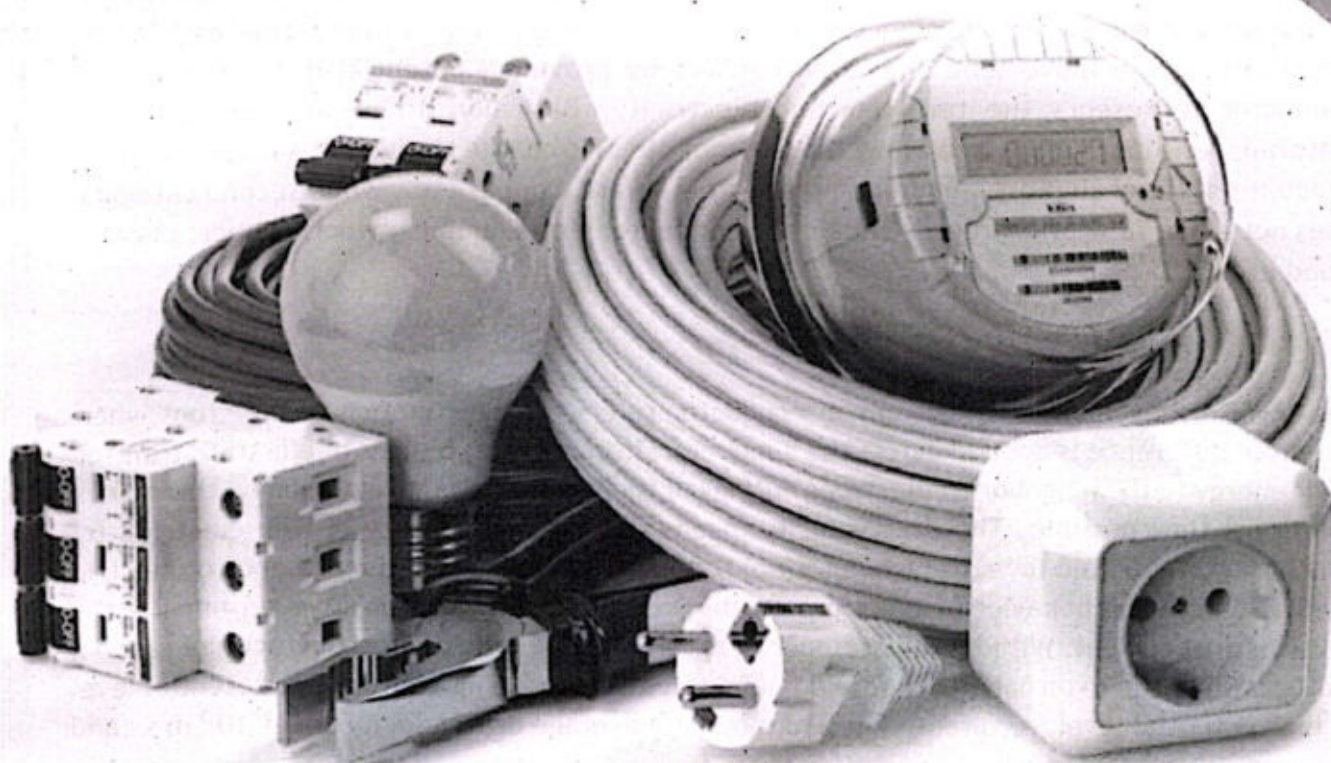


ELECTRICITY

UNIT

11



Student Learning Outcomes (SLOs)

The students will:

- Use, for a current-carrying conductor, the expression $I = Anvq$ [where n is the number of charge carriers per unit volume.]
- state and use $V = W/Q$.
- state and use $P = IV$, $P = I^2 R$ and $P = V^2/R$.
- state and use $R = \rho L/A$.
- State that the resistance of a light-dependent resistor (LDR) decreases as the light intensity increases.
- Define and use the electromotive force (e.m.f.) [of a source as energy transferred per unit charge in driving charge around a complete circuit].
- Distinguish between e.m.f. and potential difference (p.d.) in terms of energy considerations.
- Explain the effects of the internal resistance of a source of e.m.f. on the terminal potential difference.
- state Kirchhoff's first law and describe that it is a consequence of conservation of charge.
- state Kirchhoff's second law and describe that it is a consequence of conservation of energy.
- Derive, using Kirchhoff's laws, a formula for the combined resistance of two or more resistors in series.
- Derive and apply a formula for the combined resistance of two or more resistors in parallel.
- Use Kirchhoff's laws to solve simple circuit problems.
- State and use the principle of the potentiometer as a means of comparing potential differences.
- Explain the use of a galvanometer in null methods.
- Explain the use of thermistors and light-dependent resistors in potential dividers. [to provide a potential difference that is dependent on temperature and light intensity].
- Explain the internal resistance of sources and its consequences for external circuits.
- Explain how inspectors can easily check the reliability of a concrete bridge with carbon fibers as the fibers conduct electricity.

Electricity is the branch of physics in which we deal with dynamic state of charges. There are two categories of electricity; static electricity and current electricity. The discharging of electrical pulse due to imbalance of charges (Positive & Negative) refer the term Static electricity, while movement of charge carriers by providing potential difference across conductor represents the term current electricity. Basically, there are two types of materials; one category which allow the flows of electric charge through it is named as good conductors, for examples; aluminium, gold, steel, brass, copper etc., while second category does not allow flow of charges is called bad conductor, for examples; wool, rubber, plastic, wood etc.

11.1 DRIFT VELOCITY

The term drift velocity of electrons in conductor refers the slow motion of electrons when potential difference is applied across the conductor. In conductor; billions of electrons transfer their energy to the neighboring electrons during collision when they move randomly, this makes electrons' flow possible. The drift velocity means the average velocity of free electrons by which they are drifted towards high potential terminal of conductor in the presence of electric field. This drifting behavior of electrons (within conductor) towards positive terminal is the cause of drift current. Without the presence of external electric field, electrons acquire random motion, while the external electric field bounds all electrons to move from low potential level to high potential level, resultantly electrons acquire average drift velocity about 10^{-3} m s^{-1} and net current is obtained.

The term "Current" represents the dynamic state of charges and the categories of current based upon the direction of the flow of charges; means the electric and conventional flow depends upon the nature of charges. Actually, electrons do not move parallel to the conductor's length like trucks on a road. In conducting material; billions of electrons are ready to push neighboring electrons within inter-atomic spaces.

The average velocity of electrons within conducting material without the existence of an external electric field is shown in Fig. 11.1 (a). The concept of drift velocity cannot be described without the presence of an external electric field. The average value of Fermi velocity of electrons in metals is comparatively very high about 10^6 m s^{-1} (It does not depend upon current and applied voltage) while the drift velocity 10^{-3} m s^{-1} .

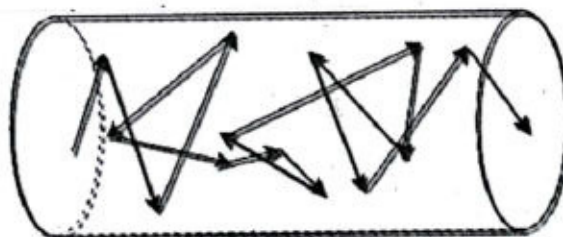


Figure 11.1 (a): Motion of electrons in the absence of electric field.

In metals, during motion, electrons acquire energy, lose it when collide with each other, as a result metal becomes hot. For the continuation of the flow of charges in metals, external electric field is mandatory which helps to drift electrons.



During the elastic collision of electrons, the thermal velocities become high in the presence of the external electric field. When the emf source is attached across the terminals of conducting material, all electrons are aligned in a well-organized way in a particular direction. Basically, emf is the measurement of work done per unit charge for conversion of non-electrical energy to electrical energy. Potential difference is the effect of that emf. The emf and potential difference can be measured with the same scales of units. The external electrical field applied can be measured in terms of potential differences across the conducting material per unit length.

