04

Linear and Quadratic Inequalities

After studying this unit students will be able to:

- Solve linear inequalities in one variable.
- Solve quadratic inequalities in one variable.
- Solve two linear inequalities with two unknowns simultaneously.
- Interpret and identify the region in plane bounded by two linear inequalities in two unknowns.

You are checking a bag at an airport. Bags can weigh no more than 60 pounds. Your bag weighs 26.5 pounds. Find the possible weights 'w' (in pounds) that you can add to the bag.

Write a model:

Weight of bag + Weight you can add ≤ Weight limit

$26.5 + w \leq 60$ (writing inequalities)

$26.5 + w - 26.5 \leq 60 - 26.5$ (subtracting 26.5 from both sides)

$w \leq 33.5$ (simplifying)

You can add less than 33.5 pounds.

Inequalities in One Variable

Furqan and Qasim are the hockey players in the school hockey team. If we compare their scoring for the season, only one of the following statements will be true:

Furqan scored fewer goals than Qasim.
Furqan scored same number of goals as Qasim.
Furqan scored more goals than Qasim.

Let 'a' represents the number of goals of Furqan and 'b' represents the number of goals of Qasim. We can compare the score using an inequality or an equation.

$$a<b, \qquad a>b, \qquad a=b$$

This is an example of trichotomy property.

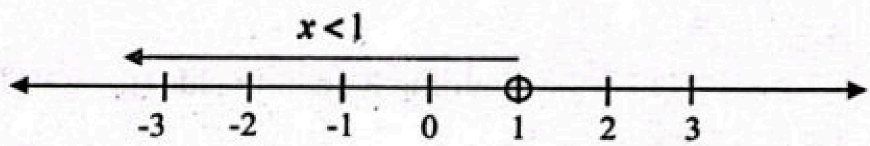
Trichotomy Property
For any two real numbers a and b, exactly one of the following statements will be true:

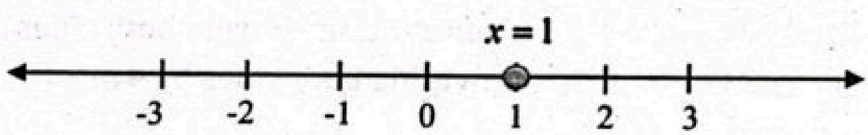
$$a<b, \qquad a>b, \qquad a=b$$

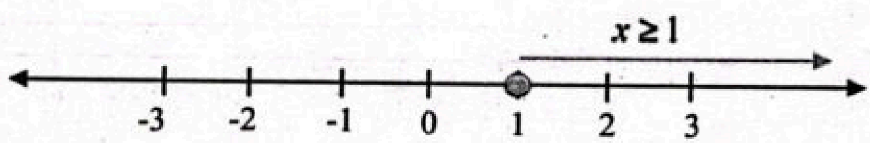
Graph of Inequality

On a number line, the graph of an inequality in one variable is the set of points that represents all solutions of the inequality. To graph an inequality in one variable, we use open circle for the symbols '<' or '>' and a closed circle for the symbols '≤' or '≥'.

The graph of $x < 1, x = 1$ and $x \geq 1$ respectively are:



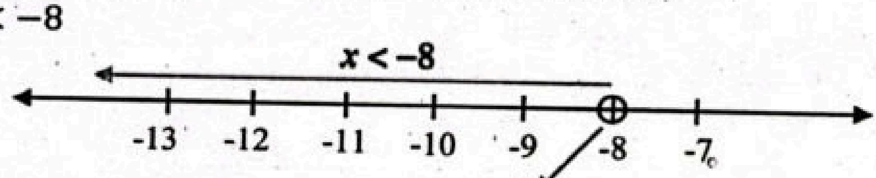




Example: Solve $9x+6<8x-2$. Graph the solution set.

Solution:

$$9x + 6 < 8x - 2$$
$$9x + 6 + (-8x) < 8x + (-8x) - 2 \quad \text{(Add } -8x \text{ to both sides)}$$
$$x + 6 < -2$$
$$x + 6 + (-6) < -2 + (-6) \quad \text{(Add } -6 \text{ to both sides)}$$
$$x < -8$$



Circle indicates that -8 is not included.

Any real number less than – 8 is solution.

Check: Substitute –8 for x in $9x+16<8x-2$, the inequalities should not be trueThen substitute a number less than –8, the inequality should be true.

Key Fact:

(i) Addition and subtraction properties of inequalities.
For any real number a, b and c:

- If $a>b$, then $a+c>b+c$ and $a-c>b-c$
- If $a<b$, then $a+c<b+c$ and $a-c<b-c$

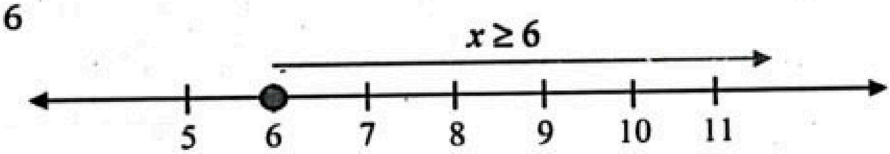
(ii) Multiplication and division properties of inequalities.

- If c is positive and $a < b$, then $ac < bc$ and $\frac{a}{c}<\frac{b}{c}$
- If c is positive and $a > b$, then $ac > bc$ and $\frac{a}{c}>\frac{b}{c}$
- If c is negative and $a < b$, then $ac > bc$ and $\frac{a}{c}>\frac{b}{c}$
- If c is negative and $a > b$, then $ac < bc$ and $\frac{a}{c}<\frac{b}{c}$

Example: Solve $6x-7\geq 2x+17$. Graph the solution.

Solution: $6x-7 \geq 2x+17$.

$$6x-7+7 \geq 2x+17+7 \quad \text{(adding 7 on both sides)}$$
$$6x \geq 2x+24$$
$$6x-2x \geq 2x-2x+24 \quad \text{(subtracting } 2x \text{ from both sides)}$$
$$4x \geq 24 \quad \text{(dividing both sides by 4)}$$
$$x \geq 6$$

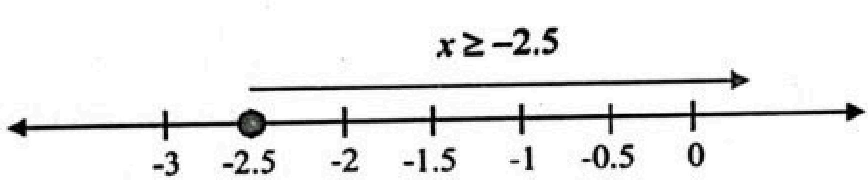


Example: Solve and graph the solution

$$10.7x-6+5.5x\geq 3(5x-3)$$

Solution: $10.7x-6+5.5x\geq 3(5x-3)$

$$16.2\ x - 6\geq 15x-9$$
$$16.2x-15x\geq -9+6$$
$$1.2x\geq -3$$
$$x\geq -2.5$$



Check Point:

