UNIT 5

# WORK AND ENERGY

## Student Learning Outcomes (SLOs)

**The students will:**

- Derive the formula for kinetic energy [Using the equations of motion].
- Deduce the work done from force-displacement graph.
- Differentiate between conservative and non-conservative forces.
- Utilize the work - energy theorem in a resistive medium to solve problems.

# UNIT 5 WORK AND ENERGY

In the sport of weightlifting, the task is to pick up a very large mass, lift it over our head, and hold it there at rest for a moment. This action is an example of doing work by lifting and lowering a mass. Weightlifting requires energy from the body's metabolic processes, specifically from the breakdown of a molecule called adenosine triphosphate (ATP). Additionally, weightlifting can also utilize stored glycogen in muscle tissue, which can be broken down into glucose to provide energy.

## 5.1 WORK

The work done on an object is the scalar (or dot) product of force **F** and displacement **d**.

$$W = \mathbf{F}.\mathbf{d}$$

or $$W = Fd\cos\theta \qquad (5.1)$$

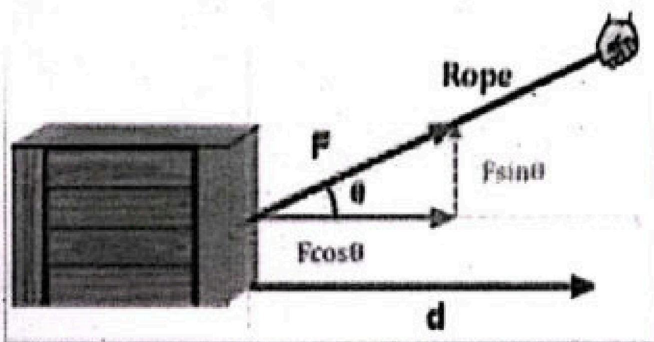



**Figure 5.1:** Work done on an object when a force 'F' produces displacement d.

In equation (5.1), 'F' is the magnitude of force, 'd' is the magnitude of displacement, and '$\theta$' is the angle between force and the displacement. From the Fig. 5.1, we can see that the force can be resolved into two components $F\cos\theta$ and $F\sin\theta$. Here, $F\cos\theta$ is the component of force along the direction of displacement which is the effective component. Whereas, $F\sin\theta$ is the component of force which is perpendicular to the direction of displacement and therefore plays no role in doing work.

**Units of Work:** The SI unit of work is the joule (J) (named in honor of the 19th-century English physicist James Prescott Joule). From above equation we see that the unit of work is the unit of force multiplied by the unit of distance. In SI the unit of force is the newton and the unit of distance is the metre, so 1 joule is equivalent to 1 newton-metre (N m).

$$1\ \text{J} = 1\ \text{N m}$$

**Dependence of Work:** The work done depends upon force 'F', displacement 'd' and the angle 'θ' between them, as shown in Fig. 5.2.

