# 09

# Chord and Arcs of a Circle

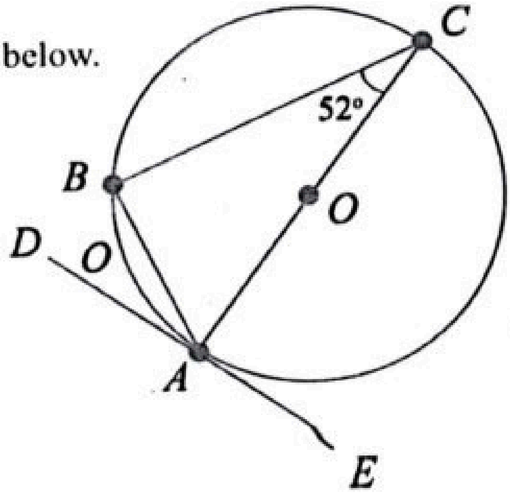
**After studying this unit students will be able to:**

- One and only one circle can pass through three non collinear points.
- A straight line drawn from the centre of a circle to bisect the chord is perpendicular to the chord.
- Perpendicular drawn from the centre of a circle on a chord, bisects it.
- Two congruent chords of a circle are equidistant from the centre.
- Two chords of a circle equidistant from the centre are congruent.
- If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
- If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semicircle) are congruent.
- Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres).
- If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding circles) are equal, the chords are equal.
- Apply concepts of chords and arcs to real life word problems such as decorative features, rainbow, bridges and roller coaster track.

Circle theorems are statements in geometry that state important results related to circles that are used to solve various questions in geometry. They have very useful applications within design and engineering.

These theorems show relationships between angles within the geometry of a circle. We can use these theorems along with prior knowledge of other angle properties to calculate missing angles, without the use of a protractor.

Can you find the value of angle in the figure below.



# Chords of a Circle

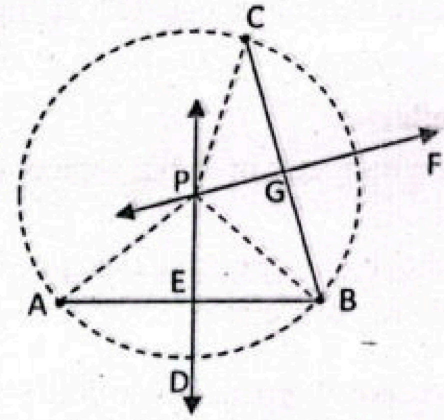
**Definition**

A line segment joining two points of a circle is called chord of that circle.

**Theorem 9.1**

**Statement:** One and only one circle can pass through three non collinear points.

**Class Activity:**




(i) Take three non-collinear points A, B and C on the paper and join them to obtain the triangle.

(ii) Draw right bisectors of the sides. They will meet at only one point P (say).

(iii) With centre P, draw a circle of radius PA.
What do you notice? Does the circle pass through B and C also?

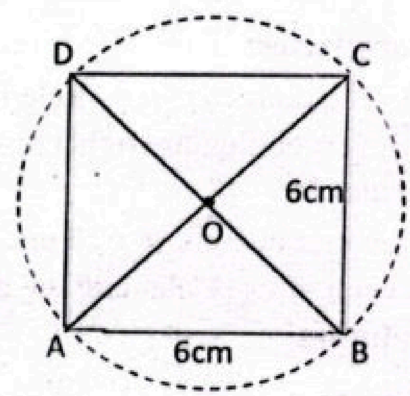
**Corollaries:** (i) One and only one circle can pass through two points lying on the same line.

(ii) Two circles cannot intersect each other at more than two points.

**Example:**

Construct a square with sides 6 cm long. Show that one and only one circle can be drawn through vertices of the square. Measure the radius of the circle.

**Solution:**




(i) Construct a square ABCD having side equal to 6 cm long.

(ii) Draw diagonals $\overline{AC}$ and $\overline{BD}$.

(iii) The two diagonals bisect each other at point O.

(iv) With centre O, construct a circle with radius OA.

(v) You will see that the circle will also pass through vertices B, C and D.

As diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other. Therefore:

$OA = OB = OC = OD$

Furthermore, there is only one centre O of the circle, therefore we can say that only one circle can be drawn through vertices of the square.

Now in right triangle ABC:

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (6)^2 + (6)^2 = 36 + 36 = 72$

Taking square root on both sides, we have:

$AC = \sqrt{72} = 6\sqrt{2}$ cm

Radius $= OA =$ half of $AC = 3\sqrt{2}$ cm

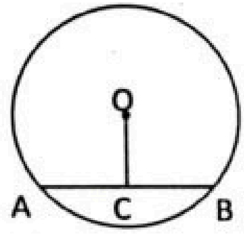
**Key Fact:**

If two chords of a circle do not pass through the centre, they cannot bisect each other.

## Theorem 9.2

**Statement:** A straight line drawn from the centre of a circle to bisect the chord is perpendicular to the chord.

**Class Activity:**




(i) Draw a chord AB of a circle with centre O.

(ii) Take a point C on the chord such that AC = BC.

(iii) Join O to C and measure $\angle OCA$ and $\angle OCB$.

Are the angles equal? Yes! they are.

$\therefore \quad \angle OCA = \angle OCB \quad$ i.e., $\overline{OC} \perp \overline{AB}$

**Corollaries:**

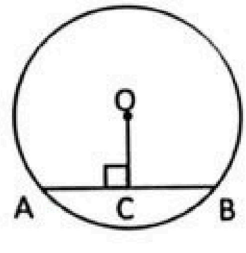
(i) If a diameter or radial segment of a circle bisects the chord, it is perpendicular to the chord.

(ii) The diameter or radial segment of a circle passes through the mid points of two parrallel chords of a circle.

## Theorem 9.3

**Statement:** Perpendicular drawn from the centre of a circle on a chord, bisects it.

**Class Activity:**




(i) Draw a chord AB of a circle with centre O.

(ii) Take a point C on the chord such that $\angle OCA = 90^\circ$.

(iii) Measure AC and BC.

Are they equal?

Yes! they are. i.e., AC = BC

**Corollaries:**

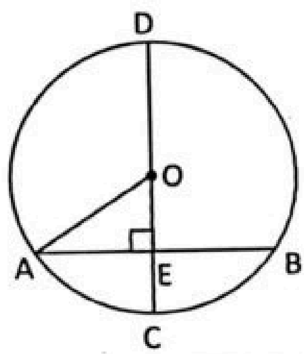
(i) If a diameter of a circle is perpendicular to the chord, it bisects the chord.

