axis (b) along y-axis (c) along z-axis (d) in xy-plane

8. Ampere's Law gives us the relationship between
(a) Force and velocity of charge (b) Force and magnitude field
(c) Current and force (d) Current and magnetic field

9. The value of permeability of free space is
(a) $10^{-7} T.m.A^{-1}$ (b) $2\pi\times10^{-7} T.m.A^{-1}$
(c) $4\pi\times10^{-7} T.m.A^{-1}$ (d) $4\pi\times10^{7} T.m.A^{-1}$

10. The magnetic field due to a current-carrying solenoid which has 'n' number of turns per unit length is
(a) $B=\mu_0 n I$ (b) $B=\mu_0 n^2 I$ (c) $B=\frac{\mu_0 n I}{\ell}$ (d) $B=\frac{\mu_0 n^2 I}{\ell}$

11. The magnetic field inside the solenoid is independent of one of the following quantities
(a) Permeability (b) Position vector
(c) Number of turns (d) Flow of current

12. What is the magnetic force on a stationery charged particle in a uniform magnetic field?
(a) Zero (b) $F=q(v\times B)$ (c) $F=qvB$ (d) $F=ILB\sin\theta$

13. An electron is moving horizontally towards east. If it enters in magnetic field directed upward then the electron will be deflected in the direction of
(a) East (b) West (c) North (d) South

14. When the direction of motion of a charged particle is perpendicular to the direction of magnetic field, then the particle follows the path of a
(a) Straight line (b) helix (c) ellipse (d) circle

15. The torque due to a current carrying rectangular coil placed in a uniform magnetic field is
(a) $\tau=IBA\sin\theta$ (b) $\tau=IAN\sin\theta$ (c) $\tau=IBN\sin\theta$ (d) $\tau=NAIB\sin\theta$

16. The working principle of a galvanometer is based upon
(a) Momentum (b) Torque (c) Force (d) Impulse

17. The current passing through the coil of a galvanometer is directly proportional to the
(a) Resistance (b) Conductance (c) Reactance (d) Angle of deflection

18. A shunted galvanometer is called
(a) Voltmeter (b) Ammeter (c) Ohmmeter (d) Potentiometer

19. A galvanometer can be converted into voltmeter by connecting it with
(a) Low resistance in parallel (b) Low resistance in series
(c) High resistance in parallel (d) High resistance in series

20. Which one of the following quantity is not measured by Avometer
(a) Charge (b) Current (c) Resistance (d) Potential difference

SHORT QUESTIONS

1. How can you determine the direction of the magnetic field due to a current carrying a conductor?
2. What is tesla and what is the relation between Tesla and Gauss?
3. Distinguish between Tesla and Weber.
4. How can you differentiate the flow of current into the page and out of the page?
5. Under what condition the magnetic flux is minimum and maximum?
6. Differentiate between magnetic flux and magnetic flux density.
7. What do you know about the magnetic force on a stationary charged particle in a uniform magnetic field?
8. Why Ampere's law is true only for a steady current?
9. What do you know about the amperean path?
10. What are the values of the magnetic field inside and outside of a current-carrying solenoid?
11. State the Fleming's left-hand rule to determine the direction of force that acts on a charged particle moving perpendicular to the magnetic field.
12. Explain the path of deflection of an electron and a proton when they enters perpendicularly in a uniform magnetic field.
13. How does an electron come under the centripetal force when its motion is perpendicular to a uniform magnetic field?
14. How can you determine $\frac{e}{m}$ by velocity selector method?
15. How can we increase the magnitude of torque due to a current carrying rectangular coil placed in a uniform magnetic field?
16. How does radial magnetic field produce in a moving coil galvanometer?
17. Why low resistance in an ammeter is called shunt resistance? Why it is connected parallel to a galvanometer?
18. How can you convert a moving galvanometer into a voltmeter?
19. Explain the method of measurement of resistance by using ohmmeter.
20. Distinguish between analogue and digital Avometer.

COMPREHENSIVE QUESTIONS

1. What is magnetic field? Explain the magnetic field produced around a current carrying conductor.
2. Define magnetic force and derive a relation for a magnetic force that is exerted on a current-carrying a conductor placed in uniform applied magnetic field.

3. What do you know about Flemming left hand rule? How can you determine the direction of magnetic force by using flaming left-hand rule?
4. Define magnetic flux, magnetic flux density and their SI units.
5. State and explain Ampere's circuital law and calculate magnetic field due to a current carrying solenoid using Ampere's law.
6. How can you determine the force that acts on a moving charged particle (proton and electron) in a uniform magnetic field?
7. Explain the torque on a current carrying rectangular coil placed in a uniform magnetic field.
8. Define and explain galvanometer, its construction and working principle.
9. How can you develop an ammeter and a voltmeter by using a galvanometer?
10. Define and explain an Avometer, its functions and its various parts.

NUMERICAL PROBLEMS

1. A current of 10A carrying conductor of length 20cm is placed in uniform magnetic field 2T. If the length is at 60^0 to the field, then calculate the magnitude of the force acting on the conductor. **(3.5N)**
2. A maximum magnetic force of 0.3N exerts on a 15cm long conductor carrying a current of 10A in a uniform magnetic field. Find the magnitude of the magnetic field. **(0.2T)**
3. Calculate the value of magnetic flux density and magnetic flux within an air core solenoid of radius 2cm and 4000 turns per unit length carrying a current of 5A. **(2.51×10^{-2}T, 3.15×10^{-5}wb)**
4. A solenoid 10cm long has 500 turns. Find the magnetic field inside the solenoid if it carries a current of 10A. **(6.28×10^{-2}T)**
5. What will be the speed of an electron if it moves at right angle to the magnetic field of 0.2T and it experiences a magnetic force of 2×10^{-12}N. **(6.25×10^{17}ms^{-1})**
6. A proton enters a magnetic field of flux density 1.5wbm^{-2} with a velocity of 2×10^7ms^{-1} at an angle of 45^0 with the field. Compute the force on the proton. **(3.4×10^{-12}N)**
7. A velocity selector has a magnetic field of 0.3T. If a perpendicular electric field of 10^4Vm^{-1} is applied, what will be the speed of the particle when it passes through the selector? **(3.3×10^4ms^{-1})**
8. A rectangular coil of sides 10cm and 6cm having 2000 turns and carrying a current of 5A is placed in a uniform magnetic field of 0.5T. Calculate the maximum torque that experiences by the coil. **(30Nm)**

9. Find the value of shunt resistance required to convert a galvanometer into ammeter of range 5A. Where the galvanometer has internal resistance 50Ω and it gives full deflection with a current of 10mA. (0.1Ω)

10. A galvanometer has an internal resistance 40Ω and deflects full scale for 4mA current. Calculate the high resistance that should be connected in series with galvanometer, in order to convert the galvanometer into voltmeter of range 100volt. (24960Ω)

Unit 14

ELECTROMAGNETIC INDUCTION

Major Concepts

(18 PERIODS)

- Induced e.m.f.
- Faraday's law
- Lenz's law
- Eddy currents
- Mutual Inductance
- Self-inductance
- Energy stored by an inductor
- Motional e.m.f.
- A.C. Generator
- A.C. motor and Back e.m.f.
- Transformer

Conceptual Linkage

This chapter is built on Electromagnetism Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

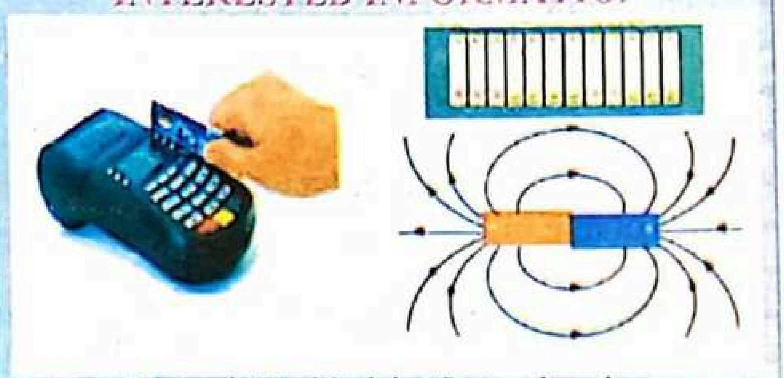
- describe the production of electricity by magnetism.
- explain that induced e.m.f.'s can be generated in two ways.
  - (i) by relative movement (the generator effect).
  - (ii) by changing a magnetic field (the transformer effect).
- infer the factors affecting the magnitude of the induced e.m.f..
- state Faraday's law of electromagnetic induction.
- account for Lenz's law to predict the direction of an induced current and relate to the principle of conservation of energy.
- apply Faraday's law of electromagnetic induction and Lenz's law to solve problems.
- explain the production of eddy currents and identify their magnetic and heating effects.
- explain the need of laminated iron cores in electric motors, generators and transformers.
- explain what is meant by motional e.m.f.. Given a rod or wire moving through a magnetic field in a simple way, compute the potential difference across its ends.
- define mutual inductance (M) and self-inductance (L), and their unit henry.

- describe the main components of an A.C. generator and explain how it works.
- describe the main features of an A.C. electric motor and the role of each feature.
- explain the production of back e.m.f. in electric motors.
- describe the construction of a transformer and explain how it works.
- identify the relationship between the ratio of the number of turns in the primary and secondary coils and the ratio of primary to secondary voltages.
- describe how step-up and step-down transformers can be used to ensure efficient transfer of electricity along cables.

INTRODUCTION

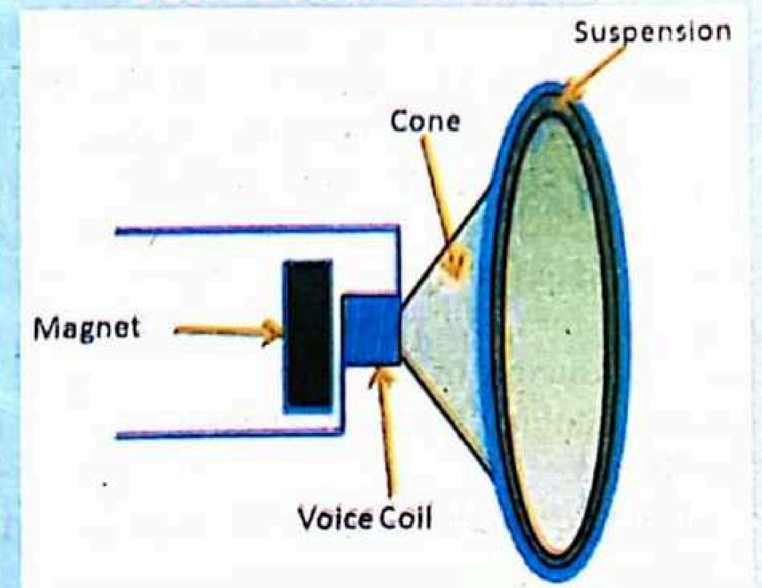
In the previous unit, we have studied that a flow of electric current produces a magnetic field. This is a the link between electricity and magnetism. Such link was discovered by Oersted in 1820. Now a question arises that can a magnetic field also produce current or electric field? A few years later, the answer of this question was sounded Yes, when the reverse effect was discovered. i.e., a changing magnetic field can also produce a current in a nearby circuit. This effect was studied by Michelson Faraday in England and Joseph Henry in America in the year 1831. The effect of inducing voltage or electromotive force due to a change in the magnetic field is known as electromagnetic induction. Induced e.m.f. can be generated either by generator effect (by relative motion between changing magnetic field and the conductor) or by transformer effect (changing B). The changing magnetic field forces the electrons

INTERESTED INFORMATION



Electronic Card Swiping System based on electromagnetic induction theory

FOR YOUR INFORMATION



When an A.C. signal is applied, a changing current in the coil of the loudspeaker produce a magnetic field in it. This magnetic field interacts with the field of permanent magnetic of the loudspeaker and it causes the magnetic force that exerts on the coil which results in vibration of the cone. This vibration produces sound waves.

to move in any conductor within the field producing an induced e.m.f. or current. The great discovery of electromagnetic induction by Faraday through his fundamental law has brought a revolution in the field of science and technology. Most of the electrical devices such as motors, generators, transformer etc. are working on the basis of principle of Faraday's law. In this unit, we will not only explain the various aspect of electromagnetic induction but also discuss its practical applications.

14.1 INDUCED E.M.F.

It has been observed experimentally that when a changing magnetic flux passes through a circuit or conductor, an e.m.f. is induced in it. This phenomena is known as induced e.m.f.. Which is explained by the an example.

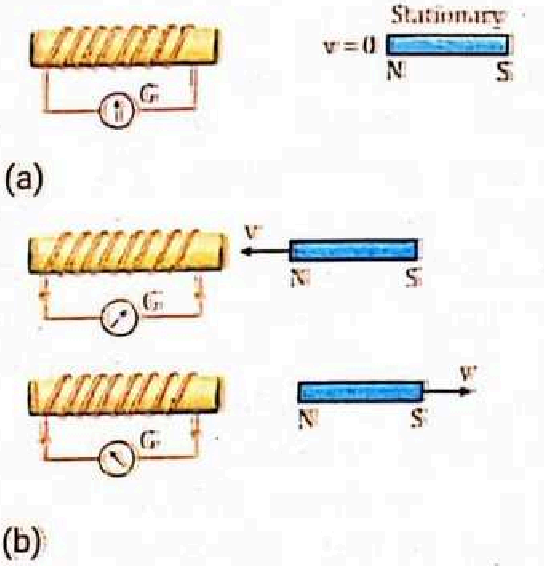


Fig.14.1(a) Magnetic bar is at rest and meter deflection is zero. (b) Magnetic bar is moving towards and away from the coil and meter shows deflection

Consider a coil connected with a sensitive galvanometer and it is placed near a stationary bar magnetic, such that the magnetic flux from its N-pole is linked with the coil as shown in Fig.14.1(a). Initially, when both are at rest i.e., there is no relative motion between coil and magnet, so the deflection of the galvanometer is zero. Now when we move the bar magnet towards the coil, the magnetic flux across the coil is increased, and galvanometer shows deflection in one side as shown in Fig.14.1(b). Similarly, when the bar magnet is moved away from the coil, this time the magnetic flux is decreased, and the galvanometer again shows deflection but on the opposite side. The same result of deflection will be observed when the bar magnet is at rest, while the coil is moved either towards or away from the magnet. It is important to notice that the deflection can be observed only when there is a relative motion between bar magnet and the coil. However, no deflection will be observed when they are stationary no matter how close they are to each other.

The deflection of the galvanometer indicates that a current is induced in the circuit in the absence of a battery or any other source of e.m.f., it is observed only due to changing magnetic flux linked the coil. Such current is called induced current. Its corresponding voltage is called induced e.m.f.. The analysis shows that the induced current depends upon the following factors:

i. Number of turns of the coil.

ii. Magnetic flux linked to the coil.
iii. Relative speed between bar magnet and coil.

There are number of methods that produce induced current, but we are going to explain two of them.

Method I: Consider a coil of uniform area connected with a galvanometer such that the coil is placed in a uniform magnetic field as shown in Fig.14.2(a). Initially, when the coil is at rest, there is no deflection in the coil. However, when the coil is rotated, the magnetic flux linked the coil changes by its rotation and galvanometer shows deflection as shown in Fig. 14.2(b). This indicates that the current is induced by the rotation of coil in uniform magnetic field. This principle is being used in electric generator.

Method II Similarly, we have another method to induce current. This method consists of two coils placed side by side. One coil is connected to a battery with a switch and is called primary coil P. The other is connected to a galvanometer and is called secondary coil S.

Initially, when the switch is open and there is no current in the primary circuit, then the deflection on the galvanometer in the secondary circuit is zero as shown in Fig.14.3(a). Now when the switch is closed then the current increases in the primary coil and it causes increasing magnetic flux in the secondary coil S linked with primary coil and thus galvanometer shows deflection as shown in Fig.14.3(b). Similarly, when the switch is again made open then the current decreases in the primary coil which causes decreasing magnetic flux in the

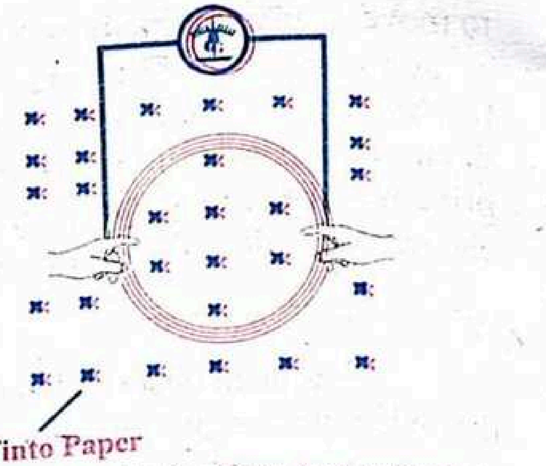


Fig.14.2(a) A coil of uniform area connected with a galvanometer in uniform magnetic field.

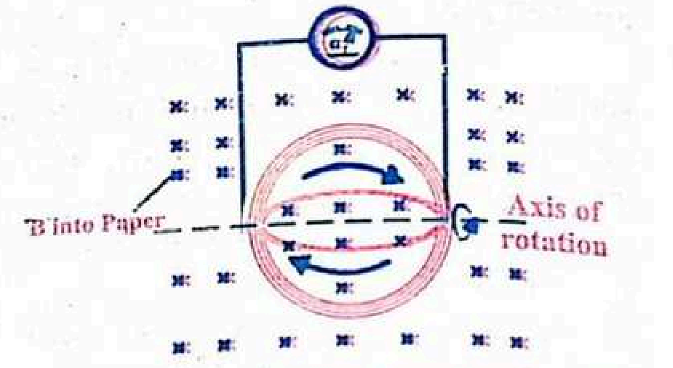


Fig.14.2(b) Induced current due to rotation of a coil in uniform magnetic field.

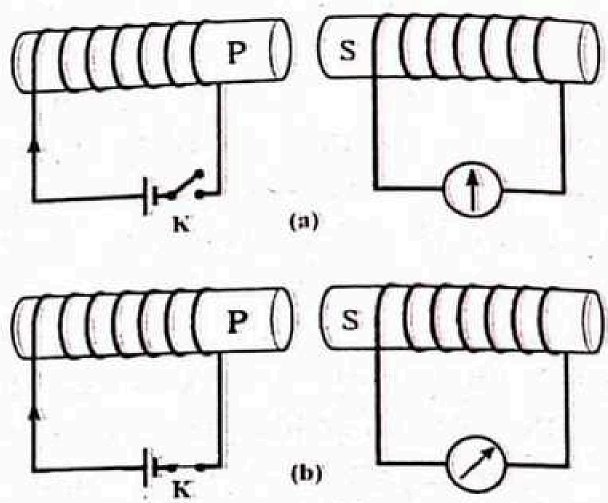


Fig.14.3(a) When the switch is opened and current is zero in the primary circuit, the deflection in meter is also zero in the secondary circuit (b) when switch K is closed current increases in primary coil and flux increases in secondary coil thus meter shows deflection.

secondary coil 'S'. Again the galvanometer shows deflection but in the opposite direction. In this process, the e.m.f. induces due to the mutual link of two coils through magnetic flux, so it is named as mutual induction. A transformer's working is based on principle of such mutual induction.

14.2 MOTIONAL E.M.F.

In the previous section, we have studied the production of induced e.m.f. by different methods. In this section, we describe **the e.m.f. induced in a conductor due to its motion across a uniform magnetic field and it is called motional e.m.f..**

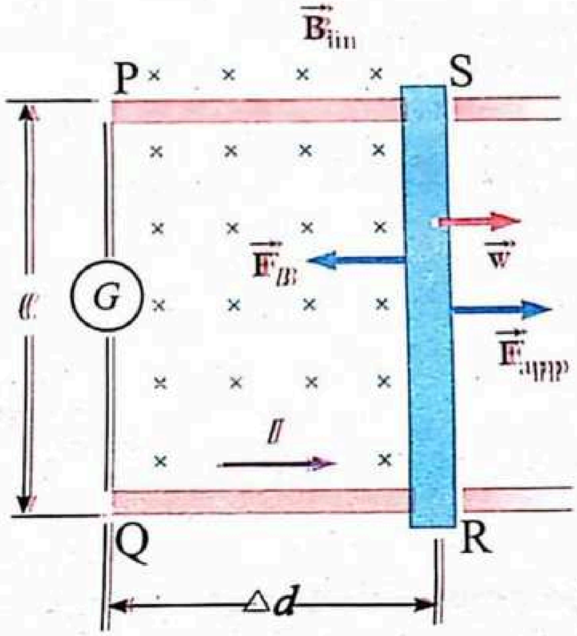


Fig.14.4(a) A conducting rod sliding with velocity v along two parallel rails produces a motional e.m.f..

The motional e.m.f. can be explained by an experimental setup which consists of two parallel conducting rails, separated by a distance 'ℓ' and a galvanometer is connected between their two ends P and Q. Let a conducting rod RS of length 'ℓ' is placed on the two parallel rails. In this way, we have a closed conducting loop PQRS as shown in Fig.14.4(a). A uniform magnetic field is also applied which directed into the page.

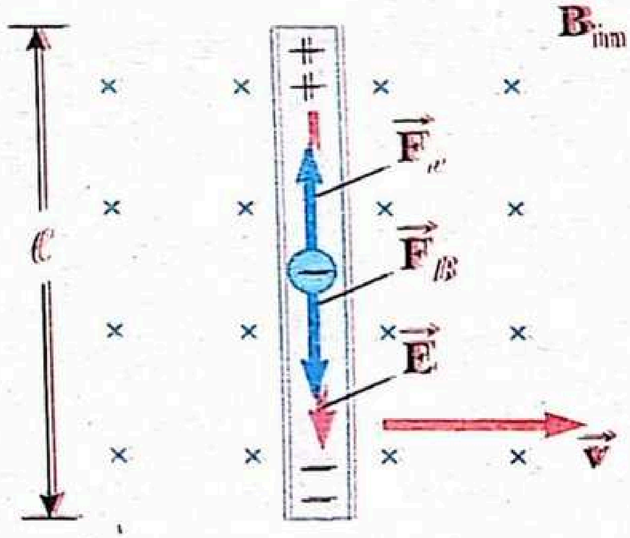


Fig.14.4(b) Separation of charges causes of electric field as well as electric potential across the ends of conductor.

Initially, when the rod is stationary, the galvanometer shows zero current in the closed loop. However when the conducting rod is pulled to the right with velocity 'v' under the action of an applied force ($\vec{F}_{app}$), the free charges (electron) of the rod experience a magnetic force $\vec{F}_B$. As a result, the free charges start to flow along the conductor and then in the closed loop thus galvanometer shows the current in the loop. This result shows that the current is induced in the closed loop due to the motion of the rod across the magnetic field.

The force that experience by the charges of the rod is given by

$$\vec{F}_B = q\left(\vec{v}\times\vec{B}\right) = qvB\sin\theta\hat{n}$$

In scalar notation

$$F_B = qvB\sin\theta$$

As the velocity 'v' is perpendicular to the magnetic field 'B' and angle θ between them is 90°. Therefore,

$$F_B = qvB\sin 90^\circ$$

$$F_B = qvB \quad ......(14.1)$$

The magnetic force causes the electrons (–ve charges) to accumulate at the one end of the rod and leaving a net the positive charge at the other end as shown in Fig.14.4(b). This separation of charges produces an electric field in the conductor where the electric force ($F_e = qE$) on the charges equals to the magnetic force ($F_B = qvB$). i.e., they are same in magnitude but in opposite direction. Thus according to Newton's third law:

$$F_e = -F_B$$

$$qE = -qvB$$

$$E = -vB \quad ......(14.2)$$

The separation of charges in the conductor also causes a potential difference (ΔV) across the ends of conductor and it is related to the induced e.m.f. (ε) in the loop. That is,

$$\varepsilon = \Delta V$$

But $$E = \frac{\Delta V}{\ell}$$

Or $$\Delta V = E\ell$$

∴ $$\varepsilon = E\ell$$

$$E = \frac{\varepsilon}{\ell}$$

Wireless charging mobile works under the principle electromagnetic induction.

Thus eq. 14.2 becomes

$$\frac{\varepsilon}{\ell} = -vB$$

$$\varepsilon = -vB\ell \quad ......(14.3)$$

This is the maximum value of motional e.m.f. when the velocity of the rod is perpendicular to the applied magnetic field. But, when there is some angle 'θ' between $\vec{V}$ and $\vec{B}$ then Eq.14.3 can be expressed as;

$$\varepsilon = B\ell v\sin\theta \quad ......(14.4)$$

This result shows that the motional e.m.f. depends upon length of the rod, speed of the rod and angle 'θ' between $\vec{v}$ and $\vec{B}$.

Example 14.1

A conducting rod of length 40cm is moving at a speed of 50 m s^{-1} in a uniform magnetic field of strength 2T. Calculate the e.m.f. induced in the rod when **(a)** the direction of motion of the rod is perpendicular to the magnetic field **(b)** and the rod moves at an angle of 60^0 to the field.

Solution:

Length of the rod = ℓ = 40cm = 0.4m

Speed of the rod = v = 50 m s^{-1}

Strength of the field = B = 2T

e.m.f. = ε = ?

(a) Angle between v and B = $\theta_1 = 90^0$

(b) Angle between v and B = $\theta_2 = 60^0$

By definition of motional e.m.f.

(a)
$$\varepsilon_1 = vB\ell\sin\theta_1$$
$$= (50\ \text{m s}^{-1})(2\text{T})(0.4\text{m})\sin 90^\circ$$
$$\varepsilon_1 = 40\text{V}$$

(b)
$$\varepsilon_2 = vB\ell\sin\theta_2$$
$$= (50\text{ms}^{-1})(2\text{T})(0.4\text{m})\sin 60^\circ$$
$$\varepsilon_2 = 35\text{V}$$

14.3 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

In motional e.m.f., we studied the induced e.m.f. by moving a conducting rod with constant velocity perpendicular to the magnetic field. Now the same phenomenon under the same experimental setup was explained by Faraday in terms of changing magnetic flux. Again considering a conducting rod of length ($RS = \ell$) which is placed on two parallel conducting rails and a galvanometer is connected between the two ends P and Q. So there is closed

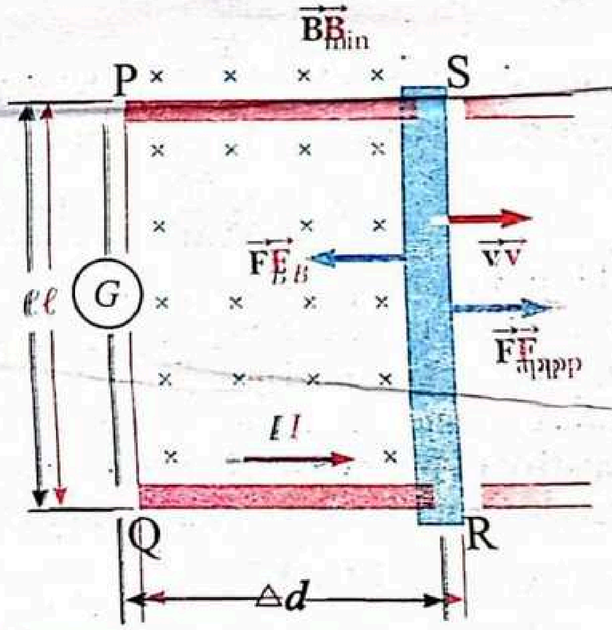


Fig.14.5 Induced e.m.f. due to changing of magnetic flux through the area of the loop PQRS.

conducting loop PQRS as shown in Fig.14.5. A magnetic field is also applied perpendicular to the plane of the conducting loop. If the rod moves through a distance Δd in time Δt then its velocity is given by:

$$v = \frac{\Delta d}{\Delta t} \quad ......(14.5)$$

Due to motion of the rod, the area of the conducting loop changes continuously. This changing area is related with the changing of magnetic flux through the loop. Thus, by definition of magnetic flux

$$\Delta\phi = B \cdot \Delta A$$

The direction of area of the loop is parallel to the direction of B and angle θ between them is 0^o. So

$$\Delta\phi = BA\cos 0^o = B\Delta A$$

$$\Delta\phi = B\ell\Delta d$$

Or

$$B = \frac{\Delta\phi}{\ell\Delta d} \quad ......(14.6)$$

> ***POINT TO PONDER***
>
> By neglecting the resistance, can a constant current in a coil setup a potential difference across the coil?

Now by definition of motional e.m.f.

$$\varepsilon = -vB\ell \quad ......(14.7)$$

Putting the value of v and B from eq. 14.5 and eq. 14.6 in eq. 14.7

$$\varepsilon = \left(\frac{\Delta d}{\ell\Delta d}\right)(\ell)$$

$$\varepsilon = -\frac{\Delta\phi}{\Delta t}$$

This result can be extended for 'N' number of loops instead of a single loop and we have indeed e.m.f. 'N' times. That is,

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t} \quad ......(14.8)$$

This is the mathematical form of Faraday law of electromagnetic induction. It shows that the induced e.m.f. depends upon the rate at which the magnetic flux through the the coil is changed. Thus, Faraday law of electromagnetic induction states that **the induced e.m.f. in a coil having 'N' number of loops is equal to N times the negative of the rate of change of magnetic flux linked with the coil.** The negative sign shows the direction of induced e.m.f. and it will be studied in Lenz's law.

14.4 LENZ'S LAW

We have studied that the magnitude of induced e.m.f. can be determined by using Faraday law of induction. i.e., the magnitude of induced e.m.f. is directly proportional to the rate of change of magnetic flux. The direction of this induced current was pointed out by a Russian Scientist H.F.E. Lenz in 1834 which states as: **"the direction of induced current is always such as to oppose the cause (direction of the action) that produces the induced current".** Thus, Lenz's law can be expressed same as that the Faraday's law but with negative sign.

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t} \qquad ......(14.9)$$

The negative sign in the Eq.14.3 implies that the induced e.m.f. opposes the rate of change of magnetic flux linked with the coil. It is further explained by an example;

Consider a coil which consists of a number of turns. When the N-pole of the bar magnet moves toward the coil, magnetic flux linking the coil increases. According to Faraday's law the induced current is setup in the circuit. Similarly, according to Lenz's law, this induced current opposes the further increase in flux through the coil. It is therefore, when the magnetic flux across the coil is changed then like a bar magnet, the one end of the coil acts as a N-pole while the other end acts as a S-pole. Hence, if the N-pole of the bar magnet move towards the coil. Now the two N-poles will then repel each other. Thus, by applying right hand rule, the direction of induced current in the circuit is anti-clockwise as shown in Fig.14.6(a).

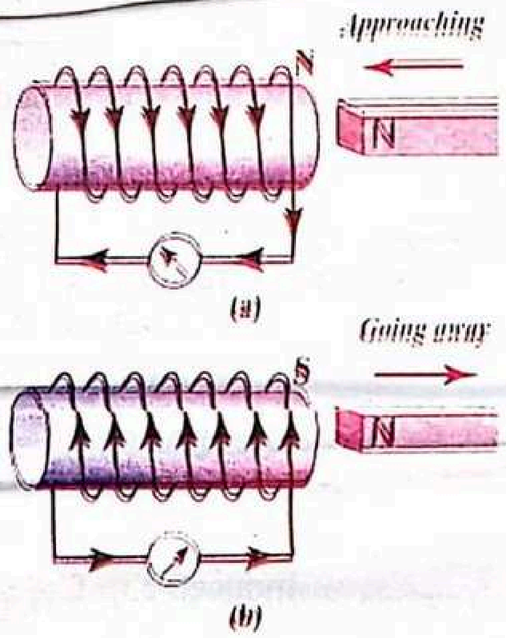


Fig.14.6(a) The induced current flows anticlockwise.
(b) The induced current flows clockwise.

If the N-pole of bar magnet moves away from the coil, the flux linking the coil decreases. The induced current in the circuit again opposes the decrease in flux. This time the end of the coil toward the magnet becomes S-pole. Now the two opposite poles will then attract each other and the direction of the induced current in the circuit is clockwise, to oppose the cause i.e., decrease in flux as shown in Fig.14.6(b).

Lenz's law is not only provided the direction of the induced current but it is also given the principle of law of conservation of energy. It is explained by considering again Fig.14.6. When N-pole of bar magnet is approaching the coil, the magnetic flux linking the coil increases and the end of the coil towards the bar magnet becomes N-pole. Now the two N-poles oppose each other. Due to this opposition, the mechanical energy is converted into electrical energy. Thus, Lenz's law is consistent with the law of conservation of energy. If the magnetic flux linking the coil increases, the direction of current in the coil will be such that it will oppose the further increase in flux and hence the induced current will be anticlockwise. If at this instant the direction of current would be clockwise then it will become self-perpetuating and this is the violation of law of conservation of energy.

Similarly, when the N-pole of the magnet moves away from the coil, the magnetic flux linking the coil decreases which induce current in the coil in such a direction that the end of coil towards the magnet becomes S-pole. Now the attraction of two unlike poles opposes the decrease in flux of the coil. This opposition to the cause of change in flux is again in accordance with the law of conservation of energy.

Example 14.2

A coil of 150 loops is pulled in 0.06s from the poles of the magnet, which decreases the magnetic flux linked with the coil from 6×10^{-4}Wb to 2×10^{-4}Wb. Determine the average e.m.f. induced in the coil due to changing of flux.

Solution:

Number of turns of coil = N = 150

Time taken = t = 0.06s

Initial flux = $\phi_1 = 6 \times 10^{-4}$ Wb

Find flux = $\phi_2 = 2 \times 10^{-4}$ Wb

Changing in flux = $\Delta\phi = \phi_2 - \phi_1$

$$\Delta\phi = 2\times10^{-4} - 6\times10^{-4} = -4\times10^{-4}\,\text{Wb}$$

Induced e.m.f. = ε = ?

According to Faraday Law,

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

$$\varepsilon = \frac{-150(-4\times10^{-4}\,\text{Wb})}{0.06\text{s}}$$

$$\varepsilon = (10000)\times10^{-4}\,\text{V}$$

$$\varepsilon = 1\text{V}$$

14.5 MUTUAL INDUCTION

When a changing current in one coil induces an e.m.f. in another nearly coil then such phenomenon is known as mutual induction. It is explained as: Consider two coils which are placed side by side and close to each other as shown in Fig.14.7. The coil which is connected to a battery, a switch and a rheostat is called primary coil P and the other coil which is connected to a sensitive galvanometer is called secondary coil S. When the switch is closed the current increases from zero to its maximum value in the primary coil P and it produces a changing magnetic flux which links with the secondary coil S and we observe momentarily deflection. After some time the magnetic field becomes steady i.e. there is no more changing flux and deflection on galvanometer becomes zero. If the current in the primary coil P is changed by changing the resistance of the rheostat, then the magnetic flux linked with secondary coil S is also changed. Hence, according to Faraday's law, the changing magnetic flux produces an induced e.m.f. in the secondary coil 'S' and galvanometer shows deflection. Since the induced e.m.f. in the secondary circuit is due to the mutual link of coil S with the coil P through magnetic flux, such induction is termed as mutual induction. The magnitude and direction this induced e.m.f. can be determined by Faraday's law and Lenz's law and its value is given by:

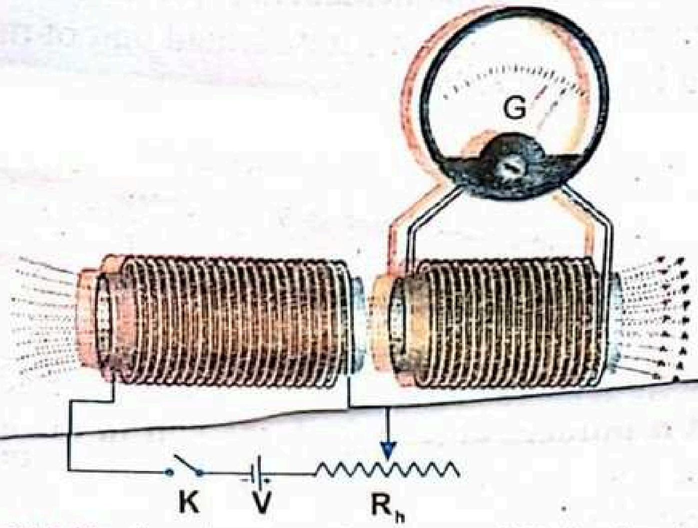


Fig.14.7 The changing current in primary coil P, the changing flux through secondary coil S produces an e.m.f. in it.

$$\varepsilon_s = -N_s \frac{\Delta\phi_s}{\Delta t} \quad ......(14.10)$$

where N_s is the number of turns of the secondary coil S. The analysis shows that the total flux $N_s\phi_s$ through the secondary coil S which causes the induced e.m.f. is directly proportional to the current I_P in the primary circuit. i.e.,

$$N_s\phi_s \propto I_P$$

$$N_s\phi_s = MI_P \quad ......(14.11)$$

where M is the constant of proportionality known as mutual inductance of the two coils. It depends upon the rate of changing of current in the primary coil, number of turns of coils, cross sectional area of the coils and closeness of the coils. Substitute the value of ϕ_s from Eq.14.11 in Eq.14.10, we get,

$$\varepsilon_s = -N_s \frac{\Delta}{\Delta t}\left(\frac{MI_P}{N_s}\right)$$

$$\varepsilon_s = -M\frac{\Delta I_P}{\Delta t} \quad ......(14.12)$$

This is the mathematical expression of induced e.m.f. due to mutual induction of the two coils. The magnitude and unit of mutual inductance can be calculated by using Eq.14.12 as;

$$M = \frac{\varepsilon_s}{\frac{\Delta I_P}{\Delta t}} \quad ......(14.13)$$

The unit of mutual inductance is Henry denoted by H. That is, **if the current in a coil P is changing at the rate of one ampere per second and it induces an e.m.f. of one volt in the coil S, then the mutual inductance is said to be one henry**, i.e.

$$H = \frac{V}{As^{-1}}$$

Swiping Induction Card acts as a key opening and closing the door.

Example 14.3

When the current in the primary coil is changing at a rate of $6As^{-1}$, it is observed that an e.m.f. of 14mV is induced in the nearby secondary coil. What is the mutual inductance of the combination?

Solution:

Rate of change of current in primary coil $= \frac{\Delta I}{\Delta t} = 6As^{-1}$

Induced e.m.f. in the secondary coil $= \varepsilon = 14mV = 1.4 \times 10^{-2}V$

Mutual inductance = M = ?

According to the relation of mutual induction

$$\varepsilon = M\frac{\Delta I}{\Delta t}$$

$$M = \frac{\varepsilon}{\left(\frac{\Delta I}{\Delta t}\right)}$$

$$M = \frac{1.4\times10^{-2}V}{6As^{-1}} = 2.3 \times 10^{-3}H$$

$$M = 2.3mH$$

14.6 SELF INDUCTION

It was first suggested by Henry that changing current in a coil induces an e.m.f. in itself. This is known as self-induction which is explained as:

Consider a coil is connected in series with a battery, a switch S and a rheostat as shown in Fig.14.8. When the switch S is closed, current flows through the coil and a magnetic field is set up in it. Now if the current is changed by adjusting the resistance of the rheostat, the magnetic flux linked with the coil also changes. This changing magnetic flux causes an induced e.m.f. in the same coil.

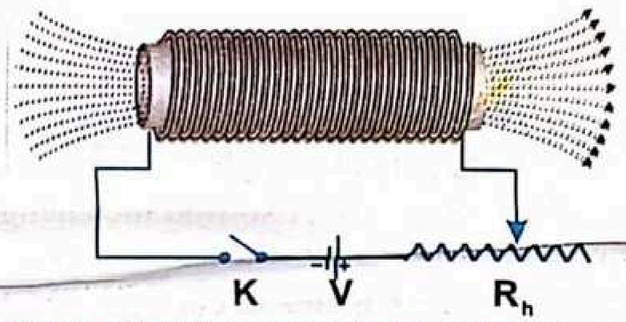


Fig.14.8 Changing current in the coil causes an induced e.m.f. in the same coil.

Such induced e.m.f. is termed as self-induced e.m.f. whose magnitude can be calculated by using Faraday's law of induction.

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t} \quad ......(14.14)$$

where 'N' is the number of turns of the coil, ϕ be the flux through a single loop of the coil. So, the total flux through the coil is $N\phi$ and the experiment shows that the total flux is directly proportional to the changing current. i.e.,

$$N\phi \propto I$$

$$N\phi = LI \quad ......(14.15)$$

where 'L' is constant of proportionality called self-inductance. The value of self-inductance depends upon number of turns of the coil, *size or shape of the coil* and the type of material on which coil is wound.

Substituting the value of magnetic flux ϕ from Eq.14.15 in Eq.14.14 we have,

$$\varepsilon = -N\frac{\Delta}{\Delta t}\left(\frac{LI}{N}\right)$$

$$\varepsilon = -L\frac{\Delta I}{\Delta t} \quad ......(14.16)$$

This is a mathematical expression of a self-induction. The magnitude and unit of self-inductance can be determined by using Eq.14.16 in the form;

$$L = \frac{\varepsilon}{\left(\frac{\Delta I}{\Delta t}\right)} \quad ......(14.17)$$

The SI unit of self-inductance is henry 'H'. It is defined as; **if the current in a coil is changing at the rate of one ampere per second and an e.m.f. of one volt is induced in the same coil then self-inductance 'L' is said to be one henry,** i.e.

$$H = \frac{V}{As^{-1}}$$

Example 14.4

Calculate change in current in an inductor of 6H in which the e.m.f. of 220V is induced in 0.09s. Also find the change in flux in the same inductor.

Solution:

Change in current $= \Delta I = ?$

Self-inductance $= L = 6H$

Induced e.m.f. $= \varepsilon = 220V$

Time taken $= \Delta t = 0.09s$

Change in magnetic flux $= \Delta\phi = ?$

According to the equation of self-inductance

$$\varepsilon = L\frac{\Delta I}{\Delta t}$$

$$\Delta I = \frac{\varepsilon \Delta t}{L}$$

$$\Delta I = \frac{(220V)(0.09s)}{6H}$$

$$\Delta I = 3.3A$$

Also

$$\varepsilon = \frac{\Delta\phi}{\Delta t}$$

$$\Delta\phi = \varepsilon \Delta t$$

$$\Delta\phi = 220V \times 0.09s$$

$$\Delta\phi = 19.8Wb$$

> ***POINT TO PONDER***
>
> Which law of Physics tells us that if a current carrying conductor produces a force on the magnet, then a magnet must produce a force on a current carrying the conductor?

14.7 ENERGY STORED IN AN INDUCTOR :

Like a capacitor which stores energy in its electric field, an inductor also stores energy in its magnetic field. It is explained as:

Consider an inductor of inductance 'L' which is connected with a battery through a switch as shown in Fig.14.9. When switch is closed then there is a growth of current from zero to its maximum value I_o in time t. This increasing of current causes of changing magnetic flux (magnetic field) linked the coil. According to Faraday's law, "this changing magnetic flux produces an induced e.m.f.". Similarly, according to Lenz's law, "the induced e.m.f. opposes the growth of current, therefore, work is needed to be done against the induced e.m.f. in the inductor during the growing current until it attains its maximum value. This work done is stored in the magnetic field of the inductor in terms of magnetic potential energy and it is calculated as under;

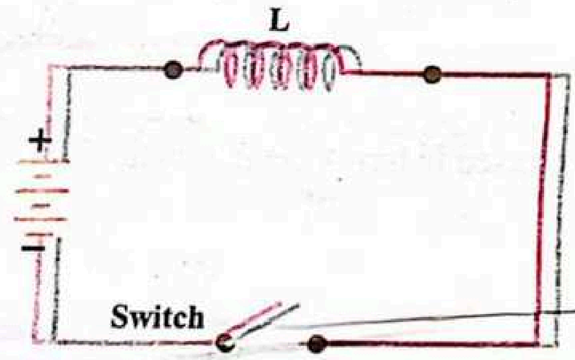


Fig.14.9 Magnetic potential energy stores in the magnetic field of an inductor.

By the definition of work on the charges to flow in the given circuit,

$$W = F \cdot d$$

But $$F = (\Delta q)E = (\Delta q)\left(\frac{V}{d}\right) \qquad \therefore E = \frac{V}{d}$$

Hence $$W = \Delta q \frac{V}{d} \cdot d = \Delta q V$$

But according to self-inductance, the magnitude of induced e.m.f. ε or V is given as:

$$V = L\frac{\Delta I}{\Delta t}$$

So $$W = \Delta q \, L\frac{\Delta I}{\Delta t}$$

$$W = \frac{\Delta q}{\Delta t} L \Delta I = LI\Delta I$$

where $I = \frac{0 + I_0}{2}$

and ΔI = change in current $= I_0 - 0$

Thus $$W = L\frac{I_0}{2} I_0$$

$$W = \frac{1}{2} L I_0^2 \quad \text{......(14.18)}$$

> *FOR YOUR INFORMATION*
>
> Shake flash light needs no battery. Shake the light for 30 second and generates upto 5 minutes of light. Electromagnetic induction occurs as a built in magnetic slides to and fro between coils that charge capacitor. When the brightness diminishes, shake again. You provide the energy to charge the capacitor.

This work is stored in the form of magnetic potential energy in the magnetic field of the inductor. Thus, according to work – energy principle, work done equals the change in energy

$$\Delta U = \text{Work}$$

$$\Delta U = \frac{1}{2} L I_0^2 \quad ......(14.19)$$

As the energy is stored in the magnetic field of the inductor, Eq.14.19 can be expressed in terms the magnetic field B.

By the definition of magnetic field inside the solenoid,

$$B = \mu_0 n I_0 = \frac{\mu_0 N I_0}{\ell} \quad ......(14.20) \qquad \therefore n = \frac{N}{\ell}$$

$$I_0 = \frac{B\ell}{\mu_0 N} \quad ......(14.21)$$

Now changing magnetic flux through an inductor of cross section area 'A' having 'N' number of turns;

$$\Delta\phi = B \cdot A$$

$$\Delta\phi = \left(\frac{\mu_0 N I}{\ell}\right) A \quad ......(14.22)$$

POINT TO PONDER

What would be the value of stored energy in an inductor if the applied current is doubled?

According to Faraday's law of induction

$$\varepsilon = -L\frac{\Delta I}{\Delta t} = -N\frac{\Delta\phi}{\Delta t}$$

$$L(I_0 - 0) = N\Delta\phi$$

$$L = \frac{N\left(\frac{\mu_0 N I_0}{\ell}\right) A}{I_0}$$

$$L = \frac{\mu_0 N^2 A}{\ell} \quad ......(14.23)$$

Thus, substituting the values of I and L from Eq.14.21 and 14.23 in Eq.14.19. We get,

DO YOU KNOW

- A capacitor stores energy in its electric field.
- An inductor stores energy in its magnetic field.

$$\Delta U = \frac{1}{2}\frac{\mu_0 N^2 A}{\ell} \cdot \frac{B^2 \ell^2}{\mu_0^2 N^2}$$

$$\Delta U = \frac{1}{2}\frac{B^2}{\mu_0}(\ell A) \quad ......(14.24)$$

$$\Delta U = \frac{1}{2}\frac{B^2}{\mu_0}(\text{Volume})$$

Now magnetic potential energy (U_m) per unit volume of the inductor is given by

$$U_m = \frac{\Delta U}{Volume} = \frac{\frac{1}{2}\frac{B^2}{\mu_0}(Volume)}{Volume}$$

$$U_m = \frac{1}{2}\frac{B^2}{\mu_0} \quad ......(14.25)$$

This is the energy density or magnetic potential energy per unit volume in the magnetic field of an inductor.

Example 14.5

A steady current of 4A in an inductor of 1000 turns causes a flux of 0.001Wb to link the loops of the inductor. Calculate (a) the e.m.f. induced in the inductor if the current is stopped in 0.06s (b) the inductance of the inductor and (c) the energy stored in the coil.

Solution:

Changing current $= \Delta I = 4 - 0 = 4A$

Number of turns $= N = 1000$

Changing flux $= \Delta\phi = 0.001Wb$

Time taken $= \Delta t = 0.06s$

(a) Back e.m.f. $= \varepsilon = ?$

(b) Inductance $= L = ?$

(c) Energy stored $= U_m = ?$

> **DO YOU KNOW**
> A steady D.C current cannot induce an induced e.m.f. because magnetic flux due to such current does not change.

According to Faraday's Law:

(a)
$$\varepsilon = N\frac{\Delta\varphi}{\Delta t}$$

$$\varepsilon = 1000\frac{(0.001Wb)}{0.06s}$$

$$\varepsilon = 16.7V$$

(b) Using the relation of self-inductance:

$$\varepsilon = L\frac{\Delta I}{\Delta t}$$

$$L = \frac{\varepsilon\Delta t}{\Delta I}$$

$$L = \frac{16.7V(0.06s)}{4A}$$

$$L = 0.25H$$

(c)

$$U_m = \frac{1}{2}LI^2$$

$$U_m = \frac{1}{2}(0.25H)(4A)^2$$

$$U_m = 2J$$

14.8 A.C. GENERATOR

An A.C. generator is a device that converts mechanical energy into electrical energy. The conversion of energy is based on the principle of induced motional e.m.f. The essential parts of an A.C. generator are as under:

Armature

It is a rectangular coil abcd of length 'ℓ' and width x having 'N' number of turns of insulated copper wire wounded on a soft iron core. It is capable of rotating about an axis perpendicular to the magnetic field, as shown in Fig.14.10(a).

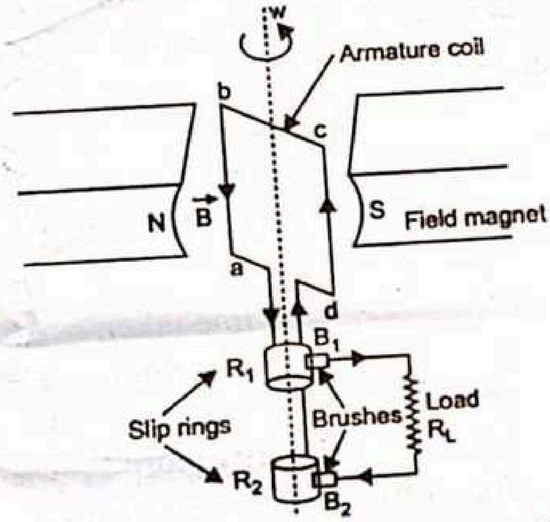


Fig.14.10(a) A schematic diagram of an A.C.

Magnetic Field

A strong uniform magnetic field exists between north and south poles of the magnets such that the armature rotates in it.

Slip Rings

The ends of the armature are connected with two slip rings R_1 and R_2. These rings rotate with the rotation of the armature.

Brushes

Two stationery carbon brushes B_1 and B_2 in contact with the slip rings are responsible to provide the induced current from the armmature to the external circuit.

Working

When the armature is rotated at constant angular speed 'ω' in the applied magnetic field between north and south poles, the magnetic flux that linked with the armature changes. This changing magnetic flux induces an e.m.f. in the armature i.e.,

$$\varepsilon = vB\ell$$

where '$\vec{v}$' is perpendicular to '$\vec{B}$'. In order to determine a general expression for such induced e.m.f., we consider the clockwise rotation of the armature. The side ab and cd are moving at velocity 'v' making angle 'θ' with magnetic field 'B', such that, the vertical component of velocity ($v\sin\theta$) is perpendicular to 'B' as shown in Fig.14.10(b). The charges in these two sides experience force along the sides of the wire, causing induced e.m.f., while the e.m.f. due to the side ad and bc is zero, because the force experience by charges in these two sides is perpendicular, not along the sides of the wires. Thus, the value of resultant e.m.f. induced in the armature having 'N' number of turns is given by

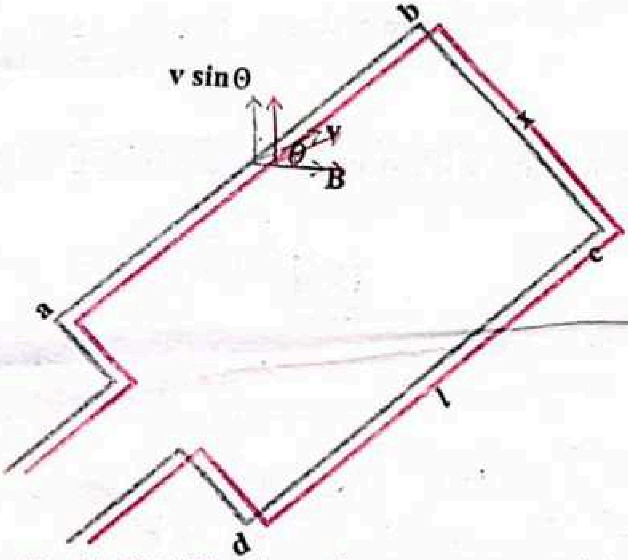


Fig.14.10(b) The plane of armature at an angle 'θ' with the magnetic field B.

$$\varepsilon = \varepsilon_{ab} + \varepsilon_{bc} + \varepsilon_{cd} + \varepsilon_{da}$$

$$\varepsilon = NvB\ell\sin\theta + 0 + NvB\ell\sin\theta + 0$$

$$\varepsilon = 2NvB\ell\sin\theta \quad ......(14.26)$$

If the armature rotates with constant angular velocity 'ω' then according to the relationship between linear velocity and angular velocities is given as

$$v = r\omega$$

$$v = \frac{x}{2}\omega \quad \text{and} \quad \theta = \omega t$$

where 'x' is the width of the armature. Thus Eq.14.26 becomes

$$\varepsilon = 2N\frac{x}{2}\omega B\ell\sin\omega t$$

As $x\ell = A$ (area of the armature)

$$\varepsilon = NAB\omega\sin\omega t \quad ......(14.27)$$

If angle 'θ' between A and B is 90^0 then ε is maximum (ε_m) and Eq.14.27 becomes

$$\varepsilon_m = NAB\omega\sin 90^\circ$$

$$\varepsilon_m = NAB\omega$$

Eq.(14.27) becomes

$$\varepsilon = \varepsilon_m \sin\omega t \quad ......(14.28)$$

Since $\omega = \frac{2\pi}{T} = 2\pi f$

Hence Eq.14.28 becomes

$$\varepsilon = \varepsilon_m \sin 2\pi f t \quad ......(14.29)$$

DO YOU KNOW

An electric generator is an electric motor with its input and output reversed.

This result shows that the e.m.f. induced in the armature varies periodically with time. If a graph between e.m.f. and time is drawn then we have a sinusoidal wave of frequency 'f'. The wave form of one cycle of A.C. due to different positions of the armature with time is shown in Fig.14.11.

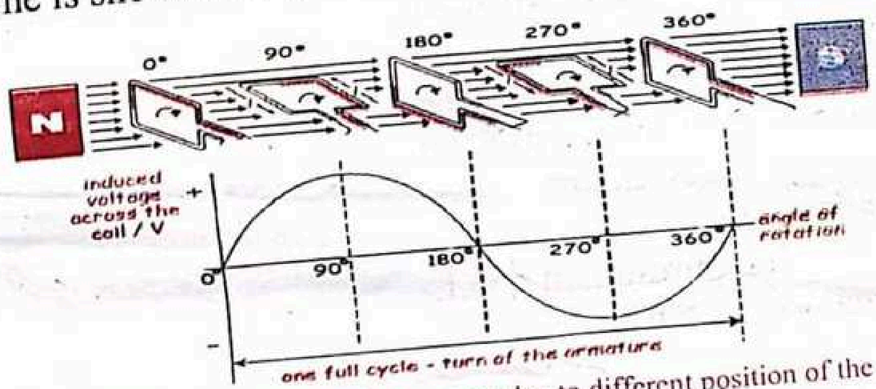


Fig.14.11 A wave cycle of alternating current due to different position of the armature.

Example 14.6

The armature of a generator has 150turns and its area is $200cm^2$. If it rotates with frequency 60Hz in 0.15T magnetic field, calculate the maximum e.m.f. induced in the coil.

Solution:

Number of turns of the coil = N = 150

Area of the coil = A = $200cm^2 = 0.02m^2$

Frequency of the coil = f = 60Hz

Magnetic field = B = 0.15T

By definition of angular frequency:

$$\omega = 2\pi f$$
$$\omega = 2(3.14)(60Hz)$$
$$\omega = 377 rad\, s^{-1}$$

Now peak value of induced e.m.f. is given by

$$\varepsilon_m = NAB\omega$$
$$= (150)(0.02m^2)(0.15T)(377rads^{-1})$$
$$\varepsilon_m = 170 volt$$

14.9 A.C. MOTOR

An A.C. motor is an electrical machine that converts electrical energy into mechanical energy. It works on the basis of principle of mutual induction, i.e., the applied alternating current in the stator winding induces current in the rotor. The magnetic field due to this induced current in the motor interacts with the stator field. This interaction between the two fields produce rotation in rotor. An A.C. motor may be either single phase or three phase. A single-phase A.C. motor converts a small amount of electric energy into mechanical energy, while, the three phase motor converts a bulk amount of electrical energy into mechanical energy.

An induction motor consists of two main parts as shown in Fig.14.12. One is the outer side called stator having coils supplied with alternating current source to produce a rotating magnetic field. While the other is rotor which rotates inside stator. The rotor is attached to the output shaft producing a second rotating magnetic field. The stator and the rotor are designed in such a way that there is a small gap between them known as air gap. Now we explain various parts of an A.C. motor.

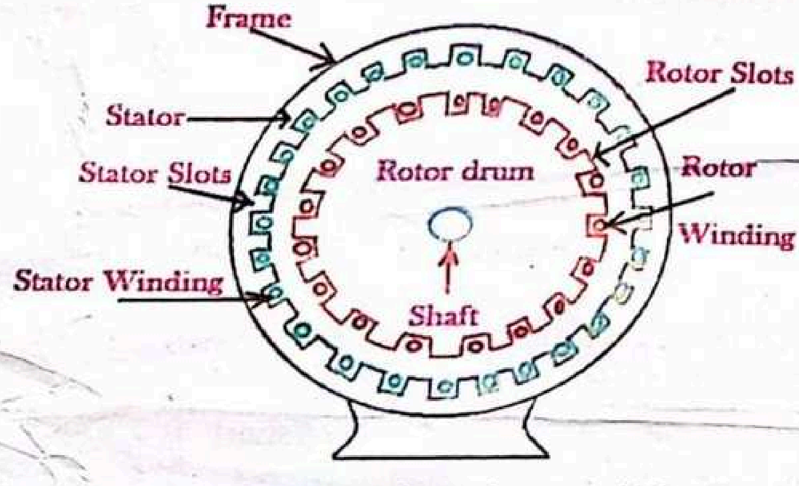


Fig.14.12 A schematic diagram of an A.C. motor with its different parts.

Stator

The word 'stator' has been derived from static, means stationary. The stator is made up of a number of stampings which are slotted to receive the winding. It is wound for a definite number of poles as shown in Fig.14.13. The number of poles are inversely related to the speed of the motor, i.e. greater the number of poles, lesser the speed and vice versa. An A.C. motor does not have any brushes, but the stator is connected directly to the A.C. source where the current alternates through the poles and it causes a rotating magnetic field.

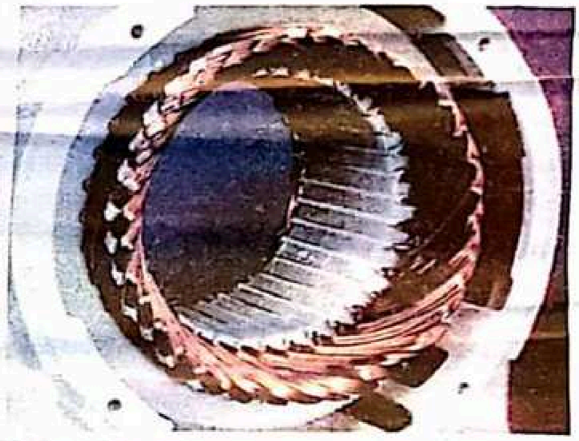


Fig.14.13 A stator winding of an A.C. motor

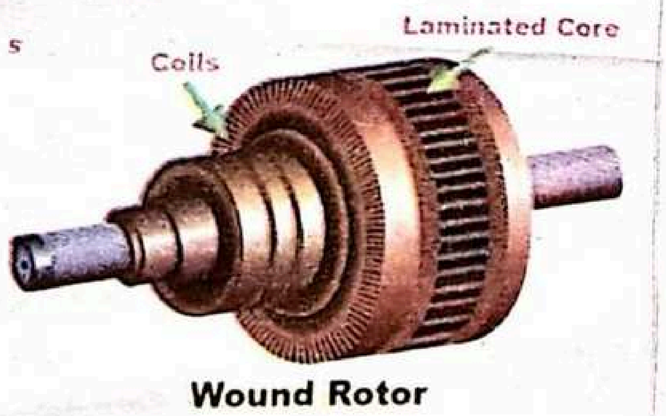


Fig.14.14 A rotor of an A.C. motor.

Rotor

The word rotor has been derived from rotation means rotational motion. A rotor is the central component of the motor. It is inserted into the stator and fixed to the shaft. The rotor consists of a cylindrical terminated core with parallel slots for carrying the rotor conductor. The rotor conductors are not simple wires, but these are heavy bars of copper or aluminum such that one bar is placed in each slot as shown in Fig.14.14. This configuration is called a squirrel-cage rotor, such rotor has the simplest and most rugged construction imaginable.

Bearings and Fan

Bearings are mounted on the shaft which supports the rotor, minimizes the friction and increases the efficiency of the motor as shown in Fig.14.15(a). A fan is also mounted on the shaft of the rotor for cooling the motor when the shaft is rotating as shown in Fig.14.15(b).

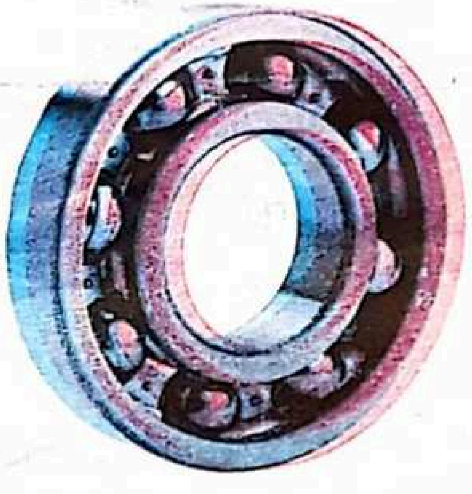


Fig.14.15(a) A bearing of an A.C. Motor

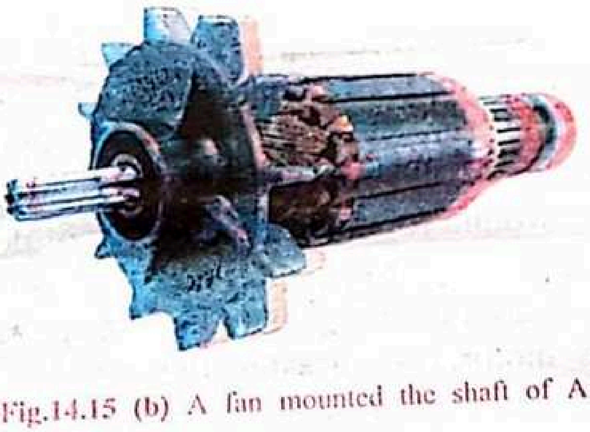


Fig.14.15 (b) A fan mounted the shaft of A.C.

Working of Motor

An A.C. motor works using the principle of electromagnetic induction. When A.C. power from a source is applied to the stator of the motor, it produces a rotational magnetic field. According to Faraday's law, this magnetic field induces a current in the rotor of motor without any physical connection between the stator and the rotor. The frequency of the induced current is same as that of applied current. The induced current in the rotor produces another magnetic field in the rotor that reacts against the stator field. Now these two magnetic fields interact with each other, their interaction produces torque and hence the rotor starts to rotate in the direction of the stator rotational magnetic field. The speed of the motor depends upon the oscillation of the rotating magnetic field of the stator.

Let us study the production of rotating field for the one applied A.C. cycle by considering the working of two phase A.C. motor. Such two phase motor consists of two pole stator having two identical windings S_1 and S_2 separated by 90^o phase difference between them as shown in Fig.14.16(a). The flux due to the applied alternating current flowing in each phase winding is sinusoidal. Let ϕ_1 and ϕ_2 be magnetic fluxes setup by two winding S_1 and S_2 respectively and let ϕ be the resultant flux of ϕ_1 and ϕ_2. i.e., $\phi = \phi_1 + \phi_2$.

Now we observe the condition of two flux ϕ_1 and ϕ_2 at different angles for one applied cycle of alternating current.

I If $\theta = 0$ then $\phi_1 = 0$ and ϕ_2 is maximum.

II If $\theta = \frac{\pi}{2}$ then ϕ_1 is maximum and $\phi_2 = 0$.

III If $\theta = \pi$ then $\phi_1 = 0$ and ϕ_2 is maximum.

IV If $\theta = \frac{3\pi}{2}$ then ϕ_1 is maximum and $\phi_2 = 0$.

V If $\theta = 2\pi$ then $\phi_1 = 0$ and ϕ_2 is maximum.

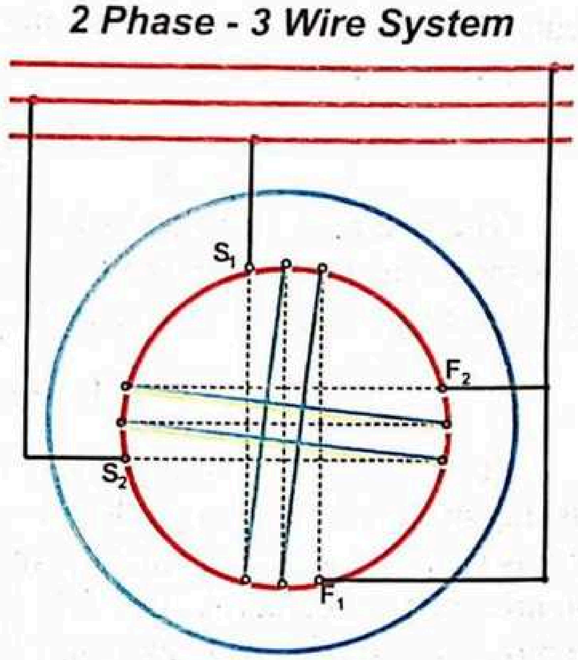


Fig.14.16(a) Circuit diagram of two phase A.C. motor contains two stator windings S_1 and S_2 at 90° apart.

Graphically, the values of flux ϕ_1 and ϕ_2 in S_1 and S_2 at different angles of the applied A.C. voltage are shown in Fig.14.16(b). The two resultant sinusoidal waves of ϕ_1 and ϕ_2 show that when the flux in one winding is minimum then at the same time the flux in other winding is maximum and vice versa but the magnitude of the resultant flux remains constant.

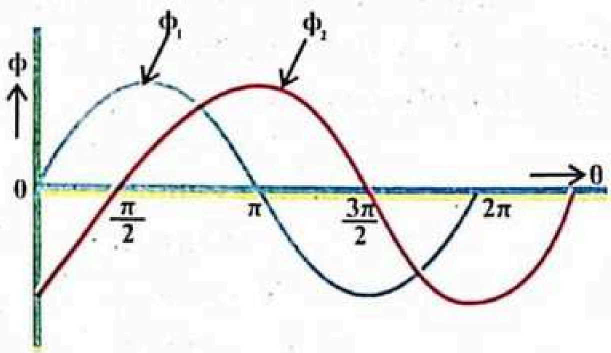


Fig.14.16(b) Wave shape of the two rotating induced magnetic fluxes.

14.10 BACK E.M.F. IN A MOTOR

When the potential difference V from a source is applied across an A.C. motor, the rotor starts rotation in the stator magnetic field and there is changing magnetic flux through the rotor. This changing magnetic flux causes of an induced e.m.f. According to Lenz's law, the induced e.m.f. opposes the applied potential difference. Due to this reason, the induced e.m.f. is called back e.m.f. of the motor. As the applied voltage (V) and induced e.m.f. (ε) act in opposite direction, the resultant e.m.f. in the circuit is $V-\varepsilon$. Similarly, if R be the resistance of the rotor then the current drawn by the motor is given by

$$I=\frac{V-\varepsilon}{R} \quad ......(14.30)$$

The magnitude of the back e.m.f. depends on the speed of the rotor. i.e., the faster the rotor, the greater the back e.m.f.. In the beginning, when the motor is just started, the back e.m.f. is zero and the current is maximum. i.e.,

$$I=\frac{V}{R} \quad ......(14.31)$$

It is our daily observations that when we start a motor of refrigerator or washing machine, the room lights near the load are affected and become dim. Because there is no ε in the starting and the motor draws a large current. At the same time the potential difference across the bulb is reduced, causing a momentary brownout. As the motor speeds up, the back e.m.f. is induced and as a result the potential difference across the motion reduces, the current drawn in it is much smaller, so the brownout ends. Now if the motor is overloaded, it turns slow. According to Eq.14.30, the back e.m.f. reduces and there is a large current drawn in the motor. If this large current maintains for a long time, the motor may burn out.

Example 14.7

An electric motor is connected across the potential difference of 220V. If the current of 50A flows through the resistance of 2Ω of the motor, then calculate the back e.m.f. produced in the motor.

Solution:

Applied voltage = V = 220V

Current = I = 50A

Resistance = R = 2Ω

Back e.m.f. = ε = ?

Using the relation

$$I=\frac{V-\varepsilon}{R}$$

$$\varepsilon = V - IR$$

$$\varepsilon = 220V-(50A)(2\Omega)$$

$$\varepsilon = 220-100$$

$$\varepsilon = 120V$$

14.11 EDDY CURRENT

As we have observed that when an armature or any other conductor rotates in a magnetic field, it changes the magnetic flux through it. According to law of electromagnetic induction, the changing magnetic flux induces an e.m.f. in the armature. Hence this induced e.m.f. sets up a current which circulate throughout the volume of the armature. Such current is known as eddy current. It is explained by an example:

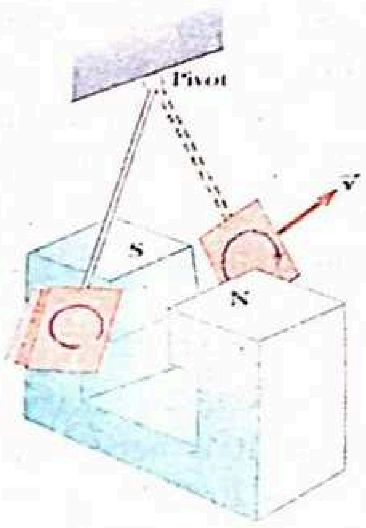
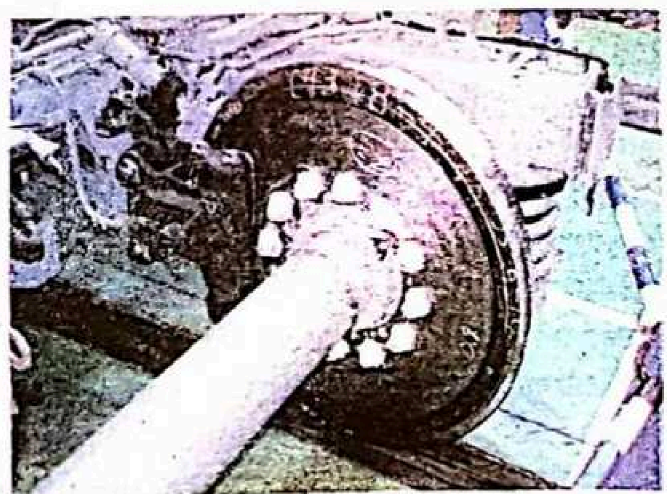


Fig.14.17 Formation of eddy currents in a conducting plate moving through a magnetic field. As the plate enters or leaves the field, the changing magnetic flux induces in e.m.f., which causes eddy currents in the plate.

Consider a copper plate attached at the end of a rigid rod to swing back and forth through a magnetic field as shown in Fig.14.17. As the plate enters the field, it cuts the magnetic flux which causes the changing magnetic flux. This changing magnetic flux induces an e.m.f. in the plate, as a result, a swirling eddy current is induced in the plate. Now according to Lenz's law, the direction of such eddy current is opposite to the direction of motion of the pivoted plate. It is therefore, the eddy current creates an opposite magnetic pole on the plate which is repelled by the poles of the magnetic field. So, there is a repulsion force that opposes the motion of the plate. In this case law of conservation of energy does not hold. On the other hand, there is power loss due to the flow of this eddy current in the body.

In order to reduce the power loss and the consequent heating, conducting parts of the rotating body are often laminated. That is, they are built-up in thin layers separated by a non-conducting material such as lacquer or a metal oxide. This layered structure prevents large current loops and effectively confines the currents to small loops in individual layers. Such a laminated structure is used in transformer cores and motors to minimize eddy currents and increase the efficiency of these devices.

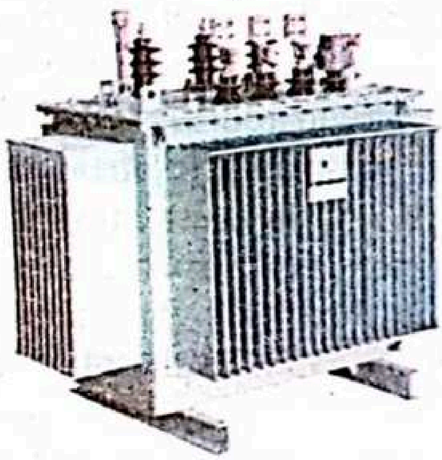


Braking system of eddy current

Conversely, eddy current in some cases are very useful, for example, the braking systems in many trains and cars make use of electromagnetic induction and eddy current. An electromagnet attached to the train is positioned near the steel rails. The braking action occurs when a large current is passed through the electromagnet. The relative motion of the magnet and rails induces eddy currents in the rails, and the direction of these currents produces a drag force at the moment acting on train. These eddy currents decrease steadily in magnitude as the train slows down, the braking effect is quite smooth. Eddy current heating is used in the unduction furnance

14.12 TRANSFORMER

For a long-distance electrical power transmission, it is necessary to use a high voltage and a low current to minimize the power (I^2R) loss in the transmission lines. Consequently, 132kV lines are used in our country (Pakistan). But in practice, the voltage should be decreased to approximately 66kV at distributing station, then to 11kV for delivery to residential areas and finally to 220V at the customer's household wires. The necessary voltage conversion is done by a device named as transformer which is defined as:



External structure of a transformer

A transformer is an electrical device that is used to change the applied alternating voltage into high or low alternating voltages. The working principle of transformer is based on mutual induction between two coils. A simple transformer consists of two coils of copper wire wound around a common soft iron core as shown in Fig.14.18.

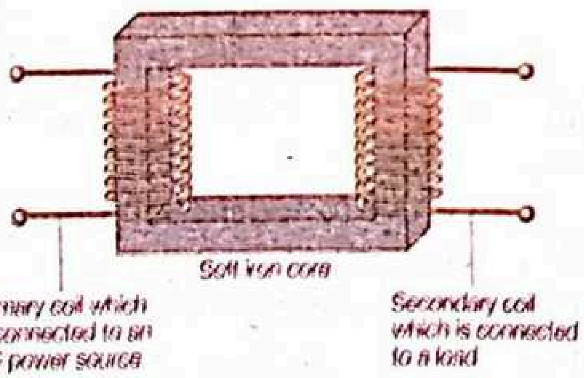


Fig.14.18. A schematic diagram for a simple transformer which consists of two coils primary and secondary

These two coils are electrically separated but are magnetically linked. The coil which is connected to the input A.C. source is called the primary coil, and has N_p number of turns. The coil which is connected to the load resistor and has N_s number of turns is called the secondary coil. Here, the function of the common iron core is to enhance the magnetic flux and to provide a medium such that all the flux through one coil passes through the other.

When the input A.C. voltage V_P is applied to the primary coil, then according to Faraday's law of electromagnetic induction, the magnitude of e.m.f. induced in the primary coil is given by

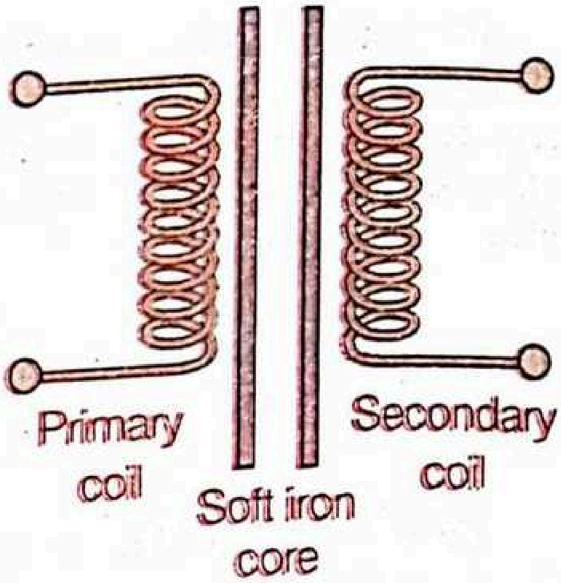
$$V_P = -N_P \frac{\Delta\phi}{\Delta t} \quad ...(14.32)$$

If there is no leakage of flux from the iron core, then the flux through each turn of the primary coil is equal to the flux through each turn of the secondary coil. Thus, voltage V_s induced in the secondary coil is given as;

$$V_S = -N_S \frac{\Delta\phi}{\Delta t} \quad ...(14.33)$$

Dividing Eq.14.33 by Eq.14.32 we get

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \quad ......(14.34)$$



A symbol of a transformer

POINT TO PONDER

If the primary coil of the transformer is connected to a D.C. source, is there an e.m.f. induced in the secondary coil?

Now let us consider an ideal transformer which has no losses. i.e., both coils are pure inductive and have no resistance. Similarly, there is no magnetic flux leakage in the core of an ideal transformer and hence it has no power losses. Thus, its input power equals to its output power. i.e.,

$$\text{Power in} = \text{Power out}$$

$$V_P I_P = V_S I_S$$

$$\frac{V_P}{V_S} = \frac{I_S}{I_P} \quad ......(14.35)$$

This relation shows that the voltage across primary or secondary coil is inversely proportional to current through the coil. This principle is being followed for long distance electrical power transmission. i.e., when high voltage from a step-up transformer is used for transmission, the current through the transmission line is decreased. Thus, the power loss I^2R due to the resistance of the transmission line is minimized.

FOR YOUR INFORMATION

In power transmission line, there are very high voltage 132kV or more and very small current are being used in order to decrease the power (I^2R) losses in the transmission line conductor

Power losses of a transformer

The power losses of an ideal transformer is always zero. The actual transformer has some power losses. The power losses are due to the following factors.

