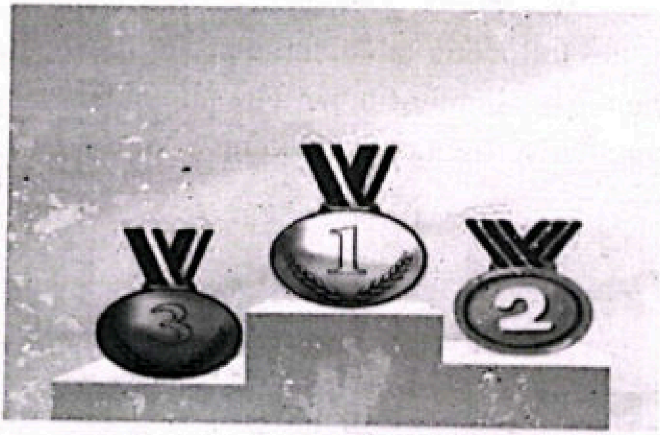
12

Basic Statistics

After studying this unit students will be able to:

- Construct cumulative frequency table, cumulative frequency polygon and cumulative frequency curve i.e. ogive
- Interpret the quartiles, deciles, percentiles and inter quartile range(IQR) from cumulative frequency curve
- Interpret and analyze box and whisker plot correlation
- Construct and interpret data from scatter diagram and also draw lines of best fit
- Measure correlation using scatter diagram
- Calculate range, standard deviation and variance for grouped data
- Use the mean and standard deviation to compare 2 sets of data
- Solve real life problems involving variance and SD of grouped data
- Apply concepts from measures of dispersion to solve real life problems
- Calculate the probability of combined events using (where appropriate):Sample space diagram, possibility diagrams, tree diagram & venn diagram
- Apply addition law of Probability to solve problems involving mutually exclusive events
- Apply Multiplication law of Probability to solve problems involving independent and dependent events

Haani and Haider participated in a Qiraat competition, with 8 other equally competent contestants. Judges have to decide about top 3 performers. Can you guess the chance of every contestant to be the top first? After decision of First, can you guess the chance of every remaining contestant to be top second? After decision of Second, can you guess the chance of every remaining contestant to be top third?



Participants	Haani	Haider	Ibraheem	Wali	Ali	Hashir	Qasim	Hassan	Muhammad	Ahmad
Chance of winning	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Frequency Distribution

In statistics it is very important to represent a data in a manageable way so that it becomes easy to understand, analyze and to draw useful results. This useful presentation of data can be made by various methods. Grouped frequency distribution is one of the most important methods to represent the quantitative data. Data can be classified in many ways for efficient usage and productivity. In previous grades we have discussed two types of data

i. raw data (ungrouped data) ii. grouped data (frequency distribution)

Here we will study the construction of cumulative frequency distribution by revising frequency polygon.

Frequency Polygon

Polygon is a many sided closed figure. To draw a frequency polygon mid points (class marks) are marked on the horizontal axis and frequencies are marked on the vertical axis. Points are plotted with class frequencies and their corresponding class marks. These points are joined by straight line segments. To complete a closed polygon we add extra classes at both ends with zero frequencies. In this way two extra points on x-axis are obtained on both ends. The points are joined by their nearer plotted points. Finally a frequency polygon is obtained.

Cumulative Frequency Distribution

A table in which cumulative frequencies of classes are written against classes is called a cumulative frequency table "The number of values of a data less than an upper-class boundary is called its cumulative frequency".

The cumulative frequency table is another way to analyze the frequency distribution. The frequency distribution tells us how many values of the data are present within each class interval, while a cumulative frequency tells us how many values of a data are less than or within each class interval.

In the following table cumulative frequencies are found and interval of 0 frequency is added at the top. Column of upper class boundary is also included which are necessary for construction of cumulative frequency polygon or ogive.

Marks	Marks(upper class boundaries)	No. of students	Cumulative frequency	c.f
0	0.5	0	0	0
1 – 10	10.5	2	2	2
11 – 20	20.5	4	4+2=6	2 + 4 = 6
21 – 30	30.5	9	9+4+2=15	6 + 9 = 15
31 – 40	40.5	6	6+9+4+2=21	15 + 6 = 21
41 – 50	50.5	7	7+6+9+4+2=28	21 + 7 = 28
51 – 60	60.5	3	3+7+6+9+4+2=31	28 + 3 = 31
61 – 70	70.5	4	4+3+7+6+9+4+2=35	31 + 4 = 35

Cumulative Frequency Polygon and Cumulative Frequency Curve (Ogive)

