

Vectors in Plane

After studying this unit students will be able to:

- Introduce rectangular coordinate system in a plane.
- Represent vector as a directed line segment.
- Express a vector in terms of two non-zero and non-parallel coplanar vector.
- Express a vector in terms of position vector.
- Express translation by a vector.
- Find the magnitude of a vector.
- Add and subtract vectors.
- Multiply a vector by a scalar.
- Solve geometrical problems involving the use of a vector.
- Apply concepts of vectors in geometrical problems such as parallel and perpendicular lines in geometrical shapes, vector projectile motion, crosswinds aviation, military usage, designing roller coasters.

Vectors are fundamental mathematical concept that have a wide range of applications in the various fields from the physics and engineering to the computer graphics and navigation systems. Vectors are utilized to assist in the localization of people, places and things, and to describe things that are acting in response to an external force being applied to them.

Do you know?

- Navigating by air and by boat is generally done using vectors.
- Planes are given a vector to travel, and they use their speed to determine how far they need to go before turning or landing. Flight plans are made using a series of vectors.
- Sports instructions are based on using vectors.
- Vectors are used in electrical engineering for analyzing and designing circuits, signals, and electromagnetic systems.



Introduction of Vectors

There are two main types of physical quantities used in Physics, Mathematics and Engineering. They are known as, 'scalars' and 'vectors'

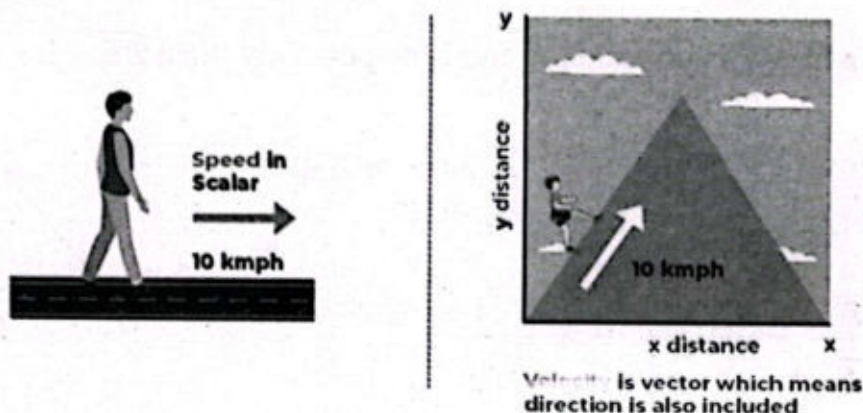
Scalar

A physical quantity that possesses magnitude only, is called a scalar quantity. It can be completely specified by a number along with unit. For example, mass, time, speed, density, work, distance, length, perimeter, area, volume, etc.

Vector

A physical quantity that possesses both magnitude and direction, is called a vector quantity. For example, weight, displacement, momentum, force, velocity, acceleration, electric field etc.

Scalar and Vector



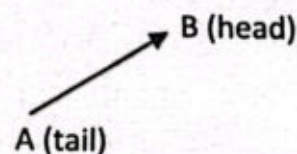
Class Activity:

Look at the table below and categorize the quantities as scalar or vector.

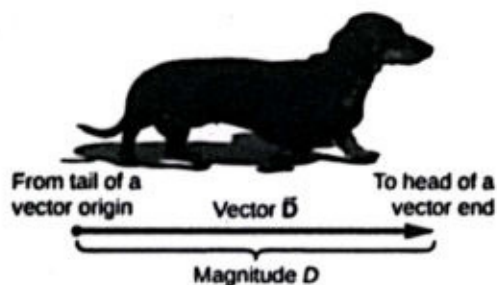
S. No	Quantity	Category
1	7 m	
2	40 m/sec, West	
3	144 square metre	
4	32 °C	
5	98 Newtons	
6	6540 Calories	
7	5 km, East	

Geometrical Representation of a Vector

Geometrically a vector is represented by a line segment with an arrow head at its one end (the ray). The length of the line segment describes the magnitude and the arrow head indicates the direction of the vector.



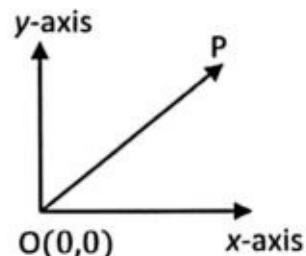
In the figure, the end A of the ray AB is called the tail or the initial point of the vector and the end B is called the terminal point with the arrow head denoted by $\overrightarrow{AB}$.



Usually, the vectors are denoted by bold face letters $\mathbf{a}, \mathbf{b}, \mathbf{c}$ or $\vec{a}, \vec{b}, \vec{c}$ or $\underline{a}, \underline{b}, \underline{c}$ etc.

Position Vector

The vector used to specify the position of a point P with respect to origin O is called position vector of P. The tail of this vector is at origin and head at the point P. Thus $\overrightarrow{OP}$ is the position vector of point P with respect to O.



Result:

If $\vec{a}$ and $\vec{b}$ are position vectors of points A and B respectively, then $\overrightarrow{AB} = \vec{b} - \vec{a}$.

Proof:

If $\vec{a}$ and $\vec{b}$ are position vectors of points A and B respectively, then:

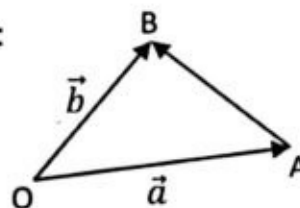
$$\overrightarrow{OA} = \vec{a} \text{ and } \overrightarrow{OB} = \vec{b}$$

Using triangle law of addition, we have:

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

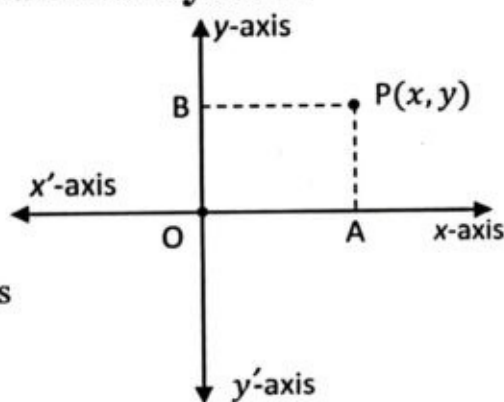
$$\Rightarrow \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$$



Rectangular or Cartesian Coordinate System

A rectangular or Cartesian coordinate system consists of two number lines which are mutually perpendicular. Their point of intersection is called the origin, usually denoted by the letter 'O'. Horizontal line is called x-axis and vertical line is called y-axis. Ordered pairs of numbers can be represented on a Cartesian plane, which is sometimes called the Coordinate plane or xy -plane. Each point in the plane corresponds to an ordered pair (x, y) of real numbers x and y , called the coordinates of the point.



Key Fact:

Cartesian plane has got this name after the French mathematician Rene Descartes (1596 – 1650). He developed the idea of the coordinate plane and the correspondence of ordered pairs of numbers to points in the coordinate plane. Each point in the plane can corresponds to only one ordered pair and vice versa.

Vector in a Plane

Let R be the set of real numbers, then the Cartesian plane is defined as:

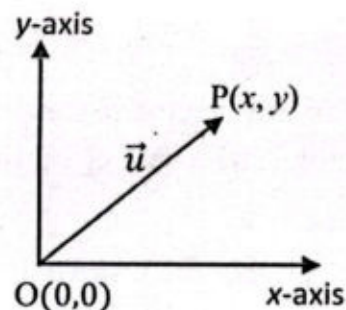
$$R^2 = \{(x, y) : x, y \in R\}$$

Any point $P(x, y)$ is uniquely determined by its coordinates x and y . For a given vector $\vec{u}$ in the plane, there exists a unique point $P(x, y)$ in the plane such that:

$$\vec{u} = \overrightarrow{OP}$$

For the point $P(x, y)$, a unique ordered pair $[x, y]$ is used for the vector $\vec{u}$ called position vector of $P(x, y)$. Thus:

$$\vec{u} = \overrightarrow{OP} = [x, y]$$

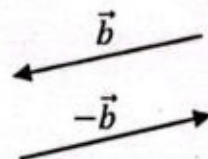


Negative of a Vector

A vector having the same magnitude but opposite in direction of a given vector $\vec{b}$ is called the negative of $\vec{b}$ and is denoted by $-\vec{b}$.

If $\vec{u} = [x, y]$, then negative vector of $\vec{u}$ is defined as:

$$-\vec{u} = [-x, -y]$$



Example:

$P(3, -4)$ is a point in the plane. Find the position vector $\overrightarrow{OP}$ and then its negative vector.

Solution:

Given is $P(3, -4)$ in the plane.

Position vector of $P = \overrightarrow{OP} = [3, -4]$

Negative vector of $\overrightarrow{OP} = -\overrightarrow{OP} = -[3, -4] = [-3, 4]$

Zero or Null Vector

If the initial and terminal points of a vector coincide then the vector has zero length.