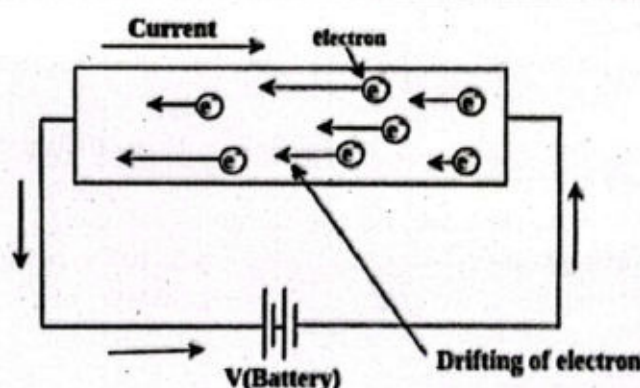


Figure 11.1 (b): Motion of electrons in the presence of an electric field.

Table: 11.1: Comparison of drift velocity and mobility.

Drift Velocity	Mobility
Average velocity acquired by charge carriers within conducting material in the presence of external electric field.	Measurement of magnitude of drift velocity of charge carriers per unit electric field.
Formula: $v = -\mu E$	Formula: $\mu = v/E$
Unit: meter per second ($m s^{-1}$)	Unit: $m^2 s^{-1} V^{-1}$

Drift velocity factors: Let's consider an emf source of potential "V" is attached across the metallic conductor (wire) having length "L" and uniform cross-section area "A" [as shown in Fig. 11.1(b)]. The external electric field intensity can be written as;

$$\text{Electric field intensity (E)} = \frac{\text{Potential across conductor (V)}}{\text{the length of conductor (L)}}$$

If Number of free electrons are "N" the free electron density within metallic conductor is:

$$\text{Free electron density (n)} = \frac{\text{Number of free electrons (N)}}{\text{Volume of the conductor (AL)}}$$

Here, number of free electrons = $N = nAL$. If charge on one electron is "e" then the charge quantization is " ne ". The total charge on conducting material becomes;

$$Q = nALe$$

As, the current "I" passes through conductor is:

$$I = nEav \quad (11.1)$$

So, the drift velocity v of electrons is:

$$v = \frac{I}{nAe}$$

Now the current density can be measured as:

$$J = nev$$

Drift velocity and current have direct relation. The particular directive flow of charge carriers depending upon provision of external electric field, otherwise electrons acquire speeds in random directions.

Example 11.1: A current of 3 A is flowing in a copper conductor with a cross-section of 1 mm^2 . Find the drift velocity of the electrons. (For copper, $n = 8.5 \times 10^{28} \text{ per m}^3$).

Given: $I = 3 \text{ A}$ $A = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$ $n = 8.5 \times 10^{28} \text{ per m}^3$

To Find: $v = ?$

Solution: Here we use the formula $I = nAvQ$

Putting values, $3 = 8.5 \times 10^{28} \times 1 \times 10^{-6} \times v \times 1.6 \times 10^{-19}$

Solving for v , we get: $v = 2.20 \times 10^{-4} \text{ m s}^{-1}$

Assignment 11.1

A wire with diameter 0.02 m contains 10^{28} free electrons per m^3 . Find the drift velocity for free electrons in the wire with an electric current 100 A.

11.2 ELECTRIC POTENTIAL

Electric Potential Energy: Any charged particle experiences a force when it moves within the region of constant electric field. i.e. $F = Eq$. It means that if we want to move any charge carrier within a constant electric field, work is required which changes the electric potential energy of the charge carrier similar to the movement of any massive object in a uniform gravitational field that requires work, which changes the potential energy of an object. When an object moves naturally, there will be no requirement for external force or in other words no need for expenditure of energy, but when an object moves against the natural forces, we must require external energy, similar concept is used in electric potential energy. Electric potential energy is basically energy required by any charge carrier to move against the electric field.

Let's consider a charge carrier q experiences a force in constant electric field E , this positive charge will move from the left plate to the right plate, i.e., from a positively charged plate to a negatively charged plate, as shown in Fig. 11.2 (a). If the distance covered by charge is S in the direction of the electric field, as shown in Fig. 11.2 (b), then work done is qES .

The electrostatic force is conservative in nature, so it means that work done on charge q does not depend upon the path followed. Basically, charged particle possesses electric potential energy when it is placed in an electric field, so electric potential energy is the measurement of work done to place a charged particle in an electric region from any infinite region.

Relation between Electric Potential energy and Electric Potential: As electric potential energy is qES , so the term electric potential refers the amount of Electric potential energy per

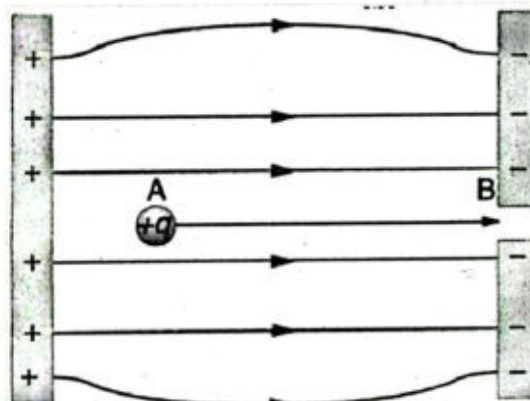


Figure 11.2 (a): A charge q experiences a force in an electric field.



unit charge to move charged object from any infinite point to the uniform or non-uniform electric field region against the directions of electric field lines.

$$\text{Electric potential} = \frac{\text{Electric potential energy}}{\text{Charge under observation}} = \frac{\text{Work}}{\text{Charge}}$$

$$V = U/q$$

So, $U = Vq$

Electric potential is measured in volt (V), and is given as $V = J\ C^{-1}$. Electric potential is work done per unit charge against the electric field. Let's consider any referenced point "a" has a distance "r" from a referenced point charge "Q". So, the electric potential with respect to the referenced point can be studied as;

$$V = \frac{w}{q}$$

$$V = \frac{F \cdot r}{q} = \frac{F \cos \theta}{q} = \frac{Eq \cos \theta}{q} = -E r$$

$\theta = 180^\circ$ represents the work done against the electric field lines. The electric potential can also be expressed as;

$$\text{Electric potential} = k \frac{q}{r}$$

Electric field intensity and electric potential are field quantities. For more than one charges electric potential can also be measured as;

$$V = k \sum_{i=1}^N \frac{q_i}{r_i}$$

Where, k is coulomb's constant; $9 \times 10^9\ Nm^2\ C^{-2}$. In the field of Atomic Physics and nuclear physics, the charged objects of interest are electrons ($q_e = 1.6 \times 10^{-19}\ C$ for one electron as well as one proton), the electrostatic potential energy of charged objects can be expressed in terms of "Electron-Volt" ($1\ eV = 1.6 \times 10^{-19}\ J$).

The electric potential is very similar to gravitational potential because both can be considered relative parameters, so electric potential cannot be measured without measuring the difference in potentials across the terminals of the battery. In any electric circuit, the term electric potential tells about the energy conversion phenomenon from electrical form to any other form of energy with respect to every point charge that moves between any two defined points within an electric circuit.

Example 11.2: What is the potential difference between two points in an electric field if it takes 600 J of energy to move a charge of 2 C between these two points?

Given: $\Delta U = 600\ J$ $Q = 2\ C$

To Find: $\Delta V = ?$

Solution: Using the relation

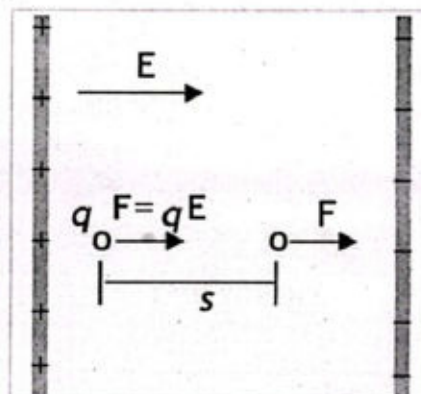


Figure 11.2 (b): Charge cover a distance S.

$$\Delta V = \frac{\Delta U}{Q}$$

Putting values, we get:

$$\Delta V = \frac{600}{2} = 300 \text{ volt}$$

Assignment 11.2

Calculate the current passing through a conducting material having cross-section area 1 cm^2 and after applying constant potential across conductor electrons acquire $1.5 \times 10^{-2} \text{ m s}^{-1}$ and free electron density is 10^{24} electrons per cubic meter.

Table 11.2: Conversion factors for energy.