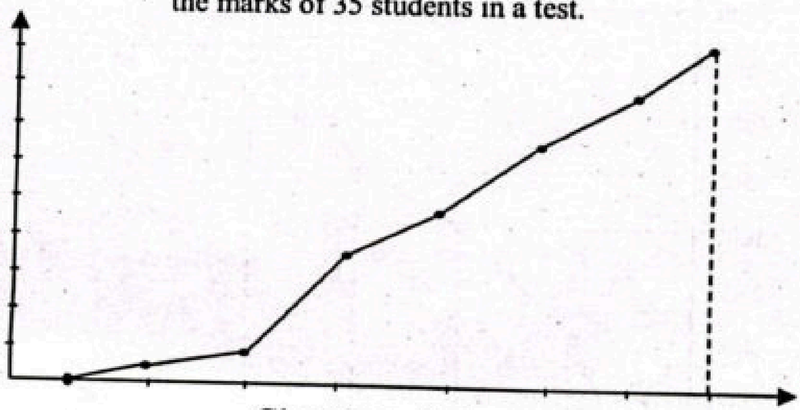
A frequency distribution can also be represented by a cumulative frequency polygon or Ogive. An ogive and a cumulative frequency polygon are related but distinct concepts in statistics and data visualization.

Cumulative Frequency Polygon

A frequency polygon made by taking cumulative frequency on y axis is called cumulative frequency polygon. It is made to visualize the cumulative distribution of a variable.

To draw a cumulative frequency polygon, the class boundaries are plotted on x-axis and cumulative frequencies on y-axis. The points are marked on the graph using the upper class boundaries and the respective cumulative frequencies. These points are joined by straight line segments and the polygon is completed by drawing a vertical line segment to join the last marked point with x-axis. Following is a cumulative frequency polygon representing the data given above.

Title: A cumulative frequency polygon showing the marks of 35 students in a test.



Example: Late arrival fine imposed to a player is shown below against the terms of 4 weeks:

weeks	1- 4	5-8	9-12	13-16
Amount of fine (Rs)	2000	1000	500	1500

a) Construct cumulative frequency table including
   i. Column of upper class boundaries
   ii. Interval for 0 frequency
   iii. Column of cumulative frequencies

b) How much fine is to be paid after 3 terms?

c) Construct cumulative frequency polygon of the above data.

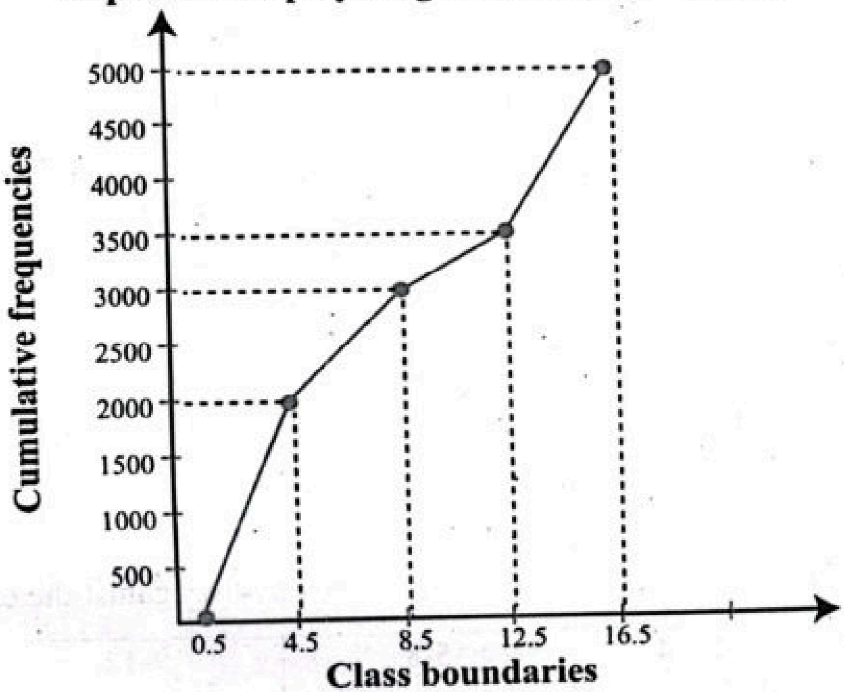
Solution: (a) Cumulative frequency table

weeks	Upper class boundaries	Amount of fine (Rs)	Cumulative frequencies
0	Less than 0.5	0	0
1-4	Less than 4.5	2000	2000
5-8	Less than 8.5	1000	3000
9-12	Less than 12.5	500	3500
13-16	Less than 16.5	1500	5000

b) After 3 terms, The player has to pay a total amount of Rs. 3500.

c) Construction of cumulative frequency polygon is given below:

Title: Cumutative freq polygon showing fine imposed on a player against terms of 4 weeks



Cumulative Frequency Curve (Ogive)

A graph showing the cumulative frequency (or cumulative relative frequency) of a variable as a curve, with the x-axis showing the values of the variable and the y-axis showing the respective cumulative frequency is called Cumulative Frequency Curve (Ogive).