(ii) Perpendicular (right) bisector of a chord of a circle passes through the centre of a circle.

**Example:**

Diameter and chord of a circle are 20 cm and 16 cm long respectively and are perpendicular to each other. Calculate the distance of chord form the centre.

**Solution:**




Diameter = CD = 20 cm

Radius = OA = 10 cm

Chord = AB = 16 cm

As the diameter and chord are perpendicular to each other.

Therefore, diameter bisects the chord.

Hence, AE = BE = 8 cm

Now in right triangle AEO:

$(AE)^2 + (OE)^2 = (OA)^2 \quad \Rightarrow \quad (8)^2 + (OE)^2 = (10)^2$

$\Rightarrow \quad (OE)^2 = (10)^2 - (8)^2 = 100 - 64 = 36$

$\Rightarrow \quad OE = \sqrt{36} = 6$ cm

$\therefore \quad$ The distance of chord from the centre = 6 cm

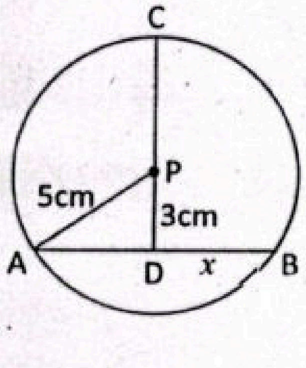
**Key Fact:**

Diameters of a circle bisect each other.

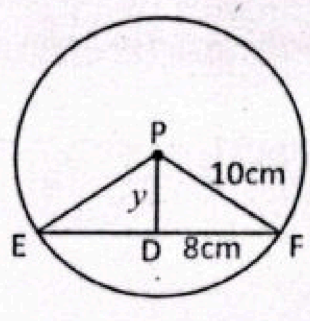
## Exercise 9.1

1. Construct an equilateral triangle with side 5 cm long. Show that one and only one circle can be drawn through the vertices of triangle. What is the radius of circle drawn?
2. Given that P is the centre of each of the circles. Find the values of unknown if line segment drawn from the centre of each circle is perpendicular to the chord.

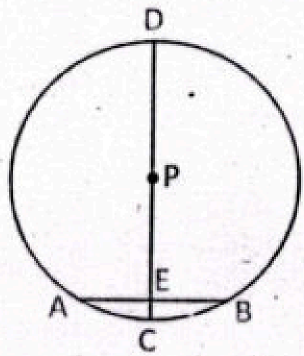
(i)



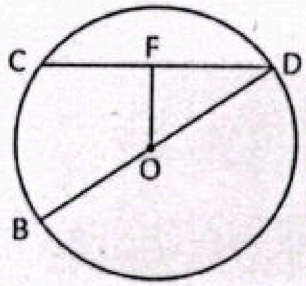

(ii)



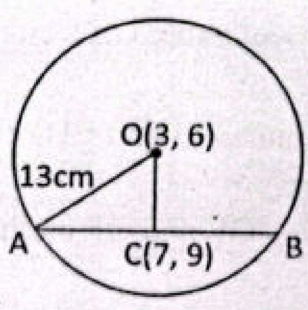

3. Find the length of diameter CD of the circle when AB = 10 cm, PE = 12 cm. Also find CE.



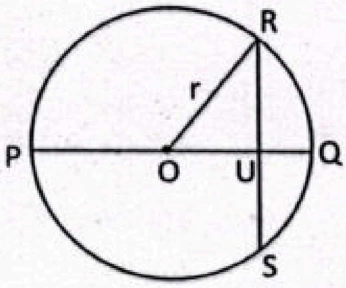

4. In the following diagram, OB = 15 cm, OF = 9 cm. Find the length of chord CD given that $\overline{OF} \perp \overline{CD}$.



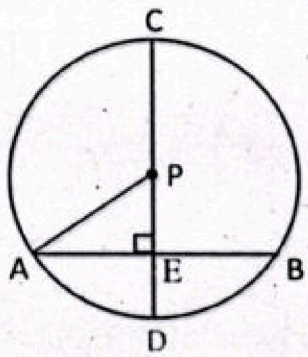

5. Calculate the length of chord AB in the following diagram if $\overline{OC} \perp \overline{AB}$.



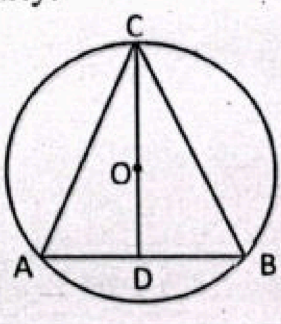
6. Construct a rectangle EFGH such that EF = 6 cm and FG = 4 cm. Draw a circle passing through its vertices and prove that it is the only circle that can be drawn through vertices.
7. Take four non collinear points A, B, C and D such that AB = BC = DB = 4cm. Draw all possible circles that can pass through A, C and D.
8. In the diagram, PU = 16 cm and RS = 10 cm.
   (i) Express OU in terms of radius r.
   (ii) Form an equation in r and solve it to find radius r.
   Hint: 16 – OU = r




9. In the following diagram, AB = 16 cm, DE 4= cm. Find the radius of the circle.




10. Given that diameter and chord of a circle are 10 cm and 8 cm long respectively. The diameter bisects the chord and the distance between chord and centre of the circle is 3 cm. Show that the diameter bisects the chord perpendicularly.
11. In the figure O is centre of the circle. AB is the chord and CD ⊥ AB passes through centre. Show that AC = BC.

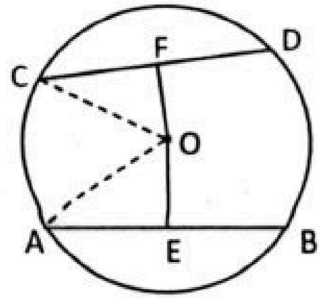



**Theorem 9.4**

**Statement:** Two congruent chords of a circle are equidistant from the centre.

**Class Activity:**

(i) Draw two equal chords AB and CD of a circle as shown in the adjoining figure.
(ii) Draw perpendiculars OE and OF on the chords AB and CD respectively.



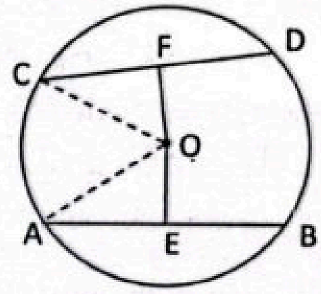
(iii) Measure OE and OF.
Are they equal? Yes! they are equal.