ENERGY UNIT	CONVERSION INTO		
	Joule	Kilo Joule per mole	Electron-volt
Joule	1	6.66×10^{20}	6.24×10^{18}
Kilo joule per mole	1.66×10^{-21}	1	0.01
Electron-volt	1.6×10^{-19}	96.5	1

11.3 ELECTROMOTIVE FORCE AND MAXIMUM POWER OUTPUT

We must require some kind of source named as emf source to accelerate charge carriers which will be able to provide potential difference across the conducting material. The emf source helps to convert chemical energy into electrical energy. The term electromotive force is not representing the mechanical concept of force rather it represents work per unit charge.

The potential difference of a cell can be expressed by the equation:

$$V = \varepsilon - Ir$$

whereas r is the internal resistance battery, as shown in Fig. 11.3. The direction of flow of charges can be taken positive or negative depending upon terminals of emf source and the load of electric circuit can be studied as;

The electric current can also be measured in term of load resistance R , emf and internal resistance r ; i.e.,

$$I = \frac{\varepsilon}{R+r} \quad (11.2)$$

In many electronic circuits and systems, it is important to have maximum transfer of power from the source to the load. For example, in radio or TV transmitting systems, we want maximum power transfer from the transmitting medium to the antenna systems. We want maximum power transfer from amplifier to speaker system. This is accomplished by proper matching of load resistance R and source resistance r .

Consider the circuit as shown in fig. If V is the P.D. across R , the loss of potential energy per second is known as power delivered to R by the current I . As for electrical power

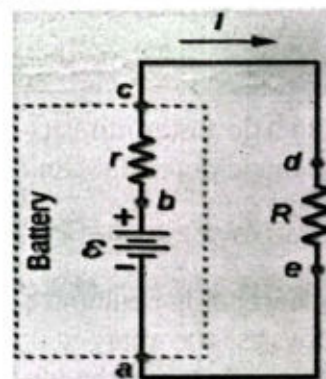


Figure 11.3: emf of source helps to convert chemical energy into electrical energy.

$$P_{\text{out}} = I^2 R$$

Using equation (11.2), we get:

$$P_{\text{out}} = \frac{\mathcal{E}^2 R}{(R+r)^2}$$

or

$$P_{\text{out}} = \frac{\mathcal{E}^2 R}{R^2 + r^2 + 2Rr}$$

or

$$P_{\text{out}} = \frac{\mathcal{E}^2 R}{R^2 + r^2 - 2Rr + 4Rr}$$

or

$$P_{\text{out}} = \frac{\mathcal{E}^2 R}{(R-r)^2 + 4Rr}$$

When $R = r$, the denominator of the expression for P_{out} is minimum and so P_{out} is maximum. Thus, it can be concluded that:

Maximum power is delivered to a load R when the internal resistance of the source of emf is equal to the load resistance.

This statement is called maximum power transfer theorem. The value of the maximum output power is

$$(P_{\text{out}})_{\text{max}} = \frac{\mathcal{E}^2}{4r} \quad \text{OR} \quad (P_{\text{out}})_{\text{max}} = \frac{\mathcal{E}^2}{4R} \quad \text{_____ (11.3)}$$

If the load resistance is less or greater than the source resistance, then the power delivered to the load will be low.

11.4 VARIATION OF RESISTANCE WITH TEMPERATURE

The following equation shows that how variation of temperature effects resistance of any conducting material;

$$R_t = R_0 [1 + \alpha (T - T_0)] \quad \text{_____ (11.4)}$$

Table 11.3: Electrical material Vs. Temperature coefficient of resistance (α)

Metals	Greater than zero; resistance increases with temperature.
Solid non-metals	Equal to zero; resistance is independent of temperature.
Semi-conductors	Less than zero; resistance decreases with increase in temperature.
Alloys	Has small value greater than zero.
Superconductors	At low temperature, the resistance of certain substances becomes exactly zero.

The resistance of conducting material basically depends upon physical factors such as length, thickness (cross-section area) of conductor and non-physical factor like temperature of the conductor. Resistance (R) of conducting material is directly proportional to length (L) of conductor and inversely proportional to cross-section area (A) of conductor. i.e.,

$$R \propto L$$

and

$$R \propto \frac{1}{A}$$

So,

$$R \propto \frac{L}{A}$$

or

$$R = \text{constant} \frac{L}{A}$$

Resistance of conducting material can also be expressed in terms of specific resistance also named as Resistivity.

$$R = \rho \frac{L}{A}$$

The SI- Unit of resistivity is Ohmmeter ($\Omega \text{ m}$). Let's consider a cylindrical shaped conductor having different dimensional structures, as shown in Fig. 11.4 (a). We can have a look at the conductor with length "l" and cross-section-area "a" with resistance R_1 , as shown in Fig. 11.4 (b).

If conducting object's length is changed from l to 2l but area of cross-section is taken constant, the resistance of conductor will be increased from R_1 to $2R_1$, as shown in Fig. 11.4 (c).

If the length of conducting object is not going to change but wire is replaced with another wire having same length but double cross-section area, then the resistance of conducting material becomes $R_3 = R_1/2$.

The reciprocal of resistivity is called conductivity, so $\rho = \sigma^{-1}$. The SI-Unit of σ is $(\text{ohm.m})^{-1}$. Another unit is "Siemens" whereas; 1 Sie = $(\text{ohm.m})^{-1}$, basically, resistivity is the physical property of conducting

materials, which shows the ability of the atoms of that particular material to impede the flow of electrons. The higher the resistivity the stronger the electrons will be attracted to the atoms, and they will experience more difficulty in moving through the conducting material. Resistivity is the intrinsic property while resistance is the extrinsic property. The resistivity of the conducting material does not depend upon the physical structure of the conductor. It does not change by changing temperature until the material phase transition occurs. In other words, we can say that resistivity is a composition function, while resistance depends upon the physical dimensions of the conducting material and the temperature variation of the conducting material which plays a very effective role in increasing and decreasing resistance.

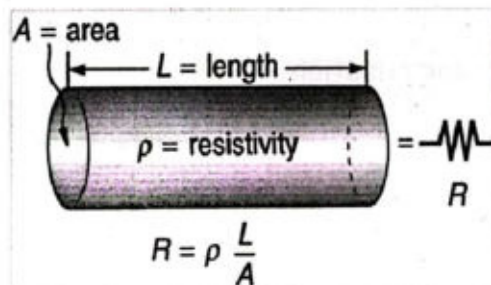


Figure 11.4 (a): A cylindrical shaped conductor.



Figure 11.4 (b): Conductor-1

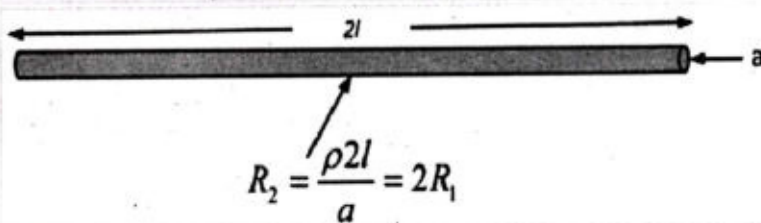


Figure 11.4 (c): Conductor-2

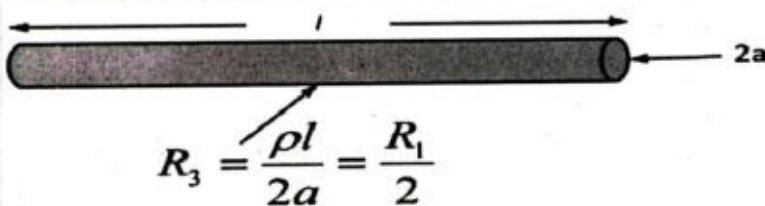


Figure 11.4 (d): Conductor-3

Table 11.4: Resistivity.

Material	Resistivity ($\Omega \text{ m}$)
Aluminum	2.7×10^{-8}
Copper	1.7×10^{-7}
Graphite	8×10^{-6}
Quartz	5×10^{16}
Silicon	2.3×10^3

It means that thicker wire resists the electric flow of charges less as compared to thinner wire. As the resistance of copper is less than the resistance of rubber; that's the reason, that copper is used for the conduction of charge carriers while for insulating purposes rubber is used.

Example 11.3: Calculate the resistance of wire 10 m long that has a diameter of 2 mm and resistivity of $2.63 \times 10^{-2} \Omega \text{ m}$.