It is also known as a cumulative distribution function (CDF)

Example:

The table below shows the cumulative frequency distribution of masses of cakes in Cocobakes made at a festival:

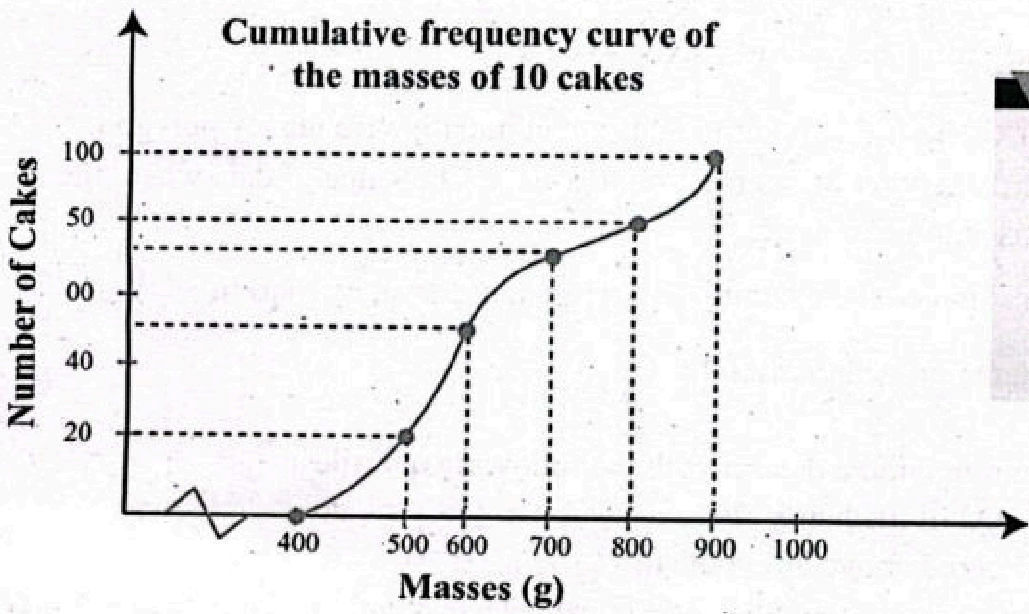
Mass of cakes (x grams)	$x \leq 400$	$x \leq 500$	$x \leq 600$	$x \leq 700$	$x \leq 800$	$x \leq 900$
No. of cakes	0	20	50	70	80	100

a) Draw cumulative frequency curve.

b) Estimate from the curve, the number of cakes having masses 750 g or less

c) Taking class intervals $400<x\leq 500$, $500 < x \leq 600$ …construct a cumulative frequency distribution .

Solution: (a) Cumulative frequency curve (Ogive)



Key Fact:

An ogive is a smooth curve, while a cumulative frequency polygon is a polygon-shaped graph

(b) From the curve, we observe that the number of cakes having masses 750 g or less is 75.

(c) Construction of the frequency cumulative distribution.

Mass of cakes (x g)	Cumulative frequency	Mass of cakes (x g)	frequency
$x \leq 500$	20	$400 \leq x \leq 500$	20
$x \leq 600$	50	$500 \leq x \leq 600$	$50 - 20 = 30$
$x \leq 700$	70	$600 \leq x \leq 700$	$70 - 50 = 20$
$x \leq 800$	80	$700 \leq x \leq 800$	$80 - 70 = 10$
$x \leq 900$	100	$800 \leq x \leq 900$	$100 - 80 = 20$

Measures of Distribution

These measures are used to understand the spread and distribution of the data, and are often used with measures of central tendency to get a better understanding of the dataset. They include:

i. Quartiles

The three values which divide the whole data into four equal parts are called quartiles. Quartiles are usually denoted by Q_1, Q_2 and Q_3. The lower quartile Q_1 divides the data in 1 : 3. The middle quartile Q_2 divides the data in 1 : 1. It is infect nothing different from median of data. The upper quartile Q_3 divides the data in 3 : 1.

$Q1 = \frac{n}{4}$ th value of data & $Q2 = (\frac{2n}{4} = \frac{n}{2})^{th}$ value of data = median

When data is arranged in some order, it can be represented by a single value that lies exactly at the centre of data. "The central or middle most value of an arranged data is called its median".

$Q3 = \frac{3n}{4}$ th value of data

The quartiles can also be estimated graphically from a cumulative frequency polygon.

- To estimate Q_1 (lower quartile), mark $\frac{n}{4}$ on y-axis.
- Draw a line parallel to x-axis from that point to intersect cumulative frequency polygon.
- Drop a perpendicular from this point of intersection to x-axis. The value of data where the perpendicular meets x-axis is lower quartile Q_1.

