TRANSLATORY MOTION

UNIT 3

Student Learning Outcomes (SLOs)

The students will:

- Derive the equations of motion [For uniform acceleration cases only. Derive from the definitions of velocity and acceleration as well as graphically].
- Solve problems using the equations of motion [For the cases of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. This also includes situations where the equations of motion need to be resolved into vertical and horizontal components for 2-D motion].
- Evaluate and analyse projectile motion in the absence of air resistance [This includes solving problems making use of the below facts:

  (i) Horizontal component (V_H) of velocity is constant.

  (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.

  (iii) The horizontal motion and vertical motion are independent of each other. Situations may require students to determine for projectiles:

  - How high does it go?

  - How far would it go along the level land?

  - Where would it be after a given time?

  - How long will it remain in flight?

  Situations may also require students to calculate for a projectile launched from ground height the

  - launch angle that results in the maximum range.

  - relation between the launch angles that result in the same range.
- Predict qualitatively how air resistance affects projectile motion [This includes analysis of both the horizontal component and vertical component of velocity and hence predicting qualitatively the range of the projectile.]
- Apply the principle of conservation of momentum to solve simple problems.
- [Including elastic and inelastic interactions between objects in both one and two dimensions. Knowledge of the concept of coefficient of restitution is not required.
- Examples of applications include:

  - karate chops to break a pile of bricks

  - car crashes

  - ball & bat

  - the motion under thrust of a rocket in a straight line considering short thrusts during which the mass remains constant]
- Predict and analyse motion for elastic collisions [This includes making use of the fact that for an elastic collision, total kinetic energy is conserved and the relative speed of approach is equal to the relative speed of separation]
- Justify why though the momentum of a closed system is always conserved, some change in kinetic energy may take place.

The cover photo of this chapter shows an anti-ship missile being fired by Pakistan Navy in the North Arabian Sea. The motion of a missile through the air can be described by its range, velocity, and acceleration. When it flies, a short patch of its path can be considered a straight line without any change in direction. However, its motion is not straight for long; instead, it is a 2-dimensional motion under the action of gravity, called projectile motion. Similarly, all the objects in this universe are in motion. Thus, understanding motion is also key to understanding other concepts in physics. For example, an understanding of velocity is crucial to the study of momentum. There are three types of motion: translational motion, rotational motion and vibrational motion. In this chapter, we will concern ourselves only with translational motion, such as motion along a straight line and projectile motion.

We begin with kinematics which is the branch of mechanics that deals with the study of motion without considering the causes of motion. This includes the terms such as displacement, velocity and acceleration in straight line. Using these terms, we study the motion of objects undergoing constant acceleration. We begin with derivation of the equations of motion and then we use them to solve problems of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. It is also extended to objects moving along curved paths, such as projectile motion under the action of gravity only and its applications. Next, we deal with dynamics, which is the branch of mechanics which deals with the study of forces that cause the objects and systems to move. This includes momentum and associated law of conservation.

3.1 EQUATIONS OF UNIFORMLY ACCELERATED MOTION

There are three equations of motion which shows the relation between initial velocity, final velocity, acceleration, displacement and time. These equations describe and predict the motion of objects under constant acceleration. To simplify the derivation of these equations, the following **assumptions** are made:

- The object is moving along the straight line.
- Acceleration is constant.
- Only magnitudes of vectors such as displacement, velocity and acceleration are considered.
- The direction of initial velocity and all the quantities in the direction of initial velocity are taken as positive.

Following are the three equations of motion:

$$v_f = v_i + a\,t$$
$$S = v_i\,t + \frac{1}{2}a\,t^2$$
$$2\,a\,S = v_f^{\,2} - v_i^{\,2}$$

Let us derive these equations of motion.

3.1.1 First Equation of Motion

Consider a body is moving with initial velocity v_i in a straight line with constant acceleration a. Its velocity becomes v_f after time t. The velocity-time graph for the motion of the body is shown in the Fig. 3.1. As we know that the slope of velocity-time graph gives the acceleration of body. From the graph:

a = slope of line AB

or $a = \dfrac{BC}{AC}$

or $a = \dfrac{BD - CD}{OD}$ ______ (3.1)

From the graph, it can be noted that BD represents the final velocity v_f, CD represents the initial velocity v_i and OD represents the time t. Hence, we can write the equation (3.1) as:

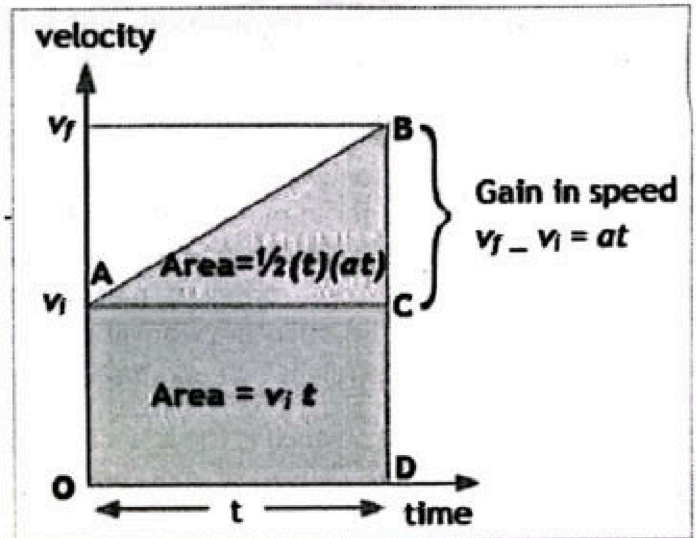


Figure 3.1: Velocity-time graph for a body moving with constant acceleration.

or $a = \dfrac{v_f - v_i}{t}$

or $at = v_f - v_i$

or $v_f = v_i + at$ ______ (3.2)

Equation (3.2) is the first equation of motion. It represents the relation between initial velocity, final velocity, acceleration and time.

3.1.2 Second Equation of Motion

Consider a body is moving with initial velocity v_i in a straight line with constant acceleration a. Its velocity becomes v_f after time t. The velocity-time graph for the motion of the body is shown in the Fig. 3.1.