Given: $L = 10 \text{ m}$ $r = \frac{2 \text{ mm}}{2} = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$ $\rho = 2.63 \times 10^{-2} \Omega \text{ m}$

To Find: $R = ?$

Solution: Using the formula $R = \frac{\rho L}{A}$ OR $R = \frac{\rho L}{\pi r^2}$

Putting values, we get: $R = \frac{(2.63 \times 10^{-2})(10)}{(3.14)(1 \times 10^{-3})^2} = 0.83758 \times 10^5 = 83,758 \Omega$

Assignment 11.3

A wire has length 10 m with resistance 100 Ω . If wire is stretched to 3 times of its original length, how much the resistance of the wire will be increased?

11.5 LIGHT-DEPENDENT RESISTORS (LDRs)

Light dependent resistor is also called Photo-resistor, this device is used to detect light levels, like; Security system based on light detection, the basic mechanism behind the working of LDR is the mutual variation between resistance of material and intensity of light. Here we can observe that when intensity of light increases the resistance of conducting material decreases.

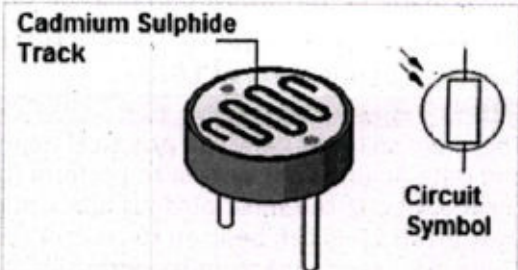


Figure 11.5 (a): LDR.

The basic principle of work of LDR is photoconductivity.

The conducting behavior of LDR can be better explained with the help of energy band theory. Actually, the material reduces its conductance when incident light intensity reduces. Declining graphical representation indicates inverse behavior of resistance at different luminance scales and darkness level, as shown in Fig. 11.5 (b).

Commonly, two types of LDR exists, i.e. Intrinsic and extrinsic photo resistors. Intrinsic LDRs are related to pure semiconducting material like germanium or silicon etc. while extrinsic LDRs are based upon modified semiconducting materials by doping process, mostly these LDRs cannot be used for shorter wavelength lights and effective only for longer wavelength. LDRs have lot of practical

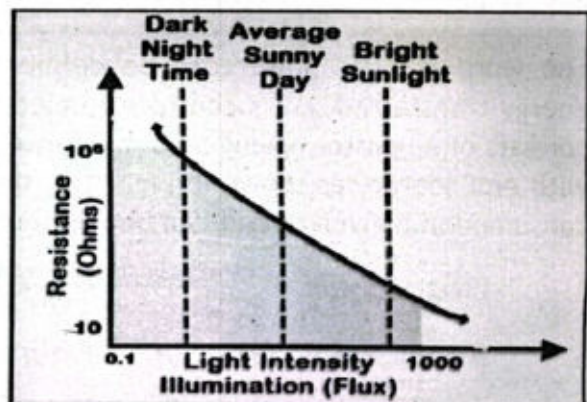


Figure 11.5 (b): Inverse behavior of resistance.

applications like electronic eye security system, street light controlling system, light switching for sunset or sunrise conditions etc.

LDR device is basically part of an automation systems, which helps to reduce the efforts of human in so many fields in daily life. Especially, for the case of power saving, the loss of electricity should be minimal. There are so many characteristics of LDR like maximum power dissipated device within given range of temperature. As LDR is a sensitive device, that's why its sensitivity changes with change of wavelength of light, this device can detect in the range from $4 \times 10^3 \text{ Å}$ to 10^4 Å .

In dark environment, resistance of LDR acquires its maximum value in $M\Omega$, while in lighted environment it reduces to few 100Ω . Photo-resistors have lower photoelectric sensitivity at high temperatures, but higher sensitivity at lower temperatures. LDRs have sharp resistance recovery rate.

In potential divider circuit, LDR can be used because its resistance can be increased or decreased by changing the intensity of that light which falls on it, as shown in Fig. 11.5 (c). It means that by increasing or decreasing intensity of light the resistance of circuit can also be changed, which affect the input potential dropped. Hence this is the one of the best practical application of LDR.

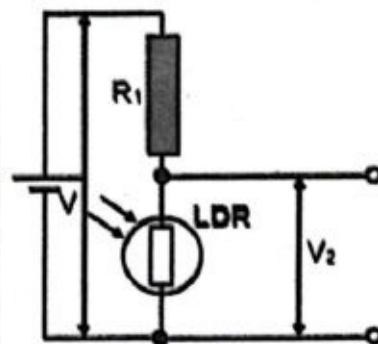


Figure 11.5 (c): LDR based potential divider circuit.

Engineering and Technology

There are so many daily life practical applications of LDRs. Used as part of a SCADA (Supervisory Control and Data Acquisition) system to perform functions such as counting the number of packages on a moving conveyor belt; the most obvious application for an LDR is to automatically turn on a light at a certain light level. LDRs can be used to control the shutter speed on a camera. These are used as light sensors. These are used to measure the intensity of light. Their latency property is used in audio compressors and outside sensing. Infrared astronomy and Infrared Spectroscopy also use photo resistors for measuring mid-infrared spectral region. Photo resistors are available in small size; it is easy to carry from one place to another place. Low cost, used in street lighting design, Alarm clocks, Burglar alarm circuits, Light intensity meters.

11.6 ELECTRIC POWER AND ELECTRIC ENERGY

The word electric power can be defined as the rate of energy transferred. Let's consider an electric circuit which consists of a resistor having load resistance (R) is connected with emf source, as shown in Fig. 11.6, the electric power consumed in a given circuit can be measured as;

$$\text{Electric power} = \frac{\text{energy consumed by the electric circuit}}{\text{time}}$$

$$P = \frac{\text{energy}}{\text{time}} = \frac{VQ}{t} = VI = I^2R = \frac{V^2}{R}$$

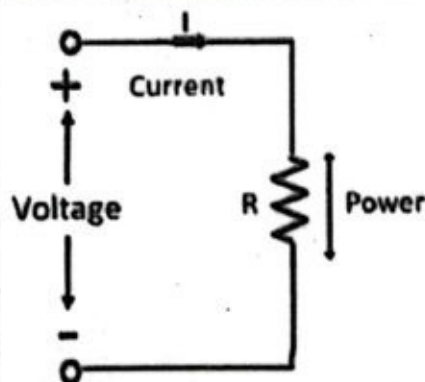


Figure 11.6: An electric circuit.

The term power dissipation in resistor is due to energy losses in term of heat energy. SI unit of electric power is watt.

$$1 \text{ watt} = 1 \text{ volt} \times 1 \text{ ampere}$$

The energy required or acquired by charge carriers to move from one position to another position is called electrical energy. When electrons move through resistor by provision of potential difference across the ends of conducting material, the concept of energy dissipation can be analyzed in term of heat energy. In other words, one coulomb charge per unit time (Q/t) drops due to potential difference (V), the rate of energy dissipation can be studied as VI . So, Energy dissipation can be calculated with the help of power dissipation.

$$\text{Energy dissipation} = VQ = VIt = I^2 R t = V^2 t/R$$

The commercial unit of electrical energy is 'kilowatt-hour'.

$$1 \text{ kilo watt hour} = 1000 \text{ watt} \times 3600 \text{ seconds} = 3.6 \times 10^6 \text{ Joule}$$

Example 11.4: Calculate 1-month cost of using 50 W energy saver for 8 hours daily in your study room. Assume that the price of a unit is Rs. 20.

Given: Power (in watt) = 50 watt Time (in hours) for 30 days = $8 \times 30 = 240 \text{ h}$

Cost per units = Rs. 20

To Find: Cost of electricity = ?

Solution: As No of units = $\frac{\text{power (watt)} \times \text{time of use in hours}}{1000}$

Putting values, we get: $= \frac{50 \times 240}{1000} = 12 \text{ unit}$

Cost of electricity = number of units $\times$ cost of 1 unit
 $= 12 \times 20 = \text{Rs. } 240$

Assignment 11.4

Estimate the cost of electricity consumed for a month, if the following devices are used as specified:

- i) 10 bulbs of 40 watts for 10 hours. iii) 40 tube lights of 25 watts for 10 hours.
 ii) 10 fridges of 250 watts for 24 hours. (Given the rate of electricity is Rs. 50/unit).