Similarly, Q_2 and Q_3 can be estimated from cumulative frequency curve, by marking $\frac{n}{2}$ and $\frac{3n}{4}$ on y-axis and following the same procedure used for Q_1.

Example:

The scores on a test were compiled into a dataset with the following quartiles:

Q1 = 40, Q2 or median = 70, Q3 = 90

What percentage of students scored above 90 on the test?

Solution: From the quartiles it is obvious that one fourth (25 percent) of the students scored above 90.

ii. Deciles

Deciles divide a dataset or a distribution into 10 equal parts, each representing 10% of the data. By looking at the deciles, we can get a sense of the shape of the distribution, including where the majority of the data points are concentrated and where the outliers are lying.

Ten Deciles of a dataset are described as follows:

D1: lowest 10% of the data

D2: lowest 20% of the data

D3: lowest 30% of the data

....

D10: highest decile under which100% of the data lies.

Key Fact:

Deciles are often used in statistics, economics, and social sciences to understand the distribution of a variable, such as income, wealth or exam scores.

Example: A company wants to analyze the salaries of its employees. The related department compiles a dataset and calculates the deciles. The deciles are as follows:

D1 = Rs.40,000 , D5 or median = Rs.60,000 , D10 = Rs.80,000

What percentage of employees earn a salary above Rs.60,000 but below Rs.80,000?

Solution: From the above data, percentage of employees earning salaries above 60,000 and below Rs. 80,000 are between D5(median) and D10 i.e 90 % – 50% = 40 %

(Remember: D10 represents highest 10 % i.e Rs. 80,000 and D9 represents below Rs. 80,000)

iii. Percentiles

Percentiles divide a dataset or a distribution into 100 equal parts, each representing 1% of the data. It's a way to understand the distribution of a dataset or a variable by looking at the percentage of data points that are below a specific value.

Some common percentiles include:

- 25th percentile (Q_1): 25% of data points are below this value.
- 50th percentile (Q_2 or median): 50% of data is below Q_2.
- 75th percentile (Q_3): 75% of data points are below this value.
- 90th percentile: 90% of data points are below this value.
- 99th percentile: 99% of data points are below this value

Maths Play Ground:

Help students to design quiz wheel or quiz machine for the reinforcement of measures of distribution.

Key Fact:

Percentiles are often used to:

understand the distribution of a variable (e.g., income, scores, masses).

identify outliers.

compare individual values to the overall distribution.

set benchmarks (e.g., top 10 %).

evaluate performance (e.g., moving up or down in percentiles).

Example: Uzair scored in the 75th percentile on a test. Find how much percentage of the students scored below him and how much percentage of students scored above him.

Solution: From the above information it is clear that the student's score is higher than 75% of the total students. It means that

- 75% of the students scored below him
- 25% scored above him

iv. Inter Quartile Range

IQR = Q3 – Q1

The IQR represents the range of values within which the middle 50% of the data points fall. In other words, it's the spread of the data within the middle half of the distribution.

IQR can describe:

Spread: A small IQR indicates less scattered, while a large IQR indicates more scattered data.

Variability: The IQR is a more efficient measure of variability than the Range.

Outliers: The IQR can help identify outliers. (data points beyond 1.5 times the IQR from the quartiles are considered outliers).

Comparison: IQR is useful for comparing the spread of different datasets.

Example:

A teacher wants to analyze the test scores of her students. She compiles a dataset as: first quartile $Q1 = 60$ and third quartile $Q3 = 95$

a) What is the Interquartile Range (IQR) of the test scores?

b) What percentage of students scored between 60 and 95 (inclusive)?

Solution:

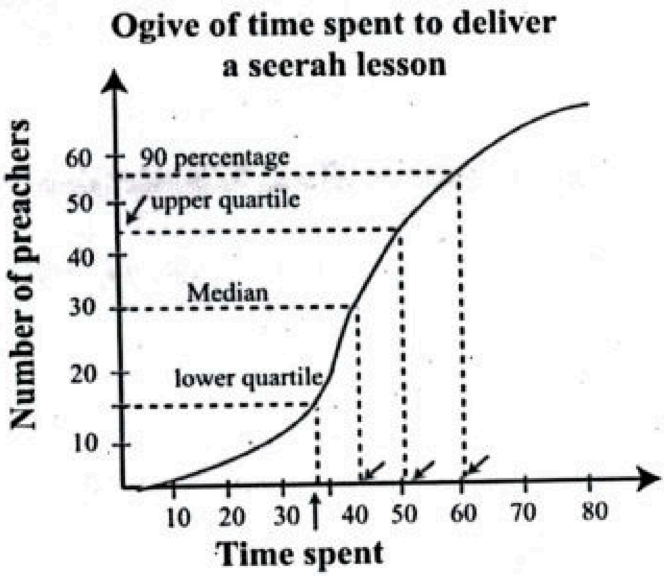
a) The Interquartile Range (IQR) of the test scores is given by the relation:

$$IQR = Q3 - Q1 = 95 - 60 = 35$$

b) As $Q3 - Q1$ represents spread of the middle half of a data set so 50% of students scored between 60 and 95 (inclusive)

Example:

The adjoining figure shows an ogive representing the distribution of time in minutes spent by 60 preachers of SEED to deliver single Seerah lecture. Find Q1, Q2, Q3 and inter quartile range.