Eddy Current (Iron Loss)

When an A.C. voltage is applied, the magnetic flux is generated in the coil. This flux also passes through the core and it induces a current in the iron core known as eddy current. This eddy current causes power dissipation and heating the core of the transformer. The eddy current can be minimized by using core having thin insulated laminations.

Hysteresis loss

The power loss that occurs during magnetization and demagnetization of the iron core in each cycle of the A.C. is known as hysteresis loss.

Resistance of the Coil

The wire that is used for winding of the coils has some finite resistance. The resistance of the coils causes power loss (I^2R) which is quite significant for large values of current. This loss can be minimized by using thick wire for winding of the coil.

Flux Leakage (magnetic loss)

The observations show that the rate of change of magnetic flux linked with the secondary coil is always less than that of primary coil. It means there is some flux leakage. It can be reduced by winding the two coils one over one another.

All these result shows that output power of a transformer is always less than its input power. Thus, the percentage efficiency (η) of a transformer can be calculated as:

$$\eta = \frac{\text{Output}}{\text{Input}} \times 100\% \quad ......(14.36)$$

Types of Transformers

There are two types of transformers:

1. **Step Up Transformer:** If the number of turns of secondary coil 'N_S' is greater than the number of turns of primary coil 'N_P' and output voltage 'V_S' is more than input voltage 'V_P' then the transformer is said to step up transformer as shown in Fig.14.19(a).

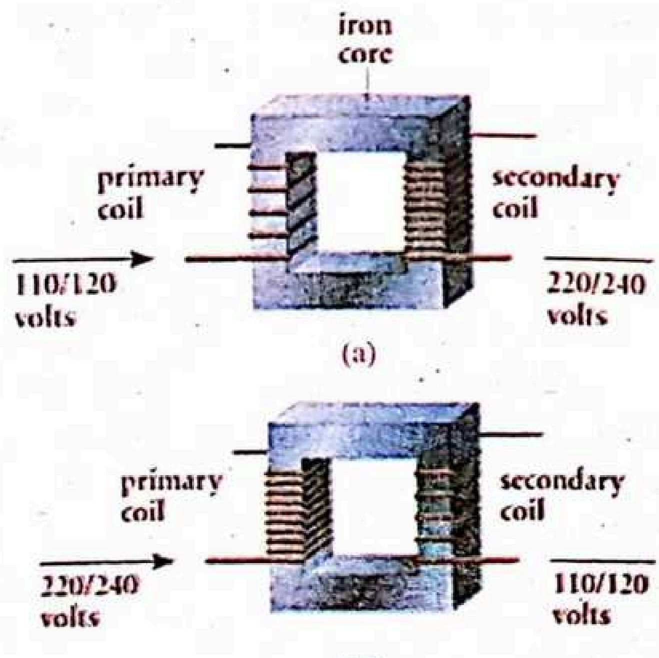


Fig.14.19(a) Step up transformer, where $N_S > N_P$ and $V_S > V_P$ (b) step down transformer, where $N_P > N_S$ and $V_P > V_S$.

2. **Step Down Transformer:** If the number of turns of secondary coil 'N_S' is less than the number of turns of the primary coil 'N_P' and the output voltage 'V_S' is smaller than input voltage 'V_P' then the transformer is said to be step down transformer as shown in Fig.14.19(b).

Example 14.8

A transformer used on a 220V line delivers 1.5A at 1800V. What current is drawn from the line (assume 100% efficiency).

Solution:

Input voltage = V_P = 220V

Output voltage = V_S = 1800V

Input current = I_P = ?

Output current = I_S = 1.5A

As the efficiency of the transformer is 100% so its input power equals to its output power

Input power = output power

$$V_P I_P = V_S I_S$$

$$I_P = \frac{I_S V_S}{V_P}$$

$$I_P = \frac{(1.5)(1800)}{220} = 12.3A$$

Example 14.9

A step-up transformer is used on a 220V line to furnish 3600V. The primary coil has 200turns. How many turns are on the secondary coil?

Solution:

Input voltage = V_P = 220V

Output voltage = V_S = 3600V

Number of turns of primary = N_P = 200

Number of turns of secondary = N_S = ?

As
$$\frac{N_P}{N_S} = \frac{V_P}{V_S}$$

$$N_S = \frac{V_S}{V_P} N_P$$

$$N_S = \frac{3600V}{220V} \times 200 = 3273 \text{turns}$$

SUMMARY

- **Induced e.m.f.:** The current that induces in a conductor due to changing magnetic flux linked with the conductor is called induced current and its corresponding e.m.f. is called induced e.m.f..
- **Faraday's Law:** This law states that the induced e.m.f. in a coil, having N-number of turns, is directly proportional to the rate of change of magnetic flux linked with the coil. Mathematically it is expressed as

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

- **Lenz's Law:** This law states that the direction of induced current is always opposite to that action which causes of induced e.m.f..
- **Motional e.m.f.:** The e.m.f. that induces due to the motion of conductor across a magnetic field is called motional e.m.f..

$$\text{Motional e.m.f. } (\varepsilon) = vB\ell \sin\theta$$

- **Mutual induction:** The phenomenon in which the rate of change of current in one coil induces an e.m.f. in another nearby coil is called mutual induction.
- **Self-Induction:** The phenomenon in which a changing current in a coil induces an e.m.f. in the same coil is called self-induction.
- **Henry:** Henry is the unit of inductance and it is defined as: if the changing of current at the rate of one ampere per second induces an e.m.f. of one volt then this inductance is called one henry.
- **A.C. generator:** An A.C. generator is a device which converts mechanical energy into electrical energy.
- **A.C. motor:** An A.C. motor is a device which converts electrical energy into mechanical energy.
- **Back e.m.f.:** The e.m.f. that induces due to the rotation of motor and it opposes the applied e.m.f. is called back e.m.f..
- **Transformer:** A transformer is an electrical device which steps-up or steps-down the input voltage at the output.

EXERCISE

○ **Multiple choice questions.**

1. The source of induced e.m.f. is
   (a) Cell (b) Battery
   (c) Interaction between stationery magnet and coil
   (d) Relative motion between magnet and coil
2. When the coil and a bar magnet are placed very closed to each other then the value of their induced e.m.f. will be

(a) Maximum (b) Positive (c) Negative (d) Zero

3. The unit of induced e.m.f. is
(a) Ampere (b) Volt (c) Watt (d) Weber

4. The law of electromagnetic induction was introduced by
(a) Maxwell (b) Faraday (c) Lenz (d) Fleming

5. According to Faraday's Law of electromagnetic induction, the induced e.m.f. depends upon
(a) Minimum magnetic flux (b) Maximum magnetic flux
(c) Change in magnetic flux (d) Rate of change of magnetic flux

6. Lenz's law of electromagnetic induction explains:
(a) The production of induced e.m.f. (b) Magnitude of induced e.m.f.
(c) Direction of induced e.m.f. (d) Both magnitude and direction of induced e.m.

7. Lenz's law is related to the law of conservation of
(a) Momentum (b) Energy (c) Mass (d) Charges

8. The mutual inductance of two coil is maximum when the coils are
(a) Parallel to each other (b) Perpendicular to each other
(c) Facing each other (d) Touching each other

9. When a current change in a coil from 4A to 6A in 0.05s and it induces an e.m.f. of 8V. The co-efficient of self-inductance is
(a) 0.1H (b) 0.2H (c) 0.4H (d) 0.8H

10. The unit of inductance is
(a) Weber (b) Tesla (c) Henry (d) Watt

11. A conducting rod of length 0.5m moves in a magnetic field of magnitude 2T with velocity 5 m s^{-1}, the e.m.f. induced in the moving rod is
(a) 5V (b) 10V (c) 20V (d) 50V

12. Which one of the following device stores magnetic potential energy in its magnetic field?
(a) Resistor (b) Capacitor (c) Inductor (d) Thermistor

13. At constant inductance of 'L' henry, the energy stored in a magnetic field is
(a) $\frac{1}{2}CQ^2$ (b) $\frac{1}{2}mv^2$ (c) $\frac{1}{2}kx^2$ (d) $\frac{1}{2}LI^2$

14. The working principle of an A.C. motor is
(a) Self-induction (b) Mutual induction
(c) Motional e.m.f. (d) None of those

15. The stator and the rotor of an A.C. motor are connected by a
(a) Copper Wire (b) Silver wire
(c) Aluminum wire (d) No physical connection between them

16. Two phase A.C. contains
(a) One wire two stators (b) Two wires two stators

(c) Three wires two stators (d) Three wires three stators

17. A transformer works when we apply
(a) A.C. voltage (b) D.C voltage
(c) A.C. as well as D.C (d) Neither A.C. nor D.C

18. The quantity that remains constant in a transformer is
(a) Voltage (b) Current (c) Power (d) Frequency

19. The soft iron core is used in a transformer in order to
(a) Enhance the magnetic flux (b) Increase the weight
(c) Increase the copper losses (d) Decrease the copper losses

20. Which relation is true for an ideal transformer?
(a) Input power $>$ output power (b) Input power $<$ output power
(c) Input power = output power (d) Output power = 0

21. When a step up transformer delivers high voltage then the current will be
(a) Remain same (b) Decreased (c) Increased (d) Maximum

22. For long distance electrical power transmission, we use
(a) Smaller current and lower voltage (b) Larger current and higher voltage
(c) Larger current and lower voltage (d) Smaller current and higher voltage

23. Lamination is used in a transformer in order to
(a) Reduce flux leakage (b) Minimize hysteresis loss
(c) Decrease coil resistance (d) Minimize eddy current

SHORT QUESTIONS

1. What is the meaning of induced current and induced e.m.f.?
2. What factor causes of induced e.m.f.?
3. Does e.m.f. induce in a coil, when a magnet is placed near to it?
4. The law of electromagnetic induction is expressed as: $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$. What is the meaning of negative sign?
5. What is the difference between Faraday's law and Lenz's law of induction?
6. What is the difference between induced e.m.f. and back e.m.f.?
7. What is the advantage and disadvantage of eddy current?
8. Distinguish between self-inductance and mutual inductance.
9. Define Henry and express its dimension.
10. How magnetic energy is stored in an inductor?
11. Explain the motional e.m.f. due to the motion of rod of length 'ℓ' with velocity 'v' which is perpendicular to the magnetic field B.
12. What is the working principle of an A.C. generator?
13. What is the role of carbon brushes in an A.C. generator?

14. Why does the bulb light become dim for a moment when we start the refrigerator or some other heavy electrical appliance at home?
15. What is the working principle of a transformer?
16. What is the difference between step up and step down transformer?
17. Why does not transformer work on D.C.?
18. What is the meaning of an ideal transformer?
19. Describe the power losses of a transformer?
20. How can you minimize the power loses of a transformer?

COMPREHENSIVE QUESTIONS

1. What is induced e.m.f.? Explain induced e.m.f. with experimental examples.
2. State and explain Faraday law of electromagnetic induction with examples.
3. What do you know about the Lenz's law of electromagnetic induction? How can you determine the direction and conservation of energy by using Lenz's law?
4. Explain motional e.m.f. and derive its mathematical relation.
5. State and explain mutual induction, co-efficient of mutual induction and its unit.
6. Define and explain self-induction.
7. How does magnetic potential energy store in an inductor? Also derive its mathematical relation.
8. What is A.C. generator? Explain its construction and working principle.
9. What is A.C. motor? Explain its construction and working principle.
10. State and explain back e.m.f. in an A.C motor.
11. Define and explain eddy current.
12. What do you know about a transformer? Describe the function, working and kinds of transformer.

NUMERICAL PROBLEMS

1. A coil 200 turns linked by a flux of 40mwb. If this flux is reversed in a time of 4ms then determine the average induced e.m.f. in the coil. **(2000V)**
2. At what rate would it be necessary for a single conductor loop to cut the flux in order that the current of 1.2mA flows through it when 10Ω resistor is connected across its ends? **($1.2 \times 10^{-2}Wbs^{-1}$)**
3. A 60m long conductor is moving in a uniform magnetic field 0.9T at a speed of $7\,ms^{-1}$. If the motion of the conductor is at right angle to the field then calculate the e.m.f. induced in the conductor. **(378V)**
4. When the current in a coil A is increased uniformly from zero to 18A in 0.3s, an e.m.f. of 6V is induced in a nearby coil B. What is the value of the mutual inductance between the coils A and B? **(0.1H)**

5. The current in a coil changes at the rate of 3A in 60ms. An e.m.f. of 10V is induced in the coil. What is the self-inductance of the coil? **(0.2H)**

6. An inductor of inductance 0.5H carries a current of 6A. Calculate the energy stored in the magnetic field of the inductor? **(9J)**

7. The frequency of the coil of an A.C. generator is 60Hz has 150 turns. If its area is 150cm^2 and it induces maximum e.m.f. of 250V during its rotation in a uniform magnetic field, then calculate the magnitude of such magnetic field? **(0.3mT)**

8. An A.C. motor is rotated by applying the potential difference of 220V. If the motor has a resistance of 4Ω and a back e.m.f. of 50V is induced in it during its rotation, then calculate (a) the current when the motor just start up (b) current when the motor is running at normal speed. **(55A, 42.5A)**

9. A step-down ideal transformer operates on a 2.5kV line and supplies a load with 80A. The ratio of the primary winding to the secondary winding is 20:1. Determine (a) the secondary voltage (b) the primary current (c) the output power. **(0.13kV, 4A, 10kW)**

10. A step-down transformer is used on 1kV line to deliver 220V. How many turns are on the primary winding if the secondary has 50 turns? **(227turns)**

Unit 15

ALTERNATING CURRENT

Major Concepts (27 PERIODS)

- Alternating current (A.C.)
- Instantaneous, peak and rms values of A.C.
- Phase, phase lag and phase lead in A.C.
- A.C. through a resistor
- A.C. through a capacitor
- A.C. through an inductor
- Impedance
- RC series circuit
- RL series circuit
- Power in A.C. circuits
- Resonant circuits
- Electrocardiography
- Principle of metal detectors
- Maxwell's equations and electromagnetic waves (descriptive treatment)

Conceptual Linkage

This chapter is built on Electricity Physics X ICT Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

- describe the terms time period, frequency, instantaneous peak value and root mean square value of an alternating current and voltage.
- represent a sinusoidally alternating current or voltage by an equation of the form $x = x_0 \sin$ (t.
- describe the phase of A.C. and how phase lags and leads in A.C. Circuits.
- identify inductors as important components of A.C. circuits termed as chokes (devices which present a high resistance to alternating current).
- explain the flow of A.C. through resistors, capacitors and inductors.
- apply the knowledge to calculate the reactances of capacitors and inductors.
- describe impedance as vector summation of resistances and reactances.
- construct phasor diagrams and carry out calculations on circuits including resistive and reactive components in series.
- solve the problems using the formulae of A.C. Power.

- explain resonance in an A.C. circuit and carry out calculations using the resonant frequency formulae.
- describe that maximum power is transferred when the impedances of source and load match to each other.
- describe the qualitative treatment of Maxwell's equations and production of electromagnetic waves.
- become familiar with electromagnetic spectrum (ranging from radiowaves to (γ rays).
- identify that light is a part of a continuous spectrum of electromagnetic waves all of which travel in vacuum with same speed.
- describe that the information can be transmitted by radiowaves.
- identify that the microwaves of a certain frequency cause heating when absorbed by water and cause burns when absorbed by body tissues.
- describe that ultra violet radiation can be produced by special lamps and that prolonged exposure to the Sun may cause skin cancer from ultra violet radiation.

INTRODUCTION

We have studied in the previous unit that an A.C. generator produces a current or voltage which varies periodically with time and is known as alternating current or alternating voltage. The alternating current has more advantages over the direct current. For example, it can be easily transformed into higher or lower voltage, it can be transmitted over long distances, reliable and can be produced at very low cost. Due to these advantages, the A.C. sources are used to power the circuits of our homes, offices, markets, industries, farms etc. In this unit, we will study the behavior of A.C. circuit, when a resistor, capacitor or an inductor is connected with the A.C. source. We will also study the combined effect of resistor, capacitor and inductor when they are connected either in series or parallel with the A.C. source.

On the other hand, we will discuss the generation, transmission and reception of electromagnetic waves. The electromagnetic waves are composed of fluctuating electric and magnetic fields. They have different forms which are being used for different purposes. For example, electromagnetic waves in the form of visible light enable us to view the world around us, infrared waves warm our environment, radio waves connect all the countries of the world through communication system in terms of video and audio signals, x-rays allow us to perceive not only the structures hidden inside our bodies, but also explore the structure of various elements. Similarly, these electromagnetic waves help us in the observation and study of solar system, stars, galaxy and other heavenly bodies.

15.1 ALTERNATING CURRENT (A.C.)

We have studied that when a rectangular coil is rotated in a uniform magnetic field with a constant angular velocity 'ω', an e.m.f. is induced in the coil. Consequently, there is a flow of current in the output of the coil. As the rotation of the coil is uniform so, its output current varies periodically both in positive and negative directions after an equal interval of time. This current is called alternating current (A.C.). Graphically, the wave shape of A.C. is shown in Fig.15.1(a). Mathematically, this alternating current can be expressed in terms of sinusoidal wave because of the continuous variation in its magnitude and direction with respect to time. That is,

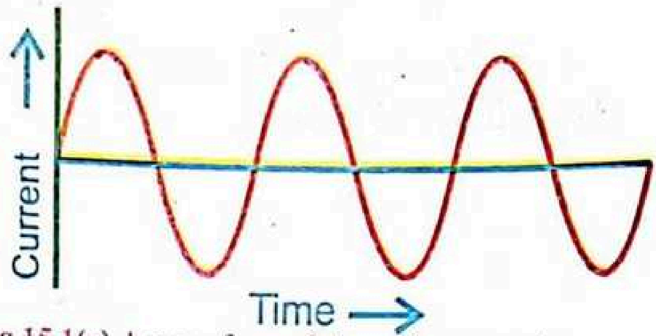


Fig.15.1(a) A wave form of alternating current

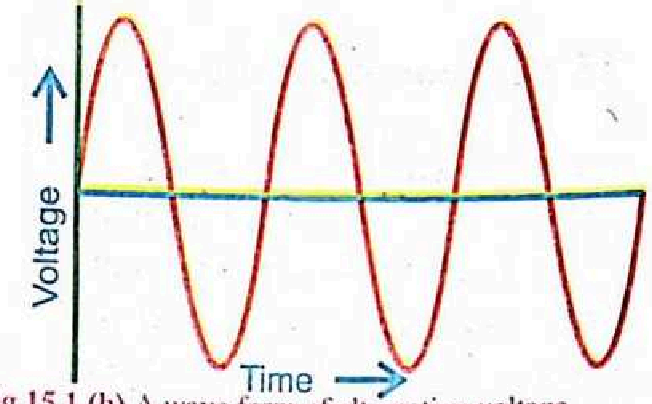


Fig.15.1 (b) A wave form of alternating voltage

> ***POINT TO PONDER***
> Why 220V of A.C. is more dangerous than 220V of D.C.?

$$i = I_o \sin \omega t$$

But $\omega = 2\pi f$

$$i = I_o \sin 2\pi f t \quad ......(15.1)$$

where 'f' is the frequency of the alternating current and it is related with the number of rotations of the coil. Thus, the frequency of A.C. is defined as the number of wave cycles of A.C. in one second. The reciprocal of the frequency is termed as the time period of A.C.. i.e., $T = \frac{1}{f}$. This is a time in which one cycle of A.C. is completed. Similarly, 'i' is the instantaneous value of current of A.C. at any instant of time 't' and 'I_o' is the maximum current of A.C.. 'I_o' is also known as either positive or negative peak value of current. Similarly, the alternating voltage can also be expressed as,

$$v = V_o \sin \omega t \quad ......(15.2)$$

Graphical representation of alternating voltage is shown in Fig.15.1(b).

Example 15.1

An alternating current of frequency 50Hz has peak value of 70A. Calculate the instantaneous value of current after 0.0015s.

Solution:

$$f = 50Hz$$

peak current(I_0) = 70A
t = 0.0015s
instantaneous current(i) = ?
According to the Eq.15.1

> **DO YOU KNOW**
> An oscilloscope is very versatile piece of equipment which is being used to display A.C. wave form, heartbeats

$$i = I_0 \sin 2\pi f t$$
$$i = 70A \sin 2(3.14)(50Hz)(0.0015s)$$
$$i = 70 \sin (0.471)$$
$$i = 70(0.00822)$$
$$i = 0.58A$$

15.1.1 Root mean square value

One cycle of alternating current or voltage consists of half positive cycle and half negative cycle. Therefore, the average value of the current or voltage over one cycle is zero and it cannot be used to specify an alternating current or voltage. To overcome this we use a power, because power is expressed in terms of i^2 and v^2. Where the magnitude of i^2 and v^2 are always positive even for their negative values. Graphically the square value of square of alternating current is shown in Fig.15.2. Where i^2varies from 0 to I_o^2 and its average value is given as;

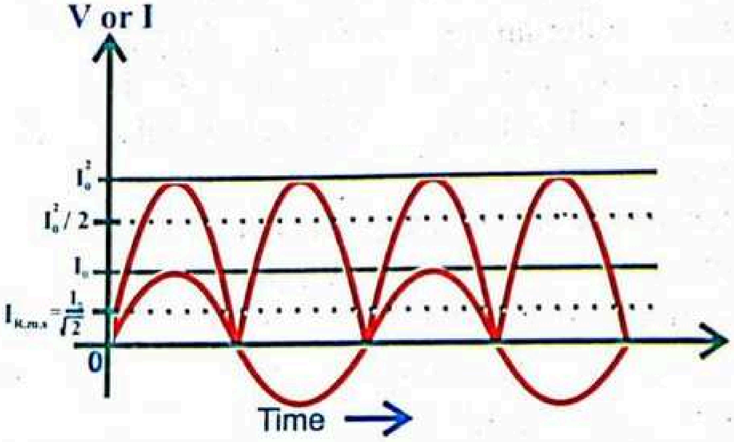


Fig.15.2 Square of the alternating current which varies with time from 0 to I_0^2 and its magnitude remains positive for complete cycle.

$$<i^2> = \frac{0 + I_o^2}{2}$$

$$<i^2> = \frac{1}{2} I_o^2 \quad ......(15.3)$$

> **DO YOU KNOW**
> The average value of an alternating current is zero but its square root of the average square value is not zero.

This result shows that the average value of A.C. is never zero. Similarly, the average of square of alternating voltage is given by;

$$<v^2> = \frac{1}{2} V_o^2 \quad ......(15.4)$$

In case of A.C. source, average power dissipation in a resistor is given by

$$<p> = <i^2> R \quad ......15.5(a)$$

Similarly, power dissipation in terms of voltage is

$$<p> = \frac{<v^2>}{R} \quad \ldots\ldots 15.5(b)$$

Now in case of D.C. source, the power dissipation is given by;

$$P = I^2R \quad \ldots\ldots 15.6(a)$$

and

$$P = \frac{V^2}{R} \quad \ldots\ldots 15.6(b)$$

The value of the direct current that dissipates in the same resistor or produces heat at the same rate as the mean rate of heat produces by the alternating current is known as root mean square value of A.C.. It is represented by $I_{r.m.s}$ and it can be calculated by comparing Eq.15.5(a) with Eq.15.6(a)

$$I^2 = <i^2>$$

Taking square root on both sides

$$I_{r.m.s.} = \sqrt{<i^2>}$$

$$I_{r.m.s.} = \sqrt{\frac{1}{2}I_o^{\,2}} = \frac{I_0}{\sqrt{2}} = 0.707\ I_o \quad \ldots\ldots (15.7)$$

Similarly,

$$V_{r.m.s.} = \sqrt{\frac{1}{2}V_o^{\,2}} = \frac{V_0}{\sqrt{2}} = 0.707V_o \quad \ldots\ldots (15.8)$$

The root mean square value of alternating current and root mean square value of alternating voltage are also known as effective current and effective voltage.

Example 15.2

The instantaneous value of current is represented by the equation $i = 25\sin 100\pi t$. Compute its frequency, maximum and rms values of current.

Solution:

We have

$$i = 25\sin 100\pi t$$

According to the Eq.15.1

$$i = I_o \sin 2\pi f\, t$$

By comparing these two equations, we get:

$$2f = 100\text{Hz}$$

$$f = 50\text{Hz}$$

$$I_o = 25\text{A}$$

$$I_{rms} = 0.707 I_o$$

$$I_{rms} = (0.707)(25)$$

$$I_{rms} = 17.7\text{A}$$

15.1.2 Phase of A.C.

We have studied that there is a continuous periodic variation of A.C. with time. Therefore, its instantaneous value at time 't' is given by:

$$i = I_o \sin \omega t \;......15.9(a)$$

$$i = I_o \sin \theta \;......15.9(b)$$

where 'θ' is the angle which specifies the instantaneous value of alternating current or voltage and it is known as phase angle or simply phase, and it depends upon time t. The instantaneous value of A.C. with respect to phase angle can further be studied graphically as well. At time $t = 0$, i.e., $\omega t = \theta = 0$ then current 'i' is also zero. The value of angle θ at $t = 0$ is known as initial phase of A.C. The instantaneous value of current is also zero when angle θ has value of π, 2π, 3π, 4π,... these are shown in Fig.15.3(a). Similarly, the instantaneous value of current 'i' is maximum positive called peak values when phase angle 'θ' is $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$ ...... and the current 'i' maximum negative when phase angle 'θ' is $\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}$ .... These values of current are also shown in Fig.15.3(a). The same wave shape as for current will be obtained for instantaneous value of voltage with respect to phase angle 'θ'. It is shown in Fig.15.3(b).

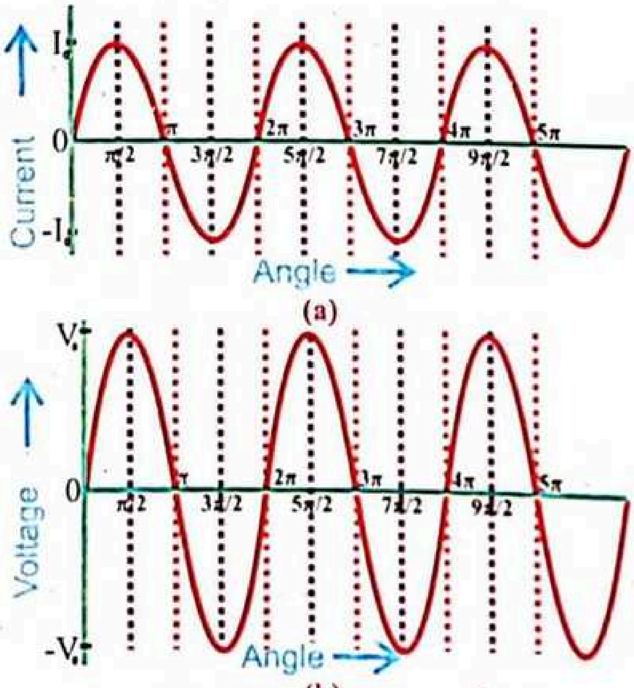


Fig.15.3(a) A graphical representation of instantaneous current with respect to phase angle 'θ' (b) A graphical representation of instantaneous voltage with respect to phase angle θ

15.1.3 Phase lag and phase lead

We know that when a coil (armature) of A.C. generator is rotated in a uniform magnetic field, then we have an induced emf (voltage) as well as a current in form of sinusoidal waves because

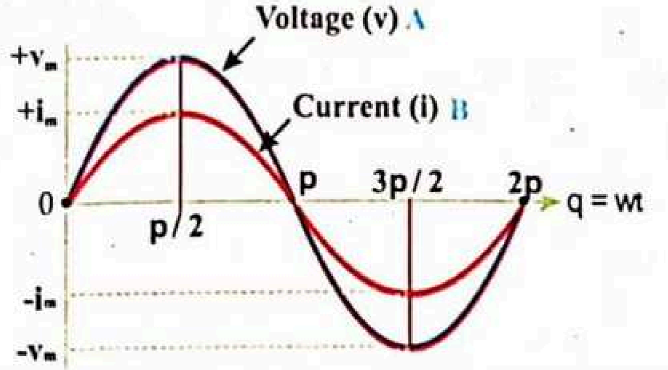


Fig.15.4 Graphical representation of voltage and current which are in phase.

both quantities are varying sinusoidally with the same angular frequency (ω). Now when both quantities reach their minimum (zero) and maximum values at the same time or the values of their initial phase are 0° at time, t = 0 then voltage and current are said to be in phase with each other. Graphically they are shown in Fig. 15.4.

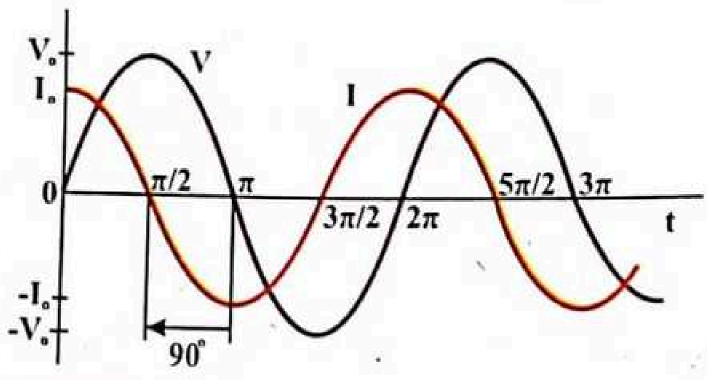


Fig.15.5 Current is leading the voltage by an angle 'e' (e = 90°)

Sometime, both quantities i.e. voltage and current do not reach their minimum and maximum values simultaneously. For example, if at time, t = 0 the value of initial phase of one quantity is 0° whereas, the initial phase of the other quantity greater than 0° then this shows that voltage and current are out of phase. In case of out of phase there are two possibilities.

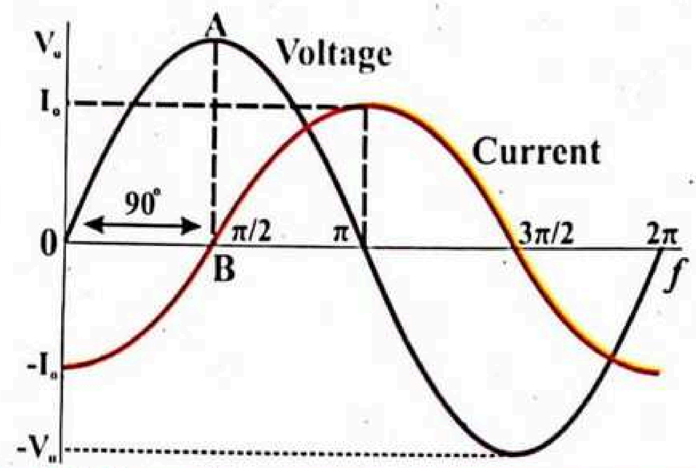


Fig.15.6 Current is lagging behind the voltage by an angle 'e' (e = 90°)

I. Let the initial phase of v is 0° but the initial phase of current i does not 0° and it had a phase of 0° earlier then it is called leading of current. Graphically, the leading of current by voltage by an angle 90° is shown in Fig.15.5.

II. Similarly, if the initial phase of voltage is 0° but current will has its phase of 0° later then it is called lagging of current. Graphically, the lagging of current behind voltage by an angle 90° is shown in Fig.15.6.

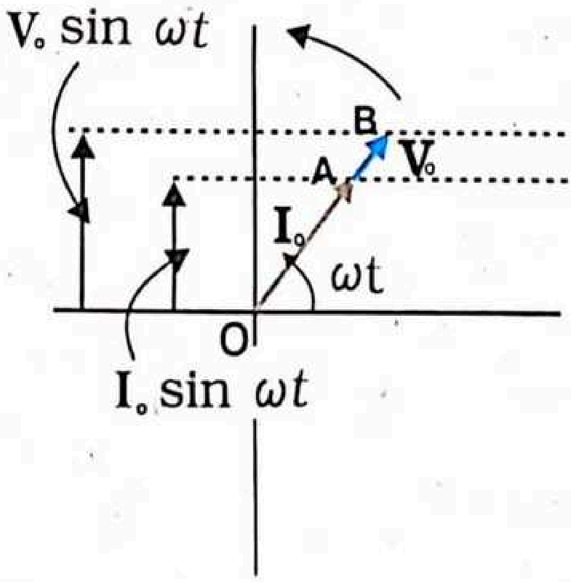


Fig.15.7 Phasor diagram for current and voltage.

The alternating current and voltage that vary sinusoidally can further be explained by a phasor diagram. A phasor is a vector which rotates about the origin with angular frequency ω. The diagram which contains such rotating vector is called phasor diagram. Let alternating current and voltage are represented by two phasors OA and OB as shown in Fig.15.7. The length of these phasors are equal to the peak value of current (I_o) and voltage (V_o) respectively. While, the projection of these phasors onto the vertical axis are equal to instantaneous value of current ($I_o \sin\omega t$) and voltage ($V_o \sin\omega t$) respectively. Current and voltage in terms of phasor diagram can be studied under three cases.

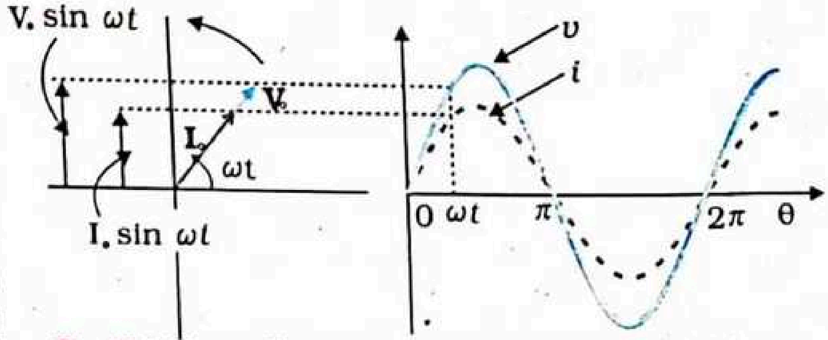


Fig.15.8 Phasor diagram for current and voltage which are in phase.

a. If voltage and current are in phase then both have same initial phase ($\theta = \omega t$) as shown in Fig.15.8. Mathematically, they are expressed as:

$$i = I_o \sin\omega t \qquad ......(15.10)(a)$$

$$v = V_o \sin\omega t \qquad ......(15.10)(b)$$

If $\omega t = \theta = 0$ then $i = 0$ and $v = 0$.

Similarly, if $\omega t = \theta = \frac{\pi}{2}$ then $i = I_o$ and $v = V_o$.

These results show that v and i are in phase.

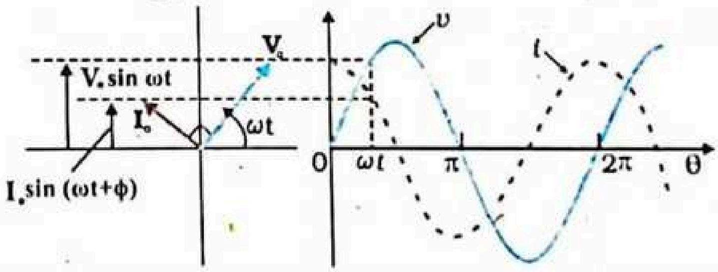


Fig.15.9 Phasor diagram for voltage and current where current is leading by voltage by an angle 'e' (e = 90°)

b. If the initial phase of voltage is 0° at time, $t = 0$ but initial phase of current is positive at the same time then mathematically they are expressed as,

$$v = V_o \sin\omega t \qquad ...15.11(a)$$

$$i = I_o \sin(\omega t + \phi) ...15.11(b)$$

At time t = 0, the value of voltage 'v' is zero but current 'i' has some positive value equal to '$I_o \sin\phi$'. This shows that 'v' and 'i' differ by phase angle ϕ. Because i had its zero value earlier by an angle 'ϕ' of 90° then 'v' as shown in Fig.15.9. Thus the current is leading the voltage by an angle ϕ of 90°.

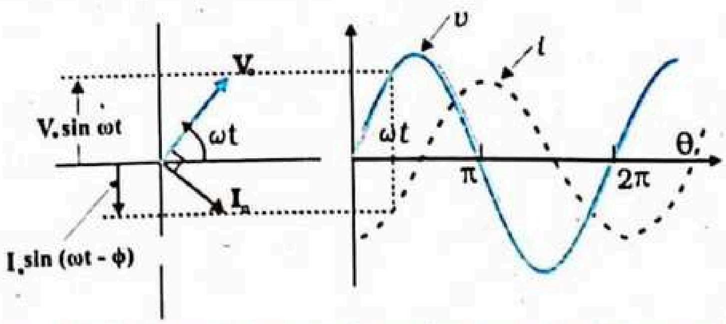


Fig.15.10 Phasor diagram for voltage and current where current is lagging behind voltage by an angle 'e' (e = 90°).

c. If initial phase of voltage is zero at time, $t = 0$ but initial phase of current is negative at the same time then mathematically they are expressed as;

$$v = V_o \sin \omega t \quad ...15.12(a)$$

$$i = I_o \sin(\omega t - \phi) \quad ...15.12(b)$$

at time $t = 0$, the value of 'v' is zero but current i has some negative value equals to '$-I_o \sin\phi$'. This shows that 'v' & 'i' are not in phase, where current 'i' is lagging behind the voltage by an angle ϕ of 90° as show in Fig.15.10.

15.2 A.C. CIRCUIT

An A.C.-circuit is an electrical network which is powered by an A.C. source and the elements such as resistors, capacitors and inductors are connected in series or parallel across it. We will study the behaviour of alternating current and voltage in each element.

15.3 A.C. THROUGH A RESISTOR

When a resistor of resistance 'R' is connected to an A.C. source as shown in Fig.15.11 then there is voltage drop across it. The instantaneous value of voltage is given by

$$v = V_o \sin \omega t \quad ......(15.13)$$

According to Ohm's law

$$v = iR$$

$$i = \frac{v}{R}$$

$$i = \frac{V_o}{R} \sin \omega t$$

$$i = I_o \sin \omega t \quad ......(15.14)$$

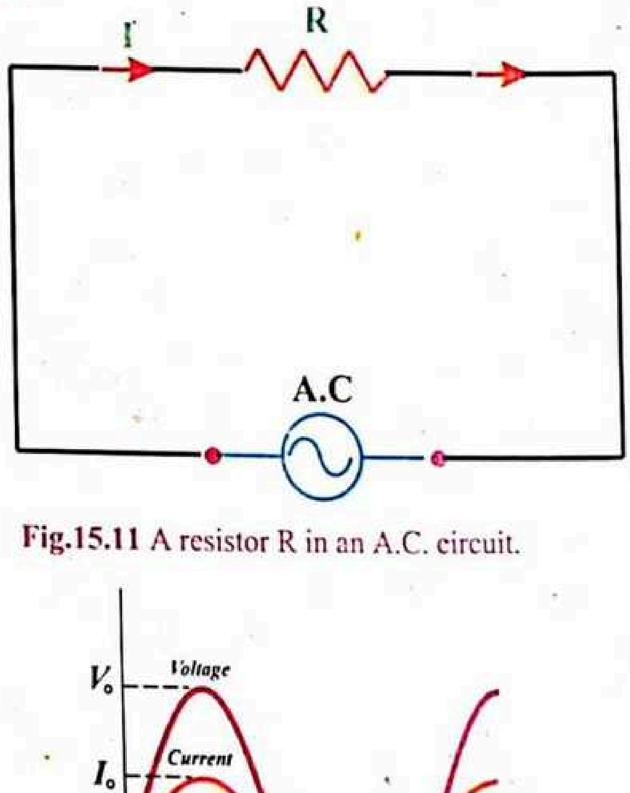


Fig.15.11 A resistor R in an A.C. circuit.

Eq.15.13 and Eq.15.14 show that the voltage and current are in phase with each other as shown in Fig.15.12(a). i.e., If $\theta = 0$ then $I = 0$ and $v = 0$.

Similarly, if $\theta = \frac{\pi}{2}$ then $i = I_o$ and $v = V_o$

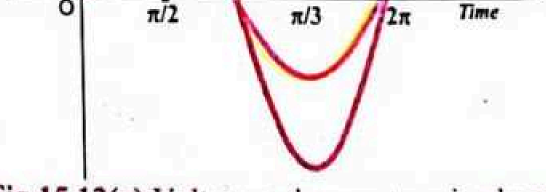


Fig.15.12(a) Voltage and current are in phase in a resistor.

These results show that when a resistor is connected with A.C. source then v and i are in phase and it is shown in phasor diagram 15.12(b).

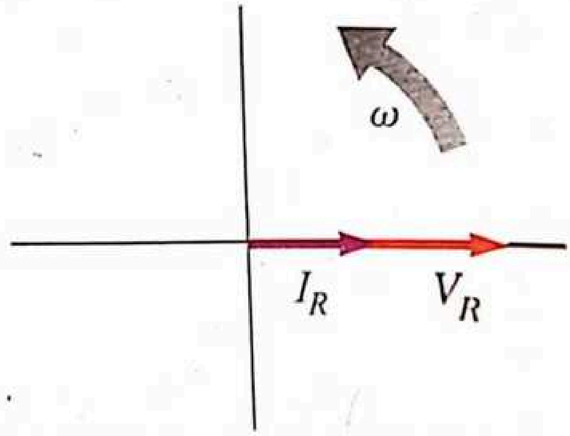


Fig.15.12(b) Phasor diagram which shows v and i are in phase.

Power dissipation

The instantaneous value of power in an A.C. circuit which contains a resistor only is given by

$$p = vi$$

$$p = V_0 I_0 \sin^2 \omega t \quad ......(15.15)$$

In case of alternating current or alternating voltage, we use average dissipation by taking limits from 0° to $\frac{\pi}{2}$

If $\omega t = \theta = 0$

Then

$$p = V_0 I_0 \sin^2 0$$

$$p = 0$$

> **FOR YOUR INFORMATION**
> In an A.C. circuit, power is always measured in terms of rms values of current and voltage.

Similarly, if $\omega t = \theta = \frac{\pi}{2}$

$$p = V_0 I_0 \sin^2 \frac{\pi}{2}$$

$$p = V_0 I_0$$

Thus, the average power is given by

$$\langle p \rangle = \frac{0 + V_0 I_0}{2}$$

$$\langle p \rangle = \frac{V_0 I_0}{2}$$

$$\langle p \rangle = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}}$$

$$\langle p \rangle = V_{r.m.s} I_{r.m.s} \quad ......(15.16)$$

This shows that the average power dissipation in a pure resistive circuit is equal to the product of r.m.s. value of voltage and current.

Example 15.2

A voltage $V = 60\sin\omega t$ is applied across a resistor of 20Ω in A.C. circuit. Calculate the root mean square value of current in the circuit?

Solution:

$$V_o = 60\text{volt}$$

$$I_{r.m.s.} = ?$$

As

$$V_{r.m.s.} = \frac{V_0}{\sqrt{2}} = \frac{60}{\sqrt{2}}$$

$$V_{r.m.s.} = 42.4V$$

$$V_{r.m.s.} = I_{r.m.s.}R$$

$$I_{r.m.s.} = \frac{V_{r.m.s}}{R}$$

$$I_{r.m.s.} = \frac{42.4V}{20\Omega}$$

$$I_{r.m.s.} = 2.12A$$

15.4 A.C. THROUGH A CAPACITOR

Let a capacitor of capacitance 'C' is connected to A.C. source as shown in Fig.15.13. There is alternating voltage drop across the capacitor and the instantaneous value of this voltage is given by

$$v = V_o \sin\omega t \quad \ldots\ldots(15.17)$$

The amount of charge stored in the capacitor due to instantaneous voltage is given by

$$q = Cv$$

$$q = CV_0 \sin\omega t$$

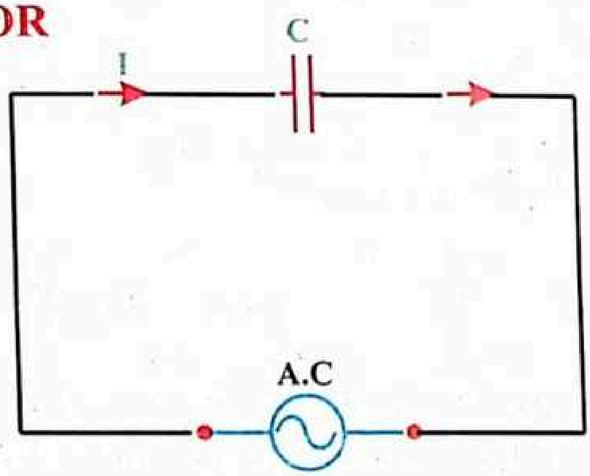


Fig.15.13 A capacitor C in an A.C. circuit.

We know that current is defined as the rate of flow of charges, therefore instantaneous value of current through capacitor is given by

$$i = \frac{\Delta q}{\Delta t}$$

$$i = \frac{\Delta}{\Delta t}(CV_0 \sin\omega t)$$

$$i = CV_0 \frac{\Delta}{\Delta t}\sin\omega t$$

The mathematical solution of $\frac{\Delta}{\Delta t}(\sin \omega t)$ is $\omega \cos \omega t$. So,

$$i = \omega C V_0 \cos \omega t$$

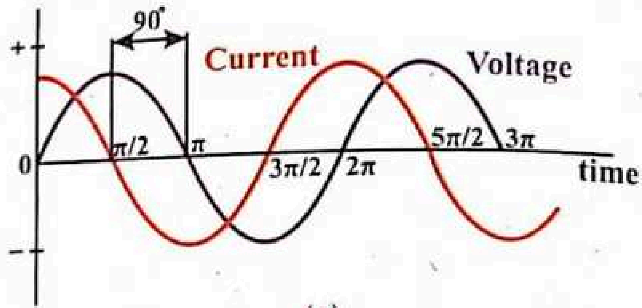
But $\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

And $\omega C V_0 = X_C V_0 = I_0$

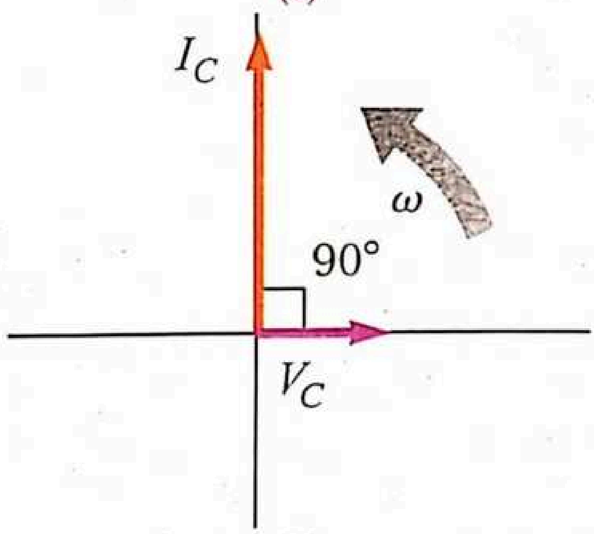
where X_C is the resistance (more precisely capacitive reactance) of capacitor. Thus,

$$i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \ldots (15.18)$$

Eq.15.17 and Eq.15.18 give us that v and i are out of phase in A.C. circuit contains a capacitor as shown in Fig.15.14(a), that is, at $t = 0$, then $v = 0$ and $i = I_0$. Similarly, at $t = T/4$; $v = V_0$ and $i = 0$, and at $t = T$ $v = 0$ again and $i = I_0$. This shows that current is leading the voltage by 90°. It is also shown in the phasor diagram 15.14(b).



(a)



(b)

Fig.15.14(a) Current and voltage are out phase in a capacitor (b) current leading the voltage by 90°.

Capacitive reactance

The opposition offered by a capacitor to the flow of A.C. is known as capacitive reactance. It is defined in terms of the ratio of root mean square values of voltage V_{rms} to the root mean square values of current $I_{r.m.s.}$. It is represented by X_c and is measured in Ohm.

$$X_c = \frac{V_{rms}}{I_{rms}}$$

$$X_c = \frac{\left(V_0/\sqrt{2}\right)}{\left(I_0/\sqrt{2}\right)}$$

$$X_c = \frac{V_0}{I_0}$$

But $I_0 = \omega C V_0$

> **DO YOU KNOW**
>
> In an A.C. circuit, the capacitive reactance X_C of a capacitor is infinite ohms for D.C. At the opposite extreme, the X_C of a capacitor is zero ohms for A.C. Therefore, a capacitor allows (filters) A.C. through it but blocks D.C.

$$X_c = \frac{V_0}{\omega C V_0} = \frac{1}{\omega C}$$

$$X_c = \frac{1}{2\pi f C} \qquad ......(15.19)$$

This shows that capacitive reactance is inversely proportional of both frequency of current and capacitance of the capacitor. In case of A.C., frequency f is large, while capacitive reactance X_C is small. In case of D.C., f is zero so X_C is infinity theoretically. In practice X_C has an extremely large value for D.C. source. It is concluded that a capacitor allows the A.C. and blocks the D.C. in an A.C. electric circuit.

Power dissipation

The instantaneous value of power dissipation in a capacitive circuit is given by

$$p = vi$$

$$p = V_o I_o \sin\omega t \cos\omega t$$

$$p = \frac{V_0 I_0}{2}(2\sin\omega t \cos\omega t)$$

$$= \frac{V_0 I_0}{2}\sin 2\omega t$$

> ***POINT TO PONDER***
> What are the reactances of capacitor and inductor, when A.C. source is applied across them?

If we integrate the above expression with respect to ωt between the limits 0 and 2π the final answer is zero.

Thus, the average power dissipation in a capacitive A.C. circuit over a complete cycle is zero.

Example 15.2

A capacitor of capacitance $100\mu F$ is connected to an alternating potential difference of 12volts and frequency 50Hz. Calculate the reactance of the capacitor and current in the circuit.

Solution:

$$C = 100\mu F = 100 \times 10^{-6} F = 1 \times 10^{-4} F$$

$$V_0 = 12V$$

$$f = 50Hz$$

$$X_C = ?$$

$$I = ?$$

As $$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{2(3.14)(50Hz)(1\times10^{-4}F)}$$

$$X_C = \frac{1}{3.14\times10^{-2}}$$

$$X_C = 31.8\Omega$$

$$V = IX_C$$

$$I = \frac{V}{X_C}$$

$$I = \frac{12V}{31.8\Omega}$$

$$I = 0.377A$$

15.5 A.C. THROUGH AN INDUCTOR

An inductor is usually a coil in the form of a solenoid of inductance 'L'. When it is connected to an A.C. source as shown in Fig.15.15, then there is a flow of alternating current through it and the instantaneous value of current is given by

$$i = I_0 \sin\omega t \text{ ......(15.20)}$$

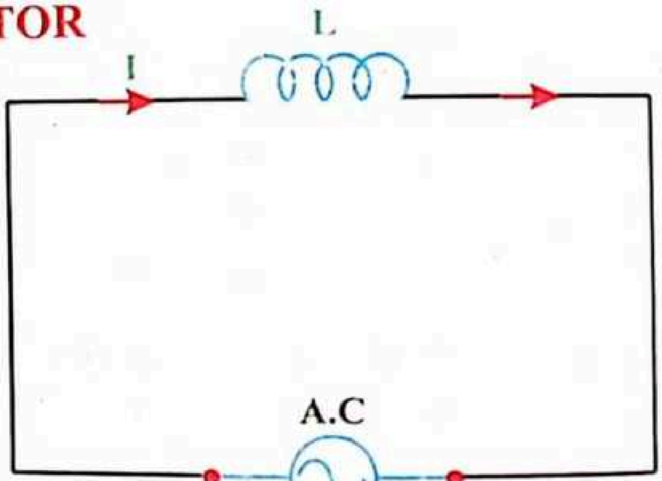


Fig.15.15 An inductor in A.C. circuit.

The growth of this current causes an induced e.m.f. in the inductor whose direction is opposite to the applied e.m.f.. Thus, the magnitude of induced emf in the inductor is given by

$$v = L\frac{\Delta I}{\Delta t}$$

$$v = L\frac{\Delta}{\Delta t}I_0 \sin\omega t$$

$$v = I_0 L\frac{\Delta}{\Delta t}\sin\omega t$$

> ***DO YOU KNOW***
> The inductive reactance X_L of an inductor is infinite ohms for A.C. Conversely, the reactance X_L of an inductor is zero ohms for A.C. Therefore, an inductor allows (filters) D.C. and blocks A.C.

The mathematical solution of $\frac{\Delta}{\Delta t}(\sin\omega t)$ is $\omega\cos\omega t$. So,

$$v = \omega I_0 L\cos\omega t$$

But $\cos\omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

and $\omega I_o L = I_o X_L = V_o$

Where X_L is the resistance of inductor

Thus, $v = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ ...(15.22)

Eq.15.21 and 15.22 show that current and voltage are out of phase in an A.C. circuit contains an inductor as shown in Fig.15.16(a). That is, at time, $t = 0$, $i = 0$ and $V = V_0$ and at time, $t = \frac{T}{4}$, $i = I_0$ and $V = 0$. The voltage is leading the current by 90° in an inductive A.C. circuit. It is shown in phasor diagram 15.16(b).

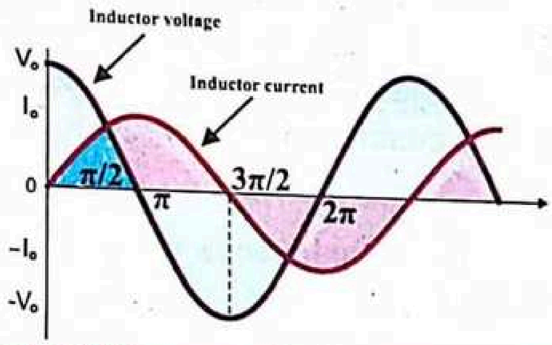


Fig.15.16(a) current and voltage are out of phase in an inductor

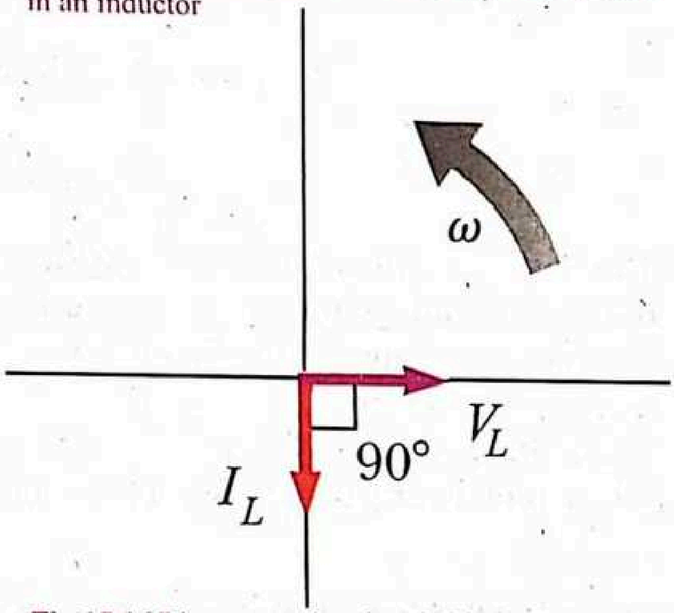


Fig.15.16(b) current lagging behind voltage by 90^0.

Inductive reactance

The opposition offered by an inductor to the flow of A.C. is known as inductive reactance. It is represented by X_L and it is measured in Ohm. The inductive reactance can be calculated by using Ohm's law.

$$X_L = \frac{V_{rms}}{I_{rms}}$$

$$X_L = \frac{\left(V_0/\sqrt{2}\right)}{\left(I_0/\sqrt{2}\right)}$$

$$X_L = \frac{V_o}{I_o}$$

But $$V_o = \omega L I_o$$

$$X_L = \frac{\omega L I_o}{I_o}$$

$$X_L = 2\pi f L \quad ......(15.23)$$

This shows that inductive reactance is directly proportional to both frequency of current and the inductance of the inductor. In case of D.C., $f = 0$, so that $X_L = 0$, while in case of A.C., frequency is large, so X_L is also large.
Thus, we conclude that an inductor allows the D.C. but blocks the A.C..

Power dissipation

The instantaneous value of power dissipation in an inductive A.C. circuit is given by

$$p = vi$$
$$p = V_0 I_0 \sin\omega t \cos\omega t$$
$$p = \frac{V_0 I_0}{2} 2\sin\omega t \cos\omega t$$
$$= \frac{V_0 I_0}{2} \sin 2\omega t$$

The average power over one cycle can be obtained by integrating above expression with respect to ωt between the limits 0 and 2π and final answer is zero.
Thus, the power dissipation in an inductive A.C. circuit over a complete cycle is zero.

> ***POINT TO PONDER***
> What are the reactances of capacitor and inductor, when D.C. source is applied across them?

Example 15.3

What is the potential difference across an inductor of inductance 15H when an alternating current of 15mA, frequency 50Hz flows through it?

Solution:

$X_L = ?$

$L = 15H$

$I_0 = 15mA = 15 \times 10^{-3}A$

$f = 50Hz$

$V = i X_L$

$V = 2\pi f L i$

$V = 2(3.14)(50Hz)(15H)(15\times10^{-3}A)$

$V = 70.65V$

15.6 IMPEDANCE

We know that a resistor offers opposition to the flow of current in a circuit. On the other hand, a capacitor and an inductor in an A.C. circuit also offer some opposition to the flow of A.C. Their opposition is called reactances which are represented by X_C and X_L respectively. Now let an A.C. circuit consists of a resistor, a capacitor and an inductor in series as shown in Fig.15.17(a) then the combined effect of resistance of a resistor and reactances of a capacitor and an inductor is termed as impedance. It is represented by Z and it is measured in Ohm. Impedance can be determined in terms of ratio between voltage to current that is,

$$Z = \frac{v_z}{i_z} \quad ......(15.24)$$

Since all the three components are in series so the current i_z through each component is the same, but the voltage drop across each component is different. The resultant voltage v_z can be calculated with help of phasor diagram as shown in Fig.15.17(b) and Fig.15.17(c).

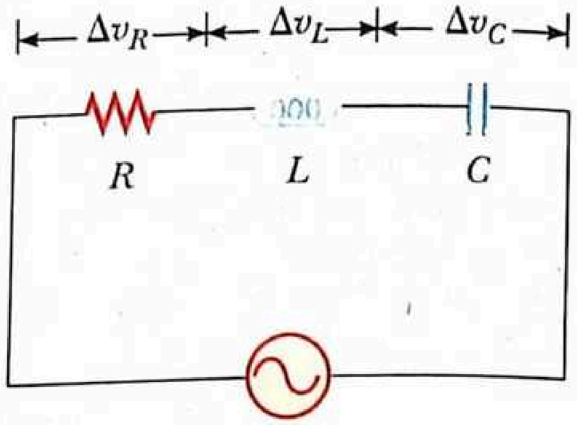
$$v^2 = v_R^2 + (v_L - v_C)^2$$

$$v = \sqrt{i_Z^2 R^2 + i_Z^2 (X_L - X_C)^2}$$

$$v = i_Z \sqrt{R^2 + (X_L + X_C)^2}$$

As $Z = \frac{v_Z}{i_Z}$

$$Z = \frac{i_Z \sqrt{R^2 + (X_L - X_C)^2}}{i_Z}$$



15.17(a) RLC-series A.C. circuit

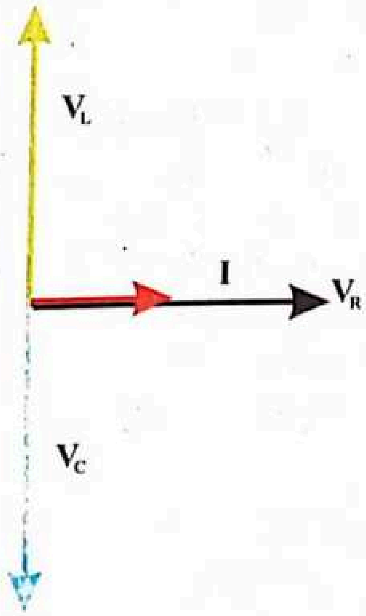


Fig.15.17(b) Phasor Diagram for RCL circuit

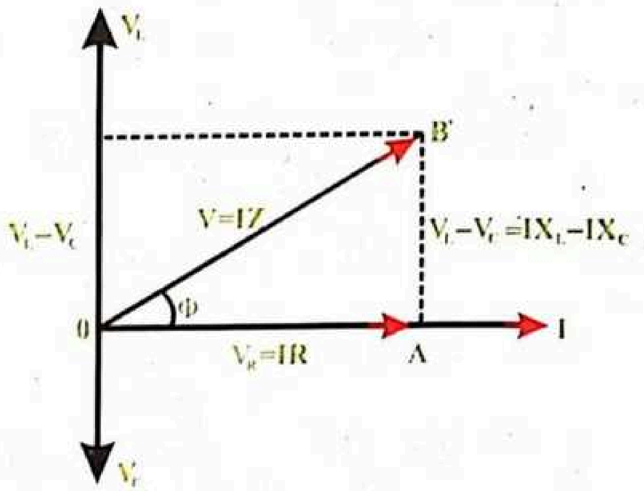


Fig.15.17(c) Resultant voltage in RCL circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad ......(15.25)$$

This is the impedance Z of RLC-series circuit.

15.7 A.C. THROUGH R-C SERIES CIRCUIT

Consider a resistor and a capacitor which are connected in series to an A.C. source as shown in Fig.15.18. As the resistor and the capacitor are in series so the current in the circuit is same at its each point and at any instant. Thus, the instantaneous value of the current is given by

$$i = I_o \sin \omega t \quad ......(15.26)$$

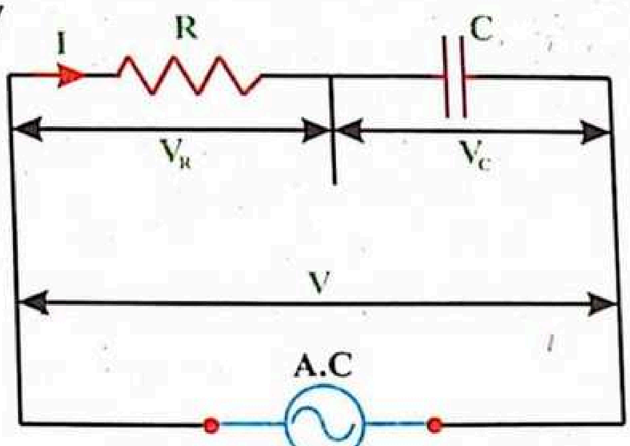


Fig.15.18 A resistor and capacitor in series in an A.C. circuit

Now the voltage can be calculated with the help of phasor diagram. As we know that the voltage drop 'v_R' across 'R' is iR and it is in phase with i. So the phasor for v_R is parallel to i as shown in Fig.15.19. On the other hand, voltage (V_C) across the capacitor is lagging behind the current by 90^0. Thus, the resultant potential difference 'v' across resistor 'R' and capacitor 'C' is equal to the vector sum of v_R and v_C.

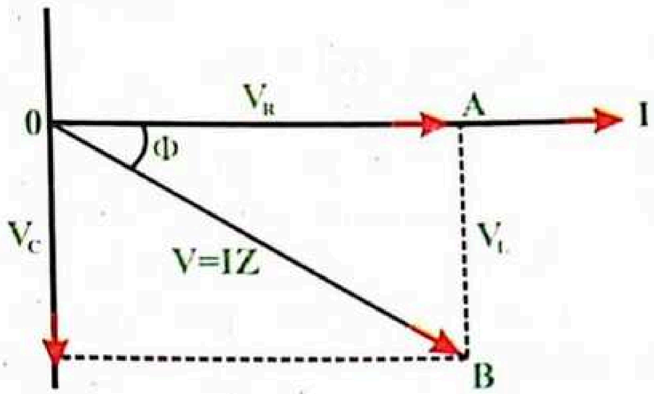


Fig.15.19 A phasor diagram for RC series circuit.

$$v^2 = v_R^2 + v_C^2$$

$$v = \sqrt{i^2R^2 + i^2X_C^2}$$

$$v = i\sqrt{R^2 + \frac{1}{(\omega C)^2}}$$

$$v = i\sqrt{R^2 + \frac{1}{(2\pi f C)^2}} \quad ......(15.27)$$

The Impedance of RC-series circuit is given by

$$Z = \frac{v}{i}$$

$$Z = \sqrt{R^2 + \frac{1}{(2\pi f C)^2}} \quad ......(15.28)$$

Fig.15.19 shows that the resultant voltage 'v' of the circuit lags behind the current i by an angle ϕ called phase difference between v and i and its value is given by

$$\tan\phi = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\phi = \tan^{-1}\left(\frac{v_C}{v_R}\right)$$

$$\phi = \tan^{-1}\left(\frac{iX_C}{iR}\right)$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

$$\phi = \tan^{-1}\left(\frac{1}{2\pi fCR}\right) \quad ...(15.29)$$

> ***FOR YOUR INFORMATION***
> The average power delivered by a generator in a RLC series circuit has maximum value when the inductive reactance equals to the capacitive reactance.

Example 15.4

An alternating current of 2mA with angular frequency 100rad s^{-1} flows through a resistor of $9k\Omega$ and a capacitor of $0.4\mu F$ connected in series. Calculate potential difference across R and C, resultant potential difference and impedance in the circuit.

Solution:

$$i = 2mA = 2 \times 10^{-3}A$$

$$\omega = 100 \text{ rad } s^{-1}$$

$$R = 9K\Omega = 9\times10^{3}\Omega$$

$$C = 0.4\mu F = 0.4\times10^{-6}F = 4\times10^{-7}F$$

$$v_R = ?$$

$$v_C = ?$$

$$v = ?$$

$$Z = ?$$

$$v_R = iR = (2 \times 10^{-3}A)(9 \times 10^{3}\Omega)$$

$$v_R = 18V$$

$$v_C = iX_C = \frac{i}{\omega C}$$

$$v_C = \frac{2\times10^{-3}A}{(100\text{rad } s^{-1})(4\times10^{-7}F)}$$

$$v_C = 50V$$

$$v = \sqrt{v_R^2 + v_C^2} = \sqrt{(18)^2 + (50)^2}$$

$$= \sqrt{324 + 2500}$$

$$v = \sqrt{2824}$$

$$v = 53V$$

$$Z = \frac{v}{i}$$

$$Z = \frac{53V}{2\times10^{-3}A}$$

$$Z = 2.65\times10^4\Omega = 26.5k\Omega$$

15.8 A.C. THROUGH R-L SERIES CIRCUIT

Consider a resistor and an inductor which are connected in series to an A.C. source as shown in Fig.15.20. The current flowing through 'R' and 'C' is same at any instant. Thus, the instantaneous value of current is given by;

$$i = I_0 \sin \omega t$$

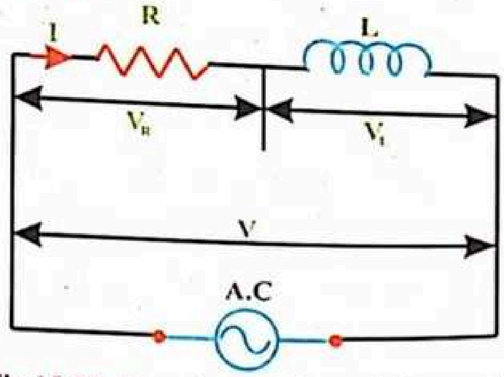


Fig.15.20 A resistor and an inductor in series in an A.C. circuit.

As R and C are in series, so the voltage drop across each component is different and resultant voltage can be calculated with the help of phasor diagram. In a resistor 'v_R' in phase with I. So, the phasor for 'v_R' is parallel to the current i as shown in Fig.15.21. Similarly, in an inductor, 'v_L' leads the current by $\pi/2$ as shown in the phasor diagram. Thus 'v' be the resultant of v_R & v_L and it is equal to their vector sum, that is

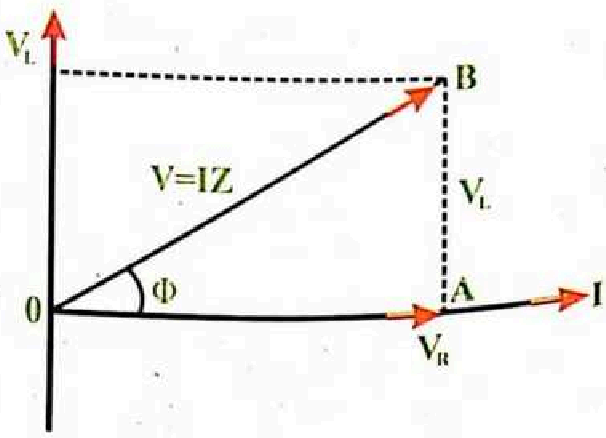


Fig.15.21 A phasor diagram for RL-series circuit.

$$v^2 = v_R^2 + v_L^2$$

$$v = \sqrt{i^2R^2 + i^2L^2}$$

$$v = i\sqrt{R^2 + (\omega L)^2}$$

$$v = i\sqrt{R^2 + (2\pi fL)^2} \quad ...(15.30)$$

Impedance of RL-series circuit is given by

$$Z = \frac{v}{i}$$

$$Z = \sqrt{R^2 + (2\pi fL)^2} \quad ...(15.31)$$

As the resultant voltage 'v' in RL-series circuit leads the current by an angle 'ϕ', so its value is given by

$$\phi = \tan^{-1}\left(\frac{v_L}{v_R}\right)$$

$$\phi = \tan^{-1}\left(\frac{iX_L}{iR}\right)$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\phi = \tan^{-1}\left(\frac{2\pi fL}{R}\right) \quad ......(15.32)$$

Example 15.5

An inductor of inductance 0.3H is connected in series with resistor of resistance 6Ω. If the voltage drop across resistor is found to be 12V, then calculate current drawn from the source of frequency 50Hz (ii) the phase angle between V & I.

Solution:

$$L = 0.3H$$
$$R = 6\Omega$$
$$v_R = 12V$$
$$f = 50Hz$$
$$i = ?$$
$$\phi = ?$$

$$\text{Current in the circuit} = i = \frac{v_R}{R}$$

$$i = \frac{12V}{6\Omega} = 2A$$

Reactance of coil (X_L) = $2\pi fL$

$$X_L = 2(3.14)(50Hz)(0.3H)$$

$$X_L = 94.2\,\Omega$$

Voltage across $L(v_L) = iX_L$

$$v_L = (2A)(94.2\,\Omega)$$

$$v_L = 188.4\text{ V}$$

Supplied Voltage $(v) = \sqrt{v_R^2 + v_L^2}$

$$v = \sqrt{(12V)^2 + (188.4V)^2}$$

$$v = 188.78V$$

$$\phi = \tan^{-1}\frac{2\pi fL}{R}$$

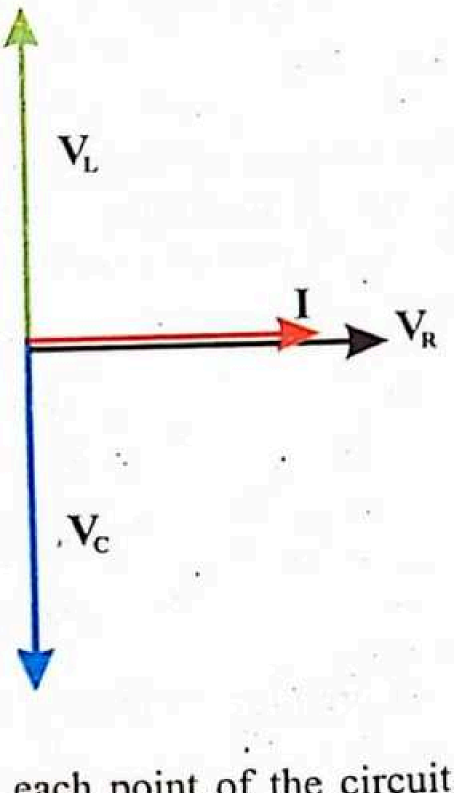
$$\phi = \tan^{-1}\frac{2(3.14)(50Hz)(0.3H)}{6\Omega}$$

$$\phi = \tan^{-1}(15.7)$$

$$\phi = 86^\circ$$

15.9 POWER IN AN A.C. CIRCUIT

We have already discussed in the previous sections that the power dissipation in a pure capacitor and a pure inductor is zero. We knows that, a pure capacitor stores the energy in terms of electrical potential energy and a pure inductor in terms of magnetic potential energy. However, there is a power dissipation in a resistor and its value is given by



$$P = i^2R$$

But according to Ohm's law

$$R = \frac{v_R}{i}$$

$$P = iv_R \;\ldots\ldots(15.33)$$

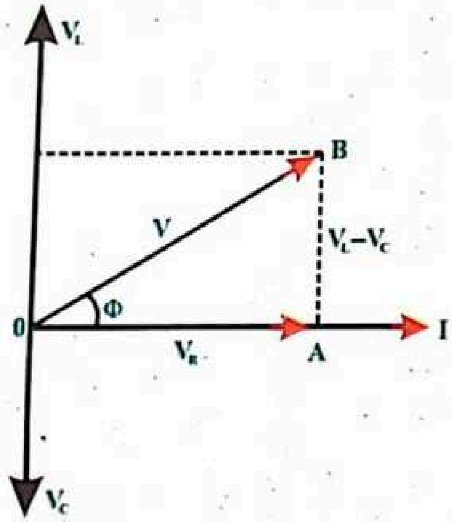
In RLC-series circuit, the current is the same at each point of the circuit, but the voltage drop across each component is different. It can be explained with the help of phasor diagram as shown in Fig.15.14(a) and Fig.15.14(b). Where v_R in phase with I but v_L leads i by $\pi/2$ and v_C lags i by $\pi/2$.

Thus $\frac{v_R}{v} = \cos\phi$

$$v_R = v\cos\phi$$

Put it in Eq.15.33

$$p = vi\cos\phi \quad ...(15.34)$$



This is known as a true power in RLC circuit and it shows that a maximum power will be dissipated when $\phi = 0$ and 'v' and 'i' are in phase with each other. It is possible only in a resistor. Thus one can say that **the power dissipation in a resistor is called true power.** However, **the power dissipation in an impedance is called apparent power (vi).**

Power factor

Power factor of an A.C. circuit is defined as the ratio of true power to apparent power

$$\text{Power factor} = \frac{\text{true power}}{\text{Apparent power}}$$

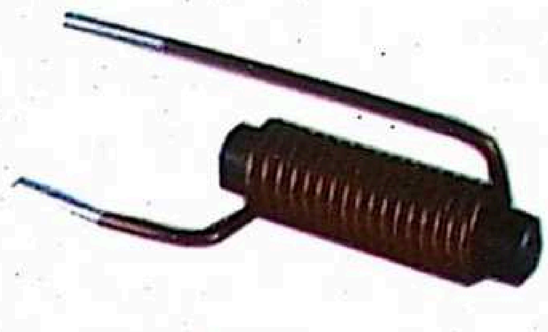
$$= \frac{vi\cos\phi}{vi}$$

$$\text{Power factor} = \cos\phi$$

15.10 CHOKE COIL

A choke coil is an inductor that presents a relatively high impedance in order to control the current in an A.C. circuit.

A choke coil consists of an insulated thick copper wire wound closely in large number of turns over a soft iron laminated core as shown in Fig.15.22. Since the wire of the coil is of thick copper, so its resistance is very small. However, due to the large number of turns, the inductance of the choke coil is very large. Thus, its power loss is extremely small. For example, a



An Inductor

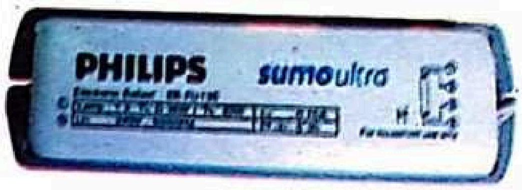


Fig.15.22 A choke coil.

fluorescent tube light requires 120V for its fluorescence, but our applied voltage is 220V. If we use a resistor to control the extra current due to potential difference of 220V, a lot of power would be dissipated by resistor in the form of heat. To overcome this problem, a choke coil is connected in series with the tube or any other electric device. It offers the reactance ($X_L = 2\pi fL$) to the flow of current. The power dissipation of a choke coil is almost zero, but it controls to extra current in the circuit.

The impedance of choke coil will be:

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{R^2 + (2\pi fL)^2}$$

In case of D.C., f = 0, then $X_L = 0$ and Z = R, this shows that a choke coil cannot be used to control D.C. because its resistance is negligible, but it is used to control A.C. only.

15.11 RESONANCE IN RLC-SERIES CIRCUIT

Consider a resistor, a capacitor and an inductor which are connected in series across the source of A.C. of constant voltage but adjustable angular frequency 'ω' as shown in Fig.15.23. As the components are in series, so there is a same current in the circuit. That is, $i = \frac{V}{Z}$, where 'Z' is the impedance of the RLC-series circuit and it depends upon the frequency. For example, if the frequency increases, then the inductive reactance $X_L = \omega L$ increases while the capacitive reactance $X_C = \frac{1}{\omega C}$ decreases. If the frequency decreases, then we have the reverse results. In between these lower and higher frequencies, there is always a frequency f_0 at which both reactances become same but in opposite direction. So both are cancelled to each other. As a result we have minimum impedance (Z) and maximum current (i). Thus, the frequency f_0 at which the impedance (Z) has its smallest value and the current amplitude reaches at its maximum value is called resonance frequency. Its value can be calculated as;

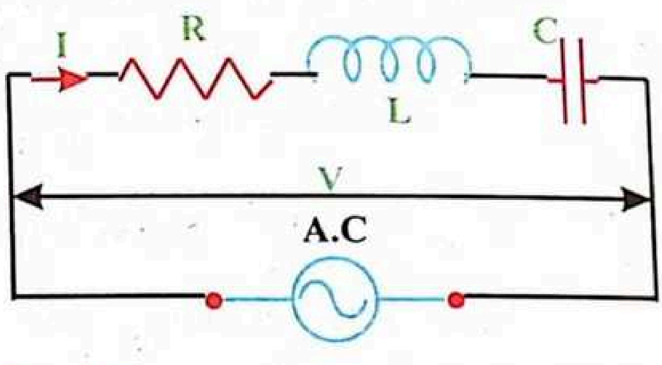


Fig.15.23 A resistor, a capacitor and an inductor connected in series in an A.C. circuit.

The resonance occurs when

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

or $$2\pi f_o = \frac{1}{\sqrt{LC}}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}} \quad \ldots(15.35)$$

This is resonance frequency. If we plot a graph between current and frequency then we have a resonance curve as shown in Fig.15.24. The graph shows that the amplitude of the current reaches its maximum value at the resonance frequency f_o.

POINT TO PONDER

If resistance remain same but capacitance and inductance are doubled, how will the resonance frequency change?