Describe and correct the error.

a. $17-3x \geq 56$

$-3x \geq 39 \Rightarrow x \geq -13$

b. $-4(2x-3) < 28$

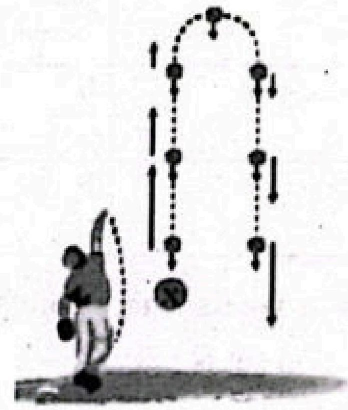
$-8x-12 < 28$

$-8x < -40 \Rightarrow x > 5$

Solving Quadratic Inequalities

We will discuss the solution of quadratic inequalities in one variable. Quadratic inequalities can be derived from quadratic equations. The word 'Quadratic' comes from the word 'Quadratum' which means square in Latin. From this, we can define quadratic inequalities on second degree equation. The general form of a quadratic equation is $ax^2+bx+c=0$. Further if the quadratic polynomial ax^2+bx+c is not equal to zero, then they are either $ax^2+bx+c>0$ or $ax^2+bx+c<0$ and are called quadratic inequalities. We explain quadratic inequalities with daily life example.

Consider a ball is thrown upward with an initial velocity of 64 feet per second. The function $h(t) = 64\,t - 16\,t^2$ gives the height of the ball after t seconds. During what time span is the ball in flight? We can find the times that the ball is in flight by solving the inequality $64t - 16t^2 > 0$.



$$64t - 16t^2 > 0$$

$$-16t^2 + 64t > 0$$

$$-16t(t-4) > 0$$

This inequality shows that the product of two numbers is positive if both of the numbers $(-16t,\ t-4)$ are positive or both are negative.

Case I:

When both numbers are positive

$-16t>0$ and $t-4>0$

$t<0$ and $t>4$

Here, $t < 0$ and at the same time $t > 4$ is never true.

Case II:

When both numbers are negative

$-16t<0$ and $t-4<0$

$t>0$ and $t<4$

Means: $0<t$ and $t<4$

$0 < t < 4$

The solution set is $\{0 < t < 4\}$. So, the ball is in flight between 0 and 4 seconds after it is thrown.

Example: Solve $x^2-4x-12>0$

Solution: $x^2-4x-12>0$

$x^2-6x+2x-12>0 \quad \Rightarrow \quad x(x-6)+2(x-6)>0$

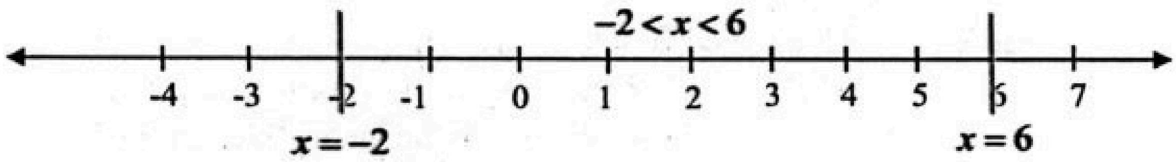
$\Rightarrow \quad (x-6)(x+2)>0$

Another way that you can solve quadratic inequalities is by using three test points.
First solve the equation $(x-6)(x+2)=0$

$x-6=0$ or $x+2=0$.

$\Rightarrow \quad x=6,\ x=-2$

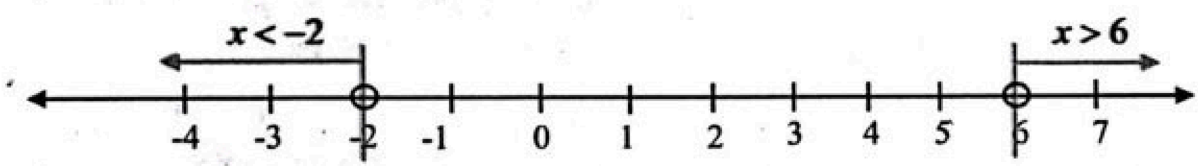
These solutions are called critical points. The points -2 and 6 separate the x-axis into three regions $x<-2,\ -2<x<6,\ x>6$.



Choose a value from each of the parts and substitute it into $(x-6)(x+2)>0$.
Construct a table to organize your results.

Part of x-axis	Point x	$(x-6)(x+2)$	Is $(x-6)(x+2)>0$?
$x<-2$	-3	$(-3-6)(-3+2)=9$	Yes
$2<x<6$	0	$(0-6)(0+2)=-12$	No
$x>6$	8	$(8-6)(8+2)=20$	Yes

The part of x-axis where points are solution of $(x-6)(x+2)>0$, belongs to the solution set.
The solution set is $\{x|x<-2 \text{ or } x>6\}$.



Example: Find the solution set of $x^2+6x-27\le 0$.

Solution: $x^2+6x-27\le 0 \quad\Rightarrow\quad x^2+9x-3x-27\le 0$

$x(x+9)-3(x+9)\le 0 \quad\Rightarrow\quad (x+9)(x-3)\le 0$

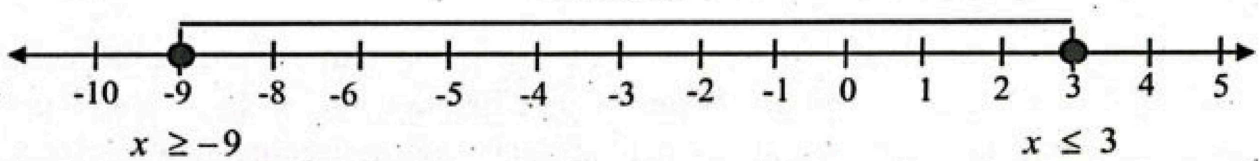
First solve the equation $(x-3)(x+9)=0$

$x-3=0$ or $x+9=0 \quad\Rightarrow\quad x=3,\ x=-9$

The point 3 and -9 are separate, its x-axis in three parts.
Make a table to organize results.

Part of x-axis	Point x	$(x-3)(x+9)$	Is $(x-3)(x+9)\le 0$?
$x\ge 3$	4	$(4-3)(4+9)=13$	No
$-9\le x\le 3$	0	$(0-3)(0+9)=-27$	Yes
$x\le 9$	-10	$(-10-3)(-10+9)=12$	No

$-9 \le x \le 3$



The part of x-axis where points are solution to $(x-3)(x+9) \le 0$ belong to the solution set.
The solution set is $\{x \mid -9 \le x \le 3\}$.