Solution:

To estimate the median time spent from ogive, we observe that 50 % (half of) preachers spent less than or equal to the median time to deliver the lecture.

Median corresponds to the 5th decile and 50th percentile, i.e.

$$\text{Median} = Q2 = D5 = P50 = \frac{1}{2} \times 60 = 30$$

From 30 on y-axis, draw a horizontal line to meet the curve and then draw a vertical line to meet x-axis as in above figure.

From the figure, we observe that median time to deliver a lecture is 43.5 minutes.

Lower quartile corresponds to 25th percentile i.e.

Q1 (the lower quartile) = P25 = one quarter (25% of) total frequency $= \frac{1}{4} \times 60 = 15$

From the curve, the lower quartile = 37 minutes.

Upper quartile corresponds to 75th percentile i.e.

Q3 (the upper quartile) = P75 = three quarters (75% of) total frequency $= \frac{3}{4} \times 60 = 45$

From the curve, the upper quartile = 50.5 minutes.

The inter Quartile Range = $Q3 - Q1 = 50.5 - 37 = 13.5$

Exercise 12.1

1. Construct cumulative frequency column for following frequency table and give answers.

Class interval	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60
Frequency	3	4	7	9	5	2

(i) What is the total number of items in the data?
(ii) In which class interval 8^{th} item of the data lies?
(iii) What is number of items having worth less than 21?
(iv) Which group contains highest number of items?
(v) What is the lower boundary of the last class?

2. The mass gained by a newly born baby is given below:

Age in months	1	2	3	4	5	6
No. of kg gained	4	3	2	3	2	1

Draw a cumulative frequency polygon (not ogive) to represent the data. Also find from the graph, how much he/she weighed at the age of 4 and half months?

3. The ages of refugees in Ghaza camp were recorded as below:

Ages in years	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
No. of refugees	5	16	12	10	8	4

Draw an ogive (not polygon) to represent the data and Find

a) First quartile. b) 7^{th} decile. c) 60^{th} percentile.

4. Scores of a Hifz e Quran test were grouped into the following frequency distribution.

scores	41- 50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
f	3	17	20	30	18	2

Draw a cumulative frequency table and cumulative frequency curve (ogive) and tell:

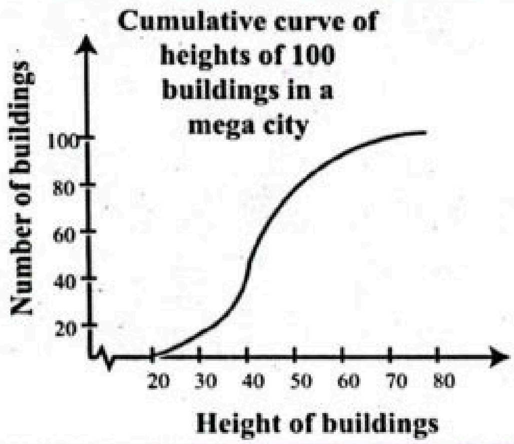
a) How many boys appeared in the test?
b) What is median score?
c) How many boys scored under median?
d) How many boys fell in first decile?
e) How many boys fell in 30^{th} percentile?
f) If excellence award is to be given to boys falling in 100^{th} percentile, how many boys got excellence award?

5. The masses measured to the nearest kg of 50 cubs are shown below:

Mass (kg)	60 – 64	65 – 69	70 – 74	75 – 79	80 – 84	85 – 89
f	2	6	12	14	10	6

(i) Construct a cumulative frequency table
(ii) Construct a cumulative frequency polygon and estimate number of cubs above 77 kg.
(iii) Construct a cumulative frequency curve and estimate number of cubs above 77 kg.
(iv) Compare the results of (ii) & (iii)

6. Study the adjoining curve showing heights (m) of 100 buildings in a megacity carefully and find:



a) Number of buildings with heights less than 50 m.
b) % age of buildings higher than 40 m.
c) Ratio of the number of buildings less than 50m and more than 50m
d) No. of Buildings falling in first quartile
e) Maximum height of buildings falling under 8^{th} decile.
f) Maximum height of buildings not falling under 30^{th} percentile.