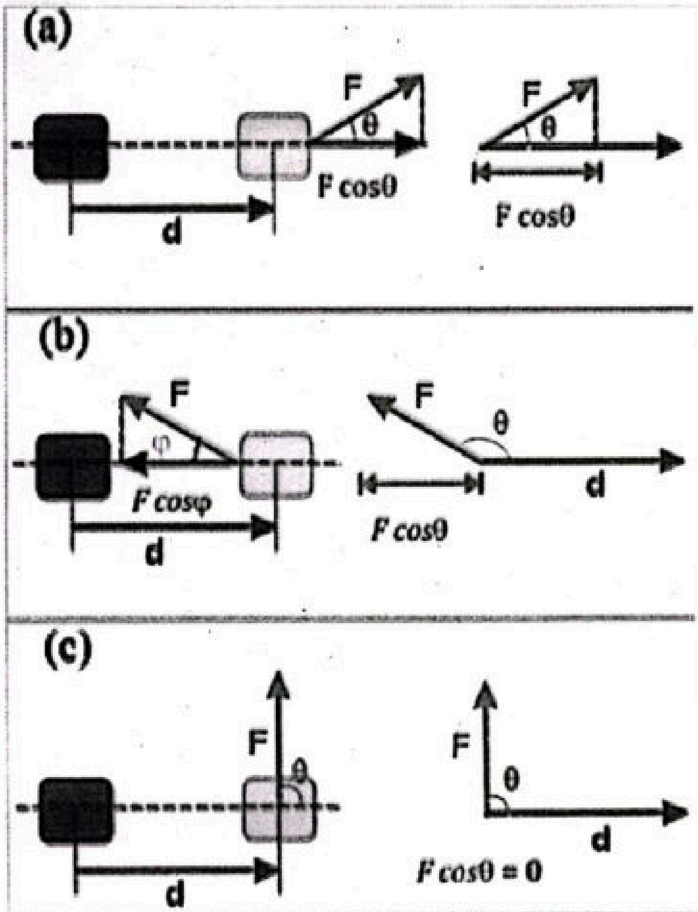



**Figure 5.2:** (a) positive work (b) negative work (c) zero work.

a) **Positive Work:** When the force has a component in the same direction as the displacement ($0^\circ \le \theta < 90^\circ$), $\cos\theta$ in above equation is positive and the work W is positive.

b) Negative Work: When the force has a component opposite to the displacement ($90^\circ < \theta \leq 180^\circ$), $\cos\theta$ is negative and work done by the force will be negative.

c) Zero Work: When the force is perpendicular to the displacement, $\theta = 90^\circ$ then the work done by the force is zero.

**Work Done from Force-Displacement Graph:** The area under the force-displacement graph gives the work done.

A. Work Done by Constant Force: In Fig. 5.3, graph for a constant force to produce net displacement is shown, here the blue shaded area of rectangle represent work done by the constant force.

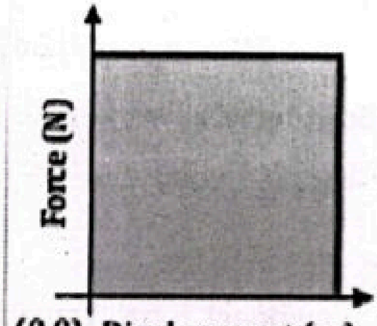



**Figure 5.3:** Work done by constant force.

B. Work Done by Variable Force: In Fig. 5.4, graph with increasing force to produce net displacement is shown, here the blue shaded area of triangle represent work done by the increasing force. However, in many situations of daily life, the force is variable in many different ways, and therefore we get different graphs for area under the curve for work done by such forces.

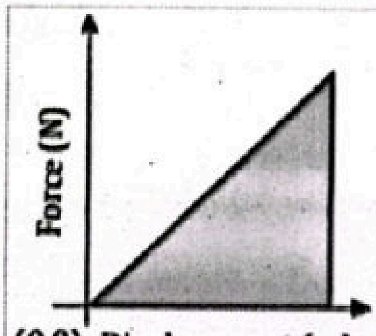



**Figure 5.4:** Work done by variable force.

For example, when we stretch a spring, the more we stretch it, the harder we have to pull, so the force we exert is not constant as the spring is stretched. When a rocket moves away from Earth the work is done against the force of gravity which varies as inverse of the square of distance from the center of Earth.

Consider a graph in Fig. 5.5, is drawn between $F\cos\theta$ and d. To find the total work done, we divide the displacement into number of small displacements $\Delta d_1$, $\Delta d_2$, $\Delta d_3$, ..., $\Delta d_n$, with corresponding effective component of forces are $F_1 \cos\theta_1$, $F_2 \cos\theta_2$, $F_3 \cos\theta_3$, ........., $F_n \cos\theta_n$.

