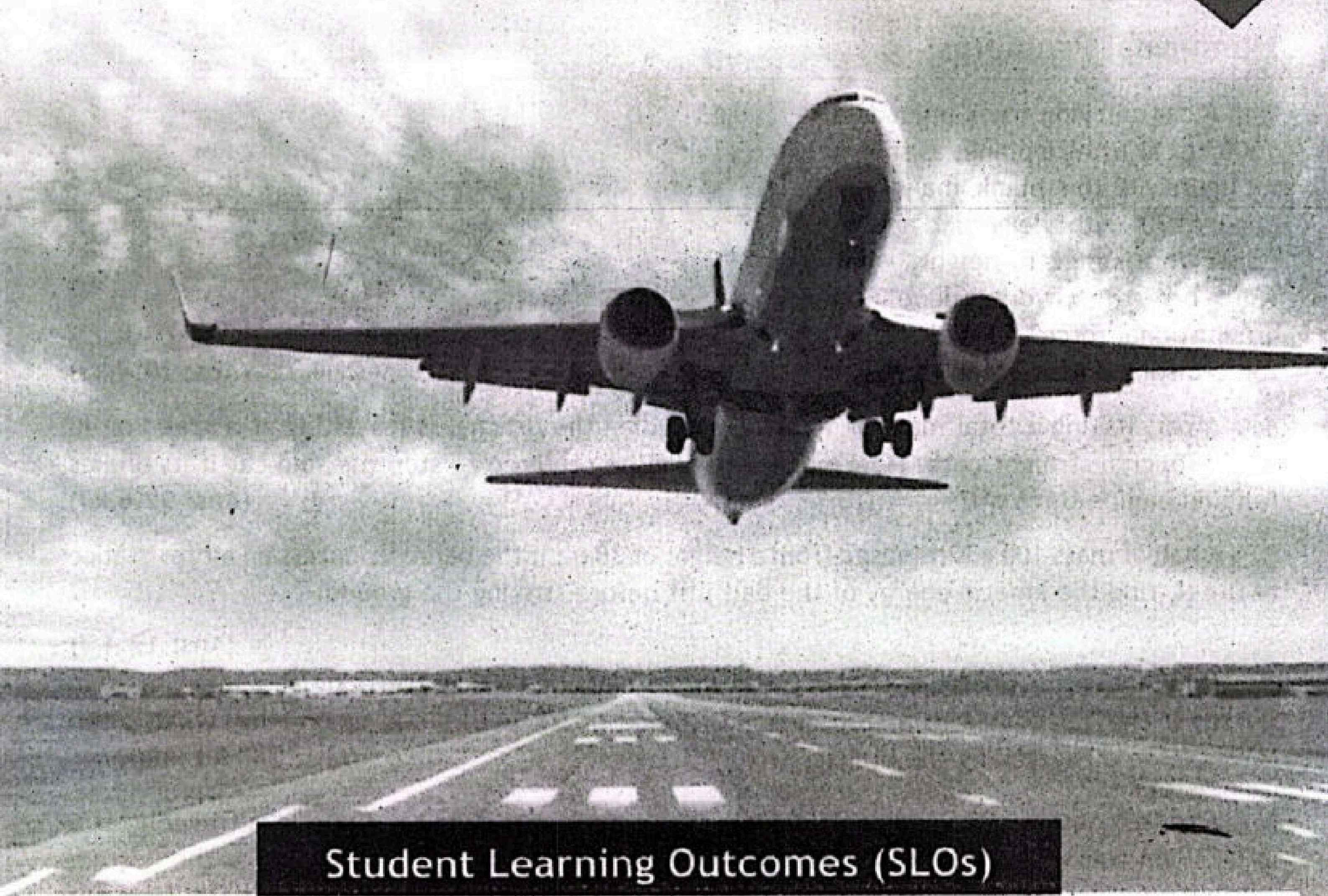
FLUID MECHANICS

UNIT 6



Student Learning Outcomes (SLOs)

The students will:

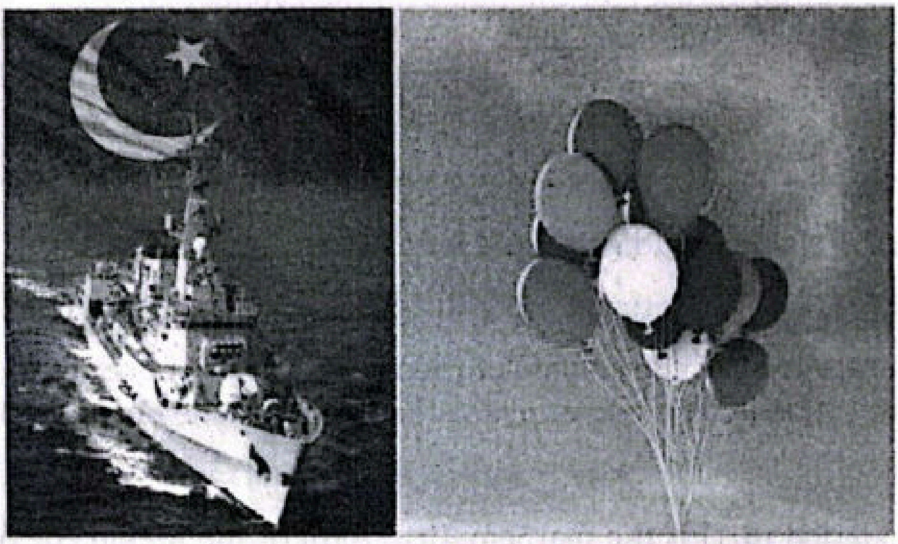
- Justify and use Archimedes's principle of flotation.
- Justify how ships are engineered to float in the sea.
- Define and apply the terms: steady (streamline or laminar) flow, incompressible flow and non-viscous flow as applied to the motion of an ideal fluid.
- Use equation of continuity to solve problems.
- Explain that squeezing the end of a rubber pipe results in increase in flow velocity.
- Justify that the continuity is a form of the principle of conservation of mass.
- Justify that the pressure difference can arise from different rates of flow of a fluid [Bernoulli effect].
- Explain and apply Bernoulli's equation for horizontal and vertical fluid flow.
- Explain why real fluids are viscous fluids.
- Describe how viscous forces in a fluid cause a retarding force on an object moving through it.
- Describe super fluidity [As the state in which a liquid will experience zero viscosity. Students should know the implications of this state e.g. this allows for super fluids to creep over the walls of containers to 'empty' themselves. It also implies that if you stir a superfluid, the vortices will keep spinning indefinitely.]
- Analyze the real world applications of the Bernoulli effect [For example, atomisers in perfume bottles, the swinging trajectory of a spinning cricket ball and the lift of a spinning golf ball (the magnus effect), the use of Ventur ducts in filter pumps and car engineers to adjust the flow of fluid, etc.

It is well known that liquids and gases belong to fluids. The air we breathe and the water we drink are fluids. An important fluid, blood is flowing in our body's veins, flow of which is essential for life. Life would not exist without fluids and without the behavior that fluids exhibit. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does an airplane gain lift while accelerating on the runway? Fluid mechanics allows us to answer these and many other questions.

In Grade-IX, we have dealt with many situations in which fluids are static. In this chapter, we will deal mainly with the flow of fluid. Fluid mechanics is the branch of physics in which we study about the fluid, either static or dynamic. Studying the fluid in motion is called fluid dynamics. The study of fluids in motion is relatively complex, but analysis can be simplified by making few assumptions such as fluid under consideration is non-viscous, incompressible and its motion is steady. The analysis is further simplified by the use of two important conservation principles, the conservation of mass and conservation of energy. The law of conservation of mass gives us the equation of continuity, while the law of conservation of energy forms the basis of Bernoulli's equation. The equation of continuity and the Bernoulli's equation along with their numerous applications in everyday life, including sports, transportation and technology are discussed in this unit.

6.1 UPTHRUST AND ARCHIMEDES PRINCIPLE

Have you ever thought that: Why do heavy ships float on water surface? How does the balloon fly in sky? Why does a mug filled with water feels lighter under water, but feel heavy as soon as we take it out of water? Why do some objects float while others do not?



(a) (b)
Figure 6.1: (a) A heavy ship float on the surface of water.
(b) Helium-filled balloons are flying off in the sky.

Answers to all these questions are based on the fact that pressure increases with depth in a fluid. So, the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward force, called buoyant force or up-thrust, on any object in any fluid. If the up-thrust is greater than the object's weight, the object will rise to the surface and float. If the up-thrust is less than the object's weight, the object will sink. If the up-thrust equals the object's weight, the object will remain suspended at that depth. The up-thrust is always present whether the object floats, sinks, or is suspended in a fluid. Due to up-thrust an air-filled balloon immediately shoots up to the surface when released under water.

Archimedes principle help us to calculate magnitude of up-thrust. It states that:

When an object is totally or partially immersed in a liquid, an up-thrust acts on it equal to the weight of the liquid it displaces.