## Theorem 9.5

**Statement:**

Two chords of a circle equidistant from the centre, are congruent.

**Class Activity:**




(i) Draw two chords AB and CD of a circle such that they are equidistant from the centre as shown in the adjoining figure.

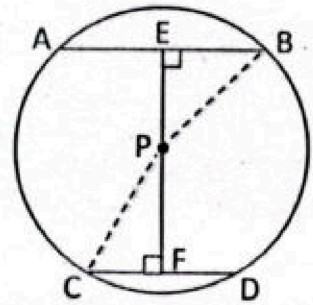
(ii) Measure chords AB and CD.
You will see that they are equal in length.

**Corollary:**

(i) If two chords of a circle are unequal, then longer chord is nearer to the centre and vice versa.

(ii) If two chords of two congruent circles are congruent, then they are equidistant from their respective centres and vice versa.

**Example:**




In the adjoining figure, $\overline{AB} \parallel \overline{CD}$ such that AB = 24 cm and CD = 10 cm. Find the radius of the circle if distance between chords is 17 cm.

**Solution:**

Let the radius of circle be r cm with centre P.

Draw $\overline{PE} \perp \overline{AB}$ and $\overline{PF} \perp \overline{CD}$

$\Rightarrow$ $BE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$

As the chords AB and CD are parallel, therefore points E, P and F are collinear and EF = 17 cm

Let PE = $x$, then PF = $17 - x$

Join P to B and C, then PB = PC = r

Now, PBE and PCF are right angled triangled. Therefore:

$(PB)^2 = (PE)^2 + (EB)^2$ and $(PC)^2 = (PF)^2 + (FC)^2$

$r^2 = x^2 + 12^2$ ......(i) and $r^2 = (17 - x)^2 + 5^2$ ......(ii)

$\Rightarrow$ $x^2 + 12^2 = (17 - x)^2 + 5^2$ ..... [comparing (i) and (ii)]

$\Rightarrow$ $x^2 + 144 = 289 + x^2 - 34x + 25$

$\Rightarrow$ $34x = 170$

$\Rightarrow$ $x = 5$

Now from (i), we have:

**Key Fact:**

If AB is chord and diameter CD is right bisector of AB, then:
AC = BC and AD = BD

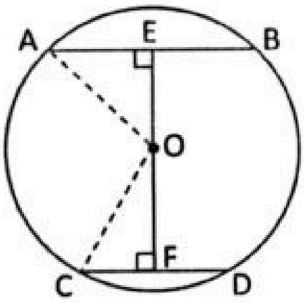
$r^2 = 5^2 + 12^2 = 25 + 144 = 169$

Taking square roots on both sides, we get $r = 13$.

$\therefore$ Radius of circle $= r = 13$ cm

**Example:**

Two parallel chords AB and CD of lengths 16 cm and 12 cm respectively are drawn on the opposite sides of centre O of the circle. Find the distance between the chords if the radius of the circle is 10 cm.




**Solution:**

Radius of circle = OA = OC = 10 cm

AB = 16 cm, CD = 12 cm

Draw $\overline{OE} \perp \overline{AB}$ and $\overline{OF} \perp \overline{CD}$

$\Rightarrow \quad AE = \frac{1}{2}AB = 8 \text{ cm} \quad \text{and} \quad CF = \frac{1}{2}CD = 6 \text{ cm}$

As $\overline{AB} \parallel \overline{CD}$, therefore points E, O and F are collinear.

Now, OAE and OCF are right angled triangles. Therefore:

$$(OE)^2 + (AE)^2 = (OA)^2 \quad \text{and} \quad (OF)^2 + (CF)^2 = (OC)^2$$

$$\Rightarrow (OE)^2 + (8)^2 = (10)^2 \quad \text{and} \quad (OF)^2 + (6)^2 = (10)^2$$

$$\Rightarrow (OE)^2 + 64 = 100 \quad \text{and} \quad (OF)^2 + 36 = 100$$

$$\Rightarrow (OE)^2 = 100 - 64 = 36 \quad \text{and} \quad (OF)^2 = 100 - 36 = 64$$

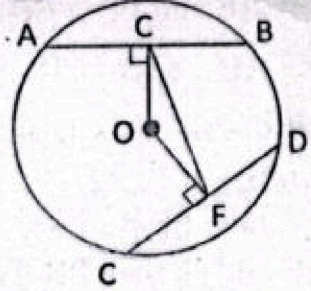
Taking square roots, we get:

$$OE = 6 \text{ cm} \quad \text{and} \quad OF = 8 \text{ cm}$$

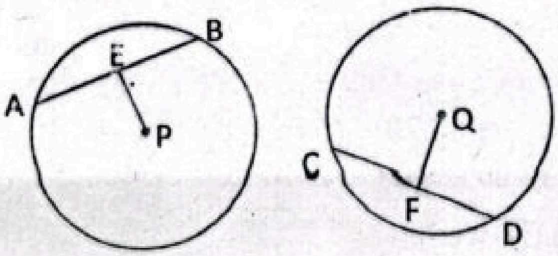
$\therefore$ Distance between chords = OE + OF = 6 + 8 = 14 cm

## Exercise 9.2

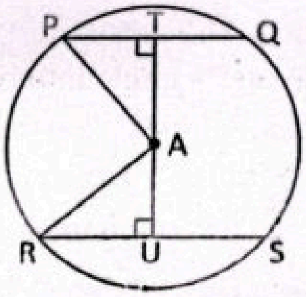
1. In the figure, O is centre of the circle and $\angle COF = 150^\circ$. Find the values of $\angle OCF$ and $\angle CFD$.




2. Two circles of radius 3 cm each are drawn with centres P and Q. AB and CD are their chords such that AB = CD = 4 cm. Find shortest distances of the chords from the respective centres. Are they equal?

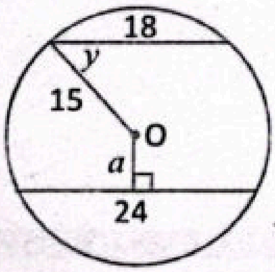


3. Find the distance between two chords PQ and RS of circle shown in the adjoining figure if: PQ = 6 cm, RS = 8 cm and radius = 5 cm

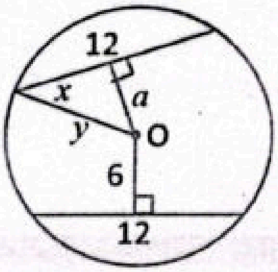



4. Given that O is the centre of each of the circles. Find the values of unknown. All dimensions are in cm.

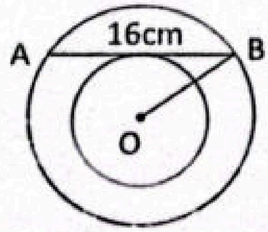
(i)




(ii)




5. Two parallel chords of lengths 24 cm and 12 cm are drawn on the opposite sides of a circle of radius 13 cm. Find the distance between the chords.
6. AB is chord of a bigger circle of radius 10 cm centered at O. Find the radius of the smaller circle passing through midpoint of $\overline{AB}$.
Also find the difference of radii of both circles.