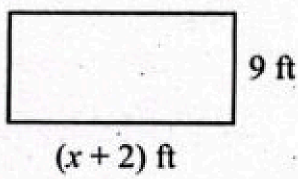
Exercise 4.1

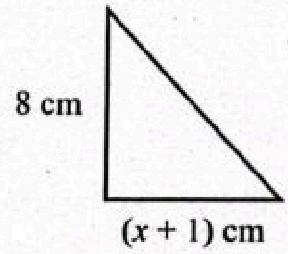
Solve the inequality and graph the solution.

1. $5x-12 \le 3x-4$
2. $1-8x \le -4(2x-1)$
3. $\frac{-2}{3}x-2 < \frac{1}{3}x+8$
4. $8-\frac{4}{5}x > -14+2x$
5. $-0.6(x-5) \le 15$
6. $\frac{-1}{4}(x-12) > -2$

Translate the phrase into an inequality. Then solve the inequality and graph the solution.

7. Four more than the product of 3 and x is less than 40.
8. Twice the sum of x and 8 appears less than or equal to -2.
   Write and solve an inequality to find the possible values of
9. Area > 81 square feet
10. Area ≤ 44 square centimeters





Solve each inequality

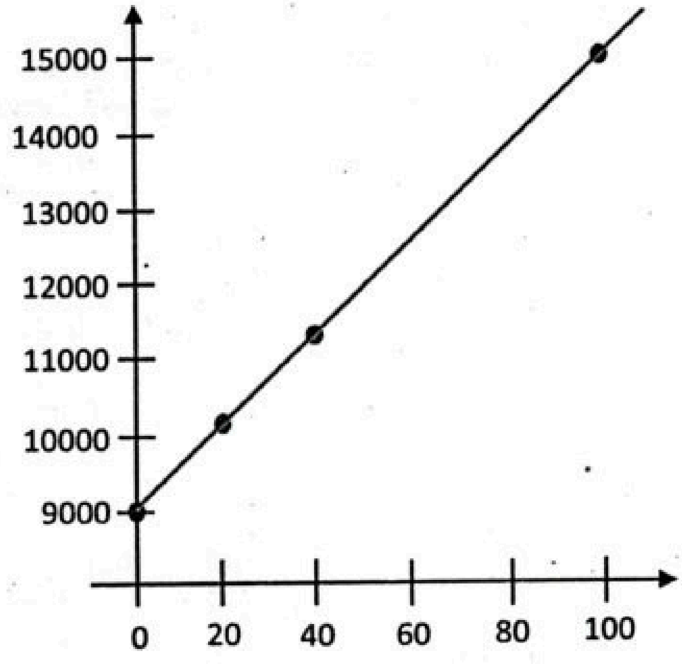
11. $(x+1)(x-3) > 0$
12. $(x-9)(x+1) < 0$
13. $x^2-x-90 > 0$
14. $x^2+4x-21 < 0$
15. $x^2+8x+16 \ge 0$
16. $9x^2-6x+1 \le 0$
17. Mr. Khalid has a field and want to make a rectangular garden with perimeter of 68ft. He would like the area of a garden to be at least 240 square feet. What would the width of the garden be?
18. Are $-2, -1, 0, 1, 2$ solution of the inequality $2x^2+3x+1 \le 0$.
19. The stopping distance $d(x)$ (in meters) of a car traveling at x km/h is modeled by $d(x) = 0.05x^2 + 0.2x$. If the maximum stopping distance allowed in a school zone is 35 meters, find the speed range that ensures the stopping distance does not exceed this limit.
20. A rectangular garden is to be enclosed with a fence. The total length of the fence is at most 60 meters and the width x of the garden is twice its length. Find the possible dimensions of the garden.

Solving System of Linear Inequalities

Mr. Waleed wants to rent a car for a business trip. A car rental company advertises that their rental rate is Rs.9000 plus Rs.60 per km. Mr. Waleed would like to compare this rate with the rates offered by other car rental agencies. First, he determines the equation containing the points that represent the relationship between the numbers of km (x) and the total cost of the rental (y). The initial cost of the car is Rs. 9000. Since this is the point where zero km are driven, it would be the y-intercept of the graph.

The slope would be the rate of change in the total cost. In this case, the rate is Rs.60 per km.

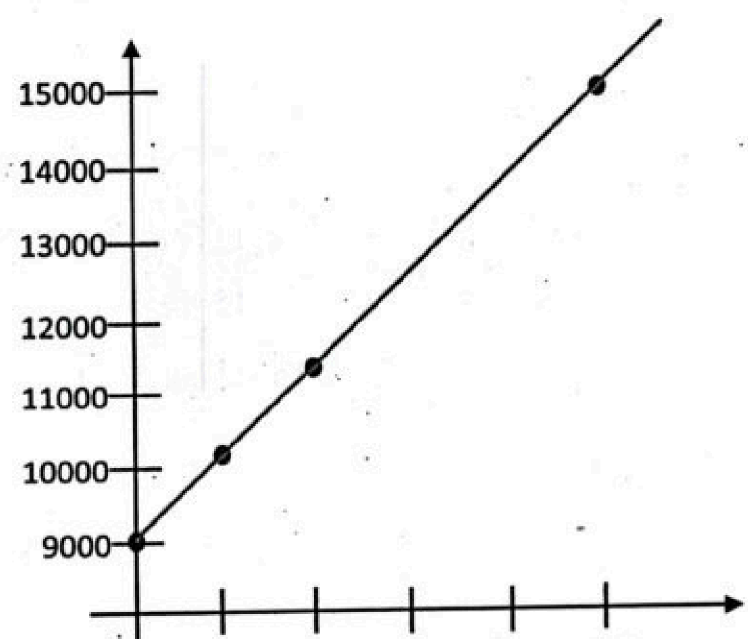
Since the slope is 60. Thus, an equation of the line is $y = 60x + 9000$.



km (x)	Cost (y)
0	9000
20	10200
40	11400
100	15000

The graph of $y = 60x + 9000$ separates the coordinate plane into two regions.

The line is called boundary of the region. To graph an inequality, first you graph the boundary and then determine which region to shade. The graph $y > 60x + 9000$ contains the points that are located above the boundary.



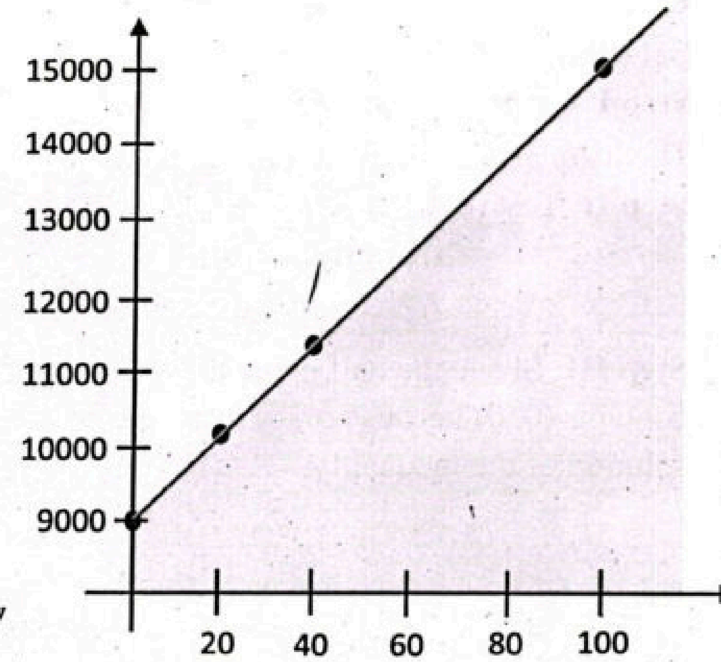
In that region, the value of the dependent variable y is greater than the value of $60x + 9000$. This graph represents car rental costs that are greater than those offered by car rental company A. For example, another company B charges 14000 for a car rental with 50km.