As the total distance S travelled by the body is equal to the total area under the velocity-time graph. Hence:

Total distance S = Area of trapezium OABD

$$S = \frac{1}{2}\left(\begin{matrix}\textit{sum of}\\ \textit{parallel sides}\end{matrix}\right) \times \left(\begin{matrix}\textit{distance between}\\ \textit{the parallel sides}\end{matrix}\right)$$

As shown in the graph that OA and BD are parallel sides, and OD is the distance between these parallel sides. Therefore, we can express the total distance in terms of the above equation as:

$$S = \frac{1}{2}(OA + BD) \times OD$$

Putting OA = v_i, BD = v_f and OD = t, we get:

$$S = \frac{1}{2}(v_i + v_f) \times t$$

Now, using first equation of motion $v_f = v_i + at$, the above equation may become:

$$S = \frac{1}{2}(v_i + v_i + a\,t) \times t$$

or $$S = \frac{1}{2}(2v_i + at) \times t$$

or $$S = \frac{1}{2}(2\,v_i t + a\,t^2)$$

Finally, we get:

$$S = v_i t + \frac{1}{2} a\,t^2 \quad \_\_\_\_\_\_ \quad (3.3)$$

Equation (3.3) is the second equation of motion, it represents the relation between initial velocity, acceleration, distance and time.

3.1.3 Third Equation of Motion

Consider a body is moving with initial velocity v_i in a straight line with constant acceleration a. Its velocity becomes v_f after time t. The velocity-time graph for the motion of the body is shown in the Fig. 3.1.

As the total distance S travelled by the body is equal to the total area under the graph. Hence:

Total distance S = Area of trapezium OABD

$$S = \frac{1}{2}\begin{pmatrix}\textit{sum of}\\ \textit{parallel sides}\end{pmatrix} \times \begin{pmatrix}\textit{distance between}\\ \textit{the parallel sides}\end{pmatrix}$$

As shown in the graph that OA and BD are parallel sides, and OD is the distance between these parallel sides. Therefore, we can express the total distance in terms of the above equation as:

$$S = \frac{1}{2}(OA + BD) \times OD$$

or $$2S = (OA + BD) \times OD$$

Multiplying both sides by $\frac{BC}{OD}$, we get:

$$2S \times \frac{BC}{OD} = (OA + BD) \times OD \times \frac{BC}{OD}$$

As, $\frac{BC}{OD} = a$, so we get:

$$2S \times a = (OA + BD) \times BC$$

Putting $OA = v_i$, $BD = v_f$ and $BC = v_f - v_i$, we get:

$$2aS = (v_i + v_f) \times (v_f - v_i)$$

or $$2aS = v_f^2 - v_i^2 \quad \_\_\_\_\_\_ \quad (3.4)$$

Equation (3.4) is the third equation of motion, it represents the relation between initial velocity, final velocity, acceleration and distance.

Example 3.1: A bus is travelling at 10 m s^{-1} accelerates uniformly at 2 m s^{-2}. Calculate the velocity of the bus after 5 s.

Given: Initial velocity of the bus = v_i = 10 m s^{-1} Acceleration = a = 2 m s^{-2}

Time = t = 5 s

To Find: Final velocity of the bus = v_f = ?

Solution: Using 1st equation of motion:

$$v_f = v_i + a\,t$$

Putting values and solving, we get:

$$v_f = 10 + 2(5)$$

$$v_f = 20\ \text{m s}^{-1}$$

Example 3.2: A car is travelling initially with a velocity of 4 m s^{-1} then it accelerates at 1 m s^{-2} for 10 s. Find the distance travelled by the car during this time.

Given: Initial velocity of the car = v_i = 4 m s^{-1} Acceleration = a = 1 m s^{-2}

Time = t = 10 s

To Find: Distance travelled = S = ?

Solution: Using 2nd equation of motion:

$$S = v_i t + \frac{1}{2} a\,t^2$$

Putting values and solving, we get:

$$S = 4(10) + \frac{1}{2}(1)\,(10)^2$$

$$S = 40 + \frac{1}{2}(100) = 40 + 50 = 90\ \text{m}$$

Assignment 3.1

1) A train slows down from 80 km h^{-1} with a uniform retardation of 2 m s^{-2}. How long will it take to attain a speed of 20 km h^{-1}?

2) A car travels with a velocity of 5 m s^{-1}. It then accelerates uniformly and travels a distance of 50 m. If the velocity reached is 15 m s^{-1}, find the acceleration and the time to travel this distance.

3.1.4 Free-Fall Motion

An example of uniformly accelerated motion is the motion of a free-falling body. In the absence of air resistance, all objects (lighter or heavier) fall freely near the surface of the Earth with same acceleration, independent of their masses. This acceleration is called acceleration due to gravity, denoted by g. It has a value of 9.81 m s^{-2} and is directed towards centre of the Earth. By substituting $a = g$ and $S = h$ in equations (3.2), (3.3) and (3.4), we get the equations of motion for free-fall as given below:

$$v_f = v_i + g\,t$$

$$h = v_i\,t + \frac{1}{2} g\,t^2$$

$$2\,g\,h = (v_f)^2 - (v_i)^2$$

Using these equations, we can solve problems for the motion of bodies falling in a uniform gravitational field without air resistance.

Example 3.3: A kangaroo can jump over an object 2.50 m high. (a) Considering just its vertical motion, calculate its vertical speed when it leaves the ground. (b) How long a time is it in the air?

Given: Height = h = 2.50 m Final velocity = v_f = 0 m s^{-1} (At highest point)

To Find: (a) Initial velocity = v_i = ? (b) Total time in the air = T = ?

Solution: (a) From 3rd equation of motion:

$$2\,g\,h = (v_f)^2 - (v_i)^2$$

$$2\,(-9.8)\,(2.5) = 0 - v_i^2$$
$$v_i^2 = 49\ m^2\,s^{-2}$$
$$v_i = 7\ m\,s^{-1}$$

(b) Now using 1st equation of motion:

$$v_f = v_i + g\,t$$

Where t is the time to reach the maximum height, g will be negative for moving up, so

$$0 = 7 - 9.8\,t$$
$$t = 0.71\ s$$

So, the total time in air = T = 2 (0.71 s) = 1.42 s

Assignment 3.2

A 1 kg ball is dropped from top of the leaning tower of Pisa. The ball reaches the ground in 3.34 s. Find the (a) height of the tower (b) velocity of the ball when it strikes the ground.

3.2 PROJECTILE MOTION

Till now, we have studied the motion of bodies in a straight line, either horizontal or vertical. There are many situations in which a body moves along a curved path in a plane having both vertical and horizontal components. For example, when a player kicks football to another player, it moves in a curved path, as shown in Fig. 3.2. Similarly, when a stone thrown horizontally in air from the top of a building, or a long jumper leaving the ground at an angle, etc., are the examples of projectile motion.