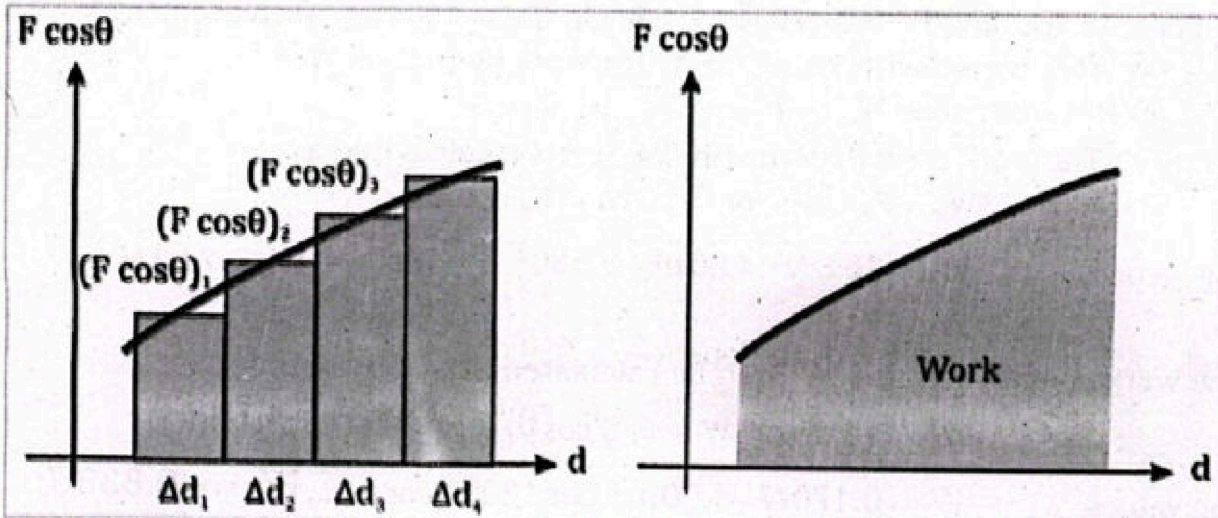



**Figure 5.5:** Graphical analysis of work done by variable force.

The total work done will now be the sum of all the individual work done.

$$W_{Total} = W_1 + W_2 + W_3 + \ldots + W_n$$

As, $W_1 = F_1 \cos\theta_1 \Delta d_1$, $W_2 = F_2 \cos\theta_2 \Delta d_2$, $W_3 = F_3 \cos\theta_3 \Delta d_3$, and $W_n = F_n \cos\theta_n \Delta d_n$.

So, the total work done is:

$$W_{Total} = F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + F_3 \cos\theta_3 \Delta d_3 + \ldots + F_n \cos\theta_n \Delta d_n \quad \text{(5.2)}$$

In compact form, the above equation can be written as:

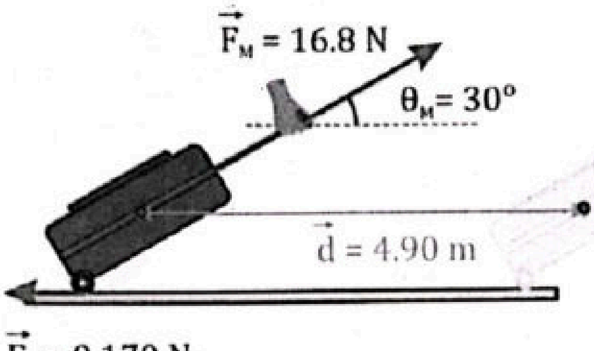
$$W_{Total} = \sum_{i=1}^{n} F_i \cos\theta_i \, \Delta d_i \quad \text{(5.3)}$$

James Prescott Joule (1818 – 1889)

Joule showed that heat was not a substance but, instead, the transfer of energy. He found that thermal energy produced by stirring water or mercury is proportional to the amount of energy transferred in the stirring.

Thus, the work done by a variable force is equal to the area under the *Fcos θ* and *d* curve.

**EXAMPLE 5.1:** Mehwish is dragging a suitcase at an airport and pulls it through a distance of 4.90 m along level ground. She is applying a constant force of 16.8 N in 30° with the horizontal. A 0.170 N friction force opposes the suitcase's motion. Find the work done by (a) Mehwish (b) frictional force and (c) net work done by all the forces acting on the suitcase.