The point (50, 14000) lies above the boundary.

The graph of $y > 60x + 9000$ contains points that are located below the boundary. In that region, the value of y is less than the value of $60x + 9000$.

The graph represents car rental costs that are less than those offered by company A.

For example, a company C charges 14000 For a car rental with 100km. the point (100, 14000) lies below the boundary.



- When graphing an inequality, the boundary you draw may be solid or dashed.
- If the inequality uses the symbol $\leq$ or $\geq$ which include equality then the boundary will be solid. Otherwise, it will be dashed.
- After graphing the boundary, you must determine which region is to be shaded. Test a point on one side of the line. If the ordered pair satisfies the inequality, that region contains solution of the inequality. If the ordered pair does not satisfy the inequality, the other region is the solution.

Key Fact:

A linear inequality in two variables: such as $x - 3y < 6$, is the result of replacing the "=" in a linear equation with $<, \leq, >$ or $\geq$. A solution of an inequality in two variables x and y is an ordered pair (x, y) that produces a true statement when the values of x and y are satisfied into the inequality.

Example: Which ordered pair is not a solution of $x - 3y \leq 6$?

a. (0, 0) b. (6, –1) c. (10, 3) d. (–1, 2)

Solution:

Test (0, 0): $x - 3y \leq 6$ (Write inequality)

$0 - 3(0) \leq 6$ (Substitute 0 for x and 0 for y.)

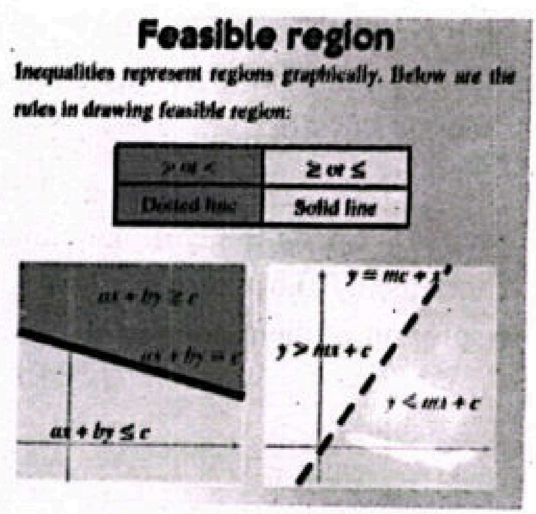
$0 \leq 6$ (True)

Test (6, –1): $x - 3y \leq 6$ (Write inequality)

$6 - 3(-1) \leq 6$ (Substitute 6 for x and –1 for y.)

$9 \leq 6$ (False)

Similarly (10, 3) and (–1, 2) also true but (6, –1) is not a solution.



Example: Graph the inequality $y > 5x - 4$.

Solution:

Step I: Graph the equation $y = 5x - 4$.

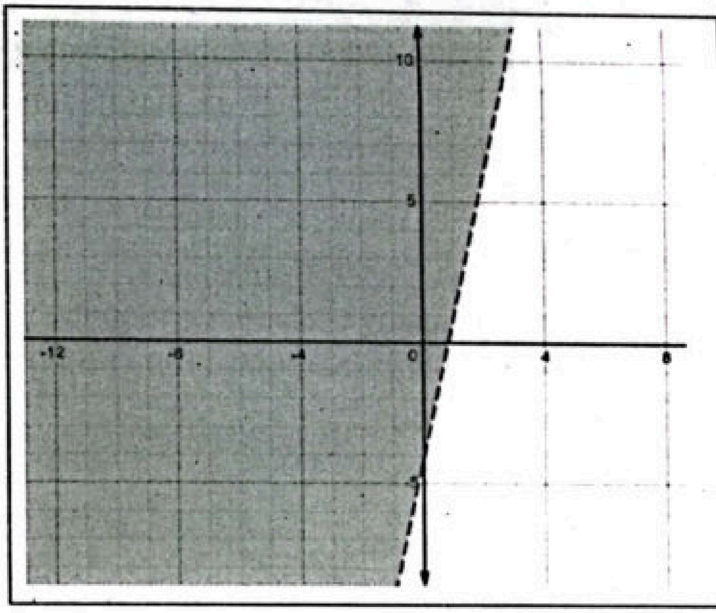
The inequality is >, use a dashed line.

Step II: Test (0, 0) in $y > 5x - 4$.

$$0 > 4(0) - 4$$

$$0 > -3 \text{ (True)}$$

Step III: Shade the half plane that contains (0, 0) because (0, 0) is a solution of the inequality.



Example: Graph the in equality $x + 2y \le 0$.

Solution:

Step I: Graph the equation $x + 2y = 0.$.

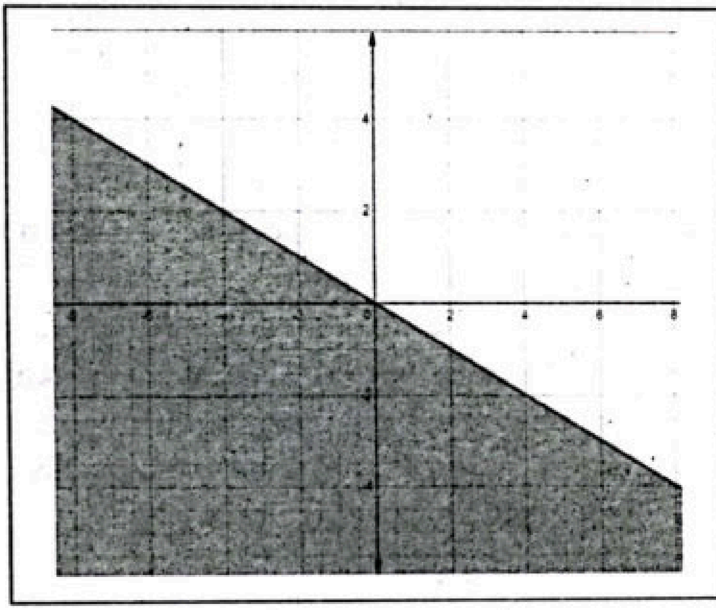
The inequality is ≤, use a solid line.