Consider a solid cylinder of cross-sectional area A and height h immersed in a liquid, as shown in Fig. 6.2. If h_1 and h_2 be the depth of top and bottom surfaces of the cylinder respectively from the surface of the liquid, then: $h_2 - h_1 = h$

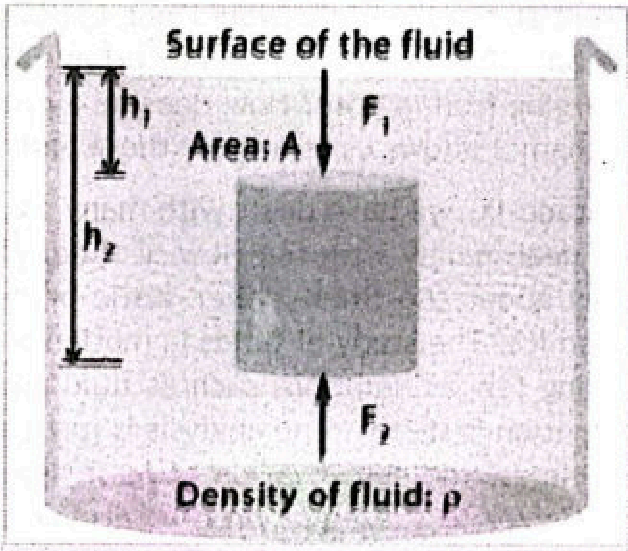


Figure 6.2: Up-thrust acts on the cylinder immersed in liquid

If P_1 and P_2 are the liquid pressure at depths h_1 and h_2 respectively, then:

$$P_1 = \rho g h_1$$

and $$P_2 = \rho g h_2$$

If F_1 and F_2 are the forces exerted by liquid on the top and bottom surfaces of the cylinder respectively, then in terms of pressure:

$$F_1 = P_1 A = \rho g h_1 A$$

and $$F_2 = P_2 A = \rho g h_2 A$$

The net up-thrust F of the liquid on the cylinder is:

$$F = F_2 - F_1$$

or $$\text{Upthrust} = \rho g h_2 A - \rho g h_1 A$$

or $$\text{Upthrust} = \rho g A (h_2 - h_1)$$

or $$\text{Upthrust} = \rho g A h$$

or $$\text{Upthrust} = \rho g V \qquad (6.1)$$

For Your Information

Nearly two thousand years ago Archimedes noticed up-thrust and found that: there is an apparent loss in the weight of the object when immersed in a liquid due to up-thrust of liquid. He stated his findings in his famous principle i.e., Archimedes principle.

Here, Ah = V is the volume of the cylinder and is equal to the volume of the liquid displaced by the cylinder. Therefore, ρgV is the weight of the liquid displaced.

So, Upthrust = weight of the liquid displaced

Equation (6.1) is the mathematical form of the Archimedes' principle. Archimedes principle is also helpful to calculate the density of an object.

Ships: Ships are engineered to float in the sea by employing several fundamental principles and design elements: buoyancy, stability and structural reliability.

a) Buoyancy: Ships are designed to displace a volume of water equal to their weight, allowing them to float. This is achieved by creating a hull shape that maximizes displacement while minimizing weight. The hull is designed to withstand water pressure, which increases with depth. The hull is shaped to displace water, creating an upward buoyant force (F_b) equal to the weight of the ship (W)., i.e.,

$$F_b = \rho V g = W$$

b) Stability: Ships are engineered to resist overturning by distributing weight low, using ballast tanks, and designing a stable hull shape.

c) Structural Reliability: The ship's structure is engineered to withstand stresses, loads, and vibrations using strong, lightweight materials like steel and aluminum.

By combining these principles and design elements, ships are engineered to float and operate safely in the sea.

Submarines: Submarine can travel over as well as under water. It floats over water when the weight of water equal to its volume is greater than its weight. Under this condition, it is similar to a ship and travel over water. It has a system of tanks which can be filled with and emptied of seawater. When these tanks are filled with seawater, the weight of the submarine increases. As soon as the weight becomes greater than the up-thrust, it sinks and remains under water. To return to the surface, the tanks are emptied from seawater.

Example 6.1: A cubic block of wood is completely dipped in water. Calculate the up-thrust of water acting on it if each side of the block is 10 cm long.

Given: $L = 10\text{ cm} = \frac{10}{100}\text{ m} = 0.1\text{ m}$

To Find: Upthrust = ?

Solution: According to the Archimedes' principle, upthrust is given by the formula:

$$\text{Upthrust} = \rho g V$$

For cube, $V = L^3$, so above equation becomes:

$$\text{Upthrust} = \rho g (L)^3$$

Here for water, we use $\rho = 1000\text{ kgm}^{-3}$, putting values, we get:

$$\text{Upthrust} = 1000 \times 9.8 (0.1)^3$$

$$\text{Upthrust} = 9.8\text{ N}$$

Assignment 6.1

An iron object with density 7.8 g cm^{-3} appears 200 N lighter in water than in air. Calculate:

(a) The volume of the object. (b) The weight of the object in the air.

6.2 VISCOUS DRAG AND TERMINAL VELOCITY

Viscosity is measure of fluid's resistance to flow. A more viscous fluid pours slowly while a less viscous pours easily. Imagine a Styrofoam cup with a hole in the bottom. If you pour honey into the cup, the cup drains very slowly. If you fill the same cup with water, the cup will drain more quickly. That is because honey's viscosity is much higher as compared to that of water. Hence, viscosity determines the *rate of flow* of a liquid.

The resistance offered by different layers of a fluid to its flow is called viscosity.

Table 6.1: Viscosity of some fluids at 20°C	
Substance	Viscosity (mPa s)
Water	1.0016
Whole milk	2.12
Honey	2000-10000
Glycerin	1410
Mercury	1.55
Methanol	0.5940
Ethanol	1.144

Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances. In most liquids, viscosity decreases as temperature increases, whereas in most gases, viscosity increases as temperature increases. Therefore, it is important to always measure the temperature of a fluid when determining its viscosity. The coefficient of viscosity is represented by η. Its unit is $N\ s\ m^{-2}$ or $kg\ m^{-1}\ s^{-1}$ or $Pa\ s$. Substances that flow easily have small coefficient of viscosity, while those that do not flow easily have large coefficient of viscosity. For example, water and air have low coefficient of viscosity, whereas honey and engine oil have large high coefficient of viscosity.

Drag Force: A drag force acts on a solid object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. If you stretch out your hand out of the window of a fast-moving car, you can easily recognize that considerable drag force has to be exerted on your hand as it moves through the air. You feel a smaller drag force when you tilt your hand. The drag force become larger as the speed of the car increases.

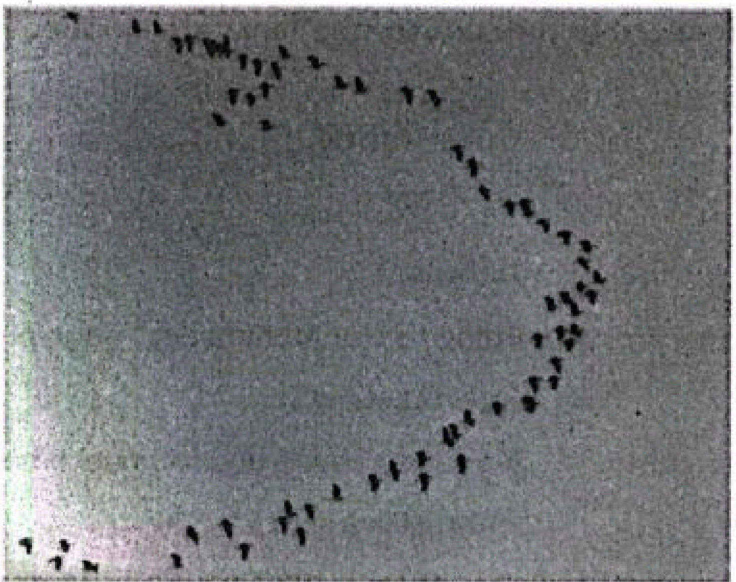


Figure 6.3: Geese fly in a V-formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds.

The retarding force (resistance) experienced by an object moving through a fluid is called drag force or viscous drag.

When the fluid is a gas (like air), it is called aerodynamic drag or air resistance. When the fluid is a liquid (like water), it is called hydrodynamic drag.

The drag force depends upon various factors such as size, shape and orientation of the object, viscosity of the fluid and the relative speed of the object with respect to fluid.

Swimmers and Skydivers

Swimmers and skydivers change the effective size and orientation of their bodies by bending, twisting or stretching their arms and legs in and out. This allows them to control their speed and direction of motion by using the drag.

For small objects, the drag force is given by Stoke's law. According to Stoke's Law, the drag force (F_d) on a spherical object is directly proportional to radius of object r, velocity v and coefficient of viscosity η of fluid. Thus, the mathematical form of Stoke's law is:

$$F_d = 6\pi\eta r v \qquad (6.2)$$

6.2.1 Terminal Velocity

The maximum constant velocity acquired by a freely falling object in a viscous medium is called terminal velocity.