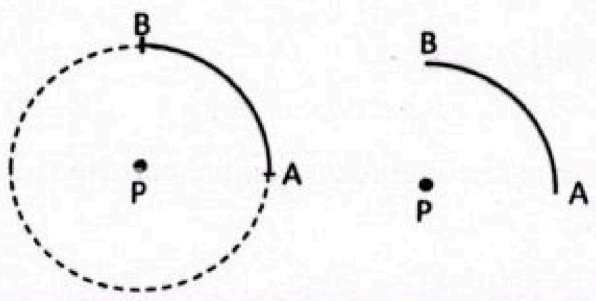



7. Two parallel chords of lengths 18 cm and 80 cm are drawn on the same side of a circle of radius 41 cm. Find the distance between the chords.
8. Two parallel chords AB and CD are 4 cm apart and lie on the opposite sides of the centre of a circle. If AB = 2 cm and CD = 6 cm, find the radius of the circle.
9. PQ and RS are two parallel chords lying on the same side of the centre of a circle and are 2 cm apart. If PQ = 8 cm and RS = 10 cm, find the radius of the circle.

## Arcs of Circles

### Arc of a Circle

An arc is a part of a circle. In the figure, AB is an arc of the circle P.
Arc AB is denoted by $\widehat{AB}$.



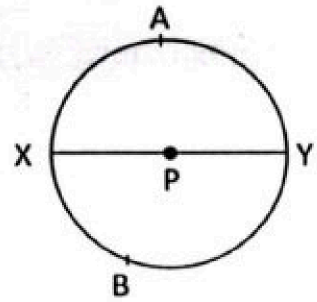

**Key Fact:**

There is a difference between a circle and a circular region. A circular region is union of circle and its interior.

**Semicircle**

A diameter divides a circle into two equal parts. Each part is called semicircle or half circle. We use three letters to name a semicircle. The first and third letter are end points of diameter.

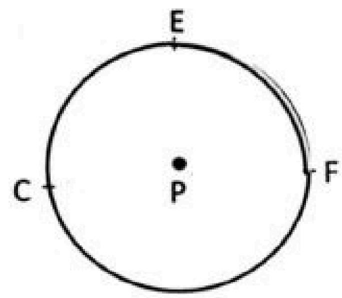
In the figure, $\widehat{XAY}$ is semicircle. Can you name the other semicircle?




**Minor and Major arcs**

Minor arc is the arc included in a semicircle while the arc which includes a semicircle is called major arc.

In the Fig. $\widehat{EF}$ is minor arc while $\widehat{ECF}$ is major arc.




**Key Fact:**

(i) Major arc of a circle and semicircle arc denoted by three letters.

(ii) Minor arc is less than and major arc is greater than the semicircle.

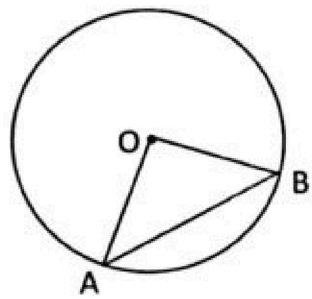
**Central Angle**

Angle subtended by an arc at the centre of a circle is called central angle of the arc. An angle whose vertex is the centre of circle and whose arms pass through end points of arc is known as central angle of arc.

**Key Fact:**

If an arc is included in a semi-circle, then its central angle is less than 180°. Central angle of semi-circle is straight angle (180°). Central angle of major arc of the circle is greater than 180°.

In the figure ∠AOB is central angle.




**Congruent Arcs**

Two arcs of a circle (or of two congruent circles) are congruent if their central angles are congruent.

**Theorem 9.6**

**Statement:**

If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.

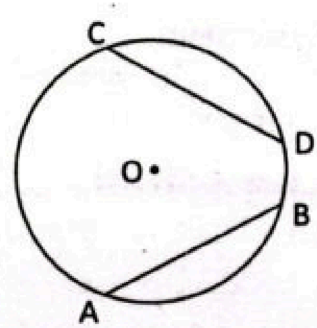
**Class Activity:**

(i) Take two congruent arcs AB and CD of a circle with centre O.

(ii) Join C to D and A to B.

(iii) Measure chords AB and CD.

You will see that AB = CD




**Key Fact:**

If two arcs of two congruent circles are congruent, then the corresponding chords of both circles are equal.

## Theorem 9.7

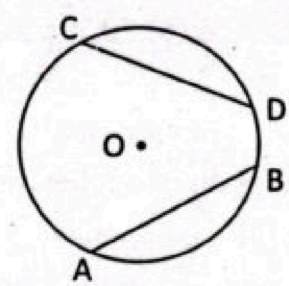
**Statement:**

If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semicircle) are congruent.

**Class Activity:**

(i) Draw two equal chords AB and CD of a circle with centre O.

(ii) Measure corresponding arcs AB and CD. They will be congruent.




**Corollary:**

(i) If two chords of a circle (or of congruent circles) are equal, then their corresponding minor or major arcs are congruent.

(ii) If two chords of a circle (or of congruent circles) are equal, then their corresponding semicircles are congruent.

## Theorem 9.8

**Statement:**

Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres).

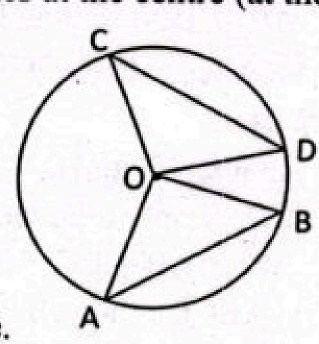
**Class Activity:**

(i) Draw two equal chords AB and CD of a circle with centre O.

(ii) Join O to A, B, C and D.

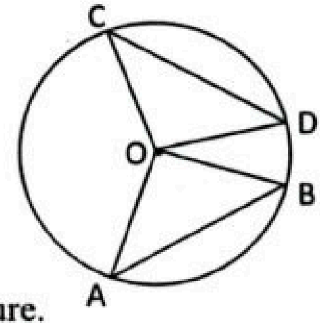
(iii) Measure corresponding angles AOB and COD.