Step II: Test (1, 0) in $x + 2y \le 0$.

$$1 + 2(0) \le 0$$

$$1 \le 0 \text{ (False)}$$

Step III: Shade the half plane that does not contain (1, 0) because (1, 0) is not a solution of the inequality.



Example: Graph the in equality $y \ge -2$.

Solution:

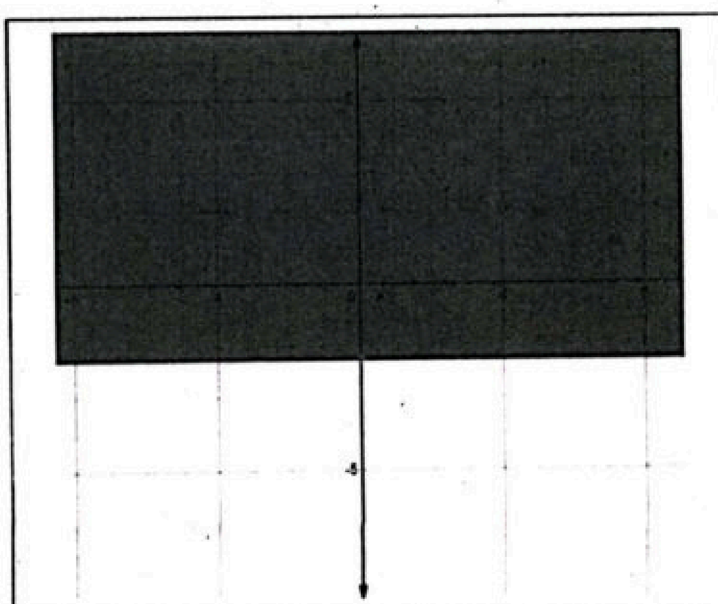
Step I: Graph the equation $y = -2$.

The inequality is ≥, use a solid line.

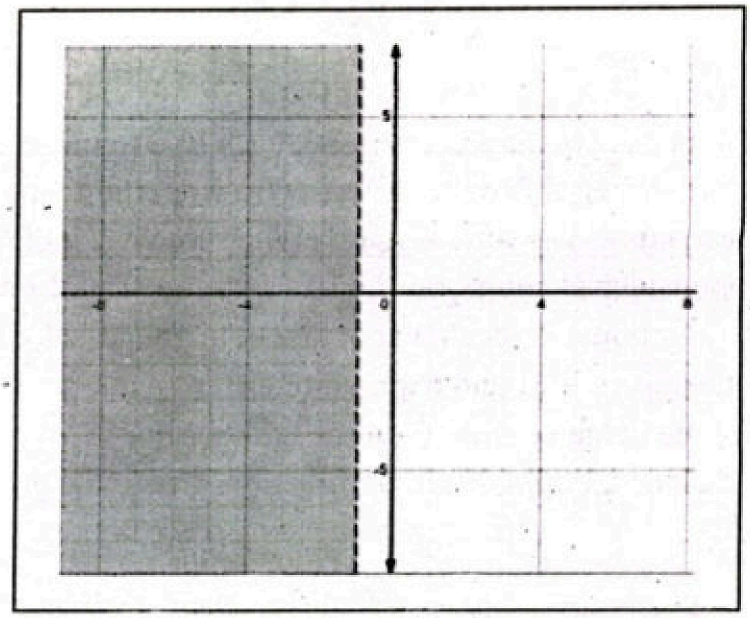
Step II: Test (0, 0) in $y \ge -2$

$$0 \ge -2 \text{ (True)}$$

Step III: Shade the half plane that contains (0, 0) because (0, 0) is a solution of the inequality.



Example: Graph the in equality $x < -1$.
Solution:



Step I: Graph the equation $x = -1$.
The inequality is $\geq$, use a solid line.
Step II: Test (0, 0) in $x < -1$
$0 < -1$ (False)

Step III: Shade the half plane that does not contain (0, 0) because (0, 0) is not a solution of the inequality.

Exercise 4.2

Tell whether the ordered pair is a solution of the inequality.

1. $(0,0)$; $x + y < -4$
2. $(-2,-3)$; $y - x > -2$
3. $(5,2)$; $2x + 3y \geq 14$
4. $(-1,5)$; $4x - 7y > 28$
5. $(-9,-7)$; $y \leq 8$
6. $(-4,0)$; $x \geq -3$

Graph the inequality.

7. $x - y < -3$
8. $-3y - 2x < 12$
9. $x - y \geq 2$
10. $2x + y \geq 8$
11. $x - y \leq -11$
12. $y < -5$
13. $x \geq 4$
14. $\frac{1}{2}(x + 2) + 3y < 8$
15. Can we use (0, 0) as a test point when graphing $x + y > 0$? Explain with reason.

Write the verbal sentence as an inequality. Then graph the inequality:

16. Three less than x is greater than or equal to y.
17. The product of -2 and y is less than or equal to the sum of x and 6.
18. The sum of x and the product of 4 and y is less than -2.

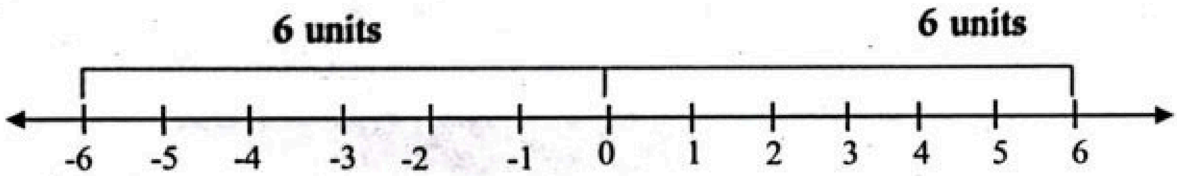
Absolute Valued Equations and Inequalities

Equations with Absolute Value

Suppose you and your friend, each live in the same street as your school, but on opposite sides of the school. You each live 6 km from the school. What can you say about your trips to school each day? Do you each travel the same distance?

Consider placing both houses and the school on a number line with the school at the origin. Your home is located at 6, and your friend is located at -6.

Certainly, -6. and 6 are quite different but they do have something in common. They have the same distance from 0 on the number line. This means that you and your friend travel the same distance, but in different direction, when you go to school.



We say that -6 and 6 have the same absolute value. The absolute value of a number is the number of units it is from 0 in the number line.

We use the symbol $|x|$ to represent the absolute value of a number x.

The absolute value of 6 is $|6| = 6$, and absolute value of -6. is $|-6| = 6$.

Absolute value →

For any real number x

- If $x \geq 0$, then $|x| = x$
- If $x > 0$, then $|x| = -x$
- Or $|x| = \begin{cases} x \text{ when } x \geq 0 \\ -x \text{ when } x > 0 \end{cases}$

In words: The definition states that the absolute value of a non-negative is the number itself and the absolute value of a negative number is the opposite of the number.