Terminal velocity is attained when the weight of the object is balanced by the upward drag force. For example, in case of a raindrop, initially, it accelerates due to the gravity (Fig. 6.4). At first, there will be no drag force. As the velocity increases, the retarding force also increases. As the object falls faster and faster, the drag force increases, So,

$$\textit{Net Force} = \textit{weight} - \textit{drag force}$$

Finally, when viscous drag is equal to the force due to gravity, the net force becomes zero and so does the acceleration. The raindrop then falls at constant velocity (i.e. terminal velocity v_t).

$$0 = w - F_d$$

$$F_d = w$$

or

$$6\pi\eta r v_t = mg$$

$$v_t = \frac{mg}{6\pi\eta r} \qquad \text{\_\_\_\_\_ (6.3)}$$

Where r is radius of droplet and η is the viscosity of the air. For sphere of uniform density ρ, mass m is given by:

$$m = \rho V$$

As the volume of sphere is $\frac{4}{3}\pi r^3$, Hence

$$m = \rho\left(\frac{4}{3}\pi r^3\right) \qquad \text{\_\_\_\_\_ (6.4)}$$

Putting equation (6.4) in equation (6.3), we get:

$$v_t = \frac{\rho\left(\frac{4}{3}\pi r^3\right)g}{6\pi\eta r}$$

$$v_t = \frac{2\rho g r^2}{9\eta} \qquad \text{\_\_\_\_\_ (6.5)}$$

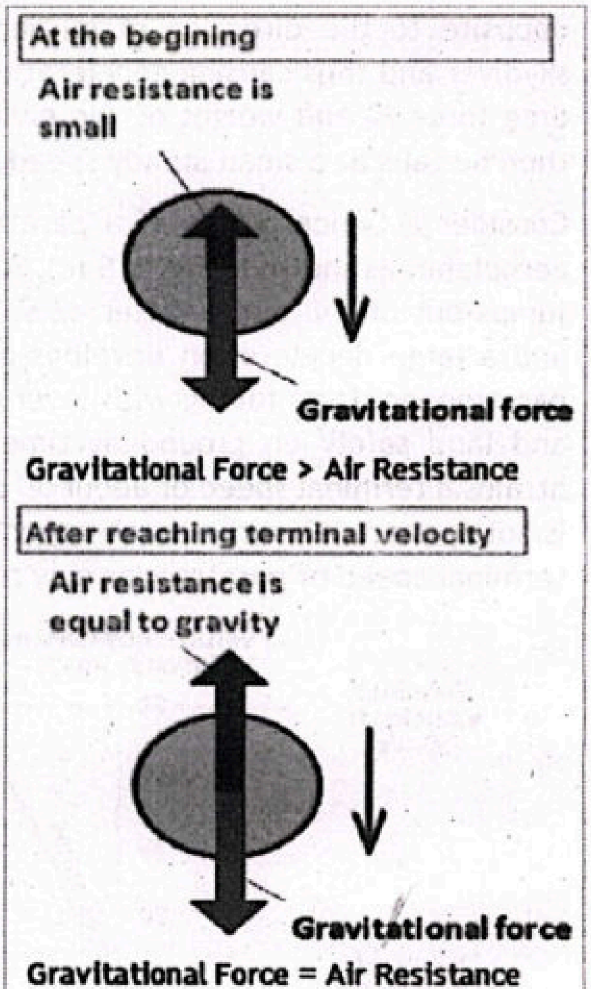


Figure 6.4: A rain droplet falling freely under the action of gravity.

As ρ, g, η are constants, so from equation (6.5) we can deduce that $v_t \propto r^2$. This indicates that the smaller mass will attain terminal speed sooner than larger mass. Thus, from equation (6.5), we can conclude that the terminal velocity depends on the square of the radius of the sphere and inversely proportional to the viscosity of the medium.

6.2.2 Paratrooper's Jump

A paratrooper initially falls with large acceleration after jumping out of the plane without opening his parachute, and attains a high terminal velocity, as shown in Fig. 6.5 (a).

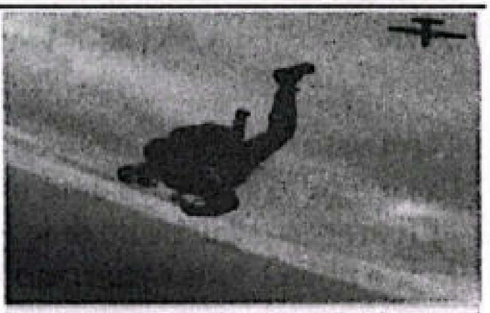


Figure 6.5 (a): A paratrooper, before opening his parachute.

To land safely on ground, he opens his parachute, as shown in Fig. 6.5 (b). Opening the parachute provides a large surface area to produces a large drag force (greater than weight) opposite to the direction of motion. This slows down the skydiver and thus causing deceleration. After some time, the drag force F_d and weight of the paratrooper become equal, then he falls at a small steady speed to the ground level.

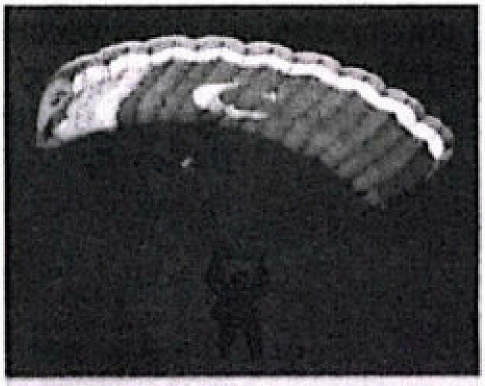


Figure 6.5 (b): A paratrooper, after opening his parachute.

Consider a typical graph of a paratrooper jumping from an aeroplane, as shown in Fig. 6.5 (c). At t = 0 s, the paratrooper jumps out of the plane. After 42 s, he opens the parachute and a large deceleration develops and within next 5 s, the paratrooper starts falling with a very moderate steady speed and land safely on ground at time 70 s. The paratrooper attains a terminal speed of about 50 m s^{-1} when the parachute is not opened. When the paratrooper opens his parachute, the terminal speed of paratrooper now reaches 8 m s^{-1}.

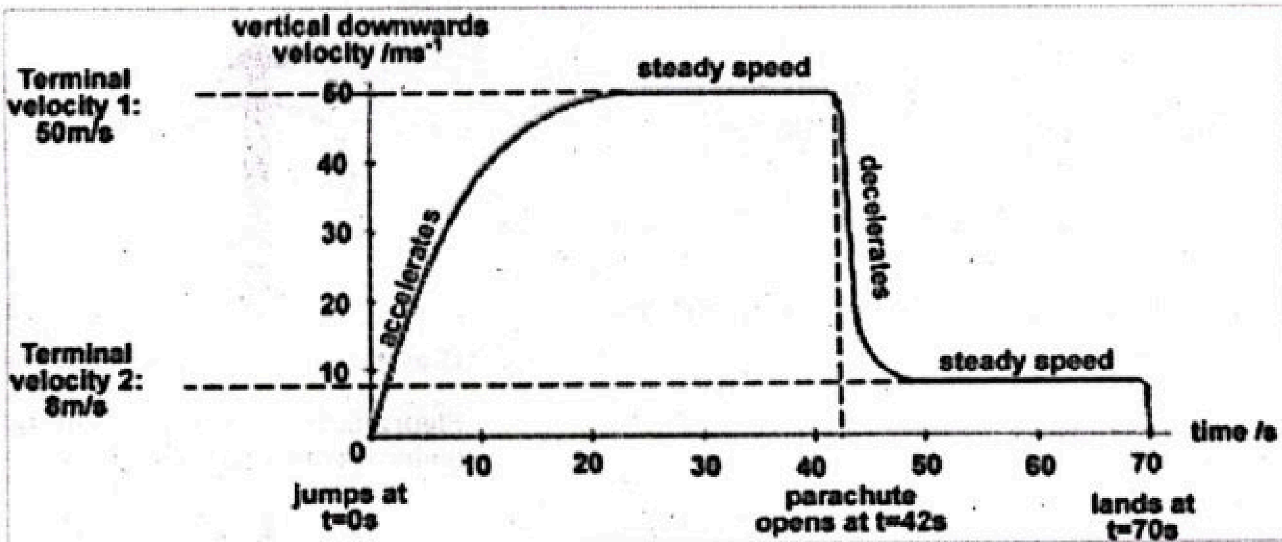


Figure 6.5 (c): A typical graph of *velocity against time* for a paratrooper.

Examples 6.2: A spherical body of radius 2 mm is passing through air with velocity 2 m s^{-1}. Find the drag force on the body due to the air. Viscosity of air is 1.9×10^{-5} kg m^{-1} s^{-1}.

Given: Radius = r = 2 mm Velocity = v = 2 m s^{-1}

Viscosity of air = η = 1.9×10^{-5} kg m^{-1} s^{-1}.

To Find: Drag force = F_d = ?

Solution: Here, we will use Stoke's law:

$$F_d = 6\pi\eta r v$$

Putting values, we get:

$$F_d = 6 \times 3.14 \times 1.9 \times 10^{-5} \times 2 \times 10^{-3} \times 2$$

$$F_d = 143.2 \times 10^{-8}\ N$$

Assignment 6.2

Find the terminal velocity obtained by a raindrop of radius 0.3 mm falling through air of viscosity 1.8×10^{-5} kg $m^{-1}s^{-1}$.