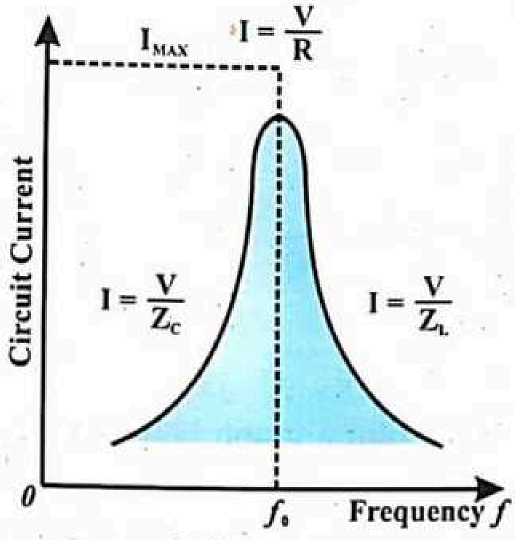


Fig.15.24 A graph between current and frequency of an A.C. in RLC-series circuit.

15.12 RESONANCE IN LC-PARALLEL CIRCUIT

Consider an inductor of inductance 'L' which is connected in parallel with a capacitor of capacitance C across an A.C. source of voltage 'v' and has adjustable angular frequency 'ω' as shown in Fig.15.25. The voltage drop across each component is the same. But the current through 'L' is i_L and through C is i_C. The phase difference between i_L and i_C is 180°, so $i = i_L - i_C$. If the frequency increases, the inductive reactance $X_L = \omega L$ also increases and the current i_L decreases while the capacitive reactance $X_C = \frac{1}{\omega c}$ decreases hence current i_C increases. Similarly, if the frequency decreases, we will observe the inverse result. The experiments show that there is a certain frequency between the lower and the higher frequencies at which $i_L = i_C$ and $i = 0$. Such current

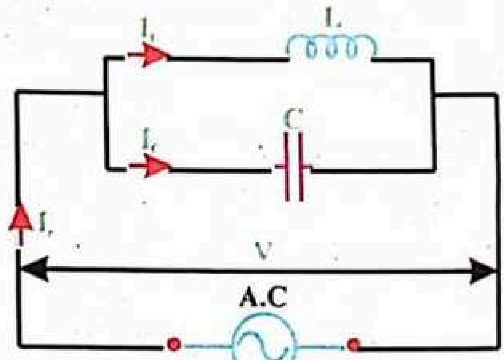


Fig.15.25 A capacitor and an inductor in parallel in an A.C. circuit.

with minimum amplitude is obtained at certain frequency frequency f_o for LC parallel circuit. Its value can be calculatec When the resonance occurs then

$$i_L - i_C = 0$$

$$i_L = i_C$$

$$\frac{V}{X_L} = \frac{V}{X_C}$$

$$X_L = X_C$$

$$2\pi f_o L = \frac{1}{2\pi f_o C}$$

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

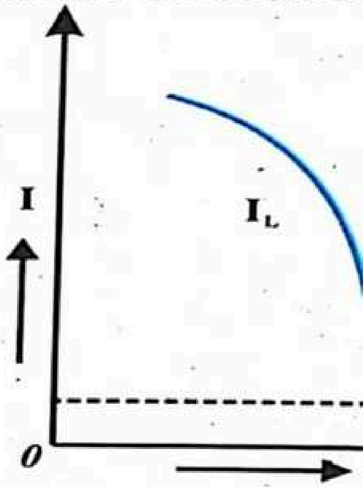


Fig.15.26 A graph of A.C. in LC-parallel c

If we plot a graph between current and frequency then curve which is inverse as that of the curve obtained in R Fig.15.26, we have observed that at resonance frequency ' f_o ',

15.13 METAL DETECTOR

The metal detector comprises of a resonating circuit and is used to detect the presence of metal nearby, hidden metals or metallic objects, buried metal underground etc. It consists of two LC oscillatory circuits A and B which are connected across the beat frequency amplifier circuit as shown in Fig.15.27. Each oscillator contains an inductor coil parallel to a capacitor to form a LC parallel circuit. A metal detector is based on the principle that the presence of metal within the range of the coil, changes its inductance L which in turn causes a change in the resonance frequency of the LC-circuit. At the normal condition, that is, when there is no metal near by the coil 'B' called search coil. The frequency of the both oscillators A and B is the same, i.e., adjusted at same resonance frequency, hence they produce zero beat When the search

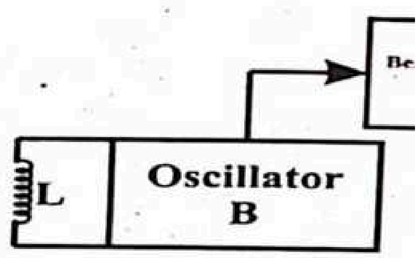


Fig.15.27 A circuit


External structu

coil approaches to the a metallic object, its inductance 'L' decreases and the frequency of the oscillator 'B' increases. So, there is difference in frequencies between the two oscillators resulting in a beat phenomenon and the loud speaker attached to the amplifier circuit is sounded as an alarm. The metal detectors are used not only for security purpose but also to detect buried and hidden metal objects. In the same way, metal detectors are used in mining and other scientific research.

15.14 MAXIMUM POWER TRANSFER IN AN A.C. CIRCUIT

In unit 12, we have studied that a D.C. source transfers its maximum power to an electric network when the internal resistance of source equals to the load resistance. This is named as "maximum power transfer theorem". Such theorem also applicable to an A.C. network which is explained as under:

Consider an A.C. circuit having a load impedance 'Z_L' which is connected across the A.C. source of impedance 'Z_g' as shown in Fig. 15.28. According to maximum power transfer theorem, the maximum power is transferred from source to load when the impedance of source Z_g and impedance of load Z_L are matching to each other, i.e.,

$$Z_L = Z_g \quad ......(15.36)$$

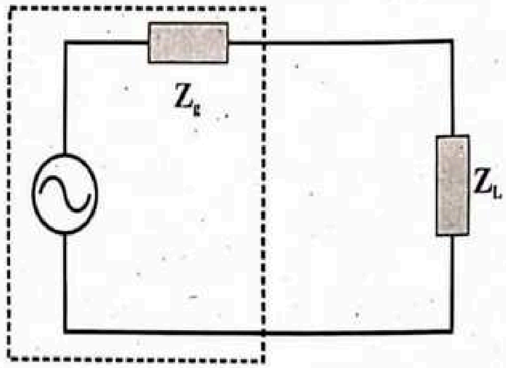


Fig. 15.28 An A.C. circuit in which the impedance of the source matches to impedance of load and source delivers its maximum power to the load.

Eq. 15.36 holds in case of a communication system where a signal transfers with its maximum power between transmitting antenna and receiving antenna when the impedances of the transmitter and the receiver match to each other. However, in case of power transmission system or any other electrical network, their efficiency in terms of maximum power transfer is about 50%, because there is large load resistance across the source.

> ***FOR YOUR INFORMATION***
> To get the maximum radiated power from an antenna in a communication system, the radiation resistance of the antenna must match the output resistance of the transmitter.

15.15 ELECTROMAGNETIC WAVES

The waves which compose of oscillating electric and magnetic fields at right angle to each other also perpendicular to the direction of their motion through space are called Electromagnetic waves, these waves do not require any medium for their propagation.

Based on Gauss's law, Faraday's and Ampere's laws, British Physicist James Clerk Maxwell in 1865 showed a closed relationship between electric and magnetic fields. He predicted that electric and magnetic fields can move through space as waves. He explained this mutual interaction between the two fields in the form of a set of four mathematical equations which are called Maxwell equations. These equations may be summarized as: a time varying or changing magnetic field produces a changing electric field. This changing electric field will in turn produce a changing magnetic field and so on.

To explain the Maxwell's hypothesis, we consider a conducting rod AB which is connected to an alternating voltage source. The charges are accelerated through rod such that a changing magnetic field produces around it in the region CD as shown in Fig.15.29(a). This changing magnetic field again set up a changing electric field and so on. This shows that each field generates the other and as a result the whole package of electric and magnetic fields start motion in forward direction through space. Such a combineed motion of electric and magnetic field is known as electromagnetic waves which consists of electric and magnetic field perpendicular to each other as shown in Fig.15.29(b). These waves travel in space with the speed of light 'c'. Its numerical value $3 \times 10^8 \text{m s}^{-1}$ can be calculated by the following equation

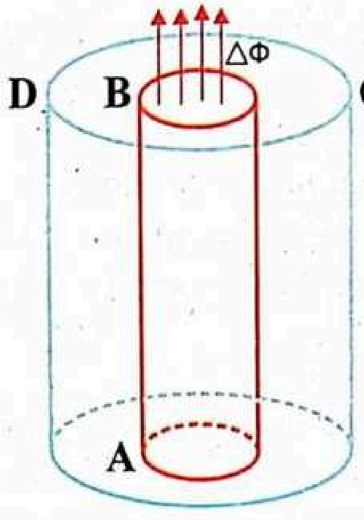


Fig.15.29(a) Changing of magnetic flux in the region AB induce an electric field in its surrounding.

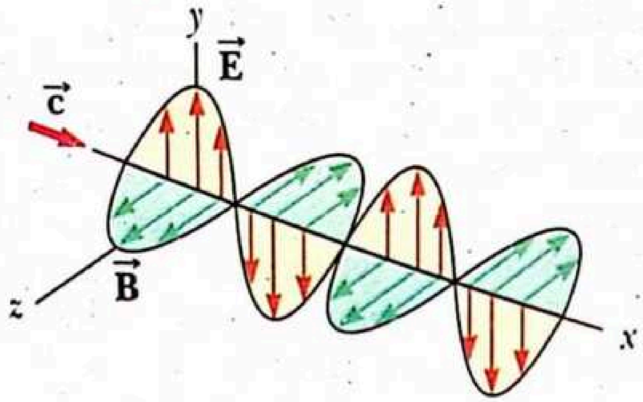


Fig.15.29(b) Propagation of electromagnetic wave that consists of electric and magnetic fields perpendicular to each other.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad ......(15.37)$$

where μ_0 and ε_0 are permeability and permittivity of free space respectively.

Electromagnetic wave has many forms such as radio waves, microwaves, infrared waves, visible light, ultraviolet light, X-rays and Gamma-rays. All these electromagnetic waves travel through free space with the same speed equal to speed of light 'c', their frequency and wave length can be study by the following relation.

$$c = f\lambda \quad ......(15.38)$$

15.15.1 The spectrum of electromagnetic waves

The speed of all the electromagnetic waves is same but their frequencies and wavelengths are different. Therefore, the electromagnetic waves can be classified on the basis of their frequencies and wavelength. The orderly classification of electromagnetic wave with respect to their frequencies and wavelengths is known as the electromagnetic spectrum. Graphically, a complete spectrum of electromagnetic waves is shown in Fig.15.30. Now all these classes of electromagnetic waves are explained as:

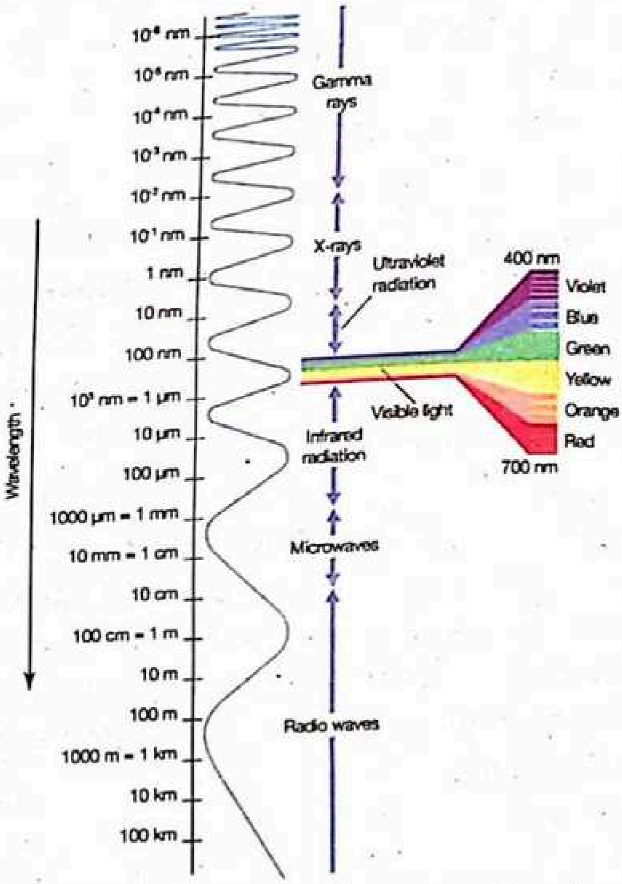


Fig.15.30 Electromagnetic waves spectrum in terms of wavelength λ and frequency ranges f.

Radio waves

Radio waves are those electromagnetic waves which have the longest wavelength. Typically, the wavelength of radio waves is longer than 1mm. They are produced by the electrical oscillation in the LC-circuits and used in the global communication system with different frequencies. e.g., AM (Amplitude Modulation) uses waves with frequency from 530KHz to 1170KHz, while FM (Frequency Modulation) radio broadcasts are at frequency from 88MHz to 108MHz and TV broadcasts use frequency from 54MHz to 890MHz. Radio waves are also used for the communication system of cellular phones with frequency from 300MHz to 3000MHz.

Microwaves

Microwaves are also generated by the LC oscillating circuit. They have wavelengths ranging from 1mm to 30cm. They are used in the radar as well as in aircraft navigation system. Similarly, the short wavelength microwaves are also used for study of atomic and molecular properties of matter. Microwave oven is a useful application of the microwaves.

Heating and burning effect of microwaves

Water is a good absorber of microwaves because the water molecules are polar and the positive and negative charges of the molecules are attracted in opposite direction. As a result, the molecules start oscillation. The frequency of rotation of water molecules is about 2.5GHz. Now if microwaves of this frequency are allowed to fall on water, the molecules of water absorb these radiations and hence water gets heated up, such heat energy is spread out in the whole region due to the rotation of molecules of water. A microwave oven is used for heating or cooking the food under this principle. Food with any moisture (i.e. free of water) cannot be heated in a microwave oven.

On the other hand, microwave radiation can heat the body living tissue in the same way that it heats the food. Therefore, prolong exposure of microwaves cause burning of body tissues.

Infrared waves

Infrared radiation IR, sometimes called infrared light or heat waves, is electromagnetic radiation with wavelengths longer than those of visible light i.e. its wavelength is lying between 700 nm and 1 mm. The large amount of infrared waves are generated by the sun, the sun gives off half of its total energy as infrared. Infrared waves have also some interesting scientific applications in vibrational spectroscopy. For example infrared is being used as short range signal wireless system between a device and a remote control.

Visible light

This is the most important part of the electromagnetic waves spectrum emitted by the sun, because it is the only radiation which is detected by the human eye. It is produced by the transitions of electrons in the atoms. The wavelengths of visible light are ranging from violet (4×10^{-7}m) to red (7×10^{-7}m).

Ultraviolet light

Ultraviolet light has a wavelength range from 4×10^{-7}m to 6×10^{-10}m. Their most important source is the sun. But they are also produced by carbon-arc lamp, electric spark, discharge tube, mercury vapor lamp, hot bodies etc. Most of the ultraviolet light from the sun is absorbed by ozone (O_3) molecules in the earth's upper atmosphere, or stratosphere. This is fortunate, otherwise prolong exposure to ultraviolet light has harmful effects on human. Such as sun burns as well as skin cancer.

Ultraviolet lamp

A ultraviolet lamp is a device that produce electromagnetic wave in the wavelength between those of visible light and x-rays. It consist of a glass tube contains a small amount of mercury. When the potential difference is applied across the tube, the molecules of the mercury are excited and they emit ultraviolet light as shown in Fig.15.31. The wavelength of emitted ultraviolet light depends upon the pressure inside the tube. The ultraviolet light emitted by the lamp is being used for the following purpose:

- Ultraviolet light produce vitamin D in the skin, therefore it is useful for Cystic fibrosis (CF) and Short Bowel Syndrome (SBS) patients, who suffer from vitamin D deficiency
- It is being used in killing and eliminating viruses, bacteria, microbes and other harmful organisms.

Fig.15.31 Excited molecules of mercury inside the tube emit ultraviolet light.

- It is also used in hospitals as well as drug and food industries to sterilize the equipments.

X-ray

X-rays are electromagnetic radiation with wavelength ranging from 10^{-8}m to 10^{-12}m. They are produced when fast moving beam of electrons strikes on the metal target. X-rays are used as diagnostic tool and also used as a treatment for certain forms of cancer. Care must be taken to avoid unnecessary exposure because X-rays can destroy living tissues and organisms. X-rays are also used in the study of crystal structure.

Gamma rays

Gamma rays have the shortest wave-lengths and their wavelength ranges from 10^{-10}m to less than 10^{-14}m. Gamma rays are highly penetrating. Therefore, they are used to destroy cancerous cells in humans. They are also used to conduct nuclear reactions. Gamma rays are emitted by radioactive nuclei in radioactive decay such as, $^{60}C_0$ and $^{137}C_S$. These rays are also emitted during the nuclear reactions.

15.16 PRINCIPLE OF GENERATION, TRANSMISSION AND RECEPTION OF ELECTROMAGNETIC WAVES

We have discussed the Maxwell's electromagnetic field theory that electromagnetic waves are generated when either an electric or a magnetic field is changing with time and these waves are capable of traveling through space. Similarly, we have also observed that an electric field due to a charged particle at rest or moving with constant velocity does not radiate in space because the magnetic flux is not changing in these cases. But the electric field only radiates through a certain region of space, when the charged particle is accelerated. Based on this principle,

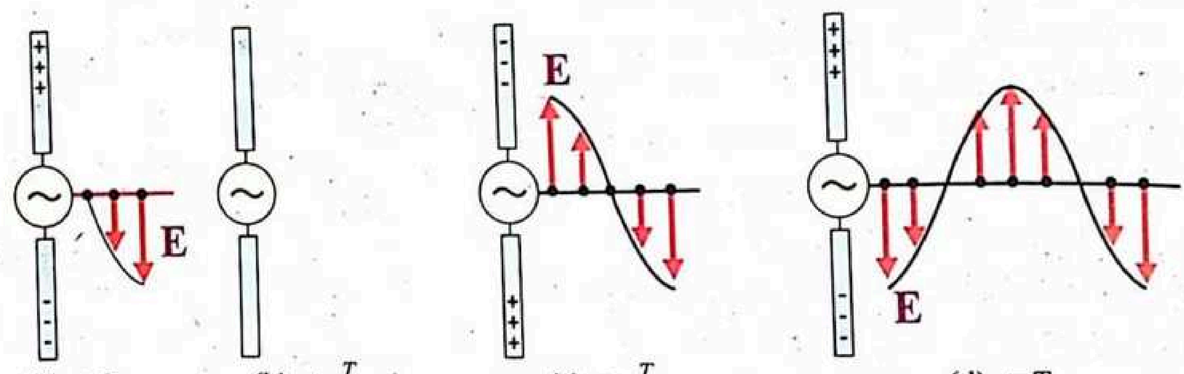


Fig.15.32 (a) An arrangement for generation and transmission of electromagnetic waves by a dipole antenna.

electromagnetic waves can be generated in space by using a radio wave transmitting dipole antenna.

A dipole-antenna consists of a long wire which is powered by A.C. source of frequency 'f' and time period T. One pole of the antenna is in space while the other is grounded as shown in Fig.15.32(a). As the source is alternating so the charges vary periodically at the ends of antenna. Now we study such variation of the charges on antenna for one cycle of A.C.

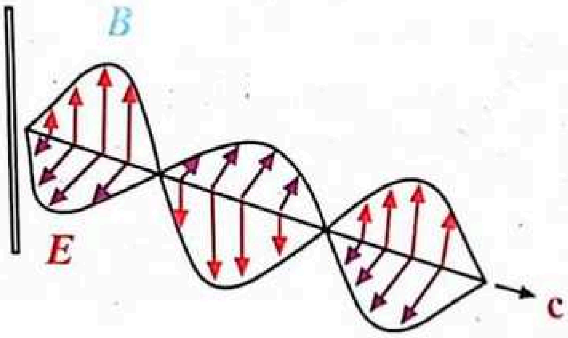


Fig.15.32(b) electric lines and magnetic lines of force perpendicular to each other.

At time t = 0; the upper part of the antenna is positively charged and the lower is negatively charged. The direction of $\overline{E}$ is downward. At $t=\frac{T}{4}$; the charges are neutralized, and $\overline{E}$ is zero. At $t=\frac{T}{2}$; upper part is negatively charged and lower is positively charged and the direction of $\overline{E}$ is upward. At t = T; $\overline{E}$ again comes at its initial position. For the next cycle, the same process of variation of charges on the antenna is repeated. On the other hand, the oscillation of these charges produces a current in the antenna, as a result a magnetic field is also generated. Now these two fields are out of phase to each other

> ***FOR YOUR INFORMATION***
>
> A resonant LC circuit generally has hundreds or thousands of signals present at its input, but only one is selected to be present at its output. e.g., the antenna for an FM receiver intercepts the signals of many different FM broadcast stations, but by turning an LC circuit to resonance, the listener can select only the station he would like listen to.

and the lines of force of $\overline{E}$ are perpendicular to the lines of force of $\overline{B}$ as shown in Fig.15.32(b). In this way, the electromagnetic waves travel through space away from the antenna with the speed of light and its frequency depends upon the frequency of the alternating voltage source.

After transmission the electromagnetic waves from the transmitting antenna, these waves can be received by an LC-parallel electrical circuit where L is inductor and C is variable capacitor. A long wire called receiving antenna is connected with this LC-parallel circuit as shown in Fig.15.33. Now when the electromagnetic waves fall on the receiving antenna, the electrons in the antenna start oscillation due to the oscillating electric field of the waves. As a result, an alternating voltage induced across the antenna. The frequency of this voltage is the same as that of the waves intercepting the antenna. There are a number of electromagnetic waves that travel through space with their own frequencies. But the receiving antenna can receive one of them at a time. It takes place only when we adjust the value of the variable capacitor such that the natural frequency $\left(f = \frac{1}{2\pi\sqrt{LC}}\right)$ of the LC-circuit is same to the frequency of the incoming waves from the transmitting station. Thus due to the resonance phenomena, the LC-circuit will respond to electromagnetic waves of particular wanted frequency and reject all the other unwanted frequencies. Such arrangement is used in the tuning circuit of a radio.

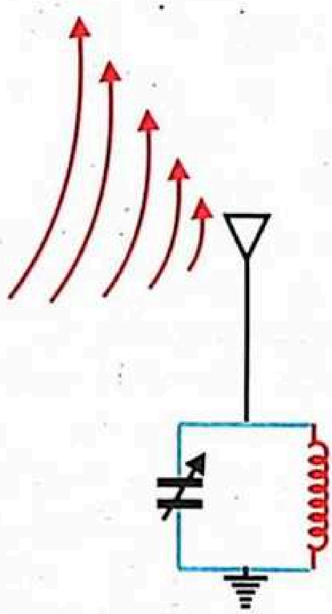


Fig.15.33 Receiving antenna with LC-circuit

15.17 TRANSMISSION AND RECEPTION OF INFORMATION

The process of transmission of information by a radio wave is shown in Fig. 15.34(a). Our information or message which is to be transmitted in the form of sound or picture, where sound has frequency ranging from 20Hz to 20KHz with speed 334ms^{-1} in air at 0^0C and it is a non-electrical signal. Therefore, microphone converts sound signal into the electrical signal and then it is fed to audio amplifier for raising the strength of the signal. This audio signal cannot be radiated out from the transmitting antenna directly. For this, a very high frequency wave called radio wave is produced by oscillator. The radio carrier wave is also known as carrier signal. The amplitude of radio wave is constant and they have high frequencies from 30kHz to 300000kHz with a speed of 3×10^8 m s^{-1}. Now the audio signal is super imposed on the radio frequency wave by using modulator. This process is called modulation. The modulation wave is finally transmitted in free space by the transmitting antenna.

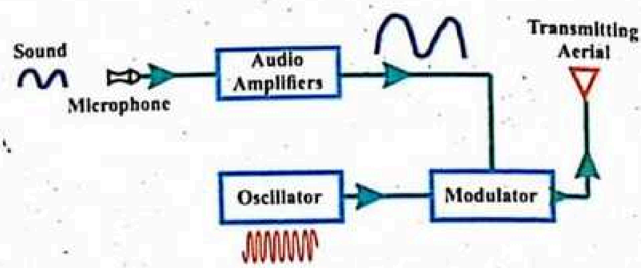


Fig.15.34(a) A process in which the information in terms of audio frequency wave is transmitted by a radio frequency wave.

When, when the modulated transmitted wave falls on the receiving antenna as shown in Fig.15.34(b), the receiving signal is first amplified by the tuned amplifier and then the modulator accepts the relevant audio signal and reject the unwanted radio frequency wave. This process by which the radio waves and audio waves are separated is known as demodulation. Finally, the audio signal is amplified by the audio amplifier and then fed to the loud speaker for producing the replica of original sound waves.

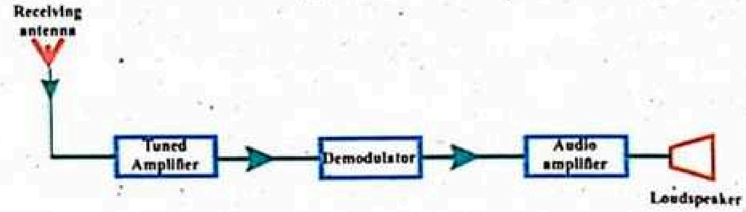


Fig.15.34(b) A process in which the information audio frequency signal is received under various stages.

15.18 ELECTROCARDIOGRAPHY (E.C.G.)

The instrument that records the voltage pulses associated with heartbeats is called electrocardiography and the pattern recorded by such instrument on a graph paper or computer is called an electrocardiogram. It is explained as under;

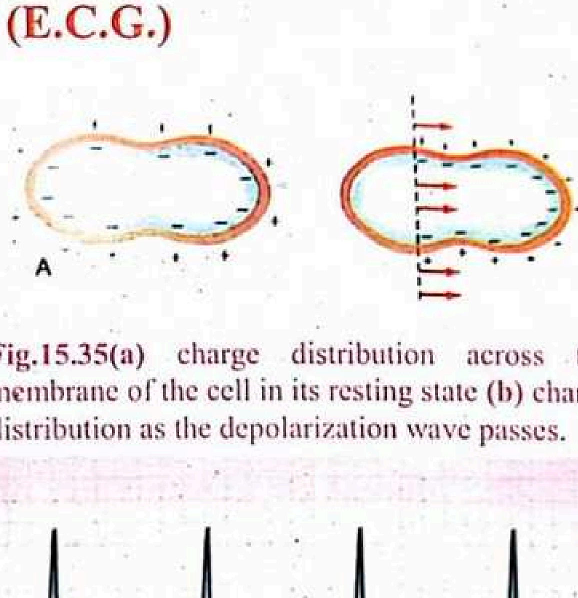


Fig.15.35(a) charge distribution across the membrane of the cell in its resting state (b) charge distribution as the depolarization wave passes.

The heart is a muscular organ made up mostly of cardiac muscles. Now we will study an individual muscle cell in its resting state. There are negative ions on the inner surface of the membrane of the cell, while positive ions on the outer surface as shown in Fig.15.35(a). This ions distribution across the membrane of the cell causes of electric potential.

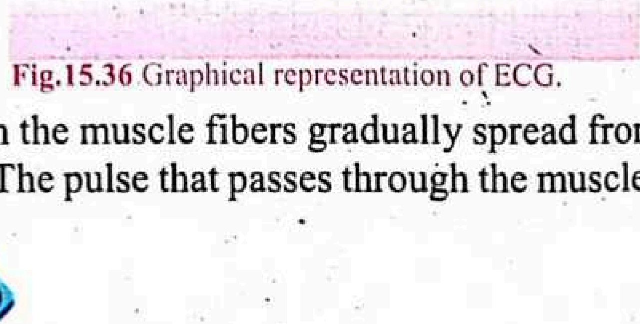


Fig.15.36 Graphical representation of ECG.

The electric impulses that originate in the muscle fibers gradually spread from cell to cell, causing the muscles to contract. The pulse that passes through the muscles cells is called a depolarization wave. The generated impulse will pump the positive ions on the outside of the cell to flow in and neutralize the negative ions on the inside of the cell as shown in Fig.15.35(b). This effect causes of neutralize the potential difference. Once the depolarization wave has passed through an individual heart muscle cell, the cell is polarized again. i.e., it recovers the resting state ions distribution in about 250ms. Thus, the depolarization and polarization of cells in the heart causes potential difference that can be measured using electrode connected to the skin. The potential difference measured by the electrodes is amplified and recorded on a graph paper or a computer as shown in Fig.15.36.

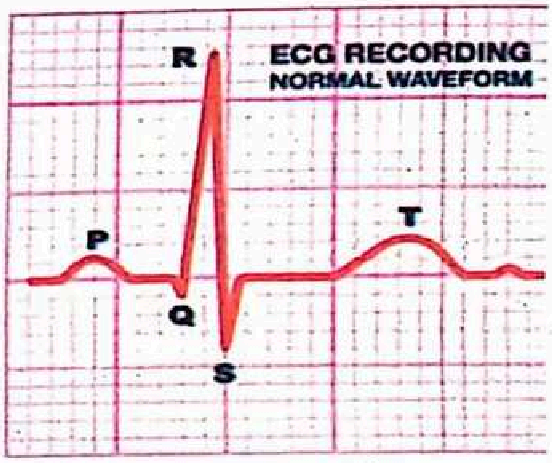


Fig.15.37 (a) The recorded ECG for a normal heart.

Let us study the recorded ECG for one beat of a normal heart as shown in Fig.15.37(a). The pulse 'P' shows the generated pulse before the muscle to contract. Similarly, when the pulse passes through the cell called polarization and it is represented by the pulse QRS. Finally, the T pulse occurs when the cells are polarized again. Similarly, the recorded ECG for an abnormal heart is shown in Fig.15.37(b). The registered result shows that there is no constant relation between the pulses due to the polarization and depolarization. On the other hand, the pulse QRS is wider than the pulse QRS of the normal one.

Fig.15.37(b) The recorded ECG for an abnormal heart.

SUMMARY

- **Alternating Current (A.C.):** A current or voltage which changes periodically with time both in positive and negative direction is called alternating current or voltage. i.e., $i = I_0 \sin \omega t$ or $v = V_0 \sin \omega t$
- **Root Mean Square Value of Current or Voltage:** The R.M.S. value is that effective value of an alternating current or voltage which produces same amount to heat in a resistor as that of produced by D.C.

$$I_{rms} = 0.707\ I_0.$$

- **A.C. Circuit:** A circuit which is powered by A.C. source and a number of components (such as R, L and C) are connected across it is known as A.C.-circuit.
- **Capacitive Reactance:** The opposition offered by a capacitor to the flow of A.C. is known as capacitive reactance $\left(X_C = \frac{1}{\omega C}\right)$.
- **Inductive Reactance:** The opposition offered by an inductor to the flow of A.C. is known as inductive reactance $\left(X_L = \omega L\right)$.
- **Impedance:** The combined resistance of a resistor, capacitor and an inductor in A.C. circuit is known as impedance.
- **Power factor:** The ratio between true power to the apparent power is called power factor, it is equal to $\cos\phi$.
- **Choke coil:** Choke coil is an inductor whose power dissipation is zero and it is being used to control the current in an A.C. circuit.
- **Metal Detector:** It is an electronic instrument which consists of resonant circuit and is used to detect metal, metallic objects, and the buried metals.
- **Maximum Power Transfer Theorem:** An A.C. source transfers its maximum power to load when its impedance equals to the impedance of load.
- **Electromagnetic waves:** Electromagnetic waves are composed of oscillating electric and magnetic fields at right angle to each other and they do not require any medium for their propagation.
- **Modulation:** A process in which audio frequency signal is super imposed on the radio frequency wave.
- **Electrocardiography:** The instrument that records the voltage pulses associated with the action of heart is called electrocardiography.
- **Electrocardiogram:** The recorded heart pulses on a paper is called electrocardiogram.

EXERCISE

Select the best option of the following questions.

1. The mean value of current over one complete cycle of A.C. is
   (a) Zero (b) One (c) I (d) I_0
2. Root mean square value of alternating current is equal to
   (a) 50% of I_0 (b) 50.7% of I_0 (c) 70% of I_0 (d) 70.7% of I_0
3. When the initial phase of A.C. is $\pi/2$ then it will complete its one cycle at the phase of
   (a) π (b) $3\pi/2$ (c) 2π (d) $5\pi/2$

4. Average power dissipation in an A.C. circuit across the resistor is
(a) VI (b) V_oI_o (c) $V_{rms}I_{rms}$ (d) V^2I^2

5. When a capacitor is connected across the A.C. source then
(a) V & I are in phase (b) I lagging behind V
(c) I leading by V (d) V leading by I

6. When the frequency of A.C. is increased then the reactance of the inductor will
(a) Remain the same (b) Decreased
(c) Increased (d) Become negative

7. The power dissipation in LC- circuit is
(a)V.I (b) V_oI_o (c) $V_{rms}I_{rms}$ (d) Zero

8. A capacitor is an A.C. circuit is working at
(a) Any frequency (b) Low frequency
(c) High frequency (d) Zero frequency

9. The unit of impedance is
(a) Ampere (b) Volt (c) Watt (d) Ohm

10. According to the phasor diagram, the phase difference between V_L and V_C is
(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) π (d) 2π

11. Power factor is equal to
(a) VI (b) $VI\cos\varphi$ (c) $\cos\varphi$ (d) $\tan\varphi$

12. In a choke and a resistor series circuit, the A.C. source delivers its maximum power when
(a) $R = 0$ (b) $Z = 0$ (c) $R = Z$ (d) $R > Z$

13. The A.C. source transfers its maximum power to load when
(a) Impedance of source is equal to 0
(b) Impedance of source less than impedance of load
(c) Impedance of source greater than impedance of load
(d) Impedance of source equals to impedance of load

14. In RLC-series circuit the amplitude of the current has its maximum value when
(a) $X_C = 0$ (b) $X_L = 0$ (c) Both zero (d) $X_L = X_C$

15. In RCL-parallel circuit, the resonance phenomenon will be observed when the net current in the circuit is
(a) Increasing (b) Decreasing (c) Reverse (d) Zero

16. The resonance frequency is
(a) $f = 2\pi\sqrt{LC}$ (b) $f = \frac{2\pi}{\sqrt{LC}}$ (c) $f = \frac{1}{2\pi\sqrt{LC}}$ (d) $\frac{\sqrt{LC}}{2\pi}$

17. The propagation of electromagnetic waves was predicted by

(a) Hertz (b) Maxwell (c) Faraday (d) Ampere

18. The phase difference between electric and magnetic line of forces in electromagnetic waves is
(a) 45^0 (b) 90^0 (c) 180^0 (d) 270^0

19. The speed of electromagnetic waves in free space is given by the equation
(a) $c = \frac{1}{\sqrt{\varepsilon\mu}}$ (b) $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ (c) $c = \frac{1}{\sqrt{\varepsilon/\mu}}$ (d) $c = \frac{1}{\sqrt{\varepsilon_0/\mu_0}}$

20. The electromagnetic waves do not transport
(a) Energy (b) Charges (c) Momentum (d) Information

21. Which one of the following waves has the shortest wavelength?
(a) Radio wave (b) Microwave (c) Ultraviolet wave (d) γ-rays wave

22. A process in which audio signal superimpose on radio frequency wave is called
(a) Rectifier (b) Amplifier (c) Modulation (d) Demodulation

SHORT QUESTIONS

1. Why we use I^2 instead of I while taking the mean value of A.C. cycle?
2. Why 220V of A.C. is more effective then 220V of D.C.?
3. At what value, the A.C. and the D.C. become equal?
4. What is the cause of the current leading by voltage in an A.C. circuit?
5. What do you know about the impedance in RLC-circuit of A.C.?
6. What will happen if the frequency of A.C. across the inductor is increased?
7. What do you know about the power factor of A.C. circuit?
8. What is a phasor and phasor diagram?
9. Why a choke cannot control D.C.?
10. Under what condition a source transfers its maximum power to a load?
11. What is the condition of the resonance phenomenon in RLC-series circuit?
12. How does the amplitude of the current becomes zero in LC parallel circuit?
13. What is the working principle of metal detector?
14. How did Maxwell predict the electromagnetic waves?
15. How does transmitting antenna transmit the electromagnetic waves?
16. What do you know about the spectrum of electromagnetic waves?
17. How does the receiving antenna receive the electromagnetic waves?
18. How can you calculate the speed of electromagnetic waves?
19. Distinguish between the process of modulation and demodulation.
20. Distinguish between electrocardiography and electrocardiogram.

COMPREHENSIVE QUESTIONS

1. Explain alternating current and alternating voltage with their mathematical and graphical representation.

2. What do you know about the instantaneous value, root mean square value and peak value of alternating current and alternating voltage?
3. What is phase of A.C.? Explain phase lag and phase lead between alternating current and alternating voltage with the help of phasor diagram.
4. Study alternating current through a resistor and calculate power dissipation in the resistor.
5. Discuss A.C. through a capacitor and power dissipation in it.
6. What are the value of instantaneous current and voltage in an inductor when it is connected across the A.C. source?
7. What is impedance? Calculate the impendence of RCL-series circuit.
8. Calculate impedance and phase angle of RC-series circuit.
9. Discuss the behaviour of A.C. through RL-series circuit.
10. State and explain power in an A.C.-circuit and power factor.
11. What do you know about choke coil, its function and application?
12. State and explain resonance in RLC-series circuit with the resonance frequency.
13. How can resonance be observed in LC-parallel circuit? Also calculate the resonance frequency in LC-parallel circuit.
14. Explain metal detector, its working principle and function.
15. What are electromagnetic waves? Discuss the process of generation, propagation and reception of electromagnetic waves.
16. What do you know about the spectrum of electromagnetic waves? State and explain all classes of electromagnetic waves.
17. Explain electrocardiography with its working principle and its function.

NUMERICAL PROBLEMS

1. The r.m.s. value of alternating current is 5A and its frequency is 50Hz which flows in a circuit through a resistor. Calculate the peak current and the value of the current after 0.002s. **(7A, 4.4A)**
2. An alternating current flows through a resistor of 10Ω and produces heat at the rate of 360W. Calculate the effective value of current and voltage. **(6A, 60V)**
3. A capacitor of capacitance $6\mu F$ is connected across the A.C. source of 220V. Calculate the current through the capacitor, if the frequency of the source is 50Hz. **(0.4A)**
4. An inductor of inductance 0.6H is connected across the A.C. source of 220V. Calculate the current through the inductor, if the frequency of the source is 50Hz. **(1.2A)**

3. What is PN junction? How can you form a PN junction?
4. Define biasing of PN junction and discuss forward biased and reverse biased of a PN junction.
5. What is rectification? How can a diode be used as a rectifier? Explain half-wave rectification.
6. State and explain full wave bridge rectification by using four diodes.
7. What is transistor? Explain the operation of NPN transistor?
8. Discuss the three configurations of a transistor.
9. What is amplification? Explain the amplification of common emitter NPN transistor.
10. How can transistor be used as an automatic switch?

NUMERICAL PROBLEMS

1. Calculate the resistance across the PN-junction silicon diode that has 0.5A of current through it with a 0.8V drop across the two terminals of the diode.
(1.6Ω)
2. How much is the emitter current with 100mA for the collector current and 800μA for base current? **(100.8mA)**
3. A transistor has $\alpha = 0.995$. If the emitter current is 80mA, what is (i) the collector current (ii) the base current (iii) current gain β?
(i) 79.6mA, (ii) 0.4mA, (iii) 199
4. The current gain β of a transistor is 200. If the base current is 0.2mA. Find (i) the collector current (ii) emitter current (iii) the value of α.
(i) 40mA (ii) 40.2mA (iii) 0.995
5. The input resistance in the common emitter amplifier circuit of a given transistor is $2k\Omega$. The output resistance of this circuit is $8k\Omega$. What would be the output voltage corresponding to an input voltage 6mV if the current gain is 100? **(2.4V)**

Unit 16

PHYSICS OF SOLIDS

Major Concepts (13 PERIODS)

- Classification of solids
- Mechanical properties of solids
- Elastic limit and yield strength
- Electrical properties of solids
- Superconductors
- Magnetic properties of solids

Conceptual Linkage

This chapter is built on
Properties of matter Physics IX
Types of Solids Chemistry XI

Students Learning Outcomes

After studying this unit, the students will be able to:

- distinguish between the structure of crystalline, glassy, amorphous and polymeric solids.
- describe that deformation in solids is caused by a force and that in one dimension, the deformation can be tensile or compressive.
- describe the behavior of springs in terms of load-extension, Hooke's law and the spring constant.
- define and use the terms Young's modulus, bulk modulus and shear modulus.
- demonstrate knowledge of the force-extension graphs for typical ductile, brittle and polymeric materials.
- become familiar of ultimate tensile stress, elastic deformation and plastic deformation of a material.
- describe the idea about energy bands in solids.
- classify insulators, conductors, semiconductors on the basis of energy bands.
- become familiar with the behaviour of superconductors and their potential uses.
- distinguish between dia, para and ferro magnetic materials.
- describe the concepts of magnetic domains in a material.
- explain the Curie point.
- classify hard and soft ferromagnetic substances.
- describe hysteresis loss.
- synthesize from hysteresis loop how magnetic field strength varies with magnetizing current.

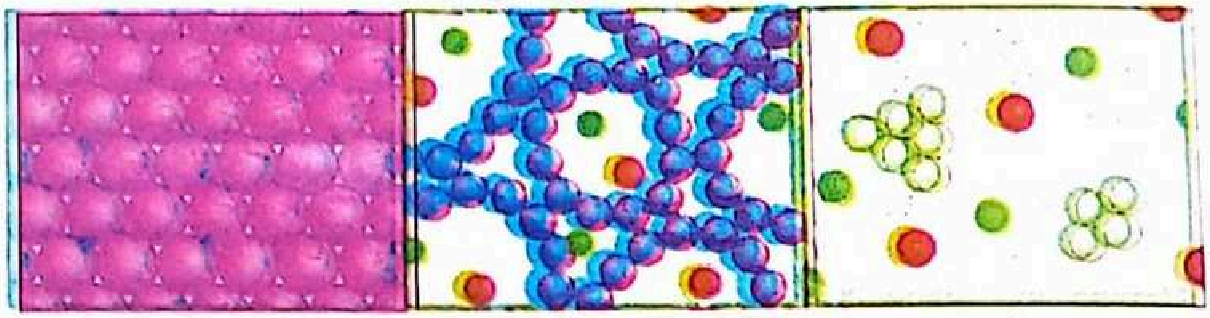
INTRODUCTION

Basically, there are three states of matter solids, liquids and gas. The liquid and gas are compressible and have ability to flow, whereas, solids are incompressible and have definite volume and shapes. These are due to the closeness of their atoms or molecules. Similarly, there are several kinds of solids, among these, each of them has its different characteristics and properties. Some solids are crystalline while others are amorphous or polymer, some are elastic while others are rigid, some are brittle while others are plastic or ductile, some have electrical conductivity while others have magnetic conductivity. For example, diamond is hard, lead is soft, steal is strong, glass is weak, copper has electrical conductivity and iron has magnetic conductivity.

Many solids can be deformed in the form of change in their length, volume or shape by a deforming force. These deformations can be studied in terms of stress and strain.

In this unit we will explain classification of solids. For example, there are three classes of solids with respect to their structure, named as: crystalline, amorphous and polymer. Electrically, there are three classes of solids such as: insulators, conductors and semi-conductors, these can be studied using band theory of solids. Magnetically, the three types of solids are paramagnetic, diamagnetic and ferromagnetic. In addition to the magnetic classes, we will also discuss the phenomenon of hysteresis and hysteresis loop by magnetizing and demagnetizing the magnetic materials.

Arrangement of Molecules in Solid, Liquid and Gas



Solid	Liquid	Gas
Molecules are tightly packed and cannot move freely	Molecules are loosely packed and can move freely	Molecules are very lossely packed and in a continuous random motion.

16.1 CLASSIFICATION OF SOLIDS

We have studied in the previous classes about the three forms of matter such as solids, liquids and gases. Plasma is often termed as the fourth state of the matter. Among all these, solid is the only form of matter which has a definite shape and volume. Normally, the distance between the adjacent atoms of the solid are the same as the diameters of the atoms themselves. Thus, there are strong interactions between the atoms of the solids. On the other hand, the distribution of atoms or molecules of the solids are not alike i.e., the atoms or molecules of some solids are arranged with orderly manner, while, the arrangement of the atoms or molecules of other solids are orderless. On the basis of such distribution of atoms or molecules, solids can be classified into three types.

Crystalline solids

The solid in which their atoms, molecules or ions are arranged in a definite order are called crystalline solids as shown in Fig.16.1. These regular arrangement of atoms, molecules or ions can be studied by using various x-rays techniques. The observations show that the arrangement of atoms, molecules or ions are repeating in a three dimensional pattern with highly ordered throughout the crystal. For example, Sodium Chloride (NaCl) crystal has a cubic structure as shown in Fig.16.2. The red spheres represent the positive sodium ion (Na^+) and blue spheres represent the negative chlorine ion (Cl^-). The distribution of the ions shows that there is regular arrangement of the ions which is repeated in three dimensions. The crystalline solids have a long range order it means that there is a regular pattern of arrangement of particles which repeats itself periodically over the entire crystal. Similarly, crystalline solids have another important property that they have sharp melting point. e.g, aluminum has melting point of 655°C, NaCl has melting point of 800°C and the melting point of copper is 1084°C.

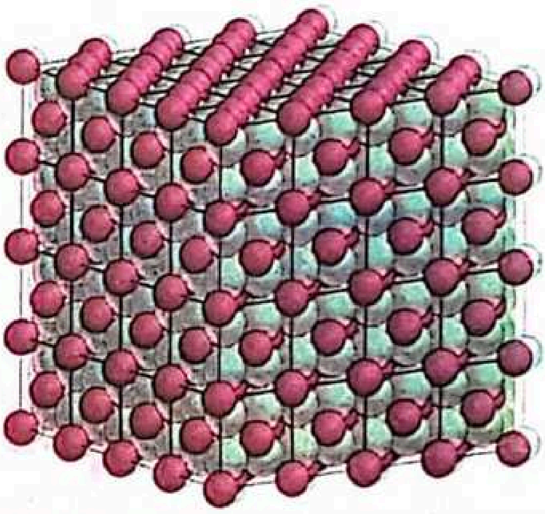


Fig.16.1 Regular arrangement of molecules of a crystalline solid.

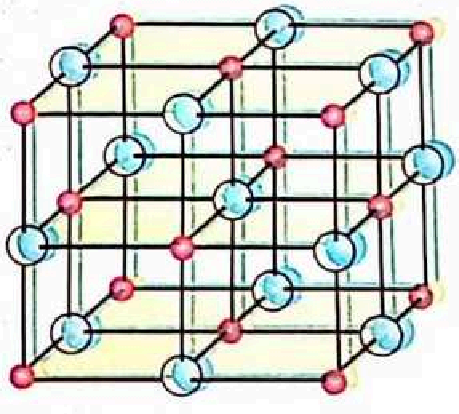
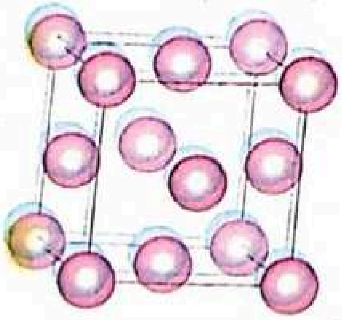
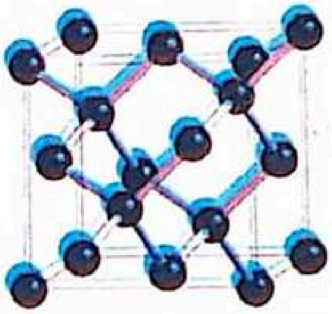


Fig.16.2 A cubic crystal structure of NaCl (Ionic Crystal).

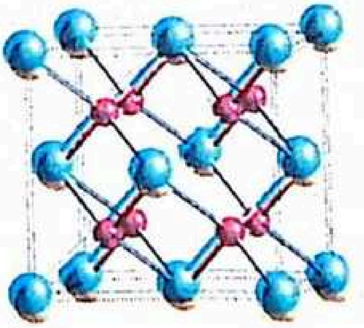
Almost all families of solid fall in the group of crystalline solids including **metals**: such as, copper, zinc and iron, **non-metal** such as, diamond, sulphar and



Crystal structure of copper (metal)



Crystal structure of diamond (non-metal)



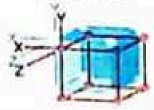
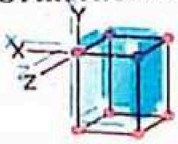
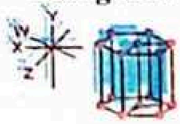
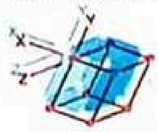
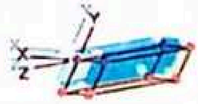
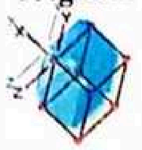
Crystal structure of zirconium (ceramic)

mica, **ionic compound**, such as, sodium chloride and copper sulphate and **ceramic**: such as zirconium.

All these crystalline solids have not only definite volume but also geometrical shapes such as, cubic, trigonal, tetragonal, hexagonal, orthorhombic, monoclinic and triclinic as shown in Table.16.1.

The study shows that the atoms, molecules or ions of the crystalline solids are not stationary, but they vibrate about a fixed point and their amplitude depends upon temperature. i.e., the amplitude of the vibration increases with increasing temperature.

Table.16.1 Different geometrical shapes of crystalline solids.

Cubic  All three axes are equal in length and they are perpendicular to one another.	**Tetragonal**  Two of the three axes are equal in length, and all the three are perpendicular to one another.
Orthorhomibic  All three axes are unequal but perpendicular to one another.	**Hexagonal**  Three axes are of equal length, the fourth axis is perpendicular to the plane of the other three.
Monoclinic  All three axes are unequal and two of them are perpendicular to each other	**Triclinic**  All three axes are unequal and are not perpendicular to another.
Trigonal  All the three axes are equal in length, but none of the axes is perpendicualr to another.	

Amorphous solids

The solids whose constituent particles i.e. atoms, molecules or ions are arranged more-or-less in a random manner are called amorphous solids. Like crystal, the atoms or molecules of the amorphous solids have no order in the long range as shown in Fig.16.3. Although amorphous solids have a definite volume, but they have not definite regular geometrical or crystalline shape. Amorphous solids have some properties as that of the frozen liquid. For example, if we heat a glass rod with a spirit lamp, we will find that the rod begins to flow more and more easily as its temperature rises, this shows that glass has no definite transition from solid to liquid, and no definite melting point. Therefore, one can say that glass at room temperature is an example of amorphous solid, which has no long range order, but only short range order. Thus, amorphous solids are also called glassy solids.

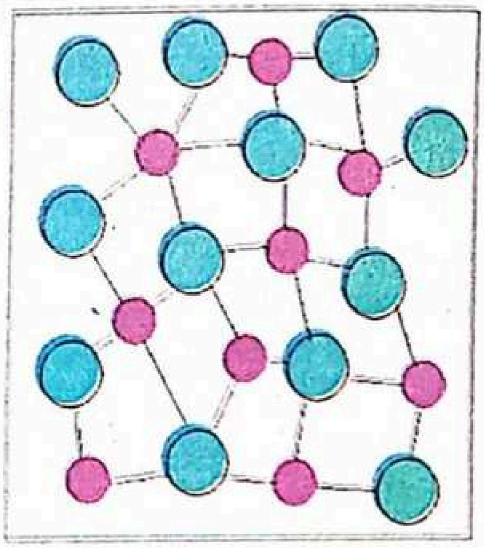


Fig.16.3 Distribution of molecules of amorphous.

Polymeric solids

The solids in which its atoms, molecules or ions are arranged neither periodically like crystalline solids nor random like amorphous solids are called polymeric solids. The distribution of atoms or molecules of polymeric solids is shown in Fig.16.4. The structure of the polymeric solid lies between order and disorder. Thus, polymeric solids may be classified as partially or poorly crystalline solids.

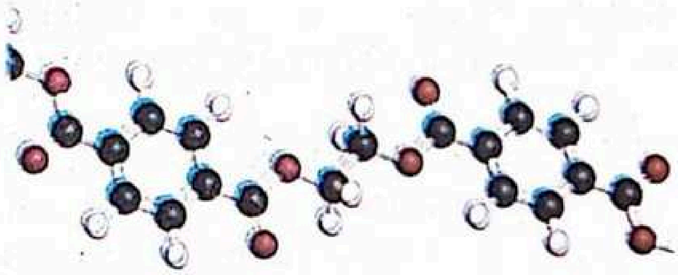


Fig.16.4 Distribution of molecules of polymeric solid.

There are two types of polymeric solids, naturally occurring and synthetic (polymeric by chemical reaction). The polymers that occur naturally are rubber, resin, cotton-wool and wood. Whereas, the synthetic polymers are polythene, polystyrene, polypropylene, polyvinyl chloride etc. Polymers, both natural and synthesized are created under the process of polymerization of many small molecules called monomers. Polymers consists of very long chain of carbon atoms bounded by oxygen, hydrogen, nitrogen and other non-metallic elements as shown in Fig.16.5. For example, plastic and synthetic rubber are formed by polymerization reaction in which relatively simple molecules are chemically combined into massive long chain molecules in the form of three dimensional structure.

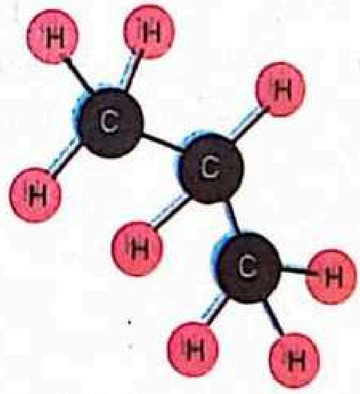


Fig.16.5 A chain of carbons bounded by other elements.

16.2 MECHANICAL PROPERTIES OF SOLIDS

Almost all bodies can be deformed more or less by applying a force called deforming force. The deformation may be in the form of change in its length, volume or shape of a body. All these are explained by some examples;

I. Let the upper end of a wire is fixed and its lower end is pulled down by the weight of the body. A change may produce in the length of the wire called tensile as shown in Fig.16.6(a)

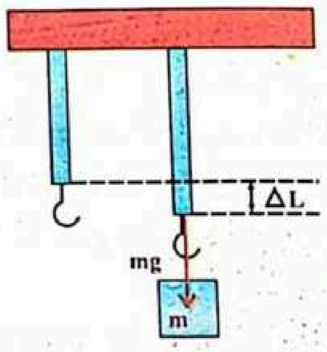


Fig.16.6(a) A change is produced in length of the wire

II. Similarly, if a rubber ball is pressed or squeezed from its all sides, its volume decreases, i.e., a change occurs in volume of the ball called volumetric as shown in Fig.16.6(b).

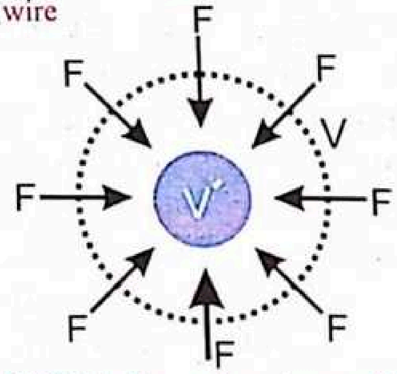


Fig.16.6(b) A change in volume of the ball

III. When a deforming force is applied on a box, the perpendicular axes of the box are displaced from their fixed position. This causes change in shape of the box called shearing as shown in Fig.16.6(c).

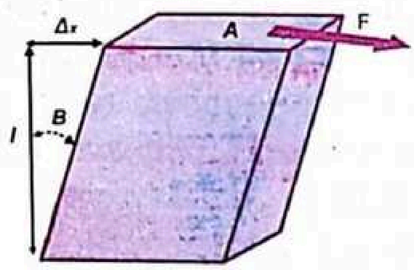


Fig.16.6(c) A change in shape of the box

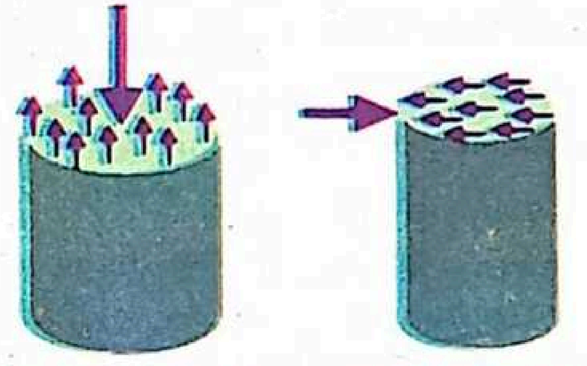
After deformation, when the deforming forces are removed, and the bodies tend to regain their original conditions, then this property of the bodies is called elasticity. Those bodies which can regain or resume completely their original shape and size, on the removal of deforming forces are called elastic bodies. On the other hand, the bodies which cannot regain their original shape and size, but a permanent change occurs in it after the removal of deforming forces are known as plastic. In practice, there are no perfectly elastic or plastic solids. All the elastic and plastic properties of solids are explained in terms of stress and strain.

Stress

The deforming force may produce a change in an object's length, volume or shape. The force in terms of its work is stored in the deformed body as elastic potential energy which helps to regain the body to its original position and its corresponding force is called elastic restoring force. Defaming and restoring forces are same in magnitude but in opposite sign. Thus, **the deforming force acting per unit area to produce any change in length, shape or volume in a body is called stress.** i.e.

$$\text{Stress}\,(\sigma) = \frac{\text{deforming force}}{\text{area}} = \frac{F}{A} \;......(16.1)$$

The SI unit of stress is newton per meter square ($N\ m^{-2}$) which is termed as Pascal (Pa) and the dimension formula of stress is $[ML^{-1}T^{-2}]$.



If the force is acting normally to the surface, then its corresponding stress is called normal stress. There are two types of normal stress, tensile stress and compressive stress. However, if the force is acting along the surface per unit area then it is called tangential stress or shear stress.

Strain

The change in length, volume or shape of a body due to the deforming force is called strain. It is always measured in terms of the ratio of change in dimension to the original dimension. i.e.,

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

As strain is defined in terms of ratio of quantities with same dimensions so it has no unit or dimensions.

If a change occurs in the length of body then the ratio of change in length (ΔL) to the original length (L) is called longitudinal strain, it is caused by longitudinal stress (tensile stress/compressive stress) as shown in Fig.16.7(a), thus

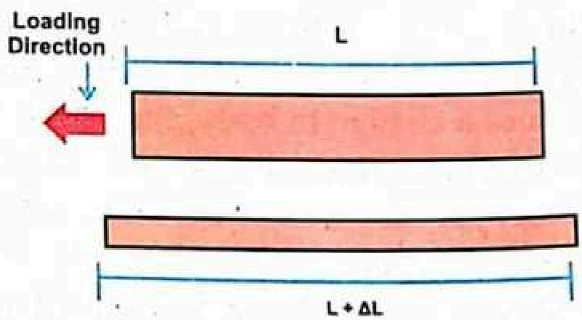


Fig.16.7(a) Longitudinal strain

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}}$$

$$\varepsilon = \frac{\Delta L}{L} \;......(16.2)$$

Similarly, **the ratio of change of volume of a body to its original volume is called volumetric strain,** as shown in Fig.16.7(b). Mathematically, it is explained as;

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}}$$

$$\text{Volumetric strain} = \frac{\Delta V}{V} \;......(16.3)$$

When the deforming force produces a change in the shape of a body (without changing its volume), it is called shear strain. In shear strain, the axes of the body are displaced through an angle 'θ' as shown in Fig.16.7(c). Thus, it is measured in terms of angle 'θ'. i.e.,

$$\text{Shear strain } (\gamma) = \frac{\Delta x}{\ell} = \tan\theta$$

For very small value of angle 'θ' in terms of radian, $\tan\theta = \theta$, so that,

$$\text{Shear strain} = \theta \;......(16.4)$$

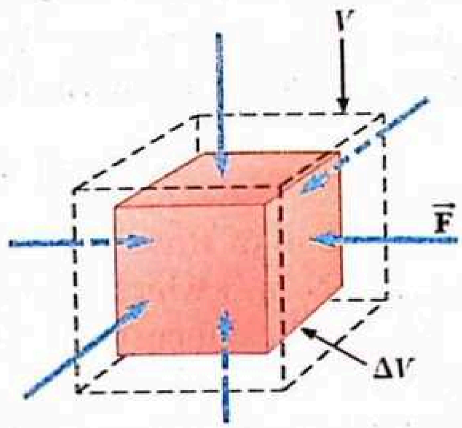


Fig.16.7(b) Volumetric strain

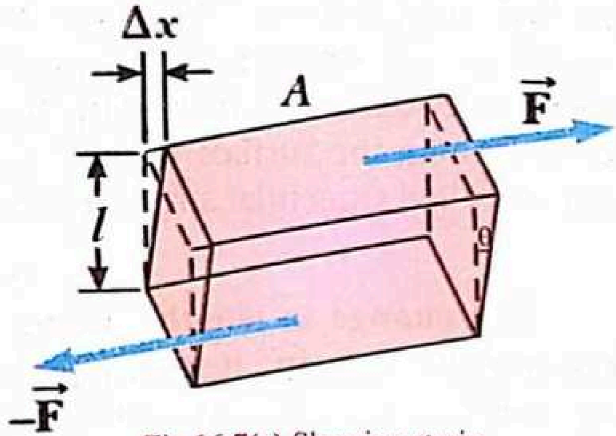


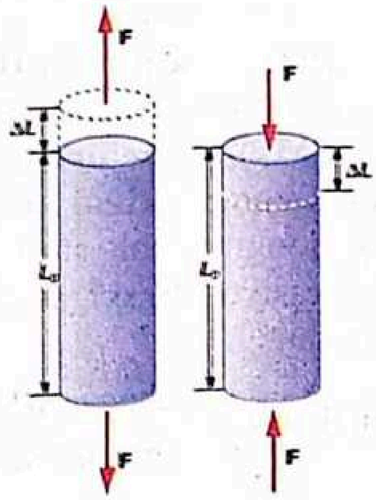
Fig.16.7(c) Shearing strain

16.3 HOOKE'S LAW

Robert Hooke studied the elastic properties of various solids and defined them in terms of stress and strain. It is explained as: when a deforming force produces a change in body, then we have both stress and strain. i.e., stress is a force that acts per unit area of the body, which produce deformation. Whereas, strain is the measurement of the amount of the deformation.

According to Hooke's law, within elastic limit, the strain is directly proportional to the applied stress. i.e.,

$$\text{Stress} \propto \text{Strain}$$



Applied Stress (F/A) is directly proportional to the strain (ΔL).

$$\frac{\text{stress}}{\text{strain}} = E \ldots\ldots(16.5)$$

where 'E' is the constant of proportionality. It is known as coefficient of elasticity or modulus of elasticity. Its value depends upon the nature of the solids. Since strain has no unit, therefore, the unit of modulus of elasticity is same as that of the stress. i.e., N m^{-2}. There are three kinds of modulus of elasticity.

I. Young's Modulus

When the deformation in a body is in the form of change in its length only then there is longitudinal stress and longitudinal strain. The ratio of longitudinal stress to longitudinal strain is called Young's modulus. It is expressed as:

$$\text{Young's Modulus (Y)} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$Y = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\,\Delta L} \ldots\ldots(16.6)$$

> ***POINT TO PONDER***
>
> A wire used to support a weight and is stretched by a millimetre. The wire is replaced by another wire of the same material having twice of cross-section area of the first wire. How much does the new wire stretched when it supports the same weight?

II. Bulk Modulus

When the applied force produces change in the volume of the body, we have the volumetric stress and the volumetric strain. The ratio of volumetric stress to the volumetric strain is called Bulk Modulus. i.e.,

$$\text{Bulk Modulus(K)} = \frac{\text{volumetric stress}}{\text{volumetric strain}}$$

$$K = \frac{F/A}{\Delta V/V}$$

$$K = \frac{FV}{A\,\Delta V} \ldots\ldots(16.7)$$

III. Modulus of Rigidity

In case of shear deformation which causes change in shape of the solid, the ratio of the shear stress to the shear strain is called Modulus of Rigidity. It is expressed as;

$$\text{Modulus of Rigidity } (\eta) = \frac{\text{shear stress}}{\text{shear strain}}$$

$$\eta = \frac{F/A}{\theta}$$

$$\eta = \frac{F}{A\theta} \quad \ldots\ldots(16.8)$$

The typical values of the three elastic constants for some selected materials are given in table 16.2.

Table.16.2 Typical Values of Elastic Moduli

Substance	Young's Modulus (Pa)	Shear Modulus (Pa)	Bulk Modulus (Pa)
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Bone	1.8×10^{10}	8.0×10^{10}	-
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Steel	20×10^{10}	8.4×10^{10}	16×10^{10}
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Glass	6.5 to 7.8×10^{10}	2.6 to 3.2×10^{10}	5.0 to 5.5×10^{10}
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Rib Cartilage	1.2×10^{10}	-	-
Rubber	0.1×10^{10}	-	-
Tendon	2×10^{10}	-	-
Water	-	-	0.21×10^{10}
Mercury	-	-	2.8×10^{10}

Example 16.1

A metal wire 80cm long and 15mm in diameter stretches 20mm when a load of 8kg is hung on its end. Find the stress, the strain and Young's modulus for the material of the wire.

Solution:

We have

Length of the wire = L = 80cm = 0.8m

Diameter of the wire = D = 15mm

Radius of the wire = r = 7.5mm = 7.5×10^{-3}m

Change in length = ΔL = 20mm = 2×10^{-2}m

Mass of the load = m = 8kg

Stress = ?

Strain = ?

Young's modulus = ?

$$\text{Stress} = \frac{F}{A} = \frac{W}{\pi r^2} = \frac{mg}{\pi r^2}$$

$$\text{Stress} = \frac{8\text{kg}\left(9.8\text{ms}^{-2}\right)}{3.14\left(7.5\times10^{-3}\text{m}\right)^2}$$

$$= 4.44\times10^{5}\,\text{N m}^{-2}$$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{2\times10^{-2}\,\text{m}}{0.8\text{m}}$$

$$\text{Strain} = 2.5 \times 10^{-2}$$

$$\text{Young's Modulus(Y)} = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{4.44\times10^{5}\,\text{N m}^{-2}}{2.5\times10^{-2}}$$

$$= 1.78 \times 10^7 \, N\,m^{-2} \text{ or Pa}$$

Example 16.2

A box has a top area of 20cm² and a height of 4cm. When a shear force of 1.5N is applied to the upper surface, the upper surface is displaced by 5mm relative to the bottom surface. What are the shear stress, shear strain and the modulus of rigidity.

Solution:

We have

$$\text{Top area of the box} = A = 20\text{cm}^2 = 0.002\text{m}^2$$
$$\text{Height of the box} = 4\text{cm} = 0.04\text{m}$$
$$\text{Shearing force} = F = 1.5\text{N}$$
$$\text{Upper surface distance} = 5\text{mm} = 5 \times 10^{-3}\text{m}$$
$$\text{Shearing Stress} = ?$$
$$\text{Shearing Strain} = ?$$
$$\text{Modulus of rigidity} = \eta = ?$$

$$\text{Shearing Stress} = \frac{\text{Force}}{\text{Area}}$$
$$= \frac{1.5\text{N}}{0.002\text{m}} = 750\text{Nm}^{-2}$$

$$\text{shearing Strain} = \frac{\text{distance}}{\text{height}}$$
$$= \frac{5 \times 10^{-3}\text{m}}{0.04\text{m}}$$
$$= 0.125$$

$$\text{Modulus of rigidity}(\eta) = \frac{\text{stress}}{\text{strain}}$$
$$= \frac{750\text{Nm}^{-2}}{0.125}$$
$$= 6\text{kPa}$$

16.4 ELASTIC LIMIT AND YIELD STRENGTH

We have studied in the previous section that there is a strong relation between stress and strain, i.e., within elastic limit the applied longitudinal stress is directly proportional to the longitudinal strain. If a graph between stress and strain is plotted, then we have a curved line with different steps. Such curved line is known as stress-strain curve as shown in Fig.16.8. It may be pointed out that this graph represents the tensile stress and the resulting tensile strain. Now we explain the behaviour of solid at various points on the given curved line.

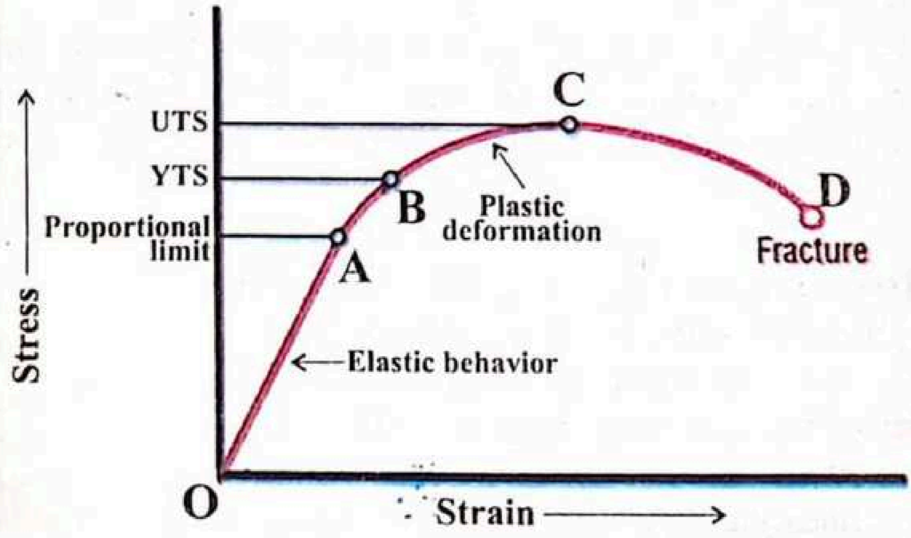


Fig.16.8 Stress vs Strain curved graph for an elastic solid under different stages.

Initially, the graph between stress and strain is a straight line as shown by the section OA of the graph. In this section, stress is directly proportional to the strain and the body obeys Hooke's law. The point 'A' is called the limit of proportionality. If the deforming force is removed at any point within the section OA, the body regains its original position.

If the applied stress exceeds the proportionality limit, the strain is no longer proportional to the stress, but it increases more rapidly. It is represented by the section AB which is a curve and it is called elastic limit. When the stress is removed, the body will return to its original position, because the elastic limit is not crossed.

Similarly, if the stress exceeds the elastic limit, the body deforms permanently. i.e., if the stress is removed the body does not come back to its original position. It is represented by the section BC and it is called plastic limit. The materials become plastic in this region. The point where the stress enters from the elastic region to the plastic region is called yield point and its corresponding stress is known as yield tensile strength (YTS).

If the applied stress is further increased such that it can cross the point 'C', a breaking region starts, i.e., the solids fracture when the stress reaches the breaking point D. On the other hand, the maximum stress that a material can withstands without necking is called the ultimate tensile strength (UTS). The ultimate tensile strength is different for different solids. The materials which continue to stretch beyond its ultimate strength without breaking is called ductile materials, such as gold, silver, copper and lead. These materials can be pulled like a toffee becoming thinner and thinner until finally reaching the breaking point.

The materials whose ultimate tensile strength and breaking point are close together are known as brittle substances e.g., glass, bones cast iron etc. The brighten materials have no plastic deformation. These substances break soon after the elastic limit is reached and they are very strong in compression.

16.4.1 Strain energy in deformed materials

When strain is produced in body by deforming force then there is work done against the elastic restoring force. This work is stored in the body as elastic strain energy and its value can be calculated as; consider a wire of length 'L', cross-section area 'A', whose upper end is fixed while, its lower end is loaded by a weight. So, there is an extension in the length of wire. Within elastic limit, the graph between deforming force and the resulting extension will be a straight line OP as shown in Fig.16.9. Thus within elastic limit, the work done against the elastic restoring force at an extension ΔL under the applied force from 0 to F is equal to area under the straight line OP. i.e.,

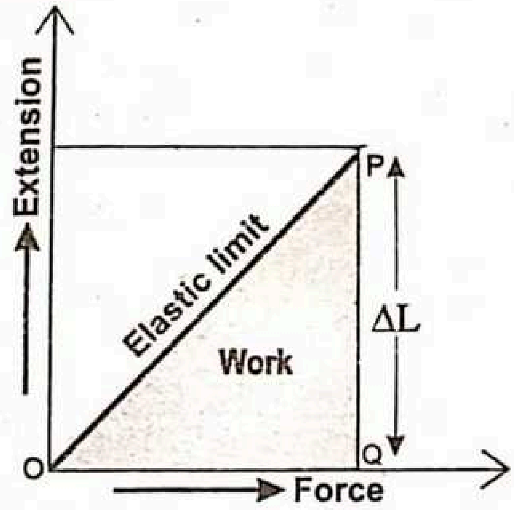


Fig.16.9 Strain energy due to extension of a wire at a distance ΔL.

$$\text{Work} = \text{Area under the straight line OP}$$

$$= \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}F\Delta L$$

$$\text{Work} = \frac{1}{2}F\Delta L$$

> ***POINT TO PONDER***
> If the strain in a wire is doubled, by what factor does the stored energy changed?

But this work is stored in the body in the form of strain energy, therefore,

$$\text{Strain energy} = \frac{1}{2}F\Delta L \quad \ldots\ldots(16.9)$$

Similarly, strain energy per unit volume (AL) called strain energy density of the given wire is given by

$$\text{Strain energy per unit volume } (\mu_0) = \frac{1}{2}\frac{F\Delta L}{AL}$$

$$\mu_0 = \frac{1}{2}\frac{F}{A} \times \frac{\Delta L}{L} \quad \ldots\ldots(16.10)$$

$$\mu_0 = \frac{1}{2}(\text{stress}) \times (\text{strain})$$

According to the definition of Young's modulus

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

Eq.16.10 becomes

$$\mu_0 = \frac{1}{2}Y\frac{\Delta L}{L} \times \frac{\Delta L}{L}$$

$$\mu_0 = \frac{1}{2}Y\left(\frac{\Delta L}{L}\right)^2$$

$$\mu_0 = \frac{1}{2}Y(\text{strain})^2$$

This is a mathematical form of strain energy per unit volume.

Example 16.3

Calculate strain energy of a metal wire of length 0.5m and cross-section area 1cm^2, when the wire is compressed with a force of 60N along its length, the value of Young's modulus for the wire is $1 \times 10^{11}\text{N/m}^2$.

Solution:

Strain energy = ?

Length of the wire = L = 0.5m

Cross-section area of the wire = $A = 1\text{cm}^2 = 1 \times 10^{-4}\text{m}^2$

Deforming force = F = 60N

Young's modulus = $Y = 1 \times 10^{11}\text{Nm}^{-2}$

by definition of strain energy

$$\text{strain energy} = \frac{1}{2}F\Delta L$$

but according to Young's modulus

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{l}}$$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$\therefore \qquad \Delta L = \frac{LF}{YA}$$

$$\text{Strain energy} = \frac{1}{2}F\Delta L$$

Thus,

$$\text{strain energy} = \frac{1}{2}F\cdot\frac{LF}{YA} = \frac{1}{2}\frac{L}{YA}F^2$$

$$= \frac{1}{2}\frac{(0.5\text{m})(60\text{N})^2}{\left(1\times10^{11}\text{Nm}^{-2}\right)\left(1\times10^{-4}\text{m}^2\right)}$$

$$= 9\times10^{-5}\text{J}$$

16.5 ELECTRICAL PROPERTIES OF SOLIDS

Electrically, the solids can be classified into three classes on the basis of their resistivity and conductivity, these are conductors, insulators and semiconductors.

The solids which have high conductivity or low resistivity are known as conductors. e.g., aluminum, copper, silver, gold, etc. The range of conductivity of the conductor is from $10^2(\Omega\text{m})^{-1}$ to $10^8(\Omega\text{m})^{-1}$ while their resistivity is from $10^{-2}\Omega\text{m}$ to $10^{-8}\Omega\text{m}$.

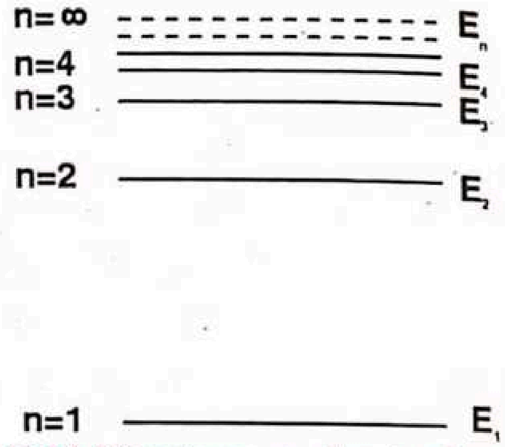
Similarly, those solids which have very small conductivity and very large resistivity are known as insulators, e.g., wood, plastic, rubber, glass, etc. The conductivity of insulators lies between $10^{-11}(\Omega\text{m})^{-1}$ to $10^{-19}(\Omega\text{m})^{-1}$ while their resistivity ranging between $10^{11}\Omega\text{m}$ to $10^{19}\Omega\text{m}$.

On the other hand, those solids which have their conductivity and resistivity in between the conductors and insulators are known as semiconductors. e.g., germanium, silicon etc. The conductivity of the semiconductors is from $10^{-6}(\Omega\text{m})^{-1}$ to $10^4(\Omega\text{m})^{-1}$ and their resistivity is from $10^{-6}\Omega\text{m}$ to $10\Omega\text{m}$.

The free electron theory based on Bohr's atomic model is used to study the electrical properties of solids, but this theory could not explain the conductivity of semiconductors and insulators. Similarly, this theory has also failed to distinguish between metals, semiconductors and insulators. Consequently, we have to use another theory named as band theory. This theory greatly helps us in the understanding of several electrical properties of solids.

Band theory of solid

An isolated atom possesses discrete energy levels such as E_1, E_2, E_3, ... E_n as shown in Fig.16.10. A significant change in the energy levels occurs when a number of identical atoms are brought close together as in solid. For example, if n number of identical atoms are brought close together, the discrete energy levels of individual atoms overlap and form group of energy levels called energy bands. Each band consists of closely spaced energy levels. The individual energies within the band are discrete but so close together, that the energy level may be considered to be a continuous energy band. In the process of formation of bands, there are three bands are formed as shown in Fig.16.11.



