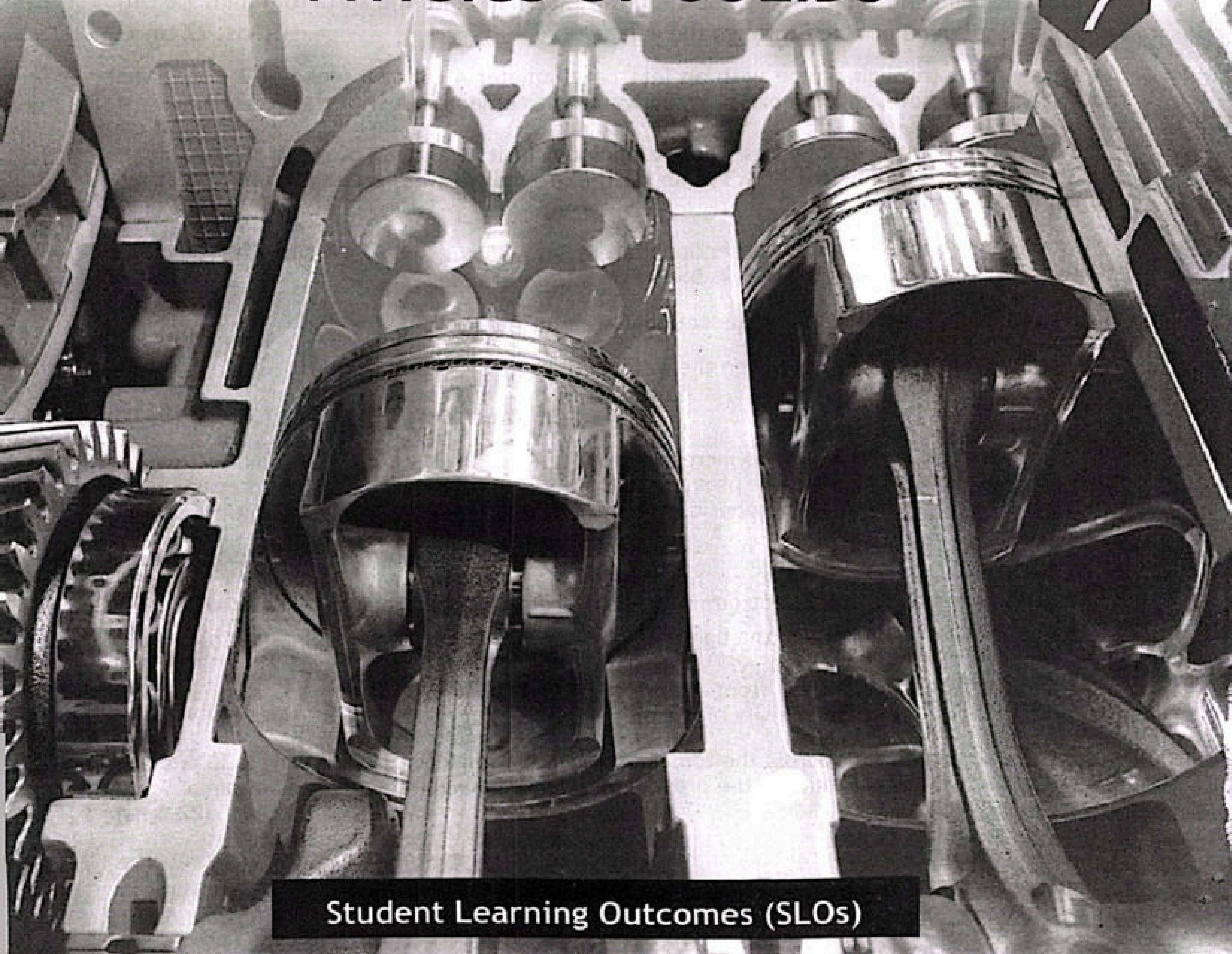
PHYSICS OF SOLIDS

UNIT 7



Student Learning Outcomes (SLOs)

The students will:

- Distinguish between the structures of crystalline, glassy, amorphous and polymeric solids.
- Describe that deformation of solids in one dimension [that it is caused by a force and that in one dimension, the deformation can be tensile or compressive].
- Define and use the terms stress, strain and the Young's modulus.
- Describe an experiment to determine the Young modulus of a metal wire.
- Describe and use the terms elastic deformation, plastic deformation and elastic limit.
- Justify why and apply the fact that the area under the force-extension graph represents the work done.
- Determine the elastic potential energy of a material [that is deformed within its limit of proportionality from the area under the force-extension graph. Also state and use $E_P = \frac{1}{2}Fx = \frac{1}{2}kx^2$ for a material deformed within its limit of proportionality].

Materials have specific uses depending upon their properties and characteristics, such as hardness, ductility, brittleness, malleability and response to the applied pressure. What makes steel hard and lead soft? It depends on the structure, the particular order and the bonding of atoms in a material. This clue has made it possible to design and create materials with unique properties for their use in the modern technology.

7.1 CLASSIFICATION OF SOLIDS

Solids are the materials which are incompressible and have a fixed shape and volume. On the basis of atomic arrangement (the structure), solids may be classified into three types: crystalline, polycrystalline (polymeric) and amorphous (glassy) solids.

7.1.1 Crystalline Solids

In solids, particles are closely packed together but their packing may have different patterns. The arrangement of particles can be studied by X-rays diffraction technique.

Crystalline solids are those in which the constituent atoms, ions or molecules, have a regular and well-defined arrangement.

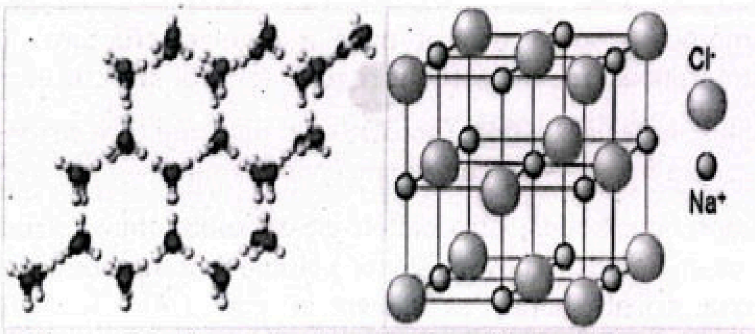


Figure 7.1: The molecular (left) and ionic (right) crystal structure of NaCl.