Given: Force by Mehwish '$F_M$' = 16.8 N  Angle '$\theta_M$' = 30°
Force of friction '$F_f$' = 0.170 N  Angle '$\theta_F$' = 180°  Displacement 'd' = 4.90 m

To Find: (a) Work by Mehwish $W_M$ = ?  (b) Work by friction $W_f$ = ?
(c) Net work done $W_{net}$ = ?

Solution: (a) The work done by Mehwish '$W_M$' can be calculated as:

$$W_M = F_M \cdot d \quad \text{or} \quad W_M = F_M\, d \cos\theta_M$$

Putting values: $W_M = 16.8 N \times 4.90 m \times \cos 30°$ or $W_M = 71.3 Nm$

Hence, $W_M = 71.3 J$

(b) The work done by friction '$W_f$' can be calculated as:

$$W_f = F_f \cdot d \quad \text{or} \quad W_f = F_f\, d \cos\theta_f$$

Putting values: $W_f = 0.170 N \times 4.90 m \times \cos 180°$ or $W_f = -0.833 J$

(c) Although there is force of gravity due to weight of the suitcase acting on it. But as it is perpendicular to the direction of motion, it does not have an effective component in work. The net work done '$W_{net}$' is the sum of work done by Mehwish '$W_M$' and friction '$W_f$'.

$$W_{Total} = W_M + W_f$$

Putting values: $W_{Total} = 71.3\ J + (-0.833\ J)$

Therefore, $W_{Total} = 70.467\ J = 70.5\ J$

**Assignment5.1**

**A box having 40 kg mass is dragged on a frictionless inclined surface to a height of 8 m. If the inclined plane makes an angle of 20° with the Earth, find the work done against gravity.**

## 5.2 CONERVATIVE & NON-CONSERVATIVE FIELDS

The region around a body where it can influence other bodies by a force associated with it is called field of force or simply force field. Examples include electric field, viscous field and gravitational field.

### 5.2.1 Conservative Field

A field is said to be conservative if it has two important properties:

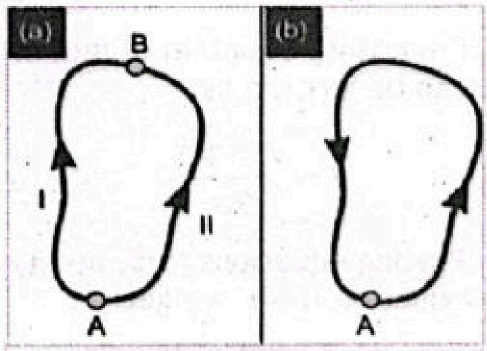



**Figure 5.6:** (a) An object moves between points A and B via two different paths 1 and 2. (b) The object makes a round trip back to A.

1. Work Done is Independent of the Path Taken: If the work done on a particle moving between any two points A and B is same for path I and II, as shown in Fig. 5.6 (a), the field will be conservative.

2. Work Done around a Closed Path is Zero: If the work done on a particle moving through any closed path (the path in which the beginning and end points are same) gives zero, as shown in Fig. 5.6 (b), such a field will be conservative.

Gravitational and electric fields are examples of conservative fields and the associated forces are conservative forces.

### 5.2.2 Non-Conservative Field

A non-conservative field is that field in which work done depends upon the path followed or the work done along a closed path is not zero.

Frictional field is a non-conservative field, and frictional or drag forces are non-conservative forces because when an object is moved in frictional field, the work done against frictional force depends upon the path followed.

Frictional force, viscous drag and air resistance are all examples of non-conservative forces and the fields where they act are termed as non-conservative fields.

## 5.3 KINETIC ENERGY

The energy possessed by a body due to its motion is called kinetic energy.