This vector is called zero vector and is denoted by $\vec{0}$. The zero vector has no direction. So, it can be assigned as convenient direction according to the situation. The zero vector is denoted by:

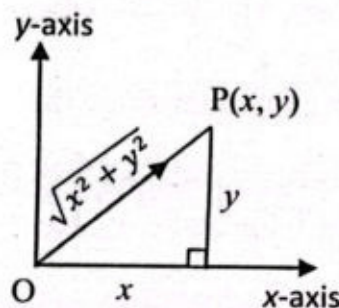
$$\vec{0} = [0, 0]$$

Magnitude (Norm) of a Vector

Given the vector $\vec{u} = \overrightarrow{AB}$. The magnitude or the length or the norm of the vector $\vec{u}$ is denoted by $|\vec{u}| = |\overrightarrow{AB}|$.

If $\vec{u} = [x, y]$, then:

$$|\vec{u}| = \sqrt{x^2 + y^2}$$



Key Fact:

- (i) If $\vec{u}$ is any vector, then $|\vec{u}| \geq 0$. (ii) $|\vec{u}| = 0$ if and only if $\vec{u} = \vec{0}$.
- (iii) $|c\vec{u}| = |c||\vec{u}|$ where c is a scalar. (iv) $|\vec{-u}| = |\vec{u}|$

Unit Vector

A vector which is in the direction of a non-zero vector $\vec{u}$ and has magnitude 1 is called a unit vector of $\vec{u}$ and is denoted by $\hat{u}$. If $\vec{u}$ is non-zero vector of arbitrary length $|\vec{u}|$ then:

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|} \Rightarrow \vec{u} = |\vec{u}|\hat{u}$$

This means any vector $\vec{u}$ can be constructed by multiplying the magnitude of the vector to its unit vector. The process of finding the unit vector of a vector $\vec{u}$ is called normalizing the vector $\vec{u}$.

If $\vec{u} = [x, y]$, then its unit vector is written as:

$$\hat{u} = \frac{[x, y]}{\sqrt{x^2 + y^2}} = \left[\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}} \right]$$

Example:

Find magnitude of the vector $\vec{u} = [-5, 12]$. Also find unit vector of $\vec{u}$.

Solution: $\vec{u} = [-5, 12]$

Here, $x = -5, y = 12$

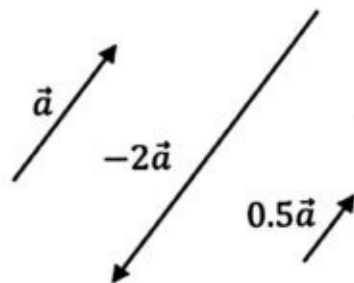
Magnitude of $\vec{u} = |\vec{u}| = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$ units

Unit vector of $\vec{u} = \hat{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{[-5, 12]}{13} = \left[\frac{-5}{13}, \frac{12}{13} \right]$

Scalar Multiplication

If λ is a non-zero scalar and $\vec{a}$ is a non-zero vector then the scalar multiple $\lambda\vec{a}$ is a vector whose magnitude is λ times the magnitude $\vec{a}$.

- $\vec{a}$ and $\lambda\vec{a}$ have the same direction if $\lambda > 0$.
- $\vec{a}$ and $\lambda\vec{a}$ have the opposite direction if $\lambda < 0$.
- If $\lambda\vec{a} = 0$ then either $\lambda = 0$ or $\vec{a} = 0$.
- If $\lambda = 0$, then $\lambda\vec{a}$ is a zero vector.



Note: If $\vec{u} = [x, y]$ then product of λ with $\vec{u}$ is $\lambda\vec{u} = \lambda[x, y] = [\lambda x, \lambda y]$.

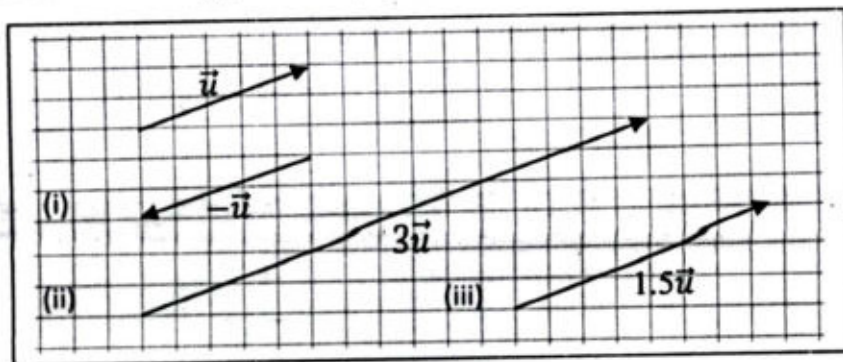
For example, if $\vec{u} = [-6, 4]$ and $\lambda = 3$, then:

$$3\vec{u} = 3[-6, 4] = [-18, 12]$$

Example:

Using graph paper draw the vectors: (i) $-\vec{u}$ (ii) $3\vec{u}$ (iii) $1.5\vec{u}$, when $\vec{u}$ is given.

Solution:

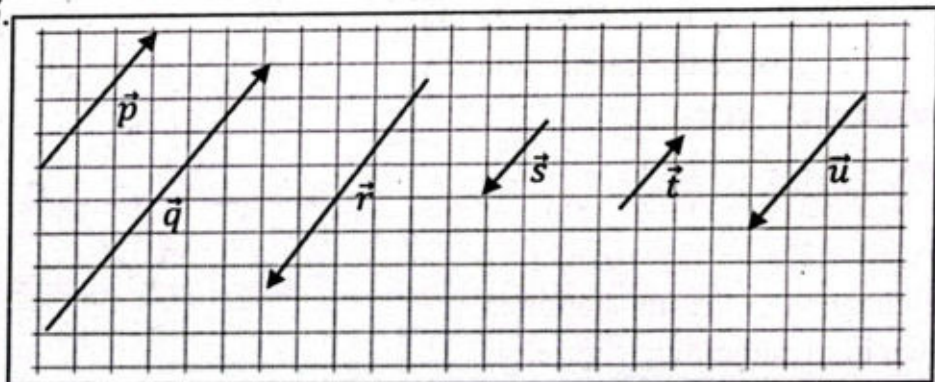


Example:

In the figure, direction of $\vec{p}$ is 3 square units to the right and 4 square units up. Find the relation of other vectors with $\vec{p}$.

Solution:

- (i) $\vec{q} = 2\vec{p}$
- (ii) $\vec{r} = -1.5\vec{p}$
- (iii) $\vec{s} = -0.5\vec{p}$
- (iv) $\vec{t} = 0.5\vec{p}$
- (v) $\vec{u} = -\vec{p}$



Translation by Vector

Translation by a vector is a process that moves a point or figure in space by a given vector. It is a type of transformation that slides a figure without rotating it, so the shape, size, and orientation of the figure do not change.

Geometrically

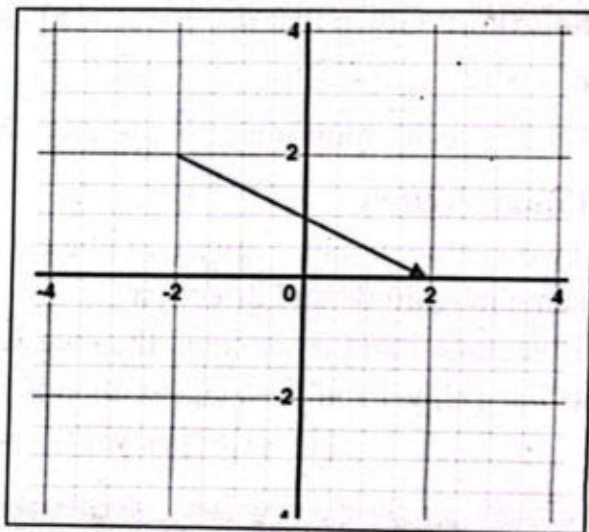
Place the tail of the arrow at the point, and the translated point is at the head of the arrow.

Algebraically

Represent the points and the vector by their coordinates. The translation vector's coordinates indicate how far to move the points along each coordinate axis.

For example, the vector $[a, b]$ moves points a units horizontally and b units vertically.

For example, a translation vector that moves a figure 4 units right and 2 units down can be represented mathematically as $[4, -2]$, or graphically as shown in the adjoining figure. It doesn't matter where the vector is positioned in the plane. In this figure, the vector starts at $(-2, 2)$ and ends at $(2, 0)$. But the initial point and terminal point of the vector is irrelevant. What matters is the length of the vector and the direction in which it points, so all you have to look at is how many units the vector moves in the y -direction and how many units the vector moves in the x -direction.

**Example:**

Use the translation vector $[2, -1]$ to find the new coordinates of triangle ABC when: $A = (-3, 2)$, $B = (-1, 0)$ and $C = (-2, -1)$

Solution:

The vector indicates a translation of 2 units to the right and 1 unit down. We can therefore add 2 to all of the x -values and subtract 1 from all of the y -values to find the vertices of the image.