6.3 FLUID FLOW

A fluid is a substance that can flow, such as a liquid or a gas. The flow rate of a fluid is the volume of fluid passing a given point in a pipe per unit time. The fluid can flow in two ways: streamline flow or turbulent flow.

If every particle that passing a particular point moves along exactly the same smooth path followed by previous particles that have passed that point earlier, then such flow is called streamline flow.

Streamline flow, also known as steady or laminar flow, is characterized by smooth flow of a fluid through a tube, as shown in Fig. 6.6 (a). By smooth flow, we mean that all particles of the fluid follow the same uniform path, called streamline. The streamlines (paths of different particles) do not cross each other and every fluid particle arriving at a given point has the same velocity. It usually occurs at lower velocities and with streamlined objects. In laminar flow, the middle layer of fluid tends to flow faster.

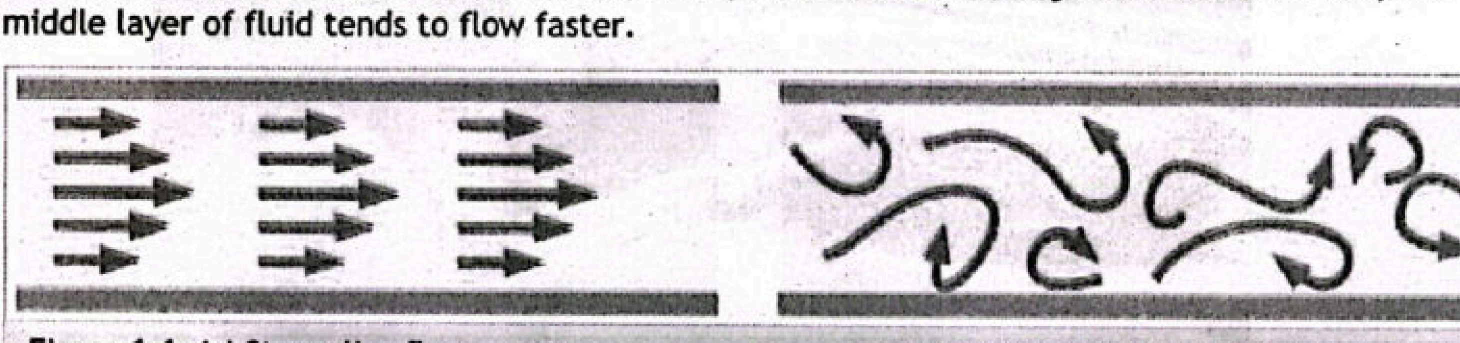


Figure 6.6: (a) Streamline flow. **Figure 6.6: (b) Turbulent flow.**

Above a certain critical speed, streamline flow becomes turbulent. Turbulent flow is irregular flow characterized by small whirlpool-like regions as shown in Fig. 6.6 (b). Turbulent flow, also known as non-laminar flow, is the unpredictable flow of a fluid resulting from excessive speed of the flow or sudden changes in direction or size of the tube or pipe. Turbulent flow occurs at higher velocities or with non-streamlined objects, where the flow lines become disordered and mixed.

Irregular flow of fluid is called turbulent flow.

Remember that the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In model of ideal fluid flow, we make the following assumptions:

- The fluid is non-viscous. In a non-viscous fluid, internal friction is neglected.
- The flow is laminar. In laminar flow, all particles passing through a point have the same velocity.
- The fluid is incompressible. The density of an incompressible fluid is constant. Liquids are generally incompressible, while gases are compressible.
- The flow is irrotational, means it travels in straight lines.

For Your Information

The streamlined bodies of dolphins assist their movement in water by reducing the pressure of water against their skin as well as reducing friction. The streamline shape of dolphins enables them to move very speedily.

In real fluids, there is always some viscosity. It means that real fluids are viscous. Ideal fluids do not have any viscosity or can be said to have zero viscosity. When a fluid is viscous, it essentially refers to the thickness of the fluid or the friction the fluid faces while it flows. Therefore, ideal fluids do not experience the opposing force and have a non-viscous flow, while real fluids have a viscous flow.

Streamlined Design of Fast-Moving Bodies

As we have studied that the drag force depends on various factors such as size, shape and orientation of the object. The drag force increases significantly with vehicle speed, which reduces the performance of vehicles. Therefore, in **Auto Engineering**, the fast-moving objects are designed as streamlined to improve their performance. A wind tunnel is used for studying the interaction between a solid-stationary model and an airstream. A wind tunnel simulates this interaction by producing a high-speed visible airstream (as shown in the picture) which flows across a model being tested.

6.4 EQUATION OF CONTINUITY

Equation of continuity is an important concept of fluid dynamics. Common applications where equation of continuity is used are pipes, tubes, ducts with flowing fluids or gases, rivers and power plants etc.

To derive this equation, consider the steady flow of the fluid through a tube of varying cross-sectional areas, as shown in the Fig. 6.7. The tube has a single entry and single exit. In short interval of time Δt, the fluid will cover a distance Δx_1 with a velocity v_1 at the lower end of the pipe. At this time, the distance covered by the fluid will be:

$$\Delta x_1 = v_1 \Delta t$$

Now, at the lower end of the pipe, the volume of the fluid that will flow into the pipe will be:

$$V = A_1 \Delta x_1 = A_1 v_1 \Delta t$$

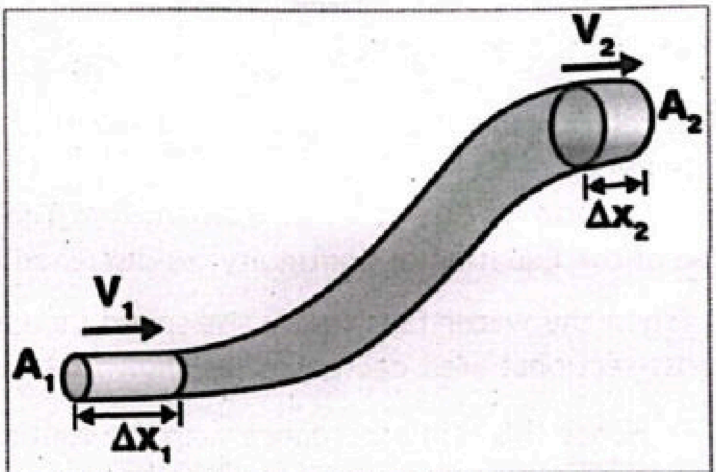


Figure 6.7: Fluid flowing through a tube of varying cross-sectional areas.

It is known that: mass (m) = Density (ρ) × Volume (V). So, the mass of the fluid in Δx_1 region will be:

$$\Delta m_1 = \rho \times V$$

$$\Delta m_1 = \rho_1 A_1 v_1 \Delta t \quad \_\_\_\_\_\_ (6.6)$$

Similarly, the mass of the fluid at the upper end in Δx_2 region will be:

$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t \quad \_\_\_\_\_\_ (6.7)$$

Here, v_2 is the velocity of the fluid through the upper end of the pipe i.e., through Δx_2, in Δt time and A_2 is the cross-sectional area of the upper end. As in case of ideal fluid flow, total mass of fluid is conserved; so equating (6.6) and (6.7), we get:

$$\Delta m_1 = \Delta m_2$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad \_\_\_\_\_\_ (6.8)$$

Moreover, if the fluid is incompressible, the density will remain constant for steady flow, i.e., $\rho_1 = \rho_2$.

Thus, equation (6.8) can be written as:

$$A_1 v_1 = A_2 v_2 \quad \_\_\_\_\_\_ (6.9)$$

Equation (6.9 a) is known as continuity equation and can be written in a more general form as:

$$A\, v = \text{Constant} \quad \_\_\_\_\_\_ (6.10)$$


Figure 6.8 (a): As the water falls, its speed increases and cross-sectional area decreases, in accordance with the continuity equation.

This equation states that for an ideal fluid, the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is always constant (if there is no source or sink present). This product is equal to the volume flow per second or simply the flow rate. From equation of continuity, we can show that:

$$v \propto \frac{1}{A}$$

This relation shows that smaller the cross-sectional area, the greater the velocity of fluid, and vice versa.

Application of Equation of Continuity: There are many phenomena in real word that make use of the Equation of Continuity, as discussed below:

i) When the water falls freely, its speed increases and so its cross-sectional area decreases (as shown by the relation $v \propto \frac{1}{A}$). Hence this is in accordance with the continuity equation.

ii) By squeezing the end of a rubber pipe (as shown in Fig. 6.8 b), its cross-sectional area (A) decreases. As a results velocity of water increases (as shown by the relation $v \propto \frac{1}{A}$). Hence according to the continuity equation, reducing the area of the opening will cause the velocity to increase.