16.10 Discrete energy levels of an

The energies of the electrons in the lower states of the atoms are affected very little when the atoms are brought very close together. These electrons remain tightly bound to their nuclei and the band is filled completely by these electrons and plays no part in the electrical conduction.

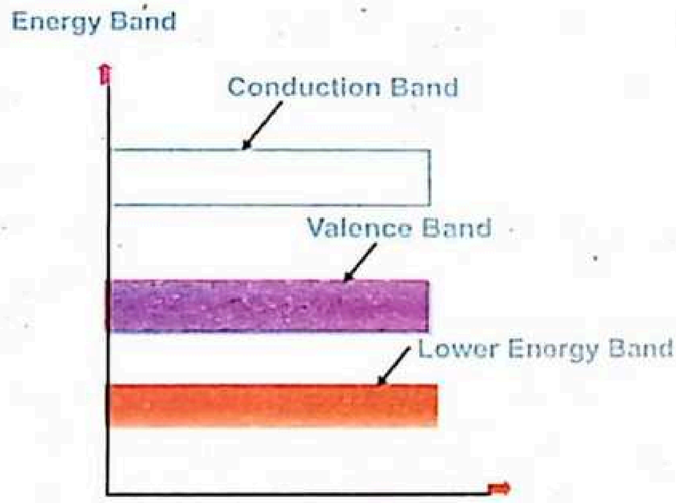


Fig.16.11 Formation of energy bands.

The outer most electrons of an atom are called valence electrons which are most affected during the formation of bands. The band of energy occupied by these valance electrons is known as valence band. It may be partially or completely filled by the electrons but never empty.

The next higher band above the valence band is called conduction band. It may be empty or partially filled with electrons. In conduction band, electrons can move freely which causes conduction in the solids. This is a reason that why such band is called conduction band.

The gap between valence and conduction bands is known as forbidden gap. It has no allowed energy state. On the basis of the band theory of solids, we can explain insulators, conductors and semiconductors, in terms of conduction and valence bands.

Insulators

Insulators are those materials in which valence electrons are bound very tightly to their atoms and they have no free electrons even at high temperature in conduction band.

In terms of band theory of solids, the valence band is completely filled whereas the conduction band is empty. There is a large forbidden gap between valence and conduction bands as shown in Fig.16.12. The forbidden energy band for an insulator is from 5eV to 10eV. So, the electron cannot jump from valence band to conduction band even at high temperatures or when a high potential difference is applied across it.

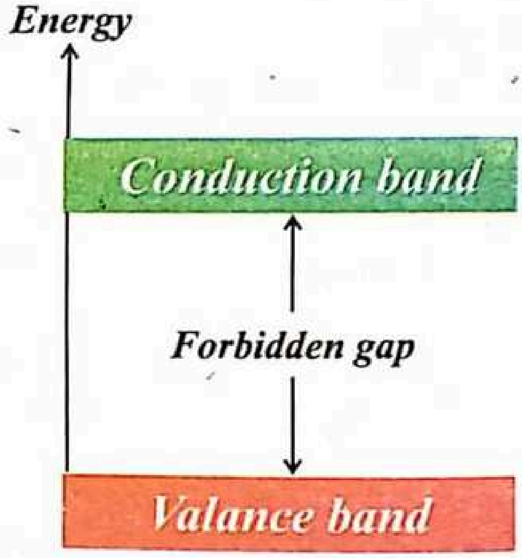


Fig.16.12 In an insulator, a large forbidden gap between filled valence band and empty condition band.

Conductors

Conductors are those substances whose valence electrons are bound loosely with their atoms and they have free electrons available for conduction even at room temperature.

In terms of energy bands of solids, the conduction band of a conductor is partially filled, and its electrons are excited. On the other hand, the valence and conduction band are overlapping. i.e., there is no forbidden energy gap between them as shown in Fig.16.13. Thus, electrons can move easily from valence band to conduction band even when a small potential difference is applied across it or temperature is increased.

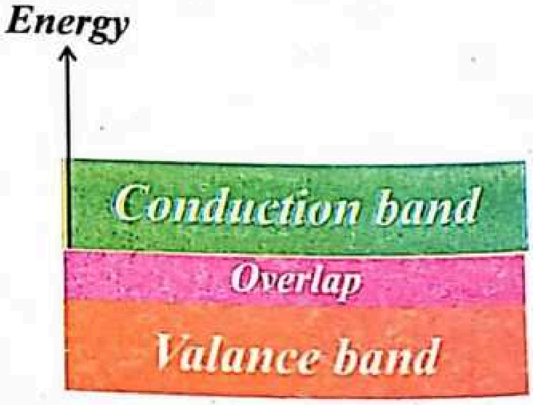


Fig.16.13 In a conductor, partially filled valence band and empty conduction band are overlap to each other.

> *FOR YOUR INFORMATION*
> **Certain materials such as mercury, tin, lead, and vanadium become superconductors when cooled by liquid helium to low temperatures.**

Semiconductors

Semiconductors are those materials which have both characteristics under different conditions. At absolute zero semiconductor have no free electrons and it behaves like an insulator. However, when the temperature is increased, they have free electrons for conduction and hence they act as conductor. In terms of energy bands, the valence band of semiconductor is filled while its conduction band is empty. On the other hand, there is a small forbidden energy gap approximately 1eV between valence and conduction bands as shown in Fig.16.14. At low temperature, the electrons cannot jump from valence band to conduction band, so semiconductors behave as insulator at this condition. When the temperature of semiconductor is raised then, the electrons gain energy and are enabled to jump from valence band to conduction band. This shows that with the increase in temperature, the semiconductors have more free electrons available for conduction. It means semiconductors are conducting more at higher temperature. Thus, semiconductors typically have negative temperature coefficient of resistance. It may be pointed that the number of temperature dependent electrons available for conduction is not sufficient for making a semiconductor device, we will study it in more detailed form in next unit.

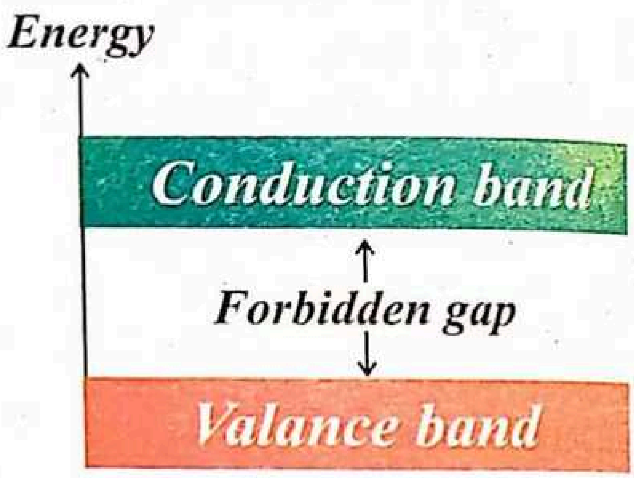
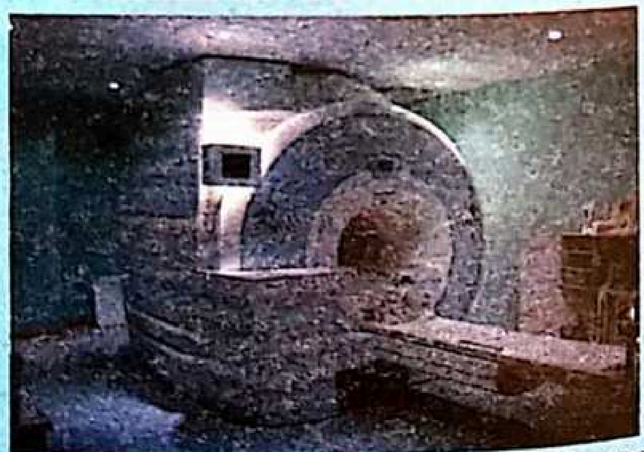


Fig.16.14 An semiconductor there is a small energy gap between filled valence band and empty conduction band.

> *DO YOU KNOW*
> Superconductors are alloys that it certain temperature conduct electricity with no resistance.

INTERESTING INFORMATION



Magnetic resonance imaging (MRI) machine uses strong magnetic field produced by superconducting material for scanning computer processing produce the image of identifying tumors and inflamed tissues.

16.6 SUPERCONDUCTORS

We have studied that the resistance of a conductor depends upon temperature i.e., the resistance of the conductors decreases by decreasing the temperature. The experiments show that resistivity of some materials fall to zero at certain temperature called critical temperature T_c. This phenomenon is known as superconductivity and the materials which exhibit superconductivity are known as superconductors. The superconductivity of a superconductor is shown in resistivity-temperature graph. The resistance at first, decreases smoothly with decreasing temperature, and then at its critical temperature its resistance suddenly drops to zero as shown in Fig.16.15. Superconductivity was discovered in 1911 by Kamerlingh Onnes. He observed that, at low temperature below 4.2K, the resistance of mercury suddenly dropped to zero. Later on, a number of other superconductors were also identified, such as aluminum, tin, zinc, lead and indium alloys. All these with their critical temperatures are listed in Table 16.3. It is very interesting to note that copper, silver and gold which are good conductor, but they do not show any superconductivity.

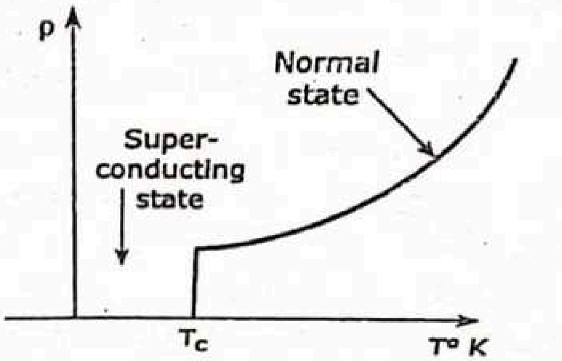
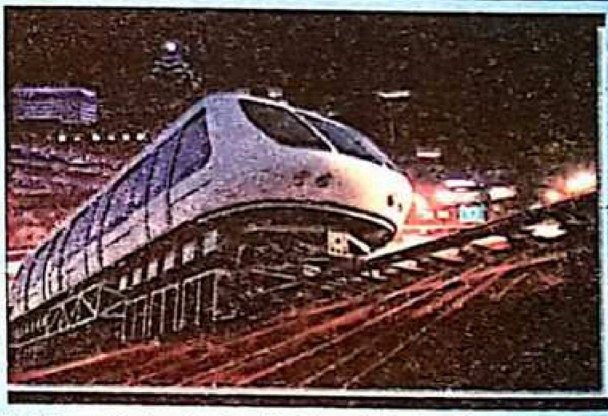


Fig.16.15 At temperature below the critical temperature resistance of the conductor becomes zero.

Table 16.3 Various Superconductors with their critical temperature.

Element	Critical Temperature T_c (K)
Zinc, Zn	0.88
Aluminum, Al	1.14
Tin, Sn	3.69
Mercury, Hg	4.15
Lead, Pb	7.26
Niobium, Nb	9.2

An important development in the field of superconductivity took place in 1986, when new superconductors with higher critical temperature were discovered. For example, the ceramic materials that become superconductor at high temperature of about 125K. Similarly, the most recent identified superconductor is a complex crystalline structure known as Yttrium barium copper oxide whose critical temperature is 163K.

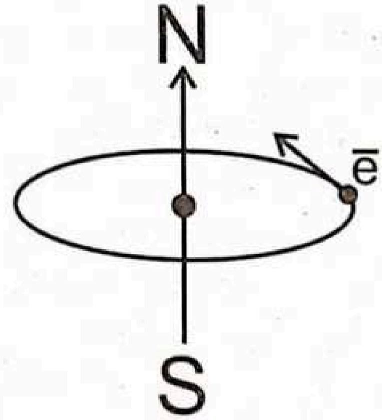


In Magnetic Levitation Train, a (Maglev) is a system of train which consists of a strong magnet that repels and pushes the train up off the track in order to reduce the friction and increase the speed of the train. Such a strong magnetic field is produced on the basis of technology of superconductivity.

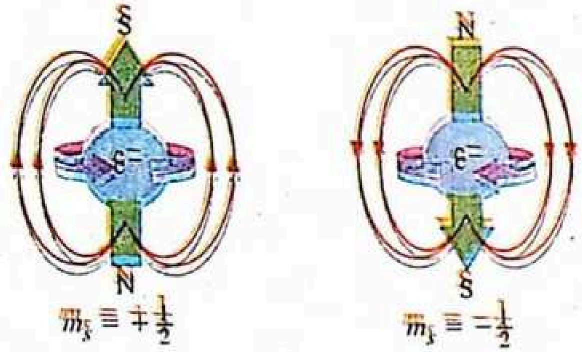
There are number of important and useful applications of superconductor. For example, the development of superconducting magnet which is being used for storage of energy as well as in magnetic resonance imaging (MRI) which is widely used in medical science. Similarly, superconducting technology can also be used for magnetic levitation trains, powerful electric motors, faster computer chips and so many others.

16.7 MAGNETIC PROPERTIES OF SOLIDS

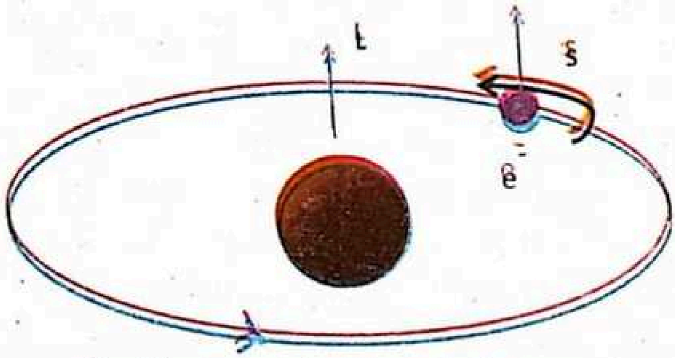
Approximately, all the solids are affected by the applied external magnetic field, i.e., some solids come under the influence of applied magnetic field strongly while the others weakly. We have studied that the electric current through a wire is due to the flow of electrons and hence there is a magnetic field produced around such current carrying the wire. Similarly, in an atom the electrons are revolving around the nucleus in a circular orbits their motion constitute a tiny current which produces small magnetic field. In most solids, the magnetic field of one electron in an atom is cancelled by that of another orbiting electron in the opposite direction. It is therefore, the magnetic field of majority of the solids produced by the orbital motion of the electrons is either zero or very small. That is why most solids do not have magnetic properties.



Orbital magnetic moment of an electron.



Spin magnetic moment of an electron


Combine magnetic moment of an atom.

Besides orbital motion and orbital magnetic field, an electron is also spinning about its own axis. So, there is also a magnetic field associated with the spin motion of electrons. The experiments show that if the spin motion of an electron in an orbit of the atom is clockwise, the motion of the other electron must be anticlockwise. Thus, the atoms which contain pair electrons, the spin magnetic field of one electron is cancelled by the magnetic field of the other electron and the net spin magnetic field of the atom is zero. However, atoms containing odd number of electrons, they have at least one unpaired electron and there is some spin magnetic field.

Thus, the net magnetic field of an atom is equal to the algebraic sum of fields produced by the orbital and spin motions of their electrons such resultant magnetic field is called magnetic dipole. On the basis this magnetic dipole the solids can be classified into three classes.

Diamagnetic materials

Diamagnetic materials are those substances in which atoms have paired electrons and their orbital as well as spin magnetic fields are zero. In other words, diamagnetic materials have no net magnetic moment. e.g., bismuth, copper, zinc, silver, gold, air, water, hydrogen etc.

When a diamagnetic material is brought near to a magnet, it is repelled by the magnet. Similarly, when a diamagnetic material is placed in an applied external magnetic field H, a weak magnetic field B is induced in it. This field inside the diamagnetic material is not only less in magnitude but also in opposite direction to that of the external magnetic field as shown in Fig.16.16. The ratio of magnetic field B inside the diamagnetic material to the external magnetic field H is termed as relative permeability (μ_r). It is expressed as

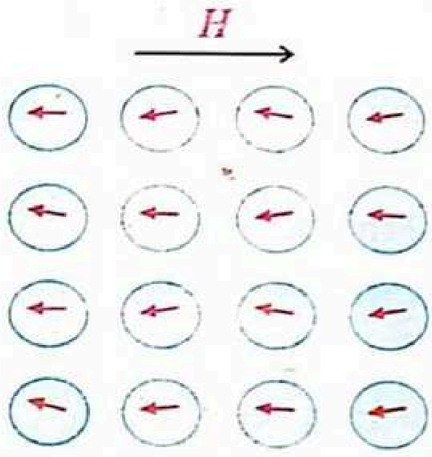


Fig.16.16 The direction of magnetic field inside the diamagnetic material is opposite to the direction of the applied field.

$$\mu_r = \frac{B}{H}$$

This shows that the relative permeability for diamagnetic material is always less than one. i.e., $\mu_r < 1$, because $B < H$. The typical value of μ_r for diamagnetic materials is 0.9998.

Paramagnetic materials

Paramagnetic materials are those in which atoms have one or more unpaired electrons and exhibit a net magnetic moment, e.g., aluminum, antimony, chromium, tungsten, lithium, sodium, oxygen etc.

Each atom of the paramagnetic materials behaves as a tiny magnet. The motions of the atoms in these materials are random as shown in Fig.14.17(a). So, at room temperature they have no net

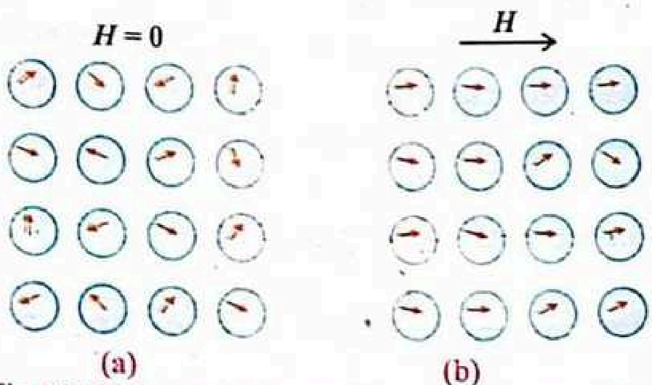


Fig.16.17(a) random motion of atoms of paramagnetic materials in the absence of field. **(b)** alignment of atoms in the presence of field.

magnetic dipole in the absence of an external magnetic field. However, in the presence of magnetic field, the magnetic dipole of the atoms of the paramagnetic materials are aligned as shown in Fig.16.17(b). When a paramagnetic material is brought near to a magnet, it is attracted towards the magnet. Similarly, when a paramagnetic material is placed in an external magnetic field H, the magnetic field B is induced in it, which is slightly greater than the external field H. i.e.,

$$\mu_r = \frac{B}{H}$$

This shows that $\mu_r > 1$, because $B > H$, the typical value of relative permeability for paramagnetic materials is about 1.001.

Ferromagnetic materials

Ferromagnetic materials are those which have one or more unpaired electrons and exhibit a strong magnetic moment. e.g., iron, cobalt, nickel, gadolinium and dysprosium.

Like paramagnetic materials, each atom of a ferromagnetic materials behaves as a tiny magnet. The atoms of ferromagnetic materials cooperate with one another in such a way so as to exhibit a strong magnetic moment. The cooperation of the atoms is in the form of a group in a microscopic region such that all the magnetic dipole are aligned called domain as shown in Fig.16.18. These domains have volumes of about $10^{-12}m^3$ to $10^{-8}m^3$ and contain 10^{17} to 10^{21} atoms.

When a ferromagnetic material is brought near to a magnet, it is strongly attracted towards the magnet. Similarly, when a ferromagnetic material is placed in an applied external magnetic field H, the field B induced inside the ferromagnetic material is stronger than the external field. Thus, the relative permeability for ferromagnetic materials is given by

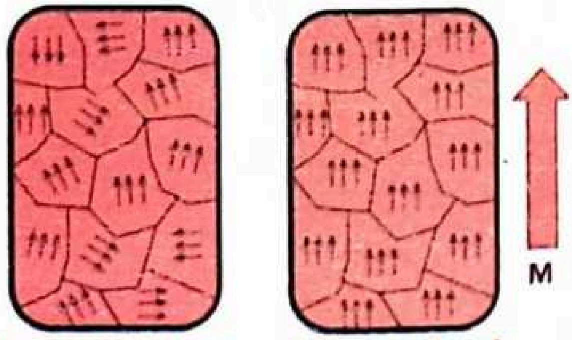
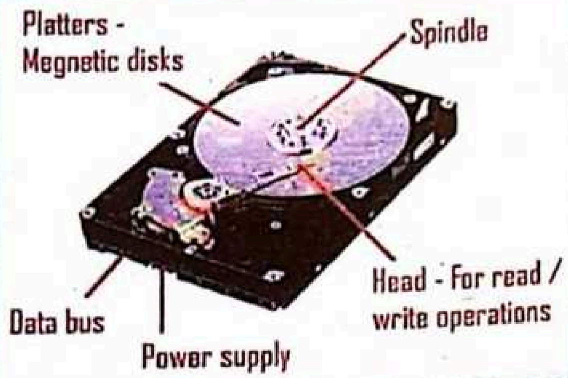


Fig.16.18 Domains of ferromagnetic materials in the presence and the absence of the magnetic field

INTERESTING INFORMATION



A computer hard drive is used to store audio, video or computer data. To record or write, an electro magnet called head is used to magnetize ferromagnetic materials in a coating on the platter of the drive. Ferromagnetic particles retain their magnetization even after the head has removed away.

$$\mu_r = \frac{B}{H}$$

Since B > H, so relative permeability for ferromagnetic material is very large. i.e., its typical value is about 10^4.

Ferromagnetic materials can be classified into two further classes. Hard and soft ferromagnetic materials where hard ferromagnetic materials are made of steel and antimony, they serve as permanent magnet, while, soft ferromagnetic materials are made of soft iron. They are mostly used in motors, fans and other electrical appliances. It has been observed experimentally that ferromagnetism decreases with increase in temperature. When ferromagnetic material is heated it loses its residual magnetism, because random thermal motion tends to destroy the alignment of domains. At certain high temperature where the ferromagnetic property of substance suddenly disappears, and the substance becomes paramagnetic is called Curie temperature. The Curie temperature of various ferromagnetic substance is listed in the table 16.4.

Table 16.4
Curie Temperature of Ferromagnetic Substances

Substances	Curie Temperature
Cobalt	1394K
Iron	1043K
Fe_2O_3	893K
Nickel	631K
Gadolinium	317K

16.8 HYSTERESIS AND HYSTERESIS LOOP

The word hysteresis is derived from Greek word hysterein means lag behind. **Hysteresis is defined as the lagging of induced magnetic flux density B behind the magnetic force 'H' in the process of magnetization or demagnetization of a ferromagnetic substances**. It is explained as;

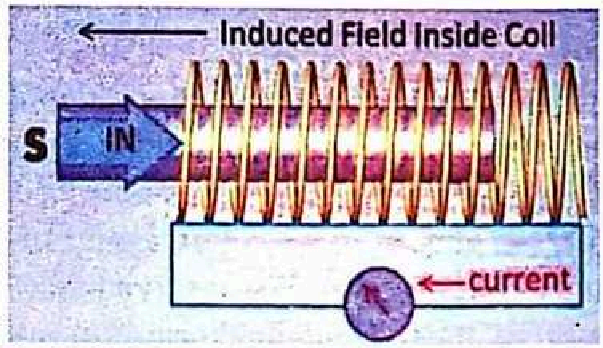


Fig.16.19 Magnetizing and demagnetizing of a ferromagnetic bar inside a solenoid

Consider a bar of ferromagnetic material which is placed inside the solenoid as shown in Fig.16.19. When switch is closed, there is a growth of current in the solenoid. This current produces a magnetic field called magnetizing force 'H' inside the solenoid. The value of 'H' can be increased or decreased by increasing or decreasing the current. On the other hand, the bar also starts magnetizing and its magnetic flux density 'B' increases by increasing the value of 'H'. If the values of 'H' and 'B' is plotted on a graph, a curve line OP is obtained, as shown in Fig.16.20. Where the material becomes magnetically saturated, i.e., it has maximum flux density for H = OM.

Now if 'H' is decreased by decreasing current, magnetic flux density 'B' also starts decreasing but it will not decrease along the line PO, it will decrease slowly, and it is along the line PQ. When H is zero at point Q, but B is not zero. i.e., the bar is not demagnetized. It has some value OQ called remanent or residual flux density B_r. In order to demagnetize the bar, we apply the reverse magnetizing force H. When the reverse value of H is increased, B is reduced to zero at point R where H = OR. This value of magnetizing force H required to wipe off residual magnetism is known as coercive force.

After demagnetization, if the reverse value of 'H' is further increased, the bar again reaches a state of magnetic saturation in opposite direction at point 'S' where H = ON. By repeating the same process, we have another curve line STUP same as that of PQRS but in opposite direction. If we again start the process from point 'U', the same curve UPQRSTU is obtained once again. In the whole process, we have observed that 'H' and 'B' did not attain their zero values simultaneously. Because 'B' always lags behind 'H' and it is named as hysteresis. The closed path PQRSTUP which is obtained during the magnetization and demagnetization of the bar is called hysteresis loop.

The area enclosed by the hysteresis loop represents the energy dissipation. We observe that when H is made zero, the domains of the ferromagnet bar do not become completely unaligned. Thus,

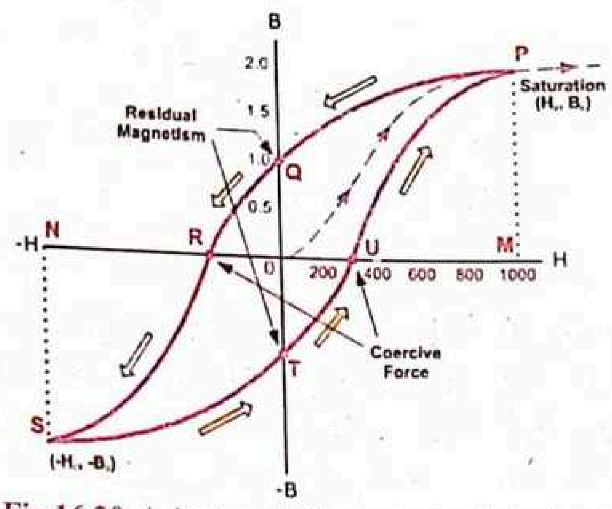


Fig.16.20 A hysteresis loop obtained during the process of magnetization and demagnetization where B is lagging behind H.

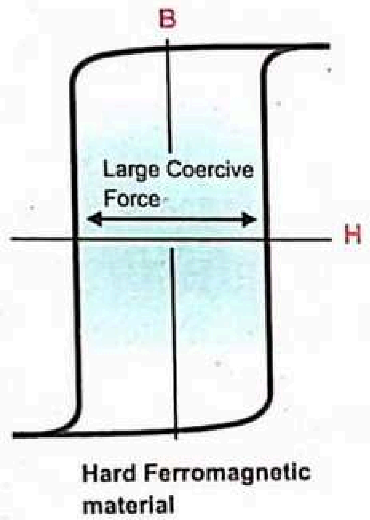


Fig.16.21 Large hysteresis loop area for steel bar.

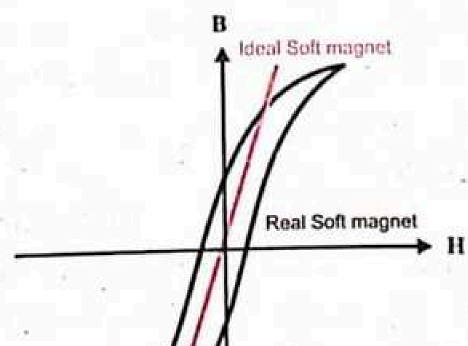


Fig.16.22 Small hysteresis loop area for soft iron.

energy is required against the residual flux density during magnetizing and demagnetizing. This energy is dissipated in form of heat called hysteresis loss.

The shape of the hysteresis loop depends upon the nature of the material. For example, the area of the hysteresis loop for a steel bar is very large due to its high remanent flux density and coercivity as shown in Fig.16.21. Therefore, the hysteresis loss of steel is also large.

On the other hand, the area of the hysteresis loop for soft iron is very small as shown in Fig.16.22. So, the hysteresis loss of soft iron is also small.

SUMMARY

- **Crystalline Solids:** Crystalline solids are those in which atoms, molecules or ions are arranged in regular pattern.
- **Amorphous Solids:** Amorphous solids are those solids which atoms or molecules are arranged in random manner.
- **Polymers:** Polymers solids are those in which atoms, molecules or ions are arranged neither regular like crystalline nor irregular like amorphous. Polymers are also called poor crystalline.
- **Deforming Force:** The applied force which produces a change in length, volume or shape of a body is called deforming force.
- **Stress:** The deforming force acting per unit area of a body is called stress.
- **Strain:** In case of deformation, strain is equal to the ratio of change in dimension to the original dimension.
- **Hooke's Law:** Within elastic limit, the strain is directly proportional to the applied stress.
- **Modulus of Elasticity:** The ratio of stress to strain is called modulus of elasticity.
- **Young's Modulus:** When a change occurs in body's length, then the ratio of longitudinal stress to longitudinal strain is called Young's Modulus.
- **Bulk Modulus:** The ratio of volumetric stress to volumetric strain is called Bulk Modulus.
- **Modulus of Rigidity:** The ratio of shear stress to shear strain is called modulus of rigidity.
- **Elastic Limit:** The limit of deformation where the body comes back to its original position after the removal of the deforming force is called elastic limit.
- **Plastic Limit:** It is the limit of permanent deformation. i.e., the body does not come back to its original position after the removal of the deforming force is called plastic limit.
- **Brittle Substances:** The substances which break down just crossing the elastic limit are called brittle substances.

- **Band Theory:** When a number of identical atoms are combined in a solid substance, than energy levels of individual atoms overlap to form band.
- **Superconductor:** The conductor whose resistance drops to zero by decreasing its temperature is called super conductor.
- **Curie Temperature:** The temperature at which the ferromagnetic materials become paramagnetic materials.
- **Diamagnetic Materials:** The materials whose resultant magnetic moment is zero are called Diamagnetic Materials.
- **Paramagnetic Materials:** The materials which exhibit a weak magnetic moment.
- **Ferromagnetic Materials:** The solids which show a strong magnetic moment.
- **Hysteresis:** The lagging of magnetic flux density 'B' behind the magnetic force H in the process of magnetization or demagnetization is called hysteresis.

EXERCISE

O Select the best option of the following questions:

1. When the temperature increases, the distance between the molecules of a crystal is
   (a) Decreased (b) Increased (c) Same (d) Zero
2. Which one of the following has low melting point?
   (a) Crystal (b) Amorphous (c) Polymer (d) Glassy Solid
3. Which one of the following is called glassy solid?
   (a) Crystal (b) Amorphous (c) Polymer (d) Diamond
4. Which one of the following is a polymeric solid?
   (a) Wool (b) Glass (c) Sodium chloride (d) Copper
5. The applied force which produces a change in a body is called
   (a) Elastic force (b) Deforming force (c) Electric force (d) Magnetic force
6. The property of a body to attain its original position after the removal of force is called
   (a) Plasticity (b) Rigidity (c) Elasticity (d) Resistivity
7. Which one of the following without any dimension and unit?
   (a) Stress (b) Strain
   (c) Tangential Stress (d) Young Modulus
8. Modulus of elasticity is dimensionally equal to
   (a) Strain (b) Shearing Strain (c) Stress (d) Surface tension
9. The Young's modulus 'Y' of an elastic body is due to the applied stress 'S', the energy stored in the body per unit volume is
   (a) $\frac{1}{2}\cdot\frac{S}{Y}$ (b) $\frac{1}{2}\cdot\frac{S^2}{Y}$ (c) $\frac{1}{2}\cdot\frac{Y}{S}$ (d) $\frac{1}{2}\cdot\frac{Y^2}{S}$

10. If the applied stress is increased, then according to Hook's law the ratio of stress to strain will be
(a) Increased (b) Decreased (c) Constant (d) Zero

11. A substance which is permanently deformed by the applied stress is called
(a) Elastic (b) Plastic (c) Ductile (d) Brittle

12. A body which breaks down just crossing the elastic limit is known as
(a) Elastic (b) Plastic (c) Ductile (d) Brittle

13. Which one of the following substance is very strong in compression?
(a) Ductile (b) Brittle (c) Monoatomic (d) Diatomic

14. Resistivity range of insulators is between
(a) $10^{2}(\Omega m)^{-1}$ and $10^{8}(\Omega m)^{-1}$ (b) $10^{-2}(\Omega m)^{-1}$ and $10^{-8}(\Omega m)^{-1}$
(c) $10^{-11}(\Omega m)^{-1}$ and $10^{-19}(\Omega m)^{-1}$ (d) $10^{+11}(\Omega m)^{-1}$ and $10^{+19}(\Omega m)^{-1}$

15. The order of the conductivity of the conductor's is
(a) $10^{2}(\Omega m)^{-1}$ to $10^{8}(\Omega m)^{-1}$ (b) $10^{-2}(\Omega m)^{-1}$ to $10^{-8}(\Omega m)^{-1}$
(c) $10^{-11}(\Omega m)^{-1}$ to $10^{-19}(\Omega m)^{-1}$ (d) $10^{+11}(\Omega m)^{-1}$ to $10^{+19}(\Omega m)^{-1}$

16. Which one of the following substance has partially filled valence band?
(a) Insulator (b) Conductor (c) Semiconductor (d) Superconductor

17. The forbidden energy gap between valance and conduction bands in semiconductor is upto
(a) 1eV (b) 2eV (c) 4eV (d) 6eV

18. When the temperature is increased, the conduction in the semiconductor material is
(a) Decrease (b) Increase (c) Same (d) Zero

19. Which one of the following does not exhibit the superconductivity?
(a) Mercury (b) Copper (c) Aluminum (d) Zinc

20. Which one of the following is not a magnetic substance?
(a) Iron (b) Nickel (c) Brass (d) Cobalt

21. Which substance has relative permeability less than 1?
(a) Diamagnetic (b) Paramagnetic (c) Ferromagnetic (d) None of those

22. When a diamagnetic material is placed in an external magnetic field , the direction of field induced in the material is
(a) Along the direction of external field
(b) Opposite to the direction external field
(c) Perpendicular to direction of external field
(d) None of these

23. The relative permeability of an iron is
(a) 0.9999 (b) 1.001 (c) 10^{2} (d) 10^{4}

24. Which substance shows a strong magnetic moment due to the co-operation of its atoms to each other?
(a) Diamagnetic (b) Paramagnetic (c) Ferromagnetic (d) Bismuth

25. The temperature at which the ferromagnetic substance becomes paramagnetic substance is known as
(a) Critical temperature (b) Curie temperature
(c) Absolute temperature (d) Normal temperature

26. When the area of hysteresis loop is small, energy dissipation is
(a) Small (b) Large (c) Uniform (d) Zero

SHORT QUESTIONS

1. Why solids have definite shape and volume?
2. How atoms, molecules or ions are arranged in a crystal?
3. Why frozen amorphous has same property as that of a liquid?
4. Why amorphous is also called glassy solids?
5. Why polymer is also called poor crystalline?
6. Is stress different from pressure?
7. Differentiate among tensile, compressive and shear strains.
8. Define Hooke's law in terms of stress and strain.
9. What do you know about the co-efficient of elasticity?
10. Differentiate between elastic and plastic limits.
11. Differentiate yield and breaking points of a solid.
12. What do you know about the ultimate strength of a solid?
13. Distinguish between brittle and ductile substances.
14. Which is more elastic rubber or iron?
15. Differentiate between valence and conduction bands.
16. What do you know about the forbidden gap?
17. What is critical temperature?
18. How spin and orbital motions of electrons cause of magnetic moment of an atom?
19. Why the magnetic moment of diamagnetic material is zero?
20. Distinguish between paramagnetic and ferromagnetic materials.
21. What is Curie temperature?
22. What do you know about hysteresis and hysteresis loop?
23. What is hysteresis loss?
24. What do you know about domains?
25. What is residual flux density?
26. What do you know about the coercive force?

COMPREHENSIVE QUESTIONS

1. Discuss the three classes of solids: crystal, amorphous and polymeric with examples.
2. What do you know about the mechanical properties of solids? Explain it with examples.
3. State and explain stress, strain and their kinds with suitable examples.
4. Define Hooke's law in terms of stress and strain. Also discuss the three kinds of elastic moduli.
5. Explain graphical representation of Hooke's law and define the terms: Elastic limit, plastic limit, yield strength and ultimate strength.
6. What is meant by the strain energy? Derive the mathematical relation for strain energy.
7. What is band theory of solids? Explain insulator, conductor and semiconductor by using band theory of solid.
8. Define superconductor and explain graphical representation of superconductivity.
9. Discuss magnetic properties of solids and explain the three classes of solids diamagnetic, paramagnetic and ferromagnetic.
10. State and explain hysteresis and hysteresis loop.

NUMERICAL PROBLEMS

1. An iron rod 4m long and $0.5cm^2$ in cross section area stretches by 0.5mm when a mass of 115kg is hung from its lower end. Compute stress, strain and Young's modulus. **(2.254×10^{11}Pa, 1.25×10^{-4}, 1.8×10^{11}Pa)**
2. Calculate change in volume of a copper cube 40mm on each edge, when subjected to a pressure of 2MPa. The bulk modulus for copper is 125GPa. **($1 \times 10^{-9}\ m^3 = 1mm^3$)**
3. A metallic wire of length 15m is stretched 6×10^{-9}m by the applied stress of $5 \times 10^8 Nm^{-2}$. Calculate the strain energy per unit volume in the wire. **(0.1J)**
4. A solid cylindrical steel column is 6m long and 10cm is diameter. What will be its increase in two length when carrying a load of 2×10^5kg(Young Modulus for steel is 1.9×10^{11}Pa) **(7.88mm)**
5. Two parallel and opposite forces, each 4000N are applied tangentially to the upper and lower faces of cubical metal block 25cm on a side. Find the angle of shear. (Shearing modulus is 80GPa) **(8×10^{-5}rad.)**
6. Calculate change in volume of a solid cube 50mm on each edge, when subjected to a stress of 25MPa. (The bulk modulus for copper is 125GPa) **($25mm^3$)**

Unit 17

ELECTRONICS

Major Concepts (16 periods)

- Intrinsic and extrinsic semiconductors
- P & N types substances
- Electrical conductivity by electrons and holes
- PN junction
- Forward and reverse biased PN junction characteristics
- Half and full wave rectification
- Uses of specially designed PN junctions
- Transistor and its characteristics
- Transistor as an amplifier (C-E configuration)

Conceptual Linkage

This chapter is built on
Introductory Electronics
Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

- distinguish between intrinsic and extrinsic semiconductors.
- distinguish between P & N type substances.
- explain the concept of holes and electrons in semiconductors.
- explain how electrons and holes flow across a junction.
- describe a PN junctions and discuss its forward and reverse biasing.
- define rectification and describe the use of diodes for half and full wave rectifications
- distinguish PNP & NPN transistors.
- describe the operations of transistors.
- deduce current equation and apply it to solve problems on transistors.
- explain the use of transistors as a switch and an amplifier.

INTRODUCTION

Electronics is the branch of Physics in which we study about the emission, flow behaviour, effects and control of electrons under the action of some devices, such as, diode, transistor etc. These devices are called semiconductor devices because the role of semiconductor materials is very important in fabrication of these devices. For example, when P-type and N-type semiconductors are prepared in the form of a single crystal such that its one half is P-type and the other half is N-type. Then the region dividing these two types is called PN-junction. The PN-junction is the first step towards the fabrication of semiconductor devices, such as diode, transistor, integrated circuits (ICs) etc. For example, a semiconductor diode consists of one PN-junction. Similarly, a transistor consists of two PN-junctions and so on. These semiconductor devices can be used as amplifier, filter, rectifier, oscillator, a switch and so many others. In this unit, we will study intrinsic and extrinsic semiconductors, various semiconductor devices and their fabrications. We will also explain the working principle, function and application of these semiconductor devices.

17.1 INTRINSIC AND EXTRINSIC SEMICONDUCTORS

Semiconductors are classified into two classes:

I. Intrinsic Semiconductor II. Extrinsic Semiconductor

I. Intrinsic semiconductors

An intrinsic semiconductor also called an undoped semiconductor is a pure semiconductor without any significant impurity or dopant species added.

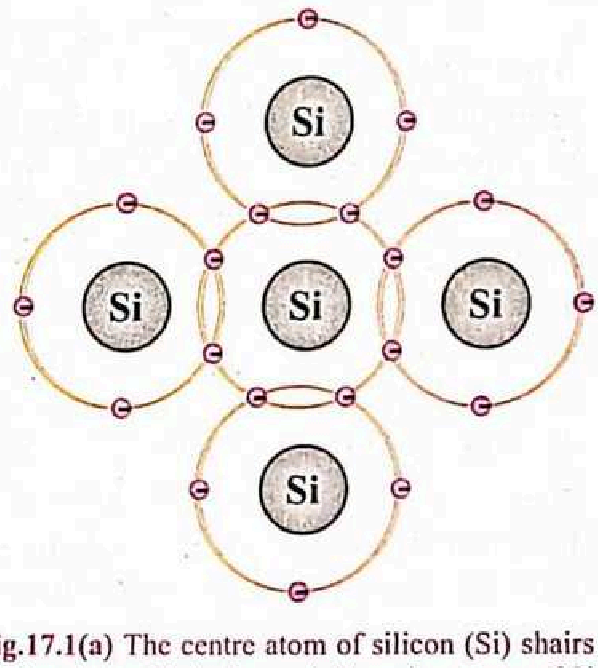


Fig.17.1(a) The centre atom of silicon (Si) shairs its electrons with its four neighbouring atoms of Si and the number of electrons in outermost shell become eight.

In the previous unit, we have studied that the resistivity of semiconductor materials lies between insulators and conductors. For example, germanium (Ge) and silicon (Si) are semiconductor materials. In pure form and at low temperature i.e., at 0K, they act as insulators. Germanium and silicon have crystalline structure and these materials are tetravalent, i.e., each atom has four valence electrons in its outermost shell. Each atom shares its four valence electrons with each of its four neighbouring atoms as shown in Fig.17.1(a).

In this way, the number of electrons in the outermost shell become eight and their atoms make covalent bond to one another as shown in Fig. 17.1(b). This arrangement gives a very stable electronic configuration to semiconductor materials. At absolute zero temperature, the covalent bonds among the atoms are very strong and there are no free electrons. Thus, in this condition the semiconductor behaves as a perfect insulator. Similarly, in terms of energy band, valence band is completely filled, and conduction band is empty. Though the forbidden gap between valence and conduction bands is very small yet, there are no free electrons available to jump from valence band to conduction bond as shown in Fig.17.2. Therefore, semiconductor behaves like insulator at low temperature.

When the temperature is raised even at room temperature, some covalent bonds in semiconductor break down the electrons become free and leaving vacancies in the valence band called holes where holes act as positive charges. Let the covalent bond is broken and an electron is free from site A and it leaves behind a hole. The electron at site B may jump into the hole at site A. Another hole is created at site B. Similarly, another electron at site C may jump into the new hole at site B and so on. Due to the movement of electrons, the hole appears at site G while the electron moves from G to A as shown in Fig.17.3. Thus, both electrons and holes are movable charges and they contribute to conduction in semiconductor material.

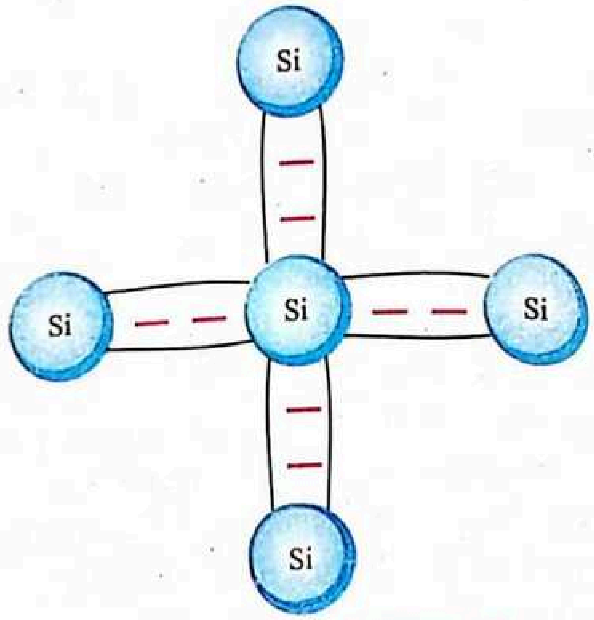


Fig.17.1(b) The atoms of silicon (Si) make covalent bond to one another.

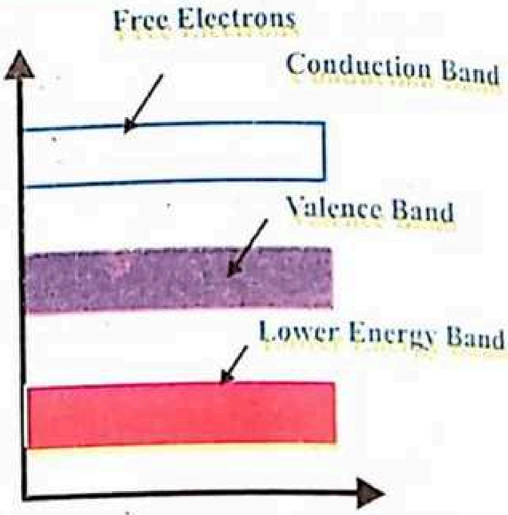


Fig.17.2 Valence band has no free electrons.

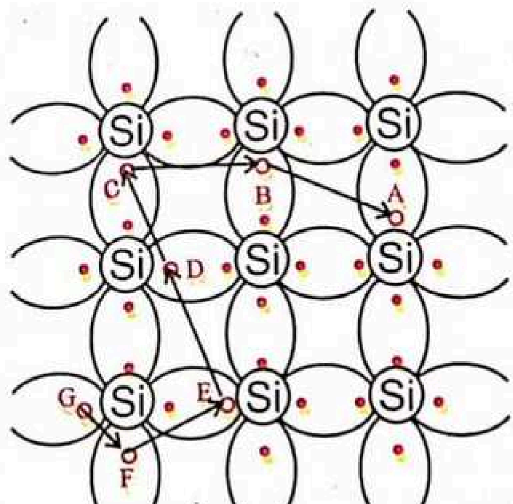


Fig.17.3 Moment of electrons and holes in a crystal structure of a semiconductor

In terms of energy band, when the temperature is increased, the valence electrons can gain enough thermal energy to jump from valence band to conduction band as shown in Fig.17.4. The number of electrons from valence band to conduction band depends upon the temperature of the semiconductor. It is a reason that semiconductor have negative temperature co-efficient of resistance.

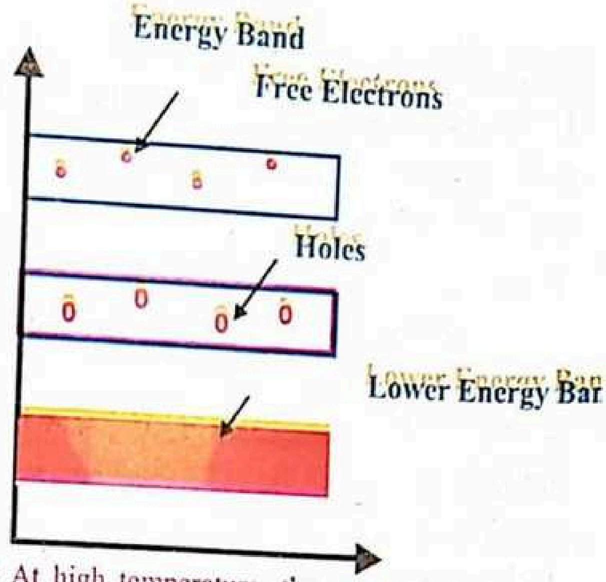


Fig.17.4 At high temperature, the energy bands have electrons and holes which cause of conduction

II Extrinsic semiconductors

The electrical conductivity of intrinsic semiconductor is small at room temperature. The conductivity of semiconductor can be improved by adding impurity of either pentavalent or trivalent atoms into a pure semiconductor. The process of adding impurity to a pure semiconductor is known as doping, and the doped semiconductors are known as extrinsic semiconductors. The doping should be done under a specific ratio of $1:10^8$, i.e., there should be only one atom of impurity in 10^8 atoms of pure semiconductor. The extrinsic semiconductors are classified into two classes.

i. N-type semiconductors ii. P-type semiconductors

N-type semiconductors

When an impurity of pentavalent (valency 5) element like arsenic (As), antimony (Sb), phosphorous (P) etc is added to a pure semiconductor (germanium, silicon) in a specific ratio then such doped semiconductor is called N-type semiconductor. An impurity of pentavalent element like phosphorous is added to a pure silicon. The phosphorus has five valence electrons in its outermost shell and silicon has four valence electrons. Therefore, the four valence electrons of phosphorus atom form covalent bonds with the four neighbouring silicon atoms. As there is no room in the Si crystal for the fifth electron of the phosphorus atom thus it becomes free as shown in Fig.17.5. It means each added phosphorus atom provides a free electron. In this way we have a number of free electrons which cause

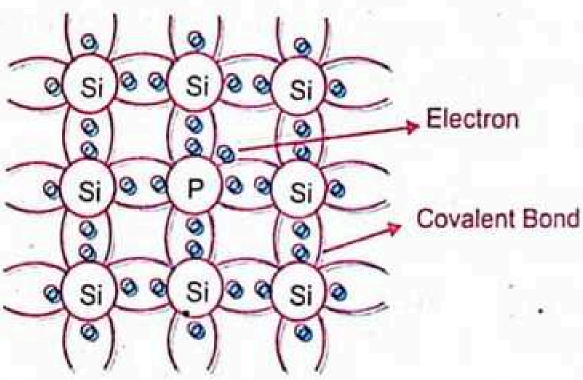


Fig.17.5 Doping of Silicon Si atoms with Phosphorus P, their covalent bonds have one free electron.

of conduction in such extrinsic semiconductor. Since pentavalent elements provide free electrons to the semiconductor crystal, so this type of extrinsic semiconductor is called donor or N-type semiconductor. In N-type semiconductors free electrons are majority charge carriers while holes are treated as minority carriers.

P-Type semiconductors

When an impurity of trivalent element (valency 3) like boron (B), gallium (Ga), Indium (In) etc. is added to a pure semiconductor (germanium, silicon) in a specific ratio then such doped semiconductor is called P-type semiconductor. To explain the formation of P-type semiconductor, consider a trivalent impurity like boron (B) which is added to a pure silicon. Boron has three valence electrons in its outermost shell while silicon has four electrons. Therefore, the three valence electrons of boron form covalent bonds with four neighbouring silicon atoms, this leaves one of the four silicon atoms with an unsatisfied bond i.e., leaves a vacancy called holes as shown in Fig.17.6. Since one added atom of boron provides one hole, so, a small amount of boron added to pure semiconductor provides a number of holes. These hole act as positive charge carrier and cause of conduction in the extrinsic semiconductor called P-type semiconductor. As the created hole accepts the electrons, so the P-type semiconductors are also called acceptor semiconductors. In P-type semiconductor holes are majority charge carriers while electrons while minority charge carriers.

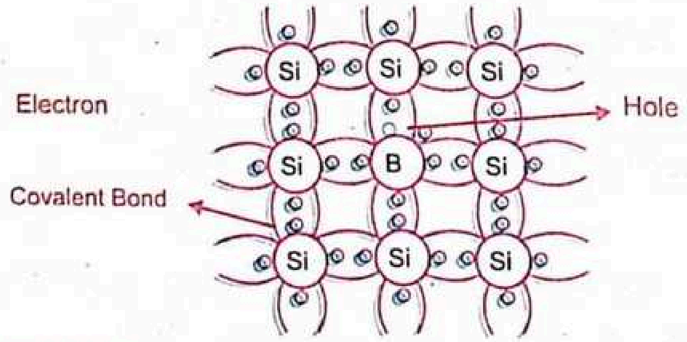


Fig.17.6 Doping of Silicon Si atoms with Boron B, their covalent bonds have one free hole.

17.2 THE PN JUNCTION

In this case semiconductor materials (Silicon or Germanium) are fabricated in such a way that its one half is doped by P-type impurities and the other half by N-type impurities. The boundary dividing the two halves as shown in Fig.17.7 is known as PN-junction. The PN-junction has important role in the fabrication of semiconductor devices such as diode, transistor, solar cell etc.

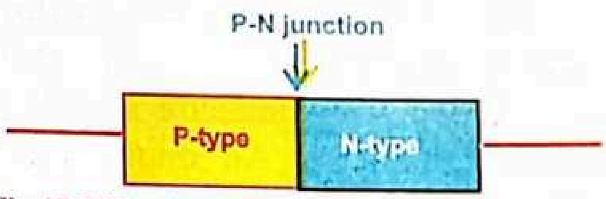


Fig.17.7 The formation of PN-Junction.

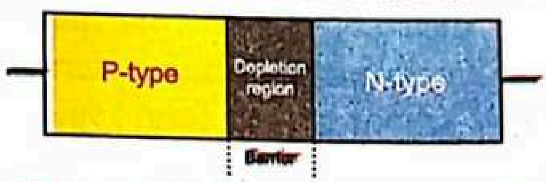


Fig.17.8 A deflection region layer at the PN-junction.

We have discussed that P-type semiconductor has holes as majority charge carriers and N-type semiconductor has free electrons as majority charge carriers. Therefore, there is a great concentration of holes in P-type than in N-type. Similarly, concentration of electrons is greater in N-type than in P-type. Thus, there is diffusion of majority carriers across the junction and they are allowed to recombine to one another. That is, holes diffuse from P-type to N-type. So P-type losses holes and this creates a region of negative charges called immobile negative charges. Similarly, electrons diffuse from N-type to P-type and N-type losses electrons. This creates a region of positive charges called immobile positive charges. These two regions of positive and negative immobile charges produce the narrow region at the junction called depletion region as shown in Fig.17.8. Once the depletion region is formed then further diffusion of charge carriers across the junction stops. Thus, one can say that depletion layer acts as a barrier to the moment of charge carrier across the junction. The width of the depletion region depends upon the concentration of the majority carriers i.e., more the carriers concentration less is the width of the depletion layer and vice versa.

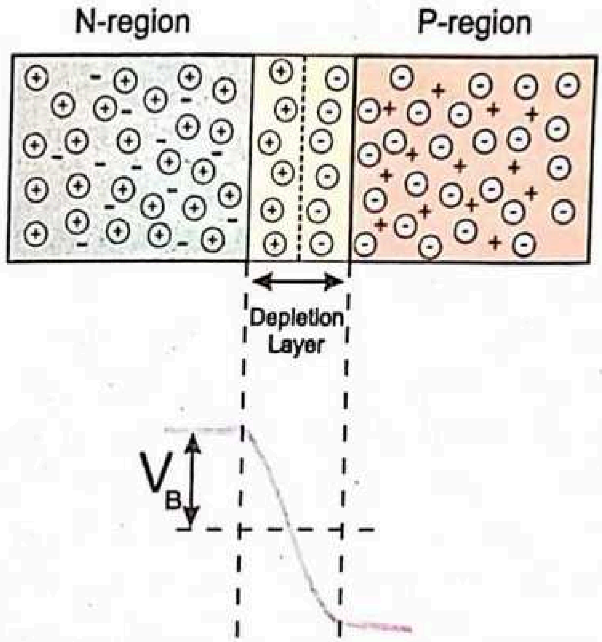


Fig.17.9 Applied potential V_o across the PN-junction

The depletion region consist of two oppositely immobile charged layers on its two sides. So the separation of these oppositely charged layers causes of potential barrier across the junction which is called junction or barrier potential as shown in Fig.17.9. Barrier potential depends upon level of doping, temperature and nature of materials. The typical value of potential barrier is about 0.3V for germanium and 0.7V for silicon.

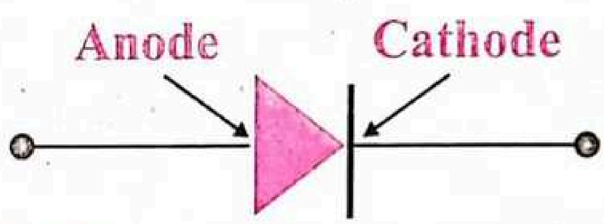


Fig.17.10 A symbol of semiconductor diode. Arrow represents P-type and bar N-type.

The PN-Junction is indeed an electronics device, named as a junction diode or semiconductor diode. The symbol of semiconductor diode is shown in Fig.17.10, which consists of an arrow and a bar. The arrow represents P-type and it is called anode, while bar represents N-type and it is called cathode.

17.3 BIASING A PN-JUNCTION

The process of applying an external voltage across the PN-Junction or any other semiconductor device is called biasing. In case of PN-Junction, there are two types of biasing.

I Forward biased II Reverse biased

I Forward biased PN junction

When a PN-Junction is connected to a battery (source) such that the positive terminal of the battery is connected to P-type and the negative terminal of the battery is connected to N-type as shown in Fig.17.11, then such biasing is known as forward biased PN-Junction. In forward biased, the majority carriers of each region are repelled by the terminals of the battery towards the junction. Thus, the width of depletion region is further decreased, which results in less resistance across the junction. When the potential difference is applied across the junction then at certain value of voltage i.e., 0.7V for Si and 0.3V for Ge, the majority charge carrier gains enough energy to cross the junction. Thus, the flow of current starts in the circuit called forward biased current whose value can be increased by increasing the applied voltage.

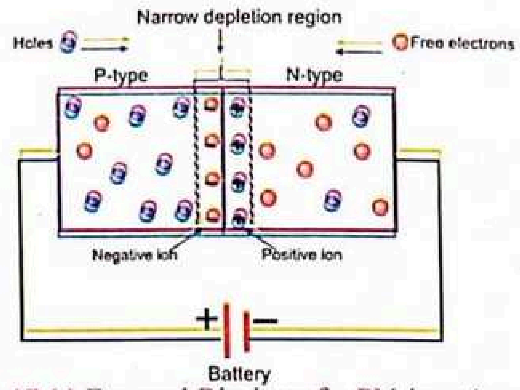


Fig.17.11 Forward Biasing of a PN-junction.

Forward biased characteristics of the PN-junction

The graph between the applied forward voltage across the PN-junction diode and flow of current through the diode is known as forward biased characteristics of the PN-junction diode.

A schematic circuit diagram for forward biased PN-Junction diode is shown in Fig.17.12. When the applied voltage across the diode is increased, the current through the diode also starts increasing which can be observed on a voltmeter and a milli-ammeter. Now when different values of the applied voltage and current are drawn on a graph then we have a curved line OPQ called forward bias

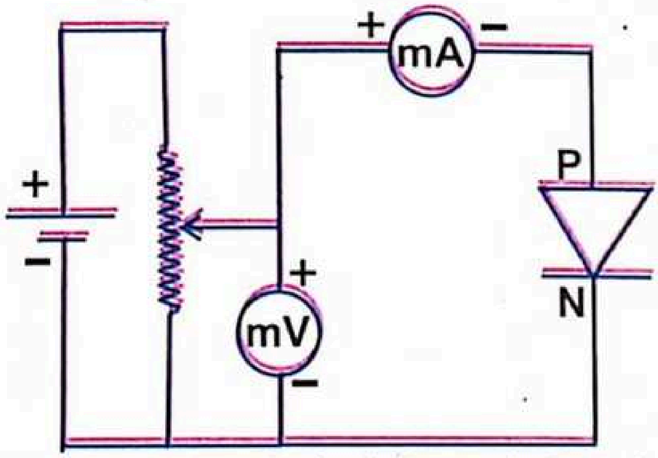


Fig.17.12 A schematic circuit diagram for forward biased characteristics of a PN-junction diode.

characteristics of a diode as shown in Fig.17.13. The reciprocal of the slope of this curved line is equal to the resistance of the diode. i.e.,

$$r = \frac{1}{\left(\frac{\Delta I}{\Delta V}\right)}$$

$$r = \frac{\Delta V}{\Delta I} \quad ......(17.1)$$

The nature of such curved line is explained as; initially when the applied voltage is increased from zero value, the current also increases but at very slow rate due to the barrier potential. It is represented by the section OP of the curved line, where the barrier potential is 0.3V for Ge and 0.7V for Si. This voltage is also called knee voltage. Once the applied voltage exceeds the knee voltage, the current through the diode increases rapidly. It is represented by the section PQ of the curved line. Below the knee voltage, we have a curve line but above the knee voltage, there is a straight line. i.e., where diode behaves like an ordinary conductor.

FOR YOUR INFORMATION

- Electrical circuit has passive components (resistor, capacitor, inductor etc.) that converts electrical energy into other form of energy such as: light, heat, sound, etc.
- Electronic circuit has active components (diode, transistor etc.) that controls the flow of electrons for particular task such as amplifier, rectifier, oscillator, etc.

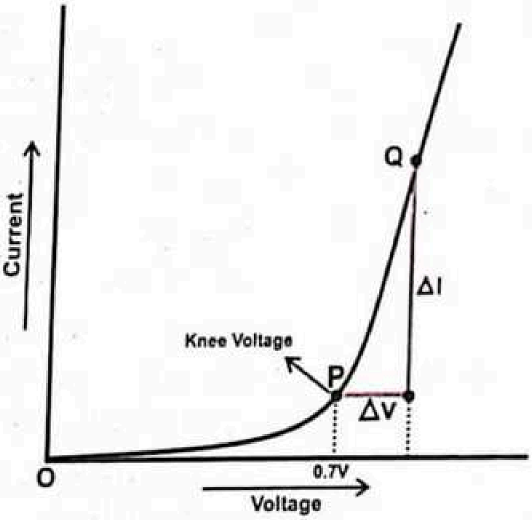


Fig.17.13 A forward biased characteristics of a diode due to the applied voltage V and flow of current I.

II Reverse biased

When PN-Junction is connected to a battery in such a way that its P-type is connected to negative terminal of the battery and its N-type is connected to positive terminal of the battery as shown in Fig.17.14 Then such biasing is known as reverse biased PN-junction. In reverse biasing, the majority carriers of each sides of the PN-Junction are attracted away from the junction by the terminals of the battery. Therefore, the width of the depletion region is further increased, and

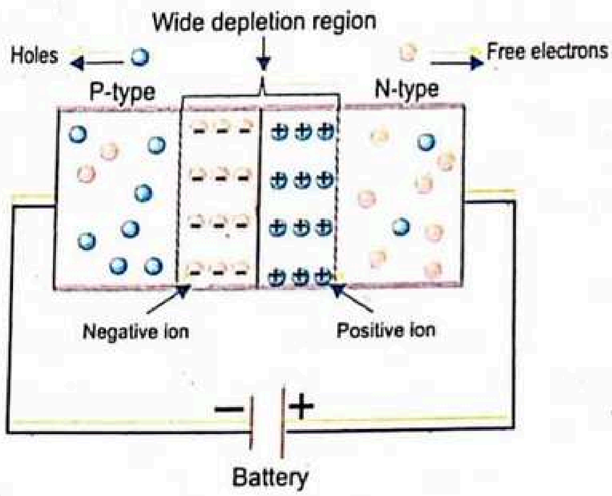


Fig.17.14 Reverse biasing of a PN-junction.

the junction offers a high resistance. Thus, there is no flow of current through a diode due to the majority carriers. However, a very small current of the order of micro-amperes flows due to minority carriers called reverse current which can be neglected.

Reverse bias characteristics

The graph between the applied reverse voltage across the PN-junction diode and the reverse current through the diode is known as reverse biased characteristics of the PN-junction diode.

A schematic circuit diagram for reverse bias PN-Junction diode is shown in Fig.17.15. The reverse voltage across the diode is increased from zero value and its corresponding value of reverse current is very small i.e., of the order of microampere. It is due to the reverse biasing of a diode, there is high resistance across the junction. This small reverse current is due to the minority carriers and graphically it is represented by the section OPQ as shown in Fig.17.16. When the reverse voltage is increased beyond the limit at point P, the minority charge carriers gain large kinetic energy and they may break the junction of the diode. If the junction of the diode breaks, the reverse current increases very rapidly. This large reverse voltage is called breakdown voltage. After breaking the junction, the reverse current increases very sharply. It is represented by the section PQ as shown in Fig.17.16.

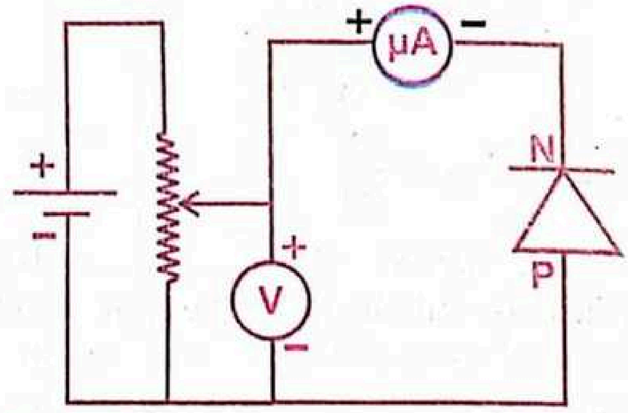


Fig.17.15 A schematic circuit diagram for reverse biased characteristics of a PN-junction diode.

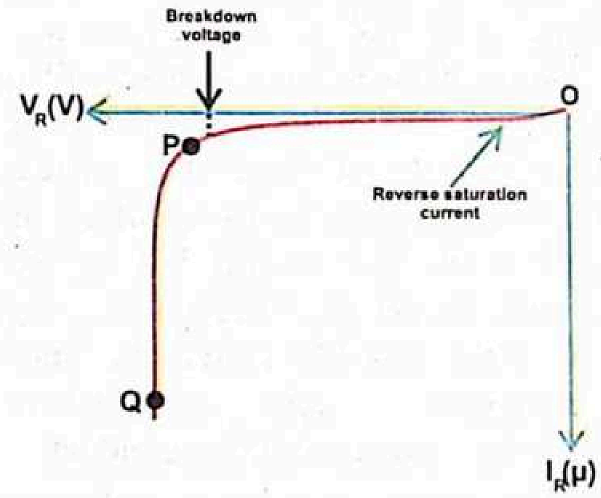


Fig.17.16 Reverse biased characteristics of a diode due to the applied reverse voltage V and reverse current I.

Example 17.1

How much is the forward current for 0.7V of forward voltage when the resistance across the PN-junction is 5Ω.

Solution:

Forward current = I_F = ?

Forward voltage = V_F = ?

Resistance across the junction = r = 5Ω

As $V_F = I_F r$

$$I_F = \frac{V_F}{r} = \frac{0.7V}{5\Omega} = 140mA$$

17.4 RECTIFICATION

Majority of the electronic devices like radio, TV, computer etc., require the D.C. sources for their operation. The D.C. sources are cells, batteries etc. These D.C. sources are not only expensive and short-range supply but their voltage are also low. On the other hand, the supply of A.C. sources are long range with high voltage and its cost is also very low. Therefore, A.C. is more useful then D.C. and it can be applied to the electronic circuit only when it is converted into D.C. In this regard, we have a rectifier circuit which is used to convert A.C. into D.C. by using the property of a diode. i.e., a diode allows current to pass only in one direction, i.e. when it is forward biased.

There are two types of rectifications

I Half wave rectification II Full wave rectification

I Half wave rectification

The schematic circuit diagram for half wave rectification is shown in Fig.17.17. The circuit consists of an A.C. source, transformer, diode D, and a load

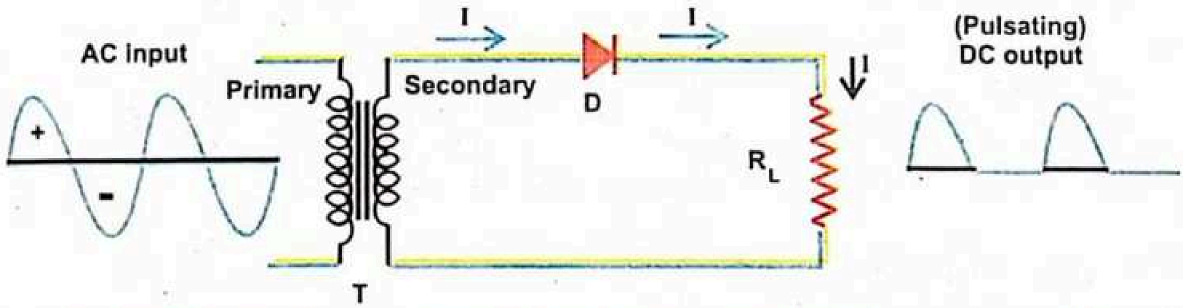


Fig.17.17 A schematic circuit diagram for half wave rectification using a single diode.

resistor R_L connected in series with the diode.

As we know that A.C. signal consists of a positive and a negative half cycle. When A.C. signal is applied to the input of rectifier circuit, during positive half cycle the anode of the diode becomes positive i.e., it makes diode forward biased and the diode conducts, so there is voltage drop across the load resistor R_L.

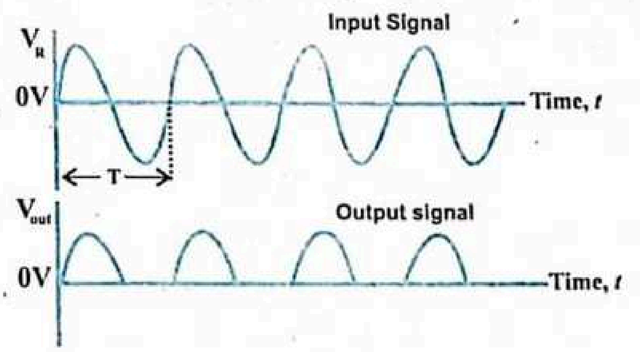


Fig.17.18 Input and output signals of half wave rectification due to a single diode.

During the negative half cycle of A.C. the anode of the diode becomes negative i.e., it makes the diode reverse biased and the diode does not conduct. Thus, there is no voltage drop across the load resistor R_L. For the next A.C. cycle the same process is repeated and so on. Hence at the output we have half wave rectification as shown in Fig.17.18. The result shows that there is no smooth D.C. signal at the output but there is a pulsating D.C. signal. Therefore, this pulsating output signal can further be smoothened by using the filter circuit.

II Full wave rectification

One of the disadvantages of the half wave rectification is that the power of half single is wasted, about 50%. To overcome this problem, we introduce a full wave bridge rectifier. It is used most frequently in electronic circuits. A schematic circuit diagram for full wave bridge rectifier is shown in Fig.17.19(a). The circuit consists of A.C. source, transformer, four diodes connected in the form of bridge loop and a load resistor R_L.

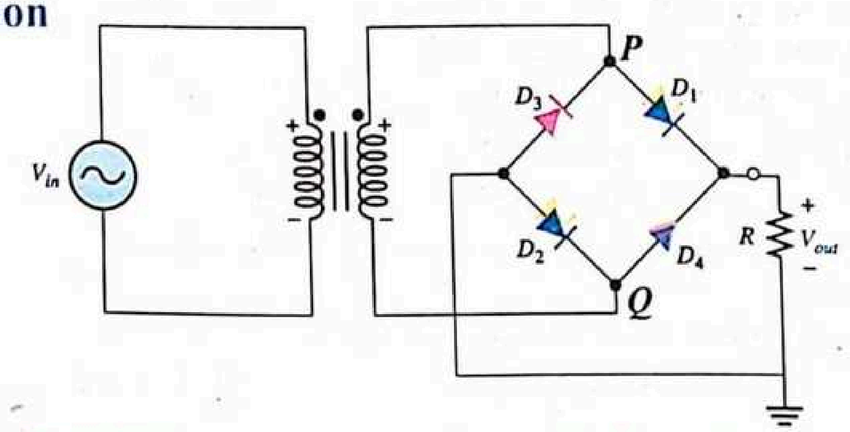


Fig.17.19(a). A schematic circuit diagram for full wave bridge rectification using four diodes.

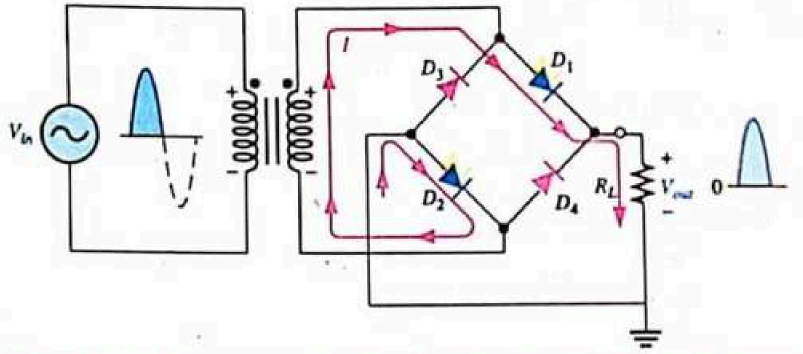


Fig.17.19(b). Due to positive half cycle, D_1 and D_2 are forward biased and conduct current, while D_3 and D_4 are reversed biased.

When A.C. is applied then due to positive half cycle node P becomes positive and node Q negative. Therefore, the diodes D_1 and D_2 become forward biased and they conduct during first half of A.C., while, the diodes D_3 and D_4 becomes revers biased and they do not conduct as shown in Fig.17.19(b). Thus, there is voltage drop across the load

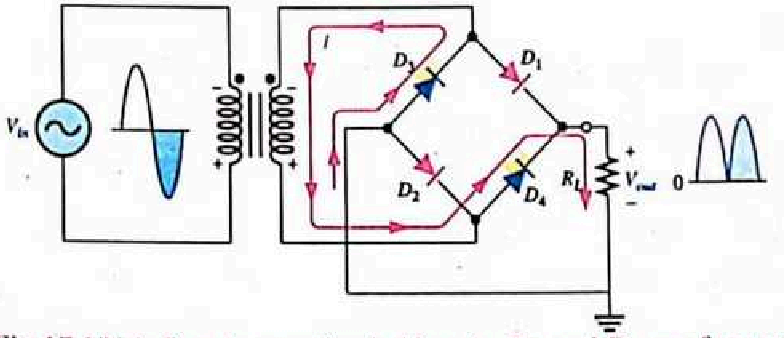


Fig.17.19(c). Due to negative half cycle, D_3 and D_4 are forward biased and conduct current, while D_1 and D_2 are reversed biased.

resistor R_L due to the diodes D_1 and D_2.

Similarly, during negative half cycle, the node P becomes negative and the node Q positive. So, this time the diodes D_3 and D_4 become forward biased and they conduct, while, the diodes D_1 and D_2 becomes reverse biased and they do not conduct as shown in Fig.17.19(c). Thus, there is a voltage drop across the load resistor R_L due to the diodes D_3 and D_4. For the next A.C. cycle, the same process is repeated. By combining these two results, we have full wave rectification. Graphically, the full wave rectifier signal is shown in Fig.17.20.

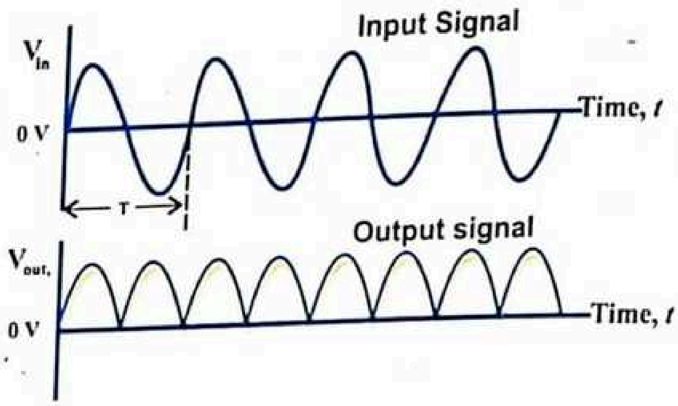


Fig.17.20 Input and output signal of full wave bridge rectifier.

17.5 TRANSISTOR

Like a diode, a transistor is also an important semiconductor device in which we study the transfer of charge carriers through a resistor. It is being used as an amplifier, switch etc.

A transistor consists of a single crystal of semiconductor in the form of two PN-junction with three electrodes named as emitter, base and collector.

In a transistor, a thin layer of one type of semiconductor is sandwiched between the two thick layers of the other type of semiconductor. For example, the Fig.17.21 shows that the thin layer of P-type semiconductor is sandwiched by the two thick layers of N-type semiconductor. It is named as NPN transistor. Similarly, Fig.17.22 shows that a thin layer of N-type semiconductor is sandwiched by the two thick layers of P-type semiconductor. Which is named as PNP transistor.

The symbols of NPN and PNP transistor are also shown in Fig.17.21 and Fig.17.22. The electrode with an arrow is emitter, the central electrode is base and the

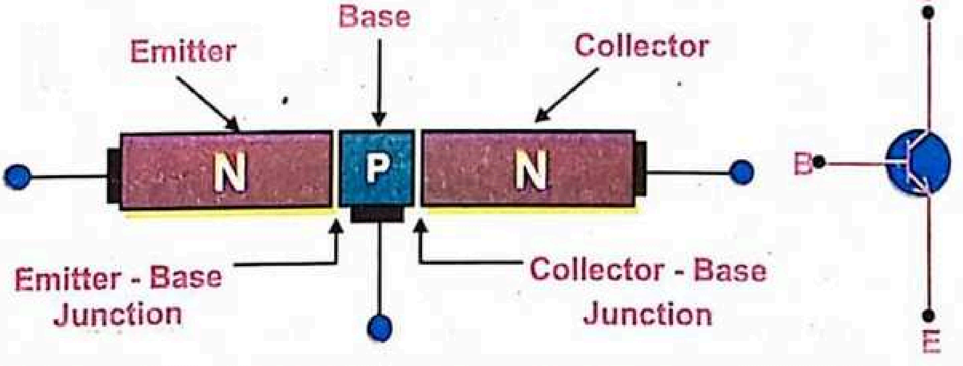


Fig.17.21 A NPN-transistor with its symbol, where the P-type semiconductor is sandwiched between two N-type semiconductors.

third one is collector. It is noted that the direction of the arrow representing the emitter of the NPN transistor is outward. And, the direction of the arrow of the PNP transistor is inward, which shows the direction of conventional current.

Now we are going to explain the functions of the three electrodes. The emitter has greater concentration of impurities as compare to the collector. Its main function is to supply the charge carriers either electrons or holes.