Molecular and ionic structures of crystals are shown in Fig. 7.1.

Examples of crystalline solids include salts (sodium chloride, and potassium chloride), metals (copper, iron and zinc), non-metals (diamond, sulphar and mica), ionic compounds (NaCl and copper sulphate), ceramic (zirconium), sand and quartz. Crystalline solids have sharp melting points. Crystalline solids are anisotropic for their physical property measurement. On cutting, they give sharp cleaves.

7.1.2 Polycrystalline Solids

The solids having structure in between order and disorder, are called polycrystalline or polymeric solids. Hence, these are partially or poorly crystalline.

A solid material made up of many small crystals with random orientation, is called polycrystalline solid.

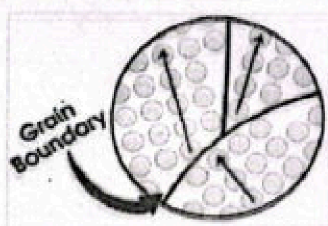


Figure 7.2: Polycrystalline structure.

The small crystals in polymeric solids are known as crystallites or grains which are oriented in different directions, and have distinct grain boundaries, as shown in Fig. 7.2. Normally, grains of the polycrystalline solids have sizes in the range of 10^2 nm to 10^3 nm. Polycrystalline materials are isotropic and exhibit the same properties in all directions.

Polymers are of two types, i.e. natural and synthetic. Polymers occur in nature include rubber with formula $(C_5H_6)_n$, resin and wood. Whereas, synthetic polymers including polythene $(C_2H_4)_n$, polystyrene $(C_8H_8)_n$, polyvinyl chloride $(C_2H_3Cl)_n$ etc. Polymers are made by the repetition of small molecules *consisting mainly* carbon, oxygen, hydrogen and nitrogen, to make large chains of molecules by a process called *polymerization*. The properties of these materials include, low specific gravity, exhibit a good strength to weight ratio, toughness, resistive to corrosion, poor conductivity (heat and electricity) and low cast.

Material Science and Space Technology

Zylon is a synthetic polymer, which is stiffer than steel, has high strength and has excellent thermal stability. It has various medical applications like artificial muscles, it is used in electric batteries and even used in Martian rover sent by NASA to the planet Mars.

7.1.3 Amorphous Solids

Amorphous solids do not have a regular structure. The term 'amorphous' means without regular form or structure.

Solid materials whose constituent particles are arranged in a random manner are called amorphous solids.

Amorphous solids, also called glassy solids, have structure like frozen liquids. They have fix volume but not definite regular geometrical shape, as shown in Fig. 7.3 (the structure of beryllium fluoride (BeF_2)). Latest technique called '*atomic electron tomography*', a type of 3-D imaging showed 85 % of atoms were in a disorder arrangement in amorphous solids but there were pockets, where a fraction of atoms found into ordered super-clusters, hence, they are called short range ordered materials. They have a range of melting temperature, i.e., if we heat a glass rod, it gradually softens into a paste like state before it becomes viscous liquid at 800 °C.

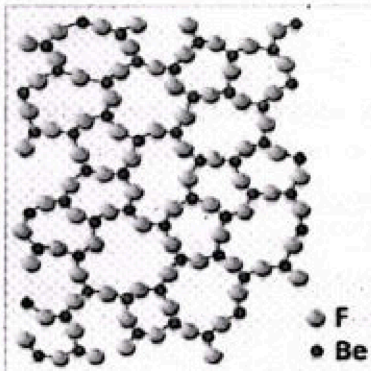


Figure 7.3: Amorphous Solid structure.

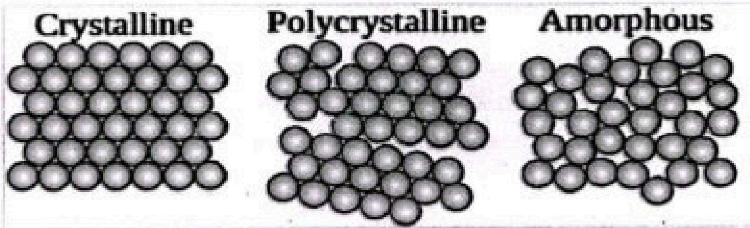


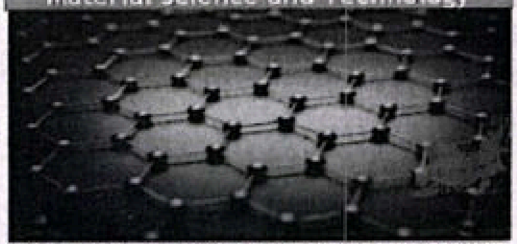
Figure 7.4: The crystalline, polycrystalline and amorphous solids.

Engineering and Technology

Atomic electron tomography (AET) has become a powerful tool for atomic scale structural characterization in 3-D and 4-D. It provides the ability to correlate structures and properties of materials at the single atom level.

Examples of amorphous solids include, glass (sometimes called super-cooled liquid), rubber, thin film systems deposited on a substrate at low temperatures, glues and many polymers. Amorphous solids can withstand higher temperatures, without affecting their *efficiency* as compared to crystalline and polycrystalline solids. If these solids are melted and then cooled slowly, they can be converted into crystalline solids. Amorphous solids are used almost in every field of life: like construction, household wares and laboratory wares, manufacturing of tyres and production of shoes. Amorphous silica is one of the best materials for converting sunlight into electricity, used in solar panels.

Material Science and Technology



Graphene, a hexagonal carbon crystalline structure, only a single atom thick, is the thinnest known material. It is the most revolutionary material to be developed in the 21st century. It is used in carbon nanotubes. In proportion to its thickness, it is the strongest material. It is an extraordinary conductor of heat and electricity and 100% transparent to light.

7.1.4 Unit Cell and Crystal Lattice

The crystal structure of solid is analyzed by using X-ray diffraction and Bragg's law. Solids are characterized by their structure, which is made up of some basic unit, that can be defined as:

A unit cell is the smallest basic portion of a crystal, which repeatedly stacked together in three dimensions, to make the entire crystal structure.