Since distance is always non-negative, we can think of a number's absolute value as its distance from zero on number line.

Example: Find the absolute value of 7 and -11.

Solution: $|7| = 7$ $\qquad |-11| = -(-11) = 11$

Key Fact:

The absolute value of a number is always positive or zero.

Example: Find the absolute value of $x - 5$.

Solution: Let's make a list of possible cases.

If x is 5 or greater, then $|x - 5| \geq 0$,

$$|x-5| = x-5$$

If x is less than 5, then $|x-5| < 0$,

$$|x-5| = -(x-5) = 5-x$$

Example: Find the solution set.

a. $|2x+5| = 11$

b. $|3-2x| = -5$

c. $|2x-3| = |x+6|$

d. $2|x+3|+2 = 16$

Solution:

a. $|2x+5| = 11$

$2x+5 = 11$ or $2x+5 = -11$

$2x = 11-5$ $2x = -11-5$

$2x = 6$ $2x = -16$

$x = 3$ $x = -8$

S.S=$\{3, -8\}$

b. $|3-2x| = -5$

The absolute value principle reminds us that absolute value is always non-negative. The equation $|3-2x| = -5$ has no solution. The solution set is ϕ.

c. $|2x-3| = |x+6|$

$2x-3 = x+6$ or $2x-3 = -(x+6)$

$2x-x = 6+3$ $2x-3 = -x-6$

$x = 9$ $2x+x = -6+3$

$x = -1$

S.S = $\{-1, 9\}$

d. $2|x+3|+2 = 16$

$2|x+3|+2 = 16$

$2|x+3| = 14-2$

$|x+3| = 7$

$x+3 = 7$ or $x+3 = -7$

$x = 4$ $x = -10$

S.S = $\{4, -10\}$

Student can verify the solution.

Example: A machine that fills the boxes of sugar is to fill the boxes with 14 kg of sugar. After the boxes are filled, another machine weighs the boxes. If the box is more than 0.3 kg above or below the desired weight, the box is rejected. What is the weight of the heaviest and the lightest box that the machine will pass?

Solution: Let w = the weight of the box, then:

$|w-14| = 0.3$ (as per condition)

$w - 14 = 0.3$ or $w - 14 = -0.3$

$w = 14.3$ or $w = 13.7$

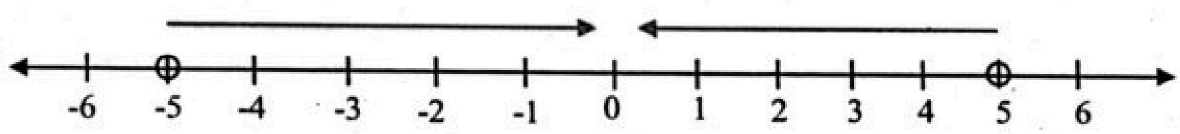
The heaviest weight allowed to pass is 14.3kg. The lightest weight allowed to pass is 13.7kg.

Inequalities With Absolute Value

The absolute value of a number is its distance from 0 on the number line. We can use this idea to solve inequalities involving absolute value.

Example: Solve $|x| < 5$.

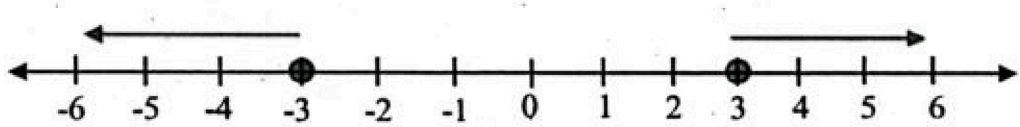
Solution: $|x| < 5$ means that the distance between x and 0 is less than five units. To make $|x| < 5$ true, we must substitute the values of x that are less than five units from 0.



All the number between – 5 and 5 are less than 5 units from zero.
The absolute solution set $= \{|x|: -5 < x < 5\}$.

Example: Solve $|x| \geq 3$. Graph the solution.

Solution: The solution of $|x| \geq 3$. are those numbers whose distance from 0 is greater than or equal to 3. In other words, those numbers x such that $x \leq -3$ or $x \geq 3$. The solution set is $\{|x|: x \leq -3 \text{ or } x \geq 3\}$.



Example: Solve and graph the solution.

a. $|2x-7| \geq 11$ b. $|3x-2| < 4$

Solution:

a. $|2x-7| \geq 11$ (Apply $|x| \geq a: x \geq a$ or $x \leq -a$)

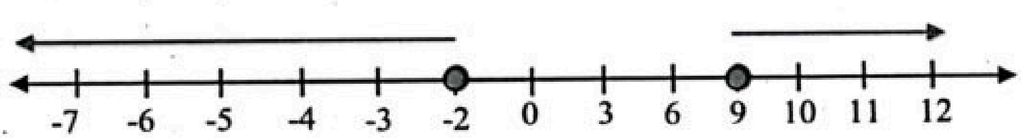
$2x - 7 \geq 11$ or $2x - 7 \leq -11$

$2x \geq 11 + 7$ or $2x \leq -11 + 7$

$2x \geq 18$ or $2x \leq -4$

$x \geq 9$ or $x \leq -2$

Solution set $= \{x|x \geq 9 \text{ or } x \leq -2\}$

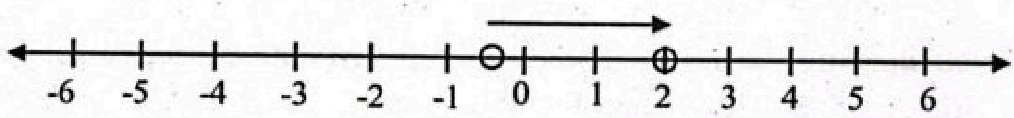


b. $|3x-2| < 4$ (Apply: $|x| < a$: $-a < x < a$)

$-4 < 3x - 2 < 4$

$$-4 < 3x - 2 \text{ and } 3x - 2 < 4$$
$$-4 + 2 < 3x \text{ and } 3x < 4 + 2$$
$$-2 < 3x \text{ and } 3x < 6$$
$$\frac{-2}{3} < x \text{ and } x < 2$$

Solution set $= \left\{x : \frac{-2}{3} < x < 2\right\}$



Example: Solve $|-4x-5|+3<9$. Graph the solution.