11.7 KIRCHHOFF'S LAWS

Gustav Robert Kirchhoff introduced laws related to two electrical physical quantities "current" and "voltage" as modified form of Ohm's work. Kirchhoff formulated a pair of laws as practical applications of law of conservation of charge and energy. These laws help in calculation of current in complex combinations of resistances with more than one emf sources.

For example, the circuit in Fig. 11.7 shows a multi-loop circuit, which consists of junctions (also known as a node). Here the three resistors are connected and three emf sources are attached to produce current. As the circuit is complex, the equivalent resistance cannot be found here. The circuit cannot be solved by

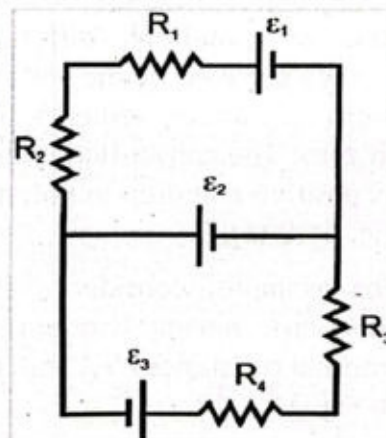


Figure 11.7: A multi-loop circuit.

applying Ohm's law, so Kirchhoff's rules can be used instead of Ohm's.

11.7.1 Kirchhoff's First Law OR Kirchhoff's Current Law (KCL)

According to the Kirchhoff's First law:

The currents entering the node always equal to currents leaving the node.

Let's consider five wires are connected at junction point, representing the currents I_1, I_2, I_3, I_4 and I_5 , where I_1, I_2, I_3 indicating the flow of charges entering the node taken positive while I_4 and I_5 indicating the flow of charges leaving from the node are taken negative, as shown in Fig. 11.8.

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0 \text{ hence } \sum I = 0$$

Since charge can neither be created nor destroyed, so the charge that has entered the junction as current must leave in equal amount from the junction. Thus, Kirchhoff's Current law is just a specific way of stating law of conservation of charge.

11.7.2 Kirchhoff's Second Laws OR Kirchhoff's Voltage Law (KVL)

According to the Kirchhoff's Second Law:

The algebraic sum of all the changes in potential around any closed path (loop) must be zero.

Mathematically, $\sum V = 0$

According to Kirchhoff's law, the algebraic sum of potential differences, including voltage supplied by the voltage sources (emf) and voltage drops in resistive elements (IR), in any loop must be equal to zero. The convention is that, rise in potential is taken as positive and drop in potential as negative, as shown in Fig. 11.9 (a).

For example, consider a simple loop ABCDA with no junctions, having two emf sources ' ϵ_1 ' and ' ϵ_2 ' with internal resistances ' r_1 ' and ' r_2 ' and a resistor R_1 , as shown in Fig. 11.9 (b).

Let's us assume clockwise current ' I ', and we travel the circuit in the direction of assumed current, starting from 'A'. At ' ϵ_1 ' the direction is from negative to positive

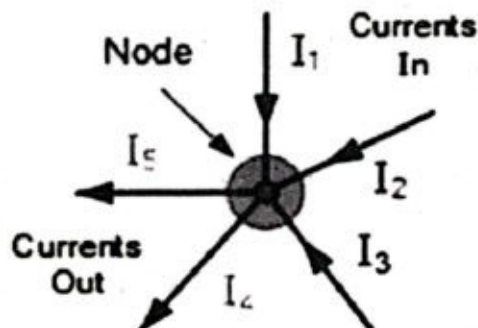


Figure 11.8: Kirchhoff's First law.

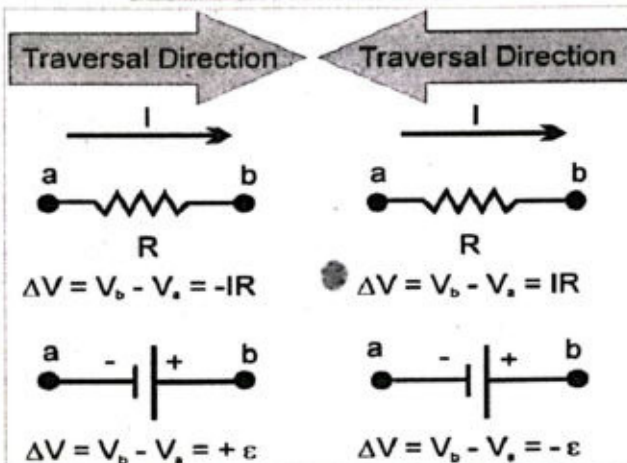


Figure 11.9 (a): Sign convention for Kirchhoff's law.

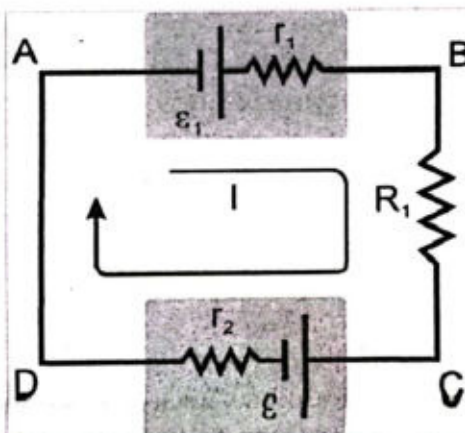


Figure 11.9 (b): A simple loop.



terminal of battery thus there is rise in potential and is taken as positive. Since at internal resistor ' r_1 ' there is drop in potential $\Delta V_{r_1} = Ir_1$ is taken as negative. At ' ϵ_2 ' the direction is from positive to negative terminal of battery and there is drop in potential and is taken as negative. Through internal resistor ' r_2 ' again there is drop in potential $\Delta V_{r_2} = IR_2$ and is taken as negative. Thus, according to Kirchhoff's Voltage law (KVL), we have:

$$\Delta V_{\epsilon_1} + \Delta V_{r_1} + \Delta V_{R_1} + \Delta V_{\epsilon_2} + \Delta V_{r_2} = 0$$

or

$$+ \epsilon_1 + (-Ir_1) + (-IR_1) + (-\epsilon_2) + (-Ir_2) = 0$$

$$+ \epsilon_1 - Ir_1 - IR_1 - \epsilon_2 - Ir_2 = 0$$

This law is based on the conservation of energy whereby voltage is defined as the energy per unit charge ($V = \Delta U/q_0$). The total amount of energy gained per unit charge must equal the amount of energy lost per unit charge, as energy and charge are both conserved. Thus, Kirchhoff's voltage law is just a specific way of stating law of conservation of energy.

11.7.3 Applying Kirchhoff's Laws

In order to solve problems by using Kirchhoff's Laws consider the circuit shown in Fig. 11.10. Let us take two closed loops:

- Loop 1 (ABCFA), assuming current ' I_1 ' to be flowing through it.
- Loop 2 (CDEFC), assuming current ' I_2 ' to be flowing through it.

The choice of loops is quite arbitrary, but it should be such that each resistance is included at least once in the selected loop.

The direction of assumed current is not important; a wrong direction of assumed current will only indicate negative sign in the result. If we go around the loop along the direction of assumed current I_1 , starting from point 'A'. The battery ' ϵ_1 ' is taken as positive because we are going from negative to positive terminal in the branch from 'A' to 'B'. At resistor ' R_1 ' the drop in potential is taken as negative $V_{R1} = -I_1R_1$. At resistor R_2 the drop in potential is taken as negative $V_{R2} = -(I_1 + I_2)R_2$. Therefore, by applying Kirchhoff's Loop rule, we get:

$$\epsilon_1 + V_{R1} + V_{R2} = 0$$

$$\epsilon_1 - I_1R_1 - (I_1 + I_2)R_2 = 0 \quad \text{--- (11.5)}$$

If we go around the loop (CFEDC) along the direction of assumed current I_2 , starting from point C. At resistor R_2 the drop in potential is taken as negative $V_{R2} = -(I_1 + I_2)R_2$. At resistor R_3 the drop in potential is taken as negative $V_{R3} = -I_2R_3$. The battery ' ϵ_2 ' is taken as positive because again we are going from negative to positive terminal in the branch from 'E' to 'D'. Therefore, by applying Kirchhoff's Loop rule. Mathematically,

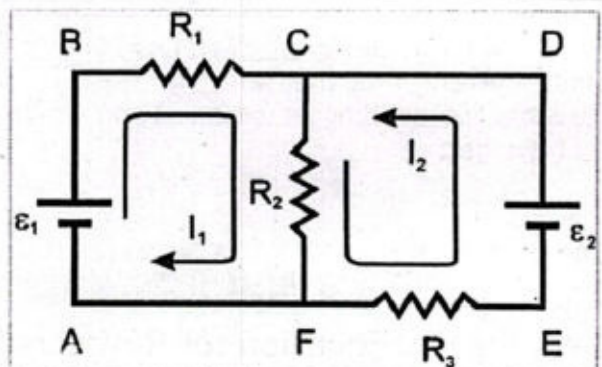


Figure 11.10: Complex circuit.

$$V_{R2} + \varepsilon_2 + V_{R3} = 0$$

$$-I_1 R_2 - I_2 R_2 + \varepsilon_2 - I_3 R_3 = 0 \quad (11.6)$$

By simultaneously solving the equations (11.5) and (11.6), the branch currents in the circuit can be determined.