A unit cell has all the characteristics of the whole crystal. The atoms, molecules or ions which make the crystal, are called basis and are arranged in regular pattern. An imaginary geometrical structure to join the basis is called lattice. A lattice basically tells us about the basic structure of the points (basis). The crystal structure is obtained by placement of atoms on basis.

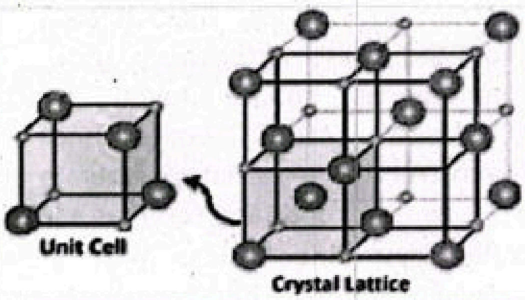


Figure 7.5 (a): Crystal lattice and unit cell.

The structure of a crystal, obtained by the repetition of the unit cell, is known as crystal lattice.

A unit cell and its corresponding crystal lattice are shown in Fig. 7.5 (a). There are three types of unit cells:

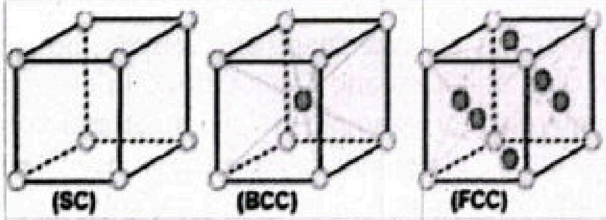


Figure 7.5 (b): Types of unit cells.

i) Simple cubic (SC) contains eight atoms at each corner. Hence, there is one atom per unit cell in SC, as shown in Fig. 7.5 (b).
ii) Body centered cubic (BCC) have eight atoms at corners and one atom at the center of body. Hence, there are two atoms per unit cell in BCC.
iii) Face-centered cubic (FCC) have eight atoms at corners and one atom at each face. Hence, there are four atoms per unit cell in FCC.

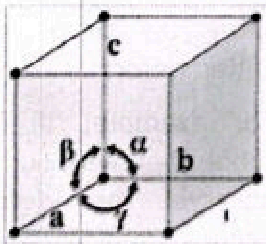


Figure 7.5 (c): Unit cell.

A unit cell has three sides for a space lattice as, (a, b, c) having three angles (α, β, γ), as shown in Fig. 7.5 (c). In terms of different relations between sides and angles of a unit cell, it can be divided into seven distinct shapes, as shown in Fig. 7.6. Some of the daily life applications of solids include:

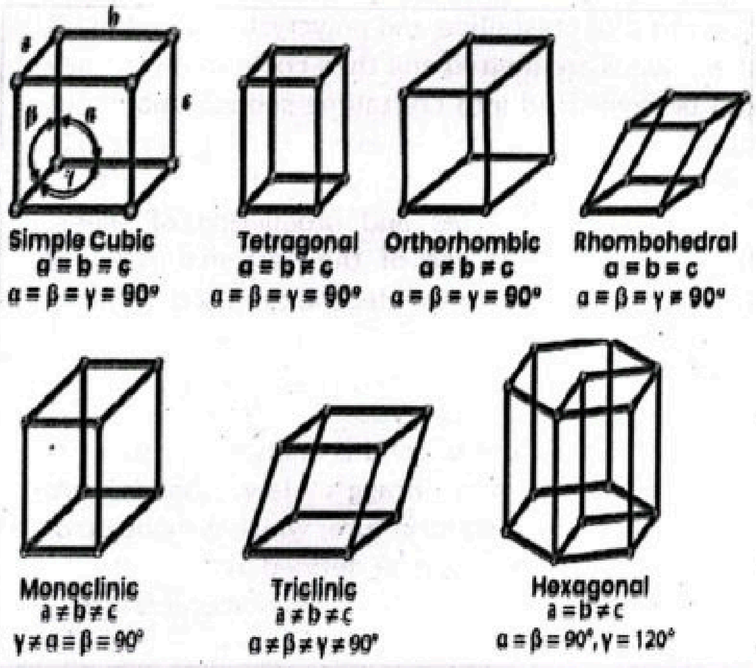


Figure 7.6: Seven shapes of unit cells.

- **Crystalline solids:** Diamond is the most decent example, which is used in jewelry and various industries. Similarly, quartz is extensively used in manufacturing of watches and clocks. Many crystalline solids are also used as a raw material in various industries.
- **Amorphous solids:** Glass is one of the most extensively used amorphous solids, found in utensils, bottles, boxes and construction material. Rubber is another widely used amorphous solid, found in tyres, footwears, ropes and serve as a material in many industries.
- **Polycrystalline solids:** Most inorganic metals, many ceramics, rocks and ice are polycrystalline, which have vast applications in our daily life.

7.2 DEFORMATION IN SOLIDS

A solid has definite shape and volume, possesses rigidity, is not compressed easily, has closely packed particles and cannot flow. In order to change the shape or size of a solid, a force is required. For example, if you stretch a spring by pulling its ends, the length of the spring increases. When you release the ends of the spring, it regains its original shape and size. To study the mechanical properties of solids, we usually check the behavior of solids under applied force. Application of force on a solid may bring change in its dimension, i.e., it deforms a solid.

The change in shape, length or volume of a solid when it is subjected to an external force is called deformation.

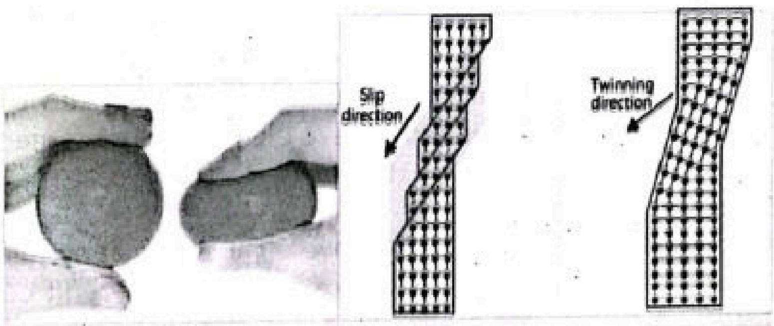


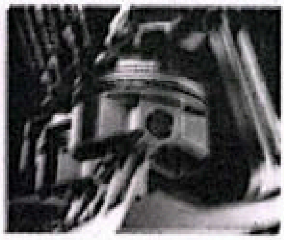
Figure 7.7: Deformation in a ball and in crystals.