First let's write down the coordinates of the pre-image, triangle ABC.

$$A = (-3, 2), B = (-1, 0), C = (-2, -1)$$

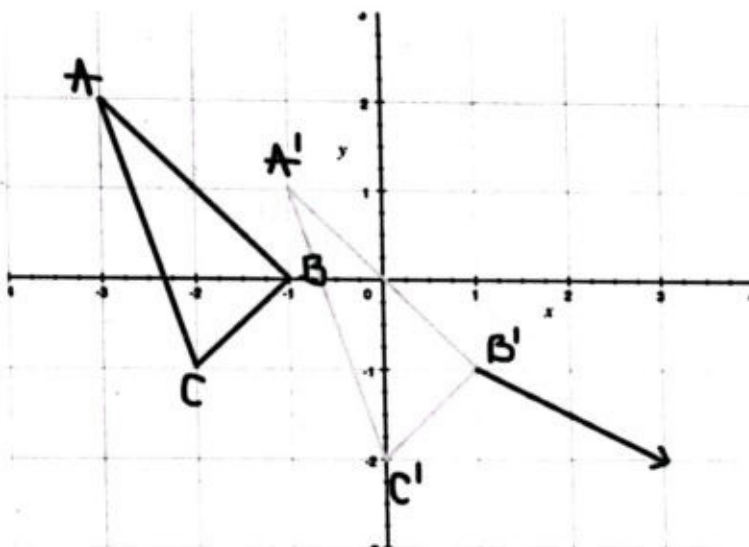
Now we can make the calculations to translate each vertex.

$$A' = (-3 + 2, 2 - 1) = (-1, 1)$$

$$B' = (-1 + 2, 0 - 1) = (1, -1)$$

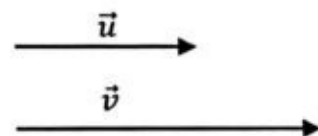
$$C' = (-2 + 2, -1 - 1) = (0, -2)$$

The image after the translation is shown in the adjoining figure.



Parallel Vectors

Two non-zero vectors $\vec{u}$ and $\vec{v}$ are said to be parallel if $\vec{u} = \lambda \vec{v}$, where λ is a scalar. If $\lambda > 0$ then both vectors have the same direction and if $\lambda < 0$ then both are in the opposite direction.



The vectors which are in the opposite direction are known as anti-parallel vectors. In component form, when $\vec{u} = [x_1, y_1]$ and $\vec{v} = [x_2, y_2]$ are parallel vectors, then:

$$\begin{aligned} \vec{u} &= \lambda \vec{v} \Rightarrow [x_1, y_1] = \lambda [x_2, y_2] \\ \Rightarrow [x_1, y_1] &= [\lambda x_2, \lambda y_2] \Rightarrow x_1 = \lambda x_2 \text{ and } y_1 = \lambda y_2 \end{aligned}$$

Example:

Prove that $\vec{a} = [-12, -15]$ and $\vec{b} = [4, 5]$ are parallel vectors.

Solution: Given that $\vec{a} = [-12, -15]$ and $\vec{b} = [4, 5]$

$$\vec{a} = [-12, -15] = -3[4, 5] = -3\vec{b}$$

As $\vec{a}$ is scalar multiple of $\vec{b}$, therefore $\vec{a} \parallel \vec{b}$.

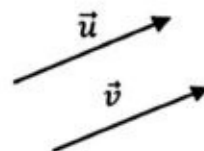
Equal Vectors

Two vectors $\vec{u}$ and $\vec{v}$ are said to be equal if both have the same magnitude and direction.

If vectors $\vec{u}$ and $\vec{v}$ are equal then we write $\vec{u} = \vec{v}$.

If $\vec{u} = [x_1, y_1]$ and $\vec{v} = [x_2, y_2]$, then $\vec{u} = \vec{v}$ implies that:

$$[x_1, y_1] = [x_2, y_2] \Rightarrow x_1 = x_2 \text{ and } y_1 = y_2$$



Key Fact:

- It is not necessary for the equal vectors to have the same position.
- Geometrically two vectors are equal if they are translation of one another.
- If in the relation $\vec{a} = \lambda \vec{b}$ and $\lambda = 1$ then $\vec{a} = \vec{b}$.
- Equal vectors are also parallel.

Example:

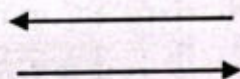
If $\vec{p} = [-c + 4, -2]$ and $\vec{q} = [4, 2d]$ are equal vectors, find the values of c and d .

Solution:

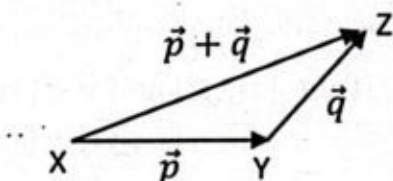
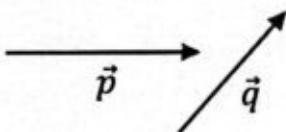
$$\begin{aligned}\text{Given that, } \vec{p} = \vec{q} &\Rightarrow [-c + 4, -2] = [4, 2d] \\ &\Rightarrow -c + 4 = 4, -2 = 2d \\ &\Rightarrow -c = 4 - 4, -2 \div 2 = d \\ &\Rightarrow c = 0, d = 1\end{aligned}$$

Check Point:

Are the vectors equal? If not why?

**Addition of Vectors****Head to Tail Rule**

To add non-zero vectors $\vec{p}$ and $\vec{q}$ join the tail of the second vector $\vec{q}$ with the head of the first vector $\vec{p}$. Now the vector obtained by joining the tail of the first vector to the head of the second vector is the vector $\vec{p} + \vec{q}$ called the resultant vector of $\vec{p}$ and $\vec{q}$.



This method for the addition of two vectors is called head to tail rule of addition. Since $\vec{p}$, $\vec{q}$ and $\vec{p} + \vec{q}$ are along the sides of a triangle XYZ, so this method of addition of vectors is also known as triangle law of addition.

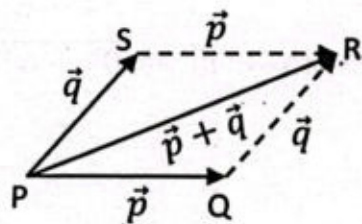
Parallelogram Law of Addition

Consider any parallelogram PQRS. Let $\vec{PQ} = \vec{p}$ and $\vec{QR} = \vec{q}$. Since the vector $\vec{QR}$ has the same magnitude and direction as that of $\vec{PS}$. Similarly, $\vec{SR}$ has the same magnitude and direction as that $\vec{PQ}$. Therefore:

$$\vec{PQ} = \vec{SR} \text{ and } \vec{QR} = \vec{PS}$$

Using triangle law of addition, we have:

$$\begin{aligned}\vec{PQ} + \vec{QR} &= \vec{PR} \\ \Rightarrow \vec{p} + \vec{q} &= \vec{PR}\end{aligned}$$



Showing that the diagonal vector $\vec{PR}$ of the parallelogram is the sum of the vectors of $\vec{p}$ and $\vec{q}$. This is known as parallelogram law of addition. Furthermore:

$$\text{From } \triangle PQR, \quad \vec{PR} = \vec{p} + \vec{q} \quad (\text{i})$$

$$\text{From } \triangle PSR, \quad \vec{PR} = \vec{q} + \vec{p} \quad (\text{ii})$$

$$\therefore \vec{p} + \vec{q} = \vec{q} + \vec{p} \quad [\text{from (i) and (ii)}]$$

This shows that vector addition is commutative.

Key Fact:

For any vector $\vec{a}$:

- $\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$
- $\vec{a} + (-\vec{a}) = (-\vec{a}) + \vec{a} = \vec{0}$

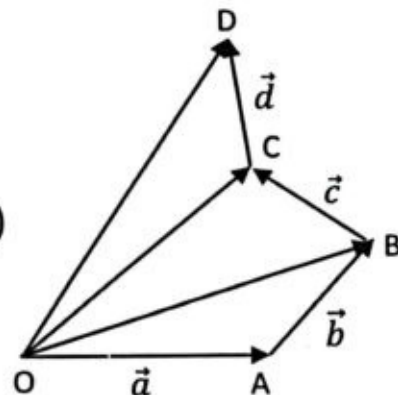
Polygon Law of Vector Addition

The process for the addition of vectors can be extended to any number of vectors. For instance, let us have four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and we need to find their sum.

From the figure,

$$\overrightarrow{OA} = \vec{a}, \overrightarrow{AB} = \vec{b}, \overrightarrow{BC} = \vec{c}, \overrightarrow{CD} = \vec{d}$$

$$\begin{aligned} \text{Now } \vec{a} + \vec{b} + \vec{c} + \vec{d} &= \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} \\ &= (\overrightarrow{OA} + \overrightarrow{AB}) + \overrightarrow{BC} + \overrightarrow{CD} \\ &= \overrightarrow{OB} + \overrightarrow{BC} + \overrightarrow{CD} \quad (\because \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}) \\ &= (\overrightarrow{OB} + \overrightarrow{BC}) + \overrightarrow{CD} \\ &= \overrightarrow{OC} + \overrightarrow{CD} \quad (\because \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}) \\ &= \overrightarrow{OD} \end{aligned}$$



Here, $\overrightarrow{OD}$ is the sum of all these four vectors. The same method is adopted to find the sum of any number of vectors. This method is called polygon law of addition of vectors.