Deriving the Equation for Resistors in Series

Consider two resistors R_1 and R_2 connected in series, as shown in Fig. 11.11. A single resistor R is equivalent to them.

From Kirchhoff's first law, the current I through each resistor is the same. Since they're connected in series (i.e., having no junctions).

From Kirchhoff's second law, the total p.d in a closed loop must equal the sum of the p.d both resistors:

$$V = V_1 + V_2 \quad (11.7)$$

From Ohm's Law, potential difference is given by the product of current and resistance, i.e.,

$$IR = IR_1 + IR_2 \quad (11.8)$$

Since current I is the same for all resistors, so dividing equation (11.8) by I , we get:

$$R = R_1 + R_2$$

The equivalent resistor R of several resistors connected in series is given by:

$$R = R_1 + R_2 + R_3 + \dots \quad (11.9)$$

Deriving the Equation for Resistors in Parallel

Consider two resistors R_1 and R_2 connected in parallel, having equivalent resistance R , as shown in Fig. 11.12. From Kirchhoff's first law, the current through each resistor will be different because it splits at the junctions. The current through the equivalent resistor R will be the total current I .

$$I = I_1 + I_2 \quad (11.10)$$

From Kirchhoff's second law, the p.d across resistors in different branches must be same and the resistor R will have the same p.d across it too:

$$V = V_1 = V_2 \quad (11.11)$$

From Ohm's Law, potential difference is given by the product of current and resistance. So, equation (11.10) becomes:

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} \quad (11.12)$$

since potential difference V is the same for all resistors, so equation (11.12) becomes:

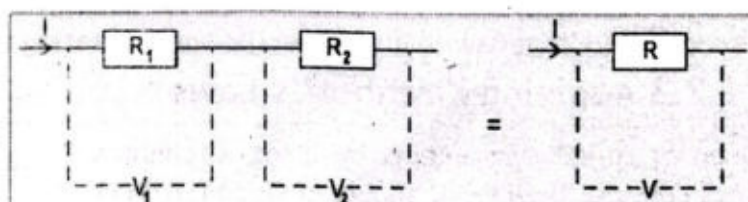


Figure 11.11: Resistors connected in series.

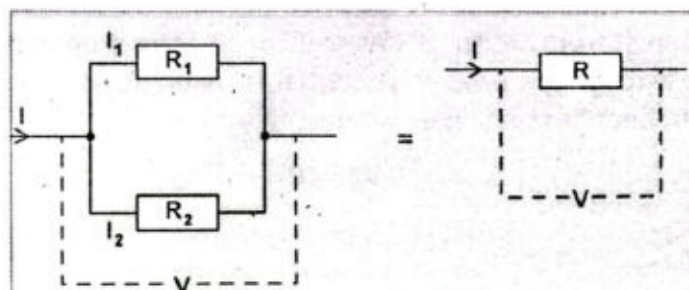


Figure 11.12: Resistors connected in parallel.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{--- (11.13)}$$

The equivalent resistor R of several resistors connected in parallel is given by:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \quad \text{--- (11.14)}$$

Example 11.5: Find current flowing through each resistor, using Kirchhoff's law, in the given circuit.

Solution:

Applying Kirchhoff's law to loop BAFEB:

$$\varepsilon_1 - I_1 R_1 - I_1 R_2 - (I_1 - I_2) R_4 = 0$$

$$3 - 3I_1 - 2I_1 - 6(I_1 - I_2) = 0$$

$$3 - 5I_1 - 6I_1 + 6I_2 = 0$$

$$11I_1 - 6I_2 = 3 \quad \text{--- (1)}$$

Now applying Kirchhoff's law to loop DCBED:

$$\varepsilon_2 - I_2 R_5 - (I_2 - I_1) R_4 - I_2 R_3 = 0 \quad \text{or} \quad 4.5 - 4I_2 - 6(I_2 - I_1) - 5I_2 = 0$$

$$4.5 - 9I_2 - 6I_2 + 6I_1 = 0$$

$$\text{or} \quad -6I_1 + 15I_2 = 4.5 \quad \text{--- (2)}$$

Multiplying equation (1) by 5 and equation (2) with 2 and adding:

$$\begin{array}{rcl} 55I_1 - 30I_2 & = & 15 \\ -12I_1 + 30I_2 & = & 9 \\ \hline 43I_1 & = & 24 \\ I_1 & = & 0.56 \text{ A} \end{array}$$

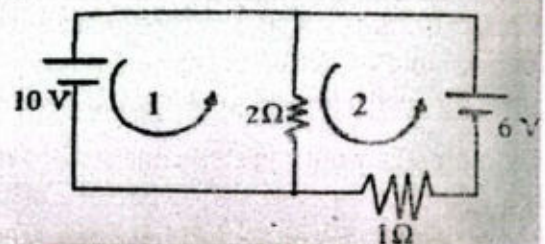
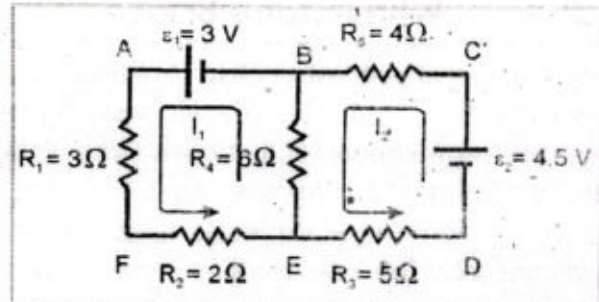
Putting value of I_1 in equation (1):

$$11(0.56) - 6I_2 = 3$$

Current flowing through resistances R_1 and R_2 is $I_1 = 0.56$ A. Current flowing through resistances R_3 and R_5 is $I_2 = 0.53$ A. Current flowing through resistance R_4 is $I_1 - I_2 = 0.56 \text{ A} - 0.53 \text{ A} = 0.03 \text{ A}$ in the upward direction. As both the values of currents I_1 and I_2 are positive, therefore the supposed directions of current are their actual directions.

Assignment 11.5

Find the current flowing through each resistor in the circuit having two loops, as shown in the figure.



11.8 NULL METHOD OF MEASUREMENT

This method basically shows null reading on galvanometer which is used for comparison between any known and unknown quantities. Null reading on galvanometer is used to find resistance in a circuit named as Wheatstone bridge. Potentiometer is also one of the electrical devices in which we use null method for finding potentials.

11.8.1 Wheatstone Bridge

Wheatstone bridge consists of four-resistors (three-known resistors while 1-unknown resistor) and a galvanometer is connected between two points, say b and d (Fig. 11.13). Wheatstone bridge becomes balance when no deflection is seen on galvanometer. The null reading measurement on galvanometer can help to calculate the value of unknown resistance.

Let I_1 and I_2 be the currents through P and R respectively when the bridge is balanced. Since there is no current through Galvanometer, the current in Q and S are also I_1 and I_2 respectively. As the Galvanometer reads zero, points B and D are at the same potential. Hence for balanced bridge:

Potential drop across ab = Potential drop across ad
i.e. $I_1 P = I_2 R$ (11.15)

Similarly,

Potential drop across bc = Potential drop across dc
 $I_1 Q = I_2 S$ (11.16)

Dividing (11.15) by (11.16), we get:

$$\frac{P}{Q} = \frac{R}{S}$$

or $R = \frac{PS}{Q}$ (11.17)

Hence, if three resistances on the right side of equation are known, the fourth resistance R can be calculated.