For example, if we squeeze a rubber ball with our fingers, we can observe the deformation that occur. Similarly, at atomic level, a crystal subjected to applied pressure, may deform mainly in two forms, a slip of one cleave over the other or twinning of crystal, both of them are shown in Fig.

7.7. The results of mechanical tests are usually expressed in terms of stress and strain, which can be defined in terms of applied force and resulting deformation.

Auto Engineering and Technology

What makes the moving parts of engine frictionless? Due to very low coefficient of friction and shine, Teflon (an amorphous solid) is used for coating the materials used in automobiles, industries and manufacturing. Teflon coating is used in high efficiency engines to reduce the friction, as it has good efficiency even at higher temperatures. It is also used in nonstick cooking utensils and moving parts of machines.



7.2.1 Stress and Strain

The internal resistance offered by a body to resist deformation is called stress. It is a physical quantity that measures the magnitude of force causing the deformation. Stress is directly proportional to the applied force and inversely proportional to the area of cross section over which the force applied. It can be defined as:

The applied force per unit area is called stress.

Mathematically, it can be written as:

$$Stress\ (\delta) = \frac{Force\ (F)}{Area\ (A)}$$

The SI unit of stress is newton per square meter ($N\ m^{-2}$), also known as pascal (Pa) and its dimensions are $[ML^{-1}T^{-2}]$. Generally, stress can be divided into three types.

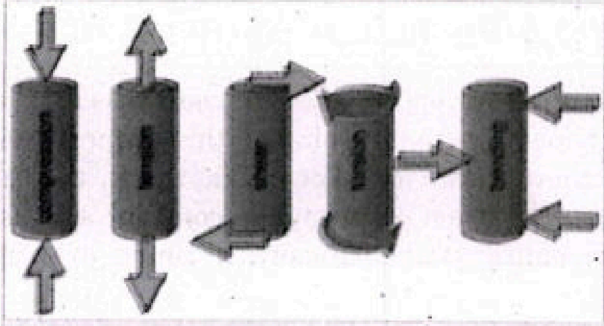


Figure 7.8: There are five types of stresses, i.e. compression, tension, shear, torsion and bending.

- The one-dimensional stretching force acting on the area of cross section of a body which produces linear deformation in it, is called *tensile stress.*
- The force which has a component acting tangentially to the area of a body produces deformation in the shape of a body, is called *shear stress.*
- The force which acts uniformly from all directions on the area producing a deformation in the volume of a body, is called *volume stress.*

For Your Information

Amorphous solids have two defining properties. They create pieces of odd, often twisted surfaces when cleaved or broken. They have poorly described patterns, when exposed to x-rays because their components are not organized in a typical sequence.

Due to action of a force on a body, a deformation is produced in it.

The quantitative measure of deformation is called the strain.

Strain can be found by the ratio of the change to the actual value of some parameter (length, shape or volume); hence it has no units and a dimensionless quantity. On the basis of stress, strain can also be subdivided into three types.

- The fractional change in length, i.e., change in length divided by the original length, is called tensile strain. It causes due to tensile stress and is given by:

$$\text{Tensile strain } (\varepsilon) = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

- The tangential force acting on an area (shear stress), produces a change in the shape of a body, called shear strain. Shear strain is given by:

$$\text{Shear strain } (\varepsilon) = \frac{\text{Displacement of sheared face}}{\text{distance from fixed face}} = \frac{\Delta X}{Y}$$

- The force acting at right angle to an area from all directions produces deformation in the volume of a body, called volume strain and can be given as:

$$\text{Volume strain } (\varepsilon) = \frac{\text{Change in volume } (\Delta V)}{\text{Original volume } (V)}$$

7.2.2 Modulus of Elasticity

After removing stress, some materials regain their original shape, length or volume; this property of materials is called '*elasticity*'. For elastic solids, the ratio of the stress to the strain is always a constant and is called '*elastic modulus*'. Mathematically, it can be given as:

$$\text{Elastic modulus} = \frac{\text{Stress}}{\text{Strain}}$$

Since strain has no units, hence modulus of elasticity has the same units as that of the stress i.e. N m^{-2} (or Pa) and dimensions as $[ML^{-1}T^{-2}]$. Depending upon type of stress, modulus of elasticity is of three types, as under:

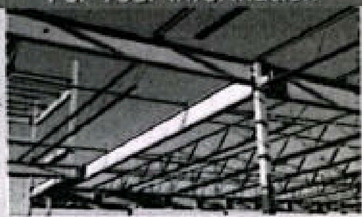
Young's Modulus: In case of linear deformation, the ratio of the tensile stress to the tensile strain is called 'Young's modulus', and is represented by 'Y'. Mathematically it can be given as:

$$Y = \frac{F/A}{\Delta L/L} \qquad \text{\_\_\_\_\_\_ (7.1)}$$

Here, 'F' is the applied force on area 'A' of the material (say a rod) of length 'L'. The force changes the length by 'ΔL', as shown in Fig. 7.9, for both increase and decrease in length.

Bulk Modulus: For three dimensional forces, the deformation occurs in all directions, as shown in Fig. 7.10. In this case, the ratio of the volume stress to the volume strain is called 'bulk modulus'. Mathematically, it is expressed as:

For Your Information



Objects can often experience both compressive and tensile stress simultaneously. Example includes a long shelf loaded with heavy books. The top surface of the shelf is in compressive stress and the bottom surface of the shelf is in tensile stress. Similarly, long and heavy beams sag under their own weight, in modern building construction, such bending strains can be eliminated with the use of L-beams.