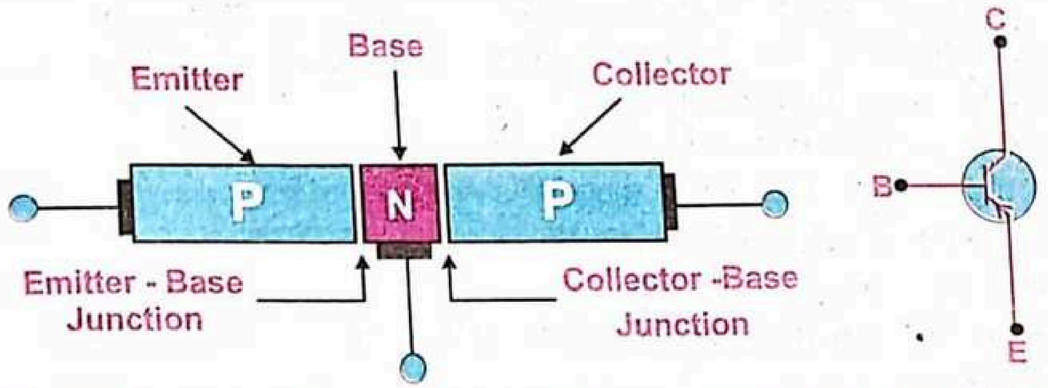


Fig.17.22 A PNP-transistor with its symbol, where the N-type semiconductor is sandwiched between two P-type semiconductors.

The central region, base of the transistor is very thin, of the order of 10^{-6}m. Its function is to control the flow of charges. The collector region of the transistor is made physically larger than the emitter region. The function of the collector is to collect the majority charge carriers coming from the emitter through the base region.

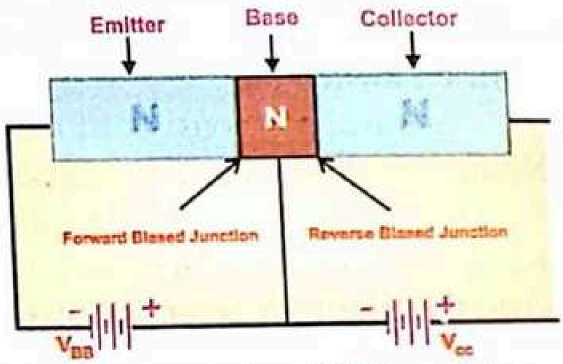


Fig.17.23 Biasing of NPN-transistor.

17.5.1 Transistor biasing

The application of voltage across the two junctions of a transistor is called biasing of a transistor. For normal operations of the transistors either NPN or PNP, their emitter-base junctions should be forward biased and collector-base junction should be reverse biased, such biasing of the transistor is taken place by using two batteries. The one battery is applied the forward voltage V_{BB} across the emitter-base junction, while the other one is applied the reversed voltage V_{CC} across the collector-base junction as shown in Fig.17.23 and 17.24.

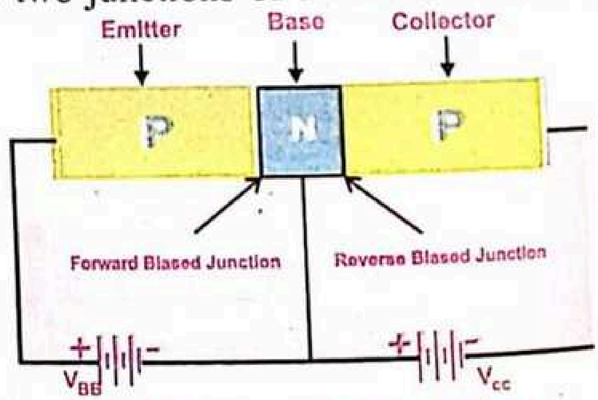


Fig.17.24 Biasing of PNP-transistor.

17.6 TRANSISTOR CONFIGURATION

A transistor has three electrodes named as; emitter, base and collector. Two batteries V_{BB} and V_{CC} are being used to operate the transistor, such that V_{bb} is applied across the input section of the transistor and V_{cc} across the output section. Thus, two electrodes are required for the input section and two for the output section, but the transistor has three electrodes. Therefore, one electrode of the transistor should be common to the input and output section of the circuit. This is named as common configuration of a transistor. There are three configurations of a transistor.

I Common emitter configuration (CEC)

In this configuration, emitter of the transistor is common to both input and output sections of the circuit as shown in Fig.17.25. In this case, the input signal is applied between the base and emitter. And, output signal is taken from the collector and emitter.

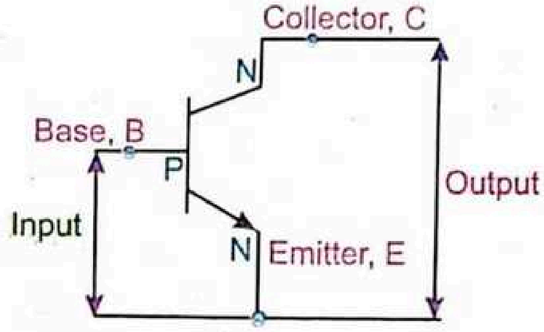


Fig.17.25 Common emitter configuration of NPN- transistor

II Common base configuration (CBC)

In this configuration, base of the transistor is common to input and output sections of the given circuit. The input signal is applied between the emitter and base. And, the output signal is taken from the collector and base as shown in Fig.17.26.

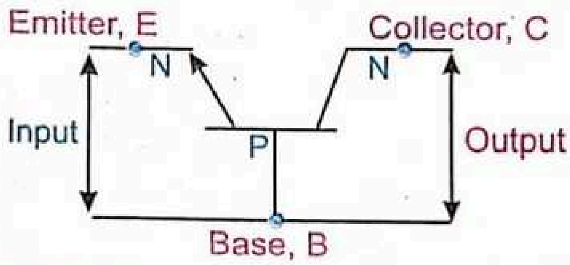


Fig.17.26 Common Base configuration of NPN-transistor.

III Common collector configuration (CCC)

In this configuration, collector of the transistor is common to both the input and output sections of the circuit. The input signal is applied between the base and collector. And the output signal is taken from the emitter and collector as shown in Fig.17.27.

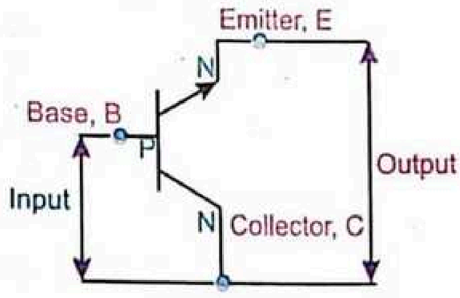


Fig.17.27 Common collector configuration of NPN-transistor.

17.7 OPERATION OF A TRANSISTOR

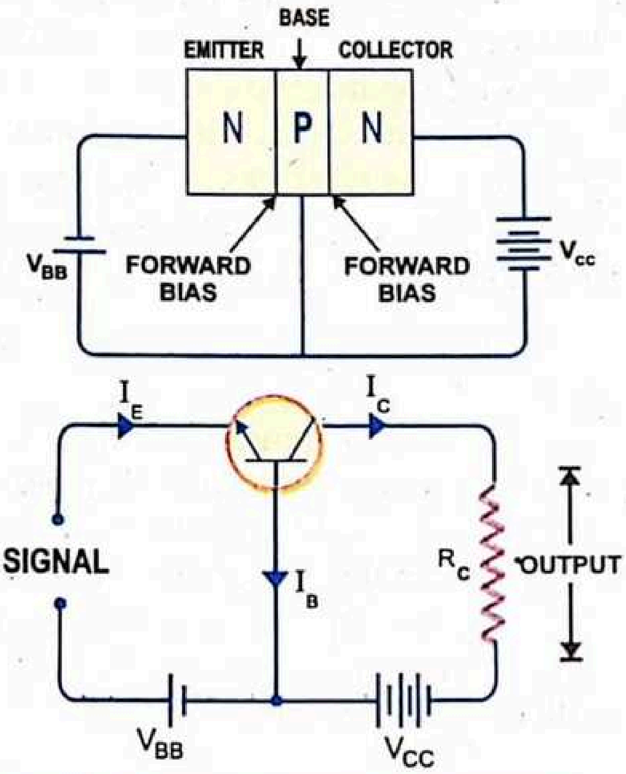


Fig.17.28 A flow of a current in PN-transistor.

In the previous section, we have studied that a transistor can operate only when its emitter-base junction is forward biased and collector-base junction is reverse biased. Let us consider the operation of NPN transistor, as shown in Fig.17.28. As the emitter-base junction of the transistor is forward biased. Due to the applied forward voltage, so the electrons from emitter cross the junction and enter the base region, where base is very thin and lightly doped region. Therefore, the concentration of electrons from the emitter is more than the hole from the base. Thus only a few electrons (about 2%) recombine with the holes to constitute the base current. While most of the free electrons (about 98%) do not combine with the holes but move through the thin base region to the collector region. As collector is already reverse biased, and its reverse voltage attract these free electrons from the base region, therefore it causes collector current. In this way, the flow of charge carriers in a transistor is possible i.e., when the emitter current 'I_E' flows into the transistor, a very small amount of it 'I_B' flows out of the base, the rest of it 'I_C' flows out of the collector. Mathematically, this relation of current in a transistor is expressed as;

$$I_E = I_B + I_C \quad ......(17.2)$$

This relation shows that the collector current 'I_C' is much greater than the base current 'I_B', but it is less than the emitter current 'I_E'. These can be expressed in terms of ratios, i.e.,

$$\alpha = \frac{I_C}{I_E} \quad ......(17.3)$$

and

$$\beta = \frac{I_C}{I_B} \quad ......(17.4)$$

where α and β are known as current gain. The typical values of α and β are 0.99 and 100 respectively for germanium transistors whereas, 0.995 and 200 respectively for silicon transistors. These results show that a small variation in the base current causes a large collector current. It is named as current amplification. It means a transistor can be used as an amplifier.

17.7.1 Characteristics of a Transistor

The curved line that is obtained due to the relationship between voltage and current by using a transistor is known as characteristics of a transistor. There are two important characteristics of a transistor named as input characteristics and output characteristics. Let us consider the operation of common emitter NPN transistor as shown in Fig.17.29. The voltage at the base with respect to emitter is V_{BE}, voltage at collector with respect to base is V_{CB} and the voltage at collector with respect to emitter is V_{CE}.

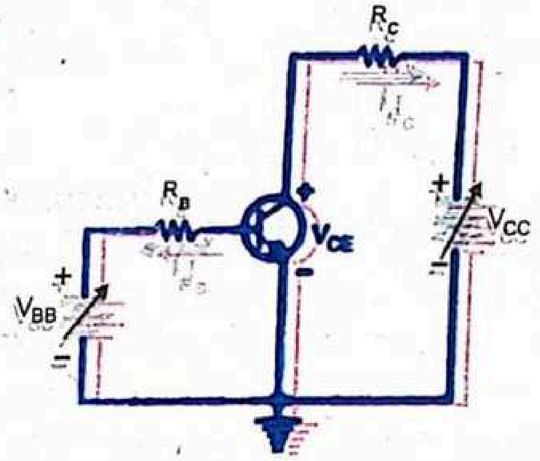


Fig.17.29 The operation of common emitter in NPN transistor.

I. Input Characteristics

This is the curved line between base current I_B and base-emitter voltage V_{BE} at constant collector-emitter voltage V_{CE}.

When V_{BE} increases by increasing V_{BB}, I_B also starts increasing keeping V_{CE} constant, if different values of V_{BE} and I_{BE} are drawn on the graph of I_B versus V_{BE} then there is curved line as shown in Fig.17.30 which is known as input characteristics of a transistor. The reciprocal of the slop of such curved line is equal to the input resistance R_B of the transistor. That is,

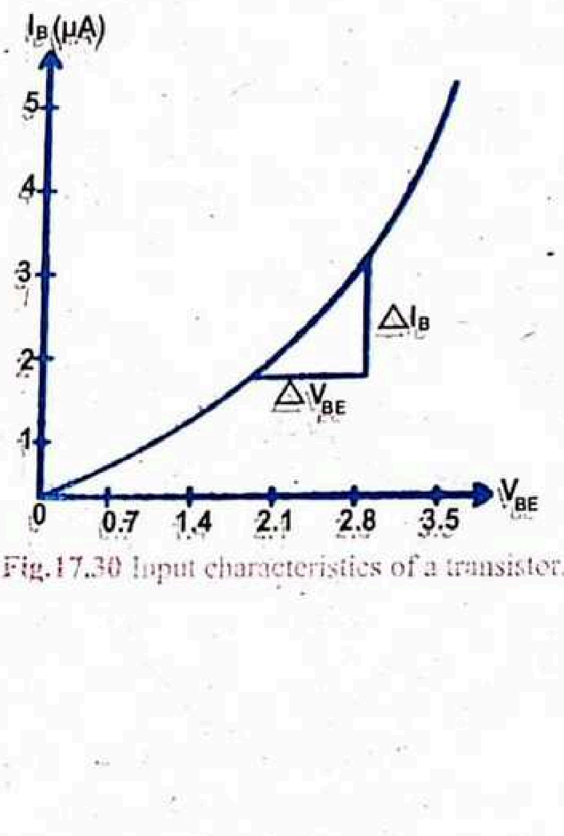


Fig.17.30 Input characteristics of a transistor.

$$R_B = \frac{1}{\left(\Delta V_{BE} / \Delta I_B\right)}$$

$$R_B = \frac{\Delta I_B}{\Delta V_{BE}} \quad \text{......(17.5)}$$

II Output Characteristics

This is the curved line that is obtained due to the relationship between collector current I_C and collector-emitter voltage V_{CE} at fixed value of base current I_B.

For output characteristics of a transistor, V_{BB} is adjusted to get a fixed value of I_B while V_{CC} is zero at this stage. Now when V_{CE} is increased by increasing V_{CC} from zero in steps, I_C also starts increasing rapidly from zero to its maximum value at fixed value of I_B. The transistor at the maximum value of I_C is known as saturation level. Graphically, the characteristics curve of a transistor where I_C varies from zero to the saturation level is shown between the two points A and B in Fig.17.31(a). It may be noted that a small amount of collector current flows even when base current I_B is equal to zero. This is termed as cutoff or non-conducting state of a transistor. When V_{CE} exceeds the saturation level, collector current becomes almost constant and operation of the transistor enters into the active or linear region. Graphically, such linear characteristics of a transistor is shown between the two points B and C in Fig.17.31(a). If V_{CE} is allowed to increase too far, collector-base junction breaks down and the collector I_C increases rapidly. It is represented by a line above the point C in Fig.17.31(a).

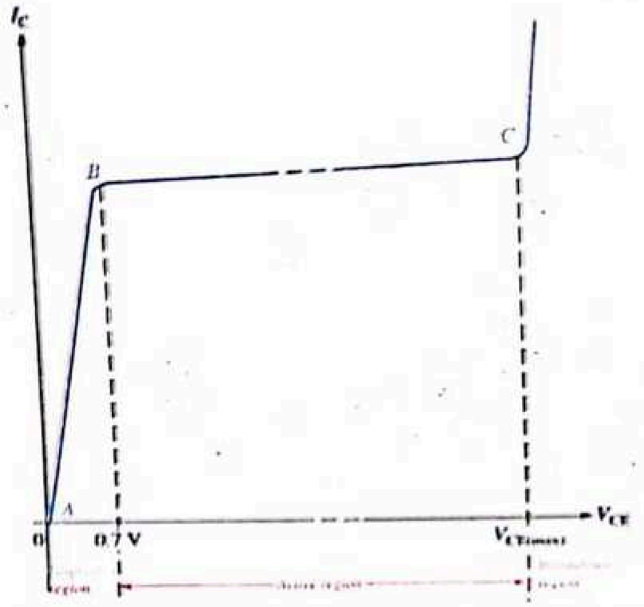


Fig.17.31(a) I_C versus V_{CE} curve for 1 value of I_B under saturation active and breakdown regions.

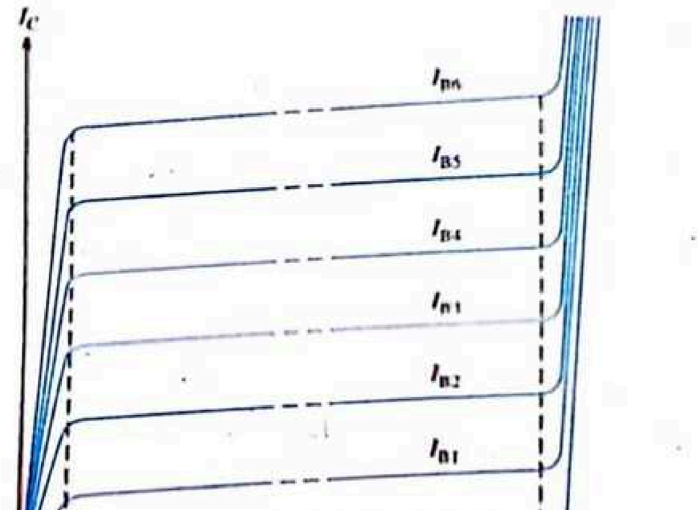


Fig.17.31(b) Output characteristics of a transistor in terms of family curves.

Again, V_{BB} is adjusted to get another fixed value of base current I_B and V_{CE} is reduced to zero. By repeating the same procedure, we have another curved characteristics of a transistor and so on. Thus, we obtain a family characteristics curves of a transistor when I_C verses V_{CE} which is plotted for several fixed value of I_B as shown in Fig.17.31(b).

Example 17.2

In a transistor I_C is 1mA and I_B is $200\mu A$, calculate I_E.

Solution:

$$I_C = 1mA$$
$$I_B = 20\mu A = 0.02mA$$
$$I_E = ?$$

According to the basic equation for flow of current in a transistor

$$I_E = I_C + I_B$$
$$I_E = 1mA + 0.02mA$$
$$I_E = 1.02mA$$

Example 17.3

In an electronic circuit, the emitter current of a transistor is 2.2mA and its collector current is 2mA. Calculate the value of α.

Solution:

Emitter current $= I_E = 2.2mA$

Coll ector current $= I_C = 2mA$

$$\alpha = ?$$
$$\alpha = \frac{I_C}{I_E}$$
$$\alpha = \frac{2mA}{2.2mA}$$
$$\alpha = 0.9$$

> ***DO YOU KNOW***
>
> Diode and transistor are active components and they do not obey Ohm's law.

Example 17.4

Calculate the emitter current in a transistor when its base current is $50\mu A$ and the value of current gain is 100.

Solution:

Emitter current $= I_E = ?$

Base current $= I_B = 50\mu A = 0.05mA$

Current gain $= \beta = 100$

The current gain is defined as

$$\beta = \frac{I_C}{I_B}$$
$$I_C = \beta I_B$$
$$= 100 \times 0.05mA$$
$$I_C = 5mA$$

> ***FOR YOUR INFORMATION***
>
> In most electronic circuit, the current I is only a small fraction of an ampere. A typical value of current I in an electronic circuit is 10mA.

$$I_E = I_B + I_C$$
$$I_E = 0.05mA + 5mA$$
$$I_E = 5.05mA$$

17.8 TRANSISTOR AS AN AMPLIFIER

An amplifier is an electronic circuit which converts a small input A.C. signal into a large output A.C. signal under the action of a transistor. The amplification takes place under the following three configurations i.e., common base amplifier, common emitter amplifier and common collector amplifier, but common emitter amplifier is used mostly.

Consider the operation of a common emitter NPN transistor as shown in Fig.17.32. The emitter-base junction is forward biased by using a battery V_{BB} and a resistor R_B called base or input resistor, whereas collector is reverse biased by using another battery V_{CC} through a resistor R_C known as collector or load resistor. The input A.C. signal is applied in the base and emitter section of the circuit. And the output signal is taken out from the collector-emitter section. In the absence of any input signal, the transistor is working in its normal mode. Now when an A.C. signal is applied at the input terminals of the circuit, then during the positive half cycle of the signal, the base of the transistor which acts as driven element becomes more positive and it increases the forward biasing across the emitter-base junction. Therefore, more electrons flow from the emitter to the collector through the base and it causes of an increase collector current I_C by β times the input base current I_B ($I_C = \beta I_B$). As a result, a greater voltage drops across the collector load resistance R_C due to the increased collector I_C. Thus, we have a large output half A.C. cycle but in

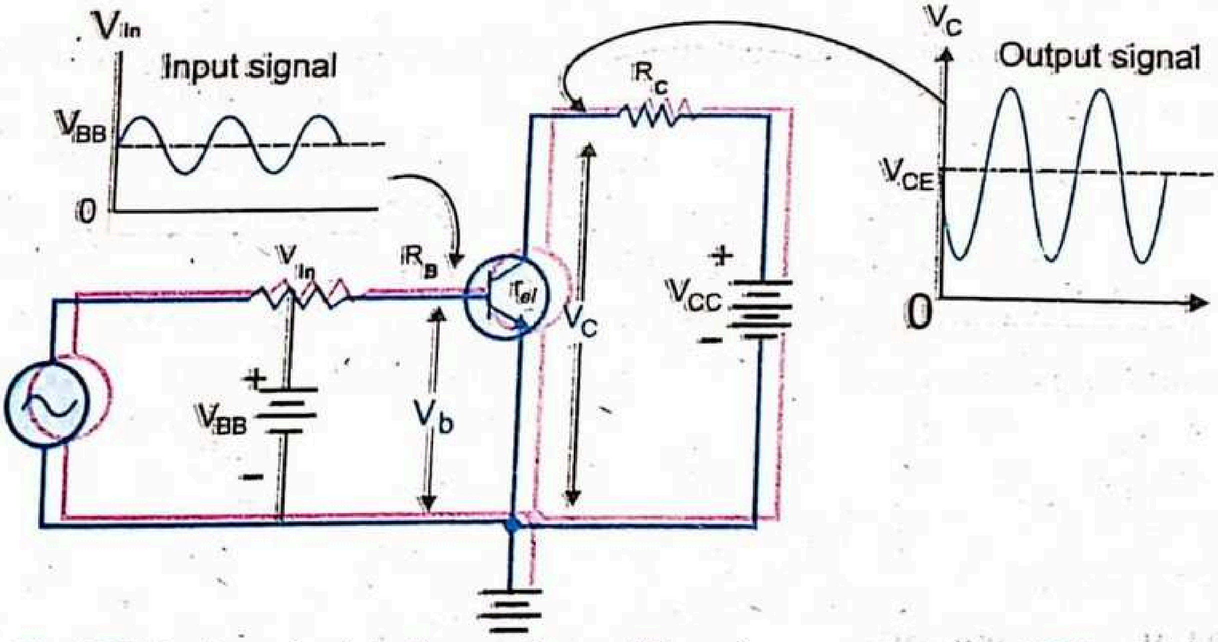


Fig.17.32 A schematic circle diagram for amplifier using common emitter NPN-transistor

the negative direction. i.e., there is a phase difference of 180° between the input and output signals.

Similarly, during the negative half cycle base becomes less positive and the forward biasing across the emitter-base junction is decreased. Therefore, there are a few electrons that flow from emitter to collector through the base. There is a very small collector current. In this condition, a large voltage drops across the load resistance R_C by V_{CC}. Thus, we have again a large half positive cycle of A.C. which is opposite in direction to the input signal as shown in Fig.17.32. For the next cycle, the same process is repeated. In this way, a transistor converts a small input A.C. signal into a large A.C. signal.

Analytically, the amplification of common emitter NPN transistor is explained as; Due to forward biasing, the base-emitter junction presents a very low internal resistance to the A.C. signal. It is represented by r_{ei} and appears in the series with the base resistance R_B. Thus, I_B in terms of r_{ei} is given as:

$$I_B = \frac{V_{BE}}{r_{ei}}$$

But $$V_{BE} = V_i \text{(Input voltage)}$$

So $$I_B = \frac{V_i}{r_{ei}} \quad ......(17.6)$$

By definition of current gain (β)

$$\beta = \frac{I_C}{I_B}$$

$$I_C = \frac{\beta V_i}{r_{ei}} \quad ......(17.7)$$

Now, the output voltage $(V_o = V_{CE})$ in the output section of the given circuit can be determined by using KVL. i.e.,

$$V_{CC} = I_C R_C + V_{CE}$$

$$V_o = V_{CC} - I_C R_C \quad ......(17.8)$$

Putting the value of I_C from Eq. 17.7 in Eq. 17.8

$$V_o = V_{CC} - \frac{\beta V_i R_C}{r_{ei}} \quad ......(17.9)$$

This is the output voltage of a transistor without applied input signal. If an input signal is applied then both input and output voltages are increased. So Eq.17.9 becomes

$$V_o + \Delta V_o = V_{CC} + \frac{\beta (V_i + \Delta V_i) R_C}{r_{ei}} \quad ......(17.10)$$

Subtract Eq. 17.9 from Eq. 17.10

$$\Delta V_o = -\frac{\beta \Delta V_i R_C}{r_{ei}} \quad ......(17.11)$$

By definition of voltage gain (A_V)

$$A_V = \frac{\Delta V_o}{\Delta V_i}$$

$$A_V = -\beta \frac{R_C}{r_{ei}} \quad ......(17.12)$$

This is the voltage amplification of a transistor. Negative signs shows that input and output signals are out of phasé. Eq.17.12 shows that voltage amplification of a transistor depends upon R_C and r_{ei}. Since R_C is always greater than r_{ei} so output voltage in a transistor is greater than its input voltage.

Example 17.5

For a germanium crystal transistor, the current gain is 200 and voltage drop across a load resistance of 2kΩ is 4volt. Find the base current for common emitter configuration.

Solution:

Current gain $= \beta = 200$

Voltage drop $= V_C = 4V$

Output resistance $= R_C = 2k\Omega = 2\times10^3\Omega$

Base current $= I_B = ?$

As $\quad V_C = I_C R_C$

$$I_C = \frac{V_C}{R_C}$$

$$I_C = \frac{4V}{2\times10^3\Omega} = 2\times10^{-3}A$$

$$I_C = 2\ mA$$

Now $\quad \beta = \frac{I_C}{I_B}$

$$I_B = \frac{I_C}{\beta}$$

$$I_B = \frac{2\times10^{-3}A}{200}$$

$$I_B = 10^{-5}A$$

17.9 TRANSISTOR AS A SWITCH

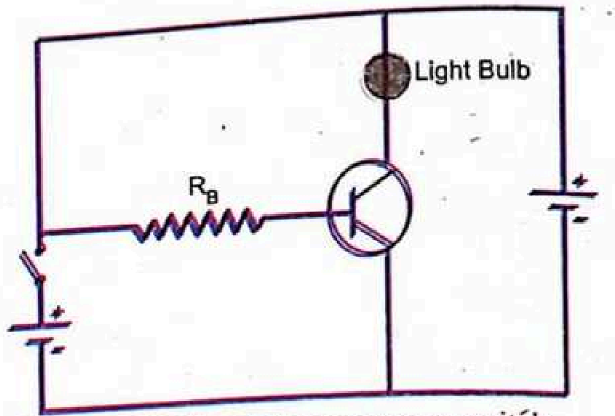


Fig.17.33 Transistor as an open switch.

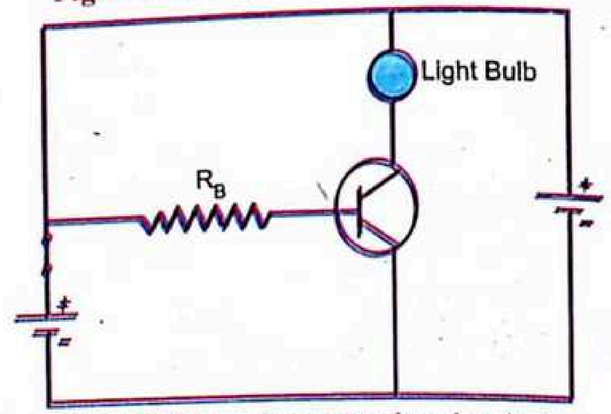


Fig.17.34 Transistor as a closed switch.

Like a diode, a transistor can also be used as switch under its cutoff and saturation regions. That is, a transistor is in cutoff region when its base-emitter is reverse biased. Similarly, it is in saturation region when its base-emitter is forward biased. These two states acts as an open and closed switch. It is explained by considering the operation of common emitter NPN transistor under two different cases.

In the first case, the emitter-base junction is not forward biased, the base current I_B is zero. Therefore, the collector current is also zero under this condition, the transistor behaves as an open switch. Thus, the bulb is switched OFF as shown in Fig.17.33.

In the second case, the emitter-base junction is forward biased, there is an increase of base current. This causes maximum collector current. Under this condition, the transistor acts as a closed switch. Thus, the bulb is switched ON as shown in Fig.17.34.

SUMMARY

- **Semiconductor:** The solids whose resistivity or conductivity lies between insulators and conductors are known as semiconductor. e.g. germanium and silicon. There are two types of semiconductor.
- **Intrinsic Semiconductor:** An extremely pure form of semiconductor is known as intrinsic semi-conductor.
- **Extrinsic Semiconductor:** When an impurity of either trivalent or pentavalent atoms is added into a pure semiconductor then this is called extrinsic semiconductor.
- **N-Type Semiconductor:** When an impurity of pentavalent atoms is added to a pure semiconductor then it is known as N-type semiconductor.
- **P-Type Semiconductor:** When an impurity of trivalent atoms is added to a pure semiconductor then it is called P-type semiconductor.
- **PN-Junction:** PN junctions are formed by joining N-type and P-type semiconductor materials in the form of a single crystal.

- **Biasing of PN-Junction:** The process in which the potential difference is applied across the PN-Junction is called biasing.
- **Forward biased PN-Junction:** When PN-Junction is connected to a battery such that P-type is connected to positive terminal and N-type is connected with negative terminal of the battery, such biasing is called forward biased.
- **Reverse biased PN-Junction:** When PN-Junction is connected to a battery such that P-type is connected with the negative terminal and N-type with the positive terminal of the battery, such biasing is called reverse biased.
- **Semi-Conductor Diode:** Diode is indeed a PN-Junction which converts alternating current (A.C.) into direct current (D.C.)
- **Rectifier:** Rectifier is an electronic circuit that converts A.C. into D.C. under the action of a diode and the process of conversion of A.C. into D.C. is known as rectification. There are two type of rectifications:

  I Half wave rectification   II Full wave rectification
- **Transistor:** Transistor is an electronic device which consists of two PN-Junctions with three electrodes named as emitter, base and collector. There are two kinds of transistors, NPN and PNP
- **Biasing of a transistor:** A transistor operates only when its emitter-base junction is forward bias and collector-base junction is reverse bias.
- **Amplifier:** Amplifier is an electronic circuit that converts a small input A.C. signal into a large output A.C. signal under the action of a transistor.

EXERCISE

Select the best option of the following questions.

1. A germanium at 0K is
   (a) Conductor   (b) Semiconductor
   (c) Insulator   (d) P&N types semiconductor
2. By increasing the temperature, the conductivity of the semiconductor is
   (a) Decreased   (b) Increased   (c) Constant   (d) Not affected
3. Which one of the following is an intrinsic semiconductor?
   (a) Boron   (b) Copper   (c) Indium   (d) Silicon
4. In extrinsic semiconductor, the doping level should be in the ratio
   (a) $1:10^{-6}$ (b) $1:10^{6}$ (c) $1:10^{-8}$ (d) $1:10^{8}$
5. In P-type semiconductor, the doping of Silicon is taken place with
   (a) Bivalent   (b) Trivalent   (c) Tetravelent   (d) Pentavalent
6. When P-type and N-type semiconductor are combined then we have a
   (a) Resistor   (b) Capacitor   (c) Diode   (d) Triode
7. The depletion region in PN-Junction carries

(a) Positively charge (b) Negatively charge
(c) Both positively and negatively charge (d) No mobile charge carrier

8. The potential difference across the silicon PN-junction is
(a) 0.3V (b) 0.5V (c) 0.7V (d) 0.9V

9. When P-type of the PN-Junction is connected with the negative terminal and N-type with positive terminal of the battery, the width of the depletion region
(a) decreases (b) increases (c) remain constant (d) vanished

10. In forward biasing, the knee voltage for Germanium PN-junction is
(a) 0.3V (b) 0.7V (c) 3V (d) 7V

11. One of the most important property of a diode
(a) Allow current in one direction (b) Allow current in bidrection
(c) Use as an amplifier (d) Use as an oscillator

12. The number of diodes used in a bridge rectifier is
(a) One (b) Two (c) Three (d) Four

13. Transistor stands for
(a) Transfer of Resistance (b) Transfer of charge carriers
(c) Transfer of power (d) Transfer of voltage

14. Which region of the transistor is most wide in terms of area?
(a) Emitter (b) Base (c) Collector (d) All have equal area

15. In biasing of a transistor, which region must be reverse biased?
(a) Emitter-Base (b) Collector-Base
(c) Emitter-Collector (d) None of these

16. The thickness of the base region of a transistor is
(a) 10^{-2}m (b) 10^{-4}m (c) 10^{-6}m (d) 10^{-8}m

17. Which one of the following relation is true for a transistor?
(a) $I_E = I_C - I_B$ (b) $I_E = I_B - I_C$ (c) $\alpha = \frac{I_C}{I_E}$ (d) $\beta = \frac{I_B}{I_E}$

18. Which one of the following relations holds for cutoff mode of transistor?
(a) $I_C = I_B$ (b) $I_C = \beta I_B$ (c) $I_C = \alpha I_B$ (d) $I_C = 0$

19. The flow of charge carriers in a transistor is controlled by
(a) Emitter (b) Base (c) Collector (d) All of them

20. If the base current of a silicon transistor is 0.01mA, its collector current is
(a) 0.99mA (b) 9.9μA (c) 0.1mA (d) 1mA

21. V_{CE} drops across the load resistor of a transistor when
(a) $I_C = 0$ (b) $I_C = I_B$ (c) $I_C = \alpha I_B$ (d) $I_C = \beta I_B$

22. When a transistor starts its working in its active region
(a) Before its saturation (b) After its saturation
(c) After its cutoff (d) At the end of its cutoff

23. A transistor is used as a switch when it operates in its
(a) Normal mode (b) Active mode
(c) Breakdown mode (d) Saturation and cutoff mode

SHORT QUESTIONS

1. Distinguish between intrinsic and extrinsic semiconductors?
2. What are the advantages of extrinsic semiconductor over the intrinsic semiconductor?
3. Under what conditions, the semiconductor materials behave as conductor and as an insulator.
4. What is meant by the process of doping?
5. What are the facts about holes?
6. What do you know about the depletion region in PN-Junction?
7. What is the diffusion in the PN-Junction?
8. How can the conductivity of semiconductor be increased by increasing the temperature?
9. How does the width of depletion region in PN-Junction increases and decreases?
10. How does a diode convert A.C. into D.C.?
11. What is the rectification of a diode?
12. Distinguish between half and full wave rectifications.
13. What is the biasing of a diode?
14. Distinguish between forward and reverse biased characteristics of a diode.
15. What is meaning of a transistor?
16. How does a transistor operate?
17. How does the biasing of a transistor take place?
18. Why is the base region of a transistor thin and lightly doped?
19. Why is the collector of a transistor anode wider than the emitter and the base?
20. Why the input resistance of a transistor is low while its output resistance is high?
21. Why the current gain α is less than β?
22. Why a common emitter configuration transistor is mostly used?
23. Under what conditions a transistor acts as an open and a closed switch?
24. What do you know about the saturation, cutoff and active regions of a transistor?

COMPREHENSIVE QUESTIONS

1. State and explain intrinsic and extrinsic semiconductors.
2. What do you know about the N-type and P-type semiconductors? Explain the development of N-type and P-type semiconductor crystals.

where p = 1, 2, 3, ... and n = p + 1

and $$R_H = \frac{E_1}{hc} = 1.0974 \times 10^7\ m^{-1} \quad ......(19.15)$$

The result of Eq.19.14 is the same as that of the empirical formula in Eq.19.6 for wavelengths of spectral lines, so it is concluded that when different transitions of electron are taking place from the higher orbits the lower orbits as shown in Fig.19.5 then we have Lyman, Balmer, Paschen, Bracket and Pfund series. All these spectral series are shown on energy level diagram (Fig.19.6).

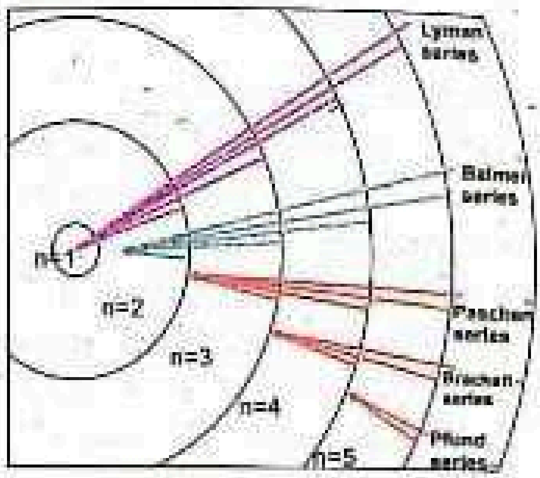


Fig.19.5 Transition of electron from various excited states in hydrogen atom.

I Lyman series

If the electrons of hydrogen atom jump from outer orbits (n = 2, 3, 4, ...) to the 1st orbit (p = 1) then we have a set of spectral lines called the Lyman series and it is the ultraviolet region of electromagnetic spectrum. The wavelength of this series is calculated as;

$$\frac{1}{\lambda} = \frac{E_1}{hc}\left(\frac{1}{p^2} - \frac{1}{n^2}\right) = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad P = 1,\ n = 2,3,4...$$

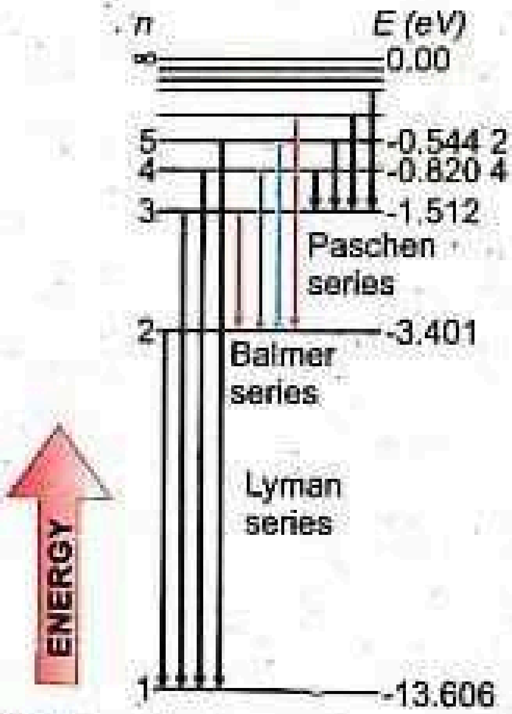


Fig.19.6 Energy level diagram of hydrogen atom.

Unit 18

DAWN OF MODERN PHYSICS

Major Concepts **(24 PERIODS)**

- Special theory of relativity
- Quantum theory of radiation
- Photoelectric effect
- Compton's effect
- Pair production and pair annihilation
- Wave nature of particles
- Electron microscope
- Uncertainty principle

Conceptual Linkage

This chapter is built on
Planck's quantum theory
Chemistry XI
Resolving power,
Magnifying power of
microscope Physics IX

Students Learning Outcomes

After studying this unit, the students will be able to:

- distinguish between inertial and non-inertial frames of reference.
- describe the significance of Einstein's assumption of the constancy of the speed of light.
- identify that if c is constant then space and time become relative.
- explain qualitatively and quantitatively the consequence of special relativity in relation to:
  - the relativity of simultaneity
  - the equivalence between mass and energy
  - length contraction
  - time dilation
  - mass increase
- explain the implications of mass increase, time dilation and length contraction for space travel.
- describe the concept of black body radiation.
- describe how energy is distributed over the wavelength range for several values of source temperature.
- describe the Planck's hypothesis that radiation emitted and absorbed by the walls of a black body cavity is quantized.
- elaborate the particle nature of electromagnetic radiation.
- describe the phenomenon of photoelectric effect.
- solve problems and analyze information using: $E = hf$ and $c = f\lambda$.

- identify data sources, gather, process and present information to summarize the use of the photoelectric effect in solar cells & photocells.
- describe the confirmation of de Broglie's proposal by Davisson and Germer experiment in which the diffraction of electrons by the surface layers of a crystal lattice was observed.
- describe the impact of de Broglie's proposal that any kind of particle has both wave and particle properties.
- explain the particle model of light in terms of photons with particular energy and frequency.
- describe Compton effect qualitatively.
- explain the phenomena of pair production and pair annihilation.
- explain how the very short wavelength of electrons, and the ability to use electrons and magnetic fields to focus them, allows electron microscope to achieve very high resolution:.
- describe uncertainty principle.

INTRODUCTION

Classical physics is the physics that evolved before the 20th century. It consists of Newton's laws of motion, gravitational laws, laws of thermodynamics, kinetic theory, Maxwell's theory of electromagnetic wave and so many others. Due to all these achievements, many scientists felt that most of the great discoveries in physics had been made. The post-Newtonian concepts in the world of physics brought a revolution in the field of physics which is known modern physics. Modern physics means physics based on the two major breakthroughs of the early 20^{th} century which are relativity and quantum mechanics. In 1900, Max Planck introduced the concept of the quantum theory and it leads to understanding the distribution of black body radiation. Later on, Einstein also explained the photoelectric effect on the basis of Planck's quantization. Bohr in 1913, introduced his model of the hydrogen atom. In 1923, Compton confirmed experimentally the particle nature of light by scattering X-rays with electrons. In 1923, de-Broglie proposed the wave nature of particles and this notion was proved experimentally by Davisson and Germer in 1927. Heisenberg and Schrodinger further explained analytically the dual nature of matter.

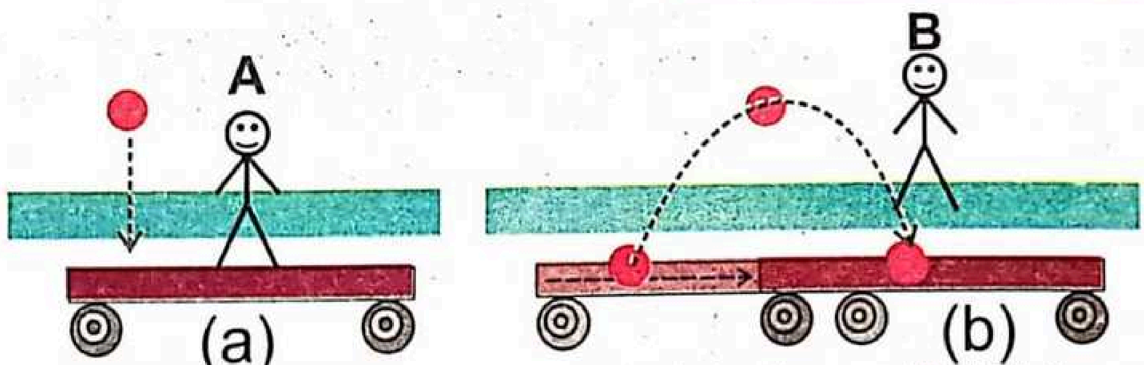
Even though the physics that was developed during the 20^{th} century has led to a multitude of important technological achievements, the story is still incomplete. Discoveries will continue to evolve during our lifetimes and beyond that, and many of these discoveries will refine our understanding of the nature of universe around us. It is still a marvelous time to be alive. In this unit, we will explain some important theories of modern physics, such as special theory of relativity and its consequences, black body radiation and Planck's law, photoelectric effect and Compton effect, De-

Broglie wave equation and Davisson-Germer experiment, pair production and annihilation of matter and Heisenberg's uncertainty principle.

18.1 RELATIVE MOTION

Our moving earth appears at rest with respect to the geostationary satellite, while appears in motion with respect to the moon. This example provides the principle of relative rest and relative motion. The experimental facts show that observations of all the events would have different meaning to different observers. It can further be explained by another example of a boy who is standing in a train which is moving with uniform velocity and is holding a ball in his hand. The ball appears stationary for an observer A, who is also standing in the train. But the same ball appears moving with the train for another observer B who is standing on the ground outside the train. Now if the boy throws the ball vertically straight upward, it will come back straight downward. This will be observed by the observer A as shown in Fig.18.1(a). But the observer B who is standing outside the train will observe that the ball is moving along the parabolic path as shown in Fig.18.1(b). Although the two observers disagree on the path followed by the ball, but both agree over the motion of the ball obeys the law of gravity and Newton's laws of motion, and they also agree on how long the ball is in the air. By

> ***POINT TO PONDER***
>
> Let you are standing at the roof the building and you are observed by two observer one is at the moon and other is at the geostationary satellite. Who will observe that you are in motion?



Observer in the moving train **Stationary observer outside the train**

The path of the ball is up and down as seen by an observer in the train moving at constant velocity relative to the ground.

The path of the ball is a parabolic as seen by a stationary observer on outside the train.

Fig.18.1

summing up these observations, it is concluded that all the motions are relative. There is no absolute rest or no absolute motion.

18.2 FRAME OF REFERENCE

In daily life, when we observe an event i.e., physical happening or measure position, or change in position of a body then we use some relative points such as east, west, north and south which are lying on the earth's surface. These points are called reference points and the earth is a frame of reference. In physics we use Cartesian coordinates as reference points for measurements. The set of coordinates with respect to which the measurements or observations are made is called frame of reference. There are two kinds of frame of reference.

INTERESTED INFORMATION

From the earth frame of reference, light takes 25,000 years to travel from the center of our milky way galaxy to our solar system. From the frame of reference of a high speed space ship flying outward from the galactic center toward earth, the trip takes less time. If a frame of reference could be attached to the light itself, the travel time could be reduced to zero.

I Inertial frame of reference

A frame of reference which is at rest or moving with uniform velocity is called inertial frame of reference. e.g., earth is taken almost as an inertial frame of reference. In inertial frame of reference, the acceleration of the body is zero and law of inertia remains valid in it. That is, if a body is at rest it will be remain at rest and if a body is moving with uniform velocity it will be continue its uniform motion unless an unbalanced force produces an acceleration in it. According to the principle of Galilean relativity, all the laws of physics must be the same in all inertial frames of reference.

II Non-inertial frame of reference

A frame of reference which is accelerating, or decelerating is known as non-inertial frame of reference. It may be pointed out that Newton's laws of motion are not valid in non-inertial frame of reference.

18.3 SPECIAL THEORY OF RELATIVITY

The Special Theory of relativity was proposed by Albert Einstein in 1905. This theory has brought a revolution in the field of the science. The scientists completely changed their notion about Ether (a hypothetical medium), speed of light, space and time coordinates and so many other. The special theory of relativity is based on the following two postulates.

1. All the laws of physics are same in all inertial frames of reference.
2. The speed of light is a universal constant. That is, the value of speed of light ($3 \times 10^8\ ms^{-1}$) remains the same in all inertial frames of reference.

The first postulate is also called the principle of relativity and it states that all the laws of physics are equally applicable in all frames of reference, if the relative motion between them is uniform. For example, let any kind of experiment such as mechanical, thermal, optical or electrical is performed in a laboratory at rest. Then according to Einstein's principle of relativity, the same result will be obtained when the same experiments are performed in a laboratory moving with uniform speed. Hence there are no preferred inertial frame of reference.

> **FOR YOUR INFORMATION**
>
> Special relativity is "special" in the sense that it deals with uniformly moving reference frames – once that are not accelerated. General relativity is "general" and deals also with accelerating reference frames. The general theory of relativity presents a new theory of gravity.

The second postulate is contrary to the Ether theory. Most of earlier physicists believed that the light travel through a hypothetical medium called Ether and the speed of light relative to ether is different in different direction. But according to Einstein, the speed of light in vacuum is the same in all inertial frames of reference. For example, let two observers measure speed of light in vacuum, such that one observer is at rest with respect to the source of light and the other is moving away from it with uniform velocity. According to Einstein's principle of relativity, the result of both observers will be the same.

> **POINT TO PONDER**
>
> If you were travelling in a space ship at a speed of C/2 relative to earth and you fired a laser beam in the direction of the spaceship's motion. What will be the light speed from the laser relative to the earth?

18.4 CONSEQUENCES OF SPECIAL THEORY OF RELATIVITY

Einstein's special theory of relativity has some very important results which are related to the base physical quantities such as length, time and mass. According to the special theory of relativity, the results of measurement of these quantities are different in different frame of reference. Therefore, we measure the distance between two points, the time interval of an event and mass of a body based on the consequence of special theory of relativity.

The relativity of simultaneity

The concept of simultaneity gives an interesting result of Einstein's second postulate. i.e., the speed of light is a universal constant. It is stated as, **when two**

events that are simultaneous in one frame of reference need not be simultaneous in a second frame of reference that moving relative to the first. It is explained by an example. Consider a light source suspended by the exact centre of the roof of the compartment of a train. When the light is switched on, it spreads out in all direction with speed 'c'. An observer inside the compartment observes that light reaches the front wall of the compartment at the same time it reaches the back wall as shown in Fig. 18.2(a). This occurs whether the train is at rest or moving at constant speed. Here the two events of falling of light on the back wall and falling on the front wall occur simultaneously for the observer inside the compartment.

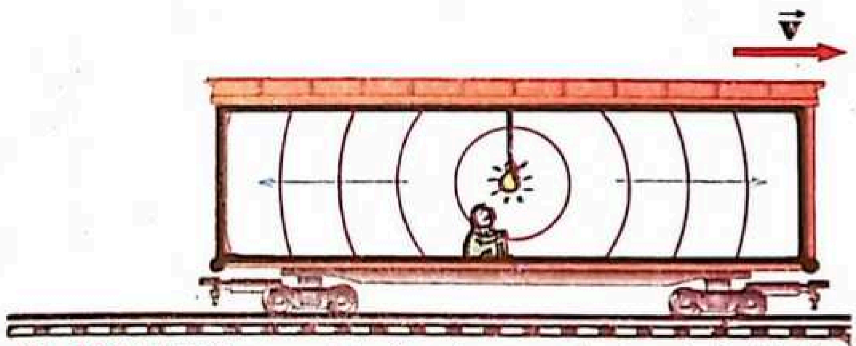


Fig.18.2(a) With respect to the observer who travels inside the compartment, light from the source travels equal distance to both walls of the compartment and therefore strikes both walls

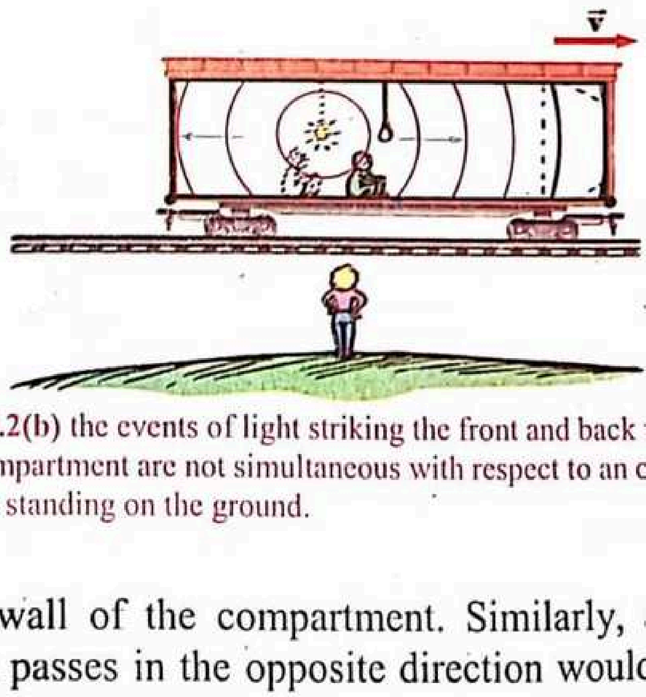


Fig.18.2(b) the events of light striking the front and back walls of the compartment are not simultaneous with respect to an observer who is standing on the ground.

On the other hand, if the same two events are observed by another observer who is standing on the ground (at rest) and outside the compartment, then these two events are not simultaneous. As light travel away from the source, this observer sees the train moves forward, so the back wall of the compartment moves toward the beam while the front moves away from it. The beam going to the back of the compartment, therefore, has a shorter distance to travel than the beam going forward, as shown in Fig. 18.2(b). Thus this outside observer sees the event of light falling the back wall of the compartment before seeing the event of light falling the front wall of the compartment. Similarly, another observer in another moving train that passes in the opposite direction would report that the light reaches the front wall of the compartment first.

We should not wonder which observer is right concerning the two events. According to Einstein's theory of relativity, both observers are correct, because simultaneously is not absolute and there is no preferred inertial frame of reference, but the two observers must find that light travel at the same speed.

Length contraction

When both ends of length of an object are measured simultaneously by an observer at rest relative to the object, then such length of the object is called its rest or proper length. According to special theory of relativity, the length of an object measured in a frame of reference which is moving with respect to the object is always less than the rest length or proper length. This effect is known as length contraction. The length contraction takes place only along the direction of motion, this effect is negligible at ordinary speeds, no such effects may be observed perpendicular to the direction of motion.

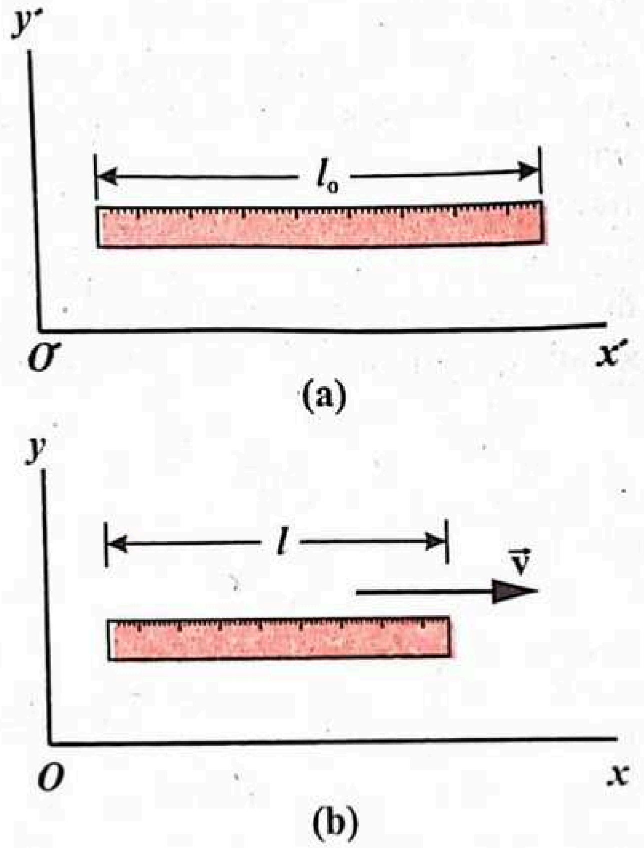


Fig.18.3 Length contraction of the body in a moving frame of reference.

Consider, the length of an object (rod) between two point which is measured by two observers, such that one observer is at rest relative to the object while the other one is moving with very high velocity 'v' with respect to the object. If the length measured by the rest observer is 'ℓ_0' and the length measured by the moving observer is 'ℓ' then according to the relativistic mechanics,

$$\ell = \ell_0\sqrt{1-\frac{v^2}{c^2}} \quad ......(18.1)$$

If the moving observer is also at rest, i.e., $v = 0$ then $\ell = \ell_0$, that is, the results of the both observers are the same. But if there is a relative motion of the observer with velocity 'v', the length ℓ is less than the proper length ℓ_0. This result shows that a moving body has a decrease in its length along the direction of motion as shown in Fig.18.3.

Time dilation

When a time interval between two events is measured by an observer who is at rest then it is called a proper time. Now according the special theory of relativity time is not absolute quantity but the time interval between the two same events would be greater than the proper time which is measured by another observer who is moving at very high-speed relative to the place of event. This effect is called time dilation i.e., to dilate means to become larger and it is explained as:

Consider two events which occurred at the same place. The time interval between these two events measured by two observers, such that one observer is at rest frame and he noted the proper time interval as 't_0' while, the other observer in other frame of reference is moving with uniform velocity v relative to the rest frame and he noted the time interval between the two events is 't'. Thus, according to the relativistic mechanics

> ### *POINT TO PONDER*
> If a rod of length 1m moves with a very high speed, will you observe its length less or greater than 1m?

> ### *POINT TO PONDER*
> Two identical constructed clocks are synchronized. One is attached with a wall of a moving spaceship while the other remains on earth. Which clock runs more fast?

$$t = \frac{t_0}{\sqrt{-\frac{v^2}{c^2}}} \quad ......(18.2)$$

1

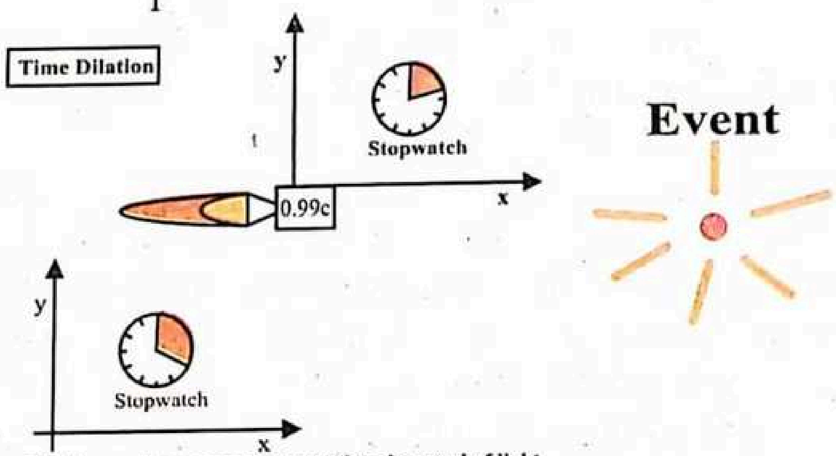


Fig.18.4 Different time interval between two events observed by two observers in two different frames of reference.

If the moving observer is also at rest. i.e., $v = 0$ then $t = t_0$. But if there is a relative motion between the two observers and the denominator of equation 18.2 is less than one thus time 't' is always greater than the proper time 't_0' as shown in Fig.18.4.

> ### *DO YOU KNOW*
> At constant distance, time and speed are inversely proportional to each other.
>
> i.e. $\quad \text{speed} \propto \frac{1}{\text{time}}$
>
> It means greater is the speed lesser is the time taken.

Mass variation.

Like length contraction and time dilation, mass of a body is also not absolute rather it is a varying quantity. It depends upon the speed of frame of reference or frame of observers. It is explained as: when a body is at rest, its measured mass is called its rest mass or proper mass. Now when the body starts motion at very high speed, then according to the special theory of relativity, the mass of the moving body will be increased as shown in Fig.18.5. This is called mass variation effect. Let m_0 be the rest mass of a body and m be the mass of the same body which is moving with velocity 'v' then the mass variation is given as

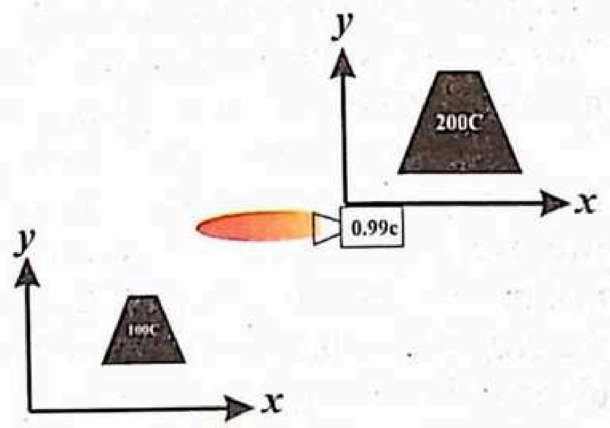


Fig.18.5 Mass dilation in a moving frame of reference.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{...(18.3)}$$

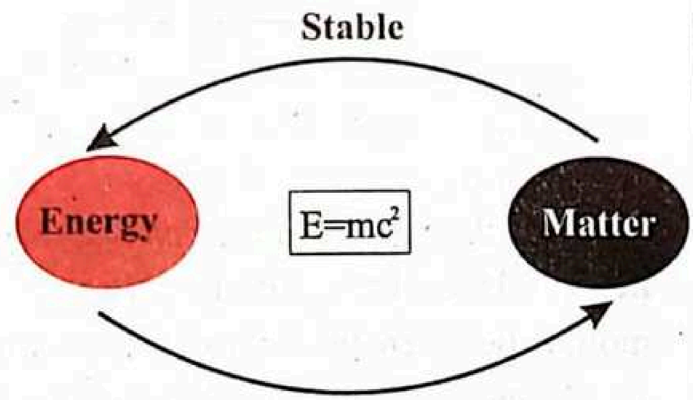
FOR YOUR INFORMATION

The global positioning system (GPS) takes account of the time dilation of orbiting atomic clocks. Otherwise, our GPS receiver would badly miss our location.

This relation shows that increase in mass of a body depends upon its speed. If the speed of mass is one-tenth of speed of light (0.1c), the increase in mass is only 0.5%. Similarly, the increase in mass of the body will be 100% if the body has a speed nine-tenth (0.9c) of the speed of light. But, in our daily life, we deal with extremely small speeds compared to the speed of light and hence $m = m_0$. This is the reason why Newton's laws of motion are valid in everyday life. However, in case of atomic particles which are moving with speeds comparable with speed of light, the relativistic effect can be observed evidently, and their experimental results cannot be explained without taking Einstein's equations into account.

Mass-Energy Relation

According to special theory of relativity, mass and energy are not two different quantities, but they are interconvertible to each other. That is, the mass 'm' of a moving body can be converted into energy 'E' and vice versa.



Mass and energy are convertible to each other.

Such conversion can be studied under the following mass-energy relation:

$$E = mc^2 \quad ......(18.4)$$

Similarly, if the body is at rest then its energy is called rest mass energy E_0 and its value is given by

$$E_0 = m_0c^2 \quad ......(18.5)$$

Mass-Energy Equivalence

$1kg \leftrightarrow 8.988 \times 10^{16}J$

$1u \leftrightarrow 931.5\ MeV_0$

$1eV_0 \leftrightarrow 1.074 \times 10^{-9}u$

According to the mass-dilation effect, change in mass causes of change in energy, since $E > E_0$, so

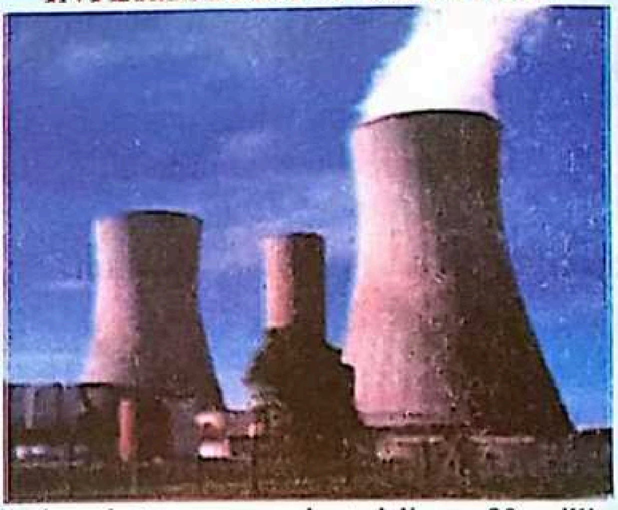
$$E - E_0 = mc^2 - m_0c^2$$

$$\Delta E = (m - m_0)c^2$$

$$\Delta E = \Delta mc^2 \quad ......(18.6)$$

This shows that a small variation in mass cause an enormous amount of change in energy, because the value of c^2 is very large. It can be observed experimentally in a nuclear power plant which produces energy by fission of uranium, it involves the conversion of a small amount of the mass of the uranium into large amount of energy.

INTERESTED INFORMATION



Saying that a power plant delivers 90 million megajoules of energy to its consumers is equivalent to saying that it delivers 1 gram of energy to its consumer because mass and energy are equivalent.

Example 18.1

A rocket of proper length 50m is moving with a speed of 0.13c. Then how much will it appear to be shortened to an observer on earth?

Solution:

Proper length = $\ell_0 = 50m$

Speed of rocket= $v = 0.13c$

Speed of light = $c = 3 \times 10^8\ ms^{-1}$

Shortened length = $\ell = ?$

According to the relation of length contraction

$$\ell = \ell_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$\ell = 50\sqrt{1 - \frac{(0.13c)^2}{c^2}}$$

$$\ell = 50\sqrt{1 - 0.0169}$$

$$\ell = 49.5\text{m}$$

Shortening the length of the rocket

$$\Delta\ell = \ell_0 - \ell$$

$$\Delta\ell = 50\text{m} - 49.5\text{m}$$

$$\Delta\ell = 0.5\text{m}$$

Example 18.2

An atom will decay in 2×10^{-6} s. What will be the decay time as measured by an observer in a laboratory if the atom is moving with a speed of 0.8c.

Solution:

Time of decay when the atom is rest = $t_o = 2 \times 10^{-6}$s

Time of decay when the atom is moving = t = ?

Speed of atom = 0.8c

According to time dilation equation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{2\times10^{-6}}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = \frac{2\times10^{-6}}{\sqrt{1-0.64}}$$

$$t = \frac{2\times10^{-6}}{\sqrt{0.36}} = \frac{2\times10^{-6}}{0.6}$$

$$t = 3.33 \times 10^{-6}\text{s}$$

Example 18.3

At what speed the mass of the body will become twice the rest mass value?

Solution:

Rest mass of the body = m_o

Mass of the moving body = m

Let us assume that at a speed of 'v', the mass of the body will be doubled. i.e.,

$$m = 2m_0$$

According to mass variation relation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$2m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$2\sqrt{1-\frac{v^2}{c^2}} = 1$$

Squaring both sides

$$4\left(1-\frac{v^2}{c^2}\right) = 1$$

$$4 = \frac{4v^2}{c^2} + 1$$

$$\frac{4v^2}{c^2} = 3$$

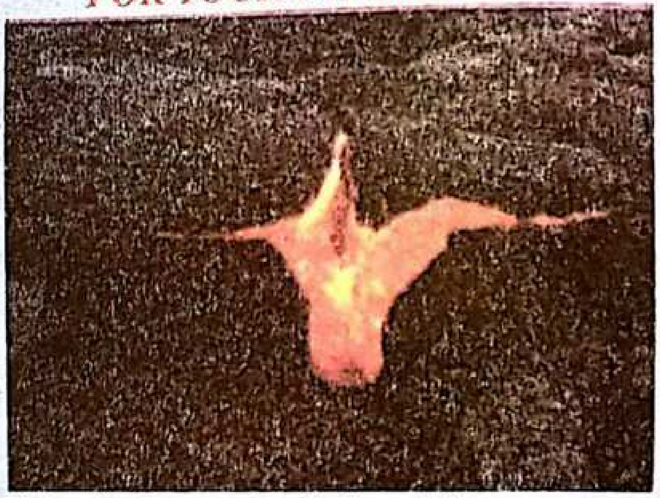
$$v^2 = \frac{3}{4}c^2 \quad \Rightarrow v = 0.866c = 0.866 \times 3 \times 10^8 \, m\,s^{-1}$$

$$v = 2.6 \times 10^8 \, m\,s^{-1}$$

18.5 BLACKBODY RADIATION

When a solid body is heated, the body glows and emits radiation. This emission of radiation depends upon temperature only. That is, when the temperature increases, the body becomes red then yellow then white. The wavelengths of their corresponding emitted radiations are decreased which are lying in the range from infrared to ultraviolet. For example, at low temperature, the emitted radiation has long wavelength, while at high temperature, the emitted radiation has short wavelength.

FOR YOUR INFORMATION



Volcanic lava emits light and is a very good example of a blackbody radiation.

The analysis show that when these radiations fall on a body, some of them may be absorbed and some of them reflected. So, we could not study well the distribution of radiation from a hot body. We consider **an ideal body that absorbs and emits the radiations of all wavelength falling on it. Such body is called Blackbody.** Blackbody is perfect absorber for all incident radiations and also a perfect emitter or radiator of all kind of radiations. When the black body is heated, it emits the radiations of all wavelengths depending upon its temperature. Thus, a black body is a perfect absorber as well as a perfect emitter of radiation. Practically, a perfect black body does not exist, but it can be constructed by making a very small hole in one of the walls of a hollow body called cavity as shown in Fig.18.6. The inner walls of the cavity are blackened with carbon soot to make them as good absorber and also as a reflector. The radiation that enters through the small hole will be trapped inside the cavity. If this body is heated to a certain temperature, it will emit radiations of all wavelengths which is called blackbody radiation. The nature of emission of radiation depends upon temperature only. That is, when the temperature of the blackbody is increased, the radiated energy emitted from the blackbody is also increased. Graphically the experimental data for the distribution of energy of blackbody radiation i.e. intensity versus wavelength are shown in Fig.18.7. These results show that wavelength of radiated energy varies with temperature and shifts towards the shorter values as the temperature of the cavity increases.

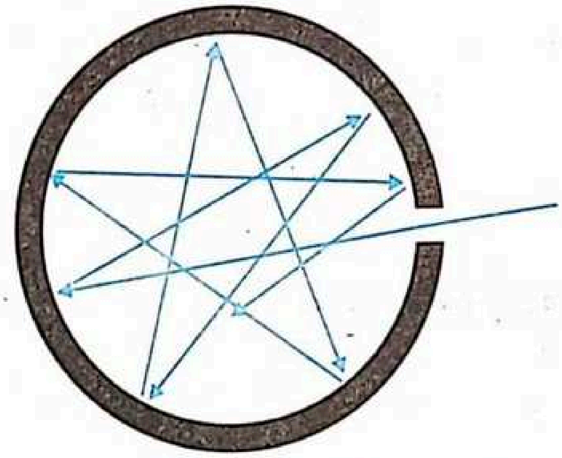


Fig.18.6 A hole in the wall of a hollow sphere is a good approximation of a black body.

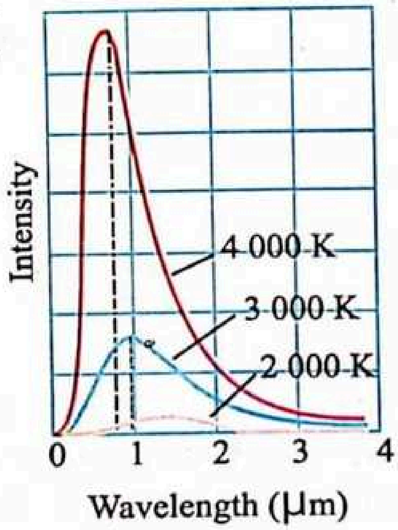


Fig.18.7 Graph between intensity of blackbody radiation and its wavelength. The energy increase with increasing temperature and hence decrease in its wavelength

Several attempts were made to explain the distribution and characteristics of these radiations. The four most important among them are given by:

> **POINT TO PONDER**
> An incandescent light bulb is connected to a dimmer switch. When the bulb operates at full power, it appears white, but as it is dimmed it looks more and more red. Explain?

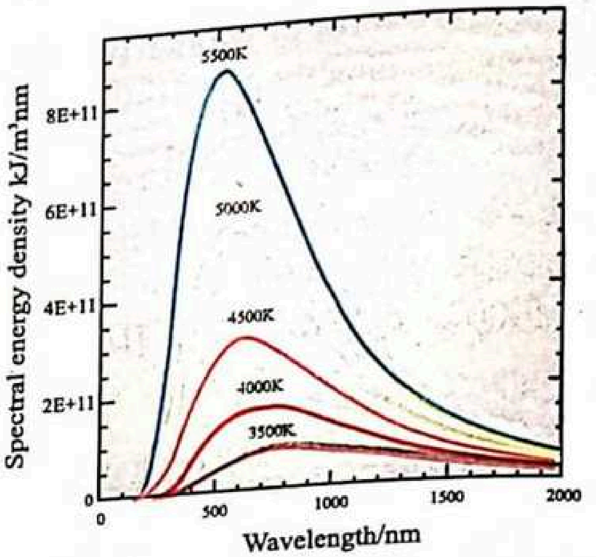
Stefan-Boltzmann law

This law states that the total energy per second per unit area (i.e. total radiant heat power) under the curve of the radiation is directly proportional to the fourth power of its absolute temperature

$$E \propto T^4$$

$$E = \sigma T^4 \quad ......(18.7)$$

where 'σ' is Stefan's constant and its value is $5.67 \times 10^{-8}\ wm^{-2}K^{-4}$.



The peak wavelengths of radiations decreasing with increasing temperature.

Wien's displacement law

According to Wien's displacement law, the wavelength with maximum intensity in the radiation emitted from a blackbody is inversely proportional to the temperature of the blackbody. That is, with increasing temperature the peak of the distribution shifts to shorter wave lengths;

$$\lambda_m = \frac{1}{T}$$

$$\lambda_m T = \text{Constant} \quad ...(18.8)$$

where the value of Wien's constant is 2.9×10^{-3}mK

Rayleigh-Jean's law

Rayleigh and Jean explained the distribution of radiation based on kinetic theory. They assumed that the radiation inside the cavity consists of standing waves. These standing waves are due to the oscillation of radiation from one wall to the other wall of the cavity. Based on this argument, they derived the following relation.

$$E = \frac{8\pi kT}{\lambda^4} \quad ......(18.9)$$

where 'k' is the Boltzmann's constant and its value is $1.3807 \times 10^{-23}\ JK^{-1}$.

Max Planck's law

Wien's displacement law and Rayleigh-Jean's law both explain only single aspect of the energy distribution along the experimental intensity and wavelength curves. Because, Wien's formula can apply for those radiations which have short wavelengths while Rayleigh-Jean's formula for radiations having long wavelengths as shown in Fig.18.8. None of these laws is able to explain the entire experimentally observed curve.

To overcome this problem, Max Planck in the year 1900 developed a formula for distribution of energy that fitted well into the experimental curve for the entire range of wavelengths. His formula for the distribution of energy based on his quantum theory. According to his theory, the absorption and emission of radiation from a black body is in the form of energy packets which he termed as 'quanta'. The energy of each quanta is directly proportional to the frequency of the radiation, i.e.,

$$E \propto f$$

$$E = hf \quad ......(18.10a)$$

where 'h' is known as Planck's constant. Its value is 6.63×10^{-34}Js.

But $f\lambda = c$ therefore eq. 18.10a becomes

$$E = \frac{hc}{\lambda} \quad ......(18.10b)$$

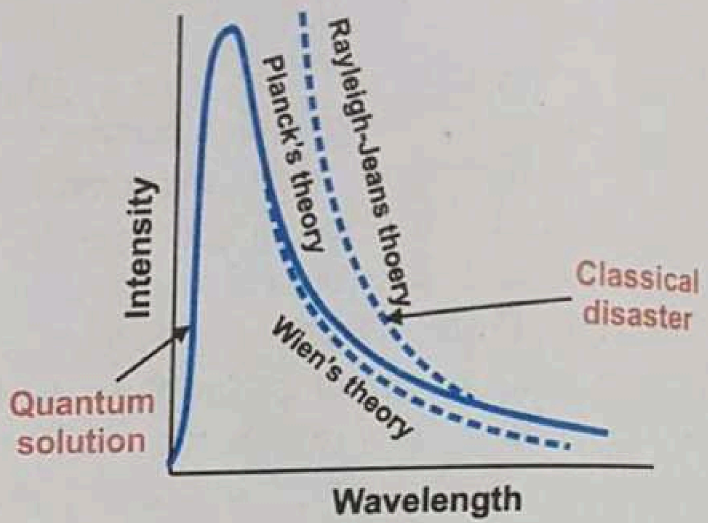


Fig.18.8. Comparison the experimental curves among Wien's, Rayleigh-Jean's and Planck's laws.

Later, Einstein extended Planck's idea that the energy is absorbed or emitted in discrete packets which are integral part of all kind of electromagnetic radiations know as photon. The energy of each photon is 'hf' and its momentum can be calculated as

Since $\quad E = hf = mc^2$

So $\quad mc = \frac{hf}{c}$

But the product of mass 'm' and velocity (velocity of light 'c') is called momentum p. Thus,

$$p = \frac{hf}{c}$$

But $f\lambda = c$ therefore

$$p = \frac{h}{\lambda} \quad ......(18.11)$$

> *POINT TO PONDER*
>
> A particle with non-zero mass can never move faster than the speed of light. Is there also upper limit on its momentum and kinetic energy?

Example 18.4

What is the wavelength of the radiation having maximum intensity when it is emitted from a blackbody at temperature 57^0C.

Solution:

Wavelength $(\lambda_m) = ?$

Temperature $(T) = 57°C = 330K$

Wien's constant $= 2.9 \times 10^{-3}$mK

According to Wien's formula

$$\lambda_m T = \text{Constant}$$

$$\lambda_m = \frac{\text{constant}}{T}$$

$$\lambda_m = \frac{2.9\times10^{-3}\text{mK}}{330\text{K}} = 8.8\times10^{-6}\text{m} = 8.8\mu\text{m}$$

Example 18.5

Compute the energy of photon of infrared light of wavelength 1240nm.

Solution:

Energy of photon (E) = ?

Wavelength of infrared light (λ) = 1240nm = 1240×10^{-9}m = 1.24×10^{-6}m

Planck constant (h) = 6.63×10^{-34}Js

The energy of the photon is given by

$$E = \frac{hc}{\lambda}$$

$$\therefore \quad E = \frac{6.63\times10^{-34}\text{Js}\times3\times10^{8}\text{ms}^{-1}}{1.24\times10^{-6}\text{m}}$$

$$E = 1.6\times10^{-19}\text{J}$$

As $\quad 1.6\times10^{-19}\text{J} = 1\text{eV}$

$\therefore \quad E = 1\text{eV}$

The energy of given infrared light is 1eV.

18.6 THE PARTICLE THEORY OF ELECTROMAGNETIC RADIATIONS

In the previous class, we have verified the wave nature of light by interference and diffraction phenomenas. In this section, we are going to explain the particle nature of electromagnetic radiations by means of the following three phenomenas:

I Photoelectric effect

II Compton effect

III Pair production

18.7 PHOTOELECTRIC EFFECT

Heinrich Hertz made the first observation of the photoelectric effect in 1887. He observed that when ultraviolet light falls on a metal surface (zinc, cadmium, magnesium etc), then there is emission of electrons from the metal surface. This phenomenon is known as the photoelectric effect and the emitted electrons are called photo-electrons. The photoelectric effect is also possible by visible light when the target material is alkali metal such as sodium, potassium, calcium etc.

A schematic diagram of a photoelectric effect is shown in Fig.18.9. It consists of photosensitive metallic plate P and collector 'C' which are enclosed in an evacuated glass tube. The plate is connected to negative terminal of the battery called cathode. The second electrode called collector or anode is connected to positive terminal of the battery through a galvanometer G. When light of certain frequency falls on the metallic plate, then there is emission of photo electrons from the plate. These photoelectrons are attracted by the anode to constitute an electric current called photo electric current which is detected by the galvanometer. The current becomes zero as soon as the light is switched off. This confirms that the light energy is converted into the electrical energy under the process of photoelectric effect.