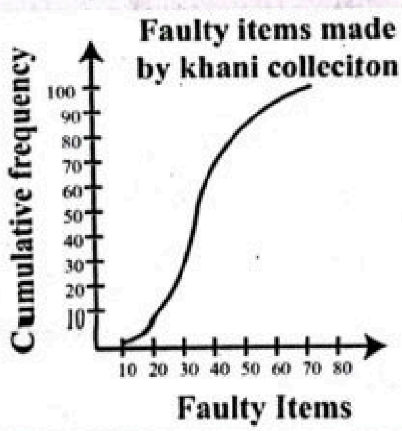
7. Mr. Rashid wants to analyze the salaries of his employees. The first quartile (Q1) is of Rs.50,000 a second quartile (Q2) of Rs.70,000, a third quartile (Q3) of Rs.90,000 and maximum and minimum salaries are of Rs.25,000 & Rs.150,000 respectively. Find range and inter quartile range of the data.

8. Khadeeja scored 85 on a test, and the teacher reported that her score was in the 75th percentile. What does this information indicate about status of Khadeeja in her class?

9. Uzair wants to price his house based on the prices of similar houses in the area. The prices have a Q1 of Rs. 20,000,000, Q2 of Rs. 25,000,000, Q3 of Rs. 30,000,000, and a maximum price of Rs. 35,000,000. What is the possible price of the house if it is in the third quartile? Also find IQR of the data.

10. The adjoining cumulative frequency curve shows the number of faulty items in a shipment of 100 items produced by Khani's Collection. Estimate

i. The median
ii. Lower quartile
iii. Upper quartile
iv. IQR
v. 7th decile
vi. 30th percentile



The Box and Whisker Plot Correlation

Box-and-Whisker Plot (or simply box plot)

A box plot typically consists of the following components:

Box: A rectangular box that represents the interquartile range (IQR). The box is divided into two parts by a vertical line, which represents the median (Q2) of the data.

Whiskers: Two horizontal lines that extend from the box to the minimum and maximum values in the dataset, excluding outliers. The whiskers represent the range of the data.

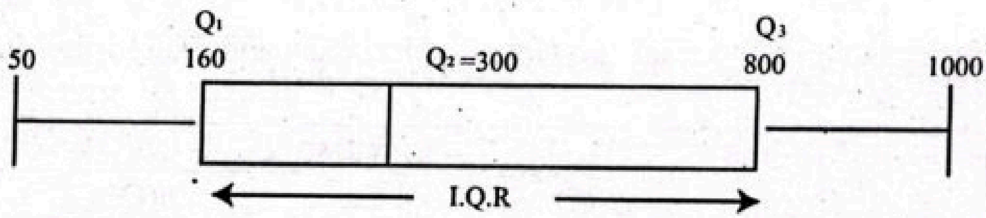
Outliers: Data points that fall beyond 1.5 times the IQR from the quartiles are considered outliers and are plotted individually.

Median: The vertical line inside the box represents the median (Q2) of the data.

Quartiles: The edges of box represent 25th percentile (Q1) and the 75th percentile (Q3).

Box plots are useful for comparing datasets, identifying outliers, visualizing distribution and summarizing data

Example: The given box and whisker plot shows the prices of stationery items, Hashir purchased yesterday.



a) What is the median price of an item?

b) what are the min & max prices of the items purchsed by Hashir?

c) What is IQR of the data? d) what %age of items is above Rs. 800?

Solution:

a) From the box plot, it is clearly seen that median price of an item is Rs. 300

b) From the box plot, it is observed that min price of any item is Rs. 50 & max price of any item is Rs. 1000

c) $IQR = Q3 - Q1 = 800 - 160 = 640$

d) As items between Q3 and Q4 are 25 % so %age of items above Rs. 800(Q3) is 25%.

Box and Whisker Plot Correlation

The box and whisker plot correlation means the patterns or relationships between the distributions of two or more variables. Box and whisker plot correlations include:

1. Positive correlation:

If the boxes and whiskers of both variables tend to move in the same direction (e.g., both increase or decrease together), it may indicate a positive correlation.

2. Negative correlation:

If the boxes and whiskers of both variables tend to move in opposite directions (e.g., one increases while the other decreases), it may indicate a negative correlation.

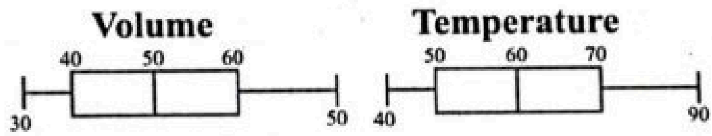
3. No correlation:

If the boxes and whiskers of both variables appear unrelated or don't show a clear pattern, it may indicate no correlation.

Key Fact:

Box and whisker plots only provide a visual representation and don't quantify the correlation.