Explanation: If $\vec{u} = [x_1, y_1]$ and $\vec{v} = [x_2, y_2]$, $\vec{w} = [x_3, y_3]$, then:

$$\vec{u} + \vec{v} + \vec{w} = [x_1, y_1] + [x_2, y_2] + [x_3, y_3] = [x_1 + x_2 + x_3, y_1 + y_2 + y_3]$$

Example:

If $\vec{a} = [4, 6]$ and $\vec{b} = [-3, 1]$, $\vec{c} = [2, -2]$ then find $\vec{a} + 2\vec{b} + \vec{c}$. Also find $|\vec{a} + 2\vec{b} + \vec{c}|$.

Solution:

Given that, $\vec{a} = [4, 6]$, $\vec{b} = [-3, 1]$ and $\vec{c} = [2, -2]$

$$\begin{aligned} \text{Now, } \vec{a} + 2\vec{b} + \vec{c} &= [4, 6] + 2[-3, 1] + [2, -2] = [4, 6] + [-6, 2] + [2, -2] \\ &= [4 - 6 + 2, 6 + 2 - 2] = [0, 6] \end{aligned}$$

$$|\vec{a} + 2\vec{b} + \vec{c}| = \sqrt{0^2 + 6^2} = \sqrt{36} = 6 \text{ units}$$

Subtraction of Two Vectors

To subtract a vector, from the other we find the negative vector of the vector to be subtracted and then add it to the other vector.

Consider two non-zero vectors $\vec{u}$ and $\vec{v}$ then:

$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v}).$$

From the figure, $\overrightarrow{AB} = \vec{u}$ and $\overrightarrow{BC} = -\vec{v}$; then

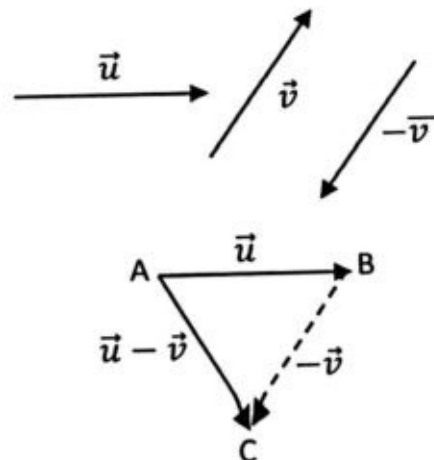
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \quad (\text{head to tail rule of addition})$$

$$\vec{u} + (-\vec{v}) = \vec{u} - \vec{v} = \overrightarrow{AC}$$

Thus, $\overrightarrow{AC}$ is the vector which represents $\vec{u} - \vec{v}$.

In component form, if $\vec{u} = [x_1, y_1]$ and $\vec{v} = [x_2, y_2]$, then:

$$\begin{aligned} \vec{u} - \vec{v} &= [x_1, y_1] - [x_2, y_2] \\ &= [x_1 - x_2, y_1 - y_2] \end{aligned}$$



Example:

If $\vec{a} = [7, 10]$ and $\vec{b} = [-3, 10]$, then find $\vec{a} - \vec{b}$ and a unit vector along $\vec{a} - \vec{b}$.

Solution:

Given that, $\vec{a} = [7, 10]$ and $\vec{b} = [-3, 10]$

Now, $\vec{a} - \vec{b} = [7, 10] - [-3, 10] = [7 + 3, 10 - 10] = [10, 0]$

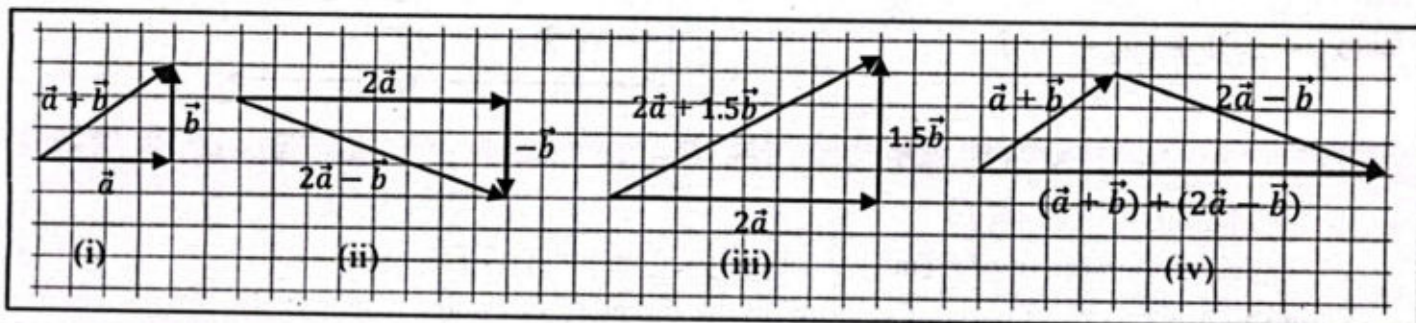
$$|\vec{a} - \vec{b}| = \sqrt{10^2 + 0^2} = \sqrt{100} = 10 \text{ units}$$

$$\text{Unit vector along } \vec{a} - \vec{b} = \frac{\vec{a} - \vec{b}}{|\vec{a} - \vec{b}|} = \frac{[10, 0]}{10} = \left[\frac{10}{10}, \frac{0}{10}\right] = [1, 0]$$

Example:

Draw vectors $\vec{a} = 4$ units east and $\vec{b} = 3$ units north on graph paper. Then, draw the following vectors on the graph paper.

- (i) $\vec{a} + \vec{b}$ (ii) $2\vec{a} - \vec{b}$ (iii) $2\vec{a} + 1.5\vec{b}$ (iv) $(\vec{a} + \vec{b}) + (2\vec{a} - \vec{b})$

Solution:**Representation of a Vector in Cartesian Plane**

We denote the unit vector along x -axis by $\hat{i}$ and along y -axis by $\hat{j}$.

Let us consider a point $P(x, y)$ in the Cartesian plane.

We want to find the position vector $\vec{OP}$ in terms of unit vectors $\hat{i}$ and $\hat{j}$.

Draw perpendicular from P on x -axis meeting the x -axis at point A . As we know that for any vector $\vec{u}$:

$$\hat{u} = \frac{\vec{u}}{|\vec{u}|} \text{ or } \vec{u} = |\vec{u}|\hat{u}$$

i.e., Every vector is equal to the product of magnitude with its unit vector. So,

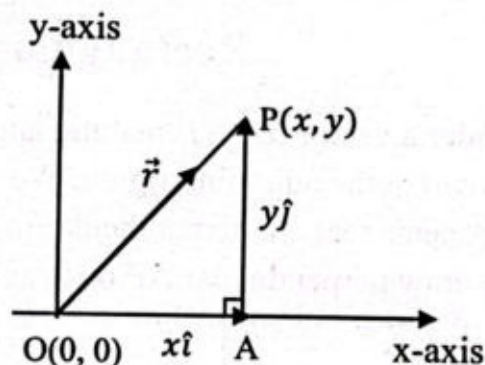
$$\vec{OA} = |\vec{OA}|\hat{i} = x\hat{i}$$

$$\vec{AP} = |\vec{AP}|\hat{j} = y\hat{j}$$

$$\text{Now, } \vec{OP} = \vec{OA} + \vec{AP}$$

$$\Rightarrow \vec{OP} = x\hat{i} + y\hat{j}$$

$$\Rightarrow \vec{r} = x\hat{i} + y\hat{j}$$



This is the position vector of the point $P(x, y)$ lying in the Cartesian plane.

Components of Vector When its Tail is not at Origin

Consider a vector $\overrightarrow{PQ}$ with end points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Draw perpendiculars from P and Q on x-axis meeting at S and T respectively.

Now from figure:

$$|\overrightarrow{PR}| = |\overrightarrow{ST}| = |\overrightarrow{OT}| - |\overrightarrow{OS}| = x_2 - x_1$$

$$|\overrightarrow{RQ}| = |\overrightarrow{TQ}| - |\overrightarrow{TR}|$$

$$= |\overrightarrow{TQ}| - |\overrightarrow{SP}| = y_2 - y_1$$

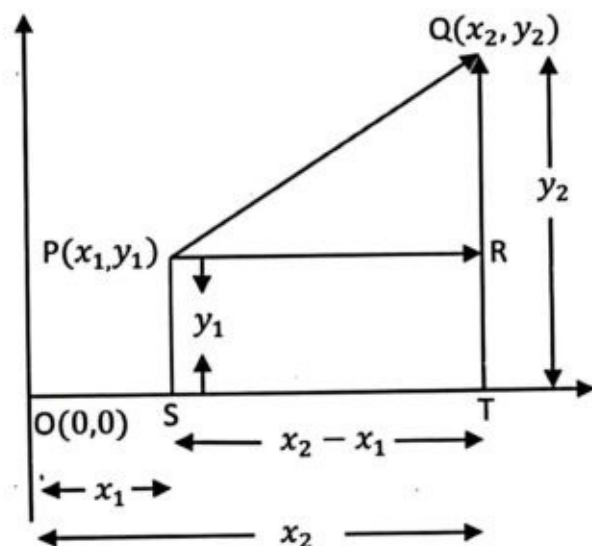
$$\therefore \overrightarrow{PR} = |\overrightarrow{PR}|\hat{i} = (x_2 - x_1)\hat{i}$$

$$\text{and } \overrightarrow{RQ} = |\overrightarrow{RQ}|\hat{j} = (y_2 - y_1)\hat{j}$$

$$\text{As, } \overrightarrow{PQ} = \overrightarrow{PR} + \overrightarrow{RQ}$$

Therefore:

$$\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$



Key Fact:

- In the relation, $\vec{r} = x\hat{i} + y\hat{j}$:
x is called x-component of $\vec{r}$ and y is called y-component of $\vec{r}$.
- $x\hat{i}$ and $y\hat{j}$ are known as parts of vector $\vec{r}$ in the direction of $\hat{i}$ and $\hat{j}$ respectively.
- When the tail of a vector is not at origin then it is not a position vector.