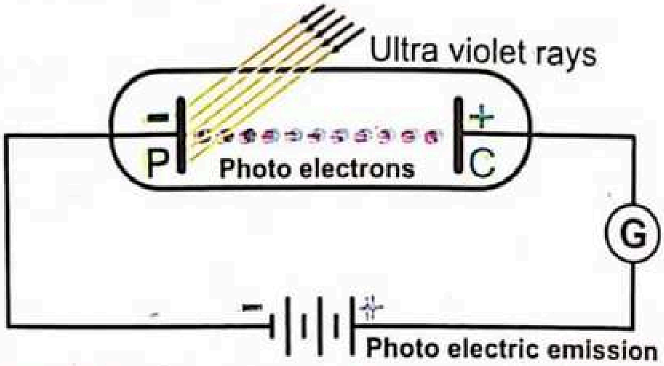


Fig.18.9. An experimental circuit diagram for photo electric effect.

Maximum kinetic energy of photoelectrons

In the experimental arrangement of photoelectric effect, anode C always collects the photoelectrons, because, it is connected to the positive terminal of the battery. When the terminals of the battery are reversed as shown in Fig.18.10, then 'C' becomes negative. In this condition, the photoelectrons are repelled by the 'C', as a result the photoelectric current starts decreasing. When the negative potential is further increased then at certain negative potential, the photoelectric current becomes zero. At this stage, no electron reaches at the anode C. This specific negative potential is called stopping potential V and it is related to the kinetic energy of photo electrons. That is,

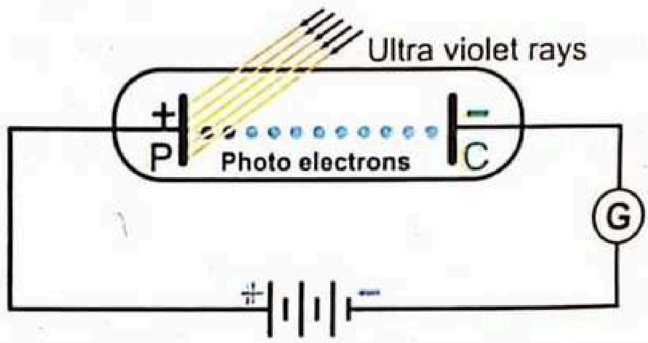


Fig.18.10. A reverse circuit diagram a photo electric effect in order to calculate the kinetic energy of photo electrons.

$$\frac{1}{2}mv^2 = V_0e \text{ ......(18.12)}$$

This is the fundamental equation of photoelectric effect.

Experimental observations or laws of photoelectric effect

The experimental observations lead different laws based upon the photoelectric effect. All these laws are summarized as:

I Threshold frequency and work function

The emission of electrons will not take place if the frequency of incident radiation or photon is below a certain minimum frequency known as threshold frequency. In other words, the minimum frequency of the incident light is required for the emission of electrons from the metal surface. The threshold frequency is denoted by f_0. The threshold frequency has a different value for different metals. The minimum energy required to eject an electron from photo emissive surface is called the work function. The work functions of different metals are given in table 18.1. If the energy of incident light is above the work function appears as the kinetic energy of photoelectrons.

II Spontaneous emission of photoelectron

When the frequency of the incident light is equal or greater than the threshold value then there will be spontaneous emission of photo electron. The time of emission is 10^{-18}s.

Table 18.1

Element	Work function (eV)
Aluminum	4.3
Carbon	5.0
Copper	4.7
Gold	5.1
Iron	4.7
Nickel	5.1
Potassium	2.3
Silicon	4.8
Silver	4.3
Sodium	2.7
Tungsten	4.6

III The photoelectric current depends upon the intensity of light

The analysis shows that the photoelectric current is directly proportional to the intensity of incident light provided that frequency if incident photon is above f_0. That is, brighter incident light will cause more emission of electrons from the metal surface and hence a large current flow in the circuit. It is noticed that the intensity of light does not increase the kinetic energy of photoelectrons. In other word, kinetic energy of photoelectrons is independent of the intensity of the incident light.

IV The kinetic energy of photoelectrons depends upon the frequency of light

The maximum kinetic energy of photoelectrons is directly proportional to the frequency of the incident light. Furthermore, the stopping potential of the photoelectrons also depends upon the frequency of the incident light.

18.7.2 Einstein's photoelectric effect equation

Einstein explained the photoelectric effect on the basis of quantum theory of light. According to Planck's quantum theory, light is absorbed or emitted in form of energy packet called quanta or photon. The energy of each photon is hf. When light falls on a metal surface then the energy of photon is absorbed by an electron of the metal surface. Thus according to Einstein, a part of this energy is used to liberate or emit the electron from the metal surface called work function ϕ and the remaining part of the incident energy is converted into the kinetic energy of the photoelectron. That is,

Energy of incident photon = work function + K.E. of photoelectrons

$$hf = \phi + \frac{1}{2}mv^2$$

$$hf = hf' + eV_0$$

$$eV_0 = hf - hf' \quad ......(18.13)$$

This is known as Einstein's equation of photo electric effect. If we plot a graph between frequency of the incident photons and kinetic energy of photoelectrons then we get a straight line as shown in Fig.18.11.

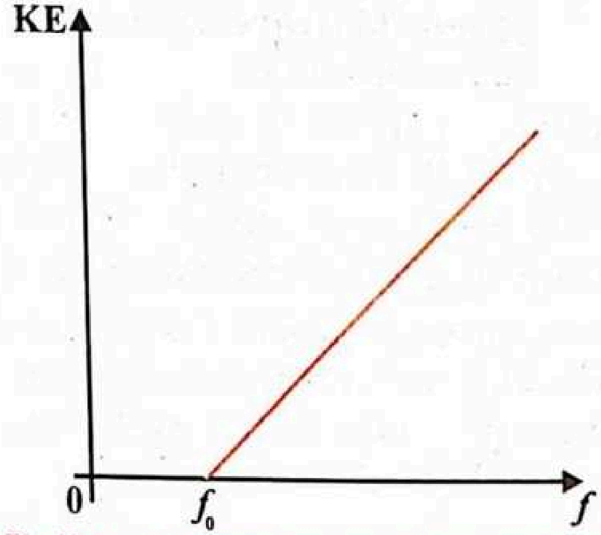


Fig.18.11. A straight line graph between kinetic energy of photo electrons and the frequency of the incident photons

18.8 PHOTOCELL

A photocell is a device that converts light energy into electrical energy. It works on the principle of photoelectric effect. The photocell consists of two electrodes, emitter and collector which are enclosed in an evacuated glass tube. The metal of emitter or cathode used must be either sodium or potassium, because these metals are very sensitive to visible light. However, if the surface of the cathode is coated with cesium then it emits electrons even for infrared light. The collector or anode is in the form of a straight rod whereas the cathode is curved to receive the maximum incident light as shown in Fig.18.12.

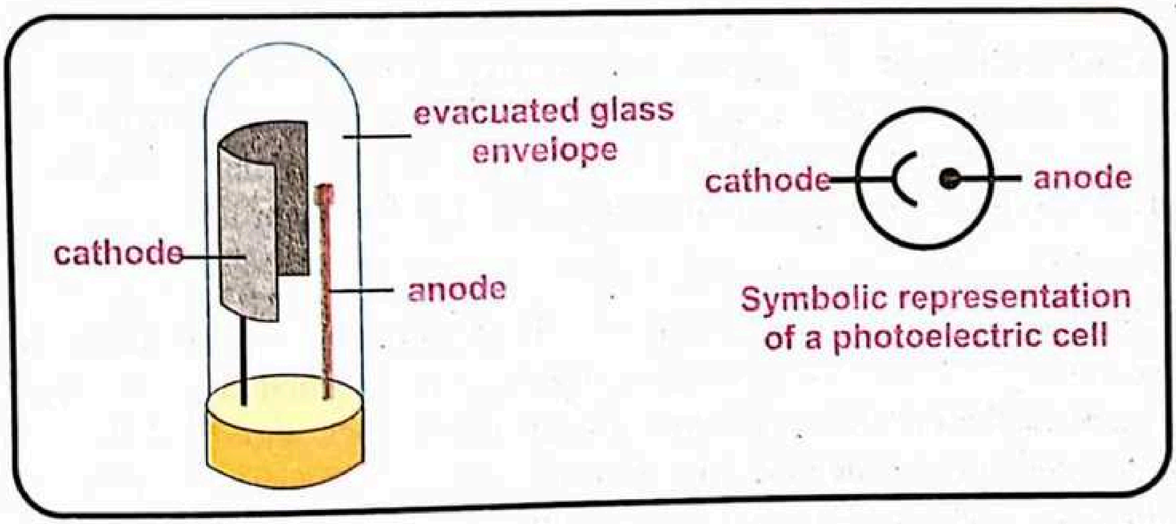


Fig.18.12. A photo cell with its symbol.

When light from a source is incident on the cathode, photoelectrons are emitted from it. These electrons are attracted by the positive anode to constitute an electric current in the external circuit. The magnitude of the current can be increased by increasing the intensity of light above threshold frequency. If the light beam is switched off, then the flow of current also stops.

There are number of applications of a photocell, some of them are listed below:

1. Counting systems
2. Security systems
3. Automatic door opening/closing
4. Automatic street lighting system
5. Soundtrack of movies
6. Burglar alarm system
7. Measurement the temperature of stars

> *POINT TO PONDER*
> Rest mass of a photon is zero. Is its momentum also zero?

18.9 SOLAR CELL

Like photocell, **a solar cell is also an electrical device which converts sun's light energy directly into electrical energy**. A crystalline silicon solar cell consists of N-type and P-type semiconductors, sandwiched in between two metal contacts that are responsible for conducting electricity out of such a device as shown in Fig.18.13(a). An anti-refractive coating is also placed on the top of the metal contact and N-type in order to absorb maximum solar energy by the cell. Finally, a thin glass sheet is placed on the top of the cell to protect it from weather and mechanical shock.

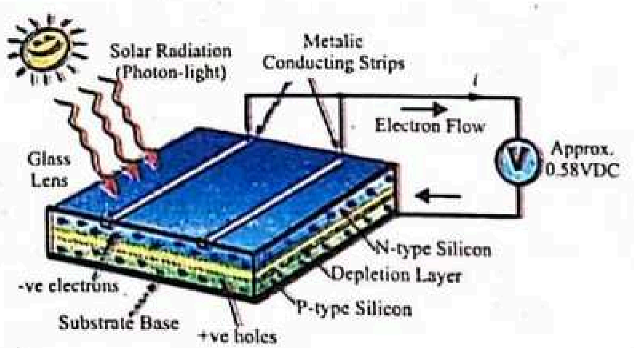


Fig.18.13(a) Schematic diagram of a solar cell with its various parts..

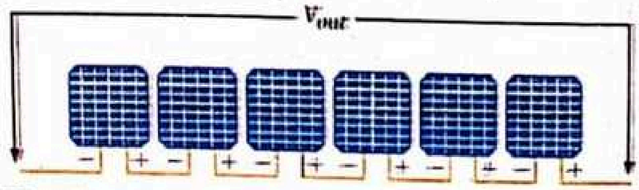


Fig.18.13(b). A series connection of solar cells panel.

When sunlight falls on the PN-junction of the solar cell then this light has sufficient energy to knock an electron out of the valance band. The electron becomes a free electron and leaves a hole in the valance band, creating an electron-hole pair. This free electron can be easily accelerated by the electric field existing naturally at the PN-junction towards the N-type and hole towards the P-type. In this way, electrons accumulated in the N-type, creating negative charges and holes accumulated in the P-type, creating positive charges. Thus an electric potential is developed between N-type and P-type contacts.

Practically, a single solar cell can produce a potential difference of 0.6V. To produce higher voltage, multiple solar cells are connected in series called solar panels as shown in Fig.18.13(b). Solar panels are generally available in 12V, 24V, 36V and 48V.

A solar cell technology is a wonderful development in the field of alternative energy sources. In the present age, its application can be observed in every walk of life, but some of these are given by:

INTERESTING INFORMATION

Noor Solar Power Plant is the world's largest solar power plant which consist of 7400 solar panels and it occupy 25000 hectares land. It is located in the Sahara Desert. The project has a 580-megawatt capacity and is expected to provide electricity for over 1 million people.

I. It is being used in the remote areas where access to the main power grid is impossible.
II. It is used to power the solar energy driven vehicles, such as: car, auto etc.
III. It is being used in boats, submarines etc. as a source of electricity.
IV. It is used in spacecraft, artificial satellite and space station to provide electricity.

Example 18.6

Electrons with a maximum kinetic energy of 3eV are ejected from a metal surface by ultraviolet light of wavelength 150nm. Determine the work function of the metal, the threshold wavelength of the metal and the negative potential required to stop the emission of electrons.

Solution:

$$\text{K.E of electrons} = 3\text{eV} = 3\left(1.6\times10^{-19}\,\text{J}\right) \qquad \therefore 1\text{eV} = 1.6\times10^{-19}\,\text{J}$$

$$= 4.8\times10^{-19}\,\text{J}$$

$$\text{Wavelength } (\lambda) = 150\text{nm} = 1.5\times10^{-7}\,\text{m}$$

$$\text{Speed of light} = c = 3\times10^{8}\,\text{m s}^{-1}$$

$$\text{Planck's constant} = h = 6.63\times10^{-34}\,\text{J s}$$

(a) Work function $(\phi) = ?$

(b) Threshold wavelength $(\lambda_0) = ?$

(c) Stopping potential $(V_0) = ?$

According to the equation of photoelectric effect

$$hf = \phi + \frac{1}{2}mv^2$$

$$\phi = \frac{hc}{\lambda} - \frac{1}{2}mv^2$$

$$\phi = \frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,ms^{-1}}{1.5\times10^{-7}\,m} - 4.8\times10^{-19}\,J$$

$$\phi = 13.26\times10^{-19} - 4.8\times10^{-19}$$

$$\phi = 8.46\times10^{-19}J = 5.28eV$$

By the definition of work function

$$\phi = hf_0 = \frac{hc}{\lambda_o}$$

$$\lambda_0 = \frac{hc}{\phi} = \frac{6.63\times10^{-34}\,Js\times3\times10^{8}\,ms^{-1}}{8.46\times10^{-19}\,J}$$

$$\lambda_0 = 2.35\times10^{-7}m = 235nm$$

According to the basic equation of photoelectric effect:

$$\frac{1}{2}mv^2 = eV_o$$

$$V_o = \frac{\frac{1}{2}mv^2}{e} = \frac{4.8\times10^{-19}\,J}{1.6\times10^{-19}\,C}$$

$$V_o = 3 \text{ volts}$$

18.10 PARTICLE (PHOTON) MODEL OF LIGHT

In quantum theory, we have studied that light consists of a small packet of energy. This packet of energy was named as 'photon' by Einstein in 1905. According to Einstein, a photon behaves as a particle, it travels with speed of light 'c' and it has both energy as well as momentum. Thus Einstein defined light in terms of photon which is called 'photon (particle) theory of light'. The particle nature of light has been observed in the Compton effect and practically it has been proved in Davisson and Germer experiment.

Salient features of photon

i. A photon behaves as a particle whose rest mass is zero and it travels with speed of light 3×10^8 m s^{-1}. In other words, a photon exists as long as it is moving. It ceases to exist when it comes to rest.

ii. Photons are electrically neutral and are not deflected in the presence of electric and magnetic fields.

iii. The energy of a photon is given as:

$$E = hf = \frac{hc}{\lambda} \quad \therefore f\lambda = c$$

This shows that the energy of photon depends upon frequency. (or wavelength)

iv. Momentum of photon is given as:

As $$E = hf = mc^2$$

So $$mc = \frac{hf}{c}$$

$$p = \frac{hf}{c}$$

v. Rest mass of a photon can be calculated by using Einstein's mass variation equation:

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m_0 = m\sqrt{1-\frac{v^2}{c^2}}$$

Since photon is moving with speed of light 'c' so $v = c$

$$m_0 = 0$$

This shows that rest mass of a photon is zero.

18.11 COMPTON EFFECT

The experiment by Arthur H. Compton is another justification of the particle theory of light. The experimental setup consists of a beam of x-rays of wavelength λ_0 which is incident on a block of graphite. This incident x-ray photon is scattered at some angle from initial direction. Compton observed that the wavelength 'λ' of the scattering x-ray is slightly longer than the wavelength of the incident x-ray. It means that the energy of the scattering x-rays is also lower than the energy of the incident x-ray photon. The increase in wavelength

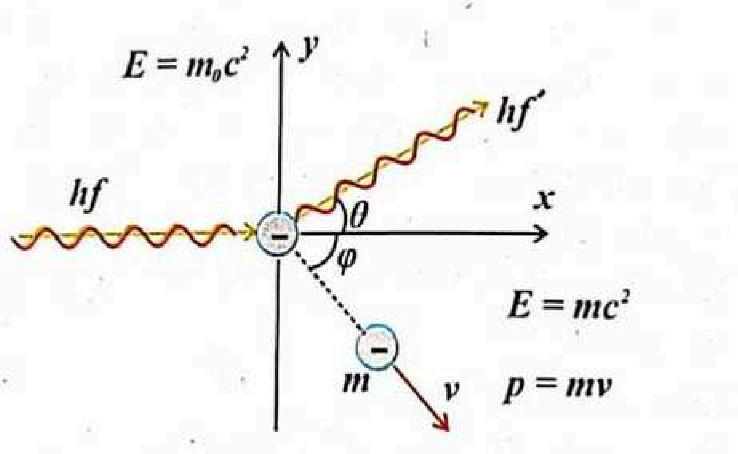


Fig.18.14. A schematic diagram for Compton effect where the elastic collision occurs between a x-ray photon and an electron at rest.

or decrease in energy of x-rays when they interact with matter or are scattered is known as Compton effect. The change in wavelength $\Delta\lambda$ between them is termed as Compton's shift.

Based on quantum theory, Compton assumed that x-rays consist of photons and these photons behave like particles. Now the collision of a single photon with an electron at rest is like a collision between two billiard balls. As x-ray photon carries both energy (hf) and momentum ($\frac{hf}{c}$). Hence during each interaction incident X-ray photon must transfers some of its energy and momentum to the electron. So, these two quantities must be conserved in an elastic collision between photon and electron at rest, where photon transfers some of its energy and momentum to the electron.

The schematic diagram of the elastic collision between a x-ray photon and an electron is shown in Fig.18.14. If 'θ' and 'ϕ' be the scattering angles of photon and electron respectively after collision then according to law of conservation of energy.

energy of a system before collision = energy of a system after collision

$$(E)_{Photon} + (E)_e = (E)_{Photon} + (E)_e$$

$$hf + m_0c^2 = hf' + mc^2$$

$$hf - hf' = (m - m_0)c^2 \quad ......(18.14)$$

Similarly, according to law conservation of momentum along x-axis

$$\frac{hf}{c} + 0 = \frac{hf'}{c}\cos\theta + mv\cos\phi \quad ......(18.15)$$

Conservation of momentum along y-axis

$$0 + 0 = \frac{hf'}{c}\sin\theta - mv\sin\phi \quad ......(18.16)$$

By simplifying Eq.18.14, Eq.18.15 and Eq.18.16 we get the following result:

$$\frac{1}{f'} - \frac{1}{f} = \frac{h}{m_0c^2}(1 - \cos\theta) \quad ......(18.17)$$

This equation shows that the photon after collision scattered at angel 'θ' with respect to its incident direction. If it is not scattered, then the value of 'θ' would be zero and hence the value of right hand side of Eq.18.17 would be zero. But angle θ has some finite value. Therefore, right hand side of Eq.18.17 is greater than zero. i.e.,

$$\frac{1}{f'} - \frac{1}{f} > 0$$

$$f > f'$$

This condition is known as Compton Effect.

POINT TO PONDER

Why does a photon that has been scattered from an electron, initially at rest have a longer wavelength than the incident photon?

Compton shift in wavelength

As

$$c = f\lambda$$

$$\frac{1}{f} = \frac{\lambda}{c}$$

and

$$\frac{1}{f'} = \frac{\lambda'}{c}$$

Therefore, Eq.18.17 becomes

$$\frac{\lambda'}{c} - \frac{\lambda}{c} = \frac{h}{m_0 c^2}(1-\cos\theta)$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) \quad ......(18.18)$$

This is known as Compton shift in wavelength and its value depends upon the scattering angle of photon.

Example 18.7

A photon with a wavelength 0.4nm strikes with an electron having rest mass 9.11×10^{-31}kg. If the x-rays are scattered at angle of 45° after the collision, then what is the wavelength of the scattering x-rays?

Solution:

Wavelength of incident photon = $\lambda = 0.4$nm $= 0.4 \times 10^{-9}$m

Angle $\theta = 45^0$

Mass of electron $= 9.11 \times 10^{-31}$kg

Wavelength scattered photon or x-rays $= \lambda' = ?$

Planck's constant $= h = 6.63 \times 10^{-34}$ Js

speed of light $= c = 3 \times 10^8$ms^{-1}

According to Compton shift in wavelength

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\theta)$$

$$\lambda' = 0.4\times10^{-9}\,\text{m} + \frac{6.63\times10^{-34}\,\text{Js}}{9.11\times10^{-31}\,\text{kg}\times3\times10^{8}\,\text{ms}^{-1}}(1-\cos45^{\circ})$$

$$\lambda' = 0.4\times10^{-9} + 0.243\times10^{-11}(1-0.707)$$

$$\lambda' = 0.4\times10^{-9} + 0.243\times10^{-11}(0.293)$$

$$\lambda' = 0.4\times10^{-9} + 0.07\times10^{-11}$$

$$\lambda' = 0.4\times10^{-9} + 0.0007\times10^{-9}$$

$$\lambda' = 0.4007\times10^{-9}\,\text{m}$$

$$\lambda' = 0.4007\ \text{nm}$$

18.12 PAIR PRODUCTION

Pair production implies the creation of an elementary particle and its antiparticle. **A process in which a high energetic gamma ray photon disappears by producing a pair i.e., an electron and a positron when it passes close to a heavy nucleus.** A positron is the antiparticle of an electron. It has same mass as that of an electron but carries opposite charge i.e. +e. In pair production energy, momentum and even charges are conserved.

Pair production process confirms the particle theory of electromagnetic radiations, because, it is a direct conversion of radiant energy into matter. Hence this process is also known as materialization of energy.

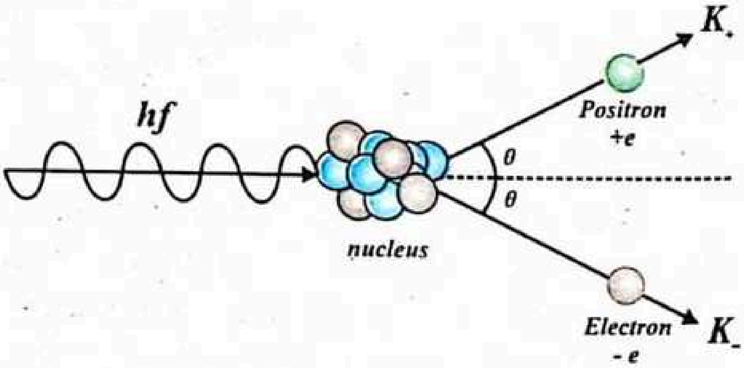


Fig.18.15. A high energetic photon interacting with a nucleus disappears into an electron and a positron pairs.

The schematic diagram of pair production is shown in Fig.18.15. The incident photon's energy hf must be equal or greater than rest mass energy of two electrons. According to Einstein's Energy-mass equation $E = mc^2$, the mass of a single electron is equivalent to 0.51 MeV of energy. Thus, for the occurrence of pair production, the energy of the incident photon must be at least equal to 1.02 MeV. Now if the energy of photon is greater than 1.02 MeV, called threshold, 1.02 MeV energy is used for pair production and the surplus energy would appear as kinetic energies of an electron and a positron. On the basis of this principle, we can develop a mathematical relation for a pair production as

Energy of incident photon = required energy for pair production + $K.E_{-e}$ + $K.E_{+e}$

$$hf = 2m_0c^2 + K.E_{-e} + K.E_{+e} \quad ......(18.18)$$

This is the basic equation for pair production.

Annihilation of matter

A process in which a particle and its antiparticle disappear during their interaction, as a result two gamma ray photons are produced is known as Annihilation of matter. It is the inverse process of pair production. For instance, when an electron and a positron are combined, they annihilate each other and give rise to two gamma ray photons as shown in Fig.18.16, which are moving in opposite directions. The energy of each gamma-ray photon is 0.51 MeV which is equal to the rest mass energy of electron or positron i.e., m_oc^2. Like in pair production, the energy and momentum are also conserved in annihilation of matter.

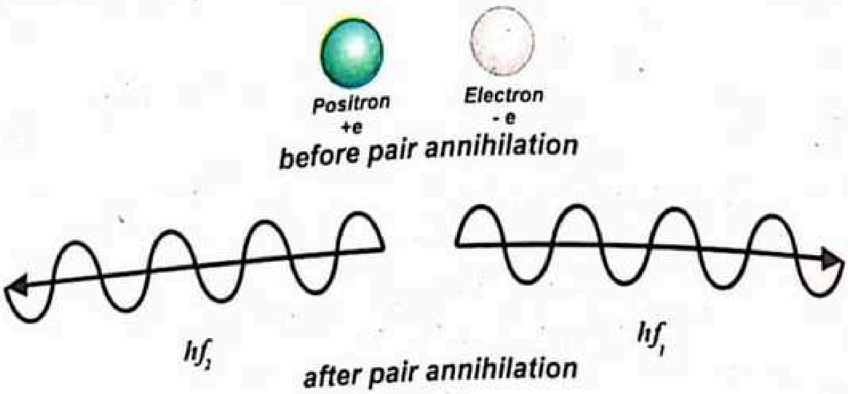


Fig.18.16. Combination of electron and positron produced two gamma ray protons in the process of annihilation of matter.

Besides of electron and positron, the process of annihilation of other anti-particles can also take place. For example, annihilation of proton and antiproton, lepton and antilepton, quark and antiquark and so on.

18.13 WAVE NATURE OF PARTICLES

It has been verified experimentally that light has dual nature or characteristics. As the results of some experiments reveal that it behaves as a particle while some other experiments reveal the wave nature of light. It must be noted that both aspects of nature of light cannot observed at the same time. Photoelectric effect, Compton effect and pair production verify the particle nature of electromagnetic radiations.

Subsequent to the confirmation of dual nature of light, Louis de Broglie in 1924 proposed that just as light has both wave-like and particle-like properties, electrons also have wave-like properties, i.e., any kind of matter has wave like properties. De Broglie proposed that 'a wave is always associated with every particle of matter of mass m moving with velocity 'v'. In this connection he proposed a mathematical relation for the photon, which is expressed as;

$$\text{Momentum of particle} = mv \quad \ldots\ldots(18.19)$$

$$\text{Momentum of photon} = \frac{hf}{c}$$

But $$c = f\lambda$$

> **POINT TO PONDER**
> If an electron and a proton have the same de-Broglie wavelength. Which particle has higher speed?

And $$\frac{1}{\lambda} = \frac{f}{c}$$

$\therefore$ Momentum of photon $= \frac{h}{\lambda}$ ...(18.20)

$\therefore$ Comparing Eq.18.19 and Eq.18.20

$$mv = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv} \quad ...(18.21)$$

This is known as de-Broglie wave equation and it shows that, if a particle of mass 'm' moving with velocity v then it has wavelength λ associated with particle. This equation illustrates that wave like properties cannot be observed for too heavy particles. If the mass of an object is very small, like sub-atomic particles such as, electron and is moving until high velocity then the wave like properties can be detected experimentally.

Davisson and Germer experiment

According to De-Broglie's hypothesis and his equation, the mass of an electron is small enough to exhibit the wave like properties, i.e., the electrons which are particles have also wave nature. This idea was confirmed experimentally in 1927 by Davisson and Germer, when they observed that scattering of electrons from crystals which act like a three-dimensional diffractions grating and shows a diffraction pattern. Their experimental setup consists of the electrons from a source (Filament)

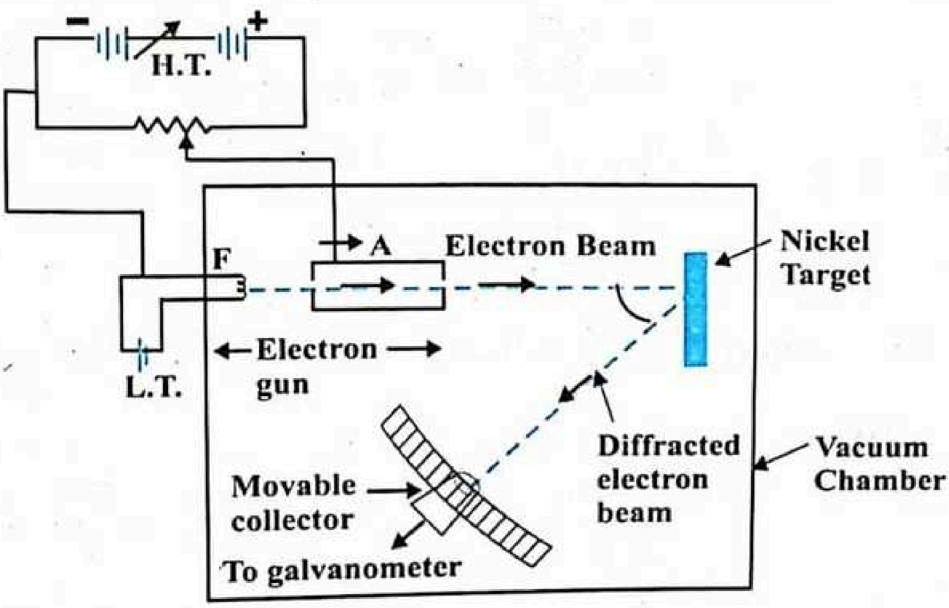


Fig.18.17. The experimental arrangement of Davisson and Germer experiment.

which are accelerated by a potential-difference V. A beam of these accelerated electrons emerges from the anode 'A' and is allowed to fall on a target which is a nickel crystal as shown in Fig.18.17. The incident electrons are scattered in different direction which are detected by a detector which moves along a circular scale. It is also observed that the intensity of scattered electrons at different angles is different and as a result, we have a diffraction pattern as shown in Fig.18.18.

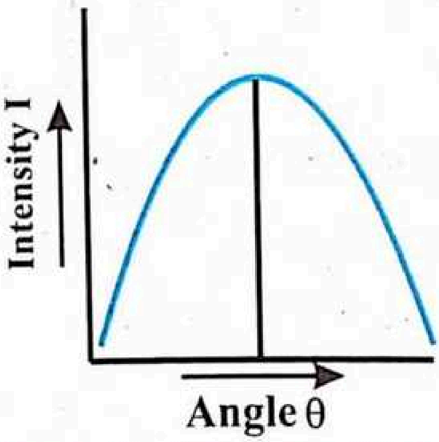


Fig.18.18. The intensity of the diffracted electrons at different angles.

The result proved that the reflected electrons have been diffracted by the crystal planes. Thus, the electrons which are particles demonstrates wave nature i.e., due to their diffraction from the crystal. The dual nature of electrons can be further verified mathematically, by using the experimental and theoretical data. The analysis show that the electrons are accelerated at 54eV and the intensity of the first order (m = 1) of the diffraction pattern is maximum at angle 50°, the spacing (d) between two adjacent planes of the Nickel crystal is 0.9×10^{-10} m. Using the data in the following two relations.

Wave nature

According to equation of diffraction from a surface of a crystal (Bragg's law)

$$m\lambda = 2d\sin\theta$$

$$(1)(\lambda) = 2(0.91\times10^{-10}\text{m})\sin(90^\circ - 25^\circ)$$

$$\lambda = 1.82\sin 65$$

$$\lambda = 1.65\times10^{-10}\text{m} \quad ......(18.22)$$

Particle nature

According to de-Broglie wave equation

$$\lambda = \frac{h}{mv}$$

According to the equation of photoelectric effect

$$\frac{1}{2}mv^2 = eV_o$$

$$v = \sqrt{\frac{2eV_o}{m}}$$

$$\therefore \quad \lambda = \frac{h}{m\sqrt{\frac{2eV_o}{m}}} = \frac{h}{\sqrt{2eV_o m}}$$

$$\lambda = \frac{6.63\times10^{-34}\,Js}{\sqrt{2(1.6\times10^{-19}\,C)(54V)(9.1\times10^{-31}\,kg)}}$$

$$\lambda = 1.66\times10^{-10}\,m \,......(18.23)$$

These two results given by Eq.18.22 and 18.23 have verified the De-Broglie hypothesis.

Example 18.8

Calculate the de-Broglie wavelength of an electron that has been accelerated through a potential difference of 9kV.

Solution:

de-Broglie wavelength (λ) = ?

Mass of an electron (m) = 9.1×10^{-31}Kg

Charge on an electron (e) = 1.6×10^{-19}C

Potential difference (V_o) = 9kV = 9×10^3 V

Planck's constant (h) = 6.63×10^{-34}J s

According to de-Broglie wave equation

$$\lambda = \frac{h}{mv}$$

But $\frac{1}{2}mv^2 = eV_o$

$$v = \sqrt{\frac{2eV_0}{m}}$$

$$\therefore \quad \lambda = \frac{h}{m\sqrt{\frac{2eV_o}{m}}}$$

$$\lambda = \frac{h}{\sqrt{2meV_0}}$$

$$\lambda = \frac{6.63\times10^{-34}\,Js}{\sqrt{2(9.1\times10^{-31}\,kg)(1.6\times10^{-19}\,C)(9\times10^3\,V)}}$$

$$\lambda = \frac{6.625\times10^{-34}}{5.12\times10^{-47}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{5.12 \times 10^{-23}}$$

$$\lambda = 1.3 \times 10^{-11} \text{m}$$

18.14 ELECTRON MICROSCOPE

An electron microscope is useful device to obtain high resolution images of extremely small biological and non-biological specimens. It uses a beam of accelerated electrons as a source of illumination instead of light, i.e., it relies on the wave nature of electrons. In electron microscope, the electrons are accelerated by applying high potential difference from 30kV to several mega volts. Such high voltage produces a high kinetic energy beam of electron of shorter wavelength. Typically, the wavelength of electrons is about 100 times smaller than that of the visible light. Due to shorter wavelength of electrons, the resolving power and magnifying power of electron microscope is about one thousand times that of the optical microscope. The beam of electrons is controlled by applied electric and magnetic fields, the electrons diverging from a small region are brought to convergence by these electric and magnetic fields.

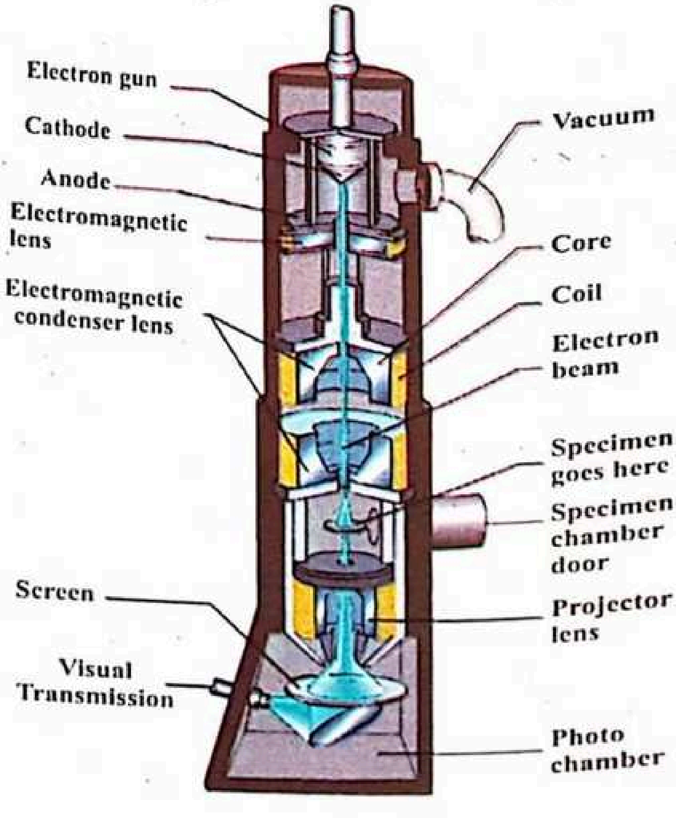


Fig.18.19. A schematic diagram of an Electron microscope

A schematic diagram of electron microscope is shown in Fig.18.19. The beam of electrons is usually focused by magnetic conducting lens and has an energy of 50-100 keV. It is directed onto the whole area of the sample under investigation and the electrons emerging are focused by second magnetic conducting lens onto a florescent screen. The screen must be florescent, otherwise, the obtained image would not be visible.

There are two main types of electron microscope the scanning electron microscope (SEM) and transmission electron microscope (TEM).

18.15 UNCERTAINTY PRINCIPLE

The fact mentioned by Werner Heisenberg in 1927 and it is stated that, **'it is impossible to know both the exact position and exact momentum of an object at**

the same time accurately'. This is known as Heisenberg's uncertainty principle and it is further explained as under:

To locate the position, speed and energy of a particles we must look at it, using a beam of light. Since light has also wave nature, so we should determine the position of the particle only within one wavelength of the used/applied light. Similarly, to reduce the uncertainty in the position of the particle, we must use the light of shorter wavelength because the light of shorter wavelength increases the accuracy of the position by its large resolution. If 'λ' be the wavelength of light which is being used to locate the particle moving along x-axis, then the uncertainty in the position measurement is given by

$$\Delta x \approx \lambda \qquad ......(18.23)$$

On the other hand, if the wavelength of the light is shorter then it would disturb more the momentum of the particle, as a result there is more uncertainty in the measurement of momentum of the particle. In order to reduce uncertainty in its momentum, we should use light of longer wavelength. If the photon of the applied light is transferred its momentum $\left(p=\frac{\lambda}{h}\right)$ to the particle then its momentum would be changed. Thus, the uncertainty in the momentum of the particle Δp is given by

$$\Delta p \approx \frac{h}{\lambda} \; ......(18.24)$$

These two relations (18.23) and (18.24) show that if we use the light of shorter wavelength then the accuracy in the measurement of position will be increased and its uncertainty in momentum will be decreased.

Likewise, by using light of long wavelength, the accuracy in the measurement of momentum will be increased while its uncertainty in position will be decreased. A general relation can be obtained by multiplying these two uncertainties i.e.,

$$(\Delta x)(\Delta p) = h \; ......(18.25)$$

This is the mathematical form of Heisenberg's uncertainty principle. It shows that the product of uncertainty in the simultaneous determination of Δx and Δp is equal to the Planck's constant 'h'. Planck's constant h is so small that the limitations imposed by the uncertainty principle are significant only in the realm of the atom. Hence Heisenberg's uncertainty principle provides a useful tool, not just a negative statement.

In the same way, the uncertainty principle can also be expressed in terms of uncertainty in the simultaneous determination of energy (ΔE) and time (Δt) as;

$$\Delta E \, \Delta t = h \qquad ......(18.26)$$

The quantity $h/2\pi$ appears often in modern physics because it turns out to be the basic unit of angular momentum and it is equal to '$\hbar$' which reduces Planck's constant. Therefore, in terms of '$\hbar$' eq. 18.25 and eq. 18.26 cannot hold more equality. Thus, these two equations now can be expressed as:

$$\Delta x \Delta P > \hbar \quad ......(18.27)$$

$$\Delta E \Delta t > \hbar \quad ......(18.28)$$

Example 18.9

An electron is found in a sphere of size 1.25×10^{-10}m which is the order of size of the atom. Estimate the velocity of the electron in the sphere along $\dot{x}$-rays.

Solution:

Position of the electron = diameter of the sphere

$$\Delta x = 1.25 \times 10^{-10}\text{m}$$

Mass of electron = 9.11×10^{-31}kg

Planck's constant = h = 6.63×10^{-34}J-s

Velocity of electron = ?

According to uncertainty principle

$$\Delta p \Delta x = h$$

$$\Delta p = \frac{h}{\Delta x}$$

$$\Delta p = \frac{6.63 \times 10^{-34}\,\text{Js}}{1.25 \times 10^{-10}\,\text{m}}$$

$$\Delta p = 5.30 \times 10^{-24}\text{kg ms}^{-1}$$

But $\Delta P_x = mv_x$

$$v_x = \frac{\Delta p_x}{m}$$

$$v_x = \frac{5.30 \times 10^{-24}\,\text{kgms}^{-1}}{9.11 \times 10^{-31}\,\text{kg}}$$

$$v_x = 5.82 \times 10^{6}\,\text{ms}^{-1}$$

Example 18.10

The life-time of an electron in an excited state is measured to be 5×10^{-7}s to an accuracy of 0.003%. Find the minimum uncertainty in determining the energy in this time.

Solution:

Lifetime of electron in excited state= 5×10^{-7} s

The uncertainty in time t is 0.003% = $\Delta t = 5 \times 10^{-7}\,\text{s} \times 0.003\%$

$$\Delta t = 5\times10^{-7}s\times3\times10^{-5}$$

$$\Delta t = 1.5\times10^{-11}s$$

$$\text{Planck's constant} = h = 6.63\times10^{-34}\,Js$$

To calculate the uncertainty in the determination of energy, we use the uncertainty principle

$$\Delta E\cdot\Delta t = h$$

$$\Delta E = \frac{h}{\Delta t}$$

$$\Delta E = \frac{6.63\times10^{-34}\,Js}{1.5\times10^{-11}s}$$

$$\Delta E = 4.42\times10^{-23}J$$

SUMMARY

- **Frame of Reference:** The set of coordinate system with respect to which observations or measurement are made.
- **Inertial frame of reference:** A frame of reference which is either at rest or moving with uniform velocity, i.e., non-accelerated frame of reference.
- **Non-inertial frame of reference:** It is an accelerated frame of reference.
- **Special Theory of Relativity:** Special theory of relativity is based upon the following postulates.
  - All the laws of physics are same applicable in all inertial frame of reference.
  - Speed of light is a universal constant.
- **Consequences of special theory of relativity:** Special theory of relativity has some important consequences such as; length contraction, time dilation, mass variation and energy-mass relation ($E = mc^2$).
- **Blackbody and Blackbody Radiation:** A perfect blackbody is one which absorbs radiations of all wavelength incident on it. When blackbody is heated then it emits all the radiation known as black body radiations.
- **Stefan Boltzmann's Law:** The energy per second per unit area is directly proportional to the fourth power of absolute temperature.
- **Wien's Law:** The wavelength having maximum intensity in the emitted radiation spectrum is inversely proportional to the temperature.
- **Max Planck's Law:** Energy exchange takes place in the form of energy packet called quanta and the energy of each quanta is directly proportional to its frequency ($E = hf$).

- **Photo electric effect:** When light of suitable frequency falls on a metal surface then emission of electrons from the metal surface take place. These electrons are called photo electrons and this phenomenon is known as photo electric effect.
- **Photocell:** It is a device which converts the light energy into the electrical energy.
- **Solar Cell:** It is a device which converts sun's light energy into electrical energy.
- **Photon:** It is a small packet of light energy and it behaves as a particle moving with speed of light.
- **Compton Effect:** A.H. Compton studied the scattering of x-ray photon from electrons in a carbon target. He observed that x-ray photons scattered by the target have a longer wavelength than the wavelength of incident photons. The increase in scattered x-ray photon wavelength resulting from the transfer of energy is known as Compton effect.
- **Pair Production:** A process in which a high energy gamma ray photon is converted into a pair of electron and a positron is called pair production.
- **Annihilation of matter:** A process in which particle moving in opposite direction and its anti-particle disappear releasing energy in the form of two γ-rays photon is known as annihilation of matter.
- **De-Broglie wave equation:** According to de-Broglie's postulate when a particle is moving with velocity 'v' then it has some wavelength 'λ' associated with it. i.e., $\lambda = \frac{h}{mv}$.
- **Electron microscope:** It is a device which has much higher magnification and resolution: power than an optical microscope. An electron microscope is a device to obtain high resolution: and magnification of extremely small specimens. It uses a beam of accelerated electrons as a source of illumination instead of light.
- **Uncertainty Principle:** It is impossible to determine simultaneously position and momentum of a particle with perfect accuracy.

EXERCISE

O Select the appropriate option of the following questions.

1. Inertial frame of reference is one which satisfies

(a) Newton's Theory (b) Einstein Theory
(c) Hertz Theory (d) Special theory of relativity

2. Non-inertial frame of reference has

(a) Zero acceleration (b) Zero velocity
(c) Uniform velocity (d) Variable velocity

3. If the source of light is moving towards the observer, then the speed of light received by the observer will be
(a) Decreased (b) Increased (c) Remain same (d) Maximum

4. The relativistic length of an moving object will be
(a) Remain same (b) Decreased (c) Increased (d) Doubled

5. If the rest mass of a particle is zero, then its speed is
(a) Equal to speed light (b) Less than speed of light
(c) Greater than speed of light (d) Not comparable with speed of light

6. Blackbody radiation depends upon
(a) Pressure (b) Volume (c) Temperature (d) Density

7. If the temperature of black body is doubled then the emitted energy from it will be increased
(a) Doubled (b) Four time (c) Eight time (d) Sixteen time

8. The dimension of a Planck's constant is
(a) $[MLT^{-1}]$ (b) $[ML^2T^{-1}]$ (c) $[ML^2T^{-2}]$ (d) $[ML^2T^2]$

9. Who did observe 1st time the photoelectric effect?
(a) Maxwell (b) Hertz (c) Einstein (d) Heisenberg

10. Photo electric effect depends upon the photon's
(a) Pressure (b) Temperature (c) Intensity (d) Frequency

11. A photon can transfer its energy into an electron, it was first explained by
(a) Maxwell (b) Hertz (c) Einstein (d) Bohr

12. The momentum of a photon is
(a) mv (b) mc (c) hf (d) $\frac{hf}{c}$

13. A change in energy of a photon occurs when it collides with an electron at rest is known as
(a) Photoelectric effect (b) Compton effect
(c) Pair production (d) Annihilation

14. Which phenomenon does not verify the particle nature of light
(a) Photoelectric effect (b) Compton effect
(c) Pair production (d) diffraction

15. The antiparticle of electron is
(a) Neutron (b) Proton (c) Photon (d) Positron

16. Davisson and Germer proved experimentally the wave nature of particle under the phenomenon of
(a) Reflection (b) Refraction (c) Interference (d) Diffraction

17. In the electron positron pair production, the speed of electron is
(a) Zero (b) Less than speed of positron
(c) Equals to speed of positron (d) Greater than speed of positron

18. If the energy of the used light is high, then the momentum of the investigated particle has
(a) Less uncertainty (b) High uncertainty
(c) High accuracy (d) Equal uncertainty

19. For small uncertainty in the measurement of position of a particle, the wavelength of the incident light should be
(a) Small (b) Large
(c) Average (d) Does not depend of wavelength

SHORT QUESTIONS

1. How inertial frame of reference is different from non-inertial frame of reference?
2. State the postulates of special theory of relativity.
3. Mention the important results of special theory of relativity.
4. Under what condition, a particle can move with a speed of light?
5. Give the formula to convert the mass of a particle into energy.
6. Does a perfect black body exist? If yes, then give an example.
7. How can you construct a blackbody?
8. How did Max Planck solve the dilemma of distribution of energy by blackbody?
9. How can you calculate the K.E of photoelectrons?
10. What is the difference between work function and threshold energy?
11. Name the metals which emit the photoelectrons for visible light.
12. What do you know about the Einstein's equation for photo electric effect?
13. What is the function of photocell?
14. How does Compton effect verify the wave theory of light?
15. What is meant by the Compton shift in wavelength?
16. What should be the minimum value of energy of photon to induce the pair production?
17. What is the difference between pair production and annihilation of matter?
18. What do you know about the de-Broglie wavelength?
19. At what angle the intensity of diffraction electrons is maximum?
20. Why position and momentum of an electron cannot be measured simultaneously with perfect accuracy?

COMPREHENSIVE QUESTIONS

1. What do you know about the relative motion? Explain the relative motion with examples.
2. Distinguish between inertial and non-inertial frame of reference.
3. State and explain special theory of relativity with its consequences.

4. What is blackbody radiation? Explain the distribution of energy from the black body under various laws.
5. Define photoelectric effect and derive its fundamental equation.
6. State and explain Einstein's equation of photoelectric effect and different laws of photoelectric effect.
7. What do you know about the Compton effect? How does it verify the particle nature of light by Compton theory.
8. Describe pair production and annihilation of matter.
9. What is the de-Broglie hypothesis? How such hypothesis was verified experimentally by Davisson and Germer.
10. State and explain electron microscope with its function and its working principle.
11. What do you know about Heisenberg uncertainty principle? Express uncertainty principle under two mathematical relations.

NUMERICAL PROBLEMS

1. How fast a rocket has to go for its length to be contracted to 80% of its rest length? **(1.8×10^8 m s^{-1})**
2. The period of a second pendulum is measured to be 2s in an inertial frame of reference of the pendulum. What is its period measured by an observer moving with a speed of 0.9c with respect to the pendulum's frame of reference? **(4.6s)**
3. Calculate the variation in the mass of a moving object with a speed of 0.85c. **($1.9m_0$)**
4. What is the energy of a photon of a blue light of wavelength 450nm (in joules and in eV). **(4.41×10^{-19}J, 2.76eV)**
5. Calculate the wavelength of light in which the photons have an energy of 650eV. **(1.9nm)**
6. Determine the maximum K.E of photoelectrons ejected from a potassium surface by ultraviolet radiation of wavelength 200nm. If the work function of the potassium surface is 2.8eV, calculate the stopping potential. **(3.4eV, 3.4V)**
7. With what speed will the fastest photoelectrons be emitted from a surface whose work function is 2eV, when the surface is illuminated with a light of wavelength 4×10^{-7}m? **(6×10^5 ms^{-1})**
8. Consider a photon that scatters from an electron at rest. If the Compton's wavelength shift is observed to be triple the wavelength of the incident photon and if the photon scatters at 60^0 then calculate the wavelength of the incident photon. **(4.05×10^{-13}m)**
9. Determine the de-Broglie wavelength of an electron accelerated from rest through a potential difference of 3.3kV volt. **(0.38 nm)**

10. An electron-positron pair, each with a K.E of 220KeV is produced by a photon. Calculate the energy and wavelength of the photon. **(1.46 MeV, 8.5×10^{-13}m)**

11. The speed of an electron is measured to be $4 \times 10^4 ms^{-1}$ to an accuracy of 0.002%. Find the minimum uncertainty in determining the position of this electron. **(0.92mm)**

Unit 19

ATOMIC SPECTRA

Major Concepts **(16 PERIODS)**

- Atomic spectra
- Emission of spectral lines
- Ionization and excitation potentials
- Inner shell transitions and characteristics x-rays
- Laser

Conceptual Linkage

This chapter is built on Atomic Structure (Bohr Model) Chemistry XI

Students Learning Outcomes

After studying this unit, the students will be able to:

- describe and explain the origin of different types of optical spectra.
- show an understanding of the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to spectral lines.
- explain how the uniqueness of the spectra of elements can be used to identify an
- element.
- analyze the significance of the hydrogen spectrum in the development of Bohr's model of the atom.
- explain hydrogen atom in terms of energy levels on the basis of Bohr Model.
- determine the ionization energy and various excitation energies of an atom using an energy level diagram.
- Solve problems and analyze information using.

  $1/\lambda = R_H[1/p^2 - 1/n^2]$
- understand that inner shell transitions in heavy elements result into emission of characteristic X-rays.
- explain the terms spontaneous emission, stimulated emission, meta stable states, population inversion and laser action.
- describe the structure and purpose of the main components of a He-Ne gas laser.

INTRODUCTION

We have studied in the previous unit that a blackbody emits all radiations of all wavelengths with different intensities depending upon temperature. The set of all the emitted radiations of different wavelength is called spectrum and the study of spectrum is called spectroscopy. But in this unit, we observe and study the spectrum by another method of an atomic gas or vapors at less than atmospheric pressure are suitably excited by passing an electric current through it, they emit radiations. The emitted radiations have a spectrum which comprises of certain specific wavelengths known as atomic spectra. Each element has a characteristic line spectrum. Atomic structure and spectra have a reflective effect on revealing the inner mysteries of the structures of atoms. The existence of line emission spectra from atomic gases is used to reveal the structure of an atom in terms of discrete energy levels in atoms. J.J. Balmer in 1885 succeeded to devise an empirical formula which could explain the existence of the atomic spectra of hydrogen. Neil Bohr in 1913 provided a theoretical reasoning to Balmer formula explaining the emission of spectral lines by presenting a semi classical model of Hydrogen atom. Following this principle, the inner shells transition in heavy atoms should give rise to the emission of high energy photons i.e., x-rays. CAT (Computerized Axial Tomography) scanner is an improved technique of x-rays which can detect tumors and other anomalies too much small to be seen with older techniques. LASER is another triumph of research in this field. The laser beam is an intense, monochromatic, unidirectional and coherent source of light, which has many applications in medical, industry, telecommunication and other fields.

19.1 ATOMIC SPECTRA

When an atomic gas at much low pressure than the atmospheric pressure is excited by passing an electric current or by electric discharge through it, the excited gas emits radiation or light. If this emitted light is investigated using a spectrometer, then a series of discrete lines may be observed on screen. Each line corresponds to a different wavelength. This series of lines called an emission spectra. In general, when the electromagnetic radiation from a source is dispersed by a prism or diffraction grating into its various component

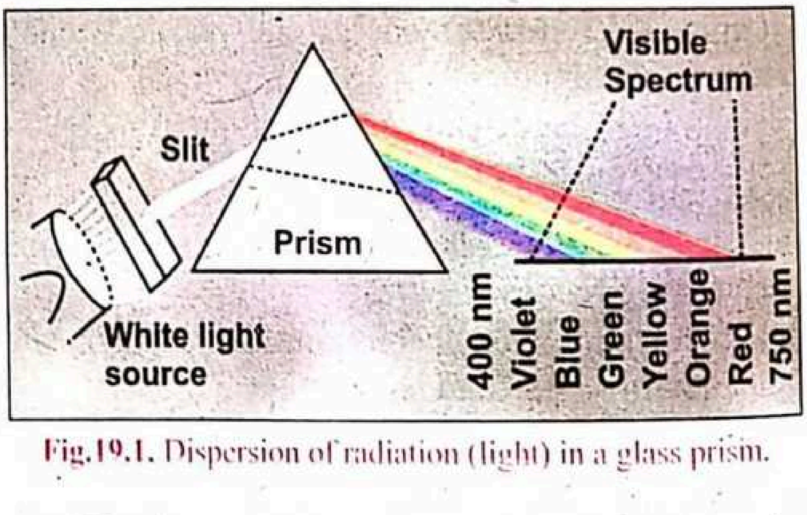


Fig.19.1. Dispersion of radiation (light) in a glass prism.

waves as shown in Fig.19.1 then the visible set of component waves have different wavelength known as visible spectrum.

Similarly, when a visible light is passed through a gas at low pressure, then we observe a spectrum with a few dark lines. These dark lines are absorbed by the molecules of the gas and such type of spectra is called absorption spectra.

The spectra can be classified into line, band or continuous as shown in Fig.19.2. Line spectra may be observed when the electrons of an excited atom, element or molecule move between energy levels, returning towards the ground state. As the spectra contains visible lines due to emission of radiation by the atoms is known as line spectra. The line spectra is not confined only to visible region. It is also observed in the ultraviolet and infrared regions of electromagnetic spectrum. If the spectra contain a series of bands closed to one another then it is called band spectra. The band spectra is formed due to the emission or absorption of radiation by the molecules. On the other hand, the spectrum in which there is a continuous region of radiation emitted or absorbed is a continuous spectrum. e.g., the spectrum of the sun or a blackbody is an example of a continuous spectrum.

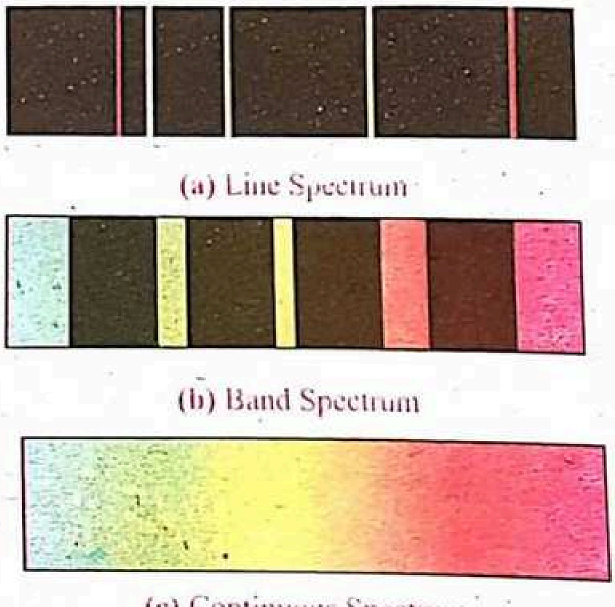


Fig.19.2. Different types of spectra of visible light.

Spectrum of hydrogen atom

Hydrogen atom produces the simplest line spectra. Balmer in 1885 identified the four prominent lines in the visible spectra of hydrogen atom i.e., H_α (red), H_β (blue-green), H_γ (blue-violet) and H_δ (violet). The wavelengths of these four lines are 656.3nm, 486.1nm, 434.1nm and 410.2nm respectively as shown in Fig.19.3. The set of these four lines is called Balmer series. The wavelength of these lines can be determined by the following equation which was modified by Rydberg of the Balmer's equation.

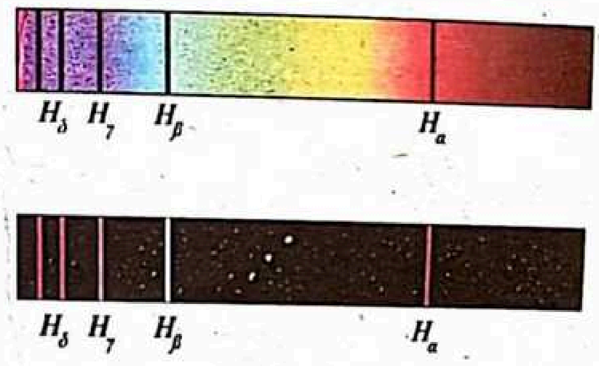


Fig.19.3. Spectrum of hydrogen atom.

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad ......(19.1)$$

where n = 3, 4, 5, ...and R_H is a Rydberg constant whose value is $1.097 \times 10^7\ m^{-1}$ for hydrogen atom.

Hydrogen spectral series of transitions resulting in visible emission lines spectra when an electron of the hydrogen atom makes a transition or jumps from $n \geq 3$ to $n = 2$.

The value of 'n' from 3 onward gives us the wavelengths of the Balmer's Series. On the other hand, the shortest wavelength in the series occurs at 364.6nm, corresponding to the series limit, $n \rightarrow \infty$.

The Balmer series contain wavelength in the visible region of the hydrogen spectrum, but the experiments show that there are also some other series in the hydrogen spectrum. i.e., the extreme ultraviolet section of the spectrum contains the Lyman series.

The remaining series, such as Paschen, Brackett and Pfund series are lying in the infrared region. All these series can be expressed in the form which is very similar to Equation.19.1.

Table 19.1 Different Wavelengths of different Spectral lines

Light Spectrum

λ - nm	Color	Group
3000-10000	IRC	
1400-3000	IRB	Infrared
700-1400	IRA	
650-700	Deep Red	
623	Red	
596	Orange	
571	Yellow	
547	Green	
524	Blue-Green	Visible
501	Turguoise	
480	Blue	
460	Indigo	
440	Violet	
422-400	Deep Violet	
400-320	UVA	
320-280	UVB	Ultraviolet
280-200	UVC	

Lyman series: $$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \quad ......(19.2)$$

where n = 2, 3, 4, ...

i.e., hydrogen spectral series will be observed in the ultraviolet region when an electron of the hydrogen atom makes a transition or jumps from from $n \geq 2$ to $n = 1$.

Paschen series: $$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right) \quad ......(19.3)$$

where n = 4, 5, 6, ...

i.e., hydrogen spectral series will be observed in the infrared region when an electron of the hydrogen atom makes a transition or jumps from $n \geq 4$ to $n = 3$.

Brackett series: $$\frac{1}{\lambda} = R_H\left(\frac{1}{4^2} - \frac{1}{n^2}\right) \quad \text{......(19.4)}$$

where n = 5, 6, 7, ... The Bracket series is also in the infrared region.

Pfund series: $$\frac{1}{\lambda} = R_H\left(\frac{1}{5^2} - \frac{1}{n^2}\right) \quad \text{......(19.5)}$$

where n = 6, 7, 8, ... The Pfund series is also in the infrared region.

The wavelength of all the hydrogen atom spectra lines can be represented by a single empirical formula as;

$$\frac{1}{\lambda} = R_H\left(\frac{1}{p^2} - \frac{1}{n^2}\right) \quad \text{......(19.6)}$$

This equation can be used for all the series of all wavelengths where $n > p$

Example 19.1

What wavelength does a hydrogen atom emit as its excited electron falls from the n = 5 to the n = 2 state.

Solution:

$$\text{Wavelength} = \lambda = ?$$

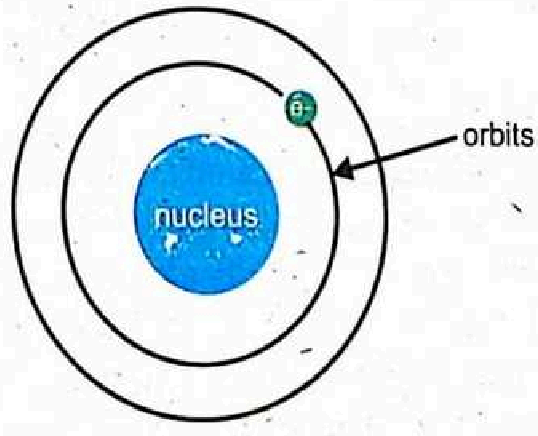
$$\frac{1}{\lambda} = R_H\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = 1.097\times10^7\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

$$\lambda = 4.33\times10^{-7}\text{m} = 443\text{nm}$$

19.2 BOHR'S ATOMIC MODEL OF THE HYDROGEN ATOM

In order to modify Rutherford's model, Niels Bohr proposed his own model of atom and explained the empirical formula of atomic spectra in 1913 by using both classical and Planck's quantum theories. It gives an accurate account of the atomic spectrum of hydrogen as well as the stability of an atom. The Bohr's model is based on the following three postulates:



An electron in its allowed orbit around the nucleus

i:- An electron in an atom can move around the nucleus in certain circular stable orbit, without emitting energy. Electrons can exist only in certain discrete orbits called allowed orbits and electrons have definite energy values such as: E_1, E_2, E_3, etc. in these orbits.

ii:- An electron can revolve around nucleus only in those circular orbits in which its angular momentum is an integral multiple of $\frac{h}{2\pi}$ or $\hbar$ i.e.,

$$L = \frac{nh}{2\pi}$$

as $$L = r \times P = mv_n r_n$$

$$mv_n r_n = \frac{nh}{2\pi} \quad ......(19.7)$$

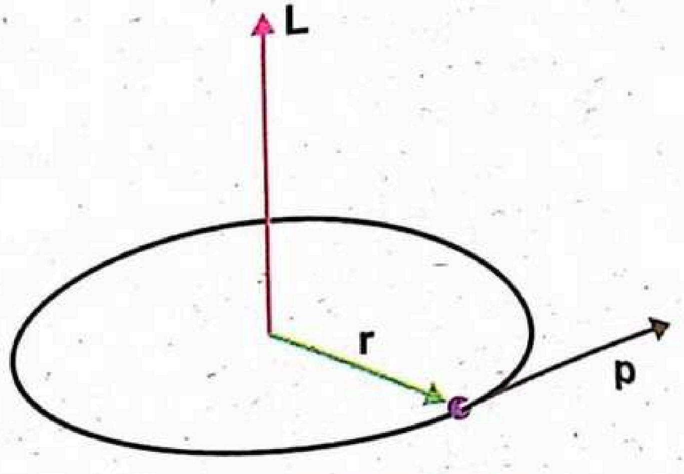
This relation is known as Bohr's quantization rule of angular momentum, where 'n' is the principle quantum number and n = 1, 2, 3, ... and 'h' is a Planck's constant.

iii:- An electron emits energy only when it makes a transition i.e., it jumps from higher allowed orbit to lower allowed orbit. If E_n be the total energy of an electron in the higher orbit and E_p be the total energy in the lower orbit then the energy emitted by an electron is given by

$$E_p - E_n = hf......(19.8)$$

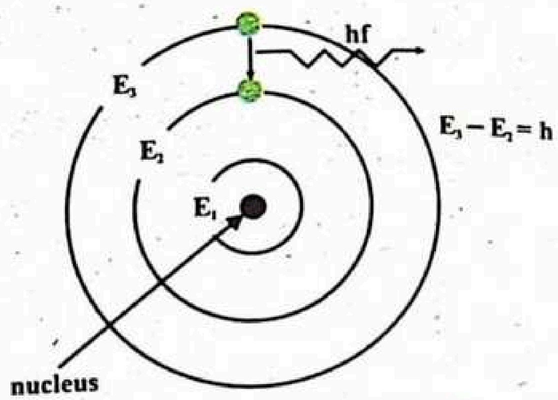
Radii of the quantized orbits

The hydrogen atom is the simplest atom. It has an electron of mass 'm' which is revolving in its allowed orbit of radius 'r_n' with velocity 'v_n' around the nucleus having a single proton as shown in Fig.19.4. The electrostatic force of attraction between the electron and proton provides the required centripetal force to the electron i.e.,



Angular momentum of an electron

Transitions between states



Electron emits energy when it jumps from higher orbit to lower.

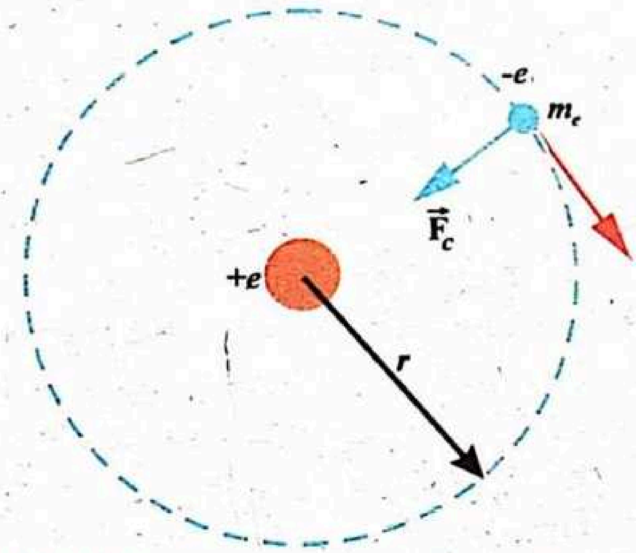


Fig.19.4 An electron is revolving in an orbit around the nucleus of hydrogen atom where the electrostatic force between nucleus and electron causes a centripetal force.

$$F_c = F_e$$

$$\frac{mv_n^2}{r_n} = \frac{ke^2}{r_n^2}$$

$$v_n^2 = \frac{ke^2}{mr_n} \quad ......(19.9A)$$

According to the 2nd postulate or Eq.19.7

POINT TO PONDER

Why is the Bohr model of the hydrogen atom incompatible with uncertainty principle?

$$mv_n r_n = \frac{nh}{2\pi}$$

$$v_n = \frac{nh}{2\pi m r_n}$$

$$v_n^2 = \frac{n^2h^2}{4\pi^2m^2r_n^2} \quad ......(19.9B)$$

Comparing the R.H.S.'s of Eq. 19.9A and 19.9B we get

$$\frac{n^2h^2}{4\pi^2m^2r_n^2} = \frac{ke^2}{mr_n}$$

$$r_n = \frac{n^2h^2}{4\pi^2kme^2} \quad ......(19.10)$$

By substituting the values of 'h', k, m and e we get

$$r_n = \frac{n^2\left(6.625\times10^{-34}\right)}{4(3.14)^2\left(9\times10^9\right)\left(9.1\times10^{-13}\right)\left(1.6\times10^{-19}\right)}$$

$$r_n = \left(0.053\times10^{-9}n^2\right)\text{m} \quad \therefore \left(10^{-9}\text{m} = \text{nm}\right)$$

$$r_n = \left(0.53n^2\right)\text{nm} \quad ......(19.11)$$

For the 1st orbit $\quad n = 1$

$$r_1 = 0.053\text{nm}$$

This is called Bohr's radius. Based on Eq.19.11, we can develop a general relation for Radii of the quantized orbits of hydrogen atom as;

$$r_n = n^2 r_1 \qquad \text{where } r_1 = \frac{h^2}{4\pi^2kme^2}$$

As $\quad n = 1,2,3$

$\therefore \quad r_n = r_1, 4r_1, 9r_1, \cdots$

Energies of electron in the quantized orbits

When an electron is revolving in its allowed orbit, it possesses both the K.E. and P.E.. The sum of K.E. and P.E. is equal to the total energy 'E_n' of the electron i.e.,

$$E_n = K.E. + P.E.$$

$$E_n = \frac{1}{2}mv^2 + F \cdot r_n$$

As angle 'θ' between F and r_n is 180° so $F.r_n = Fr_n \cos 180^\circ = -Fr_n$

and
$$F = \frac{ke^2}{r_n^2}$$

Thus
$$E_n = \frac{1}{2}m\left(\frac{ke^2}{mr_n}\right) - \frac{ke^2}{r_n^2}r_n$$

$$E_n = \frac{ke^2}{2r_n} - \frac{ke^2}{r_n}$$

$$E_n = -\frac{ke^2}{2r_n} \quad \ldots\ldots(19.12)$$

By substituting the value of r_n from Eq.19.10 we get

$$E_n = \frac{-2\pi^2k^2me^4}{n^2h^2}$$

$$E_n = \frac{2(3.14)^2(9\times10^9)^2(9.1\times10^{-31})(1.6\times10^{-19})^4}{n^2(6.625\times10^{-34})^2}$$

$$E_n = -\frac{2.17\times10^{-18}}{n^2}J$$

$$E_n = -\frac{2.17\times10^{-18}}{n^2(1.6\times10^{-19})}eV \qquad \therefore 1eV = 1.6\times10^{-19}J$$

$$E_n = \frac{-13.6}{n^2}eV \quad \ldots\ldots(19.13)$$

For the 1st orbit, n =1

$$E_1 = -13.6eV$$

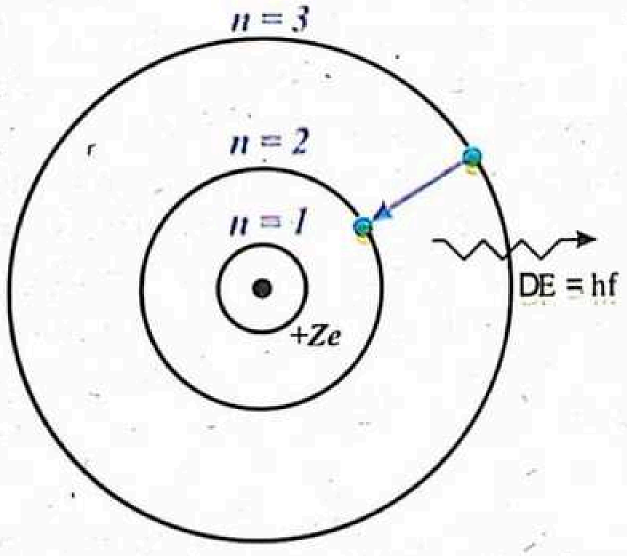
Based upon Eq.19.13, we can express a general relation for the energies of an electron in the quantized orbits as

$$E_n = -\frac{E_1}{n^2}$$

$$E_n = -E_1, \frac{-E_1}{4}, \frac{-E_1}{9}, \frac{-E_1}{16}, \ldots \frac{-E_2}{n^2}$$

Spectral lines of hydrogen atom

Another most important result of Bohr's theory is the determination of spectrum of hydrogen atom. i.e., when an electron absorbs energy equal to the difference of energy between the two states then the electron jumps from lower energy state to higher energy state. At this position, the atom is said to be in the excited state. On de-excitation i.e., when the electron jumps from higher energy state to its ground state it emits the energy during this transition in the form of spectral lines. For example, if an electron in the hydrogen atom is in the excited state 'n' with energy E_n and it makes a transition to the lower state 'p' with energy E_p, where $E_n > E_p$, then according to the 3rd postulate



When electron jumps from higher orbit to lower orbit it emits energy in form of a spectral line.

$$E_n - E_p = hf$$

$$\frac{-E_1}{n^2} - \left(\frac{-E_1}{p^2}\right) = hf$$

$$E_1\left(\frac{1}{p^2} - \frac{1}{n^2}\right) = hf$$

But $\quad c = f\lambda \Rightarrow f = \frac{c}{\lambda}$

Therefore,

$$E_1\left(\frac{1}{p^2} - \frac{1}{n^2}\right) = \frac{hc}{\lambda}$$

$$\frac{1}{\lambda} = \frac{E_1}{hc}\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{p^2} - \frac{1}{n^2}\right) \quad \ldots\ldots(19.14)$$

POINT TO PONDER

What is the energy of the photon emitted when a hydrogen atom makes a transition from the n = 2 state to n=1 state?

The relevant masses are:

$m({}_{1}^{2}H) = 2.014102u$, $m({}_{1}^{3}H) = 3.016049u$, $m({}_{2}^{4}He) = 4.002603u$ and

$m({}_{0}^{1}n) = 1.008665u$

(17.6 MeV)

7. Calculate the amount of energy released in the nuclear fission reaction.

$${}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{56}^{141}Ba + {}_{36}^{92}Kr + {}_{0}^{1}n + Q$$

The relevant masses are:

$m({}_{0}^{1}n) = 1.008665u$, $m({}_{92}^{235}U) = 235.043924u$, $m({}_{56}^{141}Br) = 140.913740u$, and

$m({}_{36}^{92}Kr) = 91.926270u$.

(174 MeV)

8. How much energy is released during following reaction?

$${}_{1}^{1}H + {}_{3}^{7}Li \rightarrow {}_{2}^{4}He + {}_{2}^{4}He + Q$$

The relevant nuclear masses are:

$m({}_{1}^{1}H) = 1.007825u$, $m({}_{3}^{7}Li) = 7.016005u$, and $m({}_{2}^{4}He) = 4.002603u$

(17.3 MeV)

II Balmer Series

If the electrons of hydrogen atom jump from the outer orbits (n = 3, 4, 5, ...) to the 2nd orbit (p = 2) then a set of spectral lines called Balmer series is obtained. The spectral lines of the Balmer series lie in the visible region. The wavelengths of the Balmer series are given by

$$\frac{1}{\lambda}=\frac{E_1}{hc}\left(\frac{1}{p^2}-\frac{1}{n^2}\right)=R_H\left(\frac{1}{2^2}-\frac{1}{n^2}\right) \qquad \begin{matrix} p=2 \\ n=3,4,5... \end{matrix}$$

III Paschen Series

When the electrons of hydrogen atom jump from the outer orbits (n = 4, 5, 6, ...) to the 3rd orbit (p = 3), the set of spectral lines called Paschen series is obtained. Paschen series lie in the infrared region and corresponding wavelengths can be calculated as;

$$\frac{1}{\lambda}=\frac{E_1}{hc}\left(\frac{1}{p^2}-\frac{1}{n^2}\right)=R_H\left(\frac{1}{3^2}-\frac{1}{n^2}\right), \qquad \begin{matrix} p=3 \\ n=4,5,6... \end{matrix}$$

IV Brackett Series

In Brackett series the transition of electrons take place from the outer orbits (n = 5, 6, 7, ...) to the 4th orbit (p = 4). Therefore, the formula for the wavelength of the lines in the infrared region in this series is given by

$$\frac{1}{\lambda}=\frac{E_1}{hc}\left(\frac{1}{p^2}-\frac{1}{n^2}\right)=R_H\left(\frac{1}{4^2}-\frac{1}{n^2}\right), \qquad \begin{matrix} p=4 \\ n=5,6,7... \end{matrix}$$

V Pfund Series

If the transition of electrons take place from the outer orbits (n = 6, 7, 8, ...) to the 5th orbit (p = 5), then there are a number of spectral lines in the infrared region and this series is called Pfund series. The wavelengths of lines in the Pfund series are given by

$$\frac{1}{\lambda}=\frac{E_1}{hc}\left(\frac{1}{p^2}-\frac{1}{n^2}\right)=R_H\left(\frac{1}{5^2}-\frac{1}{n^2}\right) \qquad \begin{matrix} P=5 \\ n=6,7,8... \end{matrix}$$

Example 19.2

An hydrogen atom is in its ground state. Using Bohr's theory calculate

(a) the radius of the orbit of the electron.

(b) the linear momentum of the electron

(c) the angular momentum of the electron

(d) the kinetic energy of the electron
(e) the potential energy of the electron
(f) the total energy of the electron

Solution:

Mass of electron = m = 9.1×10^{-31} Kg
Charge on an electron = e = 1.6×10^{-19} C
Planck's constant = h = 6.625×10^{-34} J – s
Number of orbit = n = 1
Coulomb's constant = k = 9×10^{9} Nm^{2}C^{-2}

(a) Radius (r) = ?
(b) Linear momentum = P = ?
(c) Angular momentum = $\overline{L}$ = ?
(d) K.E. = ?
(e) P.E. = ?
(f) T.E. = ?