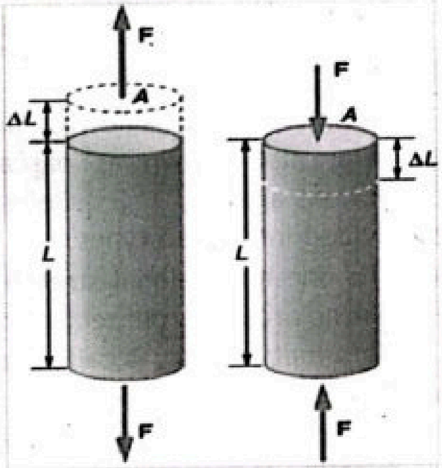


Figure 7.9: Young's modulus.

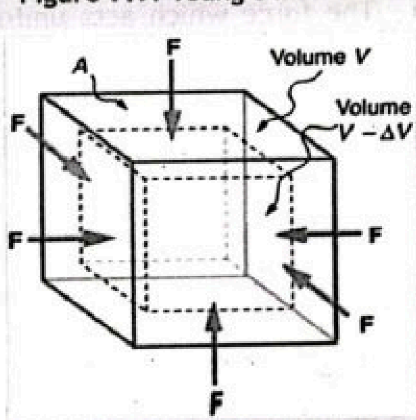


Figure 7.10: Bulk modulus.

$$B = \frac{F/A}{(-\Delta V/V)} \text{ ——— } (7.2)$$

Here, the negative sign shows the decrease in volume, but bulk modulus is always positive.

Shear Modulus: Due to tangential force, the deformation is produced in the shape of a body, as shown in Fig. 7.11. In this case, the ratio of the shear stress to the shear strain is called 'shear modulus'. Mathematically, it is expressed as:

$$S = \frac{F/A}{(\Delta X/Y)} \text{ ——— } (7.3)$$

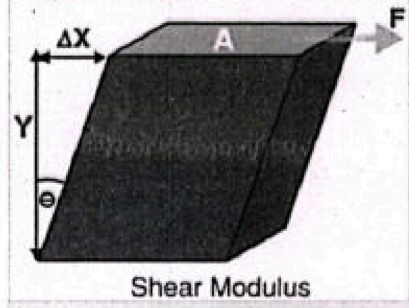


Figure 7.11: Shear modulus.

Here, strain is the ratio of the displacement of the sheared face to the height of fix face. With reference to the triangle, shown in Fig. 7.11, the shear strain it is expressed as:

$$\tan\theta = \frac{\Delta X}{Y}$$

For very small angle 'θ': $\tan\theta \approx \theta$ hence, $\theta = \Delta X/Y$. Now equation (7.3) gets the following form:

$$S = \frac{F}{A\theta} \text{ ——— } (7.4)$$

Table 7.1: Table for values of the modulus of elasticity of different materials

Material	Young's Modulus (10^{10} N m^{-2})	Bulk Modulus (10^{10} N m^{-2})	Shear Modulus (10^{10} N m^{-2})
Steel	20.0	15.8	8.0
Aluminum	7.0	7.0	2.5
Copper	12.0	12.0	4.0
Iron	19.0	8.0	5.0
Brass	1.0	14.0	3.6

Here, the angle 'θ' is called 'the angle of shear'.

Example 7.1: A Masjid's minar having area of cross-section 0.2 m^2, the upper dome's weight 10,000 N, is made up of a material, with Young's modulus of 4.5×10^{10} Pa and mass density of 2700 kg m^{-3}. Find the compressive stress at the cross-section located 3.0 m below the top of the pillar and value of compressive strain of the top 3.0 m segment of the pillar.

Given: Pillar segment height 'h' =3.0
Area of cross-section 'A' = 0.20 m^2
Density 'ρ' = 2700 kg m^{-3}
Weight of upper dome 'w_d' = 10,000 N

To Find: Compressive stress 'δ' = ?
Compressive strain 'ε' = ?

Solution: To find the mass, we have to find volume of the pillar's segment, as:

$$V = A\,h$$

$$V = (0.20\ m^2)(3.0\ m) = 0.60\ m^3$$

$$m = \rho V \Rightarrow \quad m = (2700\ kg\,m^{-3})(0.60\ m^3) = 1.60\times10^3 kg$$

The weight of the pillar segment is:

$$w_p = m\,g \Rightarrow \quad w_p = (01.60\times10^3 kg)(9.8\ ms^{-2}) = 1.568\times10^4 N$$

The normal force is the sum of weight of the upper dome and the pillar segment.

$$F = w_d + w_p \quad \Rightarrow \quad F = (1.0\times10^4 N) + (1.568\times10^4 N)$$

$$F = 2.568\times10^4 N$$

Now the compressive stress:
$$\delta = \frac{2.568\times10^4 N}{0.20\ m^2}$$

$$\delta = 128.4\ kPa$$

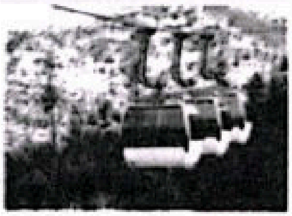
And the compressive strain:
$$Strain = \frac{Stress}{Y} = \frac{128.4\ kPa}{4.5\times10^7 kPa}$$

$$Strain = 2.85\times10^{-6}$$

Discussion: The normal force acting on the cross-sectional area is not constant along its length, but varies from the smallest value at the top to the largest value at the bottom of the pillar. So if the pillar has constant area of cross section along its lenth, the stress is the largest at the bottom. That is why the base portion of pillar has gradual increase in cross section, as can be seen from given figure.

Assignment 7.1

The suspension cable to use cable cars at Murree has an unsupported span of 3.25 km. Calculate the amount of stretch in the steel cable of diameter of 6 cm for its maximum load. The caution on weight is given by the engineers to not more than 4x10^6 N.



7.2.3 Stress-Strain Curve

To study the behavior of a material under applied stress, we use a **tensile test** machine which strains the material at a fixed linear rate and records the stress. The values are then plotted on a graph by a computer attached to the machine. Strain is plotted along horizontal axis and stress on the vertical axis. A typical stress-strain graph for a ductile material is shown in Fig. 7.12. In the initial stage of deformation, the stress increases linearly with the strain, as shown by the portion (OA) in the graph. This portion has linear proportionality between stress and strain and obeys Hooke's law. In this region, if the applied stress is *removed* the body regains its original shape.