Rectangular Components of a Vector

Consider a vector $\overrightarrow{OP} = \vec{F}$ making angle θ with x-axis as shown in the adjoining figure. We want to find its components that are perpendicular to each other.

If we draw perpendicular AP on x-axis, then from figure:

$$\vec{F} = \vec{F}_x + \vec{F}_y = F_x\hat{i} + F_y\hat{j} \quad \dots\dots (i)$$

Here, $\vec{F}_x$ and $\vec{F}_y$ are rectangular components of $\vec{F}$ while

F_x and F_y are magnitudes of $\vec{F}_x$ and $\vec{F}_y$ respectively.

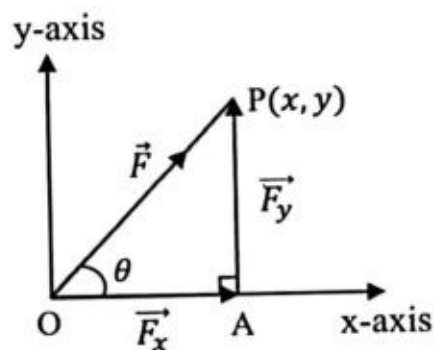
Now if the magnitude of $\vec{F}$ is given, then we can find F_x and F_y using trigonometric ratios.

From right triangle OAP:

$$\frac{F_x}{|\vec{F}|} = \cos \theta \quad \text{and} \quad \frac{F_y}{|\vec{F}|} = \sin \theta$$

$$\Rightarrow F_x = |\vec{F}| \cos \theta \quad \dots\dots(ii)$$

$$F_y = |\vec{F}| \sin \theta \quad \dots\dots(iii)$$



Key Fact:

$\vec{F}_x$ and $\vec{F}_y$ are called horizontal and vertical components of $\vec{F}$ respectively.

Equations (ii) and (iii) are used to find the rectangular components of a given vector making angle θ with x -axis.

Conversely, if the rectangular components of $\vec{F}_x$ and $\vec{F}_y$ of a vector are given then we can find the magnitude and angle of resultant vector by using following relations.

$$|\vec{F}| = \sqrt{(F_x)^2 + (F_y)^2} \quad \dots\dots (iv) \quad \text{and} \quad \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) \quad \dots\dots (v)$$

Example:

A ball is thrown with an initial velocity of 25 m/s at an angle of 45° with the horizontal. Find the horizontal and vertical components of the velocity.

Solution:

Let V represents the velocity.

Horizontal component of velocity is given as:

$$V_x = V \cos 45^\circ = 25 \times 0.707 = 17.68 \text{ m/s}$$

Vertical component of velocity is given as:

$$V_y = V \sin 45^\circ = 25 \times 0.707 = 17.68 \text{ m/s}$$



Example:

Two forces F_1 and F_2 with magnitudes 30 N and 40 N, respectively, act on an object at a point A as shown in the figure. Find the resultant force F acting at A and its magnitude.

Solution:

First, we write F_1 and F_2 in component form:

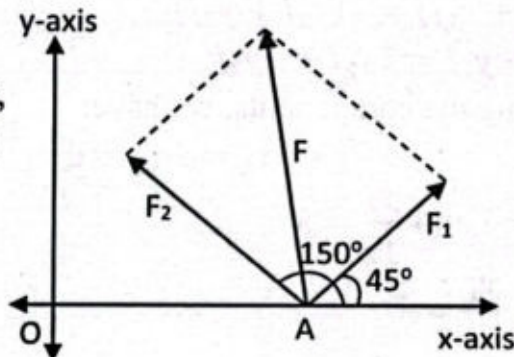
$$\begin{aligned} \vec{F}_1 &= F_1 \cos 45^\circ \hat{i} + F_1 \sin 45^\circ \hat{j} \\ &= 30 \times 0.707 \hat{i} + 30 \times 0.707 \hat{j} = 21.21 \hat{i} + 21.21 \hat{j} \end{aligned}$$

$$\begin{aligned} \vec{F}_2 &= F_2 \cos 150^\circ \hat{i} + F_2 \sin 150^\circ \hat{j} \\ &= 40 \times (-0.866) \hat{i} + 40 \times 0.5 \hat{j} = -34.64 \hat{i} + 20 \hat{j} \end{aligned}$$

So, the resultant force F is:

$$\begin{aligned} \vec{F} &= \vec{F}_1 + \vec{F}_2 = (21.21 \hat{i} + 21.21 \hat{j}) + (-34.64 \hat{i} + 20 \hat{j}) = (21.21 - 34.64) \hat{i} + (21.21 + 20) \hat{j} \\ &= -13.43 \hat{i} + 41.21 \hat{j} \end{aligned}$$

$$\begin{aligned} \text{Magnitude of resultant force} &= |\vec{F}| = \sqrt{(-13.43)^2 + (41.21)^2} = \sqrt{180.36 + 1,698.26} \\ &= \sqrt{1,878.62} = 43.34 \text{ units} \end{aligned}$$



Concept Related to Vectors Using Analytical Representation

Magnitude of a Vector

In Cartesian plane a vector $\vec{u}$ is expressed as:

$$\vec{u} = x\hat{i} + y\hat{j}$$

The magnitude of vector $\vec{u}$ is defined as:

$$|\vec{u}| = \sqrt{x^2 + y^2}$$

$$|\vec{u}| = \sqrt{(x\text{-component})^2 + (y\text{-component})^2}$$

Equal Vectors

Two vectors are said to be equal if and only if they have same x and y components.

If $\vec{u} = x_1\hat{i} + y_1\hat{j}$ and $\vec{v} = x_2\hat{i} + y_2\hat{j}$ then $\vec{u} = \vec{v}$ if $x_1 = x_2$ and $y_1 = y_2$.

Negative of a Vector

If $\vec{p} = x\hat{i} + y\hat{j}$ then negative of $\vec{p}$ is:

$$-\vec{p} = (-1)\vec{p} = (-1)(x\hat{i} + y\hat{j}) = (-x)\hat{i} + (-y)\hat{j}$$

Unit Vector

If $\vec{w} = x\hat{i} + y\hat{j}$ then $\vec{w}$ is called a unit vector if:

$$|\vec{w}| = 1 \Rightarrow \sqrt{x^2 + y^2} = 1 \text{ or } x^2 + y^2 = 1$$

Parallel Vectors

The vectors $\vec{u} = x_1\hat{i} + y_1\hat{j}$ and $\vec{v} = x_2\hat{i} + y_2\hat{j}$ are said to be parallel if for some scalar λ we have:

$$\vec{u} = \lambda\vec{v}$$

$$\Rightarrow (x_1\hat{i} + y_1\hat{j}) = \lambda(x_2\hat{i} + y_2\hat{j})$$

$$\Rightarrow x_1\hat{i} + y_1\hat{j} = \lambda x_2\hat{i} + \lambda y_2\hat{j}$$

Comparing the components, we have:

$$x_1 = \lambda x_2 \text{ and } y_1 = \lambda y_2$$

$$\Rightarrow \frac{x_1}{x_2} = \lambda \text{ and } \frac{y_1}{y_2} = \lambda$$

$$\text{or } \frac{x_1}{x_2} = \frac{y_1}{y_2} = \lambda$$

Which is the condition for the two vectors to be parallel. If λ is positive then $\vec{u}$ and $\vec{v}$ have the same direction and if λ is negative then $\vec{u}$ and $\vec{v}$ are in the opposite direction.