A boy kicks a football; it moves because it possess Kinetic energy. Now think a tennis ball and a football moving with same speed. Which possess greater ability to do work? Of course, the football with larger mass, which is difficult to stop. Similarly, now two footballs are approaching you with different speeds, which can do more work? Again, the football with greater speed is difficult to stop. Thus, the object's mass and its speed contribute to its Kinetic energy. Like all energies, Kinetic energy is also a scalar quantity.

Consider a situation in which all the work done transfers only kinetic energy to a cricket ball. Let the cricket ball is initially at rest $v_i = 0$ m/s, and a horizontal force F is applied to move it through a displacement 'd' and achieve a final velocity $v_f = v$, as shown in Fig. 5.7. This work done W appears as the kinetic energy K.E. Such that:

$$W = K.E = F\,d \quad \text{\_\_\_\_\_\_} \quad (5.4)$$

By Newton's Second Law of motion

$$F = ma \quad \text{\_\_\_\_\_\_} \quad (5.5)$$

From third equation of motion, distance d can be written as:

$$d = \frac{v_f^2 - v_i^2}{2a} \quad \text{\_\_\_\_\_\_} \quad (5.6)$$

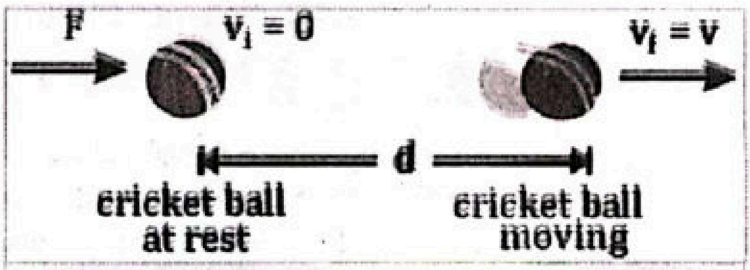



Figure 5.7: Work done transfers kinetic energy to a cricket ball.

Putting equations (5.5) and (5.6) in equation (5.4), we get:

$$K.E = \frac{1}{2}m\left(v_f^2 - v_i^2\right) \quad \text{\_\_\_\_\_\_} \quad (5.7)$$

As the object (cricket ball) started from rest therefore $v_i = 0$ and $v_f = v$.

Therefore, $$K.E = \frac{1}{2}mv^2 \quad \text{\_\_\_\_\_\_} \quad (5.8)$$

Although derived for cricket ball, the equation (5.8) in general shows the relation for the kinetic energy of any moving object. For example, an iron ball of 1.0 kg moving with a speed of 2.0 m $s^{-1}$ has a Kinetic energy of 2 J.

Equally important, it demonstrate the work kinetic energy theorem which states that work done on an object is equal to change in energy i.e. $W = \Delta E$, where 'W' is the work done and $\Delta E$ is the change in energy.

## 5.4 WORK - ENERGY PRINCIPLE

Since work is defined as the movement of an object through a distance, energy can also be described as the ability to move an object through a distance. The net work done on an object is equal to the change in the object's kinetic energy, i.e.,

$$W_{net} = K.E_f - K.E_i = \Delta K.E \quad \text{\_\_\_\_\_\_} \quad (5.9)$$

Where, the change in the kinetic energy is due to the object's change in speed.

Work-Energy Theorem is also valid for potential energy. However, potential energy cannot be defined only for conservative forces.

In situations where potential energy can be defined, change in potential energy is exactly equal to the **negative** of change in kinetic energy, in which case the Work-Energy theorem becomes

$$W_{net} = \Delta K.E = -\Delta U \quad \_\_\_\_\_ \quad (5.10)$$

## 5.4.1 Work - Energy Theorem in Resistive Medium

Why is a skydiver quite confident when using a parachute? Probably she relies on balance between resistive forces and her weight. A resistive force on a moving object opposes the motion of the object or prevent a stationary object from moving, for example friction, viscous force etc.