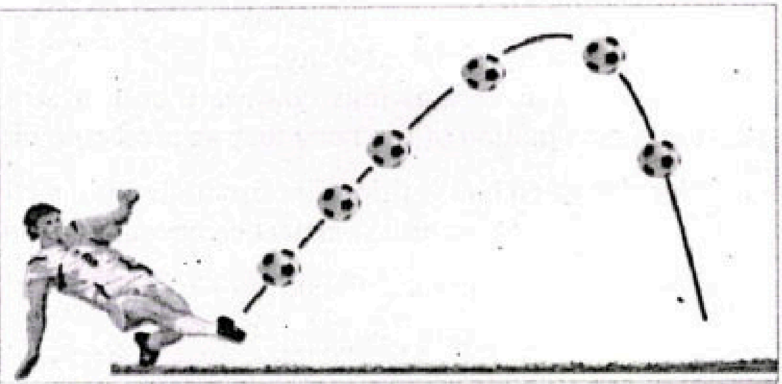


Figure 3.2: Two-dimensional motion of a football.

Projectile motion is a two-dimensional motion of an object thrown in the air under the action of gravitational force only.

In our discussion of projectile motion, the effects of other forces, such as air friction and rotation of the Earth, are neglected. The object that is thrown is called projectile and its path is called its trajectory. A football, a cricket ball or a baseball when thrown in air are examples of projectiles.

Experimentally, it can be proved that the horizontal and vertical motions are completely independent of each other.

Consider the motion of two different coloured balls, blue and green, as shown in Fig. 3.3. The blue ball is dropped vertically downward while the green ball is thrown horizontally at the same time from a cliff. Neglecting air resistance, both the balls hit the ground at the same time. The key to analyse two-dimensional projectile motion is to break the motion into two components, one along the horizontal axis and other along vertical axis.

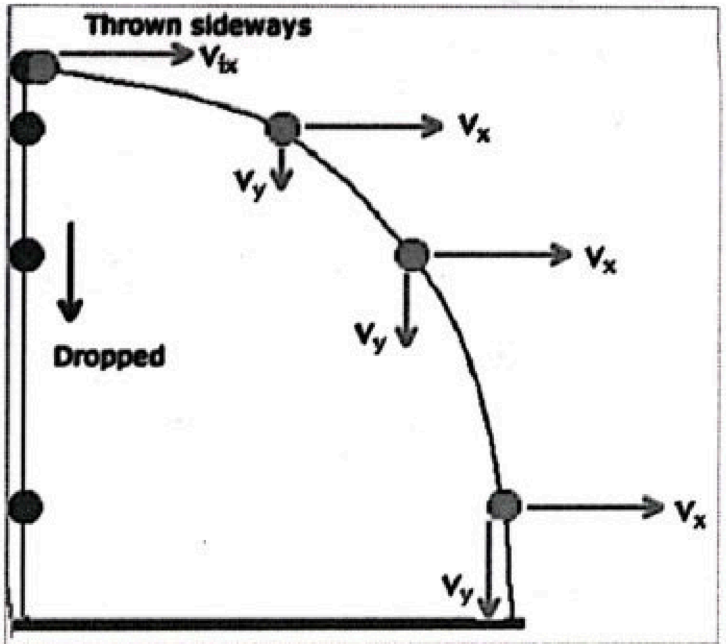


Figure 3.3: The two balls, one is thrown horizontally and other is thrown vertically down, hit the ground at the same time.

3.2.1 Projectile Motion for an Object Launched Horizontally

In many situations, when a projectile is thrown horizontally from a certain height with some initial velocity, it travels forward as well as falls downward until it strikes the ground. Hence, neglecting air resistance, the motion of the body follows projectile motion.

Consider the green ball is thrown horizontally from a cliff of certain height with velocity v_i, as shown in Fig. 3.3. There is no vertical component of the initial velocity, i.e., $v_{iy} = 0$. Hence:

Initial horizontal component of velocity = $v_{ix} = v_i$

Initial vertical component of velocity = $v_{iy} = 0$

Thus, there is **no horizontal acceleration,** while the **vertical acceleration is the acceleration due to gravity,** i.e.,

$$a_x = 0, \qquad a_y = g$$

Acceleration due to gravity g is taken positive when the ball is falling downward and negative when the ball is moving upward.

The horizontal component of velocity remains constant, while the vertical component of velocity increases. So, at any instant t, its velocity components are:

Horizontal component of velocity is:

$$v_{fx} = v_{ix} = v_i \cos\theta$$

Vertical component of velocity is:

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = 0 + g\,t$$

$$v_{fy} = g\,t$$

Hence, the magnitude of instantaneous velocity t is given by:

$$v_f = \sqrt{(v_{fx})^2 + (v_{fy})^2} \quad \_\_\_\_\_ (3.9)$$

The direction θ of instantaneous velocity v_f is determined as:

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right) \quad \_\_\_\_\_ (3.10)$$

At instant t, the **horizontal displacement** X covered by the body is given by:

$$X = v_{fx}\,t \quad \_\_\_\_\_ (3.11)$$

And the **vertical displacement**, as the body moves downward from the height is given by:

$$Y = \frac{1}{2}gt^2 \quad \_\_\_\_\_ (3.12)$$

3.2.2 Projectile Motion for an Object Launched at Some Angle with Horizontal

There are many situations in which a projectile (object) is thrown with velocity v_i at an angle θ with the horizontal. After given an initial push, it moves under the action of force of gravity, as shown in Fig. 3.4. Here, the air resistance is neglected.

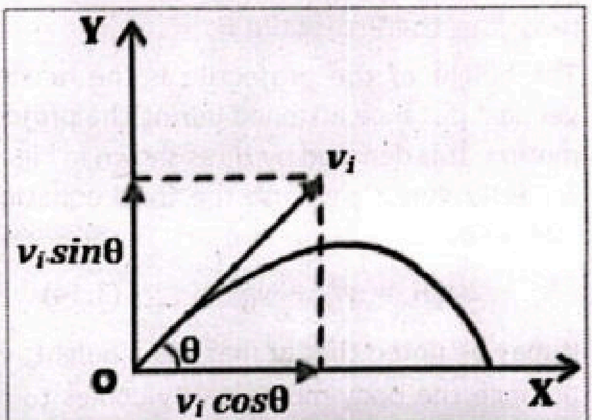


Figure 3.4: Rectangular components of initial velocity.