Addition and Subtraction of Vectors

If $\vec{u} = x_1\hat{i} + y_1\hat{j}$ and $\vec{v} = x_2\hat{i} + y_2\hat{j}$ are any two vectors then:

$$(i) \quad \vec{u} + \vec{v} = (x_1\hat{i} + y_1\hat{j}) + (x_2\hat{i} + y_2\hat{j}) = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j}$$

$$(ii) \quad \vec{u} - \vec{v} = (x_1\hat{i} + y_1\hat{j}) - (x_2\hat{i} + y_2\hat{j}) = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$$

Unit Vector in the Direction of another Vector

Let we have a non-zero vector $\vec{u} = x\hat{i} + y\hat{j}$

$$\therefore |\vec{u}| = \sqrt{x^2 + y^2}$$

Now the unit vector in the direction of $\vec{u}$ is

$$\hat{a} = \frac{\vec{u}}{|\vec{u}|} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$

$$\hat{a} = \left(\frac{x}{\sqrt{x^2 + y^2}} \right) \hat{i} + \left(\frac{y}{\sqrt{x^2 + y^2}} \right) \hat{j}$$

Example:

If $\vec{a} = 2\hat{i} - \hat{j}$ then find unit vector along the vector $\vec{a}$.

Key Fact:

A vector having both x -component and y -component zero is called a null vector denoted by $\vec{0}$.

$$\vec{0} = 0\hat{i} + 0\hat{j}$$

Key Fact:

If λ is a scalar and $\vec{u} = x\hat{i} + y\hat{j}$ then product of λ with $\vec{u}$ is
 $\lambda\vec{u} = \lambda(x\hat{i} + y\hat{j}) = \lambda x\hat{i} + \lambda y\hat{j}$

Solution:

$$|\vec{a}| = \sqrt{2^2 + (-1)^2} = \sqrt{4 + 1} = \sqrt{5}$$

The unit vector in the direction of $\vec{a}$ is:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i} - \hat{j}}{\sqrt{5}} = \frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{j}$$

Example: If $\vec{u} = 5\hat{i} + \hat{j}$ and $\vec{v} = 2\hat{i} - 9\hat{j}$, then find (i) $2\vec{u} + 3\vec{v}$ (ii) $4\vec{u} - 2\vec{v}$

$$(i) \quad 2\vec{u} + 3\vec{v} = 2(5\hat{i} + \hat{j}) + 3(2\hat{i} - 9\hat{j}) = 10\hat{i} + 2\hat{j} + 6\hat{i} - 27\hat{j} = 16\hat{i} - 25\hat{j}$$

$$(ii) \quad 4\vec{u} - 2\vec{v} = 4(5\hat{i} + \hat{j}) - 2(2\hat{i} - 9\hat{j}) = 20\hat{i} + 4\hat{j} - 4\hat{i} + 18\hat{j} = 16\hat{i} + 22\hat{j}$$

Example: Check whether the vectors $\vec{u} = 3\hat{i} + 2\hat{j}$ and $\vec{v} = 6\hat{i} - 4\hat{j}$, are parallel or not.

Solution:

Given that $\vec{u} = 3\hat{i} + 2\hat{j}$ and $\vec{v} = 6\hat{i} - 4\hat{j}$

$$\vec{v} = 6\hat{i} - 4\hat{j} = 2(3\hat{i} - 2\hat{j}) \neq 2\vec{u}.$$

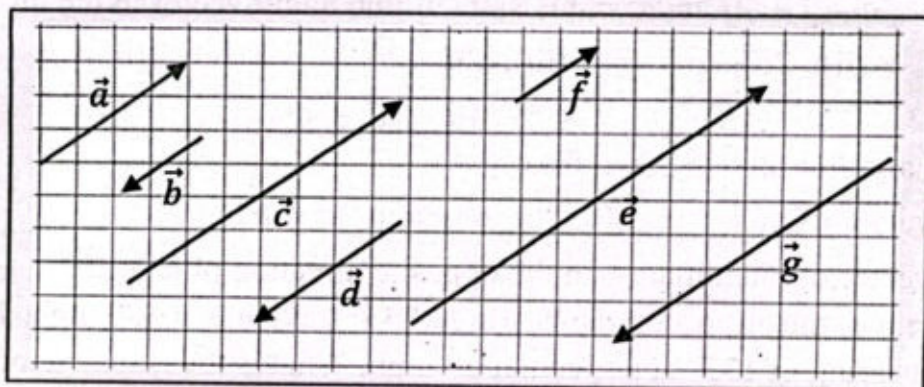
Therefore $\vec{u}$ and $\vec{v}$ are not parallel.

Check Point:

If $\vec{a} = x\hat{i} + 2\hat{j}$ and $\vec{b} = 6\hat{i} - (x + y)\hat{j}$ are equal vectors, find the values of x and y .

Exercise 7.1

- Draw the following vectors.
 - 10 N force along x-axis
 - 50 m/s velocity at 150° with x-axis
 - 220 m displacement towards north
 - 24 m/s² acceleration at 45° with x-axis
- Draw the vectors on the graph paper when $\vec{p}$ is 4cm long making an angle of 45° with x-axis.
 - $2\vec{p}$
 - $-\vec{p}$
 - $0.5\vec{p}$
 - $-1.5\vec{p}$
 - $-0.5\vec{p}$
- Draw vectors $\vec{a} = 3$ units west and $\vec{b} = 3$ units north on graph paper. Draw the following vectors on the graph paper.
 - $2\vec{a} + \vec{b}$
 - $\vec{a} - 2\vec{b}$
 - $3\vec{a} + 1.5\vec{b}$
 - $(2\vec{a} + \vec{b}) + (\vec{a} - 2\vec{b})$
 - $0.5(\vec{a} + \vec{b})$
 - $3\vec{a} - 2\vec{b}$
 - $2\vec{a} - 2.5\vec{b}$
 - $(2\vec{a} + \vec{b}) - (\vec{a} - 2\vec{b})$
- In the figure, direction of $\vec{a}$ is 4 square units to the right and 3 square units up. Find the relation of other vectors with $\vec{a}$.



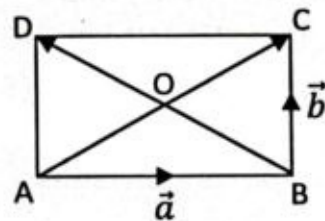
- A point with coordinates (5, -7) have been translated by the vector $[0, -3]$. Find new position of the point.

6. A vector $[-5, 4]$ have been translated by another vector $[4, -3]$. Find new location of the given vector.
7. Triangle ABC is a pre-image with its vertices at $A = (-4, 6)$, $B = (-1, 4)$ and $C = (-6, 1)$. Find the coordinates of the image if:
- (i) the triangle is translated by $[5, 0]$ (ii) the triangle is translated by $[-3, -4]$
8. What translation by vector would be needed to bring the point $E(-6, 5)$ to the origin?

9. ABCD is a rectangle as shown in the adjoining figure.

Represent the following vectors in terms of $\vec{a}$ and $\vec{b}$.

- (i) $\overrightarrow{DC}$ (ii) $\overrightarrow{DA}$ (iii) $\overrightarrow{AC}$ (iv) $\overrightarrow{BD}$
 (v) $\overrightarrow{AO}$ (vi) $\overrightarrow{BO}$ (vii) $\overrightarrow{AB} + \overrightarrow{AC}$



10. Given that $\overrightarrow{OP} = \vec{p} + \vec{q}$, $\overrightarrow{OQ} = \vec{p} - \vec{q}$ and M is mid-point of $\overline{PQ}$. Find in terms of $\vec{p}$ and $\vec{q}$.
- (i) $\overrightarrow{PQ}$ (ii) $\overrightarrow{PM}$ (iii) $\overrightarrow{QM}$ (iv) $\overrightarrow{OM}$
11. Express $\overrightarrow{AB}$ in the form of $[x, y]$ and $x\hat{i} + y\hat{j}$, when:
- (i) $A(1, 3)$, $B(4, 6)$ (ii) $A(-2, 5)$, $B(2, -5)$ (iii) $A(4, -7)$, $B(-1, -9)$
12. Find the unit vectors of the following vectors.
- (i) $\overrightarrow{PQ}$ when $P(1, 4)$, $Q(4, 8)$ (ii) $\vec{u} = 5\hat{i} - 12\hat{j}$ (iii) $\vec{b} = -6\hat{i} - 8\hat{j}$
13. If $\vec{a} = 3\hat{i} - 2\hat{j}$ and $\vec{b} = \hat{i} + 4\hat{j}$, then find x and y such that $x\vec{a} + y\vec{b} = 4\hat{i} - 12\hat{j}$.
14. Given that $D = (3, -2)$, $E = (-3, -5)$, $F = (0, 5)$ and $G = (3, 6)$. Find the following vectors.
- (i) $\overrightarrow{DE}$ (ii) $\overrightarrow{DE} + \overrightarrow{FG}$ (iii) $3\overrightarrow{EF} - 2\overrightarrow{EG}$
 (iv) $\frac{3}{2}\overrightarrow{DF} + \frac{1}{2}\overrightarrow{EG}$ (v) $\overrightarrow{DF} - \overrightarrow{EF} + 2\overrightarrow{FG}$
15. If $\vec{a} = 3\hat{i} + 5\hat{j}$ and $\vec{b} = 2\hat{i} - 4\hat{j}$, then find:
- (i) $\vec{a} + 2\vec{b}$ (ii) $2\vec{a} - 3\vec{b}$ (iii) $5(2\vec{a} + \vec{b})$ (iv) $0.5(\vec{a} - \vec{b})$
 (v) $|\vec{a} + \vec{b}|$ (vi) $|\vec{a} - \vec{b}|$ (vii) $|\vec{a}| + 2|\vec{b}|$ (viii) $|\vec{a}| - |\vec{b}|$
16. Find the value of p for which the vector $\vec{u} = 3\hat{i} + 6\hat{j}$ is parallel to $\vec{v} = \hat{i} + p\hat{j}$.
17. If $\vec{a} = 2\hat{i} - 4\hat{j}$, $\vec{b} = \hat{i} - 2\hat{j}$ and $\vec{c} = [1, 3]$, then find a unit vector in the direction of:
- (i) $\vec{a} + \vec{b}$ (ii) $2\vec{a} - \vec{c}$ (iii) $2\vec{a} - \vec{b}$ (iv) $3(\vec{a} + \vec{c})$
 (v) $\vec{a} + \vec{b} + 2\vec{c}$ (vi) $4\vec{a} - 2\vec{b} + \vec{c}$
18. If $\vec{a} = \hat{i} - 3\hat{j}$ and $\vec{b} = 2\hat{i} + 4\hat{j}$ then find:
- (i) A vector of magnitude 4 in the direction $2\vec{a} - \vec{b}$.
 (ii) A vector of magnitude 6 in the opposite direction of $\vec{a} + 2\vec{b}$.
19. An airplane leaves the airport on the bearing of 45° traveling at 380 mph. The wind is blowing at a bearing of 135° at a speed of 50 mph. What is the actual velocity of the airplane?
20. A truck leaves dry port on a bearing of 30° and travels 15 km. It then turns due west and travels 8 km. How far is the truck from the dry port and what is its bearing?