You will observe that the corresponding angles are equal in measure.



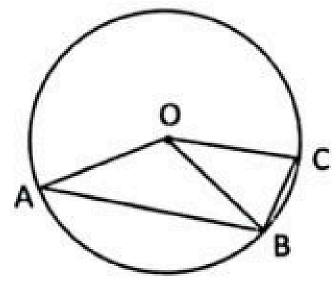
## Theorem 9.9

***Statement:***
If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding circles) are equal, the chords are equal.




**Class Activity:**
(i) Draw two chords AB and CD of a circle such that
$\angle AOB = \angle COD$
(ii) Measure corresponding chords AB and CD.
What do you notice?
You will observe that the corresponding chords are equal in measure.

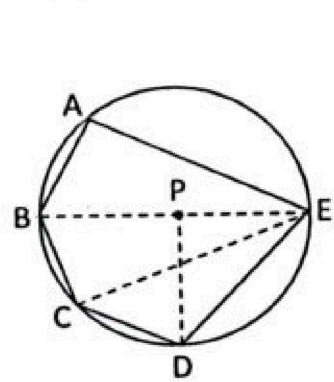
**Example:**
In the figure, $\widehat{AB} = 4\,\widehat{BC}$ and $\angle AOB = 120^\circ$, find:




(a) $\angle BOC$ (b) $\angle OBC$ (c) $\angle OBA$
(d) $\angle OAB + \angle ABC + \angle BCO + \angle COA$

**Solution:**
(a) Let $\angle BOC = x$, then $\angle AOB = 4x$ ... $(\widehat{AB} = 4\,\widehat{BC})$
But $\angle AOB = 120^\circ$ .... (Given)
$\therefore\ 4x = 120^\circ \Rightarrow x = 30^\circ$
Hence, $\angle BOC = 30^\circ$
(b) $\Delta OBC$ is an isosceles triangle. $(OB = OC = \text{radii})$
$\Rightarrow\ \angle OBC = \angle OCB$ (opposite angles of congruent sides of triangle)
$\therefore\ \angle OBC = \frac{1}{2}(180^\circ - 30^\circ) = 75^\circ$
(c) $\Delta OAB$ is an isosceles triangle. $(OA = OB = \text{radii})$
$\Rightarrow\ \angle OAB = \angle OBA$ (opposite angles of congruent sides of triangle)
$\therefore\ \angle OBA = \frac{1}{2}(180^\circ - 120^\circ) = 30^\circ$
(d) $\angle OAB + \angle ABC + \angle BCO + \angle COA = 30^\circ + (30^\circ + 75^\circ) + 75^\circ + (30^\circ + 120^\circ)$
$= 30^\circ + 105^\circ + 75^\circ + 150^\circ = 360^\circ$

**Example:**
Diagram shows a pentagon inscribed in a circle with centre P.
If $\widehat{AB} = \widehat{BC} = \widehat{CD}$ and $\angle ABC = 140^\circ$, then find:




(a) $\angle AEB$ (b) $\angle AED$ (c) $\angle BPD$

**Solution:**
(a) Given that, $\widehat{AB} = \widehat{BC} = \widehat{CD}$, then:
Corresponding chords are equal in length.
i.e., $AB = BC = CD$
ABCE is a cyclic quadrilateral.
$\therefore\ \angle ABC + \angle AEC = 180^\circ$

$\Rightarrow \angle AEC = 180^\circ - \angle ABC = 180^\circ - 140^\circ = 40^\circ$

$\Rightarrow \angle AEB + \angle BEC = 40^\circ$ (angle sum postulate)

$\Rightarrow 2\angle AEB = 40^\circ$ $(\angle AEB = \angle BEC)$

$\Rightarrow \angle AEB = 20^\circ$

(b) As, $AB = BC = CD$ (given)

$\angle AEB = \angle BEC = \angle CED = 20^\circ$

(Equal chords subtend equal angle at a point on the circumference.)

Now, $\angle AED = \angle AEB + \angle BEC + \angle CED = 20^\circ + 20^\circ + 20^\circ = 60^\circ$

(c) $\angle BPD = 2\angle BED$ (Angle at the centre is double the angle at a point on circumference.)

$= 2 \times 40^\circ = 80^\circ$

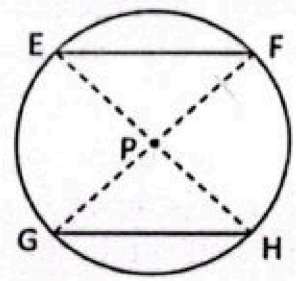
**Key Fact:**

If two arcs of a circle are unequal, then chord and angle corresponding to greater arc are greater.

**Example:**

In the figure, chords $\overline{EF} \parallel \overline{GH}$. Prove that:

$\widehat{EG} = \widehat{FH}$




**Solution:**

Draw $\overline{EF}$ and $\overline{FG}$ passing through centre of circle P.

$\angle EPG = 2\angle EFG$ (Angle at the centre is double the angle at a point on the circumference.)

$\therefore \quad \angle EFG = \frac{1}{2}\angle EPG \quad$ (i)

Similarly, $\angle FGH = \frac{1}{2}\angle FPH \quad$ (ii)

But, $\angle EFG = \angle FGH$ (alternate angles)

$\therefore \quad \angle EPG = \angle FPH$ [from (i) and (ii)]

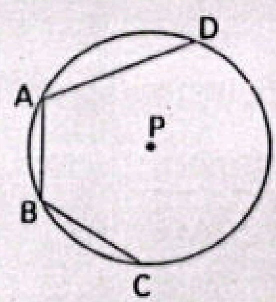
Hence, $\widehat{EG} = \widehat{FH}$ (Arcs subtending equal angles at the centre of circle are equal.)

**Check Point:**

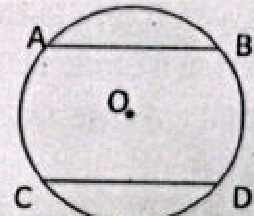
Prove that non-parallel sides of a cyclic trapezium are equal.

## Exercise 9.3

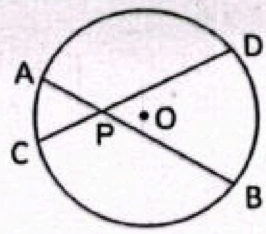
1. In the figure, chord AB = chord BC.
   (i) Is $\widehat{AB} = \widehat{BC}$ ?
   (ii) Is $\angle APB = \angle BPC$ ?
   (iii) If $\widehat{AB} < \widehat{AD}$, then what is relation between corresponding chords?