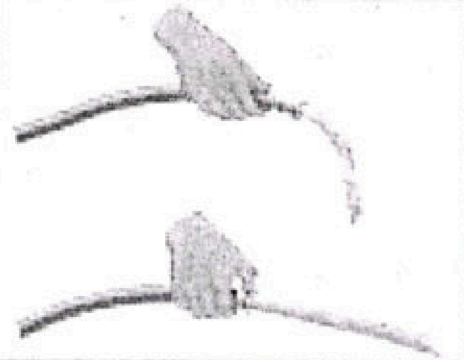


Figure 6.8 (b): Squeezing the end of a rubber pipe results in increase in flow velocity.

Example 6.3: In a normal adult, the average speed of the blood through the aorta (radius r = 0.8 cm) is 0.33 m s^{-1}. From the aorta, the blood goes into major arteries, which are 30 in number, each of radius 0.4 cm. Calculate the speed of the blood through the arteries.

Given: Radius of aorta = r_1 = 0.8 cm = 0.8×10^{-2} m

Velocity of blood in aorta = v_1 = 0.33 m s^{-1}

Radius of artery = r_2 = 0.4 cm = 0.4×10^{-2} m

To Find: Velocity of blood in arteries = v_2 = ?

Solution: We use equation of continuity at the aorta and arteries, such as:

$$A_1 v_1 = 30 A_2 v_2$$

The factor of 30 on R.H.S appears because blood from aorta distributed into 30 equal-sized arteries, each with an internal cross-sectional area A_2. As $A = \pi r^2$ (internal cross-sectional area being circular), thus above equation becomes:

$$\pi r_1^2 v_1 = 30 \pi r_2^2 v_2$$

$$r_1^2 v_1 = 30 r_2^2 v_2$$

$$v_2 = \frac{1}{30}\left(\frac{r_1}{r_2}\right)^2 v_1$$

Putting values, we get:

$$v_2 = \frac{1}{30}\left(\frac{0.8\,cm}{0.4\,cm}\right)^2 0.33$$

$v_2 = 0.044\ m\ s^{-1}$

Thus, the blood flows through the arteries at a speed of $0.044\ m\ s^{-1}$. This speed is much lower than the speed of blood in aorta. This is due to the fact that the total combined internal cross-sectional area of 30 arteries is greater than that of the aorta. The slower speed of the blood in arteries is favorable for the exchange of gasses.

Assignment 6.3

Why does the deep water flows slowly as compared to shallow water in rivers?

6.5 BERNOULLI'S EQUATION

The Bernoulli's equation is an approximate relation between pressure, velocity and elevation for flow of an ideal fluid. In this section, we will derive the Bernoulli's equation by applying the conservation of energy principle for flowing fluids.

Let us consider the two different regions, as shown in Fig. 6.9. An ideal fluid flows through the pipe in time interval Δt. If the speed of fluid at lower point is v_1 and at upper point is v_2, then the distance Δx_1 covered by fluid in time Δt is $v_1 \Delta t$.

$$\Delta x_1 = v_1 \Delta t \qquad (6.11)$$

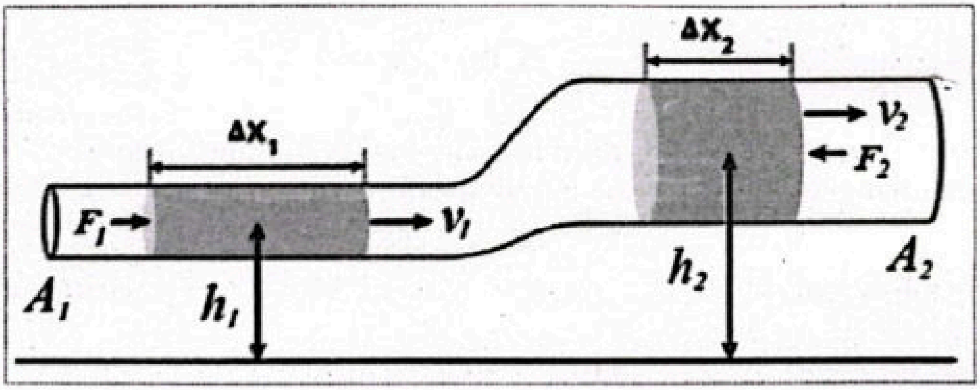


Figure 6.9: Fluid flowing through a tube of varying cross-sectional area.

Similarly, during the same interval of time Δt, the distance Δx_2 travelled by fluid is $v_2 \Delta t$.

$$\Delta x_2 = v_2 \Delta t \qquad (6.12)$$

At lower end, the work done on the fluid in moving through a distance Δx_1, will be:

$$W_1 = F_1 \Delta x_1 \qquad (6.13)$$

using $P = F/A$ or $F = PA$ in (6.13), we get:

$$W_1 = P_1 A_1 \Delta x_1 \qquad (6.14)$$

The work done on the fluid at the upper end is negative because F_2 is opposite to F_1. Thus:

$$W_2 = -P_2 A_2 \Delta x_2 \qquad (6.15)$$

Net work done W is obtained by adding equation (6.14) and equation (6.15), i.e.

$$W = W_1 + W_2$$

$$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \quad \_\_\_\_\_\_ (6.16)$$

If v_1 and v_2 are the velocities of fluid at lower and upper ends respectively, then:

$$W = P_1 A_1 v_1 t - P_2 A_2 v_2 t \quad \_\_\_\_\_\_ (6.17)$$

Moreover, if we consider the equation of continuity, then:

$$A_1 v_1 = A_2 v_2 = \frac{V}{t}$$

Where V is the volume of the fluid that flows in time t. Hence:

$$A_1 v_1 t = A_2 v_2 t = V,$$

So, the equation (6.17) becomes:

$$W = P_1 V - P_2 V$$

$$W = (P_1 - P_2) V \quad \_\_\_\_\_\_ (6.18)$$

As $V = \frac{m}{\rho}$, so equation (6.18) becomes:

$$W = (P_1 - P_2)\frac{m}{\rho} \quad \_\_\_\_\_\_ (6.19)$$

Daniel Bernoulli (1700-1782)

The name Bernoulli's equation was set on the name of a Swiss physicist Daniel Bernoulli (1700-1782). He derived this equation in 1738.

A part of this work W is utilized by the fluid in changing its K.E and a part is used in changing its P.E, so applying conservation of energy, we get:

$$W = \Delta K.E + \Delta P.E$$

$$(P_1 - P_2)\frac{m}{\rho} = \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + (mgh_2 - mgh_1)$$

Where h_1 and h_2 are the heights of pipe at lower and upper ends, respectively. Cancelling m from both sides we get:

$$(P_1 - P_2)\frac{1}{\rho} = \left(\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2\right) + (gh_2 - gh_1)$$

Rearranging, we get:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \_\_\_\_\_\_ (6.20)$$

The equation (6.20) is the-well-known Bernoulli's equation. However, the subscripts with various parameters on both sides of the equation represent two different points along the pipe. Thus, the general equation can be written as:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \quad \_\_\_\_\_\_ (6.21)$$

Thus, Bernoulli's equation states that sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume will remain constant. Bernoulli's equation is an important tool in investigating many technical systems such as power generation stations, plumbing systems and flight of an aeroplane.

6.6 APPLICATIONS OF BERNOULLI'S EQUATION

According to Bernoulli's equation, slow air flow creates a high pressure while fast air flow creates low pressure. This effect is used in many fields of life, such as in filter pump, venturi meter, atomizers, the flow of air over an aerofoil and in blood flow. In this section, we will discuss all of these applications.

Activity: Bernoulli's Theorem

Take two A4 size sheets and position them as shown in the picture. When a continuous stream of air is blown between them what do you think will happen to the positions of the papers? Will the sheets separate or move closer to one another? Test your prediction by doing this activity. Was your prediction correct?

The sheets will move closer to one another. This is due to the Bernoulli's principle, which states that fast airflow creates low pressure between the two sheets. Thus, sheets will move towards the low-pressure central region.

6.6.1 Atomizer

Atomizer is a device that is used to emit liquid droplets as fine spray. It works on Bernoulli's principle.

When you squeeze the rubber bulb (Fig. 6.10), high speed air passes horizontally over a vertical tube and creating a lower pressure than that inside the container. This pressure difference pushes the liquid from the reservoir up through the vertical narrow tube and into the moving stream of air. Atomizer has a nozzle at the end of the horizontal tube, which causes the liquid to break up into small drops, mixing it with the air and carried away with the stream of air.

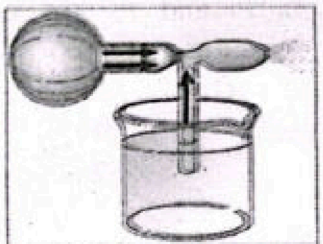


Figure 6.10: A simple Atomizer.

Atomizer nozzles are used for spraying perfumes, applying paint and in engine carburetor etc.

6.6.2 Filter Pump

A filter pump is designed and can be explained on the basis of Bernoulli's effect. A filter pump has a jet in the middle (Fig. 6.11). When water from tap reaches the jet, its speed increases and hence causes a pressure drop near it. According to Bernoulli's Principle, the pressure of the moving air decreases as the speed of the air increases.