When forces are acting on an object, energy transformations occur, which means the work is being done. The work done '$W$' on an object by an applied force 'F' is the sum of the gain in kinetic energy '$\Delta K.E$' of the object and the work done by the object against the resistive force '$W_r$'. That is:

$$W = \Delta K.E + W_r$$

The work done by the object against the resistive force takes energy away from the object, decreasing its kinetic energy. Mathematically:

$$W_r = W - \Delta K.E \quad \_\_\_\_\_ \quad (5.11)$$

Both Newton's laws and work energy theorem can be used to solve problems. However, if the forces are not constant, Newton's laws will be difficult to apply.

## 5.4.2 Implications of Energy Loses in Practical Devices and Efficiency

**Input of Mechanical Machine:** The energy supplied to a mechanical machine is called input. Input is equal to the product of effort 'p' and the distance 'd', through which effort acts (Fig. 5.8).

$$input = p \times d$$

**Output of Mechanical Machine:** The work done by a mechanical machine is called output. Output is the product of load 'W' and the distance 'D' through which the load lifts.

$$output = W \times D$$

**Efficiency:** The efficiency is the ratio of work output to the work input, and can be expressed in percentage as:

$$efficiency = \frac{\text{useful energy output}}{\text{energy input}}$$

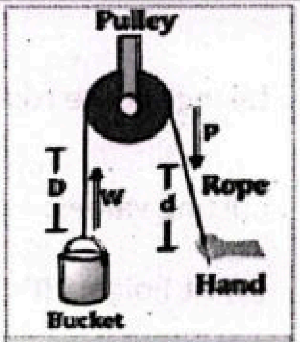


Figure 5.8: Input and out of pulley.

Efficiency is the ratio of similar quantities and therefore has no unit.
The efficiency cannot be greater than 1 (or 100%), in fact it cannot be equal to 1 (or 100%) for real machines. In case of pulley system, the efficiency is only 40%, rest of the 60% goes into waste and converted into unwanted forms of energy.

**Ideal Machine:** For an ideal machine only, the input will be equal to output, this means that no energy is wasted and all energy is converted into useful work. Therefore, the efficiency of an ideal machine is 100 %.

$$output = input \quad \text{or} \quad efficiency = 1$$

Almost all energy transformation technologies operate at efficiencies less than 100 %, most of the wasted energy becomes thermal energy.

Table 5.1: Typical efficiencies of energy transformation technologies.

| Device | Efficiency (%) |
|---|---|
| Electric generator | 98 |
| Hydroelectric power plant | 95 |
| Large electric motor | 95 |
| Home gas furnace | 85 |
| Wind generator | 55 |
| Fossil fuel power plant | 40 |
| Automobile engine | 25 |
| Fluorescent light | 20 |
| Incandescent light | 5 |

For example, *automobiles* are highly inefficient. Suppose that an amount of fuel containing 1000 J of chemical potential energy is used by an automobile's engine. Only 10 % of the energy is available to do work.

$$\text{Efficiency} = \frac{\text{energy output}}{\text{energy input}} \times 100$$

or

$$\text{Efficiency} = \frac{100\,\text{J}}{1000\,\text{J}} \times 100 = 10\,\%$$

Only about 10% (or 100 J) of useful work is done in producing the kinetic energy of the moving car; rest of the energy is wasted in engine and cars transmission. Some other examples of energy losses in practical devices are given in Table 5.1.

**EXAMPLE 5.2: In a science experiment, a 3.00 kg water rocket is launched from ground. The rocket's total energy at the top of its flight is 2352 J. (a) What was the rocket's launching speed? (b) What height did the rocket reach? (c) What are the kinetic energy and potential energy of the rocket 2.5 s after its launch?**

Given: Mass of rocket 'm' = 3.00 kg; Total energy '$E_T$' = 2352 J

Acceleration due to gravity 'g' = 9.8 m $s^{-2}$; Time 't' = 2.5 s

To Find: (a) Initial speed 'v' = ? (b) Height 'h' = ?

(c) 'K.E' of rocket after 2 s = ? 'P.E' of rocket after 2 s = ?