11.8.2 Potentiometer

A potentiometer is a null type resistance network device for measuring potential differences. Its principle of action is that an unknown emf or P.D. is measured by balancing it, wholly or in part, against a known potential difference.

A simplest potentiometer consists of wire AB of uniform cross-section, stretched alongside a scale and connected across battery of potential V, as shown in Fig. 11.14. A standard cell of known emf $\mathcal{E}_1$ is connected between A and terminal 1 of a two-way switch S.

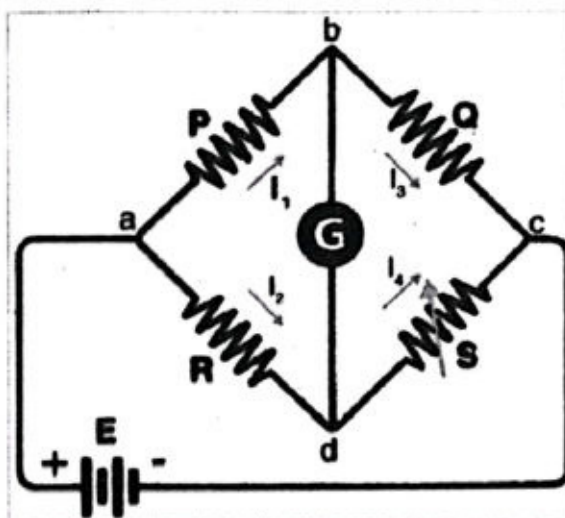


Figure 11.13: Wheatstone bridge circuit.



Slider N is pressed momentarily against wire AB and its position is adjusted until the galvanometer deflection is zero. Let l_1 be the corresponding distance between A and N. The fall of potential over length l_1 of the wire is then the same as the emf $\mathcal{E}_1$, i.e.

$$\mathcal{E}_1 \propto l_1 \quad \text{--- (11.18)}$$

Then move the switch to 2, thereby replacing the standard cell by another cell, the emf $\mathcal{E}_2$ of which is to be measured. Adjust the slider N again to give zero deflection on G. If l_2 be the new distance between A and N, then

$$\mathcal{E}_2 \propto l_2 \quad \text{--- (11.19)}$$

From the equations (11.18) and (11.19), we get:

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{l_2}{l_1}$$

$$\mathcal{E}_2 = \left(\frac{l_2}{l_1}\right) \mathcal{E}_1 \quad \text{--- (11.20)}$$

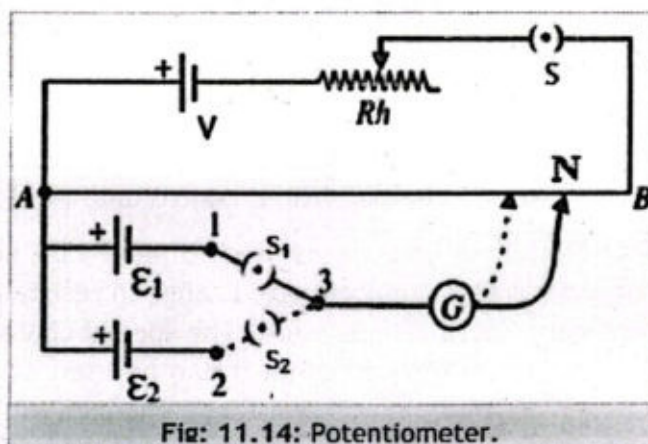


Fig: 11.14: Potentiometer.

Basic purposes of this electrical instrument are;

- Determination of electromotive force.
- Measurement of internal resistance.
- Comparison of potential of two emf sources.
- Better output in measuring potential difference as compare to voltmeter.

11.9 THERMISTOR

A thermistor (short for thermal resistor) is a heat sensitive resistor; it means that the change of temperature has direct relation with change of resistance. Samuel Ruben was the first achiever who invented the first thermistor, which later on used for commercial purposes.

Thermistors are usually made of a semiconductor material (semiconductor oxides of iron, nickel and cobalt). Pair of platinum leads is attached at the two ends for electrical connections. The arrangement is enclosed in a very small glass bulb and sealed. They are generally in the form of discs, rods, beads, etc., as shown in Fig. 11.15.

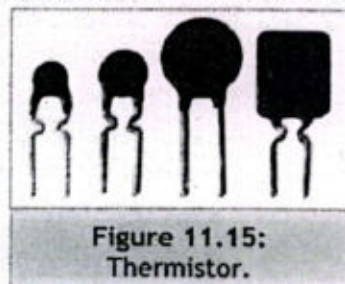


Figure 11.15: Thermistor.

Thermistor can be categorized on the basis of numeric values of temperature co-efficient as: negative temperature coefficient (NTC) and positive temperature coefficient (PTC). The difference between NTC and PTC is inverse and direct relation of temperature and resistance respectively. Positive temperature co-efficient of resistance means that the resistance

increases with the rise in temperature, whereas negative temperature co-efficient of resistance that the resistance decreases with the rise in temperature.

There are so many practical applications of thermistor like; temperature's compensation, controlling, etc.

Polycrystalline material is used to make LDR because this material is very supportive to change in resistance even by small change of temperature. The special characteristic of this electrical device is that it can be used as a potential divider. As thermistor's resistance changes with change of temperature, if this resistor is connected with potential divider circuit with known value of resistance, then desired voltage can be measured on the basis of temperature variation.

In potential divider circuit, thermistor can be used because its resistance can be increased or decreased by changing temperature (as shown in Fig. 11.16). By increasing temperature, the resistance of thermistor falls and by decreasing temperature the resistance of thermistor increases depending upon which type of thermistor is connected NTC or PTC. If we feel thermistor's temperature high, then it means potential will be dropped because its resistance will also be decreased and vice versa.

For Your Information

Metals (e.g. copper, aluminum) have positive temperature co-efficient of resistance because the resistance of metals increases with the rise in temperature.

Electrolytes, insulators (e.g. glass, mica, rubber etc.) and semiconductors (e.g. germanium, silicon etc.) have negative temperature co-efficient of resistance because their resistance decreases with the rise in temperature.

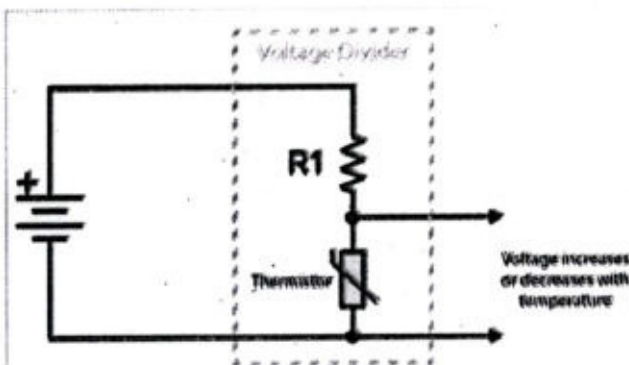


Figure 11.16: Temperature based potential divider circuit.

11.10 CARBON FIBERS IN CONCRETE BRIDGE

Carbon fibre-reinforced concrete has been used in construction projects due to its ability to improve compressive strength, tensile strength, elasticity modulus, chemical stability, abrasion resistance and corrosion resistance; reduce shrinkage cracks and brittleness; light weightiness; good thermal conductivity. Carbon fiber was introduced into concrete bridges (like shown in Fig. 11.18) to improve the flexural strength and reduce the crack width. Carbon fibers contain mainly carbon atoms and are commonly used in civil engineering works.

The optimum carbon fiber amount should be 0.3 % by volume of concrete. Inspector can easily check the reliability of concrete bridge made by carbon fibres because fiber conducts electricity



Figure 11.18: Carbon fibre-reinforced concrete.



if sensor shows the electric resistance increasing over time the fibres are separating because of cracks.

Carbon is a very competence material as conductor because its molecular structure is very similar to graphite. There are so many electrical applications of carbon fibres, for example: carbon electrodes for electrolysis, electromagnetic interference shielding, sensing, electrical switching etc.