There is no horizontal acceleration, while the **vertical acceleration is the acceleration due to gravity**, i.e.,

$$a_x = 0, \qquad a_y = -g$$

Resolve the initial velocity into its horizontal and vertical components:

$$v_{ix} = v_i \cos\theta$$

$$v_{iy} = v_i \sin\theta$$

The horizontal component of velocity remains constant, while the vertical component of velocity changes. So, at any instant t, its velocity components are expressed as:

Horizontal component of velocity is:

$$v_{fx} = v_{ix} = v_i \cos\theta$$

Vertical component of velocity is:

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = v_i \sin\theta - g\,t$$

Hence, magnitude of instantaneous velocity at any instant t is given by:

$$v_f = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$

The direction θ of instantaneous velocity v_f is determined as:

$$\theta = \tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)$$

At instant t, the **horizontal displacement** x covered by the body is given by:

$$x = v_{fx}\,t$$

$$x = v_i \cos\theta \times t$$

And the **vertical displacement**, as the body moves upward, is given by:

$$y = v_{iy}\,t + \frac{1}{2} a_y t^2$$

$$y = (v_i \sin\theta)\,t - \frac{1}{2} g t^2 \quad \text{\_\_\_\_\_\_ (3.13)}$$

We will describe the maximum height, the range and time of flight of a projectile in the coming sections.

Height of the Projectile H:

The height of the projectile is the maximum vertical distance attained during the projectile motion. It is denoted by H, as shown in Fig. 3.5. To determine H, we use the third equation of motion as;

$$2\,a_y H = v_{fy}^2 - v_{iy}^2 \quad \text{\_\_\_\_\_\_ (3.14)}$$

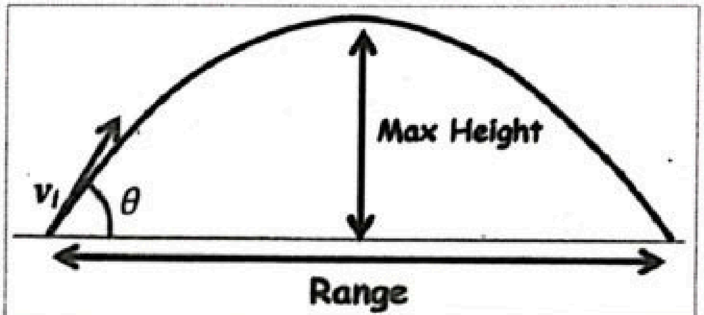


Figure 3.5: Height and range of a projectile.

It may be noted that at maximum height, $v_{fy} = 0$, since the body momentarily comes to rest. Also, for a projectile moving upward, $a_y = -g$, and equation (3.14) becomes:

$$2(-g)H = 0 - v_{iy}^2$$

Since, $v_{iy} = v_i \sin\theta$ so, we get: $\quad -2\,g\,H = 0 - v_i^2 \sin^2\theta$

By arranging, we get:

$$H = \frac{v_i^2 \sin^2\theta}{2g} \quad \text{\_\_\_\_\_\_ (3.15)}$$

Using this equation, we can find the height of the projectile if the magnitude and direction of the initial velocity is known.

Time of flight T:

The time taken by the projectile from the point of projection to the point where it hits the ground at same level is called the time of flight. It is denoted by T. To determine T, we use the second equation of motion:

$$s = v_i t + \frac{1}{2} a t^2 \qquad \_\_\_\_\_\_ (3.16)$$

For y-direction, equation (3.16) becomes:

$$H = v_{iy} t + \frac{1}{2} a_y t^2$$

Since, the projectile returns to ground (same height), so we take $H = 0$. Substituting $a_y = -g$, $v_{iy} = v_i \sin\theta$ and $t = T$ in above equation, we get:

$$0 = (v_i \sin\theta)\, T - \frac{1}{2} g T^2$$

By rearranging, we get:

$$T = \frac{2v_i \sin\theta}{g} \qquad \_\_\_\_\_\_ (3.17)$$

If magnitude and direction of the initial velocity is known, then this equation can be used to find the time of flight of the projectile.

The time taken by the projectile to reach the highest point is called the time of summit. It is denoted by T′ and is given by:

$$T' = \frac{T}{2} = \frac{v_i \sin\theta}{g} \qquad \_\_\_\_\_\_ (3.18)$$

Range of the Projectile R:

The horizontal distance travelled by a projectile is called range. It is denoted by R. To determine R, we use the relation, $S = v t$. As the range is horizontal distance, so we can write:

$$X = v_x t$$

Substituting $X = R$, $v_x = v_i \cos\theta$ and $t = T = \dfrac{2v_i \sin\theta}{g}$ in above equation, we get:

$$R = v_i \cos\theta \times \frac{2v_i \sin\theta}{g}$$

or

$$R = \frac{v_i^2 \sin 2\theta}{g} \qquad \_\_\_\_\_\_ (3.19)$$

This equation can be used to find range of a projectile, if magnitude and direction of initial velocity are known. Thus, by knowing two quantities; magnitude and direction of initial velocity, we can find height, range and time of flight for the projectile.

Maximum Range R_{max}: The greater the initial speed, the greater the range. For a given value of initial velocity v_i, the range of the projectile is maximum if $\sin 2\theta = 1$, which occurs when

$2\theta = 90^\circ$ or $\theta = 45^\circ$. By substituting the value of $\theta = 45^\circ$ in equation (3.19), we get maximum range, i.e.,

$$R_{max} = \frac{v_i^2}{g} \qquad \text{———} \quad (3.20)$$

Same Range: If the initial speed of the projectile v_i and acceleration due to gravity g remain constant, then there are always two angles for which the projectile has the same range, as shown in Fig. 3.6 (a). These angles are complementary to each other i.e., θ and $90^\circ - \theta$. Hence, a projectile has the same range for pairs of angles such as (75°, 15°), (60°, 30°) and (70°, 20°), etc.

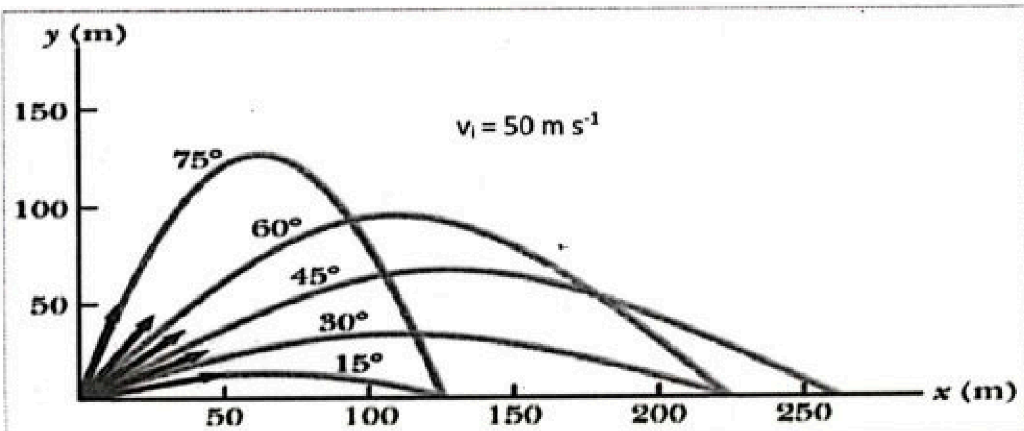


Figure 3.6 (a): Height verses range at different angles of projectile.