Solution: (a) By law of conservation of energy, when the rocket is launched it K.E is converted into P.E, and at height 'h' this potential energy is equal to the total energy.

$$K.E = E_T \quad \text{or} \quad \frac{1}{2}mv^2 = E_T \quad \text{or} \quad v^2 = \frac{2 \times E_T}{m}$$

taking square root on both sides:

$$v = \sqrt{\frac{2 \times E_T}{m}}$$

putting values:

$$v = \sqrt{\frac{2 \times 2352}{3.00}} \quad \text{or} \quad v = 39.6\,\text{m s}^{-1}$$

(b) At height 'h', the potential energy is equal to the total energy.

$$P.E = E_T \quad \text{or} \quad mgh = E_T \quad \text{or} \quad h = \frac{E_T}{mg}$$

Putting values:

$$h = \frac{2352}{3.00 \times 9.80} \quad \text{or} \quad h = 80\,m$$

(c) To find 'K.E' after 2.5 seconds we will first have to calculate the speed of rocket by using first equation of motion along y-axis

$$v_f = v_i - g \times t$$

Putting values: $v_f = 39.6 - 9.8 \times 2.5$

Therefore, $v_f = 15.1\, ms^{-1}$

Putting this value in the kinetic energy equation $K.E = \frac{1}{2}mv_f^2$

Putting values: $K.E = \frac{1}{2} \times 3.00 \times (15.1)^2$ or $K.E = 342J$

To find the potential energy, we will use law of conservation of energy

$E_T = K.E + P.E$ Or $P.E = E_T - K.E$

putting values: $P.E = 2352 - 342$

Hence, $P.E = 2010\ J$

Assignment 5.2

**A crane holding 5000 kg of mass at a height of 12 m. Suddenly, the crane un-holds the mass. Find the kinetic energy of the mass just before striking the ground.**

## SUMMARY

- **Work:** The work done on a body by a constant force is defined as the product of the displacement and the components of the force in the direction of the displacement.
- The energy possessed by a body due to its motion is called **Kinetic energy.**
- **Gravitational field as conservative field:** Gravitational field is conservative field as work done is independent of the path followed and work done along the closed path is zero.
- A **non-conservative field** is that field in which work done depends upon the path followed or the work done along a closed path is not zero.
- **Efficiency:** The efficiency is the ratio of work output to the work input.

## EXERCISE

### Multiple Choice Questions

**Encircle the correct option.**

1) If the unit of force and displacement travelled each be increased five times, then the unit of work will be increased by:

A. 25 times B. 10 times C. 5 times D. 0 times

2) The force that acts on a body but does no work is:

A. gravitational force B. frictional force C. elastic force D. centripetal force

3) The odd force from the following is:

A. gravitational force B. elastic force C. frictional force D. electric force

4) The work done by a body while covering a vertical height of 10 m is 500 J. By how much amount does the energy of the body change?

A. 500 J B. - 500 J C. 50 J D. - 50 J

5) Two objects, A and B, have the same mass. Object A is moving at twice the speed of object B. The kinetic energy of object A as compared to object B is

A. $K.E_A = 4\ K.E_B$. B. $K.E_A = 2\ K.E_B$ C. $K.E_A = K.E_B$ D. $K.E_A = \frac{1}{2}\ K.E_B$

6) If velocity of a body reduces to half of its initial value, then the kinetic energy of the body compared to its initial value is:
A one fourth B. double C. four times D. half

7) A moving hockey ball is hit, such that its speed doubles. Its kinetic energy will become
A. *double* B. quadruple C. halve D zero

8) An engine converts 600 J of input energy into 150 J of useful work. The efficiency of the engine is
A. 25 % B. 50 % C. 75 % D. 100 %

9) Two engines, A and B, are rated for the same power output. Engine A has a higher efficiency than engine B. What can you conclude?
A. Engine A requires less fuel to produce the same power.
B. Engine A is physically larger and heavier than engine B.
C. Engine B produces more waste heat than engine A.
D. The efficiency rating is unrelated to fuel consumption.