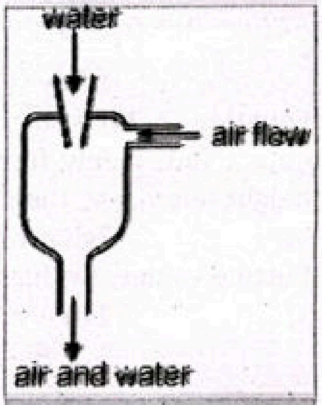


Figure 6.11: Filter pump.

The air thus flows in from the side tube to which the vessel is connected. The air and water together are forced to the bottom of the filter pump.

6.6.3 Torricelli's Theorem

Torricelli's Theorem states that

The speed of flow of fluid from an orifice at depth h below the top surface of a liquid is equal to speed gained by the fluid in free falling through the height h.

Consider a tank filled with some fluid and with an orifice (slit) near the bottom, as shown in Fig. 6.12. The top surface of fluid is at height h_1 and the opening near bottom is at height h_2. According to Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{——— (6.22)}$$

Here velocity v_1 at top surface is negligible as compared to velocity of efflux v_2. By neglecting second term from L.H.S in the equation (6.22), we get:

$$P_1 + \rho g h_1 + 0 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \quad \text{——— (6.23)}$$

If we put $v_2 = v$, then equation (6.22) becomes as:

$$\frac{1}{2}\rho v^2 = P_1 - P_2 + \rho g h_1 - \rho g h_2$$

As P_1 and P_2 are equal being the atmospheric pressures at top surface and at the orifice respectively, so $P_1 - P_2 = 0$. Hence, we get:

$$\frac{1}{2}\rho v^2 = \rho g (h_1 - h_2)$$

or

$$\frac{1}{2} v^2 = g (h_1 - h_2)$$

$$v = \sqrt{2g(h_1 - h_2)}$$

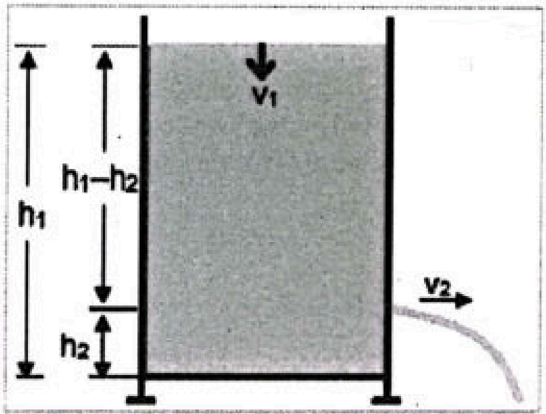


Figure 6.12: A tank filled with water and has an orifice near the bottom. The top surface of water comes down with almost zero speed.

As $h_1 - h_2 = h$, in general, we can write:

$$v = \sqrt{2gh} \quad \text{——— (6.24)}$$

Equation (6.24) also true for the velocity gained by an object in falling from height h. If an object falls freely from height h, its initial velocity $v_i = 0$ and its final velocity after falling height h is $v_f = v$, then using third equation of motion, we get:

$$2gh = v_f^2 - v_i^2$$

Putting values, we have:

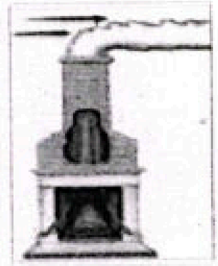
$$2gh = v^2 - 0^2$$

$$v = \sqrt{2gh}$$

This is the same result as in equation (6.24), indicating that the speed of flow of fluid from an orifice at depth *h* below the top surface of a liquid is equal to the speed gained by the fluid in free fall from the height *h*.

Do You Know?

In the summer you can enjoy by getting heat from the fireplace without the room filling up with smoke! This is again due to the Bernoulli's effect. Can you explain how?



6.6.4 Venturi Meter

A venturi meter (or flow meter) is used to measure the flow speed of liquid through a tube. The liquid having high pressure and low velocity gets converted to the low pressure and high velocity at a particular point and again reaches to high pressure and low velocity. The point where the fluid has low pressure and high velocity is the place where the venturi flow meter is used.

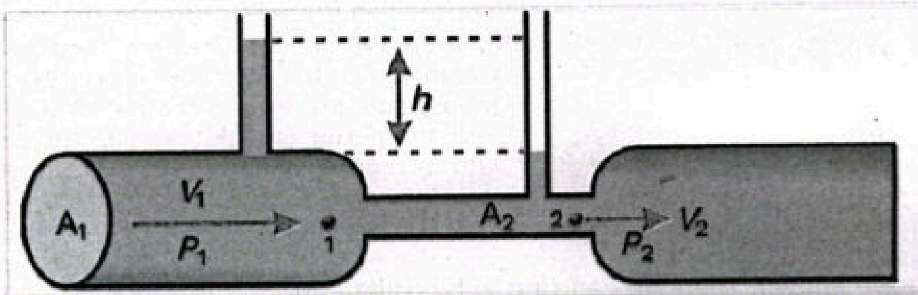


Figure 6.13: Venturi meter.

The venturi meter is constructed, as shown in Fig. 6.13. Venturi meter is so designed as to ensure the stream line flow of liquid through a pipe section with area A_1, a flow velocity v_1 and pressure P_1. In the narrow section (the throat) with area A_2, the fluid flows with flow speed v_2, and has pressure P_2. As a result, the pressure difference at two points 1 and 2 is appeared as a height difference h.

To derive an expression for the pressure difference, we use the Bernoulli's equation and the continuity equation. First applying Bernoulli's equation to the portion 1 and 2, we have:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

As $h_1 = h_2$, so $\rho g h$ term will cancel out from both sides and we get:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \qquad \text{\_\_\_\_\_\_ (6.25)}$$

From equation of continuity, we have:

$$A_1 v_1 = A_2 v_2$$

or $$v_2 = \frac{A_1}{A_2} v_1 \qquad \text{\_\_\_\_\_\_ (6.26)}$$

Putting (6.26) in (6.25), we get:

$$P_1 - P_2 = \frac{1}{2}\rho\left[\left(\frac{A_1}{A_2}v_1\right)^2 - v_1^2\right]$$

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) \qquad \text{\_\_\_\_\_\_ (6.27)}$$

Also, $P_1 - P_2 = \rho g h$ ______ (6.28)

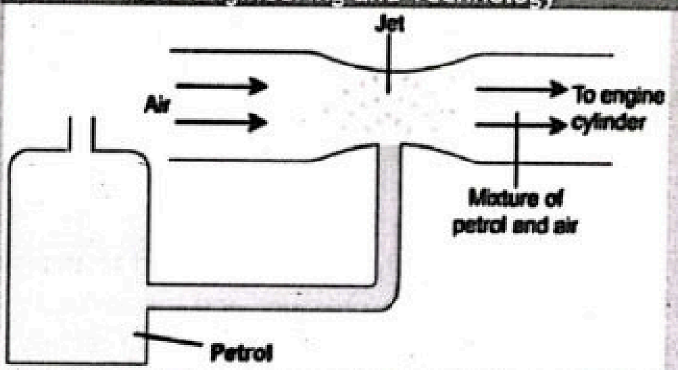
Putting equation (6.28) in (6.27), we get:

$$\rho g h = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$2 g h = v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}} \qquad \text{\_\_\_\_\_\_ (6.29)}$$

Auto Engineering and Technology



The carburetor used in engines contains a venture tube. When the engine is switched on, air is sucked into the venture tube. Fast moving air in the narrow throat of the tube causes a low pressure at this region according to the Bernoulli's principle. The higher atmospheric pressure in the petrol compartment will push the petrol into the region. The petrol will mix with air before it flows into the cylinder engine for combustion.

If A_1, A_2 and h are known, then we can find v_1 by using equation (6.29) and hence rate of flow can be calculated using equation of continuity.

6.6.5 Aerofoil

An aerofoil is the term used to describe the cross-sectional shape of an object that creates a lift, when moved through a fluid such as air.

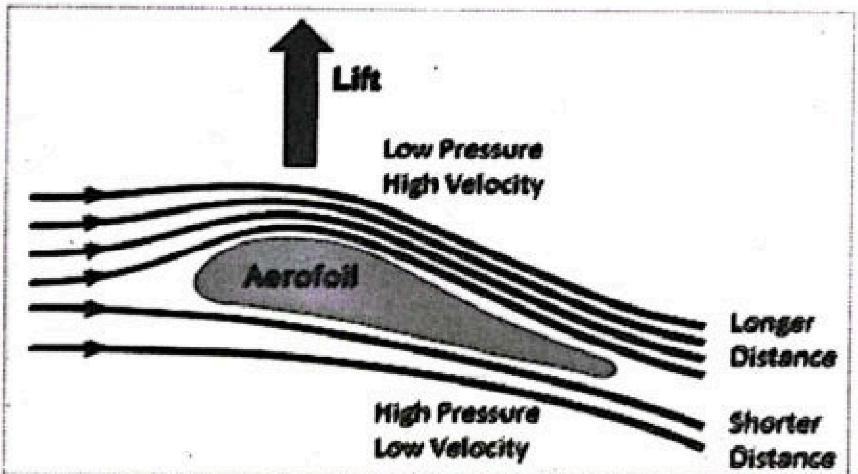


Figure 6.14: Cross-section of aeroplane's wing.