Effect of Air Resistance on Projectile Motion:

Air resistance affects the motion just like friction. Due to air resistance, particle's energy decreases. Generally, air resistance decreases the velocity of projectile. As a result, both the horizontal and vertical components of velocity decreases.

Air resistance affects the parabolic motion of a projectile by reducing its range and maximum height. Hence, air resistance can significantly alter the trajectory of the motion, as shown in Fig. 3.6 (b). Air resistance also increases the time of flight of the projectile.

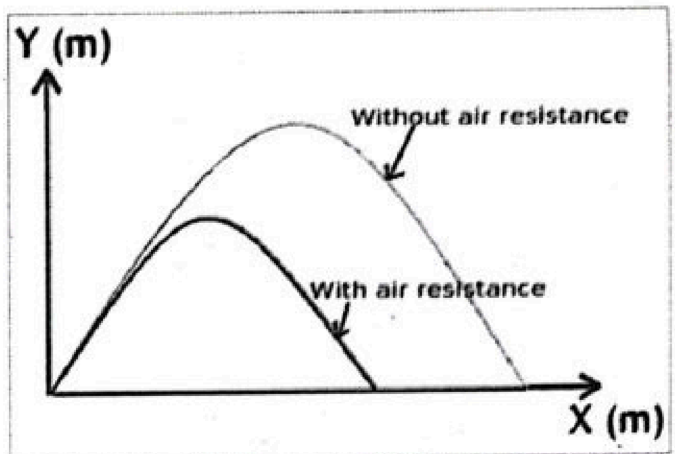


Figure 3.6 (b): Trajectory of a projectile with and without air resistance.

Example 3.4: If a projectile is launched horizontally with a speed of 12.0 m s^{-1} from the top of a 24.6 m high building. Determine the horizontal displacement of the projectile.

Given: $v_{ix} = 12.0$ m s^{-1} $\qquad$ $y = 24.6$ m

To Find: $x = ?$

Solution: First, we need to find time t, for this we use second equation of motion in y-direction.

$$y = v_{iy}\,t + \frac{1}{2}a_y t^2$$

For a projectile launched horizontally:

$v_{iy} = 0$, $a_y = g = 9.8\ m/s^2$, so we get:

$$24.6 = (0)t + \frac{1}{2}(9.8)\,t^2$$

$$t = 2.24\ s$$

Now using second equation of motion in x-direction, we get:

$$x = v_{ix}\,t + \frac{1}{2}a_x\,t^2$$

For a projectile launched horizontally $a_x = 0\ m/s^2$, so we get:

$$x = (12)\,(2.24) + \frac{1}{2}(0)t^2$$

$$x = 26.9\ m.$$

Example 3.5: A projectile is launched with an initial speed of 200.0 m s^{-1} at an angle of 30° with the horizontal.
(a) Determine the time of flight of the projectile.
(b) Determine the peak height of the projectile.
(c) Determine the horizontal displacement of the projectile.

Given: $v_i = 200\ m\ s^{-1}$ $\theta = 30^\circ$

To Find: (a) Time of flight = T = ? (b) Maximum height = H = ?
(c) Range of projectile = R = ?

Solution: (a) Time of flight = T = ?

$$T = \frac{2v_i \sin\theta}{g}$$

$$T = \frac{2(200)\sin 30^\circ}{9.8} = 20.41\ s$$

(b) Maximum height = H = ?

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

$$H = \frac{(200)^2 \sin^2 30^\circ}{2(9.8)} = 510.2\ m$$

(c) Range of projectile = R = ?

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R = \frac{(200)^2 \sin 2(30^\circ)}{9.8} = 3534.7\ m$$

Assignment 3.3

A ball is thrown with a speed of 30 m s^{-1} at an angle of 30° with the horizontal. Determine: (a) the height to which it rises, (b) the time of flight and (c) the horizontal range.

3.3 LAW OF CONSERVATION OF MOMENTUM

If a system consists of particles, each moving with different velocities, then the total momentum of an isolated system is the sum of the momenta of the individual particles. A system is said to be isolated if and only if the net external force, such as the gravitational force or friction, acting on the system is zero. There is no ideally isolated system in the universe, but we consider an isolated system that does not interact with its environment. The law of conservation of momentum states as:

The total momentum of an isolated system of interacting particles is conserved.

If p_i and p_f are initial and final momenta of an isolated system, then according to law of conservation of momentum:

$$P_f = P_i$$

or

$$P_f - P_i = 0$$

or

$$\Delta p = 0 \quad \text{\_\_\_\_\_\_ (3.21)}$$

The law of conservation of momentum is useful in collision problems. When a collision occurs in an isolated system due to internal forces, the momentum of each particle changes. Such forces do not contribute to the net force, which remains zero. Hence, the total momentum of the system does not change with the passage of time. It is also applicable in explosions.

3.3.1 Applications of Conservation of Momentum

In an explosion, chemical energy (stored in the bonds of the atoms) is transformed into the kinetic energy of the fragments. Using the principles of conservation of momentum, numerical values can be predicted for the fragments after the explosion.

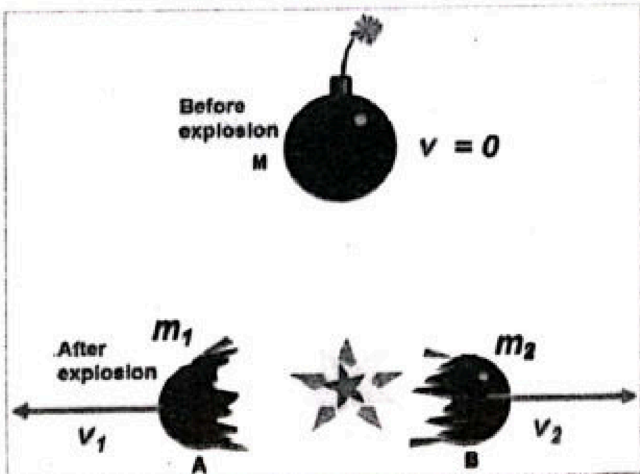


Figure 3.7 (a): Explosion of a bomb into two fragments A and B.

All objects before an explosion are generally considered at rest. After and during the explosion, the objects fly away in different directions with different speeds. Momentum is always conserved; in this case, the initial momentum of everything is zero. Knowing this, the final momentum should be zero too. Consider the explosion of a bomb into two fragments identified as A and B, as shown in Fig. 3.7 (a). The initial momentum of the system before the explosion is zero. As the momentum must be conserved, so sum of the final momentum of the two fragments must be zero.