Solution: $|-4x-5|+3<9 \Rightarrow |-4x-5| < 6$

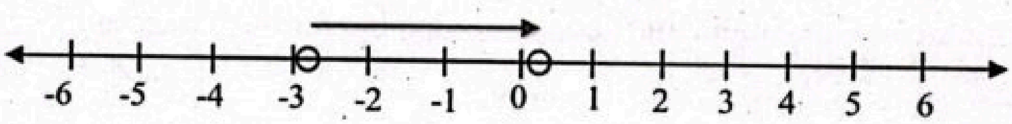
$$-6 < -4x - 5 < 6$$
$$-6 < -4x - 5 \text{ and } -4x - 5 < 6$$
$$-1 < -4x \text{ and } -4x < 11$$
$$\frac{1}{4} > x \text{ and } x > \frac{-11}{4}$$

We can write as: $-\frac{11}{4} < x < \frac{1}{4}$

Solution set $= \left\{x : \frac{-11}{4} < x < \frac{1}{4}\right\}$

Key Fact:

- $|x| \le a$ implies that $-a \le x \le a$
- $|x| \ge a$ implies that $x \le -a$ or $x \ge a$



Example:

You are buying a new calculator and find 10 models in a shop. The prices are Rs. 980, Rs. 570, Rs. 470, Rs. 820, Rs. 560, Rs. 770, Rs. 580, Rs. 390, Rs. 680 and Rs. 550.

a. Find the mean of the calculator price.

b. You are willing to pay the mean price with an absolute deviation of at most Rs. 100. How many of the calculator prices meet your condition?

Solution:

a. Find the mean first:

$$\text{Mean} = \frac{980+570+470+820+560+770+580+390+690+550}{10}$$
$$= \frac{6380}{10} = 638$$

b. Write and solve an inequality.

An absolute deviation of at most Rs.100 from the mean 638, is given by the inequality.

$$|x-638| \leq 100$$

$$-100 \leq x - 638 \leq 100$$

$$-100 \leq x - 638 \text{ and } x - 638 \leq 100$$

$$538 \leq x \text{ and } x \leq 738$$

We write as: $538 \leq x \leq 738$

The price of calculator must be at least 538 and at most 738. Five prices meet your condition that is Rs. 570, Rs. 560, Rs. 580, Rs. 690 and Rs. 550.

Exercise 4.3

Solve the absolute valued equations (1-6).

1. $8|x-3|=88$
2. $|2x+9|=30$
3. $|x-3|=-26$
4. $3|x+6|=9x-6$
5. $|x-3|=2x$
6. $\left|\frac{4-5x}{6}\right|=7$
7. Explain, why the equation $|3x-6|+7=4$ has no solution.
8. Before the start of a professional basketball game, a basketball must be inflated to an air pressure of 8 pounds per square inch (psi) (Absolute error is the absolute deviation). Find the minimum and maximum air pressure acceptable for the basketball.

Solve the inequality and graph the solution (9-14).

9. $|9x-1| \leq 10$
10. $|2x-7|<1$
11. $5\left|\frac{1}{2}x+3\right|>5$
12. $3|14-x|>6$
13. $\left|\frac{3x-2}{5}\right| \leq 2$
14. $\left|\frac{2-5x}{4}\right| \geq \frac{2}{3}$
15. An essay contest requires that essay entries consists of 500 words with an absolute deviation of at most 30 words. What are the possible number of words that the essay can have?

Solving Systems of Linear Inequalities

A system of linear inequalities in two variables or simply a system of inequalities consists of two or more linear inequalities in the same variable. As:

$$\left.\begin{array}{l} x-y>9 \text{ Inequality I} \\ 2x+y<9 \text{ Inequality II} \end{array}\right\} \text{A}$$

A solution of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system. For example, (7, – 6) is a solution of the system A.

The graph of a system of linear inequalities is the graph of all solutions of the system.

Example: Graph the system of inequalities:

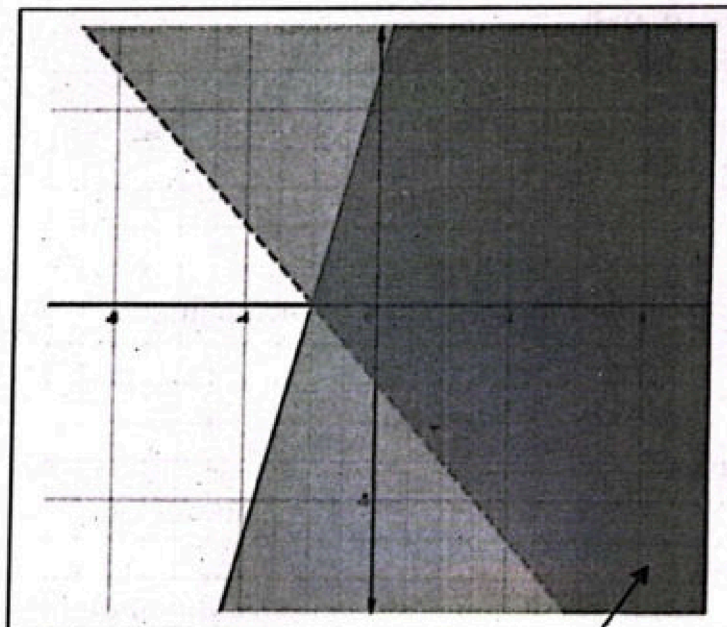
$$x+y>-2$$
$$3x-y\geq -6$$

Solution:

Step I: Graph both inequalities in the same coordinate plane.

Step II: Find the solution of each inequality. Preferably taking (0, 0) as test point to decide the solution region.

Step III: The graph of the system in the intersection of the two half planes. Which is shaded red.



This dark shaded region is a solution of these inequalities.

Check: Choose a point in the region such as (0, 1). To check this solution, substitute 0 for x and 1 for y into each inequality.

Inequality I:

$$x+y>-2$$
$$0+1>-2$$
$$1>-2 \text{ True}$$

Inequality II:

$$3x-y\geq -6$$
$$3(0)-1\geq -6$$
$$-1\geq -6 \text{ True}$$

Key Fact:

- Graph each inequality in the cartesian plane.
- Find the intersection of the half planes.
- The graph of system is the intersection.

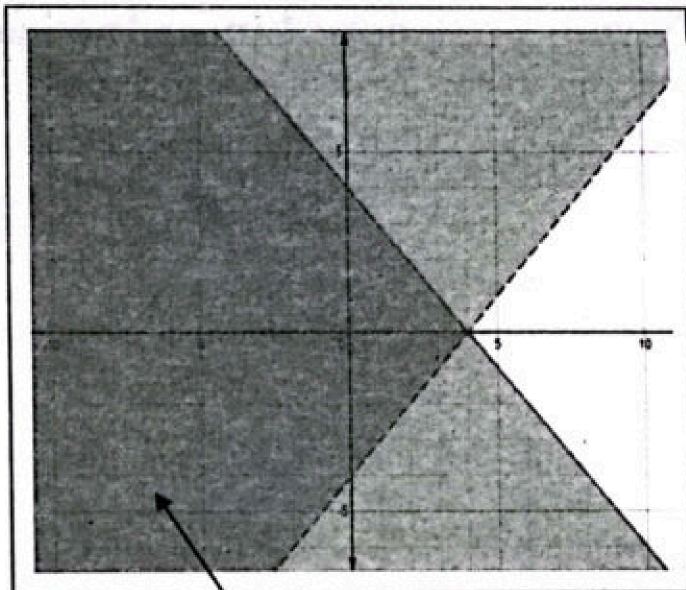
Example: Graph the system:

$$x+y\le 4$$
$$x-y<4$$

Solution:

To graph the inequality $x+y\le 4$, we graph $x+y=4$ using a solid line. We then consider (0, 0) as a test point and find that it is a solution, we shade all points in that region.