Aerofoils are employed on aircraft as wings to produce lift perpendicular to the air flow (Fig. 6.14). When a wing shaped like an aerofoil, moves in air, the flow of air over the top travels faster creating a region of low pressure. The flow of air below the wing is slower resulting in a region of higher pressure. The pressures difference between the upper-side and lower-side of the wing causes a net upward force, called lift, which helps the plane to take-off.

6.6.6 The Magnus Effect

The Magnus effect is a phenomenon related to a spinning object moving through a fluid. In Magnus effect, a spinning ball curves away from its path of flight, as shown in the Fig. 6.15. The path of the spinning object is deflected in a manner that is not present when the object is not spinning. The Magnus Effect depends on the rotational speed of the object. The deflection is caused by the pressure difference of the fluid on opposite sides of the spinning object due to the varying fluid velocities.

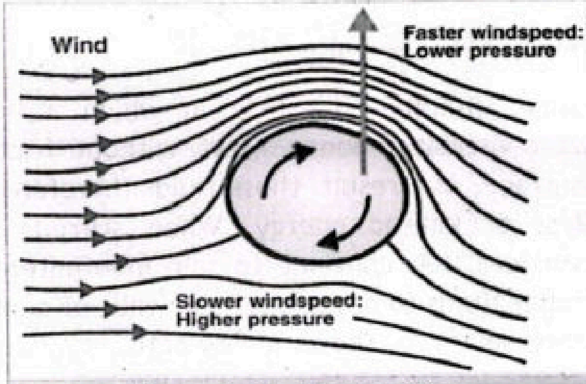


Figure 6.15: Magnus Effect

In the case of a ball spinning through the air, the spinning ball drags some of the surrounding air with it. A spinning object moving through a fluid deviates from its straight path because of pressure differences that develop in the fluid as a result of velocity changes induced by the spinning body.

Example 6.4: Water is flowing streamline through a closed pipe system. The speed of water at one point is 4 m s^{-1} and the pressure is 47.1 kPa, while at another point 3 m lower, the speed is 3 m s^{-1}. Find the pressure at the lower point.

Given: At higher point:

Speed = v_1 = 4 m s^{-1} — height = h_1 = 3.0 m

Pressure = P_1 = 47.1 kPa = 47.1×10^3 Pa

At lower point:

Speed = v_2 = 3 m s^{-1} — height = h_2 = 0 m

To Find: Pressure = P_2 = ?

Solution: Apply Bernoulli's theorem:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

By rearranging above equation, we get:

$$P_2 = P_1 + \frac{1}{2}\rho (v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

Putting the given values, and taking ρ = 1000 kg/m^3 for water, we get:

$$P_2 = (47.1\times10^3) + \frac{1}{2}(1000)\,[(4)^2-(3)^2] + (1000)(9.8)(3-0)$$

After using calculator, we get:

$$P_2 = 47.1 + 3.5 + 29.4 \times 10^3$$

P_2 = 80,000 Pa or 80 k Pa

Assignment 6.4

The speed of air across the top and bottom of an aeroplane's wing is 450 m s^{-1} and 410 m s^{-1} respectively. Calculate the lift on the wing if the wing is 15 m long and 3 m wide (ρ = 1.29 kg m^{-3}, for air).

6.7 SUPER FLUIDITY

Super fluidity is a state in which a liquid experiences zero viscosity, hence flows without friction through any surface. As a result, these fluids therefore flow without any loss of kinetic energy. When stirred, superfluid forms vortices that continue to spin indefinitely. This allows for super fluids to creep over the walls of containers to 'empty' themselves, as shown in Fig. 6.16.

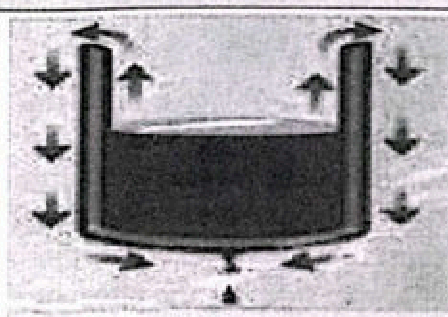


Figure 6.16: Super fluids creeps over the walls of containers to 'empty' themselves.

Super fluidity is observed in liquid helium at temperatures near absolute zero, as well as electrons within a superconducting solid. The neutron fluid in a neutron star may also be a superfluid. The unusual behavior of superfluid arises from quantum mechanical effects.

To create superfluid states, helium gas cooled to a few degrees above absolute zero, as shown by the graph in Fig. 6.17. This is achieved by compressing the gas, and then expelling it through a small nozzle. As the gas expands, it rapidly cools.

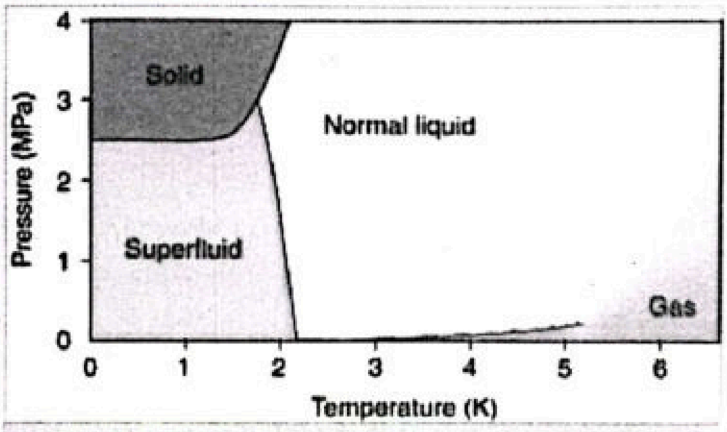


Figure 6.17: Pressure-Temperature graph for He.

If a superfluid is placed in a rotating container, instead of rotating uniformly with the container, the rotating state consists of quantized vortices. That is, when the container is rotated below a certain velocity (known as the critical angular velocity), the liquid remains perfectly stationary. Once the critical angular velocity is achieved, the superfluid will form a vortex.

The vortex strength of super fluid is quantized, i.e., a superfluid can only spin at certain 'allowed' angular velocities. Rotation in a normal fluid (like water) is not quantized. If the rotation speed is increased more and more quantized vortices will be formed which arrange in wonderful patterns.

For Your Information

Helium-4 (^4He) becomes a superfluid at a temperature below 2.17 K. Helium-3 (^3He) becomes a superfluid below 0.0025 K.

Application of Superfluids: Superfluids are used in high-precision devices, such as gyroscopes, which allow the measurement of some theoretically predicted gravitational effects. Recently, superfluids have been used to trap and slow the speed of light.

Fluid Mechanics and Medical Technology

Medical technology uses heartbeat and sound of blood flow to find the blood pressure. A fluid exerts pressure on the walls of its container. As a fluid, blood also exerts pressure on the walls of the heart, arteries, vessels, and capillaries that make up the circulatory system. The sphygmomanometer is a well-known medical equipment used for the measurement of blood pressure of a person. The sphygmomanometer cuff (air bag) is placed around the upper arm of a person, inflated, and then deflated while the meter measures the pressure of blood passing through those arteries in the arm. The external pressure is increased by pressing the air bulb repeatedly. The sound of the blood flow is heard by the stethoscope.

The gradual increase in pressure squeezes the arm and compresses the blood vessels under the air bag. When the pressure in the air bag becomes greater than the systolic pressure (120 torr), then the vessels collapse, cutting off the flow of blood. At this instant stethoscope detect no sound. The pressure in the bag is then decreased by slowly opening the release valve of air bulb. When the external pressure becomes equal to systolic pressure, the blood flows (turbulent) with high speed through narrow contracted vessel. Stethoscope detects sound at this instant and dial of the barometer gives systolic pressure. By further opening the release valve, the pressure in the air bag drops enough that the vessels no longer remain compressed and the blood is now in laminar flow. At this instant there are no gurgle's sound heard by stethoscope, so this is the signal to record diastolic pressure.

For a normal person the blood pressure varies between systolic (120 torr) to diastolic (75-80 torr). These values change with age.

SUMMARY

- **Archimedes principle** states that: When an object is totally or partially immersed in a liquid, an up-thrust acts on it equal to the weight of the liquid it displaces.
- The resistance offered by different layers of a fluid to its flow is called **viscosity**. Its unit is $N\,s\,m^{-2}$ or $kg\,m^{-1}\,s^{-1}$ or Pa s.
- The retarding force (resistance) experienced by an object moving through a fluid is called **drag force or viscous drag.**
- The maximum constant velocity acquired by a freely falling object in a viscous medium is called **terminal velocity.**
- If every particle that passes a particular point moves along exactly the same smooth path followed by previous particle passing that has passed that point, then such flow is called **Streamline.**
- Irregular flow of fluid is called **turbulent flow.**
- **Equation of continuity** is defined as: For an ideal fluid the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is always constant. This product is equal to the volume flow per second or simply the flow rate.
- **Bernoulli's equation** states that sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume will remain constant. According to Bernoulli's equation slow air flow creates a high pressure, while fast air flow creates low pressure.