10) Consider two machines, A and B. Machine A has an efficiency of 40 % and Machine B has an efficiency of 60 %. If both machines are provided with 500 J of energy, how much more useful work does Machine B perform compared to Machine A?

A. 50 J B. 100 J C. 200 J D. 300 J

11) A wind turbine converts 30 % of the wind's kinetic energy into electrical energy. If the kinetic energy of the wind is 900 J. The electrical energy produced by the turbine is
A. 90 J B. 270 J C. 600 J D. 700 J

## Short Questions

**Give short answers of the following questions.**

5.1 Is it possible that a force is acting on a body and the body is in motion due to this force but the work done after certain time is zero?

5.2 Some non-conservative forces are acting on a body. Can they change the total mechanical energy of the body?

5.3 Kinetic energy and work are related. Can kinetic energy ever be negative? Can work ever be negative?

5.4 Differentiate between conservative and non-conservative forces.

5.5 What is the work done by the moon as it revolves around the Earth?

## Comprehensive Questions

**Answer the following questions in detail.**

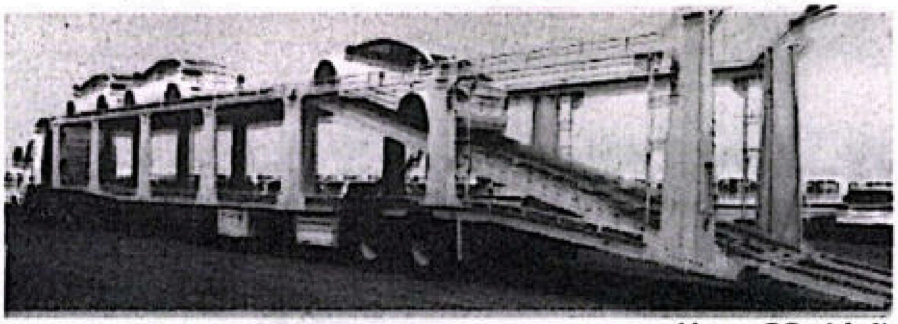
5.1 Derive the formula for kinetic energy by using the equations of motion.

5.2 Explain work done by a constant and variable force using force-displacement graph.

5.3 What is work-energy theorem? Explain in detail. Also, write some implications of energy loses in practical devices and efficiency.

## Numerical Problems

5.1 A car carrying truck unloads a 1500 kg car using a plank, as shown in figure. If the plank makes an angle of $30^0$ with the ground and its upper end is at 2 m height, what will be the work done by gravitational force? Also, draw the force-displacement graph for.



**(Ans: 29.4 kJ)**

5.2 A car, having a total mass of 1500 kg (including the driver), is travelling at a speed of 40 kph through a straight path. How much work will be required to stop the said car if its brakes fail and engine turns off? **(Ans: 92.6 kJ)**

5.3 A ball of mass 100 g is released from a height of 30 m. If the ball encounters an air resistance of 0.4 N, find the kinetic energy of the ball just before striking the ground.

**(Ans: 17.4 J)**

5.4 (a) What is the kinetic energy of a car with a mass of 1200 kg traveling at a speed of 20 m $s^{-1}$? (b) If the car comes to a stop over a distance of 50 meters due to braking, what is the average force exerted by the brakes? **(Ans: 240 kJ, 4800 N)**

5.5 A roller coaster cart weighing 500 kg is at the peak of a 30 m tall hill and is traveling at 5 m $s^{-1}$. Find the total mechanical energy at the top of the hill. If it descends to the bottom without any friction losses, what will be its speed at the bottom?**(Ans:153,250 J, 24.75 m $s^{-1}$)**

5.6 A car engine operates with 25 % efficiency and produces 50 kJ of useful work. How much fuel energy did the car consume? **(Ans: 200 kJ)**

5.7 A refrigerator uses 600 W of electrical power and has an efficiency of 15 %. How much cooling power (useful energy) does the refrigerator produce in one hour? **(Ans: 324 kJ)**