2. In the figure,
   $\widehat{AC} = \widehat{BD}$
   Prove that: $\overline{AB} \parallel \overline{CD}$

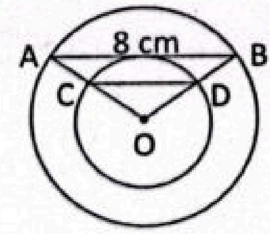


3. In the given figure, chords $\overline{AB}$ and $\overline{CD}$ intersect at P.
If AB = CD, then prove that:
$\widehat{AB} = \widehat{CD}$

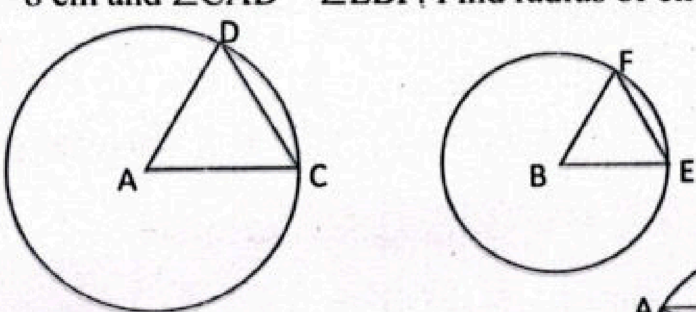



4. Consider two circles of radii 2 cm and 4 cm centered at P and Q respectively. $\widehat{AB}$ and $\widehat{CD}$ are their arcs such that $\angle APB = \angle CQD$.
   i. Are $\widehat{AB}$ and $\widehat{CD}$ congruent?
   ii. What is relation between the lengths of $\widehat{AB}$ and $\widehat{CD}$?
   iii. Show that $\Delta APB \sim \Delta CQD$

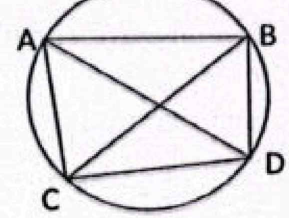
5. O is centre of two concentric circles of radii 3 cm and 6 cm. OA and OB are radii of larger circle and they meet smaller circle at C and D respectively.
   i. Are $\overline{AB}$ and $\overline{CD}$ parallel?
   ii. If AB = 8 cm, then find CD.
   iii. Find distance between $\overline{AB}$ and $\overline{CD}$.



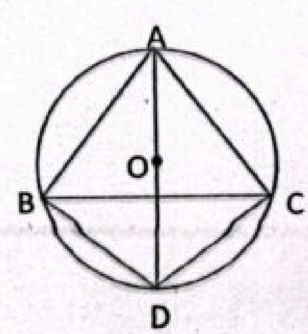

6. The length of an arc of a circle is 4 cm and its central angle is of measure 60°. Find central angle of the arc whose length is 8 cm.
7. Given that M is a point on a circle centered at P. If M is equidistant from the two radii PE and PF, show that $\widehat{ME} = \widehat{MF}$.
8. Consider two circles centered at A and B respectively. $\overline{CD}$ and $\overline{EF}$ are their chords such that CD = 12 cm, EF = 8 cm and $\angle CAD = \angle EBF$. Find radius of circle A if radius of circle B is 4 cm.



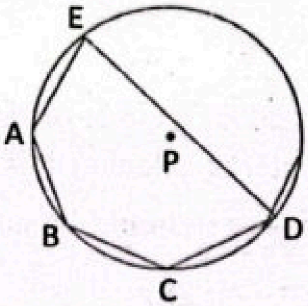

9. In the figure, ABDC is cyclic quadrilateral such that:
   AB = CD.
   Prove that:
   AC = BD



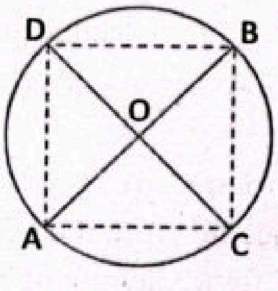

10. In the given figure, ABC is an isosceles triangle inscribed in a circle centered at O.
   Prove that $\overline{AD}$ bisects $\angle BDC$.



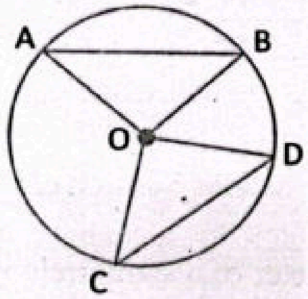
11. ABCDE is a pentagon inscribed in a circle.
If AB = BC = CD, $\angle BCD = 130^\circ$ and $\angle BAE = 120^\circ$
Find:
(a) $\angle ABC$ (b) $\angle CDE$
(c) $\angle AED$ (d) $\angle EAD$



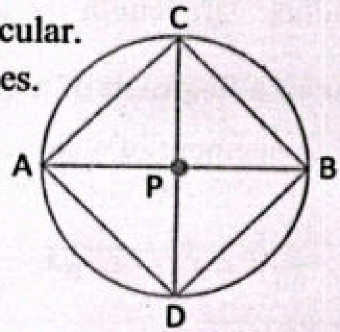

12. Given that O is the centre of a circle. Diameters $\overline{AB}$ and $\overline{CD}$ are perpendicular to each other. Prove that the line segments joining in order the end points of diameters form a square.



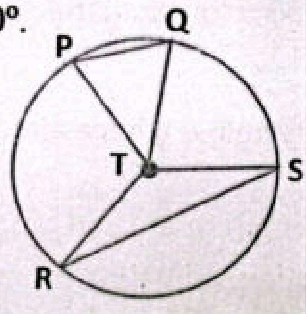

13. In the figure, O is centre of the circle.
AB = CD and $\angle OAB = 40^\circ$.
Find $\angle AOB$ and $\angle COD$.
Are the angles equal? If yes, then why?



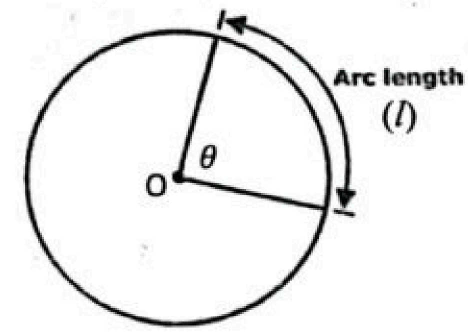

14. In the figure, diameters AB and CD of circle are perpendicular.
(i) Prove that all the four chords make equal central angles.
(ii) Find $\angle PCB$ and $\angle ACB$.
(iii) Find other angles that are equal to $\angle ACB$.
(iv) What is the name of inscribed polygon?
(v) How many circles can be drawn through the vertices of polygon formed?