Ratio Formula for Finding Position Vector of a Point on a Vector

Let $\overline{AB}$ be any line segment and P be the point which divides this line segment in the given ratio $m : n$ internally. The position vectors of the given points A and B are $\vec{a}$ and $\vec{b}$ respectively. Let $\vec{r}$ be the position vector of point P .

Given that:

$$|\overline{AP}| : |\overline{PB}| = m : n$$

$$\Rightarrow \frac{|\overline{AP}|}{|\overline{PB}|} = \frac{m}{n} \Rightarrow n|\overline{AP}| = m|\overline{PB}|$$

Because $\overline{AP}$ and $\overline{PB}$ have the same direction, so

$$n\overline{AP} = m\overline{PB} \quad (i)$$

From figure, $\overline{OA} + \overline{AP} = \overline{OP}$

$$\Rightarrow \vec{a} + \overline{AP} = \vec{r} \Rightarrow \overline{AP} = \vec{r} - \vec{a}$$

Similarly, $\overline{OP} + \overline{PB} = \overline{OB}$

$$\Rightarrow \vec{r} + \overline{PB} = \vec{b} \Rightarrow \overline{PB} = \vec{b} - \vec{r}$$

Substituting the values in equation (i), we get:

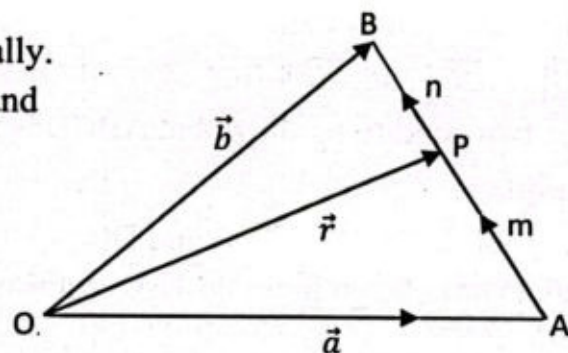
$$n(\vec{r} - \vec{a}) = m(\vec{b} - \vec{r}) \Rightarrow n\vec{r} - n\vec{a} = m\vec{b} - m\vec{r}$$

$$\Rightarrow n\vec{r} + m\vec{r} = n\vec{a} + m\vec{b} \Rightarrow (n + m)\vec{r} = m\vec{b} + n\vec{a} \Rightarrow \vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Particular Case:

If $m = n$, then P will be the midpoint of $\overline{AB}$ and position vector of P in this case is:

$$\vec{r} = \frac{n\vec{b} + n\vec{a}}{n + n} = \frac{n(\vec{b} + \vec{a})}{2n} = \frac{\vec{a} + \vec{b}}{2}$$



Key Fact:

When the point P divides the line segment $\overline{AB}$ in the ratio $m : n$ externally then:

$$\vec{r} = \frac{n\vec{a} - m\vec{b}}{n - m}$$

Example:

The position vectors of points A and B are $2\hat{i} - \hat{j}$ and $3\hat{i} + 2\hat{j}$ respectively. Find the position vector of point P dividing the line segment joining A and B in the ratio $3 : 4$ internally.

Solution:

Let $\vec{a} = 2\hat{i} - \hat{j}$, $\vec{b} = 3\hat{i} + 2\hat{j}$ and $m : n = 3 : 4$

If $\vec{r}$ be the position vector of P , then by using ratio formula, we have:

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m + n} = \frac{3(3\hat{i} + 2\hat{j}) + 4(2\hat{i} - \hat{j})}{3 + 4} = \frac{9\hat{i} + 6\hat{j} + 8\hat{i} - 4\hat{j}}{7} = \frac{17\hat{i} + 2\hat{j}}{7} = \frac{17}{7}\hat{i} + \frac{2}{7}\hat{j}$$

Application to Geometry

There are many theorems and geometrical problems that can be solved by using vector methods.

We solve some example here..

Example:

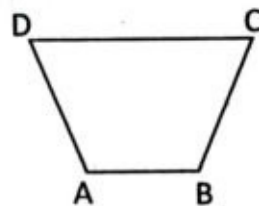
What type of a quadrilateral $ABCD$ is if $2\overline{AB} = \overline{DC}$?

Solution:

$2\overrightarrow{AB} = \overrightarrow{DC}$ implies that:

$$2|\overrightarrow{AB}| = |\overrightarrow{DC}| \text{ and } \overrightarrow{AB} \parallel \overrightarrow{DC}$$

These two conditions show that ABCD is a trapezium.

**Example:**

A(x, y), B(-2, 3), C(-3, -4) and D(4, -5) are vertices of a parallelogram taken in order. Use vector method to find vertex A.

Solution:

Position vectors of vertices of parallelogram are:

$$\overrightarrow{OA} = x\hat{i} + y\hat{j}, \quad \overrightarrow{OB} = -2\hat{i} + 3\hat{j}$$

$$\overrightarrow{OC} = -3\hat{i} - 4\hat{j}, \quad \overrightarrow{OD} = 4\hat{i} - 5\hat{j}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-2\hat{i} + 3\hat{j}) - (x\hat{i} + y\hat{j}) = (-2 - x)\hat{i} + (3 - y)\hat{j}$$

$$\overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = (-3\hat{i} - 4\hat{j}) - (4\hat{i} - 5\hat{j}) = (-3 - 4)\hat{i} + (-4 + 5)\hat{j} = -7\hat{i} + \hat{j}$$

As, ABCD is a parallelogram, therefore:

$$\overrightarrow{AB} = \overrightarrow{DC} \Rightarrow (-2 - x)\hat{i} + (3 - y)\hat{j} = -7\hat{i} + \hat{j}$$

$$\Rightarrow -2\hat{i} - x\hat{i} + 3\hat{j} - y\hat{j} = -7\hat{i} + \hat{j} \Rightarrow -2\hat{i} + 7\hat{i} + 3\hat{j} - \hat{j} = x\hat{i} + y\hat{j}$$

$$\Rightarrow 5\hat{i} + 2\hat{j} = x\hat{i} + y\hat{j} \Rightarrow x = 5, y = 2$$

$\therefore$ Fourth vertex of parallelogram is A(5, 2).

Example:

Use vectors to prove that the diagonals of a rhombus bisect each other.

Solution:

Consider any rhombus PQRS. Let $\vec{p}, \vec{q}, \vec{r}$ and $\vec{s}$ be the position vectors of the vertices P, Q, R and S respectively.

Let M and N be the mid points of diagonal vectors $\overrightarrow{PR}$ and $\overrightarrow{QS}$ respectively.

$$\text{p. v. of M} = \frac{\vec{p} + \vec{r}}{2} \quad (i) \quad \text{p. v. of N} = \frac{\vec{q} + \vec{s}}{2} \quad (ii)$$

Since PQRS is a rhombus, therefore:

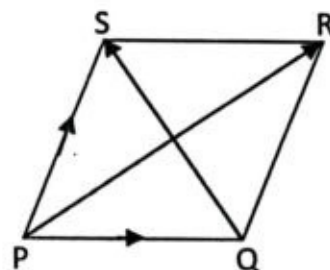
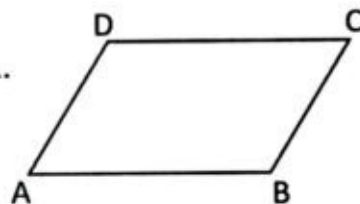
$$\begin{aligned} \overrightarrow{PQ} &= \overrightarrow{SR} \\ \Rightarrow \vec{q} - \vec{p} &= \vec{r} - \vec{s} \\ \Rightarrow \vec{q} + \vec{s} &= \vec{p} + \vec{r} \end{aligned} \quad (iii)$$

Using equation (iii) in (i) and (ii), we get:

$$\text{p. v. of M} = \text{p. v. of N}$$

Since the position vectors of the mid-points of both the diagonals are same.