(a) As
$$r_n = \frac{n^2h^2}{4\pi^2kme^2}$$
$$r_1 = \frac{(1)^2\left(6.63\times10^{-34}\right)^2}{4(3.14)^2\left(9\times10^{9}\right)\left(9.1\times10^{-31}\right)\left(6.1\times10^{-19}\right)^2}$$
$$r_1 = \frac{46.9\times10^{-68}}{8.3\times10^{-57}}$$
$$r_1 = 5.3\times10^{-11}\text{m} = 0.053\text{nm}$$

(b)
$$p = mv$$

As
$$v^2 = \frac{ke^2}{mr} \Rightarrow v = e\sqrt{\frac{k}{mr}}$$
$$p = me\sqrt{\frac{k}{mr}} = e\sqrt{\frac{mk}{r}}$$
$$p = 1.6\times10^{-19}\text{C}\sqrt{\frac{9.1\times10^{-31}\text{kg}\times9\times10^{9}\text{Nm}^2\text{C}^{-2}}{5.3\times10^{-11}\text{m}}}$$
$$p = 1.6\times10^{-19}\sqrt{15.45\times10^{-11}}$$
$$p = 1.6\times10^{-19}\sqrt{1.545\times10^{-10}}$$
$$p = 1.6\times10^{-19}\times1.243\times10^{-5}$$
$$p = 1.99\times10^{-24}\text{kg m s}^{-1}$$

(c) $L = rp\sin\theta$

If 'θ' between $\bar{r}$ and $\bar{p}$ is 90^0, then

$$L = 5.3\times10^{-11}\,m\times1.99\times10^{-24}\,kg\ m\ s^{-1}\times90^0$$

$$L = 1.05\times10^{-34}\,kg\ m^2 s^{-1}$$

(d)
$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\cdot\frac{ke^2}{mr} = \frac{1}{2}\frac{ke^2}{r}$$

$$= \frac{1}{2}\times\frac{(9\times10^9\,Nm^2C^{-2})(1.6\times10^{-19}\,C)^2}{5.3\times10^{-11}\,m}$$

$$= \frac{23.04\times10^{-29}}{10.6\times10^{-11}}J$$

$$K.E. = 2.17\times10^{-18}\,J = 13.56eV$$

(e)
$$P.E. = \frac{-ke^2}{r}$$

$$= \frac{(-9\times10^9\,Nm^2C^{-2})(1.6\times10^{-19}\,C)^2}{5.3\times10^{-11}\,m}$$

$$= \frac{-14.4\times10^{-10}\times1.6\times10^{-19}}{5.3\times10^{-11}}$$

$$= -2.717\times10\times1.6\times10^{-19}$$

$$P.E. = -27.17eV$$

(f)
$$T.E = K.E. + P.E.$$

$$= 13.56eV - 27.17eV$$

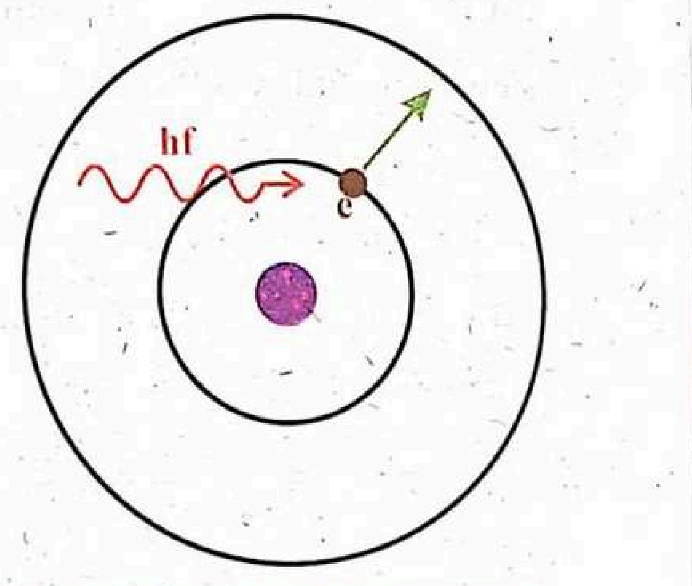
$$T.E = -13.61eV$$

19.3 EXCITATION ENERGY AND EXCITATION POTENTIAL

We have studied an atom of hydrogen has quantized orbits called allowed orbits with discrete energy levels. The electrons can revolve around the nucleus without radiating energy in these allowed energy orbits. For example, hydrogen atom has a single electron and it is revolving in its lowest orbit called ground state. Let the electron absorbs some energy and it is excited from its ground state to the higher allowed orbits. The energy that absorbed by the electron to jump from the ground state to the excited states is known as excitation energy and it is equal to the difference of energies of the electrons in the two states. It is measured in terms of eV_o, where

'V_o' is the applied potential which provides the excitation energy to an electron. Therefore, it is called excitation potential.

As we know that in case of hydrogen atom, the total energy of an electron in its ground state is ($E_1 = -13.6eV$). Similarly the total energy of an electron in the second state is ($E_2 = -3.4eV$). Thus, the excitation energy of an electron from ground state to the 2nd state is given by



The electron excited by a photon's energy hf.

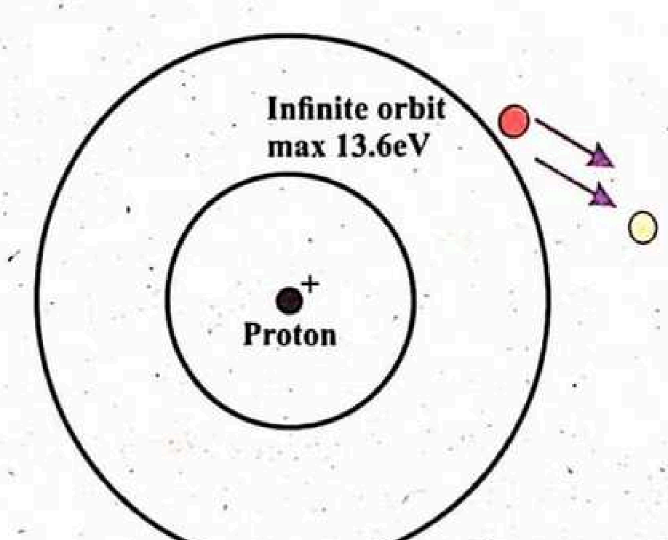
$$E = E_2 - E_1$$
$$E = -3.4 - (-13.6)$$
$$= 10.2eV$$

Similarly, the excitation energy of an electron from the ground state (E_1) to the 3rd state (E_3) is given by

$$E = E_3 - E_1$$
$$E = -1.51 - (-13.6) = 12.1eV$$

19.4 IONIZATION ENERGY AND IONIZATION POTENTIAL

When an electron is boosted from its ground state to infinity, such that the electron becomes free from the electrostatic force of the nucleus, then the atom is said to be ionized. The energy required to escape an electron completely from its ground state to infinity is known as ionization energy. It is measured in terms of eV_o, where 'V_o' is the applied potential which provides ionization energy to an electron so, it is called ionization potential. For example, in case of hydrogen atom, the total energy of an electron in its ground state is ($E_1 = -13.6eV$). Consider the case in which the electron has moved to infinity, the energy of the orbit at the infinity is zero ($E_\infty = 0$). Thus, the ionization energy of hydrogen atom is given by



Ionization of an atom by a photon of energy greater than $13eV_0$

$$E = E_{\infty} - E_1$$
$$E = 0 - (-13.6)$$
$$E = 13.6 eV$$

This shows that the ionization energy of the hydrogen atom is 13.6eV and the ionization potential is 13.6 volts. It may be noted that there is only one value of ionization potential i.e. 13.6V for hydrogen atom because it has a single electron. There can be more than one value of ionization potentials for the atoms with several electrons.

19.5 INNER SHELL TRANSITIONS AND X-RAYS

We have discussed that when electrons make a transition from their higher orbits to lower orbits then they emit radiations in form of spectral lines in the range of infrared, visible or ultraviolet light.

In heavy atoms, electrons are arranged in concentric shells which are named as K, L, M, N, O...... shells, as shown in Fig.19.7(a). The electrons in the outer shells are loosely bound with the nucleus due to large distance and weak electrostatic force between electrons and the nucleus. Therefore, a small amount of energy is required for excitation of the electrons from the outer shells of such atoms. When the electrons return to their original states, they emit radiation of longer wavelength, which lie in the infrared region.

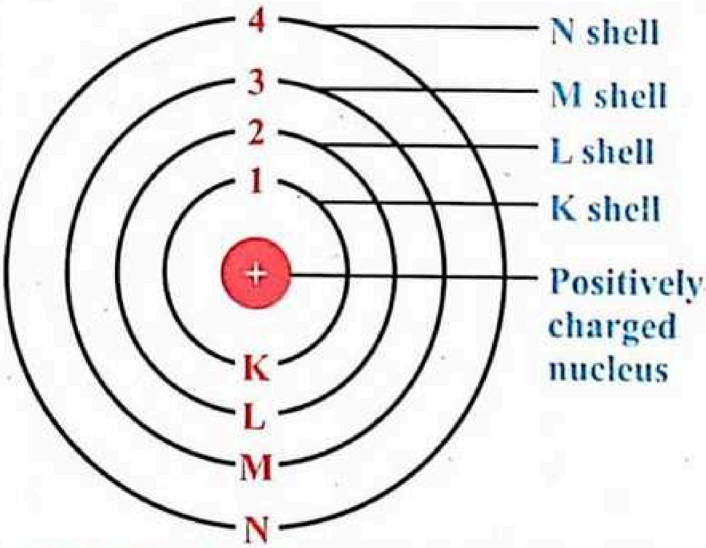


Fig.19.7(a). An isolated atom possessed a number of concentric shells.

On the other hand, the electrons in the inner shells are tightly bound with their nucleus due to a small distance and strong electrostatic force between the electrons and the nucleus. Therefore, a large amount of energy is required for excitation of these electrons. When the electrons return to their ground states, they emit highly energetic radiations of shorter wavelength, which are lying in the ultraviolet region. These highly energetic radiations are named as x-rays which are also known as characteristic x-rays.

In other words, characteristic x-rays are emitted from heavy elements due to inner shell transition, i.e., when their electrons make transitions between the different energy levels. As each element has a unique set of atomic energy levels, it emits a unique set of x-rays which are characteristic of that element. X-rays originate from atomic electrons and also from free electrons decelerating in the vicinity of heavy atoms (i.e., Bremsstrahlung).

Production of x-rays:

X-rays are electromagnetic waves which have extremely short wavelengths. They were discovered by Rontgen in 1895 when he was investigating cathode rays.

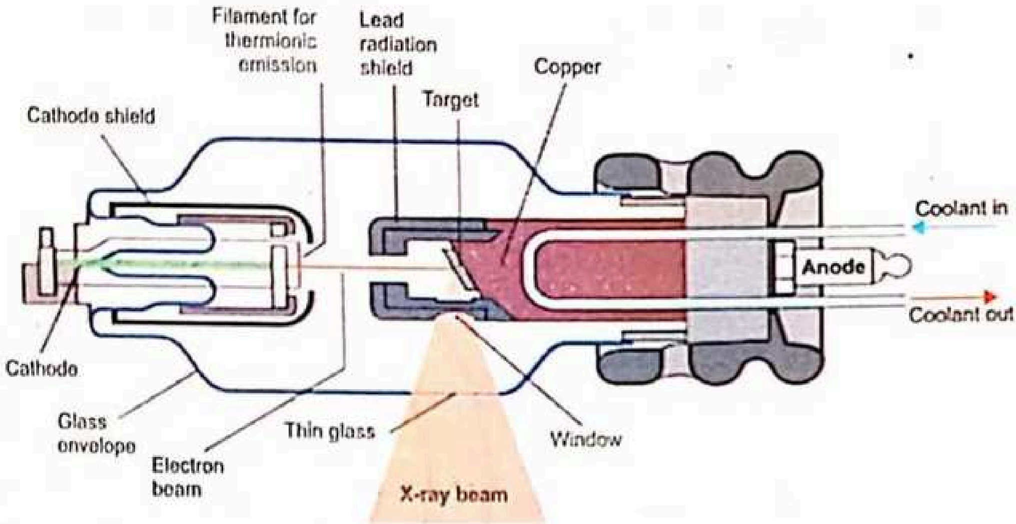


Fig.19.7(b). A schematic diagram for the production of x-rays, where electrons emitted by the hot filament are allowed to hit the target. The target then emits x-rays. .

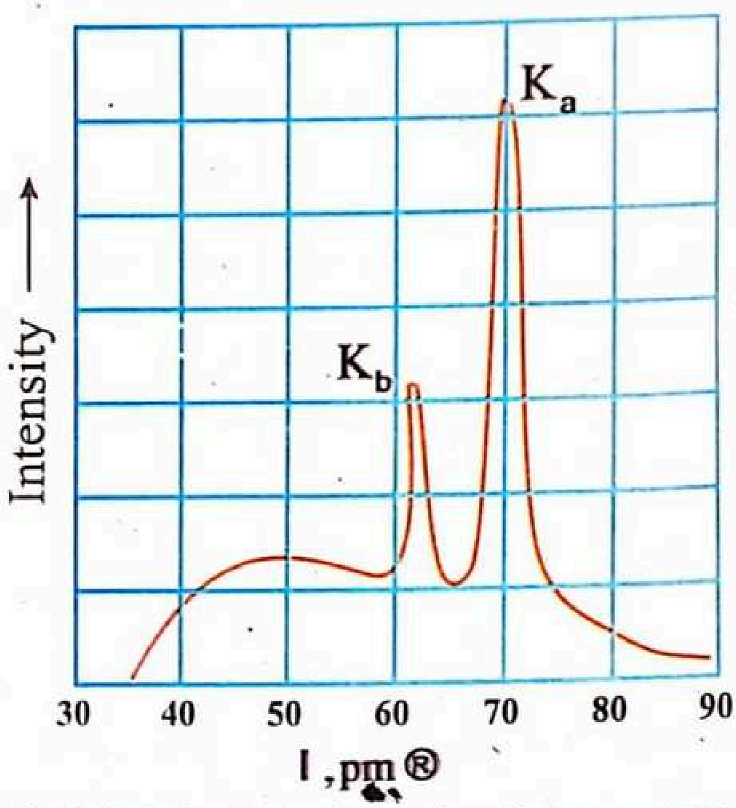


Fig.19.7(c). Graphical representation of x-rays spectrum.

In modern age, a tube called x-rays tube is used to produce x-rays as shown in Fig.19.7(b). Such tube consists of cathode and anode, where cathode is a heated filament and it acts as a source of electrons. The electrons emitted from the heated filament are accelerated through potential difference of the order 10^5V. When these high energy electrons strike the target anode (usually tungsten, molybdenum or copper), only a small fraction about 1% of the kinetic energy of the incident electrons is converted into x-rays while the remaining energy of the electrons is converted into heat at the anode. It is therefore, the target should be metal of high melting point. The target is cooled using cooling fans or a specially designed cold water summing system. The x-rays emitted from x-ray tube consist of various wavelengths and forms a spectrum. Graphically, it is shown in Fig.17.7(c). The x-ray spectrum produced by x-ray sources consists of continuous spectra and the line or characteristic x-ray spectra but no band spectra. **Continuous spectra or continuum radiation also called white radiation or bremsstrahlung.** Bremsstrahlung refers to "braking radiation" derived from German word bremsen "to brake" and Strahlung "radiation". Line spectra or characteristic X ray corresponds to quantum energies of inner shell transition of electrons.

x-rays are most commonly produced by bombardment of a metal target with a beam of high energy electrons. When highly energetic electrons from cathode strikes the target they knocks out one of the electrons from innermost K-shell. The atom is then raised to an excited state, producing a vacancy in the K-shell. If the vacancy in the K shell is filled by an electron from L-shell, then a photon of x-ray K_α is emitted. If the vacancy in the K shell of atom is filled by an electron from M-shell, then K_β is emitted. The series of x-ray produced due to transition of electron from L shell (K_α) or M shell (K_β) is known as K-characteristic x-ray.

The characteristic L series are produced when vacancies in the L-shell are filled by electrons from the higher energy states. The shorter-wavelength group is called the K series (from 1s orbital) and the L series (from 2s or 2p), even longer then L lines are M-series. Elements with atomic numbers smaller than 23 produce only a K series.

19.6 LASER

The term LASER is an acronym for 'light amplification by stimulated emission of radiation'. A laser is a device which produces intense, monochromatic and coherent beam of visible or infrared light. The fundamental principle of laser is that, the excited atoms are stimulated by the incident photon in order to emit another photon of same frequency as that of the frequency of incident photon. Thus, the emitted and the incident photons having same frequencies will travel away in phase. In order to understand the working of a laser, we explain some phenomenon related to it.

Spontaneous and stimulated emissions

Consider an atom has an electron in its ground state as shown in Fig.19.8(a). Suppose a photon of energy hf equal to the energy difference between two energy levels is incident on this atom then the photon can be absorbed by the atom. This process is called stimulated absorption because the photon stimulates the atom to transfer the electron from its ground state to the excited state as shown in Fig.19.8(b).

After the excitation, the electrons make a transition back to a lower energy state, because the electrons can remain in an excited state typically for 10^{-8}s. Moreover, during the transition when the electrons come back to the ground state, then it emits a photon of energy hf as shown in Fig.19.9. This process is known as spontaneous emission.

Now consider an atom which is in an excited state E_2 with lifetime 10^{-3}s, this lifetime is much longer than 10^{-8}s. Therefore, the excited state E_2 is called metastable state. Metastable state is an abnormally excited state of an atom with a longer lifetime than the other ordinary excited states. Thus, for an electron in its metastable state E_2 if a photon of energy $hf = E_2 - E_1$ is incident, it induces the electron returns to the ground state by emitting a second photon with energy $hf = E_2 - E_1$. **The phenomenon when an incident photon induce transition electron to make downward transition with the release of photon of same frequency is known as stimulated emission.** These two photons i.e., incident and emitted are in phase and travel in the same direction as shown in Fig.19.10.

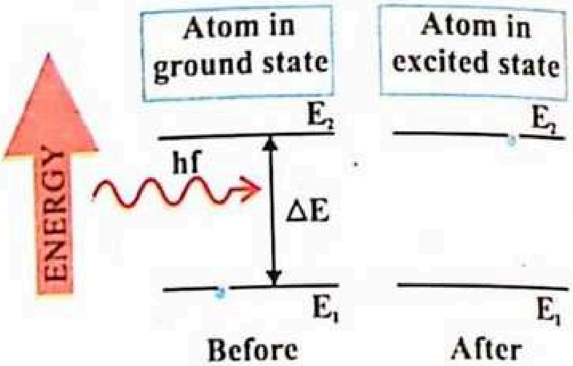


Fig.19.8(a). the electron of an atom in its ground state (b) the process of stimulated absorption, where the transition of electron from the ground state to the excited state when the atom absorbs the photon of energy hf.

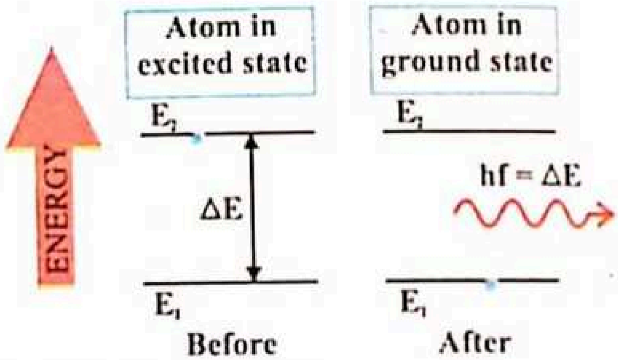


Fig.19.9. the process of spontaneous emission of photon due to the transition of electron from the excited state to the ground state.

Population inversion

When there are more atoms in the ground state than in the excited state this is known as normal population as shown in Fig.19.11(a). On the other hand, when there are more atoms in their excited states than the ground state as shown in Fig.19.11(b), this condition is called population inversion. If the number of atoms in the excited state becomes more than number of atoms in the ground state, then more stimulated emissions occur. Population

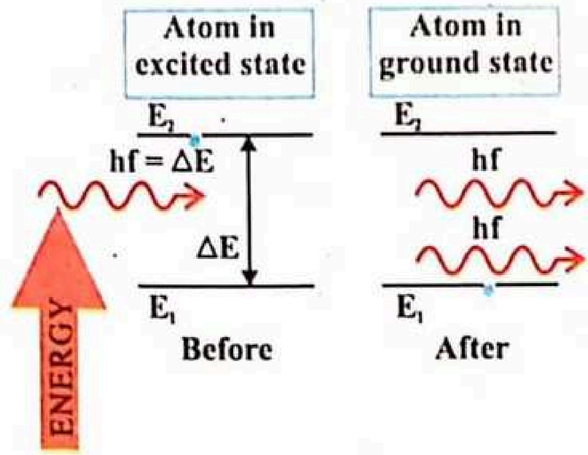


Fig.19.10. Stimulated emission of photon by incident of photon.

inversion can be achieved if there exists metastable state. This is the most important principle that involves in the action of a laser. One method of achieving population inversion is 'optical pumping' i.e., illuminating the laser material with light. The process of light stimulated emission is fundamental to laser operation.

Laser is produced by an active medium or gain medium inside the laser optical cavity. The active medium is a collection of atoms, or molecules that can undergo stimulated emission. The active medium can be in a gaseous, liquid or solid form.

Hence the essential components of a LASER are medium, pump (source of energy) and resonant cavity.

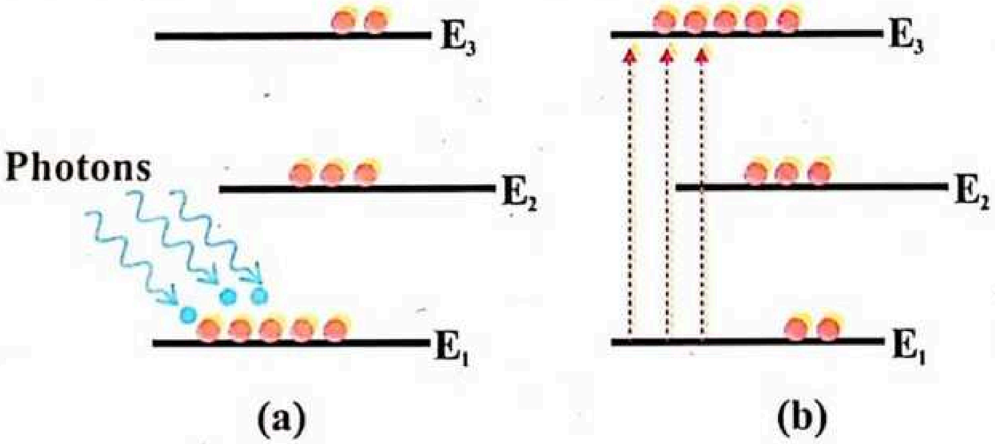


Fig.19.11(a). Normal population (b) Population inversion.

LASER action

Suppose population inversion has been achieved by some means and the atoms are gathered in the excited states E_2 and E_3 as shown in Fig.19.12. After the process of excitation, a spontaneous emission occurs due to the transition of atoms from the excited state E_3 to the excited state E_2 (metastable) because the lifetime of E_3 is only 10^{-8} s, and the lifetime of metastable (E_2) is 10^{-3} s which is much longer than 10^{-8}s.

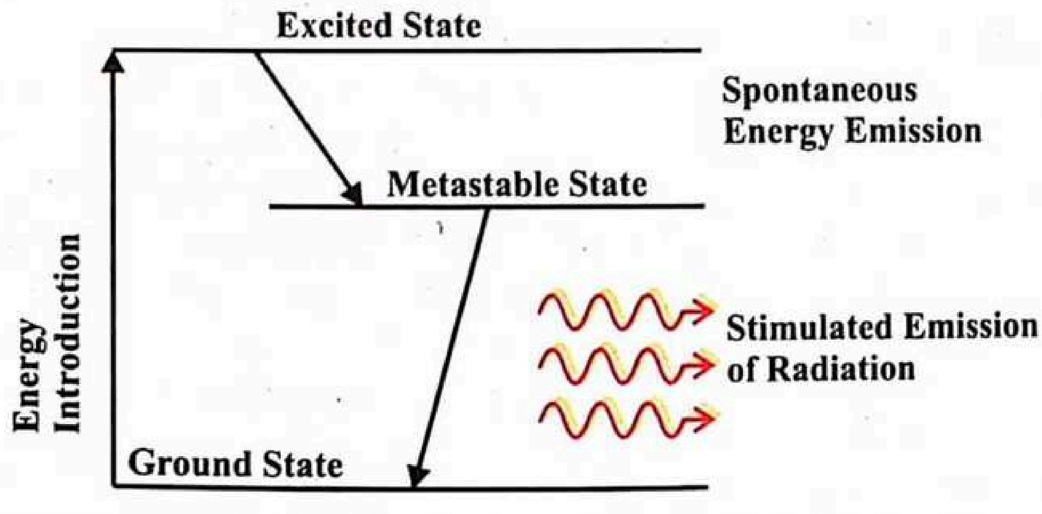


Fig.19.12. Energy level diagram showing stimulated absorption, spontaneous emission and stimulated emission.

In order to get the stimulated emission, a photon of energy $hf = E_2 - E_1$ is incident. This incident photon induces the stimulated emission. As a result, another photon is emitted having the same energy hf and travelling in the same direction. These two photons are in phase and they can stimulate other atoms to emit photons in a chain to similar processes. Thus, a chain of these emitted photons causes an intense, monochromatic and coherent beam of light i.e., laser.

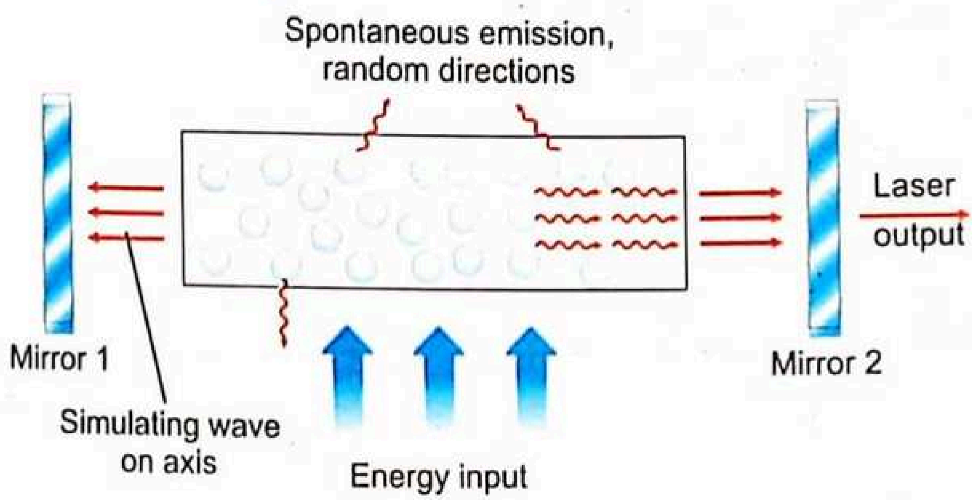


Fig.19.13. Schematic diagram of LASER action, the two mirrors keep the photons confined to the system.

The emitted photons must be confined in an optical cavity long enough to enable them to stimulate further emission from other excited atoms. It can be achieved by using two mirrors at the ends of this optical cavity as shown in Fig.19.13. One end of this assembly is made totally reflecting and the other is partially reflecting. The photons are reflecting back and forth between the two mirrors from the end of this optical cavity, so a very intense, monochromatic and coherent beam is setup and a small fraction of the beam passes through the partially reflecting mirror, producing the beam of the laser light.

Helium-neon laser

A familiar example of a laser is the helium-neon laser. It is a common and inexpensive laser that is available in physics laboratories. The helium-neon laser usually consists of a gas discharge tube containing a mixture of 85% helium and 15% neon gas at low pressure. The metastable states of helium and neon are identically located at 20.61eV and 20.66eV respectively, which are much closer to each other. When the helium-neon laser is electrically pumped then the electrical discharge excites the atoms of helium from the ground state to its metastable state with energy 20.61eV as shown in Fig.19.14.

Now these excited atoms of helium make inelastic collisions with the atoms of neon in the ground state. Therefore, there is transfer of energy from the excited helium atoms to the neon atoms in the ground state. As a result, the neon atoms are excited from the ground state to the metastable state with energy 20.66eV while the helium atoms return to their ground state. In this way, we have the necessary population inversion in neon. After population inversion, stimulated emission occurs during the transition of neon atoms from its metastable state of energy 20.66eV to the excited state of energy 18.70eV. Thus, it causes emission of highly coherent laser light of wavelength 632.8nm. Finally, spontaneous transition quickly takes place during the transition of neon atoms from the excited energy state 18.70eV to the ground state.

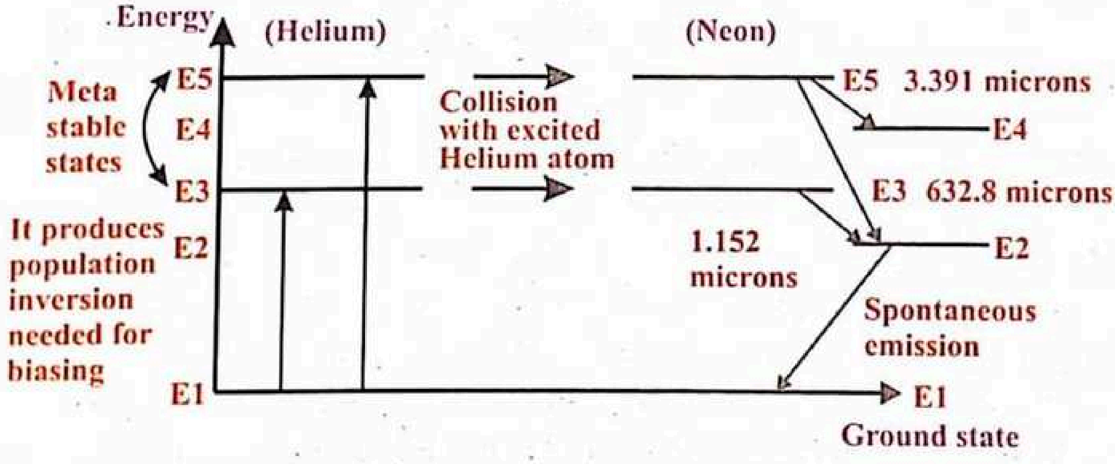


Fig.19.14. A schematic energy level diagram for Helium and Neon gas LASER.

Laser have several usage and applications, some of them are listed below:

i Lasers are used to destroy tumors and to fragment kidney stones and gallstones.

ii A detached retina of the eye can be welded back into place by using a laser beam.

iii Laser surgery is used to reshape the cornea of the eye to correct near sightedness (myopia).

iv Laser is used in the process of endoscope.

v Intense laser beam is being used for ranging i.e., to determine far distance between two objects accurately, such as, the distance between earth and moon.

vi A high intensity laser beam can drill a very small hole in hand objects such as diamond.

FOR YOUR INFORMATION

A patient undergoes laser eye surgery to correct his vision by modifying the shape of the surface of the eyeball. This process is called LASIK (laser assisted in situ keratomileusis).

vii A laser light is being used as a communication signal through a fiber optics and it can transmit both video and audio information.

viii Laser plays a dynamic role in the system of compact disc (CD) and digital video disc (DVD) players.

ix A laser beam is being used to draw three dimensional photography called holography.

x A laser light also used in the printing process called laser printing.

xi A laser beam has a potential to induce and control the fusion reaction.

xii Laser is being used for weapons guided system and submarine tracking.

SUMMARY

- **Spectrum:** A set of wavelengths or frequencies is known as spectrum (spectra). It can be classified into line, band and continuous spectra.
- **Emission Spectra:** The spectra due to the emission of light from excited atoms (gas) is called emission spectra, such spectra contain a series of visible lines.
- **Absorption Spectra:** The spectra due to the passing of visible light through a gas is called absorption spectra, such spectra contain few dark lines.
- **Bohr's Atomic Model:** Bohr's atomic model can be explained under the following postulates:
  i An electron does not radiate energy when it is revolving in its allowed orbit.
  ii Electrons are revolving in the allowed orbits for which their angular momenta is an integral multiple of $\frac{h}{2\pi}$.
  iii An electron radiates energy only when it jumps from higher orbit to lower orbit.
- **Spectrum of hydrogen atom:** When an electron in an excited hydrogen atom makes transition from higher orbits to lower orbits then it emits radiations in the form of spectral lines series.
- **Excitation energy and excitation potential:** The amount of energy which can raise an electron from its normal state to higher energy state is called excitation energy, and the potential which provides such energy is called excitation potential.
- **Ionization energy and ionization potential:** The amount of energy required to make an electron free from the electrostatic force of the nucleus is called ionization energy and the potential which provides such energy is known as ionization potential.
- **Characteristic X-rays:** Characteristic x-rays are emitted from heavy elements due to inner shell transition, i.e., when their electrons make transitions between the different energy levels.

- **LASER:** Laser is a device stands for light amplification by stimulated emission of radiation and it produces intense, monochromatic and coherent light.
- **Stimulated absorption:** Absorption of photon by electron that electron becomes excited from ground state to higher state energy level.
- **Spontaneous Emission:** Emission of photon by an atom during the transition from the higher orbit to the lower orbit.
- **Metastable State:** Second energy state (E_2) is known as metastable state because it has longer lifetime of 10^{-3}s.
- **Population Inversion:** If there are more atoms in a higher energy state or excited state than the ground state then this condition is called population inversion.
- **Stimulated Emission:** A process in which a photon is incident with the excited electron, causing the electron to jump from excited state to de-excited state by emitting a photon of same frequency is that of the incident photon.

EXERCISE

Select the best option of the following questions.

1. When light is passed through a gas at low pressure then we observe
   (a) Emission spectra (b) Absorption spectra
   (c) Band spectra (d) Molecular spectra
2. The spectra due to absorption or emission of radiation by atoms of a gas is known as
   (a) Line spectra (b) Band spectra
   (c) Continuous spectra (d) Molecular spectra
3. The band spectra is produced due to emission or absorption of radiation by
   (a) Atoms (b) Molecules (c) Electrons (d) Photons
4. Dark lines correspond to
   (a) Emission spectra (b) Absorption spectra
   (c) Band spectra (d) Continuous spectra
5. Name of the scientist who identified the four lines spectrum series for the first time
   (a) Lyman (b) Balmer (c) Paschen (d) Brackett
6. Which one of the following spectrum series is found in the visible region?
   (a) Brackett (b) Pfund (c) Balmer (d) Paschen
7. Lyman series lies in which region
   (a) Visible (b) Infrared (c) Violet (d) Ultraviolet
8. The dimensions of Rydberg constant are
   (a) $[M^0L^0T]$ (b) $[M^0LT^0]$ (c) $[M^0LT]$ (d) $[M^0L^{-1}T^0]$

9. When an electron in hydrogen atom is raised from the ground state to the 2nd energy state, how many times its radius is greater than the radius of its ground state
(a) Same (b) Half (c) Twice (d) Four time

10. When an electron in hydrogen atom is raised from the ground state to the excited state then its K.E. and P.E. will be
(a) K.E. increased and P.E. decreased (b) K.E. decreased and P.E. increased
(c) Both K.E. and P.E. increased (d) Both K.E. and P.E. decreased

11. The diameter of hydrogen atom is
(a) 0.35A° (b) 0.53A° (c) 0.70A° (d) 1.06A°

12. The ionization potential of hydrogen atom is
(a) 6.13eV (b) 6.13V (c) 13.6V (d) 13.6eV

13. Laser action cannot occur with the process of
(a) Spontaneous absorption (b) Spontaneous emission
(c) Normal population (d) Population inversion

14. Stimulated emission occurs only when the population inversion is in
(a) Ground state (b) Metastable state (c) 3rd energy state (d) Any energy state

15. Lifetime of metastable state is about
(a) 10^{-2} s (b) 10^{-3} s (c) 10^{-5} s (d) 10^{-8} s

16. Lifetime of higher excited states is
(a) Equal to the metastable state (b) Shorter than the metastable state
(c) Longer than the metastable state (d) Is not compare with metastable state

SHORT QUESTIONS

1. How can you produce atomic spectra?
2. What is the difference between absorption spectra and emission spectra?
3. How can you distinguish among line, band and continuous spectra?
4. What is the reason of dark lines in the line spectra?
5. Calculate the value of Rydberg constant.
6. How the stability of an atom is explained by Bohr's postulates?
7. What do you know about the quantization of orbits?
8. Under what condition an electron can emit energy?
9. What do you know about the Bohr's radius?
10. What is the significance of negative energy of electron in an orbit?
11. What do you know about excitation energy and excitation potential?
12. Calculate ionization energy and ionization potential for hydrogen atom.
13. What is the working principle of a laser?
14. Distinguish between spontaneous and stimulated emission.
15. What is the difference between normal population and population inversion?

16. What do you know about the metastable state?
17. Why population inversion in metastable state is necessary for the action of laser?

COMPREHENSIVE QUESTIONS

1. State and explain atomic spectrà with its different kinds.
2. What do you know about spectrum of hydrogen atom? Discuss the Balmer's series with empirical formula.
3. State and explain Bohr's atomic model of the hydrogen atom with postulates. Also derive the empirical formulas for radii and energies of the quantized orbits of the atom.
4. Discuss the energy level diagram and various series due to the different transition of electrons in the excited atoms.
5. What do you know about the excitation and ionization of an atom? Also discuss the potentials which are being used for excitation and ionization of atoms?
6. Explain characteristic x-rays due to the transition of electrons in the excited atoms. Also explain the production of x-rays.
7. What is LASER action? Explain the various process which are related with laser, such as spontaneous and stimulated emission and population inversion.
8. Define Helium-Neon laser. Also write down the various applications of laser.

NUMERICAL PROBLEMS

1. Calculate the shortest wavelength of the Balmer series. **(3646A°)**
2. Determine the longest wavelength in the Lyman series. **(1216A°)**
3. Calculate the speed of an electron of hydrogen atom in the second orbit and also find its kinetic energy also **(1.23×10^{12} ms^{-1}, 3.4eV)**
4. Show that the Paschen series of spectral lines is entirely in the infrared region.
5. Determine the time period of the first Bohr's orbit in the hydrogen atom. **(1.53×10^{-16} s)**
6. An electron jumps from a level $E_i = -2.2$eV to $E_f = -7.5$eV. What is the wavelength of the emitted radiation? **(234nm)**
7. The ionization energy of hydrogen atom is 13.6eV. Calculate the wavelength of the 1st line of the Lyman series. **(1212A°)**
8. The wavelength of K X-ray from copper is 1.377×10^{-10} m. What is the energy difference between the two levels from which this transition occurs? **(9.025keV)**

Unit 20 NUCLEAR PHYSICS

Major Concepts **(30 PERIODS)**

- Composition of atomic nuclei
- Isotopes
- Mass spectrograph
- Mass defect and binding energy
- Radioactivity (properties of α, β and γ rays)
- Energy from nuclear decay
- Half-life and rate of decay
- Interaction of radiation with matter
- Radiation detectors (GM counter and solid state detector)
- Nuclear reactions
- Nuclear fission (fission chain reaction)
- Nuclear reactors (types of nuclear reactor)
- Nuclear fusion (nuclear reaction in the Sun)
- Radiation exposure
- Biological and medical uses of radiations (radiation therapy, diagnosis of diseases, tracers techniques)
- Basic forces of nature
- Elementary particles and particle classification (hadrons, leptons and quarks)

Conceptual Linkage

This chapter is built on Nuclear Physics X

Students Learning Outcomes

After studying this unit, the students will be able to:

- describe a simple model for the atom to include protons, neutrons and electrons.
- determine the number of protons, neutrons and nucleons it contains for the specification of a nucleus in the form $^{A}_{Z}X$.
- explain that an element can exist in various isotopic forms each with a different number of neutrons.
- explain the use of mass spectrograph to demonstrate the existence of isotopes and to measure their relative abundance.

- define the terms unified mass scale, mass defect and calculate binding energy using Einstein's equation.
- illustrate graphically the variation of binding energy per nucleon with the mass number.
- explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission.
- identify that some nuclei are unstable, give out radiation to get rid of excess energy and are said to be radioactive.
- describe that an element may change into another element when radioactivity occurs.
- identify the spontaneous and random nature of nuclear decay.
- describe the term half-life and solve problems using the equation (9=0.693/$T_{1/2}$.
- determine the release of energy from different nuclear reactions.
- explain that atomic number and mass number conserve in nuclear reactions.
- describe energy and mass conservation in simple reactions and in radioactive decay.
- describe the phenomena of nuclear fission and fusion.
- describe the fission chain reaction.
- describe the function of various components of a nuclear reactor.
- describe the interaction of nuclear radiation with matter.
- describe the use of Geiger Muller counter and solid state detectors to detect the radiations.
- describe the basic forces of nature.
- describe the key features and components of the standard model of matter including hadrons, leptons and quarks.

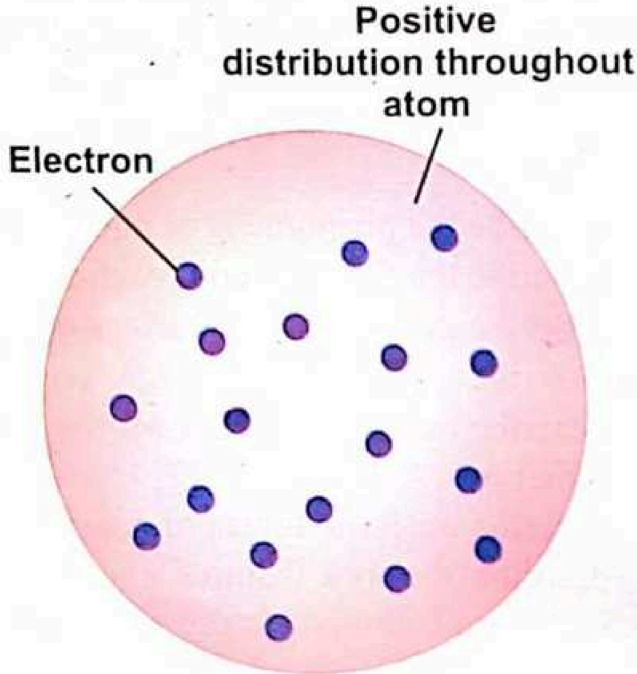
INTRODUCTION

The discovery of radioactivity by Henri Becquerel while investigating phosphorescence in uranium salts had prompted other scientists to describe the details of radioactivity and structure of the atomic nucleus. In this regard, Ernest Rutherford studying the properties of radioactive decay named the emitted radiations as alpha, beta particles and gamma rays. Also Rutherford discovered atomic nucleus by performing the alpha particle scattering experiments. He explained that the nucleus is a small, dense region at the centre of the atom. It consists of positive protons and neutral neutrons, so it has an overall positive charge. These experiments provided the basic properties of the nucleus of an atom such as; charge number, mass number etc.

In this unit we will not only discuss the structure of the nucleus but also explain its properties such as: charge number, mass number, isotopes, mass defect, binding energy etc. Similarly, we will study the radioactive nuclei i.e., unstable nuclei which emit radiations called radioactivity. We will also study the artificial radioactivity under the two chemical reactions i.e., fission reaction and fusion reaction. Some of the fundamental conservation laws are used in these nuclear reactions. The characteristics of nuclear fission reactions are discussed and applied to the example of a nuclear reactor used for the generation of electrical power. Energy can also be produced by nuclear fusion. Reference is made to the fusion reactions in stars, and some advantages and disadvantages of fusion as a future source of energy. In the last of this unit we will deal with forces of nature, various elementary particles and their classifications like Hadrons, Leptons and Quarks etc.

20.1 COMPOSITION OF ATOMIC NUCLEI

Rutherford designed an experiment to use the alpha particles emitted by a radioactive element to investigate atomic structure. He demonstrated that the atom has a central massive core which he called the nucleus. It has very small and very dense structure. It occupies only 10^{-12} of the total volume of the atom, i.e., its diameter is 10^{-14} m whereas the diameter of the atom is 10^{-10} m. The nucleus is positively charged, and its mass is about 99.9% of the total mass of the atom.

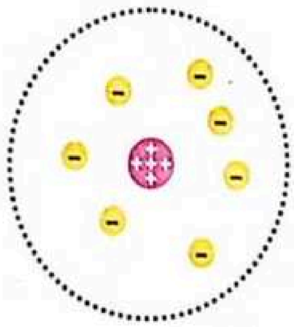


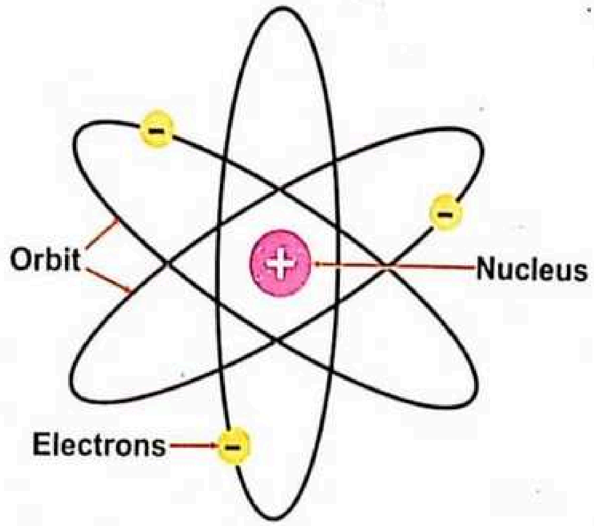
Early model of the atom.

After the discovery of neutron in 1932, Dmitri and Heisenberg developed the model of a nucleus which is composed of protons and neutrons. That is an atom is composed of a positively-charged nucleus, with a cloud of negatively-charged electrons surrounding it, bound together by electrostatic force. The protons were also discovered by Rutherford, which carries a positive charge of magnitude 1.6×10^{-19}C with mass 1.673×10^{-27} kg. Similarly, the other particle of the nucleus is the neutron. It was discovered by Chadwick. It is a neutral particle i.e., it carries no charge and its mass is 1.675×10^{-27} kg. The mass of proton is almost equal to the mass of the neutron but it is approximately 1836 times as massive as the electron.

FOR YOUR INFORMATION

After performing the alpha particles scattering experiment, Rutherford concluded that most part of an atom is empty and that mass is concentrated in a very small region called nucleus.





Rutherford's Atomic Model

The number of protons in a nucleus is called the atomic number or the charge number. It is represented by 'Z'. The total charges of a nucleus is Ze. Where 'e' is a charge on a proton or an electron.

Similarly, **the total number of protons and neutrons, called nucleons in a nucleus is termed as mass number.** It is denoted by 'A'. It is measured in terms of unified atomic mass unit (u) and this unit is defined as follows, the atom of carbon-12 is being considered the value of exactly 12u where 1u is equal to 1.6605×10^{-27} kg. On the basis of this scale, the proton rest mass and the neutron rest mass are both approximately 1u or 1a.m.u. The masses of some other important sub-atomic particles in terms of atomic mass unit are given in table 20.1. If both mass number "A" and charge number 'Z' are known, then the number of neutrons in the nucleus is (A – Z).

Table.20.1

Obeject	Mass (kg)	Mass (u)	Mass (MeV/c²)
Neutron	1.674929×10^{-27}	1.008664	939.57
Proton	1.672623×10^{-27}	1.007276	938.28
Electron	9.109390×10^{-31}	0.0054858	0.511

The observations show that the simplest nucleus of the hydrogen atom contains only one proton and it has no neutron. Similarly, the number of protons and the number of neutrons in the initial light elements of the periodic table are almost equal. However, the number of neutrons in the heavy elements are greater than the number of protons. For example, in helium ${}^{4}_{2}He$, the number of neutrons is 2 and number of proton is also 2. In ${}^{235}_{92}U$, the number of protons 92 and number of neutrons 143.

In order to classify nuclei in terms of their atomic number and mass number, the symbol X is being used to represent a nucleus as;

$$^{\text{Mass number}}_{\text{atomic numbers}}X = {}^{A}_{Z}X$$

For example, the nucleus of helium atom is represented by ${}^{4}_{2}He$ similarly the nucleus of Hydrogen atom is represented by ${}^{1}_{1}H$ where Z=1 and A =1. The proton is also represented by ${}^{1}_{1}H$ and the neutron is represented by ${}^{1}_{0}n$.

20.2 ISOTOPES

Atoms of the same element that have the same number of protons but different numbers of neutrons are known as isotopes. In such cases, their mass numbers are also different. Hence, the nuclei of an element having identical values of atomic number (Z) but different values of mass number (A) are called isotopes. For example, hydrogen has three isotopes, i.e., ${}^{1}_{1}H$ -ordinary hydrogen or protium has only one proton in its nucleus, ${}^{2}_{1}H$ -heavy hydrogen or deuterium contains one proton and one neutron and ${}^{3}_{1}H$ -tritium contains one proton and two neutrons in its nucleus as shown in Fig.20.1.

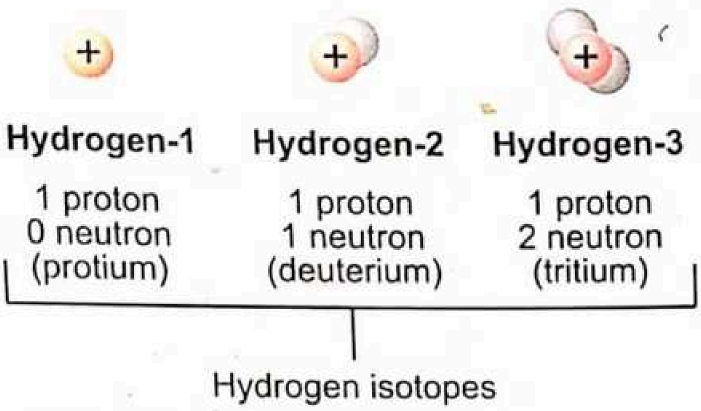


Fig.20.1. Three isotopes of hydrogen.

The natural abundance of isotopes can differ substantially. For example, carbon has four isotopes, ${}^{11}_{6}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$ and ${}^{14}_{6}C$. The natural abundance of the ${}^{12}_{6}C$ isotopes is approximately 98.9%, whereas that of the ${}^{13}_{6}C$ isotope is only about 1.1%, the rest isotopes, such as ${}^{11}_{6}C$ and ${}^{14}_{6}C$ are not natural occurring but can be produced by nuclear reaction in the laboratory or by cosmic rays.

The nuclei that have the same mass number, but have different atomic number are called isobars, e.g. ${}^{40}_{18}Ar$ and ${}^{40}_{20}C$. Similarly, the nuclei having the same number of neutrons are known as isotones (${}^{13}_{6}C$, ${}^{14}_{7}N$). The nuclei having identical atomic number and mass number but having different half-lives are called isomers. For example, ${}^{58}_{27}CO$ and $({}^{58}_{27}CO)^{m}$ both have same atomic number and mass number but the half-life of ${}^{58}CO$ is 71 days while the half-life of $({}^{58}_{27}CO)^{m}$ is 9 hours.

> ***POINT TO PONDER***
> - What is the function of in the atomic nucleus?
> - What is the fate of neutron when it is along are distant from one or more protons?

20.3 MASS SPECTROGRAPH

Mass spectrograph is used to determine the existence of isotopes and to measure their relative abundance or it is a device used to separate electrically charged particles according to their masses. A process in which the isotopes of any element can be separated, and their masses can be determined is called mass-spectrography. The mass of ions are determined from their deflection in a magnetic field. A mass spectrometer essentially consists of an ion source, a mass analyzer, and a detector as shown in Fig 20.2. Other minor components are two slits (S_1 and S_2), high voltage battery, applied magnetic field (B) in a vacuum chamber and a photographic plate.

In order to achieve the ionization of the given element in vapour from inside the source, one electron is removed from the particle, leaving with a net positive charge +e. The positive ions escaping the slits are accelerated through potential difference (V_o) applied between the two slits, the gain in K.E of the accelerated ions is given by

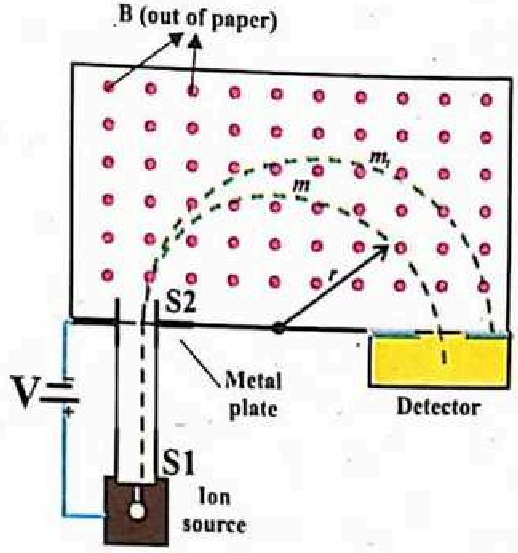


Fig.20.2. A schematic diagram for mass-spectrograph in which ions are deflected along the circular path by magnetic applied field.

$$\frac{1}{2}mv^2 = eV_o \quad ......(20.1)$$

When this narrow beam of accelerated ions passes through slit S_2 and enters into a vacuum chamber then it will be exposed to a perpendicular uniform magnetic field B. As a result, the beam of ions is deflected into a circular path of radius 'r' as shown in fig 20.4. The centripetal force of this circular path is equal to the magnetic force due to the applied magnetic field. i.e.,

$$\frac{mv^2}{r} = evB$$

$$m = \frac{eBr}{V} \quad ......(20.2)$$

The value of v can be obtained from Eq. 20.1;

$$v^2 = \frac{2eV_0}{m}$$

$$v = \sqrt{\frac{2eV_0}{m}}$$

Thus $$m = \frac{eBr}{\sqrt{\frac{2eV_o}{m}}}$$

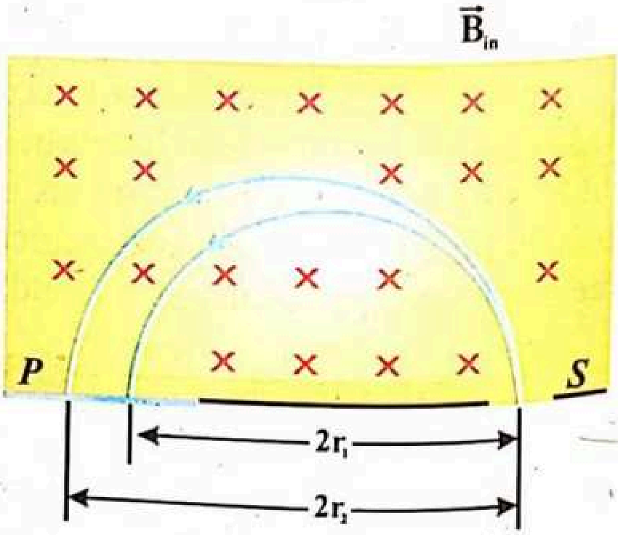
$$m^2 = e^2B^2r^2\left(\frac{m}{2eV_o}\right)$$

$$m = \left(\frac{er^2}{2V_o}\right)B^2 \quad ......(20.3)$$

By using Eq.20.3, we can calculate the mass of the ion if the values of B, r, e and V_o are known. Finally, when the ions fall on the photographic plate, they cast the images on it. The brightness images on the photographic plate give us a very interesting result. i.e. when an element gives rise the two or more different beams in the process of spectrograph then we will observe the same number of radii on the photographic plate. Equation 20.3 also shows that the beams of different radii have different masses. Thus, it is concluded that nuclei of the same element may have different masses. For example, when a beam of ions obtained from pure chlorine is passed through the mass–spectrograph then we will observe two images on the photographic plate. This shows that chlorine consists of two types of nuclei with respect to their masses, m_1 and m_2 where m_1 = 34.97u and m_2 = 36.97u. Natural abundance of m_1 is 75.4% and m_2 is 24.6 %.

Example 20.1

In a mass spectrograph, the masses of ions are determined from their deflection in a magnetic field. Suppose that singly charged ions of chlorine are shot perpendicularly into a magnetic field 0.15T with a speed of 5×10^4 m s^{-1}. Chlorine has two major isotopes, of masses 34.97u and 36.97u. What would be the radii of the circular paths, described by the two isotopes in the magnetic field as shown in figure.



Solution:

Magnetic field = 0.15T

Speed (v) = 5×10^4 ms^{-1}

Mass of the 1^{st} ion (m_1) = 34.97u = $(34.97\times1.66\times10^{-27})$kg

m_1 = 5.81×10^{-26} kg

Mass of the 2^{nd} ion (m_2) = 36.97u = $(36.97\times1.66\times^{-27})$kg

m_2 = 6.14×10^{-26} kg

Radius of the 1^{st} ion (r_1) = ?

Radius of the 2^{nd} ion (r_2) = ?

To find the radii we have the following relation

$$r = \frac{mv}{eB}$$

Thus
$$r_1 = \frac{m_1 v}{eB}$$

$$r_1 = \frac{5.81\times10^{-26}\,kg \times 5\times10^{4}\,m\,s^{-1}}{1.6\times10^{-19}\,C\times0.15T}$$

$$r_1 = \frac{2.905\times10^{-21}}{2.4\times10^{-20}}\,m$$

$$r_1 = 0.12m$$

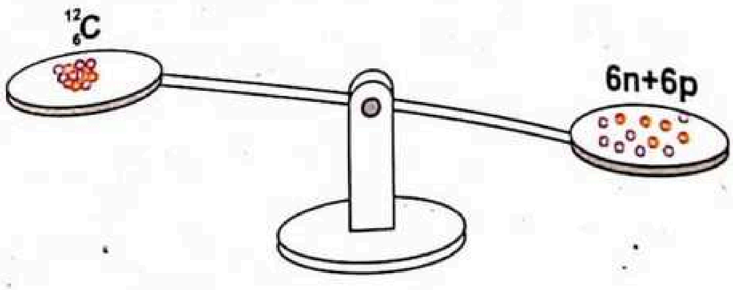
Similarly,
$$r_2 = \frac{6.14\times10^{-26}\,kg \times 5\times10^{4}\,m\,s^{-1}}{1.6\times10^{-19}\,C\times0.15T}$$

$$r_2 = \frac{3.07\times10^{-21}}{2.4\times10^{-20}}\,m$$

$$r_2 = 0.13m$$

20.4 MASS DEFECT AND BINDING ENERGY:

It has been observed experimentally that the mass of the nucleus is always less than the sum of masses of its free constituent i.e., protons and neutrons. The difference between mass of nucleus and the sum of masses of the nucleons of which it is composed is known as mass defect. It is represented by Δm and expressed as

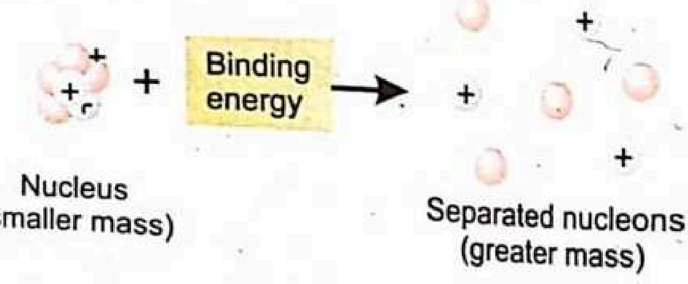


The mass of a nucleus is less than the total mass of its constituents i.e. protons and neutrons.

$$\Delta m = Zm_p + Nm_n - M_{nucleus}$$

$$\Delta m = Zm_p + (A - Z)m_n - M_{nucleus} \quad \ldots\ldots(20.4)$$

where 'Z' is the total number of protons and m_p is the mass of a proton. Therefore, Zm_p is the total mass of the protons. Similarly, (A–Z) is the total number of the neutrons and m_n is the



Binding Energy

mass of a neutron. So $(A-Z)m_n$ is the total mass of the neutrons and $M_{nucleus}$ is the mass of the nucleus.

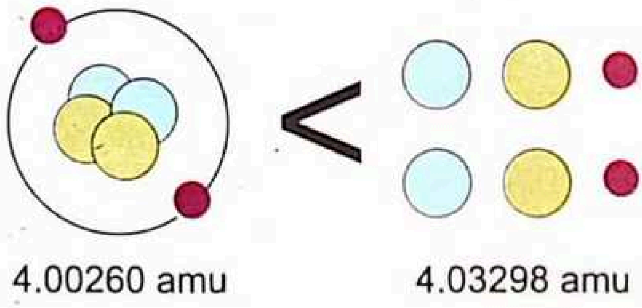
The mass defect 'Δ m' is due to the conversion of the mass into energy when nucleons combine to form a nucleus. This energy is called binding energy of the nucleus and it is obtained by Einstein's mass-energy relation.

$$E = \Delta mc^2 \text{ ......(20.5)}$$

Substituting the value of Δm from equation 20.4 in equation 20.5 we get.

$$B.E = \left[Zm_p + (A-Z)m_n - M_{nucleus}\right]c^2 \text{ ......(20.6)}$$

This is a mathematical from of binding energy and it is defined as the amount of energy that holds the nucleons together with in the nucleus against the repulsive coulomb's force. In other words, **the minimum energy that would be required to break the nucleus of an atom into its constituent nucleus i.e. protons and neutrons is called its binding energy.**



Mass Defect

In order to study the stability of a nucleus of an atom, we use the concept of binding energy per nucleon E_b/A usually called binding fraction. It has been observed experimentally that the greater is the binding energy per nucleon, the more stable is

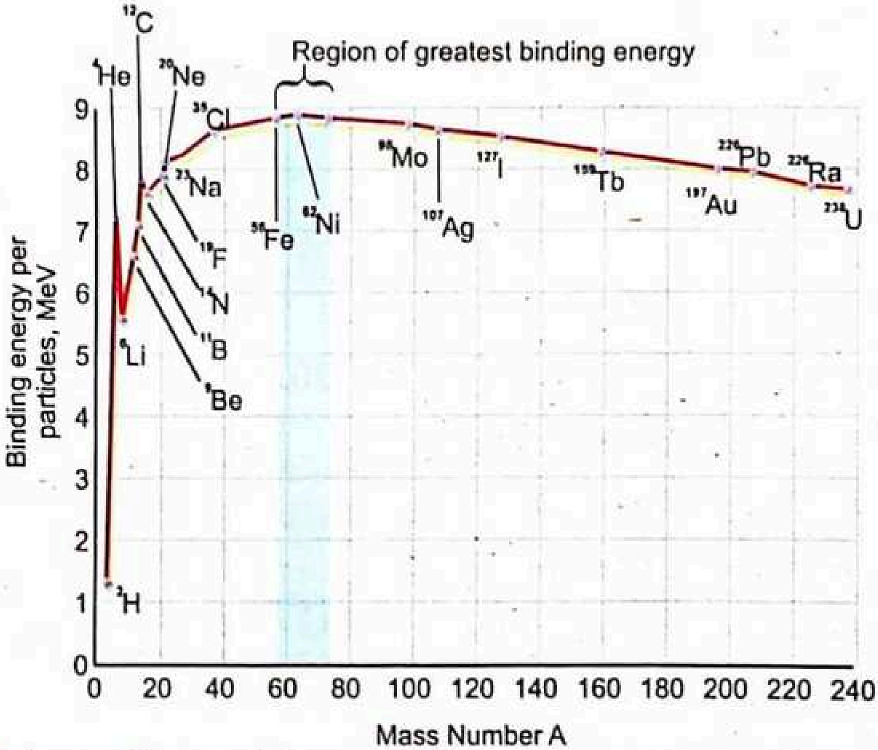


Fig.20.3. A curved line graph between binding energy per nucleon and mass number.

the nucleus and vice versa. This can be explained by drawing a graph between binding energy per nucleon and the mass number A.

The graph in the Fig 20.5 shows that when the mass number 'A' increases, the binding energy per nucleon increases rapidly up to the mass number of 50-60 i.e., for iron, nickel etc., and then decreases slowly. It means, the nucleons are bound most strongly in nuclei having mass numbers of the order of 50-60, the binding energy per nucleon for these nuclei reaches to its maximum value. i.e., 8.7 MeV per nucleon. On the other hand, the binding energy per nucleons is small for both light nuclei ($A < 30$) and heavy nucleis ($A > 170$).

Another important characteristic revealed by the graph 20.3 is that the binding energy per nucleon is approximately constant at around 8MeV per nucleon for nuclei having intermediate mass number (A = between 30 and 170). For these nuclei, the nuclear forces are said to be saturated.

Example 20.2

Calculate mass defect, binding energy and binding energy per nucleon of the $^{56}_{26}Fe$ nucleus. The mass of $^{56}_{26}Fe$ is 9.288×10^{-26} kg.

Solution:

$$\text{Mass of proton } (m_p) = 1.673 \times 10^{-27}\text{kg}$$
$$\text{Mass of neutron } (m_n) = 1.675 \times 10^{-27}\text{kg}$$
$$\text{Mass of } {}_{26}Fe^{56} \text{ nucleus } (m) = 9.288 \times 10^{-26}\text{kg}$$
$$Z = 26$$
$$A = 56$$
$$A\text{-}Z = 56 - 26 = 30$$

Mass defect
$$\Delta m = Zm_p + (A - Z)m_n - m_{nucleus}$$
$$\Delta m = 26\,(1.673 \times 10^{-27}) + 30\,(1.675 \times 10^{-27}) - 9.288 \times 10^{-26}$$
$$\Delta m = 4.3498 \times 10^{-26} + 5.025 \times 10^{-26} - 9.288 \times 10^{-26}$$
$$\Delta m = 8.68 \times 10^{-28}\text{kg}$$
$$B.E. = \Delta mc^2 = (8.68 \times 10^{-28})(3 \times 10^8)^2$$
$$B.E. = 7.812 \times 10^{-11}\text{J}$$
$$B.E. = \frac{7.812 \times 10^{-11}}{1.6 \times 10^{-19}}\text{eV} = 4.88 \times 10^8\text{eV} \quad \therefore 1\text{eV} = 1.6 \times 10^{-19}\text{J}$$
$$B.E. = 488 \text{ MeV}$$
$$\text{Binding energy per nucleons} = \frac{BE}{A} = \frac{488\,\text{MeV}}{56}$$
$$\text{Binding energy per nucleon} = 8.71 \text{ MeV per nucleon}$$

20.5 RADIOACTIVITY

Henri Becquerel in 1896 accidentally discovered radioactivity. He observed that a uranium salt spontaneously emit radiations in the absence of any source of energy (light). He placed a photographic plate close to the uranium salt such that the plate was wrapped in a black paper to keep light out. He observed that the photographic plate was blackened, which indicates that the radiation has been emitted from the uranium salt. Thus, the term radioactivity is used to describe the spontaneous emission of radiation from the uranium salt or other substances called radioactive substances. These emissions of radiations are independent of physical conditions of the radioactive substance such as temperature and pressure.

Later, Marie curie and Pierre curie discovered two new radioactive elements which they named as radium and polonium. Similarly, Rutherford's experiment on α -particle scattering suggested that radioactivity is the result of the decay or disintegration of the radioactive elements. The work done on radioactivity by various scientists revealed that those elements whose atomic number (Z) is greater than 82

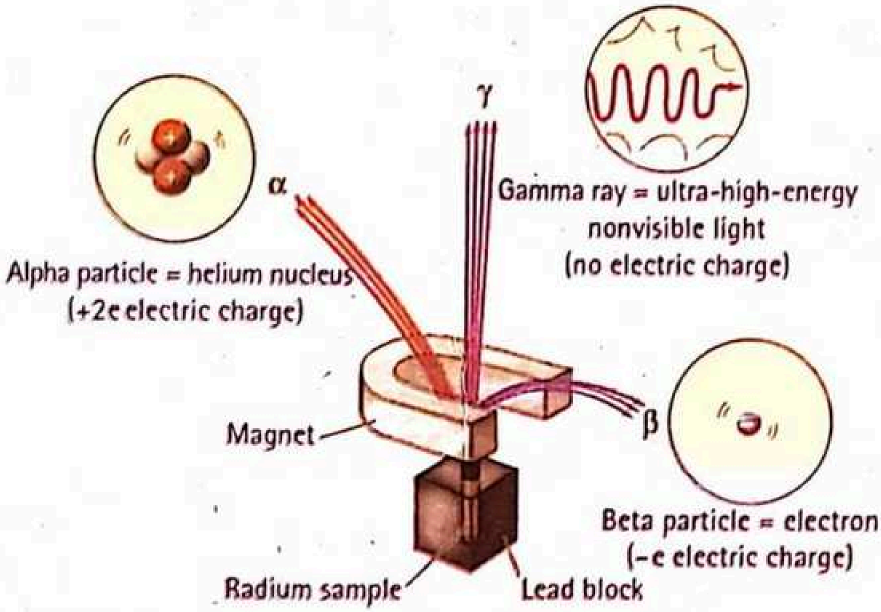


Fig. 20.4 The three emitted radioactive radiations, alpha, beta and gamma.

are unstable and they emit radiation spontaneously. The emissions consist of positively and negatively charged particles and neutral rays as shown in Fig.20.4, These three radiations were named as alpha (α), beta (β) and gamma (γ) respectively.

In the decay process, the original nucleus is usually called the parent nucleus. This parent nucleus is converted into the daughter nucleus by the emission of radiations. The daughter nucleus may also be unstable. In this connection a series of successive decays occurs until a stable configuration is reached i.e. the final nucleus is not radioactive. During the radioactive decay or nuclear decay all the laws of conservation such as: mass, energy momentum and charge must be conserved. These are termed as conservation of nucleons and it is stated as neutrons may be converted into protons and vice versa but total number of nucleons must remain constant.

20.5.1 Properties of α-Particles, β - Particles and γ – rays:

The three radiations α, β and γ have some most important properties which are explained as under:

Table.20.2

Type of Radiation	Nature	Mass amu	Charge
Alpha (α)	Helium ion (nucleus)	4	+2
Beta (β)	Fast moving electron	$\frac{1}{1800}$	-1
Gamma (γ)	High energy shortwave length light	0	0

Identity

α-Particles are identical to the nucleus of helium atom consisting of two protons and two neutrons having a mass of 4 units and charge of +2 units. The β-particles are fast moving electrons i.e., β-particles has the same mass and charge as an electron. γ-rays are form of electromagnetic radiations, having photon energies above 100keV. All these are explained in Table 20.2.

DO YOU KNOW

Once alpha and beta particles are slowed by collisions, they combine to become harmless helium atoms.

Charges

α -particles carry positive charges equal the charges of two protons whereas, β-particles carry negative charges equal to charge on an electron and γ-rays carry no charge. A summary of these characteristics is given in Table 20.2.

Effect of electric and magnetic fields

Both α-particles and β-particles are deflected in the presence of electric and magnetic fields whereas the γ-rays do not deflect. The deflection of the alpha particles, beta particles and gamma rays in an electric field is shown in Fig.20.5.

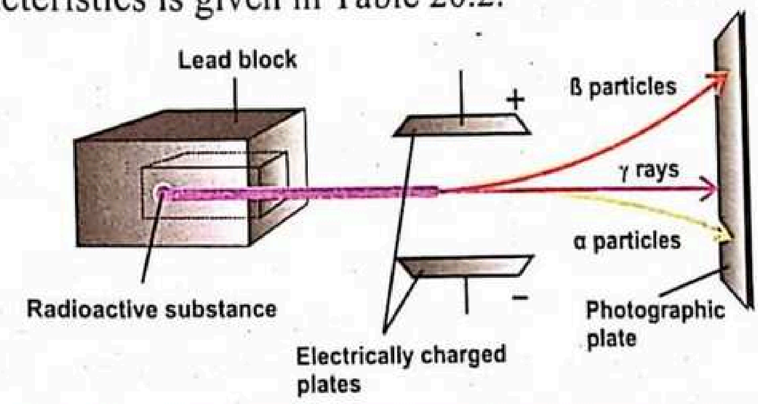


Fig.20.5. Deflection of alpha particles, beta particles and gamma rays in the presence of electric field.

Speed

As α-particles are the heaviest, so they are slow moving particles and their speed is almost $\frac{1}{200}$ of speed of light. The speed of β-particles is almost $\frac{2}{3}$ of speed of light. As γ-rays are electromagnetic waves therefore they move with the speed light.

Penetrating power

The penetrating power of α-particles is very small. They can penetrate only a few millimetres thickness of paper as shown in Fig 20.6. Similarly, β-particles can penetrate a few centimetres thickness of an Aluminum. On the other hand, the penetrating power of γ- rays is very large, it can penetrate more than 20 cm thickness of a lead.

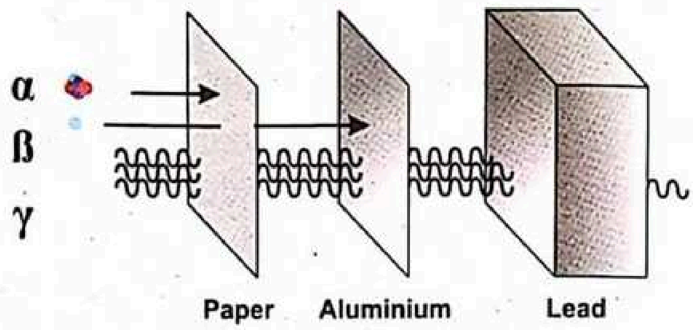


Fig.20.6. Penetrating power of alpha-particles, beta-particles and gamma-rays.

Ionization power

α-particles have highest ionization power because of their double positive charge and large mass. One alpha particle can ionize 10,000 atoms. The ionization power of β- particles is about $\frac{1}{100}$ that of the α-particles and practically the ionization power of γ- rays is zero

Photographic effect

All the three radiations α-particles, β-particles and γ-rays can blacken the photographic film.

Fluorescence

Though all the three radiations produce fluorescence when they fall on the fluorescence materials like zinc-sulphide but the fluorescence ability of γ-rays is less than the others two.

20.5.2 Laws of radioactive disintegration

We have studied that when a radioactive element disintegrates with time, it emits α-particles, β-particles and γ-rays. Rutherford and his colleague. Frederick introduced the disintegration theory of radioactivity and summarized it in the form of the following laws.

i The radioactive decay is spontaneous, and it does not depend upon the physical and external conditions.

iii The decay has different radiations such as: alpha, beta and gamma but all of them are not emitted simultaneously.

iii The rate of decay is different for different radioactive elements.

iv The rate of decay is directly proportional to the initial number of atoms of radioactive element.

v No element practically decays completely, when its atomic number is less than 81 i.e. these elements are stable.

vi When an element emits an α-particles ($^{4}_{2}He$), the nucleus of its atom loses four nucleons, i.e. two protons and two neutron.

Therefore, the charge number (Z) of the nucleus decreases by two and mass number (A) decreases by four. Hence the parent element $^{A}_{Z}X$ is converted into the daughter element $^{A-4}_{Z-2}Y$. This can be expressed by following equation.

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

For example, when an α-particles is emitted from uranium $^{234}_{92}U$ then we have thorium $^{230}_{90}Th$ and this change can be represented by the following equation.

$$^{234}_{92}U \rightarrow {}^{230}_{90}Th + {}^{4}_{2}He$$

In this reaction we can observe the law of conservation of nucleons.

vii Similarly, when an element emits a β-particles, then there is no change in the number of nucleons i.e. its mass number (A) remains the same. However, its charge number (Z) increases by one. Hence, the parent element $^{A}_{Z}X$ is converted into a daughter element $^{A}_{Z+1}Y$ by the emission of β-particles. This change can be represented by the following reaction as,

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$$

As we know that electrons do not exist within the nucleus. However, the emission of electron from the nucleus is being thought that a neutron is converted to proton and an electron. It means that β-particles is formed at the time of its emission. It is a reason that why at the time of emission of β-particles the charge number of the nucleus increases by one but no change occurs in its mass number.

For example, if a β-particles is emitted from carbon $^{14}_{6}C$ then we have $^{14}_{7}N$ which may be written as:

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$$

viii If there is emission of γ-rays from an element then there is no change in its charge number (Z) and its mass number (A), because a γ- rays is simply a photon that having neither charge nor any mass. The nucleus is in the excited state i.e. ($^{A}_{Z}X$) and acquires ground state after the emission of γ-rays. Thus, the decay of γ-rays from the radioactive element is represented by this equation.

$$\left({}^{A}_{Z}X\right)^{*} \rightarrow {}^{A}_{Z}Y + \gamma$$

20.6 RADIOACTIVE DECAY EQUATION

Consider a radioactive element which has 'N' radioactive atoms at the starting point. After some time Δt the number of radioactive atoms of the element decreases by 'ΔN' due to radioactive disintegration process. If ΔN be the number of atoms which decay in time Δt, then according the law of decay, the rate of decay is directly proportional to the initial number of atoms of the element, i.e.,

$$\frac{\Delta N}{\Delta t} \propto N$$

$$\frac{\Delta N}{\Delta T} = \lambda N \quad ......(20.6)$$

> ***FOR YOUR INFORMATION***
> - Any quantity that decreases by half over equal time intervals is said to decay exponentially.
> - Any quantity that increases by twice over equal time interval is said to grow exponentially.

where 'λ' is a constant of proportionality and it is called decay constant or disintegration constant. Since the number of radioactive atoms decreases with time therefore a negative sign has to be introduced on the right hand side of Eq. 20.6.

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

or $$\frac{\Delta N}{N} = -\lambda \Delta t \quad ......(20.7)$$

Eq.20.7 can be solved using a mathematical technique called integration and we have the following relation

$$N = N_0 e^{-\lambda t} \quad ......(20.8)$$

where N_o is the number of atoms of radioactive element at $t = 0$. Equation 20.7 is known as radioactive decay equation and it shows that the number of atoms of a radioactive element decreases exponentially with time. Now if we draw a graph between undecayed number of atoms N and time 't' then we have a curved line as shown in Fig 20.7, which illustrates the exponential nature of the decay of a radioactive element.

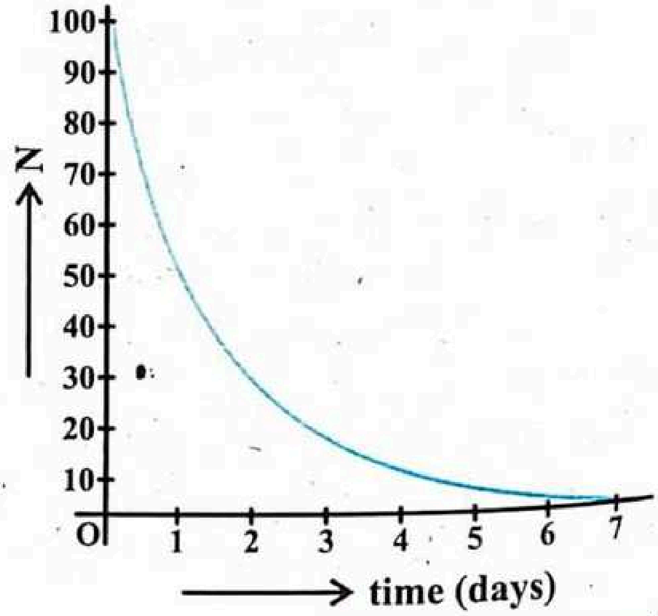


Fig.20.7. A graph between undecayed number of nucleons and time showing an exponential nature of decay.