Therefore, the fragments of same masses must move in opposite directions with equal speed for their momentum to be conserved.

An explosion in a cannon also follows conservation of momentum. For example, consider a cannon on a frictionless ground shooting a cannon ball, as shown in Fig. 3.7 (b).

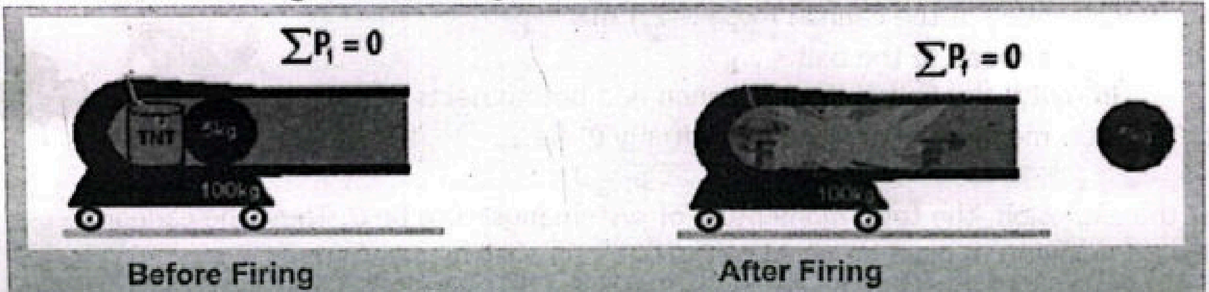


Figure 3.7 (b): Momentum conservation in cannon.

The sum of initial momentum is zero, because nothing is moving. After explosion inside the cannon, the cannon ball will be shot forward at very high speed, while the cannon recoils at a much slower speed due to its heavy mass. Sum of the final momentum will be zero, this makes the initial momentum and the final momentum equal.

Figure 3.8 (a): A karate player strikes the pile with his hand very fast and break it with a single blow.

In explosions, objects move apart instead of coming together like in collision.

A karate player can break a pile of tiles with a single blow because he strikes the pile with his hand very fast, as shown in Fig. 3.8 (a). In doing so, the large momentum of his hand is reduced to zero in a very short time interval. This exerts a large force on the pile of tiles which is sufficient to break them apart.

Conservation of momentum is also applied on **ball and bat**. When a ball hits on the bat, the total momentum before and after the collision must be equal (You have to add up the momentum of ball and bat). So, the momentum of ball and bat before and after the collision must be equal, as shown in Fig 3.8 (b).

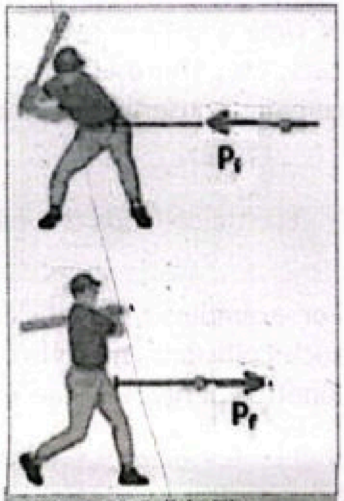


Figure 3.8 (b): The momentum of ball and bat before and after the collision must be equal.

Rockets and jet engines also work on the law of conservation of momentum. In these machines, hot gases produced by burning of fuel rush out with large momentum. As a result, the machines gain an equal and opposite momentum. This enables them to move with very high velocity.

Conservation of momentum is also applied when two car hits each other. Net momentum of cars before and after the collision must be equal.

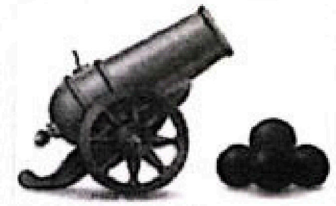
Example 3.6: A 46 g tennis ball is launched from a 1.35 kg homemade cannon. If the cannon recoils with a speed of 2.1 m s^{-1}, determine the muzzle speed of the tennis ball.

Given: Mass of the ball = m_b = 46 g = 0.046 kg

Mass of the cannon = m_c = 1.35 kg

Recoil speed of the cannon = v_c = −2.1 m s^{-1}

To Find: Muzzle speed of the ball = v_b = ?



Solution: Initially, the ball is in the cannon and both objects are at rest. The total momentum of system is initially 0. i.e.,

$$p_i = 0$$

After the explosion, the total momentum of system must also be 0. Thus, the cannon's backward momentum must be equal to the ball's forward momentum.

$$p_f = 0$$

$$m_c v_c + m_b v_b = 0$$

$$(1.35)(-2.1) + (0.046)v_b = 0$$

$$-2.8 + (0.046)\, v_b = 0$$

Solving for v_b, we get:

$$v_b = 2.8/0.046 = 60.87 \text{ m s}^{-1}$$

Assignment 3.4

A bullet of mass 20 g is fired from a gun with a muzzle velocity of 100 m s^{-1}. Find the recoil of the gun if its mass is 5 kg.

3.4 ELASTIC AND INELASTIC COLLISIONS

A collision occurs when two bodies come in physical contact with each other for a short interval of time and then separate. For example, the collision of ball with a bat, the collision of two cars, etc. There are two types of collisions: Elastic collision and inelastic collision. Momentum remains conserved in all types of collisions, but kinetic energy may change.

Elastic Collision

Elastic collision is a collision in which both the momentum and the kinetic energy of the system are conserved.

For example, the collisions between atomic and subatomic particles are elastic in nature. In such collisions, the two objects collide and return to their original shapes with no loss of total kinetic energy, i.e. the kinetic energy does not change into other types of energy.

Inelastic Collision

Inelastic collision is a collision in which the momentum of system is conserved but kinetic energy is not conserved.

For example, a meteorite falls on the Earth. During inelastic collisions, the kinetic energy is transformed into other forms of energy, such as heat energy, sound energy, and material deformation etc.

3.4.1 Elastic Collision

Consider an isolated system consisting of two spherical bodies such as billiard balls of masses m_1 and m_2 initially moving towards right with velocities u_1 and u_2 along a straight line without rotation. The speed of body having mass m_1 is greater than speed of body having mass m_2 and is approaching to it. So, the two bodies encounter head-on elastic collision. After collision they move with velocities v_1 and v_2 along the same straight line, as shown in Fig. 3.9.