15. In the adjoining figure, RS = 3PQ and $\angle PTQ + \angle RTS = 180^\circ$.
(i) Find $\angle PTQ$ and $\angle RTS$.
(ii) What is the ratio between measures of $\angle PTQ$ and $\angle RTS$?



# Applications of Chords and Arcs



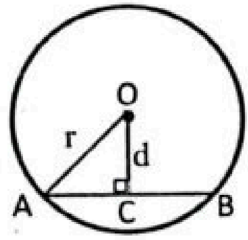

### Arc Length

The length of an arc of a circle of radius '$r$' that subtends an angle $\theta$ at the center is calculated by the following formula:

Arc length $= l = \frac{\theta}{360^o} \times$ circumference of circle

$= \frac{\theta}{360^o} \times 2\pi r$

Where 360° is the interior angle subtended by a circle.




### Chord Length

Length of a chord of a circle of radius '$r$' lying at a distance '$d$' from the centre is:

In the adjoining figure, length of chord AB is:

$AB = 2\sqrt{r^2 - d^2}$

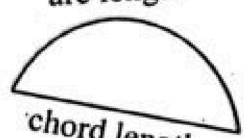
Note that AC is half of AB.

### Perimeter of a of Semicircle

The perimeter of a semicircle is given by the following formula.

P = 2 × radius + arc length $= 2r + \pi r$

### Perimeter of a Segment of a Circle




Perimeter of segment of a circle is defined as:

P = arc length + chord length

$= \frac{\theta}{360^o} \times 2\pi r + 2\sqrt{r^2 - d^2}$

### Area of Sector of Circle

Area of the sector of a circle having radius $r$ and central angle $\theta$ is:

$A = \frac{\theta}{360^o} \times \pi r^2$

Where the angle $\theta$ is measured in degrees.

**Example:**

Find the length of arc and area of sector a circle with central angle of 60° and radius 14cm.

**Solution:** Given that:

$\theta = 60^o$ and $r = 14$cm

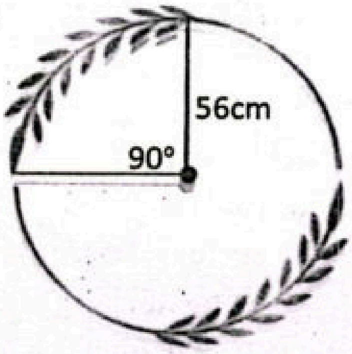
Arc length $= \frac{\theta}{360^o} \times 2\pi r$

$= \frac{60^o}{360^o} \times 2 \times \frac{22}{7} \times 14 = 14.67$ cm

Area $= \frac{\theta}{360^o} \times \pi r^2 = \frac{60^o}{360^o} \times \frac{22}{7} \times (14)^2 = 102.67$ cm$^2$

**Example:**

A circular decoration piece consists of two equal parts as shown in the figure. Find the length of one part of the piece. What is the cost of decoration piece @ Rs.8 per centimetre?




**Solution:**

Given that $r = 56\text{cm}$, $\theta = 90^\circ$, then:

$l = \frac{\theta}{360^\circ} \times 2\pi r = \frac{90^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 56$

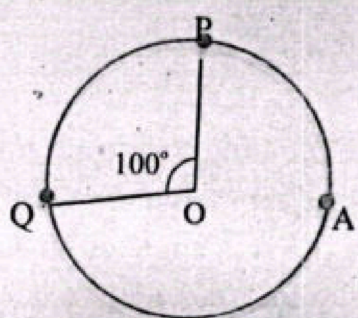
$l = 88$ cm

Length of one part of the decoration piece is = 2 × 88 = 176 cm

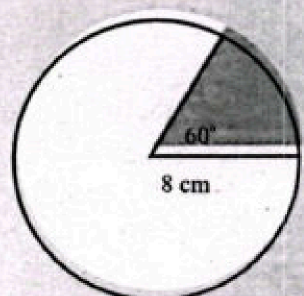
Cost of decoration piece @ Rs.8 per cm = Rs. 2 × 176 × 8 = Rs. 2,816

## Exercise 9.4

1. In the figure, radius of circle is 7cm. Find:
   (i) the length of minor arc $\widehat{PQ}$ and major arc $\widehat{PAQ}$.
   (ii) the circumference of circle.
   Is the sum of lengths of minor arc and major arc equal to circumference of circle?

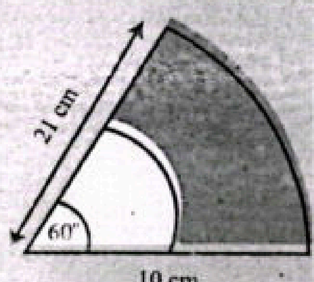



2. Given the radius of circle is 8cm. Find:
   (i) the area of minor sector and major sector.
   (iii) the area of circle.
   Is the sum of areas of minor and major sectors equal to area of circle?

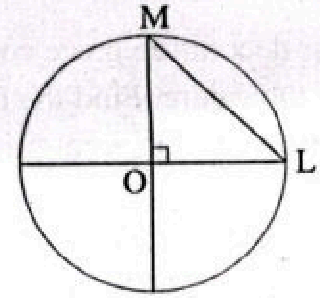



3. Find the distance covered by the tip of hour hand of a clock in 5 hours if the length of hour hand is 10cm.

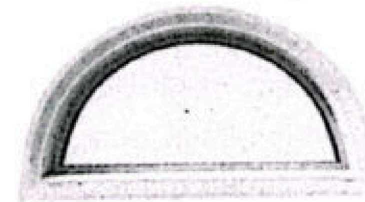
4. Given is the sector of circle. Find:
   (i) the length of outer minor arc.
   (ii) the length of inner minor arc.
   (iii) the difference of lengths of both arcs.
   (iv) the area of minor sector of bigger circle.
   (v) the area of minor sector of smaller circle.
   (vi) the area of shaded part of sector.