- The speed of flow of fluid from an orifice at depth h below the top surface of a liquid is equal to speed gained by the fluid in free falling through the height h. This statement is called **Torricelli's Theorem.**
- A **venturi meter (or flow meter)** is used to measure the flow rate of liquid through a tube.
- An **aerofoil** is the term used to describe the cross-sectional shape of an object that creates a lift, when moved through a fluid such as air.
- In **Magnus effect,** a spinning ball curves away from its path of flight.
- **Super fluidity** is a state in which a liquid experiences zero viscosity, hence flows without friction past any surface.

EXERCISE

Multiple Choice Questions

Encircle the correct option.

1) Principle of floatation helps us to find:

A. Density B. Velocity C. Area D. Pressure

2) According to Archimedes' principle, the up-thrust on an object equal to:

A. weight of the displaced liquid
B. volume of the displaced liquid
C. mass of the displaced liquid
D. density of the displaced liquid

3) In the equation for Stoke's law, the sphere must be:

A. moving with a low, non-zero acceleration.
B. moving with a constant velocity.
C. at rest in the fluid.
D. of the same density as the fluid.

4) Two identical spherical drops of water are falling through air with a steady velocity of 20 cm s^{-1}. If the drops combine to form a single drop, what would be the terminal velocity of the single drop?

A. 10 cm s^{-1} B. 20 cm s^{-1} C. 32 cm s^{-1} D. 40 cm s^{-1}

5) Spherical balls of radius 'r' are falling in a viscous fluid of viscosity 'η' with a velocity 'v'. The retarding viscous force acting on the spherical ball is:

A. inversely proportional to 'r' but directly proportional to velocity 'v'
B. directly proportional to both radius 'r' and velocity 'v'
C. inversely proportional to both radius 'r' and velocity 'v'
D. directly proportional to 'r' but inversely proportional to 'v'

6) Two rain drops reach the Earth with their terminal velocities in the ratio 4:9. The ratio of their radii is:

A. 4 : 9 B. 2 : 3 C. 3 : 2 D. 9 : 4

7) A liquid flows through a pipe with a diameter of 10 cm at a velocity of 9 cm/s. If the diameter of the pipe decreases to 6 cm, the new velocity of the liquid will be:

A. 12 cm s^{-1} B. 15 cm s^{-1} C. 21 cm s^{-1} D. 25 cm s^{-1}

8) If a pipe with flowing water has a cross-sectional area nine times greater at point 2 than at point 1, what would be the relation of flow speed at the two points?

A. The flow speed at point 2 is nine times that at point 1
B. The flow speed at point 2 is three times that at point 1
C. The flow speed at point 1 is nine times that at point 2
D. The flow speed at point 1 is three times that at point 2

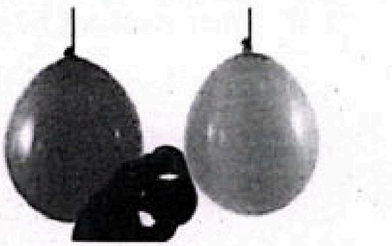
9) With an increase in temperature, the viscosity of liquids and gases, respectively will

A. increase and increase B. increase and decrease
C. decrease and increase D. decrease and decrease

10) According to equation of continuity, when water falls its speed increases, while its cross-sectional area ________

A. remains same B. decreases C. increases D. can't be predicted

11) The figure shows two balloons are hung by a string. When the air is blown harder through the straw, the balloons will:



A. remain stationary
B. move closely to each other
C. move far apart to each other
D. burst

12) Which will produce the greatest increase in flow velocity through a tube?

A. halving the tube radius B. doubling the viscosity of the liquid
C. doubling the tube area D. doubling the tube radius

13) Where does the Venturi effect specifically occur?

A. Where a stream is expanded
B. Where a stream flows downwards with gravity
C. Where a stream constricts
D. In the exact center of a flowing stream

14) The end of a hose has a diameter of 4 cm. If one wants the velocity of the water coming out to be 4 times higher, what should be the diameter of the nozzle on the end?

A. 1/4 cm B. 4 cm C. 2 cm D. 1/2 cm

Short Questions

Give short answers to the following questions:

6.1 Why do athletes, such as swimmers and bicyclists, wear body suits in competition.
6.2 Distinguish between turbulent and streamline flow.

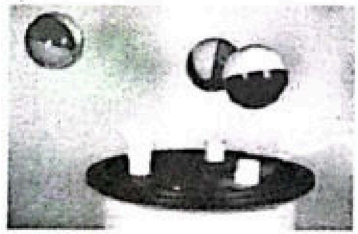
6.3 When there is a change in the width of the river. The speed of the water decreases in wider regions whereas the speed of water increases in the narrower regions. Why?

6.4 It is dangerous to stand close to rail tracks when a rapidly moving train passes. Explain why?

6.5 Verify that pressure has units of energy per unit volume.

6.6 A perfume bottle or atomizer sprays a fluid from inside the bottle when it is pressed. How does the fluid rise up in the vertical tube in the bottle?

6.7 If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?



6.8 Explain how an upthrust is produced when an aeroplane is running on runway?

6.9 Why does a ball placed in a vertical air jet become suspended (as shown in figure)?

6.10 Can you increase the flow velocity of water in a rubber pipe by squeezing? Explain briefly.

6.11 What is meant by superfluid? How is superfluid made?

Comprehensive Questions

Answer the following questions in detail:

6.1 What is up thrust? Explain with the help of examples.

6.2 Explain Archimedes' principle of flotation.

6.3 What is viscosity? Explain with the help of examples.

6.4 What is viscous drag? Explain with the help of examples.

6.5 Explain Stoke's law. Derive its mathematical formula by using dimensional analysis.

6.6 Explain terminal velocity and derive its equation by using Stoke's law.

6.7 State the equation of continuity and derive its mathematical form. Also write one example from daily life, where equation of continuity applies.

6.8 Derive the Bernoulli's equation. Also, discuss its application in daily life.

6.9 What is Torricelli's theorem. Explain.

6.10 What is venturi meter? Explain.

6.11 Describe super fluidity.

Numerical Problems

6.1 The settling speed of a small fog droplet in air is found to be 2.98 mm s^{-1}. Find out the radius of the droplet. The coefficient of viscosity of air is 1.9×10^{-5} kg m^{-1} s^{-1}. **(Ans: 5.1×10^{-6} m)**

6.2 Water is flowing at a velocity of 2.00 m s^{-1} through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m s^{-1}. What is the nozzle's inside diameter? **(Ans: 0.402 L s^{-1}, 0.584 cm)**

6.3 Blood is pumped from the heart at a rate of 5.0 L min^{-1} into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta. **(Ans: 27 cm s^{-1})**

6.4 Water flows through a pipe with an internal diameter 4 cm at a speed of 2 m s^{-1}. What should be the diameter of the nozzle, if the water is to emerge out with a speed of 4 m s^{-1}. **(Ans: 2.8 cm)**

6.5 A hose lying on the ground has water coming out of it at a speed of 5.4 m s^{-1}. You lift the nozzle of the hose to a height of 1.3 m above the ground. At what speed does the water now come out of the hose? **(Ans: 1.9 m s^{-1})**

6.6 A pipe has two different cross-sectional areas, A_1 = 25 cm^2 and A_2 = 4 cm^2. The volumetric flow rate of water through the pipe is 5×10^{-3} m^3 s^{-1}. Determine:

(a) the speed of water in both cross-sectional areas of the pipe.

(b) the pressure difference between them. (ρ = 10^3 kg m^{-3})

(Ans: 2 m s^{-1}, 12.5 m s^{-1}, 7.6×10^4 Pa)

6.7 The speed of water in a hose increases from 1.96 m s^{-1} to 25.5 m s^{-1} as it moves from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is 1.01×10^5 N m^{-2} (atmospheric, as it must be) and assuming level and frictionless flow.

(Ans: 4.24×10^5 N m^{-2})

6.8 A hole is drilled at the bottom of a full bucket of water of height h. What will be the velocity of water exiting from the hole? **(Ans: $\sqrt{2gh}$)**

6.9 The efflux speed of oil from a narrow hole of a tank is 33.8 m s^{-1}, at pressure of 4.12×10^5 Pa. Calculate the density of the oil? **(Ans: 0.72 kg m^{-3})**

6.10 If the speed of air across the top and bottom of a small aeroplane's wing is 30 m $s^{-1}$and 20 m s^{-1} respectively. Calculate the pressure difference between the top and bottom of wings if ρ = 1.29 kg m^{-3}, for air. **(Ans: 322.5 Pa)**

6.11 An empty metrological balloon has a weight of 80 N. How much maximum contents the balloon can lift besides its own weight if it is filled with 10 m^3 of hydrogen? (Density of hydrogen = 0.09 kg m^{-3}, Density of air = 1.3 kg m^{-3}). **(Ans: 38.58 N)**