Example shows volume of a gas is shown in first box-and-whisker plot and increase in temperature is shown in second box-and-whisker plot in the adjoining figures.



In this example, we observe that the boxes and whiskers of both variables tend to move in the same direction (both increasing together), suggesting a positive correlation.

Example:

The following datasets show 2 Variables. Hours of Exercise and Body Mass Index, (BMI) of a gym members.

variables	Min/max	Q1/Q3	median	Q3/Q1	Max/min
Hours	0(min)	2(Q1)	4	6(Q3)	8(max)
BMI	30(max)	28(Q3)	25	22(Q1)	20(min)

Draw box and whisker plots.

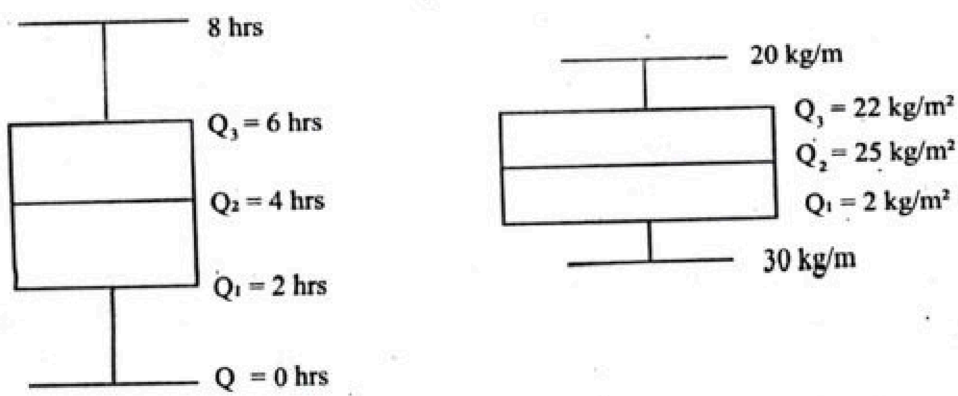
a) Find type of correlation b) Comment on type of correlation

Solution:

a) As the hours of exercise (Variable A) increase, the BMI (Variable B) tends to decrease. The boxes and whiskers of both variables move in opposite directions, indicating a negative correlation.

b) The negative correlation suggests that exercise may lead to weight loss, as higher exercise hours are associated with lower BMI values.

Box-and-whisker_plosts
Time (hrs) of exercise & BMI (kg/m)



Construction of Scatter Diagram, Lines of Best Fit and Interpretation of Data

Construction of a scatter diagram consists of:

i. Plotting the data points on a coordinate plane.

ii. Representing each data point by a mark (e.g., dot, circle, or square).

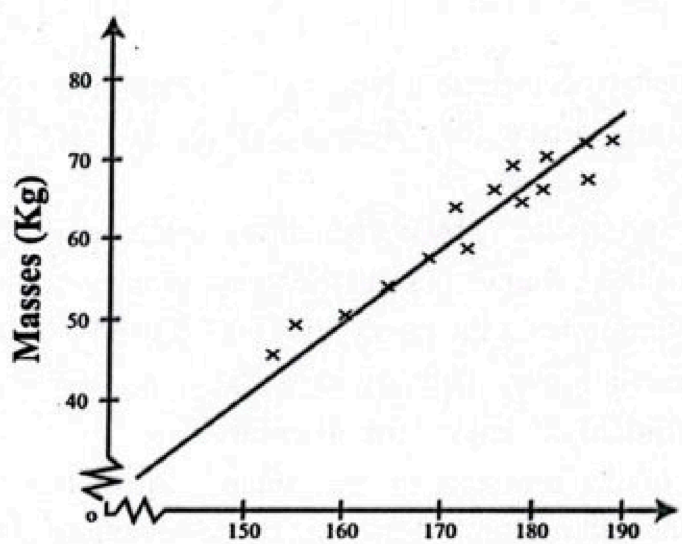
We solve an example to understand the concept.

Example: Table below shows the masses and heights of 11 boys.

Heights(cm)	167	165	184	153	180	170	179	174	160	176	157	177
Mass(kg)	60	54	69	48	64	58	68	64	51	67	46	63

a) Draw a scatter diagram for the above data.
b) What type of correlation is observed in the scatter diagram?
c) Draw a line of best fit on the scatter diagram.
d) Use this line of best fit to estimate mass of a boy whose height is 150cm.

Solution (a):



(b) From the graph, it is observed that there is a strong positive correlation between masses and heights of boys. More height, more the mass of boys.

(c) Line of best fit is drawn having almost half the data points on either sides of it.

(d) From scatter diagram, the boy with height 150cm should have a mass of 39 kg app.

Interpretation of a Scatter Diagram

Pattern: There are several types of patterns:

- Positive correlation: As x increases, y also increases.
- Negative correlation: As x increases, y decreases.
- No correlation: No apparent relationship between x and y.