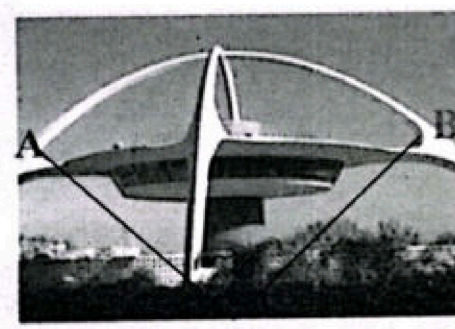
5. Radius of the given circle is 14cm. Find:
   (i) length of $\overline{LM}$.
   (ii) length of $\widehat{LM}$.
   (iii) perimeter of segment along chord $\overline{LM}$.
   (iv) area of triangle OLM.
   (v) area of sector OLM.
   (vi) area of segment along chord $\overline{LM}$.




6. In a circle whose diameter is 12 cm, there is a central angle whose measure is 120°. A chord joins the endpoints of the arc cut off by the angle. Find the length of arc along the chord.
7. The diameter of a circle is 10 cm long and a chord parallel to it is 6cm long. Find the distance between the chord and the diameter of the circle
8. Find the area and perimeter of semicircular window if its radius is 2.8feet.



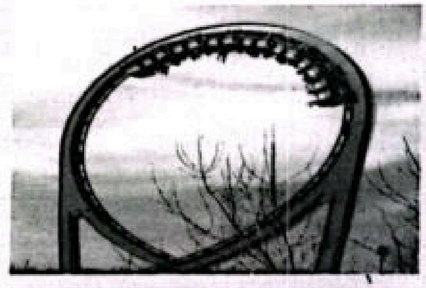
9. The building shown resembles a flying saucer that has landed on its four legs. Find the length of supporting beam AB if it makes an angle of 98° with center and the radius of the beam is 21 m.



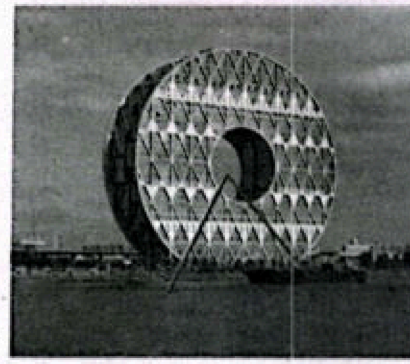

10. Figure shows a bridge of semi-circular shape with arc length of 44 m.
Find the length of road constructed below.

11. Find the perimeter of lower portion (below the line PQ) of circular window if:
radius of window = 2.1 feet,
$\angle POQ = 105°$,
and shortest distance between centre O and $\overline{PQ}$ = 1.4 feet.
Also find the area of upper portion (above line PQ) of window.

12. A circular roller coaster is shown in the figure. Find the distance covered by driver of train between two points making an angle of $80^\circ$ with the centre of roller coaster if its radius is 21ft.



13. Building shown in the figure is located in Guangzhou province, China. The building's height is 138 m with 33 stories. It has an empty circular core almost 59 m in diameter.
    (i) What is the covered area of the building?
    (ii) Find the circular length of the part of building that touches ground if it makes an angle of $54^\circ$ with the centre.



## I have Learnt

1. Solving the problems by using the properties of a circle that:
    - One and only one circle can pass through three non collinear points.
    - A straight line drawn from the centre of a circle to bisect the chord is perpendicular to the chord and vice versa.
    - Two congruent chords of a circle are equidistant from the centre and vice versa.
    - If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal and vice versa.
    - Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres) and vice versa.
    - Applying concepts of chords and arcs to real life word problems such as decorative features, rainbow, bridges and roller coaster track.

## MISCELLANEOUS EXERCISE-9

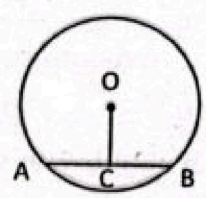
1. Encircle the correct option in the following.

i. One and only one circle can pass through ...... non collinear points.
(a) 2 (b) 3 (c) 4 (d) 5

ii. ............ number of circles can pass through a point.
(a) 1 (b) 2 (c) 3 (d) infinite

iii. Diameter of circle which bisects the chord is ........... to the chord.
(a) collinear (b) parallel (c) perpendicular (d) equal

iv. In the figure, OC = 3 cm, AB = 8 cm.
The radius of circle is:




(a) 4 cm (b) 4.5 cm (c) 5 cm (d) 10 cm

v. Diameter of circle perpendicular to the chord ............ the chord.

(a) intersects (b) bisects (c) trisects (d) touches

vi. Two chords which are equidistant from the ......... are congruent.

(a) centre (b) diameter (c) circle (d) chord

vii. Two ........ which are equidistant from the centre are congruent.

(a) circles (b) diameters (c) segments (d) chords

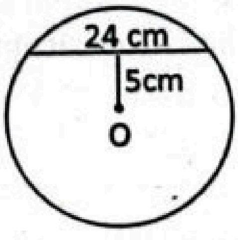
viii. Length of chord of a circle of radius 5 cm is 8 cm. The distance of chord from centre is:

(a) 3cm (b) 4cm (c) 5cm (d) 6cm

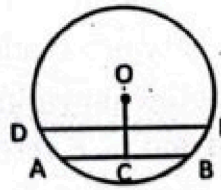
ix. What is the radius of circle in the figure?

(a) 26cm (b) 10cm

(c) 5cm (d) 13cm




x. In the figure, AB = 4cm, OC = 3cm. Chord DE passes through mid of OC and is parallel to AB. What is the length of DE?




(a) 4.5 cm (b) 5.6 cm

(c) 6.6 cm (d) 3.3 cm

xi. An angle whose vertex is centre of circle and whose arms pass through end points of an arc is known as____________ angle.

(a) inscribed (b) central (c) interior (d) exterior

xii. Corresponding arcs of two congruent chords of a circle are ___________.

(a) unequal (b) major (c) congruent (d) minor

xiii. Length of two chords of a circle are in the ratio 1 : 3, and central angle of one arc is 60°, the second arc is ___________.

(a) minor (b) major (c) semicircle (d) circle

xiv. The central angle of quadrant of a circle is ____________.

(a) 30° (b) 45° (c) 60° (d) 90°

xv. If a circle is divided into ten equal arcs, then central angle of each arc is _____.

(a) 10° (b) 36° (c) 60° (d) 90°

xvi. How many central angles of an arc can be drawn?

(a) one (b) two (c) finite (d) infinite

xvii. Two congruent chords of two congruent circles have ........ central angles.

(a) different (b) same (c) proportional (d) acute

xviii. Central angle of an arc which includes a semicircle in it is ________.

(a) $< 90°$ (b) $> 90°$ (c) $< 180°$ (d) $> 180°$