Next, we graph $x-y<4$. We graph $x-y=4$ as a dashed line and consider (0, 0) as a test point. Again (0, 0) is a solution, we shade that region. Finally, the solution set of the system is the region that is commonly shaded in both inequalities.



This dark shaded region is a solution of these inequalities.

Example: Graph the system of inequalities.

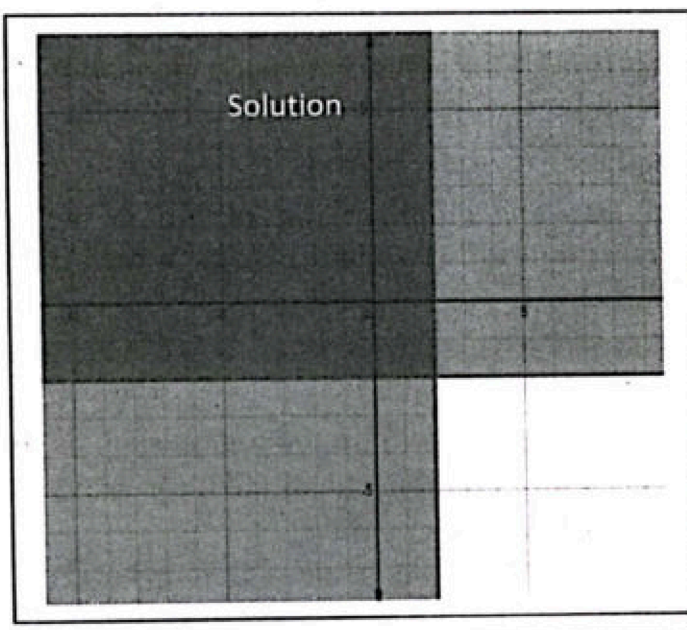
$$x\ge -2$$
$$x\le 2$$

Solution:

Graph two inequalities in the same coordinate system.

$y=-2$ (The region is above the line)

$x=2$ (The region is left of the line)



Exercise 4.4

1. Describe the steps you would take to graph the system of inequalities shown.

$$x-y<6\text{......(Inequality I)}$$
$$y\ge 3\text{......(Inequality II)}$$

2. Tell whether the ordered pair is a solution of the system of inequalities.
   a. $(3,1),(0,0),(-3,1),(4,1);\ x+y<1, x-y<2$
   b. $(1,-1),(4,1),(2,0),(3,2);\ 2x-y\le 5, x+2y>2$

3. Graph the system of inequalities.

a. $-x+y<0$
$x+y>3$

b. $x-y<0$
$x+y<1$

c. $y-2x\geq 3$
$y-2x\leq 3$

d. $y\leq -2$
$x\geq 2$

e. $x>-2$
$2x+y<3$

f. $x+y\leq 3$
$x-y\leq 4$

g. $y\geq 2x=1$
$y<-x+4$

h. $x<8$
$x-4y\leq -8$

4. Does the system of inequalities have any solution?

$$x-y>5$$
$$x-y<1$$

5. Open-Ended Question: Describe a real-world situation that can be modeled by a system of linear inequalities. Then write and graph the system of inequalities.

I have Learnt

- On a number line, the graph of an inequality in one variable is the set of points that represent all solutions of the inequality.
- To graph an inequality in one variable are an open circle for < or > and a closed circle for ≤ or ≥.
- Dividing / multiplying each side of an inequality by a positive number produces an equivalent inequality.
- Dividing / multiplying each side of an inequality by a negative number and reversing the direction of the inequality symbol produces an equivalent inequality.
- A linear inequality in two variables, such as $ax+by<c$ is the result of replacing the "=" sign in a linear equation with <, ≤, > or ≥.
- A solution of an inequality in two variables *x* and y is an ordered pair (*x*, y) that produces a true statement when the values of *x* and y are substituted into the inequality.
- The boundary line for an inequality in one variable is either vertical or horizontal.
- If an inequality has only the variable *x*, substitute the x-coordinate of the test point into the inequality.
- If an inequality has only the variable y, substitute the y-coordinate of the test point into the inequality.
- A solution of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system.
- The graph of a system of linear inequalities is the graph of all solutions of the system.

1. Tick the correct option.

i. The solution of the inequality $6x-7 \geq 2x+17$ is:

a. $x<6$ b. $x \leq 6$ c. $x>6$ d. $x \geq 6$

ii. The solution of the inequality $x<3$ for $x \in N$ is:

a. $\{0, 1, 2\}$ b. $\{-2, -2, 0, 1, 2\}$ c. $\{1, 2\}$ d. $\{1\}$

iii. In general, we use a test point when graphing the inequality:

a. $(-1, -1)$ b. $(0, 0)$ c. $(2, 2)$ d. $(-3, -3)$

iv. The solution of the $\frac{x}{-2} < 3-x$ is:

a. $x<2$ b. $x<6$ c. $x>2$ d. $x<-2$

v. Which ordered pair is a solution of the inequality $4x-y \geq 3$? ?

a. $(0, 0)$ b. $(-1, 2)$ c. $(1, 1)$ d. $(0, -2)$

vi. Which ordered pair is a solution of the system $2x-y \leq 5$ and $x+2y>2$?

a. $(1, -1)$ b. $(4, 1)$ c. $(2, 0)$ d. $(3, 2)$

vii. The solution for $-5x \geq -80$ is:

a. $\{x \mid x<16\}$ b. $\{x \mid x \leq 16\}$ c. $\{x \mid x>16\}$ d. $\{x \mid x \geq 16\}$

viii. $(7, -2)$ is a solution of the inequality;

a. $x-y<-4$ b. $2x+y<10$ c. $x+10y<1$ d. $-x-y>-3$

Solve the inequalities, if possible, graph the solution.

2. $\frac{2}{3}x-4 \geq 1$

3. $1-3x \leq -14+2x$

4. $7(x-1)>-8+7x$

5. $-3(2x-1) \geq 1-8x$

Graph on a plane.

6. $y>2x$

7. $y<3x$

8. $y \geq -5$

9. $x \leq 6$

Graph the system of inequalities:

10. $3x+4y \geq 12$
$5x+6y \leq 30$

11. $y-x \geq 1$
$y-4 \leq 4$

12. $8x+5y \leq 40$
$x \geq 0$

13. $2x+y \leq 12$
$y \geq 0$

Write a system of inequalities for the shaded region described.

14. The shaded region is a rectangle with vertices (2, 1), (2, 4), (6, 4) and (6, 1).

15. The shaded region is a triangle with vertices at (3, 0), (3, 2) and (0, –2).