20.6.1 Half-life of a radioactive element

Half-life of a radioactive element is a method which is being used to determine the rate of radioactivity and it is defined as; '**the time taken for the activity of a given amount of a radioactive substance to decay to half of its initial value' or the time interval during which half of the given number of radioactive nuclei decay.** It is represented by $T_{1/2}$ and its relation can be expresses as;

Let N_o be the initial number of radioactive atoms of an element. i.e. at $t = 0$ which reduces to $\frac{N_o}{2}$ after $t = T_{\frac{1}{2}}$ i.e. after one half-life. Mathematically

$$N = N_o, \text{ at } t = 0$$

and

$$N = \frac{N_o}{2}, \text{ at } t = T_{1/2} \text{ (half life).}$$

Thus Eq.20.8 becomes

$$\frac{N_o}{2} = N_o e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{\frac{1}{2}}}$$

$$2 = e^{\lambda T_{\frac{1}{2}}}$$

$$\ln 2 = \lambda T_{1/2}$$

$$\lambda T_{1/2} = \ln 2$$

$$T_{1/2} = \frac{0.693}{\lambda} \quad \text{......(20.9)}$$

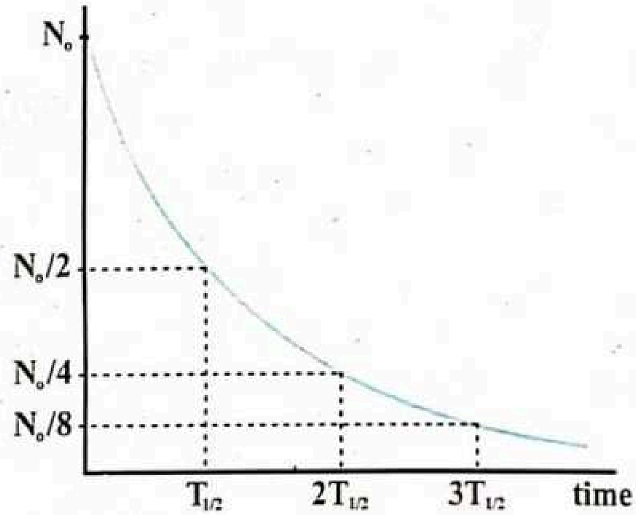


Fig.20.8. The decay of radioactive element in terms of half life.

Equation 20.9 gives the half-life of a radioactive element and it shows that the half-life of an element is inversely proportional to its decay constant.

Similarly, after two half-lives, the half-life of the rest nuclei has decayed and $\frac{N_o}{4}$ radioactive nuclei are left; after three half-lives, $\frac{N_o}{8}$ are left and so on. Graphically, the decay of a radioactive element in terms of half-life is shown in Fig 20.8.

The SI unit of a radioactivity is the Becquerel (Bq). That is, $1\text{Bq} = 1\text{decay s}^{-1}$ (Radioactive decay per second). The radioactivity is also being measured in terms of curie (Ci), where $1\text{Ci} = 3.7\times 10^{10}$ decay s^{-1}

Thus $1\text{Ci}=3.7\times10^{10}\text{Bq}$...(20.10)

It may be noted that the half-life of each radioactive element is different as given in Table 20.3.

Table.20.3 Half-life of some radioactive nuclei.

Radioactive nuclide	Nuclide notation	Half-life
Lithium-8	${}^{8}_{3}Li$	0.838s
Krypton-89	${}^{89}_{36}Kr$	3.16 minutes
Sodium-24	${}^{24}_{11}Na$	15 hours
Iodine-131	${}^{131}_{53}I$	8 days
Cobalt-60	${}^{80}_{27}Co$	5.27 years
Radium-228	${}^{226}_{88}Ra$	1600 years
Uranium-235	${}^{235}_{92}U$	703 million years

Example 20.3

The half-life of carbon ${}^{14}_{6}C$ is 5.7×10^3 years, what fraction of a sample of ${}^{14}_{6}C$ will remain undecayed after a period of five half-life times.

Solution:

Half-life of ${}^{14}_{6}C$ $T_{1/2} = 5.7 \times 10^3$ years

Time for five half–lives = $t = 5\ T_{1/2} =$

$5(5.7\times10^3 y) = 2.85\times10^4 y$

According to the relation for half life

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{5.7\times10^3 y}$$

$$\lambda = 1.216\times 10^{-4} y^{-1}$$

According to exponential decay law

$$N = N_o\ e^{-\lambda t}$$

$$\frac{N}{N_o} = e^{-\lambda t}$$

Substitute the values of λ and t in above equation

$$\frac{N}{N_o} = e^{-(1.216\times10^{-4} y^{-1})(2.85\times10^4 y)}$$

$$\frac{N}{N_o} = e^{-3.4656} = 0.03125$$

This is the desired fraction.

Alternate Method:

Times	Activity
0	1
T½	0.5
2 T½	0.25

Time	Activity
3T½	0.125
4T½	0.0625
5 T½	0.03125

Hence 0.03125 (or 3.125%) of sample of ${}^{14}_{6}C$ will remain undecayed after a period of five half-lives.

20.7 RADIATIONS DETECTORS

Radiation cannot be detected directly by human senses. The radiations detection is possible only when they interact with the matter and there are various methods by which we can detect the interaction of radiation with matter, such as, ionization of atoms, scattering from atoms or absorbing by atoms. In this regard, there are several devices which have been developed for detecting radiations, but we will explain two of them which are being used most commonly.

(1) GM – Counter

(2) Solid State detector.

Geiger – Muller counter

G.M. counter is a device designed by Geiger and Muller, used for the detection and measurement of all types of radiations. It detects ionizing radiations such as alpha, beta particles and gamma rays using the ionization effect produced in a G.M. tube. The apparatus consists of two parts, the tube and the counter. The G.M. tube consists of a pair of electrodes surrounded by a gas. It consists of a sealed metal tube whose boundary acts as a cathode. The anode is in the form of a thin wire lying along the control axis of the tube as shown in Fig.20.9. The tube is filled with a suitable mixture of Argon gas and Bromine vapours at low pressure i.e., at about one tenth of the atmospheric pressure. The Argon gas is called the detecting gas and Bromine act as a quenching gas. A high potential difference of about 1000V is applied across the anode and cathode through a high resistance. This potential difference is slightly less than the potential difference which would produce continuous discharge in the tube.

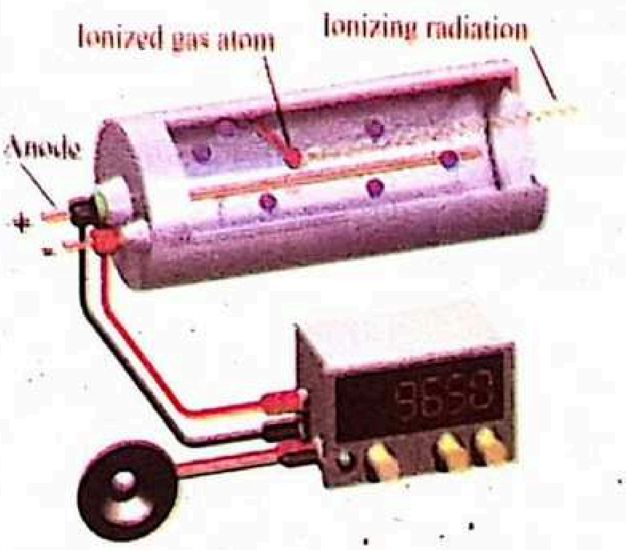


Fig.20.9. A schematic diagram of Geiger-Muller Counter.

When a radiation (or high energy particles) from a radioactive element enters into the tube through a thin window at the end, some of gas atoms are ionized. The electrons removed from these ionized gas atoms are attracted to the anode and positive ions towards cathode. The high potential difference between the anode and cathode accelerate the electrons and they collide with other atoms of gas, which produce further ionization. This secondary ionization results in an avalanche of electrons. The process produces a current pulse and is counted by a pulse counter.

On the other hand, further incoming particles cannot be counted, it is therefore, when the positive ions strike the cathode, secondary electrons are emitted from the surface. These electrons would be accelerated to give further spurious count. But it is represented by molecules of bromine gas, when they absorb the energy of the positive ions moving towards the cathode. Hence, bromine gas acts as a quenching agent and it is called quenching gas. The quenching gas must have an ionization potential lower than that of the principal gas (Argon gas).

G.M. counter can be used to determine the range or penetration power of ionizing particles. The reduction in the count rate by inserting metal plates of varying thickness between the source and the tube helps to estimate the penetration power of the incident radiation. Though Geiger Counter has ability to measure the accurate count, but it is not suitable for fast counting, because of its relative long dead time of the order of more than a million second which limit the counting rate to a few hundred count per second. If the particles are allowed to enter the tube at a faster rate, not all of them will be counted since some will arrive during the dead time. Alternately, we have another device such as a solid state detector which is fast enough, more efficient and accurate.

Solid state detector

A solid state detector is a semiconductor detector. It is used to detect, and fast counting of incident charged particles or photons. Its working is based on revere biasing in which electron hole pairs are formed by the incident particles which causes a current pulse to flow through the external circuit. It consists of PN- junction diode such that its two opposite sides are coated with a thin layer of gold as shown in fig 20.10.

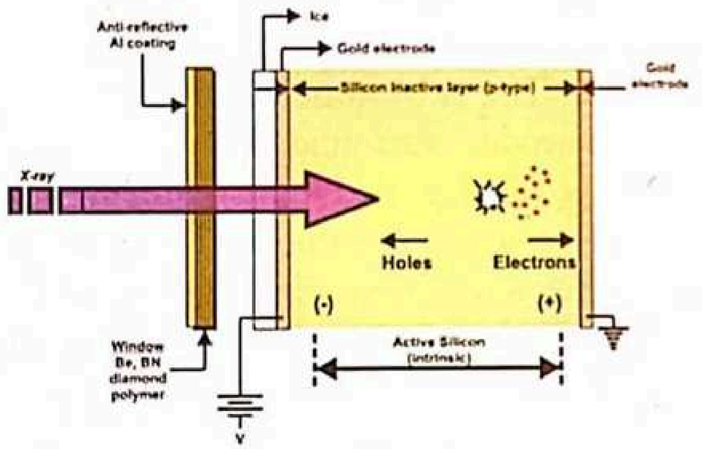
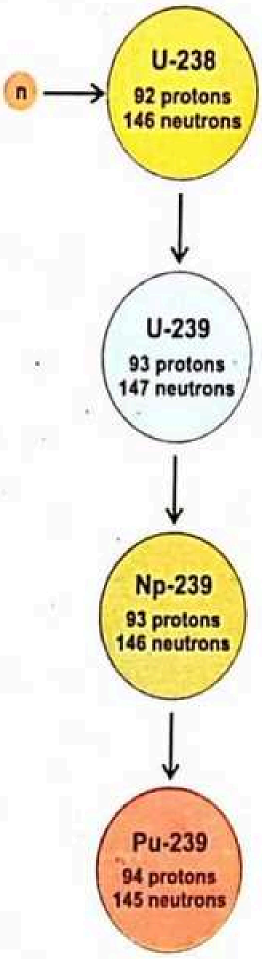


Fig.20.10. A schematic diagram of Solid State detector.

The thin gold layers make a good conducting contact with the external circuit. For radiation detection this device is connected in reverse biasing of the PN-junction. When the voltage is applied through the two conducting layers of gold then there is a large depletion region around the junction. The small current is due to only the flow of minority carries.

Now when the ionizing incident particles enter into PN-junction through the depletion region, then they form electron–hole pairs. These mobile charge carriers move towards their respective sides of the junction. This motion of charge carries (holes and electrons) produces a pulse of current which is amplified and measured with an electronic counter called scalar. In a typical device, the duration of the pulse is 10^{-8}s. The size of the pulse is found proportional to the energy of the incident particles. The energy needed to produce an electron hole pair is about 3eV to 9eV, which makes the device useful for detecting low energy particles. The collection time of the electrons and hole is much less than the gas filled counters and hence, a solid state defector can count very fast. It is small in size and operates at low voltage. The solid state detectors are more useful, for detecting α or β particles but specially designed detector can also be used for detecting high energy γ - rays.

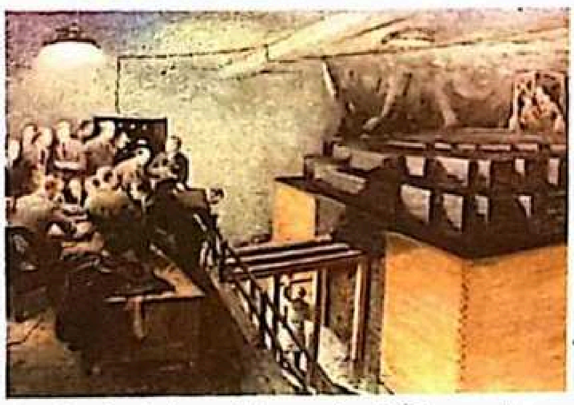


A transmutation from the nucleus of $_{92}U^{238}$ into the nucleus of $_{94}Pu^{239}$ occurs in an induced nuclear reaction.

20.8 NUCLEAR REACTION

A nuclear reaction is a process in which two nuclear particles (two nuclei or a nucleus and a nucleon) interact to produce two or more nuclear particles or γ -rays (gamma rays). Or nuclear reaction is a process that occurs in an atomic nucleus, where a change occurs in a nucleus, such as a transformation of at least one nuclide to another. In the process of a nuclear reaction, when an element is converted to another by changing of its nucleus then such a conversion is called nuclear transmutation.

FIRST NUCLEAR REACTOR



An artist depiction of the setting in the sqush court beneath the stands at the University of Chicago's Stagg Field, where Enrico Fermi and his colleagues constructed the first nuclear reactor.

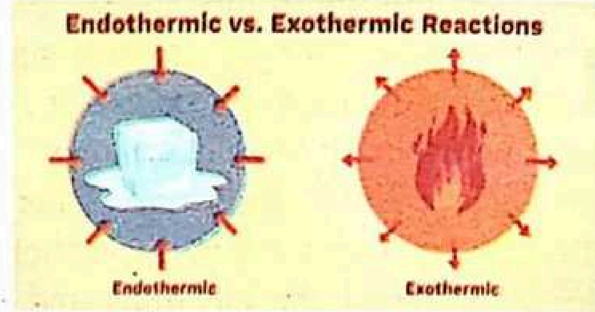
If 'a' is a nuclear particle 'X' is a target nucleus, 'Y' is resultant nucleus and 'b' is a emitted particle then the nuclear reaction can be represented by the following equation;

$$a + X \rightarrow Y + b$$

Rutherford, was the first who observed the nuclear reaction in 1919. He bombarded α-particles on a nitrogen ($^{14}_{7}N$) then he got oxygen ($^{17}_{8}O$) and emitted particle proton ($^{1}_{1}H$) i.e.,

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

In a nuclear reaction charge number, mass number, energy, momentum all are conserved. The minimum energy required for a reaction to take place is called the threshold energy.



In nuclear reaction, energy is either absorbed or emitted which is called Q- value and it is equal to the mass defect. The value of Q is taken as positive when the energy is released, and its corresponding reaction is called exothermic. Similarly, the value of Q is taken as negative when the energy is absorbed, and its corresponding reaction is called endothermic. For example;

$$^{1}_{1}H + ^{7}_{3}Li \rightarrow ^{4}_{2}He + ^{4}_{2}He + 17.3MeV$$

In this reaction Q = + 17.3 MeV. Similarly,

$$^{4}_{2}He - ^{17}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}H + 1.19MeV$$

In this reaction Q = - 1.19 MeV

There are two main kinds of nuclear reaction:

I Nuclear fission reaction

II Nuclear fusion reaction

20.9 NUCLEAR FISSION

A process in which a heavy nucleus splits into two smaller nuclei with the release of energy is known as nuclear fission.

Nuclear fission was discovered in 1938 by Otto Hahn and Fritz strassman pursuing earlier work by Fermi. They bombarded uranium nucleus $^{238}_{92}U$ with a slow moving neutron and observed that the uranium nucleus had split into two lighter nuclei Barium ($^{141}_{56}Ba$) and Krypton ($^{92}_{36}Kr$) with emission of slow neutrons (typically two or three). On average, 2.5 neutrons are released per event as shown in Fig.20.11. They also observed that approximately 200MeV of energy was released in each fission reaction. The equation of above fission reaction is expressed as:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + energy$$

In nuclear fission, the absorption of the incident neutron creates an unstable nucleus and can change to a lower–energy configuration by splitting into two lighter nuclei.

The combined mass of the daughter nuclei is less than the mass of the parent nucleus and such difference in mass is called the mass defect. If we multiply this mass deflect by ${}^2C$ then it becomes equal to the binding energy of the nucleus. This binding energy must be released when the parent nucleus is splitted into two daughter small nuclei.

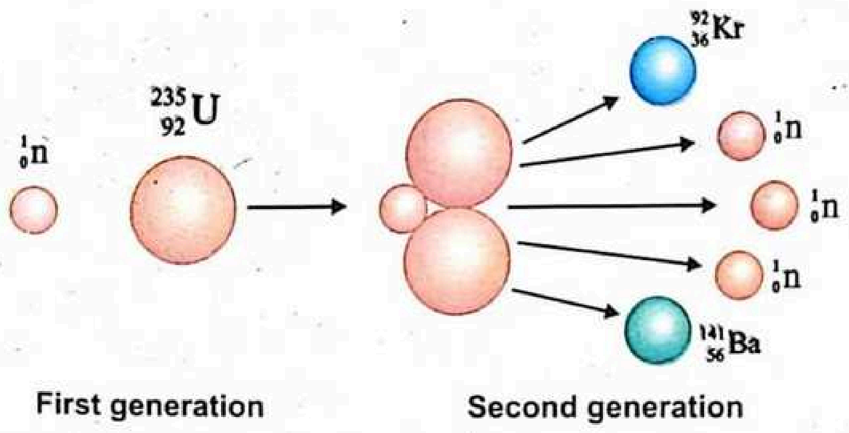


Fig.20.11. A schematic diagram for a nuclear fission reaction.

The experimental results show that such reaction releases large amount of energy. We have studied the graph between the binding energy per nucleon and mass number which shows that the intermediate elements in the periodic table have higher binding energy per nucleon. Whereas, the binding energy per nucleon of the heavy elements is little less i.e. the binding energy per nucleon for ${}^{235}_{92}U$ is 7.6 MeV per nucleon and 8.5 MeV per nucleon for Barium (${}^{141}_{56}Ba$) and Krypton (${}^{92}_{36}Kr$). It means that if the uranium ${}^{235}_{92}U$ nucleus is splitted into two nuclei of ${}^{141}_{56}Ba$ and ${}^{92}_{36}Kr$ then the amount of energy released will be $8.5 - 7.6 = 0.9$ MeV per nucleon. Thus, total energy $235 \times 0.9 = 200$ MeV is released in uranium fission reaction.

20.9.1 Chain Reaction

A process in which an induced fission reaction continues till all the atoms of the radioactive material have gone through the fission reaction is called chain reaction.

We have studied that when a slow moving neutron is bombarded at the nucleus of uranium-235 it undergoes fission. Beside the daughter nuclei, two or three neutrons are also released in this fission reaction. These neutrons can further induce fission in the other nuclei. This is the basis of the self-sustaining chain reaction as shown in Fig.20.12. If at least one neutron on average, results in another fission, the chain reaction is said to be critical. Because a sufficient amount of mass is required to increase the probability of a neutron being absorbed, a critical mass of fissionable material must be present. Similarly, if less than one neutron, on average, produces another fission, the reaction is termed as sub critical. If more than one neutron, on average, produces another fission, the reaction is said to be supercritical. An atomic bomb is an extreme example of a supercritical fission chain reaction.

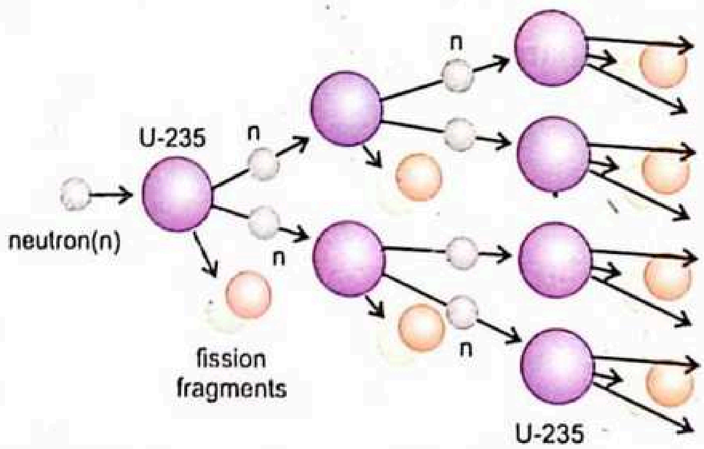


Fig.20.12. A schematic diagram of Chain fission reaction.

In chain reaction, the process proceeds very quickly and in very short interval of time, the whole radioactive element undergoes fission. The observations show that if the chain reaction is not controlled, it can result in a violent explosion with the sudden release of an enormous amount of energy. However, when the reaction is controlled, the released energy can be used for constructive purpose for example energy released during the process is converted into electrical energy.

20.9.2 Nuclear Reactor

A nuclear reactor is a system used to initiate and control nuclear chain reaction to produce heat. This heat energy is used to generate steam, which operates a turbine and turns an electrical generator. For example, when fission reaction is induced, then

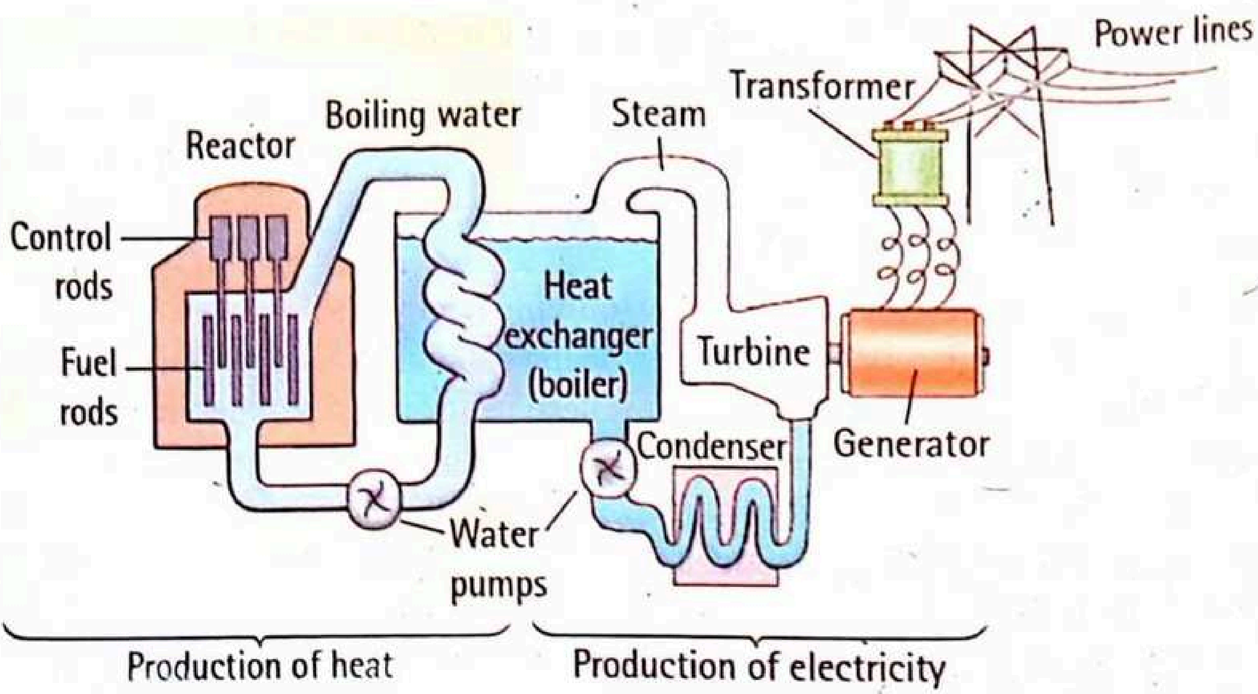


Fig.20.13. A schematic arrangement of nuclear power plant.

energy at the rate of 200 MeV per fission is produced. This energy appears in the form of kinetic energy of the fission fragment, these fast moving fragments besides colliding with one another also collide with the uranium atoms. In this way, their kinetic energy gets transformed into heat energy. This heat energy is used the produce steam which in turn rotates the turbine and then turbine operates generator to produce electricity. The whole process of the nuclear power plant is shown in Fig.20.13.

The nuclear reactor was introduced in 1942 by Enrico Fermi and his colleague. They used uranium as the fuel. A controlled nuclear reactor has the following important parts.

Fissionable fuel

Most reactors in operation use uranium as fuel. Natural uranium contains 0.7% of the $^{235}_{92}U$ isotope which is fissile. However, the remaining 99.3% of $^{238}_{92}U$ which does not contribute directly to fission process. It is noted that if the neutrons are slowed down, then they are much more likely to be captured by $^{235}_{92}U$ and induce fission. For this reason, the quantity of fuel $^{235}_{92}U$ is increased a few percent. The nuclear fuel is sealed in long, narrow metal aluminum tubes called fuel rods.

Moderator to slow down neutrons

The neutrons emitted by fission are moving very fast. At this high speed, the chance of a neutron being captured by another nucleus $^{235}_{92}U$ is very small, therefore, the high speed neutrons are slowed down by collision with the nuclei in the surrounding material called the moderator, so they are much more likely to cause further fissions. In nuclear power plants, heavy water or graphite is often used as moderators.

Control rods

Besides the moderator, there is an arrangement of control rods made of cadmium or boron which are very efficient in the absorption of fast moving neutrons without undergoing any additional reaction. Thus, the rate of reaction is controlled by inserting or withdrawing control rods in the core of the reactor.

Coolant

The fission reaction produces heat in the core of the reactor. This heat causes rise of temperature of the water contained in the primary loop which is maintained at high pressure to refrain the water from boiling. The hot water is pumped through a heat exchanger, where the internal energy of the water is transferred by conduction to the water contained in the secondary loop. The hot water in the secondary loop is converted to steam, which does work to drive a turbine generator system to produce electric power.

Protective shield

In a nuclear reactor, there are many types of harmful radiations emitted which are dangerous for all living things. In order to protect from these radiations, the reactor is surrounded by a massive biological shield.

Kinds of reactors

Basically, there are two kinds of nuclear reactors such as:

(i) Thermal or slowdown reactor (ii) Fast reactor

The neutrons which are slowed down by using moderator called thermal neutrons. These thermal neutrons can easily capture by the nucleus of $^{235}_{92}U$ to induces a fission reaction. Such reaction is called thermal or slowed down reaction, its corresponding reactor is called thermal reactor.

On the other hand, the fast moving neutrons can capture by the nucleus of $^{238}_{92}U$ to create a fission reactions, such reaction is called fast reaction. Some neutrons are allowed to escape and under suitable conditions they will be captured by nuclei of $^{238}_{92}U$ which are converted to plutonium ^{239}Pu i.e.,

$$^{1}_{0}n + ^{238}_{92}U \rightarrow ^{239}_{93}Np + ^{0}_{-1}e$$

$$^{239}_{93}Np \rightarrow ^{239}_{94}Pu + ^{0}_{-1}e$$

As more plutonium can be produced then the required to enrich the fuel in the core, so these are called fast reactors. In fast reactor, the moderator is being used. Because the nuclei of plutonium are fissioned by fast moving neutrons. The core of fast reactor contains a mixture of plutonium and uranium dioxide surrounded by a blanket of $^{238}_{92}U$.

20.10 NUCLEAR FUSION

A process in which two lighter nuclei fuse together to form a heavier nucleus and release a large amount of energy is known as nuclear fusion reaction. In this process, the mass of the resultant nucleus is always less than the sum of masses of the original nuclei which are fused. The missing mass is converted into energy which is released during the reaction. Though fusion

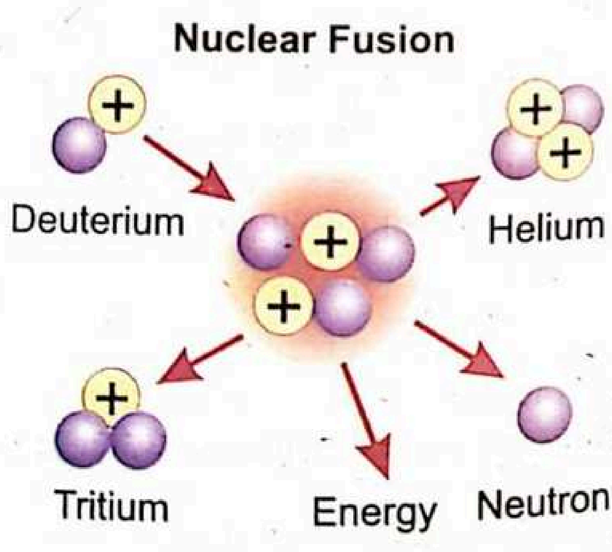


Fig.20.14. A schematic diagram of nuclear fusion reaction.

reactions release energy for the same reason as fission reaction, but the process of fusion reaction is essentially the opposite of the nuclear fission. The energy released per unit mass in nuclear fusion is more than the energy released per unit mass in nuclear fission. For example we have studied the graph between binding energy per nucleon and mass number, where the nuclei near A=56 have the highest binding energy per nucleon, therefore, when two light nuclei are fused together to form a heavy nucleus whose mass number 'A' is less than 50, then there will be huge amount of energy released. The observation shows that the most energy is released if two isotopes of hydrogen, ${}_1H^2$ and ${}_1H^3$ are fused together as shown in Fig.20.14 under the following reaction;

$$ {}_1^2H + {}_1^3H \rightarrow {}_2^4He + {}_0^1n + 17.6MeV $$

In this reaction Q-values is 17.6 Mev. Due to the strong binding of ${}_2^4He$, there is release of energy of about 3.5 MeV per nucleon in fusion reaction. On the other hand, 0.7 MeV per nucleon is released in fission reaction. Thus, this result shows that in nuclear fusion process, lighter nuclei fusing to form a heavier nucleus is a more prolific source of energy. But it is comparatively more difficult to produce fusion because of large electrostatic repulsive force between the two nuclei. The force becomes stronger, when the nuclei are brought very close for fusion. To overcome this repulsive force, the two nuclei must have high speed or kinetic energies. This high speed can be achieved by increasing the temperature. Typically, the temperature of order 10^8K is required for fusion to induce. At such a high temperature, a substance is a completely ionized plasma. This means a fusion reactor's fuel is in the form of plasma.

> ***FOR YOUR INFORMATION***
>
> The energy released in fusing a pair of hydrogen nuclei is less than in fissioning a uranium nucleus. But because there are more atoms in a gram of hydrogen than in a gram of uranium, gram for gram, the fusion of hydrogen releases several times more energy as the fission of uranium.

The induced fusion by high temperature is called thermonuclear fusion. Such a high temperature can be achieved by explosion of an atom bomb for short time.

Nuclear reaction in the sun

The sun is a star and is mostly composed of the elements hydrogen (75%), helium (25%) and other element (less than 0.1 %). The sun continuously generates its energy in the form of light and heat by nuclear fusion of hydrogen nuclei into helium nucleus. In its core, the Sun fuses 500 million metric tons of hydrogen each second. The temperature of its core is about 20 million degree Celsius. While its surface temperature is about 5 million degree Celsius. Most of its energy is due to the fusion of hydrogen to a nucleus of deuteron. Such reaction can be expressed as;

$$ {}_1^1H + {}_1^1H \rightarrow {}_1^2H + {}_1^0e + \text{energy} $$

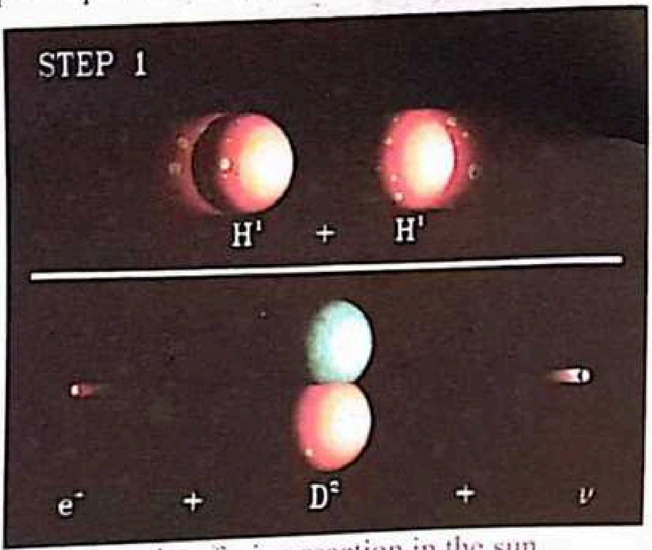
Similarly, the fusion reaction of deuteron (^2_1H) with the hydrogen (^1_1H) forms a helium (^3_2He) nucleus i.e.,

$$^2_1H + ^1_1H \rightarrow ^3_2He + \text{energy}$$

Now at the final stage, the fusion reaction of two nuclei of helium (^3_2He) is is represented by the following nuclear reaction.

$$^3_2He + ^3_2He \rightarrow ^4_2He + ^1_1H + ^1_1H + \text{energy}$$

This reaction shows that there are six hydrogen atoms take part in the reaction. Four hydrogen atoms have formed a helium nucleus and two hydrogen atoms are surplus. It has been measured that in the hydrogen-hydrogen reaction, there is a 25.7MeV energy is generated at the rate 6.4 MeV per nucleon which is quite a large amount of energy as compared to the energy obtained per nucleon from a fission reaction.



Nuclear fusion reaction in the sun.

20.11 RADIATION EXPOSURE

We have observed the detection of radiations using various instruments, like G.M tube etc. The radiations can also be detected in the open environment even when there is no radioactive source present near the instrument. This is due to background radiations i.e., the radiation which exist around us in the absence of deliberate radiation sources. Natural sources of background radiation include:

> **FOR YOUR INFORMATION**
> Background radiation refers to radiation that is always present. It comes from sources such as the Sun, space, soil, living organisms, medical procedures and the materials used in buildings.

cosmic rays - radiation that reaches the Earth from space,
rocks and soil - some rocks are radioactive and give off radioactive radon gas,
living things - plants absorb radioactive materials from the soil and these are passed onwards through the food chain.

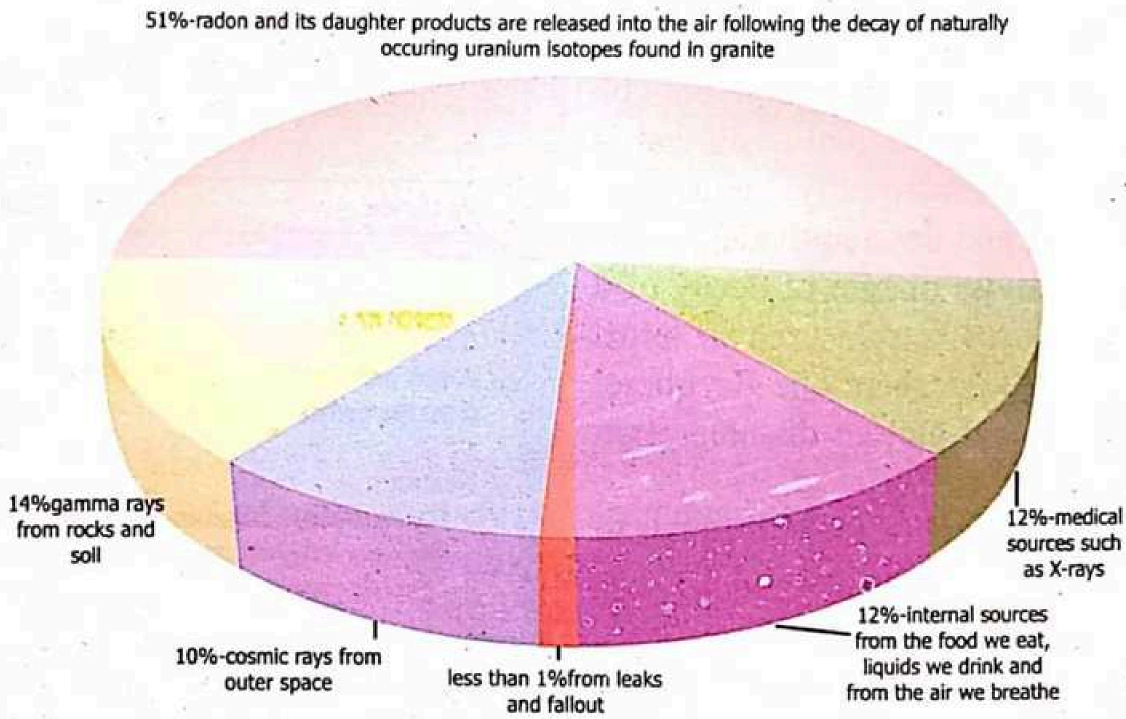
There are both low and high level background radiations which are due to the natural sources. Radiation exposure is a measure of the ionization of air due to ionizing radiation from high-energy photons (i.e. X-rays and gamma rays).

Instead of natural sources, the radiations exposure is also due to the human activities, such as the radiations from some materials used in building which contains some radioisotopes.

The X-rays and other radiation used in the medical diagnosis and therapeutic, radiation from burning of toxic materials or fuel in industries etc. are also sources of background radiation.

Low level background radiations from natural sources are being considered to be harmless, whereas, high level radiation exposure causes damage to any material, including the material that compose our bodies. Radiation damage is the effect of ionizing radiation on living things i.e. humans, animals and plants. Radiation exposure to even small amounts over a long time, raises your risk of cancer. One of the most common types of radiation damage to humans is due to the ultraviolet rays in sunlight. These lead to sunburn and tanning of skin. Though most of the sun's ultraviolet rays are absorbed by the ozone in the upper atmosphere. But it has been observed that excessive release of chemicals in atmosphere such as chlorofluorocarbon (CFC) deplete the ozone layer.

If the exposure is measured in terms of Sievert (Sv), then it has been estimated that, each person experiences a background radiation dose of 1mSv in a year, a lifetime exposure would reduce our life expectancy about 40 days.



Explaining the radiation from various sources that absorbed by a person.

20.12 BIOLOGICAL EFFECT OF RADIATIONS

Under the term radiation we include (α, β particles) and electromagnetic radiations (γ-rays x-rays, UV etc). These radiations have a great interaction with living organisms. When these radiations interact with the atoms of the matter, they may cause ionization which can damage the cells of the living tissues. The degree and type of damage depend on several factors, such as; strength and energy of the radiations as well as the property of the matter. For example, alpha particles cause extensive damage but have small penetration power. The neutrons penetrate deeper, causing significant damage. Gamma rays are high energy photons that can cause severe damage.

It is a well-known fact that excessive exposure to radiations can cause very severe illness or death by a variety of mechanism including alternations of genetic material and destruction of the components in bone marrow that produce red blood cells, the different biological effects of radiations can be classified into two classes; somatic effect and genetic effect.

Somatic effect is a radiation damage to any cell such as, skin cells, lung cells etc. but the productive cells are exempted from it. Somatic effect causes of cancer or seriously alter the characteristics of specific organisms. On the other hand, the genetic effect is a radiation damage to only reproductive cells. Due to damage the genes in the reproductive cells, genetic effect causes defective offspring or mutation.

20.13 MEASUREMENT OF RADIATIONS EXPOSURE AND DOSE

The effects of radiation can be measured using interrelated units that is in terms of radioactivity, exposure, absorbed dose, and dose equivalent.

The activity or rate of decay of a radioactive source is measured in terms of Becquerel (Bq), where one Becquerel is defined as, one atomic disintegration per second.

Its larger unit is curie (ci) which equals to 3.7×10^{10} atomic disintegration per second.

Table.20.4 Relative biological effectiveness (RBE)

Radiation	RBE
X-rays	1
Gamma rays	1
Beta particles	1
Alpha particles (into the body)	10 to 20
Neutrons:	
For immediate radiation injury	1
For cataracts, leukemia and genetic changes	4 to 10

Similarly, the exposure of X-rays and γ - rays are measured in term of roentgen (R), where the quantity of radiation which produces of 2.08×10^9 ion pairs in 1 $cm^3$of dry air is known as one roentgen.

On the other hand, the effect of radiation on an absorbing body, which relates to a quantity called absorbed dose (D). It is the amount of energy absorbed from ionizing radiation per unit mass 'm' of the absorbing body, i.e.,

$$\text{Absorbed dose(D)} = \frac{\text{absorbed energy(E)}}{\text{Unit mass(m)}}$$

The SI unit of absorbed dose is gray (Gy) equals to the absorption of one joule of radiation energy per kilogram (1J per kg) of matter.

Another common unit for absorbed dose is rad. (radiation absorbed dose) 1rad = 0.01J kg^{-1} = 0.01 Gy.

The analysis shows that different kinds of radiation cause different biological effects, even if the absorbed dose is the same. For example, for the same absorbed dose, α-particles are 20 times more damaging than x-rays. This variation can be described by introducing the quality factor (QF). Some time, it is called the relative biological effectiveness (RBE) which is assigned to each type of radiation.

To measure the biological effect caused by exposure to radiation, we calculate the biologically equivalent dose (De) and it is defined as the product of absorbed dose and RBE. i.e.,

$$De = D \times RBE$$

Table.20.5 Average radiation doses from a number of common sources of ionizing radiation.

Types of Exposure	mSv
Watching television for a year	10
Radiation from nuclear power stations for a year.	10
Wearing a radioactive luminous watch for a year (now not very common)	30
Having a chest X-ray	200
Radiation from a brick house per year	750
Maximum dose allowed to general public from artificial sources per year	1000
Working for a month in a uranium mine	1000
Typical dose received by a member of the general public in a year from all sources	2500
Maximum dose allowed to workers exposed to radiation per year.	50000

The SI unit for biologically equivalent dose is Sievert (Sv)

where $1\ Sv = 1\ Gy \times RBE$

Another unit of biologically equivalent dose is rem (roentgen equivalent in men)

$$1\ rem = 0.01\ Sv$$

20.14 BIOLOGICAL AND MEDICAL USES OF RADIATIONS

Radiations are widely used in medicine, diagnostic examination, biological research and education for a range of purposes. We will explain a few of them, such as radiation therapy, diagnosis of diseases and tracer techniques.

I Radiation therapy

Radiation therapy is one of the most useful application of nuclear physics. It is used in cancer treatment as it works by destroying cancer cells and damaging a cancer cell's DNA so that it stops dividing and growing. The idea of radiation therapy is to supply enough radiation to destroy the intentional

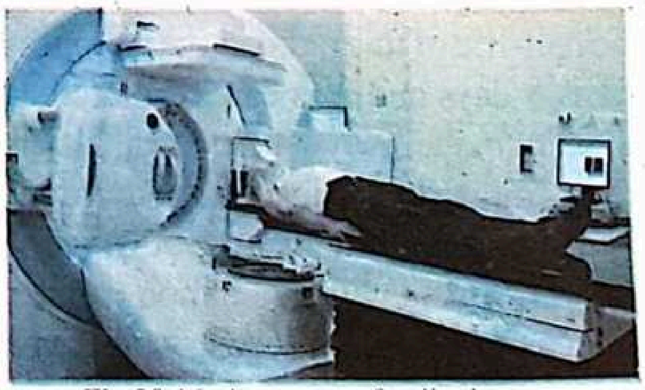


Fig.20.16. A process of radio therapy.

selective tissue such as tumors. The radiations can be applied internally or externally by using various mechanism. In case of the internal treatment, the radiation sources are placed very close to the tumor site, or sometimes they may be injected into the tumor. Similarly, in case of external treatment the sources are placed at a distance from the body and the radiation are directed toward the cancer site of the body. In this method a narrow beam of X- rays or γ- rays from Co^{60} is widely used as shown in Fig.20.16.

II Diagnosis of diseases

There are several nuclear medicine procedures for diagnostics and treatment of diseases. For example, some chemicals are absorbed by the organs, i.e iodine is absorbed by thyroid, phosphorus and strontium by bones, cobalt by liver and so many other. All these can serve as tracer, and they provide information about the functioning of a body's specific organs, or to treat diseases, for example, iodine is being used to check whether a person's thyroid gland is working properly. Similarly, radioactive isotope sodium 24 is being used to monitor the circulation of blood in the body. Liver gallbladder problems can be diagnosed using hepatobiliary iminodiacetic acid, technetium-99 is being used to diagnose cancer, embolisms (heart diseases) and other pathologies.

III Tracer techniques

Many radioactive isotopes are used as tracers, to follow the path that various chemicals take in the body. For example, it has already been explained that iodine, a nutrient needed by the human body is obtained largely through the intake of iodized salt and seafood. Nearly 70-80% of the iodine is stored in the thyroid. A small quantity of radioactive sodium iodide containing I-131 is fed or injected into the patient. The half-life of this radioactive isotopes I-131 is 8 days and it is carried directly to the point in the body where its radiation is needed in the treatment of thyroid cancer.

Radiation Counter

Similarly, radioactive sodium in the form of a solution is injected into a vein of an arm or leg and the time at which the radio isotope reaches at another part of the body is detected with a radiation counter. The elapsed time is a good indication of the absence or presence of constrictions in the circulatory system.

In the field of agriculture, tracers are being used to determine the best method of fertilizing a plant. A certain element in a fertilizer such as nitrogen can be tagged with one of its radioactive isotopes.

To measure pesticide levels, a pesticide can be identified with a radioisotope, such as chlorine-36. The tracer technique has also helped to explain the process of photosynthesis.

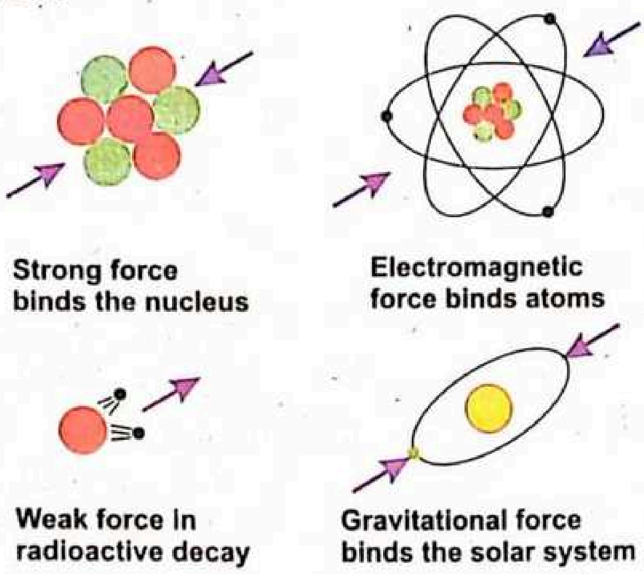
20.15 BASIC FORCES OF NATURE

All the natural phenomena or interaction can be described in terms of by the existed forces of nature. The four fundamental forces of nature are Gravitational force, Weak nuclear force, Electromagnetic force and Strong nuclear force.

> ***DO YOU KNOW***
> Without the nuclear strong force – strong interaction – there would be no atoms beyond hydrogen.

The strong nuclear force is an attractive force between nucleons, and it holds the neutrons and protons together in the nucleus. The nuclear force has a very short range and is negligible for separation distances between nucleons greater than approximately 10^{-15} m.

The electromagnetic force is the unification of the electric force and the magnetic force. These two forces were unified by Faraday and Maxwell. The electromagnetic force binds atoms and molecules together to form ordinary matter. It has a long range. It has a strength of approximately 10^{-2} times that of the nuclear force.



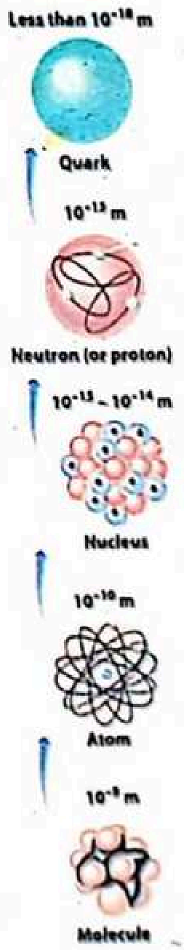
Four basic natural forces.

The weak nuclear force has a short range and it is much weaker than the strong nuclear force. Its strength is only about 10^{-5} times that of the strong nuclear force. The weak force produces instability in certain nuclei and hence it is responsible for decay processes.

The gravitational force is an attractive force that acts on all forms of masses. It is a long range force. i.e., it has a strength of only about 10^{-39} times that of the nuclear force. The gravitational force holds the planets in their orbit around the Sun, stars and galaxies together in the universe. Its effect on elementary particles is negligible.

20.16 ELEMENTARY PARTICLES AND THEIR CLASSIFICATION

The idea that all matter is composed of elementary particles has a long history. In the early age about 400 B.C., the Greek philosophers believed that matter is made of indivisible particles. They were named them as atoms. But at the end of the 19^{th} century, when the electron, proton and neutron were discovered, then these particles were considered as the fundamental constituents of matter. After 1935, the observations of several experiments pointed out that protons and neutrons are made up of extremely tiny particles called quarks. So the scientists had recognized that the protons and neutrons could not be the fundamental particles of the matter. Until the 1960s, there were a great number of subatomic particles including the two elementary particles were also discovered. Thus, according to the current theory, all matters are constructed from only the two families of particles: quarks and leptons. They are considered as elementary particles. Now all the sub-atomic particles can be classified into two classes.



The building block of matter.

Fermions (Quarks, Leptons)
Boson (gauge bosons and the Higgs bosons)

All these particles can be explained on the basis of the fundamental force between them as well as on the value of their spin momentum. e.g., the particles with half integral spin are called fermions and those with integral spin are called bosons.

The word Leptons is derived from the Greek word 'Leptos' which means thin or light. The leptons are the elementary particles and have no measurable size or internal structure. There are six types of leptons: electron, electron neutrino, muon, muon neutrino, tau and tau neutrino. For each of these, the neutrino brand carries a neutral charge, while their counterparts all have a negative charge. All leptons have weak and electromagnetic interactions. Since the spin of all leptons is equal to ½, so they are also called fermions.

Table.20.6. Classification of elementary particles.

Particle	Generic Name	Spin	Occupation Numbers
Electron	Fermion	1/2	1
Positron	Fermion	1/2	1
Proton	Fermion	1/2	1
Neutron	Fermion	1/2	1
Muon	Fermion	1/2	1
Quark	Parafermion	1/2	3
α particle	Boson	0	∞
He atom (ground state)	Boson	0	∞
π meson	Boson	0	∞
Photon	Boson	1	∞
Deutron	Boson	1	∞

Boson is a particle that has a whole number spin and it carries energy. A photon is an example of a boson as it has a spin of 1.

> *POINT TO PONDER*
>
> Which quarks and laptons are found in an atom?

Hadrons derived from the Greek word 'hadros' means strong or robust. They are not considered elementary particles. Because they are composed of two or more quarks, i.e., hadron is a composite particle. The protons and neutrons in the nucleus of an atom are composite particles, as they are composed of quarks. However, electrons orbiting the nucleus are not composite particles. Hadrons have strong nuclear interaction. There are two classes of hadron's i.e., mesons and baryons. Mesons derived from the Greek word "meso" means middle. i.e., the mass of mesons lies between the masses of the electrons and the protons. Mesons have not only weak but also strong nuclear interaction. The spin of all the mesons is equal to 0 or 1, so that they are called bosons. Charged mesons decay into electrons and neutrinos while uncharged mesons may decay to photons.

> *FOR YOUR INFORMATION*
>
> 
>
> The Big European Bubble Chamber (BEBC) at CERN, near Geneva typical of the large bubble chambers used in the 1970s to study particles produced by high-energy accelerators.

Baryons, the name baryon means heavy in Greek. All baryons have strong nuclear interaction and their spin is equal to $\frac{1}{2}$ or $\frac{3}{2}$, so that baryons are fermions. e.g., protons and neutrons are Baryons. All Baryons except protons are unstable and they decay in such a way that the end products include a proton.

SUMMARY

- **<u>Atomic number</u>:** The number of proton, inside the nucleus or the number of electrons in the allowed orbits around the nucleus is called atomic number. It is represented by 'Z'
- **<u>Mass number:</u>** The total number of protons and neutrons inside the nucleus is called atomic mass number it is represented by A.
- **<u>Nucleons:</u>** The protons and neutrons in a nucleus are collectively called nucleons.
- **<u>Isotopes:</u>** The nuclei of an element that have the same atomic number Z, but have different mass number A are called isotopes.

- **Isobars**: The nuclei that have the same mass number A, but have different the atomic numbers 'Z' are called Isobars.
- **Mass Spectrograph:** A process in which the isotopes of any element are separated is called mass spectrograph.
- **Mass defect:** The difference in mass between the mass of the nucleus and the sum of masses of its constituent nucleons is known as mass defect.
- **Binding energy:** The amount of energy that holds the nucleons in a nucleus or energy needed to break down the nucleus into its constituents is called binding energy
- **Radioactive elements:** The elements whose atomic numbers are greater than 82 are unstable and emit radiations. These elements are called radioactive elements.
- **Radio activity:** The process of spontaneous emission of radiations $(\alpha, \beta \text{ and } \gamma)$ from a radioactive element is called radioactivity.
- **Half-life:** The time in which half of the given number of radioactive nuclei decay is known as half-life of radioactive Elements.
- **Geiger – Muller Counter:** It is a device used to detect and count the ionizing particles.
- **Nuclear reaction:** A process in which a change occurs in a nucleus by approaching a nuclear particle is called nuclear reaction.
- **Fission reaction:** A reaction in which a heavy nucleus splits into two lighter nuclei with release of energy is called fission reaction.
- **Fusion reaction:** A reaction in which two light nuclei are fused to form a heavy nucleus with release of energy is called fusion reaction.
- **Elementary Particles:** The particles which made up the other sub atomic particles are known as elementary particles, these particles can be classified into two classes
  (1) Fermions (2) Bosons

EXERCISE

Multiple Choice Question.

1. Proton was discovered by
   (a) J.J Thomson (b) Chad wick (c) Ruther ford (d) Neil Bohr
2. Which of the following particle was discovered by Chadwick?
   (a) Electron (b) Proton (c) Neutron (d) Photon
3. Which of the following particle has the greatest mass?
   (a) Electron (b) Positron (c) Proton (d) Photon
4. The energy equivalent of one atomic mass unit (a.m.u) is
   (a) 1.6×10^{-19}J (b) 1.6×10^{19}J (c) 931 MeV (d) 9.31 MeV

5. The number of neutrons in the nucleus of $^{235}_{92}U$ is

(a) 92 (b) 143 (c) 235 (d) 327

6. The Nuclei $^{22}_{11}Na$ and $^{22}_{10}Ne$ are known as

(a) Isotopes (b) Isobars (c) Isotones (d) Isomers

7. Which of the following nucleus has the highest value of binding energy?

(a) $^{4}_{2}He$ (b) $^{56}_{26}Fe$ (c) $^{141}_{56}Ba$ (d) $^{235}_{92}U$

8. The binding energy per nucleon remains constant at

(a) 6 MeV per nucleon (b) 7 MeV per nucleon
(c) 8 MeV per nucleon (d) 9 MeV per nucleon

9. The nuclei have maximum binding energy per nucleon whose mass number lies between

(a) 50-60 (b) 100-150 (c) 150-200 (d) 200-235

10. Curie is the unit of

(a) Energy (b) Intensity (c) Radioactivity (d) Half life

11. The rate of decay depends upon

(a) Pressure (b) Temperature
(c) Intensity (d) Number of nuclei

12. Which rays have highest ionization power?

(a) α - rays (b) β- rays (c) γ- rays (d) X - rays

13. A radioactive element will decay when its atomic number is

(a) Equal to 82 (b) Less than 82
(c) Greater than 82 (d) does not depend upon number

14. When a radioactive element emits a β-particle the mass number of the atom will be

(a) increased by 1 (b) decreased by 1
(c) increased by 2 (d) remain the same

15. The unit of decay constant 'λ' is

(a) m (b) m^{-1} (c) s (d) s^{-1}

16. The energy released per fission reaction is

(a) 0.85 MeV (b) 28 MeV (c) 150 MeV (d) 200 MeV

17. Moderator used in a nuclear reactor is

(a) Ice (b) heavy water (c) boron rods (d) cadmium rods

18. The energy released in a fusion reaction is

(a) 0.85 MeV (b) 28 MeV (c) 200 MeV (d) 400 MeV

19. On average, the number of neutrons emitted per fission is

(a) 2 (b) 2.5 (c) 3 (d) 3.5

20. Which force has the negligible effect on elementary particles?
    (a) Strong nuclear force   (b) Weak nuclear force
    (c) electromagnetic force   (d) gravitational force
21. The particles with half integral spin are known as
    (a) Fermion   (b) Photon   (c) Bosons   (d) Meson
22. Which of the following particles have the highest value of mass?
    (a) Photons   (b) leptons   (c) mesons   (d) Baryons

SHORT QUESTIONS

1. Distinguish between atomic number and mass number.
2. What do you know about the atomic mass unit?
3. Why carbon atom is being used as a reference for atomic mass unit?
4. Differentiate between
   (a) Isotopes and Isobars   (b) Isotones and Isomers
5. Why the mass of the nucleus is always less than the sum of masses of its nucleons?
6. Describe briefly binding energy.
7. How does stability of a nucleus depend upon the binding energy per nucleon?
8. Express the relation between Curie and Becquerel.
9. What do you know about the radioactive elements?
10. Explain the conversion of parent nucleus into the daughter nucleus.
11. Why α - particles, β - particles and γ - rays are not emitted simultaneously?
12. What is the decay constant?
13. What is meant by nucleus transmutation?
14. What is the role of moderator in the nuclear reactor?
15. Why fusion reaction releases more energy per nucleon than fission reaction?
16. Why a fast reactor does not require moderator?
17. Why fusion reaction is more difficult than fission reaction?
18. What is process by which the Sun generates its energy?
19. How many forces of nature are there?
20. Distinguish between elementary and sub atomic particles.
21. What is the difference between Fermions and Bosons?
22. How many hadrons are there?

COMPREHENSIVE QUESTIONS

1. State and explain the composition of a nucleus of an atom with all its properties.
2. What do you know about the isotopes, isobars, isotone and isomers? Explain all them with examples.
3. Define mass spectrograph. How can we separate the isotopes of an element by this method?

4. What do you know about mass defect and binding energy? Discuss graphically, the relation between mass defect and binding energy.
5. State and explain radioactivity with all its properties.
6. Discuss various laws of radioactivity. Also derive equation for decay of a radioactive element.
7. State and explain the functions and working principles of the following two radiations detectors.
   1) GM-counter 2) Solid state detector
8. Discuss nuclear reaction with all its kinds.
9. State and explain nuclear fission and chain reaction. Also discuss the various parts of the nuclear reactor.
10. What is nucleus fusion reaction? Explain nuclear fusion reaction in the Sun.
11. State and explain radiation exposure and various methods used for its measurement.
12. How can we diagnose and treat the fatal diseases like cancer by using radioactive radiations?
13. What do you know about the basic forces of nature? Explain their interaction
14. Discuss the elementary particles and their classifications.

NUMERICAL PROBLEMS

1. Calculate mass defect, binding energy and binding energy per nucleon of $^{14}_{7}N$. The mass of $^{14}_{7}N$ nucleus is 14.003074u. **(0.108513u, 101MeV and 7.2MeV)**
2. The half-life of $^{92}_{36}Kr$ is 3.16 minutes. Find its decay constant. **(3.66×10^{-3} s^{-1})**
3. The half-life of radium is 1600 years. How many radium atoms decay in 1s in a 1g sample of radium? (Mass number of radium is 220 kg/k mol) **(Practically no atom of radium-220 will decay in 1s)**
4. The half-life of $^{14}_{6}C$ is 5700 years. What fraction of a sample of ^{14}C will remain unchanged after a period of five half-lives? **(0.0315)**
5. Determine the energy associated with the following nuclear reaction.

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

The relevant nuclear masses are:

$m\left(^{4}_{7}N\right) = 14.003074u$, $m\left(^{4}_{2}He\right) = 4.002603u$, $m\left(^{17}_{8}O\right) = 16.999131u$ and $m\left(^{1}_{1}H\right) = 1.007825u$

(–1.191MeV)

6. Calculate the energy associate with the given nuclear fusion reaction.

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

APPENDIX A

Some useful fundamental constants

Name	Symbol	Numerical Value
Speed of light	c	2.9979×10^{8} ms^{-1}
Charge of electron	e	1.602×10^{-19} c
Mass of electron	m_e	9.109×10^{-31} kg
Mass of neutron	m_n	1.675×10^{-27} kg
Mass of proton	m_p	1.673×10^{-27} kg
Planck's constant	h	6.626×10^{-34} J s
Planck's constant	$\hbar = h/2\pi$	1.05×10^{-34} J s
Boltzmann constant	k	1.381×10^{-23} J K^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8} W m^{-2} k^{-2}
Electron volt	eV_0	1.602×10^{-19} J
Electron rest energy	m_0c^2	0.5109989 MeV$_0$
$\frac{e}{m}$ for electron	$\frac{e}{m}$	1.7588×10^{11} kg^{-1}C
Permittivity of free space	ε_0	8.854×10^{-12} C^2 N^{-1} m
Coulomb constant	$k = \frac{1}{4\pi\varepsilon_0}$	8.99×10^{9} N m^2 C^{-2}
Permeability of free space	μ_0	$4\pi\times10^{-7}$ wb A^{-1} m^{-1}
Atomic mass unit	a.m.u (1u)	1.66×10^{-2} kg
Rydberg constant	R_H	1.097×10^{7} m^{-1}
Bohr radius	r	5.29×10^{-11} m
Electron Compton wavelength	h/m_ec	2.43×10^{-12} m

APPENDIX B

Particles of Atomic Physics

Particle	Charge	Mass (electron mass)	Mean Life (sec)
Electron	−e	1	Stable
Proton, $_1H^1$	+e	1,836	Stable
Antiproton	−e	1,836	
Neutron, $_0n^1$	0	1,837	~1,000
Antineutron	0	1,837	
Positron	+e	1	Stable
α-Particle, $_2He^4$	+2e	7,270	Stable
Duerron, $_1D^2$ or $_1H^2$	+e	3,630	Stable
Photon, γ	0	0	Stable
Neutrino, ν	0	0	Stable
Antineutrino	0	0	
Mesons:			
Mu meson, $\mu^{\pm}$	±e	207	$(2)(10^{-6})$
Pi meson, $\pi^{\pm}$	±e	273	$(2)(10^{-6})$
Pi meson, π^0	0	264	$\sim 10^{-16}$
Tau meson, $\tau^{\pm}$	±e	965	$\sim 10^{-9}$
Kappa meson, $\kappa^{\pm}$	±e	920 – 960	$\sim 10^{-9}$
Theta meson, θ^0	0	965	$(1.5)(10^{-10})$
Theta meson, $\theta^{\pm}$	±e	955	$\sim 10^{-9}$
Hyperons:			
Λ^0	0	2,180	$(3)(10^{-10})$
Σ^+	+e	2,300	$\sim 10^{-10}$
Σ^-	−e	2,300	$\sim 10^{-10}$
Ξ^-	−e	2,600	$\sim 10^{-10}$

GLOSSARY

Alternating Current	A current which changes periodically with time both in positive and negative directions.
A.C-Circuit	An electrical network which is powered by A.C Source.
A.C Generator	A device which converts mechanical energy into electrical energy
A.C Motor	A device which converts electrical energy into mechanical energy.
Ammeter	A device which is being used for the measurement of electric current.
Amorphous	The solids whose atoms, molecules and ions are not arranged in a regular manor.
Ampere	It is the SI unit of electric current.
Artificial Radioactivity	The emission of radiations or energy during the nuclear reaction either fission or fusion.
Atomic Number	The number of protons in the nucleus of an atom.
Avalanches	A process that the particles ionize the molecules of the gas.
Annihilation of matter	A process in which two anti-particles disappear to form two γ-rays photon.
Avometer	A multimeter which is being used for the measurement of current, voltage and resistance.
Absorb dose	The quantity of radiation absorbed per unit mass.
Back emf	The emf that induces during the rotation of motor and it opposes the applied emf.
Becquerel	It is the SI unit of radioactivity.
Binding energy	The energy which keeps the nucleons in a nucleus or the energy which breaks down the nucleus into its constituent protons and neutrons.
Blackbody	A body which absorbs all the radiations of all wavelength that fall on it and vice versa.
Black body radiation	Emission of radiation from a black body.
Baryons	A class of hadrons. These are heavy particles and their spins are half.
Bohr's radius	The radius of the first orbit of hydrogen atom.
Bosons	Those particles whose spin is a multiple integer e.g. electrons, protons etc.
Bulk Modulus	The ratio of volumetric stress to volumetric strain

Brittle substance	The solids which break down just passing the elastic limits.
Band Theory	Discrete energy levels of atoms that overlap to one another in the form of bands. These bands explain insulators, conductors and semiconductors of solids.
Capacitive reactance	Resistance of a capacitor is called capacitive reactance.
Capacitor	A device which is being used for storage of charges.
Chain reaction	A series of nuclear fission reactions initiated by a single neutron with ${}^{235}_{92}U$.
Charges	A property of a body such that attracts or repels the other charged particles.
Choke	It is a coil (inductor) that controls the current in an A.C. circuit.
Circuit	An electrical network which consists of a source of emf with a number of components connected across it.
Current	The rate of flow of charges.
Coherent light	The light with same frequency and phase.
Compass needle	A device which detect the presence of magnetic field.
Compressional strain	When a change occurs in a body in its volume due to deforming force then its corresponding strain is compressional strain.
Compton effect	An interaction of x-rays photon with an electron at rest, where photon transfer a fraction of its energy to the electron. So the energy of photon is decreased.
Compton shift	The wavelength of photon is increased when it interacts with an electron at rest.
Conservation of charges	In an isolated system, the total amount of charges remains constant.
Coulomb	It is the SI unit of charge.
Coulomb's law	The electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of charges and inversely proportional to square of the distance between them.
Crystalline solids	The solids whose atoms, molecules and ions are arrange in a regular manner.
Curie	It is the unit of radioactivity.
Collector	It is a terminal of the transistor which collects the charge carries.

Conductor	The solids which allow the passage of current through them.
Conduction	The transfer of electric effect through a medium.
Conductance	The reciprocal of resistance.
Conductivity	The reciprocal of resistivity.
Conventional current	The current that flows from positive to negative terminal of the battery.
Critical mass	The minimum mass of a fissile material that will sustain a chain reaction.
Critical temperature	The temperature at which the resistance of the conductor becomes zero.
Curie Temperature	The temperature at which the ferromagnetic materials become paramagnetic materials.
Direct current	The current that flows in one direction with constant amplitude.
D.C. generator	A device which converts mechanical energy into direct current.
Daughter nucleus	A nucleus left behind after a decay process
Davisson-Germer experiment	An experiment whose result verifies the de-Broglie's hypothesis.
Decay constant	The probability per unit time of the radioactive decay of an unstable nucleus.
Deuterium	It is an isotope of hydrogen whose nucleus contains a single proton and a single neutron.
Dielectric	The insulating medium in an electric field is called dielectric.
Diffraction	The bending of light through a slit or through a crystal.
Diffusion	The flow of majority charge carries toward the junction.
Diode	A semiconductor device which allows the current in one direction.
Drift velocity	The average velocity of moving charges through a conductor.
Depletion region	The region at the PN-junction where the majority carriers recombine to one another.
Diamagnetic substance	The materials whose resultant magnetic moment of their atoms is zero.
Ductile substance	The substances which have ability (property) to roll like a wire.

Eddy current	The current that induces during the rotation of motor or generator.
Electrocardiography	The instrument that records the voltage pulses of heart.
Electrocardiogram	The recorded heart pulses pattern on a paper.
Elastic limit	The limit of deformation where the body comes back to its original position.
Electric field	The region around a charged particle in which another charged particle can experience force of attraction or repulsion.
Electric intensity	Electric force acting per unit charge at a point in an electric field.
Electric dipole	Two charges of same magnitude but of different signs separated by a small distance.
Electric potential	Work on a charge against the direction of electric field.
Electromagnetic waves	The wave which consists of electric and magnetic fields which are oscillating at right angle.
Electron volt	It is the unit of electrical energy.
Electric flux	The number of electric lines of force passing through an area held perpendicular.
Electrostatic force	It is a force of attraction or repulsion between two point charges.
Electrolytes	It is the ionized liquid.
Electromotive force	The work on the charges inside the source.
Electron microscope	A device which is used to get a highly magnified and resolved image of biological and non-biological specimens. It uses a beam of acceleration electrons as a source of illumination instead of light.
Endothermic reaction	A nuclear reaction in which the energy is absorbed
Exothermic	A nuclear reaction in which the energy is released.
Ether	An ideal (hypothetical) medium
Excited state	The orbit in which electron is excited and it cannot revolve too long in it.
Exponential charging	When a capacitor or other device is not charging with the same rate.
Exponential decay	When radioactive elements decay with different rates.
Electric polarization	The insulator in an electric field, such that the molecules of the insulator become in form of dipole.

Emitter	It is an electrode of a transistor which supplies the charge carriers.
Extrinsic semiconductor	When an impurity of either 3[rd] or 5[th] group is added into pure semiconductor.
Farad	It is the SI unit of capacitance
Faraday law of induction	The induced emf is directly proportional to the rate of change of magnetic flux.
Fermions	The elementary particles whose spin are half.
Ferromagnetic material	The solids which show a strong magnetic moment.
Fission reaction	A process in which a heavy nucleus is splited into small nuclei with releases of energy
Frame of reference	Co-ordinates system which is being used to describe the relative motion of body.
Forbidden gap	The gap between valance and conduction bands.
Galvanometer	It is an electrical device which detects a small current.
Gauss's law	The total flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times of the total charges enclosed in it.
Gaussian surface	The surface which encloses the charges.
Geiger-Muller counter	It is a device which detects and counts ionized particles.
Germanium	It is a semiconductor material. The barrier potential for Germanium is 0.3v.
Generator	A device which converts mechanical energy into electrical energy.
Gravitational force	The force of attraction between two masses.
Hadron	Massive fundamental particle of matter.
Half life	A direction of time in which half number of atoms of radioactive elements decay
Heisenberg uncertainty principle	There is great uncertainty in the accurate and simultaneous determination of position and momentum of a particle.
Hook's law	Within elastic limit, the applied stress is directly proportional to the strain.
Hydrogen atom	The simplest atom which contains a single proton and a single electron.
Henry	It is the SI unit of inductance

Hysteresis	The lagging of magnetic field density 'B' behind magnetizing field 'H' in the process of magnetization or demagnetization.
Hysteresis loop	A closed path which is obtained due to the process of magnetization or demagnetization.
Impedance	The combined opposition of resistor, capacitor and inductor to flow of current in a circuit.
Induced emf	The emf that induces due to the changing of magnetic flux.
Inductance	The phenomenon in which changing current in a coil produces on emf in it.
Inductive reactance	The opposition of an inductor.
Inductor	It is a coil which stores magnetic potential energy.
Inertial frame of reference	A frame of reference which is at rest or moving with uniform velocity.
Internal resistance	The resistance inside the source. It is due to the electrolyte.
Ionization energy	The energy that ionized the atom.
Ionization potential	The potential which provides the ionization energy.
Isotopes	The nuclei that have same atomic number but different mass number.
Isobar	The nuclei which have same mass number but different atomic number
Isotones	The nuclei that have the same number of neutrons.
Isomers	The nuclei have same atomic number and same mass number but their half-lives are different.
Intrinsic semiconductor	A pure form of semiconductor (Ge & Si)
Junction diode	When P and N types semiconductors are combined such that there is junction between them.
Kirchhoff's current law	The sum of currents flowing towards the node is equal to sum of currents flowing away from the node.
Kirchhoff's voltage law	The total voltage drops across the closed loop is equal to zero.
LASER	Light amplification by stimulated emission of radiation: It is a coherent and monochromatic intense light.
Length contraction	The length of a moving object is decreased.
Leptons	Light fundamental particles of matter.
Load resistance	A resistance in the output of the circuit to check the presence of current.

Lenz's law	The direction of induced emf is always opposite to the action of emf.
Loop analysis	It is based upon Kirchhoff's voltage law
Magnetic domain	The volume in which the magnetic moments of a group of atoms are aligned in the same direction.
Magnetic field	The region around a magnet or region around the current carrying a conductor.
Magnetic flux	The magnetic lines of force passing through area held perpendicular
Magnetic flux density	The magnetic lines of force passing through a unit area.
Mass number	The number of protons and neutrons in the nucleus of an atom.
Mass defect	The mass of the nucleus is always less than the sum of masses of its separated nucleon
Mass-spectrography	A process in which the isotopes of an element is separated.
Mass variation	The increase in mass of a body in its velocity.
Mesons	The fundamental particle of matter which has an intermediate mass.
Metastable state	It is a 2nd energy level in a laser and the life time of electron in it is 10^{-3}s
Metal detector	A device which detects the hidden metal.
Moderator	The materials used to slow down the fast neutrons for fission reaction.
Motional emf	The emf that induces due to the motion of a conductor in a magnetic field.
Modulus of elasticity	The ratio between stress to strain
Mutual inductance	A phenomenon in which a changing current in one coil produces emf in the other coil.
Monochromatic light	The light that have a same wavelengths
Natural radioactivity	The simultaneous emission of radiation from radioactive elements
Non-ohmic devices	The devices which do not obey the ohm's law
Node-analysis	It is based upon Kirchhoff's current law
Nuclear reaction	The interaction of nuclear particles with a nucleus
Nuclear force	It is a force between nucleon
Nuclear transmutation	A process in which a change occurs in a nucleus

N-type semiconductor	When an impurity of 5^{th} group is added into semiconductor material
Ohm's law	The applied voltage across the conductor is directly proportional to the current at constant temperature.
Ohm	It is the SI unit of resistance.
Ohmic devices	The devices which obey ohm's law
Parent nucleus	The original nucleus without any decay from it.
Photo cell	A device which converts light energy into electrical energy.
Phase angle	The angle between voltage and current in case of alternating current.
Phasor diagram	The graph between voltage and current in case of alternating current.
Photo electric effect	A phenomenon in which photo electrons are emitted from the metal surface due to light falling on it.
Photon	It is a small packet of energy and it behaves as a particle which is moving with a speed of light.
Planck's law	The energy of photon/quanta is directly proportional to its frequency
Plasma	A fourth state of matter e.g. ionized gas
Population inversion	When a large number of electrons are accumulated in the excited state than the ground state of a laser.
Positron	Anti-particle of electron
Potential difference	The work on a charge to displace against the direction of electric field.
Power factor	The ratio between real power to apparent power.
Projectiles	The nuclear particles i.e. proton, neutron, photon and α-particles which exist in a nucleus of an atom and they are known as projectiles.
Pair production	The conversion high energy γ-rays photon into electron and positron pair.
P-type semiconductor	When an impurity of 3^{rd} group is added into a pure semiconductor material
Plasticity	When permanent change occurs in a body by deforming force.
Polymeric solids	The solids whose atoms, molecules or ions are arranged neither like crystal nor like amorphous
Potentiometer	A device which measures the potential difference

Proportional limit	The limit in which the stress is propositional to the strain.
Protium	It is an ordinary hydrogen which contains a single proton in its nucleus.
Quarks	Quarks are the basic building block of matter
Q-values	It is the amount of energy which is released or absorbed in a nuclear reaction.
Resistance	Opposition to the flow of charges
Resistivity	The resistance of one metre cube of the conductor.
Roentgens	It is the unit of exposure of x-rays.
Reverse bias	When P-type is connected with negative terminal and N-type with positive terminal of the battery.
Radioactive elements	The elements whose atomic number are greater than 82 and they are unstable.
Rectification	A process in which A.C is converted into D.C by using diode.
Rem	It is the unit for biologically equivalent
Rheostat	It is a variable resistance used to regulate the current in the given circuit.
Self-inductance	The phenomenon in which emf is induced in the same coil in which the current is changing.
Semiconductor	Semiconductor materials have dual characteristics. They behave as insulators at low temperature as well as conductor at high temperature.
Shearing modulus	The ratio between shearing stress to shearing strain.
Shearing strain	When a change occurs the shape of the body.
Sinusoidal waves	These waves are either sine waves or cosine waves.
Solar cell	A device which converts solar energy into electrical energy.
Spectrum	When visible light is split into its seven continent colours. The group of seven colours is called spectrum.
Spontaneous emission	The emission of photon during the transition of electron from higher orbit to lower orbit.
Stefan-Boltzmann law	The energy emitted from a blackbody is directly proportional to the fourth power of its absolute temperature.
Step down transformer	When $N_P > N_S$ then $V_P > V_S$
Step up transformer	When $N_S > N_P$ then $V_S > V_P$

Stimulated emission	The emission of photon due to the transition of electron from metastable state to the ground state.
Stopping potential	The applied negative potential which is equal to the K.E of photo electrons.
Strain	A change that occurs in a body in its length, volume or shape.
Stress	The applied force or deforming force per unit area of the object.
Superconductor	The conductor whose resistance is zero at low temperature.
Sievert	It is the SI unit of equivalent dose.
Spectroscopy	The study of wavelengths of the spectral lines of the spectrum.
Tensile strain	The change in length per unit original length of a body.
Tesla	It is the SI unit of magnetic field density.
Thermocouple	A process in which e.m.f. is produced by heat.
Threshold frequency	The minimum value of frequency of light at which the photoelectrons are emitted from metal surface.
Transistor	A semiconductor device which changes a small input A.C signal into a large A.C signal.
Transmutation	A nuclear reaction in which a change occurs in the nucleus
Tritium	It is the isotope of hydrogen which contains two neutrons and a single proton in its nucleus.
Ultimate strength	The limiting stress, where the body reaches to its breaking point.
Volts	The SI unit of potential difference.
Voltmeter	An electrical device which is being used for the measurement of voltage.
Work function	The minimum energy required for emission of photo electrons from a metal.
Wien's displacement law	The wavelength having maximum intensity in the emitted radiation is inversely proportional to the temperature.
Wheat stone bridge diagram	An electrical network in a diamond shape used for the measurement of unknown resistance.
Wilson cloud chamber	A device that detects the ionized particles
Young's modulus	The ratio between tensile stress and tensile strain.

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قومی احتساب بیورو بلوچستان

According to the National Curriculum 2006 under the National Textbook and Learning Material Policy 2007. N.O.C No. 1137-41/CB Dated 22/07/2020. Office of the Director Bureau of Curriculum & Extension Centre, Balochistan, Quetta.

قومی ترانہ

پاک سَرزمین شاد بَاد کِشورِ حَسِین شاد بَاد
تُو نشانِ عزمِ عالی شان اَرضِ پاکِستان
مرکزِ یقین شاد بَاد
پاک سَرزمین کا نِظام قُوّتِ اُخُوّتِ عوام
قوم، مُلک، سَلطنت پائندہ تابندہ باد
شاد بَاد منزلِ مُراد
پرچم سِتارہ و ہِلال رہبرِ ترقّی و کمال
ترجمانِ ماضی شانِ حال جانِ اِستقبَال
سَایۂ خدائے ذُوالجلال

Serial No. Code No. Phy-XII/468(NP-2007)2020

Year	Edition	Copies	Price
2021	First	15000	244/-