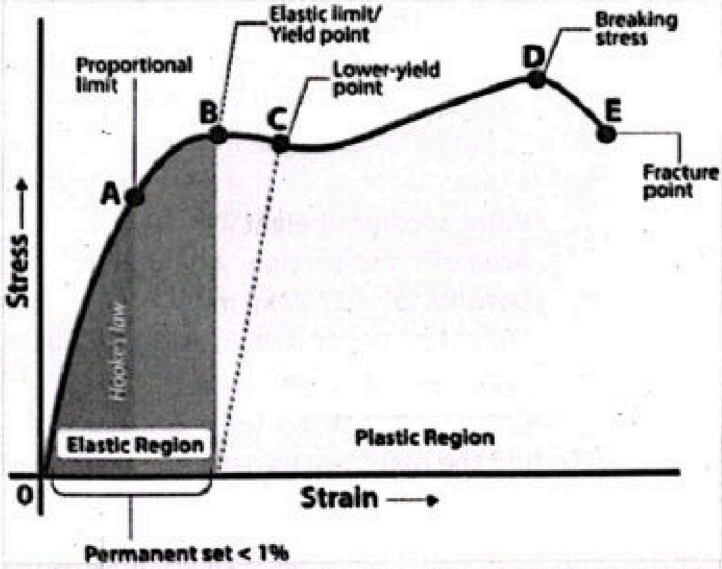
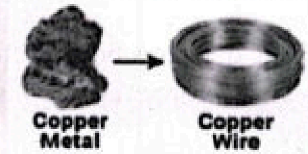


Figure 7.12: Stress-strain curve for a ductile material

Point 'A' is called the proportional limit corresponding to stress δ_p and defined as the greatest stress, which a material can endure without losing straight line proportionality between stress and strain. After crossing point 'A' the region is shown by the portion (AB) in the graph, where straight line proportionality between stress and strain ends, but still if stress is removed, the body will regain its original shape. Hence, the portion (O-B), where the body has the ability to regain its original shape, is called '*elastic region*'. The maximum limit of the stress for elasticity of a material, is called yield point or elastic limit of the material, corresponding to point 'B' in the graph whose stress can be given as δ_y. Some material has lower yield point represented by 'C'. If stress is increased beyond the yield point, the body gets permanently changed and does not recover its original shape, even if stress is removed. This behavior of materials is called 'plasticity', where if the stress is zero still strain is not zero. This region is represented by the portion after point 'B' and onward. Stress is increased further at point 'D', called ultimate tensile strength (δ_u), the maximum stress that a material can withstand. After crossing point 'D' the body breaks at point 'E' representing the fracture stress (δ_f). Substances, which undergo plastic deformation until they break are known as ductile materials. Lead, copper and wrought iron are examples of ductile materials. Other materials, which break just after elastic limit are called brittle materials, like glass and high carbon steel.

For Your Information



Metals, with high ductility such as, gold copper and steel can be drawn into long and thin wires without breaking. Gold is the most ductile material. A wire of about 2 km in length can be drawn from one gram of gold.

Example 7.2: Calculate the change in length of the upper leg bone of a soccer player, having mass of 70.0 kg, supporting 62.0 kg of his mass on this bone, assuming the bone acts as a uniform rod, 40.0 cm long and with a radius of 2.00 cm.

Given: Mass 'm' = 62.0 kg Young's modulus for bone 'Y' = 9×10^9 Pa

Radius of the bone 'r' = 0.020 m Length of the bone 'L' = 0.400 m

To Find: Change in length 'ΔL' = ?

Solution: The force is equal to the weight supported by the bone:

$$F = m\,g \qquad F = (62.0\ \text{kg})(9.80\ \text{ms}^{-2}) = 607.6\ \text{N}$$

The area of the cross-section is:

$$\pi r^2 = (3.14)(0.020\ \text{m})^2 = 1.257\times10^{-3}\text{m}^2$$

Now by using equation: $$\Delta L = \frac{F\,L}{Y\,A}$$

Putting values, we get: $$\Delta L = \frac{(607.6\ \text{N})(0.400\ \text{m})}{(9\times10^9\text{N/m}^2)(1.257\times10^{-3}\text{m}^2)} = 2\times10^{-5}\text{m}$$

Discussion: This change in length is very small, consistent with our experience that bones are rigid. Even the large forces during haevy physical activities (like weight lifting) of a person do not compress or bend bones significantly.

Assignment 7.2

Find the strain in your own leg's bone while lifting a 50 kg box. Relate your finding with the example given above. Do the strain in your leg is greater or lesser than the strain in above case? Explain the reason.

7.3 ELASTIC POTENTIAL ENERGY

As works done on bodies store some amount of energy in them, similarly work done on elastic bodies for deformation also stores energy in the body which is known as elastic potential energy.

7.3.1 Modulus of Metallic Wire

To study the work done on a body to produce strain, consider a metallic wire which can be strained by the application of force on it. Measure the length and area of cross section of the wire and connect a vernier scale to it. Take another wire as reference wire with a scale on it for precise measurement of change in length of the metallic wire upon adding the load (weight) with it, as shown in Fig. 7.13. We will find the extension in the wire with the help of vernier scale due to the increasing force on test wire. By using equation 7.1 with average force, original length, change in length and area of the metallic wire, we can find the Young's modulus of the metallic wire.

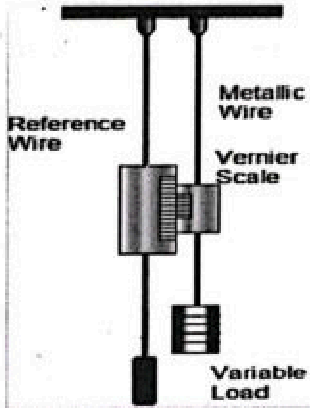


Figure 7.13: Young's modulus for metallic wire.