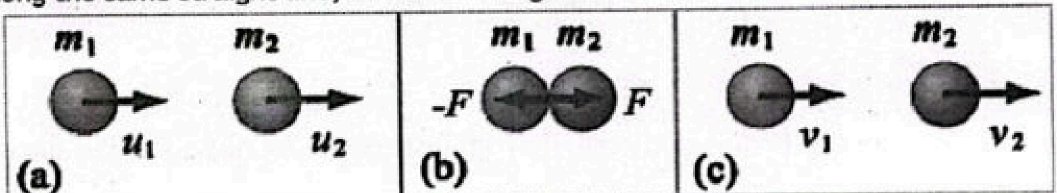


Figure 3.9: Two balls: (a) before collision (b) during collision (c) after collision.

We take the velocity as positive if a body is moving towards right and negative if it is moving towards left. Since the collision is elastic, therefore, we have two conservation laws in this case. According to the **law of conservation of momentum:**

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{\_\_\_\_\_\_} (3.22)$$

After rewriting the above equation, we have:

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \qquad \text{\_\_\_\_\_\_} (3.23)$$

Similarly, according to the **law of conservation of kinetic energy**, we get:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

After cancelling the factor 1/2, rewrite the above equation:

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \qquad \text{\_\_\_\_\_\_} (3.24)$$

Dividing equation (3.24) by (3.23), we get:

$$u_1 + v_1 = v_2 + u_2$$

or $$u_1 - u_2 = v_2 - v_1$$

or $$u_1 - u_2 = -(v_1 - v_2) \qquad \text{\_\_\_\_\_\_} (3.25)$$

This equation shows that relative speed of two bodies before collision is equal but opposite to relative speed after collision. Hence, for two bodies colliding elastically, the relative speed of approach before collision is equal to the relative speed of separation after collision. From equation (3.25), we get: $v_2 = u_1 + v_1 - u_2$ ______ (3.26)

By putting the value of v_2 from equation (3.25) into the equation (3.22), we get:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$

or $$m_1u_1 + m_2u_2 = m_1v_1 + m_2u_1 + m_2v_1 - m_2u_2$$

or $$m_1v_1 + m_2v_1 = m_1u_1 + m_2u_2 - m_2u_1 + m_2u_2$$

or $$(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2u_2$$

or $$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2 \qquad \text{\_\_\_\_\_\_} (3.27)$$

Similarly, from equation (3.25)

$$v_1 = u_2 + v_2 - u_1 \qquad \text{\_\_\_\_\_\_} (3.28)$$

By putting the value of v_1 from equation (3.28) into the equation (3.22), we get:

$$m_1u_1 + m_2u_2 = m_1(u_2 + v_2 - u_1) + m_2v_2$$

or $$m_1u_1 + m_2u_2 = m_1u_2 + m_1v_2 - m_1u_1 + m_2v_2$$

or $$m_1v_2 + m_2v_2 = m_1u_1 + m_2u_2 - m_1u_2 + m_1u_1$$

or $$(m_1 + m_2)v_2 = 2m_1u_1 - (m_1 - m_2)u_2$$

or $$v_2 = \frac{2m_1}{(m_1 + m_2)}u_1 - \frac{(m_1 - m_2)}{(m_1 + m_2)}u_2 \quad \text{------ (3.29)}$$

Equations (3.27) and (3.29) give the velocities of two bodies after collision.

Case 1: When the two colliding bodies have the same mass i.e., $m_1 = m_2$, then from equations (3.27) and (3.29), we get:

$$v_1 = u_2 \quad \text{and} \quad v_2 = u_1$$

This shows that velocities of bodies exchange during elastic collision of two bodies having equal masses.

Case 2: When bodies have the same mass i.e., $m_1 = m_2$, and the second body (target) is at rest ($u_2 = 0$), then from equations (3.27) and (3.29), we get:

$$v_1 = 0 \quad \text{and} \quad v_2 = u_1$$

This shows that when the first body comes to rest the second body moves with the initial velocity of the first body.

Case 3: When a lighter body (m_1) collides with a massive body ($m_2 \gg m_1$) at rest ($u_2 = 0$), then under such condition m_1 can be neglected i.e., $m_1 = 0$, so from equations (3.27) and (3.29), we get:

$$v_1 = -u_1 \quad \text{and} \quad v_2 = 0$$

Hence the first body (which is lighter) rebounds with the same initial velocity but in opposite direction. The second body (which is heavier) remain at rest even after collision. For example, if a ball is thrown at a fixed wall, the ball will bounce back with the same velocity, it had initially, but in the opposite direction.

Case 4: When a massive body (m_1) collides with a lighter body ($m_1 \gg m_2$) at rest ($u_2 = 0$), then under such condition m_2 can be neglected i.e., $m_2 = 0$, so from equations (3.27) and (3.29), we get:

$$v_1 = u_1 \quad \text{and} \quad v_2 = 2\,u_1$$

Hence, the first body (which is heavier) continues to move with the same initial velocity. While the second body (which is lighter) will move with twice the initial velocity of the first body.

Example 3.7: An object of mass 5 kg moving at 10 m s^{-1} collides with another object of mass 10 kg moving in the same direction at 5 m s^{-1}. Assume that the collision is a one-dimensional elastic collision. What will be the speed of both objects after the collision?

Given: $m_1 = 5$ kg $\quad u_1 = 10$ m s^{-1} $\quad m_2 = 10$ kg $\quad u_2 = 5$ m s^{-1}

To Find: $v_1 = ?$ $\quad v_2 = ?$

Solution: Using the relation:

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)}u_1 + \frac{2m_2}{(m_1 + m_2)}u_2$$

Putting values: $$v_1 = \frac{(5-10)}{(5+10)}10 + \frac{2(10)}{(5+10)}5$$

$$= 50/15 = 3.3 \text{ m s}^{-1}$$

Similarly,
$$v_2 = \frac{2m_1}{(m_1+m_2)}u_1 - \frac{(m_1-m_2)}{(m_1+m_2)}u_2$$

$$v_2 = \frac{2(5)}{(5+10)}10 - \frac{(5-10)}{(5+10)}5 = 8.33\ \text{m s}^{-1}$$

Hence, velocity of the lighter object will decrease to 3.3 m s^{-1} and heavier object will move with the velocity i.e., 8.33 m s^{-1}.

SUMMARY

- **Equations of motion** show the relation between initial velocity, final velocity, acceleration, displacement and time.
- **Projectile motion** is a two-dimensional motion of an object thrown in the air under the action of gravitational force only. In projectile motion horizontal component of velocity remains constant, while the vertical component of velocity changes.
- The **height of the projectile** is the maximum vertical distance attained by it during projectile motion.
- The time taken by the projectile to reach the maximum height and then return to the ground is called the **time of flight.**
- The horizontal distance travelled by a projectile is called **range.** A projectile has the same range for complementary angles.
- The **law of conservation of momentum** states, the total momentum of an isolated system of interacting particles is conserved.
- An **elastic collision** is a collision in which both the momentum and the kinetic energy of the system are conserved.
- An **inelastic collision** is a collision in which the momentum is conserved but kinetic energy is not conserved.