Therefore, the diagonals of a rhombus bisect each other.

**Check Point:**

Prove that the diagonals of parallelogram, square and rectangle bisect each other.

Exercise 7.2

1. What type of a quadrilateral EFGH is if $\overrightarrow{EF} = \overrightarrow{HG}$ and $\overrightarrow{EH} = \overrightarrow{FG}$?
2. Use vectors to find the length of diagonals of a parallelogram having adjacent sides $2\hat{i} + \hat{j}$ and $\hat{i} - 3\hat{j}$.
3. (i) Show that $A(0, 2), B(\sqrt{3}, -1), C(0, -2)$ are the vertices of a right-angled triangle.
(ii) Show that $A(3, 1), B(-2, -3), C(2, 2)$ are the vertices of an isosceles triangle.
4. The position vectors of points C and D are $3\hat{i} + 2\hat{j}$ and $\hat{i} - 4\hat{j}$ respectively. Find the position vector of point O dividing the line segment joining C and D in the ratio 2 : 3 internally.
5. If $M(2, 4)$ is mid-point of $\overline{XY}$ such that $X(-1, 3)$ and $Y(x, y)$. Find the point Y.
6. If $\overrightarrow{OA} = -\hat{i} + 3\hat{j}$, $\overrightarrow{OB} = 6\hat{i} + 2\hat{j}$, find the point trisecting the join of $\overline{AB}$.
7. Find x, such that the points $P(-1, x), Q(3, 2)$ and $R(7, 3)$ are collinear.
8. Find the coordinates of vertex D of a parallelogram ABCD if $A(-3, 0), B(1, -2)$ and $C(5, 0)$ are its three vertices.
9. If $\overrightarrow{AB} = \overrightarrow{CD}$ and $A(0, 2), C(-2, 4), D(-1, 5)$, then find vertex B.
10. Vertices of a quadrilateral are $U(9, 4), V(-7, 7), W(-4, -7)$ and $X(5, -5)$. Find mid points of its sides and prove that the figure obtained by joining mid points consecutively is a parallelogram.
11. Prove that the line segments joining the midpoints of two sides of a triangle are half in length of third side.
12. Prove that the joining the midpoints of the two non-parallel sides of a trapezium is parallel to its parallel sides.
13. Prove that the line segment joining mid points of adjacent sides of a square or rectangle or parallelogram divides the corresponding diagonal in the ratio 1 : 3.

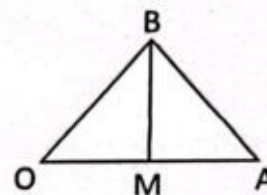
I have Learnt

- Introduction of rectangular coordinate system in a plane.
- Representing vector as a directed line segment.
- Expressing a vector in terms of two non-zero and non-parallel coplanar vector.
- Expressing a vector in terms of position vector.
- Expressing translation by a vector and finding the magnitude of a vector.
- Adding and subtracting vectors, and multiplying a vector by a scalar.
- Solving geometrical problems involving the use of a vector.
- Applying concepts of vectors in geometrical problems such as parallel and perpendicular lines in geometrical shapes, vector projectile motion, crosswinds aviation, military usage, designing roller coasters.

1. Chose the correct option.
 - i. Which of the following is a scalar quantity?
 - a. velocity
 - b. speed
 - c. torque
 - d. force
 - ii. Which of the following is a vector quantity?
 - a. velocity
 - b. speed
 - c. distance
 - d. work
 - iii. If $\vec{a}$ and $\vec{b}$ are position vectors of points A and B respectively, then $\overrightarrow{AB}$ is:
 - a. $\vec{b} + \vec{a}$
 - b. $\vec{a} - \vec{b}$
 - c. $\vec{b} - \vec{a}$
 - d. $-\vec{b} - \vec{a}$
 - iv. Which of the following is not a symbol of vector a ?
 - a. $\vec{a}$
 - b. $\underline{a}$
 - c. $\mathbf{a}$
 - d. $|\vec{a}|$
 - v. If $\overrightarrow{OP} = [-6, 7]$, then $-\overrightarrow{OP}$ is equal to:
 - a. $[-6, 7]$
 - b. $[6, 7]$
 - c. $[6, -7]$
 - d. $[-6, -7]$
 - vi. If $\vec{u} = -5\hat{i} + 12\hat{j}$, then $|\vec{u}|$ is equal to:
 - a. 17
 - b. 7
 - c. 169
 - d. 13
 - vii. Given that $\vec{u}$ is any vector. Which of the following is true?
 - a. $|\vec{u}| = |\vec{u}|$
 - b. $|\vec{u}| = |\vec{u}|$
 - c. $|\vec{u}| + |-\vec{u}| = 0$
 - d. $|\vec{u}| = 0$
 - viii. The unit vector of the vector $\vec{u} = 6\hat{i} + 10\hat{j} - 2\hat{k}$ is:
 - a. $\frac{3\hat{i}}{5} - \frac{4\hat{j}}{5}$
 - b. $\frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}$
 - c. $-\frac{3\hat{i}}{5} - \frac{4\hat{j}}{5}$
 - d. $-\frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}$
 - ix. If $\vec{a} = \lambda\vec{b}$, $\vec{a} = 12\hat{i} - 18\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{j}$, then λ is equal to:
 - a. 3
 - b. -3
 - c. 6
 - d. -6
 - x. If $\vec{u} = [-5x, 8]$ and $\vec{v} = [10, 4y]$ are equal vectors, then:
 - a. $x = 2, y = 2$
 - b. $x = -2, y = -2$
 - c. $x = -2, y = 2$
 - d. $x = 2, y = -2$
 - xi. If $\vec{p} = [5, -6]$ and $\vec{q} = [2, 6]$, then $\vec{p} - 2\vec{q}$ is:
 - a. $[1, -18]$
 - b. $[9, 18]$
 - c. $[1, 18]$
 - d. $[-9, -18]$
 - xii. If $\vec{u} = 5\hat{i} + 10\hat{j}$ and $\vec{v} = 4\hat{j}$, then $|\vec{u} - \vec{v}|$ is:
 - a. $\sqrt{123}$
 - b. $\sqrt{61}$
 - c. $\sqrt{11}$
 - d. $-\sqrt{61}$
 - xiii. Which of the following vectors represents a position vector?
 - a. $\overrightarrow{OP}$
 - b. $-\overrightarrow{OP}$
 - c. $\overrightarrow{PO}$
 - d. $\overrightarrow{PQ}$
 - xiv. What type of a quadrilateral ABCD is, if $\overrightarrow{AB} = \frac{2}{3} \overrightarrow{DC}$?
 - a. kite
 - b. rectangle
 - c. trapezium
 - d. rhombus
2. Given that $\vec{p} = 3\hat{i} - 4\hat{j}$ and $\vec{q} = -3\hat{i} - 4\hat{j}$. Prove that $|\vec{p}| = |\vec{q}|$. Is $\vec{p} = \vec{q}$?
3. If $\vec{a} = 2\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + x\hat{j}$, then find the value of x if $|\vec{a} + 2\vec{b}| = 6$.

4. In $\triangle OAB$, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and M is mid-point of $\overline{OA}$. Write the following vectors in terms of $\vec{a}$ and $\vec{b}$.

(i) $\overrightarrow{OM}$ (ii) $\overrightarrow{AM}$ (iii) $\overrightarrow{BM}$ (iv) $\overrightarrow{OB} + \overrightarrow{BA}$



5. Two tractors are pulling a truck stuck in the mud. First tractor is pulling with a force of 250 N at an angle of 50° with the horizontal while the second tractor is pulling with a force of 300 N at an angle of 40° with the horizontal. What is the magnitude and direction of the resultant force?
6. Ammar and Javed are playing with a ball. Ammar throws two balls for Javed to catch. The path of the two balls is described by the vectors $\mathbf{V}_1 = 120\hat{i} + 12\hat{j}$ and $\mathbf{V}_2 = 90\hat{i} - 30\hat{j}$ where the distances are expressed in meters.
- (i) How much farther did the first ball travel as compared with the second ball?
- (ii) What is the distance between the two balls thrown?
7. Ahmad can swim in still water with a speed of 6 m/s. He goes swimming in a river Chenab which has a current flowing towards west with a speed of 1.5 m/s. Assuming his speed is the same, what is his resultant velocity if:
- (i) He tries to swim due west along the current?
- (ii) He tries to swim due east against the current?
- (iii) He tries to swim north across the river?
8. A plane is travelling north with a speed of 150 km/h. A steady wind is blowing due east with a speed of 50 km/h.
- (i) What is the resultant speed of the plane?
- (ii) How far is the plane from its starting point after 10 hours?