7.3.2 Work from Force-Extension Graph

To find the work done by the variable load on the metallic wire to produce extension, draw a graph by taking weight on x-axis and extension on y-axis. Initially, no weight is attached to the wire and it has zero extension. By continuously increasing the weight, the wire got extension, as shown in the Fig. 7.14. The area under the curve from O to A can be found by finding the area of triangle AOF, which can be given as:

$$Area = \frac{1}{2} F x \quad \text{\_\_\_\_\_} \ (7.5\ a)$$

The average force applied on the wire is given as,

$$F_{Avg} = \frac{0 + F}{2} = \frac{1}{2} F$$

We know that the work done can be found by the product of average force and displacement. Therefore, work done is:

$$W = F_{Avg}\, x \quad \Rightarrow \quad Work = \frac{1}{2} F x \quad \text{\_\_\_\_\_} \ (7.5\ b)$$

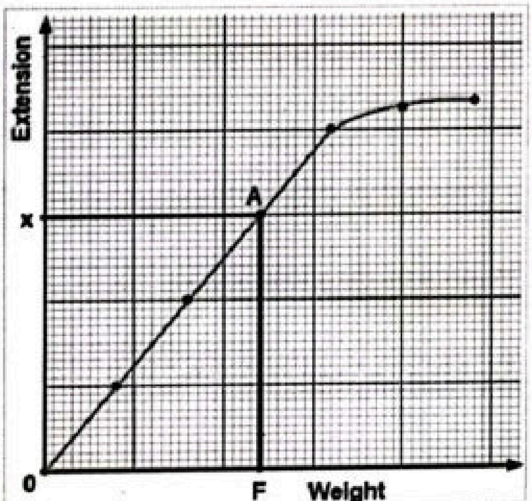


Figure 7.14: Force-extension graph.

From equations (7.5 a) and (7.5 b), it is clear that the work done during the application of a load for extension in elastic material, within elastic limit, is equal to the area under the force-extension graph.

7.3.3 Derivation of Elastic Potential Energy

The energy stored in a material due to strain produced in it is called the elastic potential energy. This energy can be calculated by using the relation for work done during extension. For an elastic material, the force can be given by the Hook's law as:

$$F = k\,x$$

The elastic potential energy in deformed material, within elastic limit is equal to the work done in producing that deformation. Hence, the potential energy of a deformed material is expressed as:

$$\text{Elastic Potential Energy} = \text{Work Done} = \frac{1}{2}F\,x$$

$$P.E_{elastic} = \frac{1}{2}(k\,x)x$$

or $$P.E_{elastic} = \frac{1}{2}k\,x^2 \quad \text{\_\_\_\_\_\_ (7.6)}$$

The energy of deformed material can also be written in terms of Young's modulus. If we use the value of force from equation (7.1) with extension 'x' then it gets the form:

$$F = \frac{YAx}{L} \qquad \because y = \frac{F/A}{x/L}$$

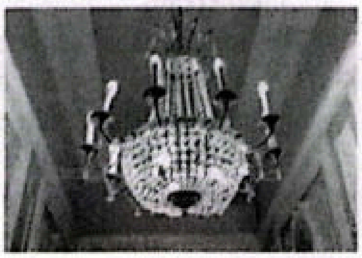
Civil Engineering

Engineers while making the suspension bridges test the materials for their elastic properties. They use some mechanical testing machines apply stress and calculate strain. This helps them to choose the best material to be used. The wires used in suspension bridges possess large strain energy and can withstand the huge load of traffic.

Where 'A' is the area of cross-section and 'L' is the original length of the wire. substituting above value of force in equation (7.5) we get:

$$P.E = \frac{1}{2}\left[\frac{YA}{L}\right]x^2 \quad \text{\_\_\_\_\_\_ (7.7)}$$

This elastic potential energy is also called the strain energy.



Example 7.3: Calculate the strain energy stored in a 1 m long steel wire connected to the roof, holding a chandelier of weight 980 N in the main lounge of your home. The diameter of steel wire is 0.004 m and the Young's modulus of steel is 2.0×10^{11} N m^{-2}.

Given: Length of steel wire 'L' = 1 m
Young's modulus for steel 'Y' = 2.0×10^{11} Pa
Diameter of the steel wire 'r' = 0.004 m
Weight of chandelier 'W' = 980 N

To Find: The change in length 'ΔL' = ?
The Strain energy stored in wire 'U' = ?

Solution: The force on the wire is equal to the weight supported by it.

$$F = W = 980\ N$$

The area of the cross-section of the steelwire is:

$$\pi r^2 = \pi \left(\frac{d}{2}\right)^2 = (3.14)(0.002\ m)^2 = 1.257\times10^{-5} m^2$$

Now by using equation (7.1):

$$\Delta L = \frac{F\,L}{Y\,A} = \frac{(980\ N)(1\ m)}{(20\times10^{10} N/m^2)(1.257\times10^{-5} m^2)}$$

$$\Delta L = 3.90\times10^{-4} m$$

Now, the strain energy U stored in the steel wire can be found by using the formula.

$$P.\ E = \frac{1}{2}\left[\frac{Y\,A}{L}\right] x^2$$

$$U = \frac{1}{2}\left[\frac{(20\times10^{10} N\ m^{-2})\ (1.257\times10^{-5} m^2)}{1m}\right](3.90\times10^{-4} m)^2$$

$$U = 0.191\ J$$

This change in length in steel wire is very small, and hence the strain energy stored in the wire is also small, which shows the suitability of material (steel) for holding heavey masses.

Assignment: 7.3

Calculate the work done in stretching a wire of length 5 m with cross-sectional area 1 mm^2. When the force applied by you on the edges of wire produces change in the length of wire by 1 mm. Young's modulus of the wire is 2x10^{11} Pa.

SUMMARY

- Solids can be classified into crystalline, polycrystalline and amorphous.
- **Crystalline solids** have a regular three dimensional structure. Polycrystalline solids have a structure that is intermediate between order and disorder, while amorphous solids have an irregular structure.
- **A unit cell** is the basic building block of the crystalline solids whose repetition in all directions makes a whole crystal.
- **Stress** is the force applied on unit area of a solid and the deformation produces due to it is called strain.
- **The modulus of elasticity** of a solid is a constant, and is the ratio of stress to strain.
- **Tensile stress** produces a change in length and the ratio of this change in length to the original length is called tensile strain. The ratio of the tensile stress to the tensile strain is called Young's modulus.
- **Volume stress** produces a change in volume of a solid and the ratio of this change in volume to original volume is called the volume strain. The ratio of the volume stress to the volume strain is called bulk modulus.
- **Shear stress** produces change in shape and the ratio of the sheared face to the fix face is called shear strain. The ratio of the shear stress to the shear strain is called shear modulus.
- **Strain energy** of a material is represented by the area under the stress-strain curve.
- **Elastic materials** break shortly after elastic limit, while those who break after ultimate tensile strength, are called plastic materials.