SUMMARY

- ❖ **Drift velocity:** The average velocity with which electrons 'drift' in the presence of an electric field.
- ❖ **Resistance:** A measurement of the opposition to flow of charge carriers in an electrical circuit. Its unit is ohms.
- ❖ **Resistivity:** It is a fundamental specific property of a material that measures its electrical resistance.
- ❖ **Conductivity:** It is the reciprocal of resistivity its unit is MHO.m^{-1}
- ❖ **Electromotive force:** The electric potential produced by either an electrochemical cell or by changing the magnetic field.
- ❖ **Potential gradient:** The slope of potential distance graph is called potential gradient.
- ❖ **Kirchhoff's first rule (KCL):** The algebraic sum of all currents inward and outward with respect to node must equal zero.
- ❖ **Kirchhoff's second rule:** For any closed loop in a circuit, the sum of the potential differences across all components present in a closed loop is zero.
- ❖ **Light dependent resistor (LDR):** A special type of resistor that works on the photoconductivity principle, it means that resistance increases or decreases with the variation of intensity of light.
- ❖ **Thermistor:** A special kind of thermometer, which shows the inverse relation between temperature and resistance.
- ❖ **Potential divider:** A voltage divider is a circuit that takes a larger voltage and divides it down by a fixed ratio according to the electronic components to give a smaller output voltage. Thermistor and LDR can be used within potential divider circuit.
- ❖ **Null Measurement Method:** Measuring method which can be used in Wheatstone bridge electric circuit arrangements, in which the quantity to be measured is balanced by an opposing known quantity that is varied until the resultant of the two is zero, which can be seen on electrical device like galvanometer.

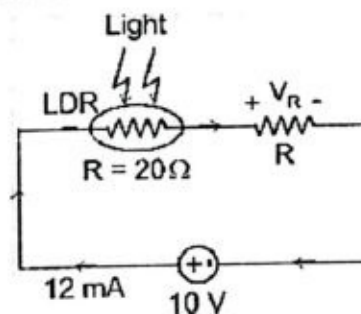
EXERCISE

Multiple Choice Questions

Encircle the Correct option.

- 1) On which of the following factor the drift velocity does not depend: _____.
- A. length of the wire. B. cross-section of the wire.

- C. number of free electrons. D. magnitude of the electric field.
- 2) Drift velocity of electrons is independent of _____.
 A. time consumed. B. the length of the wire.
 C. the number of free electrons. D. the Strength of the electric field.
- 3) A $10\ \Omega$ resistor is connected across the terminals of a 10 V battery. The power dissipation of the resistor is
 A. 100 W B. 50 W C. 25 W D. 10 W
- 4) A heavy duty refrigerator is used about 1000 W, if it is allowed to run continuously for 10 hours daily, how many kilowatt-hours of energy does it consume in 10 days?
 A. 100 kWh B. 200 kWh C. 300 kWh D. 350 kWh
- 5) What will be the value of emf if 3A current within 5mins perform work about 900J?
 A. 10 V B. 3 V C. 1 V D. 5 V
- 6) Which one of the following electrical devices helps us to measure the accurate value of emf?
 A. meter Bridge B. voltmeter C. multimeter D. potentiometer
- 7) The unit of dynamic charge is:
 A. coulomb B. ohm C. ampere D. volt
- 8) The resistance of a steel wire of particular length and thickness is $100\ \Omega$. If the length and the diameter of the wire both are doubled, then the resistivity in $\Omega\ m$ will be:
 _____.
 A. $1 \times 10^{-7}\ \Omega\ m$ B. $2 \times 10^{-7}\ \Omega\ m$ C. $4 \times 10^{-7}\ \Omega\ m$ D. $5 \times 10^{-7}\ \Omega\ m$
- 9) How much power dissipation will be increased if current passing through the circuit is increased 100 % (let keeping temperature constant)?
 A. 100 % B. 200 % C. 300 % D. 500 %
- 10) There are 2 resistors of having values $2\ \Omega$ and $4\ \Omega$ respectively are connected in series to a 6 V battery. The heat dissipated by the $4\ \Omega$ resistor in 5 s will be:
 A. 5 J B. 10 J C. 20 J D. 30 J
- 11) The algebraic sign of an IR drop primarily depends upon the
 A. direction of flow of current. B. battery connections.
 C. magnitude of current flowing through it. D. value of resistance.
- 12) What will be the nearest value of protected resistance, if LDR having resistance $20\ \Omega$ is operated at specific intensity of light is to be protect through a series resistance in such a way that up to 12 mA current in the provision of 10 V potential across the circuit as shown in following figure?
 A. 681 Ω B. 773 Ω
 C. 813 Ω D. 973 Ω
- 13) Which of the following electrical instrument can be used as a null detector in the Wheatstone bridge?
 A. galvanometer B. ammeter C. voltmeter D. multimeter
- 14) Potentiometer can be used for _____





- A. comparing two voltages. B. comparing two currents.
 C. measuring a voltage. D. measuring a current.
- 15) In order to achieve high accuracy, the slide wire of a potentiometer should be:
 A. as long as possible. B. as short as possible.
 C. neither too small not too large. D. very thick.
- 16) There are _____ types of thermistor.
 A. one B. two C. three D. four
- 17) Thermistor is special kind of resistor named as:
 A. laser resistor B. photo resistor C. thermal resistor D. electric resistor

Short Questions

Give short answers of the following questions.

- 11.1 Why it is not possible to measure the drift speed for electron by timing their travel along a conductor?
- 11.2 What is the difference between e.m.f. and potential difference?
- 11.3 Why we use Kirchhoff's law for circuit problems solution.
- 11.4 The Kirchhoff's current rule is based on conservation of charge. Explain how?
- 11.5 While analyzing a circuit, the internal resistance of e.m.f. sources is ignored. Why?
- 11.6 Why rise in temperature of a conductor, with positive temperature coefficient, is accompanied by a rise in the resistance?
- 11.7 Under what circumstances can the terminal potential difference of a battery exceed its e.m.f.?
- 11.8 What is the working principle of Galvanometer?
- 11.9 How inspectors can easily check the reliability of a concrete bridge with carbon fibers.
- 11.10 Could electronic devices charge themselves without being plugged into an electricity source?
- 11.10 Does a source of electricity ever run out of electrons?
- 11.11 Two wires have equal length, one is made of copper and the other of manganin and they have the same resistance. Which wire here will be thicker between these two given wires?

Comprehensive Questions

Answer the following questions in detail.

- 11.1 What is LDR? Discuss in detail.
- 11.2 Explain the use of thermistors and light-dependent resistors in potential dividers.
- 11.3 Explain the following terms. (A) Drift velocity, (B) emf, (C) internal resistance.
- 11.4 What is potential divider? Explain.
- 11.5 What is Wheatstone bridge? Explain in detail.
- 11.6 What is a potentiometer? Explain.
- 11.7 Explain Kirchhoff's first law and describe that it is a consequence of conservation of charge.
- 11.8 Explain Kirchhoff's second law and describe that it is a consequence of conservation of energy.

11.9 Using Kirchhoff's laws, derive a formula for the combined resistance of two or more resistors in series.

11.10 Using Kirchhoff's laws, derive a formula for the combined resistance of two or more resistors in parallel.

11.11 Explain the electronic current in a metallic wire due to the drift of free electrons in the wire.

Numerical Problems

11.1 If 1A current flows through a copper wire having 1 cm^2 area of cross-section and 10 km length; calculate the time required by charge carrier to travel from one end to other end of the conductor (Free electron density of copper is 8.5×10^{28} per m^3). (Ans: 431 years)

11.2 Calculate the flow of charges per unit time inside the emf source having potential 2 V and terminals are connected with each other having internal resistance 0.02Ω ? (Ans: 100 A)

11.3 Calculate the terminal potential of the emf source if internal resistance of the battery is 10Ω and current and internal potential are 1 A and 10 V respectively? (Ans: 0 V)

11.4 There is a copper coil having 2000 number of turns with 0.8 mm^2 cross-section are, the length per turn is 80 cm. Calculate the resistance of the coil? (Resistivity of copper is $0.02 \mu\Omega\text{m}$)? (Ans: 40Ω)

11.5 A piece of silver wire has a resistance of 1Ω . What will be the resistance of magnum wire of one third of the length and one third of diameter, if the specific resistance of wire is 30 times that of silver? (Ans: 90Ω)

11.6 Calculate the current that flows in the 1Ω resistor in the following circuit. (Ans: 0.13 A)