EXERCISE

Multiple Choice Questions

Encircle the correct option.

1) A projectile thrown upward moves in its parabolic path, the velocity and acceleration vectors for the projectile are perpendicular to each other at:

A. no where B. the highest point C. the launch point D. the landing point

2) A truck driving along a highway road has a large amount of momentum. If it moves at the same speed but has twice as much mass, its momentum is _______.

A. zero B. quadrupled C. doubled D. unchanged

3) A 5 N force is applied to a 3 kg ball to change its velocity from 9 m s^{-1} to 3 m s^{-1}. This impulse causes the momentum change of the ball to be ____ kg m s^{-1}.

A. −2.5 B. −10 C. −18 D. −45

4) Which of the following statements is true about the projectile motion?

A. Projectile motion is the motion of an object projected vertically upward into the air and moving under the influence of gravity.

B. Projectile motion is the motion of an object projected into the air and moving independently of gravity.

C. Projectile motion is the motion of an object projected into the air and moving under the influence of gravity.

D. Projectile motion is the motion of an object projected horizontally into the air and moving independently of gravity.

5) The vertical component of velocity of a projectile is smallest at:

A. The instant it is thrown. B. Halfway to the top.
C. The top. D. The landing point.

6) A 4 kg object has a momentum of 12 kg m s^{-1}. The object's speed is:

A. 3 m s^{-1} B. 4 m s^{-1} C. 12 m s^{-1} D. 48 m s^{-1}

7) A bomb of mass 9 kg explodes into 2 pieces of masses 3 kg and 6 kg. The velocity of mass 3 kg is 1.6 m s^{-1}, the kinetic energy of mass 6 kg is:

A. 3.84 J B. 9.6 J C. 1.92 J D. 2.92 J

8) What is the force experienced by a projectile after the initial force that launched it into the air, in the absence of air resistance?

A. The gravitational force B. The nuclear force

C. The contact force D. The electromagnetic force

9) If a projectile is launched on level ground, what launch angle maximizes the range of the projectile?

A. 0° B. 30° C. 45° D. 90°

Short Questions

Give short answers of the following questions.

3.1 What are the conditions for using the equations of motion?

3.2 You through a small ball vertically up in the air. How are the velocity and acceleration of the ball oriented with respect to one another (a) when the ball is moving upward (b) when the ball is moving downward?

3.3 For a projectile motion, is the velocity ever zero? Is the acceleration ever zero?

3.4 Draw a diagram showing the velocity and acceleration of a projectile at several points along its path, assuming (a) the projectile is launched horizontally and (b) the projectile is launched at an angle θ with the horizontal.

3.5 An aeroplane while flying horizontally drops a bomb when reaches exactly above the target, but misses it. Explain why?

3.6 How air resistance affects projectile motion?

3.7 Why do a slow-moving loaded truck and a speeding rifle bullet each have a large momentum?

3.8 What is the difference between elastic and inelastic collision?

3.9 An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

3.10 Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

3.11 For any specific velocity of projection, prove that the maximum range is equal to four times of the corresponding height.

3.12 Is momentum conserved when a bat hits a ball? How?

Comprehensive Questions

Answer the following questions in detail.

3.1 Derive the equations of motion for uniformly accelerated objects.

3.2 Define the law of conservation of momentum. Explain how it applies to a handball bouncing off a wall.

3.3 Explain elastic collision and prove that for two bodies colliding elastically, relative speed of approach before collision is equal to relative speed of separation after collision.

3.4 Discuss the motion of a projectile in the absence of air resistance.

3.5 Derive mathematical equations for (a) Maximum height attained, (b) time of flight, (c) range of a projectile.

3.6 Discuss that why though the momentum of a closed system is always conserved, but some change in kinetic energy may take place.

Numerical Problems

3.1 On dry concrete, a car can decelerate at a rate of 7.00 m s^{-2}, whereas on wet concrete it can decelerate at only 5.00 m s^{-2}. Compare the distances necessary to stop the car moving at 30.0 m s^{-1} (about 110 km h^{-1}) (a) on dry concrete and (b) on wet concrete.

(Ans: 64.3 m / 90.0 m)

3.2 Find the angle of projection of a projectile for which the maximum height and corresponding range are equal. **(Ans: 76°)**

3.3 A jet plane comes in for a landing with a speed of 100 m s^{-1}, and its acceleration can have a maximum magnitude of 5.00 m s^{-2} as it comes to rest.

(a) From the instant the plane touches the runway, what is the minimum time interval needed before it can come to rest?

(b) Can this plane land on a small tropical island airport where the runway is 0.8 km long? Explain your answer.

(Ans: 20 s, it cannot, it would require a runway of minimum length 1 km)

3.4 A projectile is launched with an initial speed of 21.8 m s^{-1} at an angle of 35°above the horizontal. Determine:
(a) the time of flight of the projectile.
(b) the peak height of the projectile.
(c) the horizontal displacement of the projectile. **(Ans: 2.55 s, 7.98 m, 45.6 m)**

3.5 A projectile is launched horizontally from the top of a 45.2 m high cliff and lands a distance of 17.6 m from the base of the cliff. Determine the magnitude of the launch velocity.

(Ans: 5.79 m s^{-1})

3.6 Two equal-mass carts roll towards each other on a level, low-friction track. One cart rolls rightward at 2 m s^{-1} and the other cart rolls leftward at 1 m s^{-1}. After the carts collide, they couple and roll together. Ignoring resistive forces, find their combined speed.

(Ans: 0.5 m s^{1})

3.7 A 0.5 kg ball traveling at a speed of 4 m s^{-1} to the right collides elastically with another ball of 3.5 kg which is initially at rest. Find velocities of both the balls after collision?

(Ans: −3.0 m s^{-1}, 1.0 m s^{-1})

3.8 A 17.5 g bullet is fired at a muzzle velocity of 582 m s^{-1} from a gun with a mass of 8.0 kg and a barrel length of 75.0 cm.
(a) How long is the bullet travelled in the barrel?
(b) What is the force on the bullet while it is in the barrel?
(c) Find the impulse exerted on the bullet while it is in the barrel.
(d) Find the bullet's momentum as it leaves the barrel.

(Ans: 0.00258 s, 3950 N, 10.2 kg m s^{-1}, 10.2 kg m s^{-1})