EXERCISE

Multiple Choice Questions

Encircle the correct option.

1) Glue is an example of:

A. crystalline solid B. amorphous solid C. poly-crystalline solid D. ductile solid

2) Which of the following properties is generally exhibited by amorphous solids?

A. anisotropy B. glass-transition C. orderliness D. geometry

3) Silicon is found in nature in the forms of:

A. SC structure B. BCC structure C. FCC structure D. network structure

4) One end of a uniform wire of length 'L' and of weight 'W' is attached rigidly to a point in the roof and a weight 'W_1' is suspended from its lower end. If 'S' is the area of cross-section of the wire, the stress in the wire at a height '3L/4' from its lower end is:

A. $\frac{W_1}{S}$ B. $\frac{W_1+(W/4)}{S}$ C. $\frac{W_1+(3W/4)}{S}$ D. $\frac{W_1+W}{S}$

5) On suspending a weight 'mg', the length 'l' of an elastic wire with area of cross-section 'A' becomes double of its initial length. The instantaneous stress on the wire is:

A. $\frac{mg}{2A}$ B. $\frac{mg}{A}$ C. $\frac{2mg}{A}$ D. $\frac{4mg}{A}$

6) The diameter of a brass rod is 4 mm and Young's modulus of brass is 10×10^{10} N m^{-2}. The force required to stretch by 0.1% of its length is:

A. 36π N B. 40π N C. 360π N D. 400π N

7) A cantilever beam has a load at the free end. The strain energy in the beam is due to:

A. bending B. shearing C. stretching D. elongation

8) When a pressure of 100 atmospheres is applied on a spherical ball of rubber and its volume reduces to 0.01%. The bulk modulus of the material of rubber in dynes cm^{-2} is:

A. 1×10^{12} B. 10×10^{12} C. 20×10^{12} D. 100×10^{12}

9) The property of a material to store strain energy is called:

A. ductility B. hardening C. resilience D. stiffness

10) Gradient of the force-extension graph has:

A. increasing value B. decreasing value C. variable value D. constant

Short Questions

Give short answers to the following questions:

7.1 Why do most of the solids prefer to be in the crystalline state? What is glass transition in amorphous solids?

7.2 Why the window glasses of old buildings show milky appearance over time?

7.3 In the elastic body, which one is more fundamental: the stress or the strain? Explain.

7.4 Can tensile stress produce volumetric strain? Explain.

7.5 What does the slope of the force-extension graph represents? Explain.

7.6 Is strain energy always positive or can it be negative? Justify your answer.

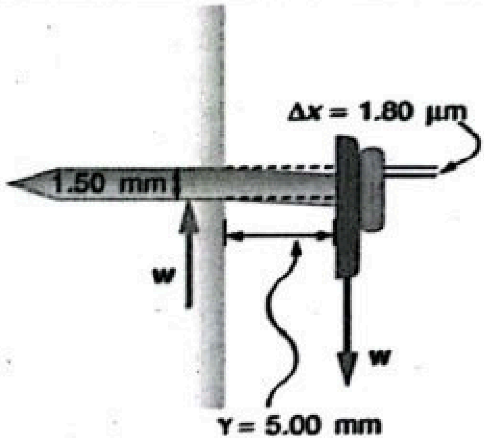
7.7 On which types of loads the strain energy depends? Explain.
7.8 Why are suspension bridges assign a period of use? Is it dangerous to use the bridge after that period?
7.9 If we want to break a wire, why we use repeated bending? How does it affect the wire?
7.10 What is the significance of modulus of elasticity?
7.11 What is meant by crystal lattice.

Comprehensive Questions

Answer the following questions in detail.
7.1 Explain the three types of solids: crystalline, poly-crystalline and amorphous solids.
7.2 What is unit cell? Explain its types.
7.3 Explain the terms: stress, strain and modulus of elasticity.
7.4 Draw and discuss the stress-strain curve for a ductile material.
7.5 Explain plastic and elastic deformation by drawing stress-strain curve.
7.6 Justify that 'the area under the force-extension curve is equal to the work'.
7.7 Derive a relation for the elastic potential energy of a material.
7.8 Describe an experiment to determine the Young's Modulus of metallic wire.

Numerical Problems

7.1 Calculate the mass of picture frame hanging from a steel nail ($Y_S = 20 \times 10^{10}$ Pa), which is sufficient to produce a force providing shear strain, as shown in figure here. Given that the nail bends only 1.80 μm. Also discuss the result.
(Ans: 5.2 kg, Discussion: This is a fairly massive picture, and it is impressive that the nail flexes only 1.80 μm, amount undetectable to the unaided eye, which shows strength of the material of nail i.e. the steel.)



7.2 A 5 m long aluminum wire ($Y_A = 7 \times 10^{10}$ Pa) of diameter 3 mm, supports a 40 kg mass. To achieve same elongation in a copper wire ($Y_C = 12 \times 10^{10}$ Pa) of the same length under the same weight, find the diameter of copper wire. **(Ans: 2.29 mm)**

7.3 A 4 kg mass is hanging with the help of a 1.5 m long steel wire ($Y_S = 20 \times 10^{10}$ Pa) another mass of 6 kg is hanging from the 4 kg mass using 1 m long brass wire ($Y_B = 1.0 \times 10^{10}$ Pa). Both wires have diameter 0.25 cm. Calculate the total elongation in the whole system of wires, also calculate the total strain energy. **(Ans: 2.79×10^{-4} m, 11.02 mJ)**

7.4 A rod with a cross-sectional area of 90 mm^2 and a length of 3 m. If a stress of 300 MPa is applied to stretch the rod, then find the strain energy if Young's modulus of the rod is 200 GPa. **(Ans: 12.15 J)**

7.5 A force of 500 N is applied to one end of a cylindrical steel rod of diameter 50 cm. What is the tensile stress? **(Ans: 2.5×10^5 N m^{-2})**