Strength: Observe the strength of the relationship. A stronger relationship means the points are closer to a straight line.

Outliers: Identify data points that don't fit the pattern.

Drawing Lines of Best Fit

A line of best fit helps to predict the value of a variable y for a given x. It is a line that appears to fit the pattern. It may or may not pass through any data points however almost equal number of points should lie on either side of it. The line of best fit can be represented by $y = mx + c$.

Interpretation of a Line of Best Fit includes following attributes:

Slope (m): The rate of change of y with respect to x.

Intercept (c): The value of y when x is 0.

Prediction: Use the equation to predict the value of y for a given x.

Key Fact: A line of best fit is not always a perfect fit and it's essential to consider the strength of the relation and the presence of outliers when interpreting the results.

Measuring Correlation using Scatter Diagram

Measuring correlation using a scatter diagram involves analyzing the pattern and strength of the relationship between two variables.

It includes following steps:

1. Plot the data
2. Identify the pattern (Positive correlation, Negative correlation and No correlation)
3. Assess the strength (Strong correlation, Moderate correlation and Weak correlation)
4. Calculate the correlation coefficient r by:
   - Identifying the data points (x, y) by coordinates of each point on the scatter plot.
   - Finding the mean of the x-values ($\bar{x}$) and the mean of the y-values ($\bar{y}$).
   - Calculating the deviation from the mean for x ($x_i - \bar{x}$) and y ($y_i - \bar{y}$).
   - Multiplying the x-deviation by the y-deviation $(x_i - \bar{x})(y_i - \bar{y})$.
   - Adding up the multiplied deviations for all points.
   - Dividing the sum of the products by the square root of the product of the sum of the squared x-deviations and the sum of the squared y-deviations. i.e.

     $$r = \sum\{(x_i - \bar{x})\times(y_i - \bar{y})\} \,/\, \sqrt{(\sum(x_i - \bar{x})^2 \times \sum(y_i - \bar{y})^2)}$$

Where r is the correlation coefficient, ranging from –1 (perfect negative correlation) to 1 (perfect positive correlation).

5. Interpret the results high r value i.e. close to 1 or – 1 means Strong correlation, Low r value i.e. close to 0 means weak correlation & sign of r indicates the direction of the correlation i.e. positive or negative.

Example:

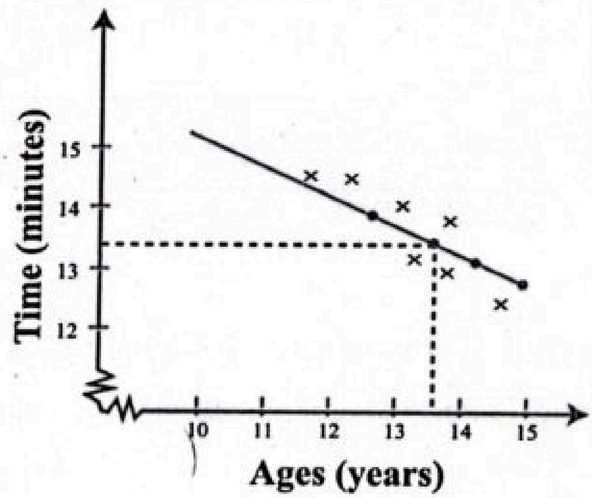
The table below shows the reading time of 10 children aged between 11 and 16 years to win the 50 pages reading activity in 10 minutes.

Age (yrs)	13.2	14.9	13.3	15.3	13.6	15.5	14.0	14.3	12.8	14.8
Time (min)	13.3	12.1	13.9	12.3	13.2	11.6	13.1	12.3	13.9	13.1

a) Draw a scatter diagram for the above data.
b) What type of correlation is observed in the scatter diagram?
c) Draw a line of best fit on the scatter diagram.
d) Use your line to estimate the time that a participant aged 14.5 years is expected to complete 50 pages reading activity.
e) Measure the correlation coefficient.
f) Is your line of best fit reliable to estimate the time taken by a child of age 1 year?

Solution:

a)



b) From the data it is observed that older the age of child corresponds to greater speed in reading activity. The line is sloping down from left to right showing negative correlation (and neg. slope as well)
c) .... drawn on the graph
d) From the graph reading for 14.5 years age, expected time to complete 50 pages reading activity is 12.7min.
e) For measuring correlation coefficient, let's calculate the means:

Age ($\bar{x}$): 14.2 Time ($\bar{y}$): 12.9

Next, we calculate the deviations, their products and squared deviations.

age	time	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
13.2	13.3	−1.0	0.4	− 0.4	1	0.16
14.9	12.1	0.7	−0.8	−0.56	0.49	0.64
13.3	13.9	−0.9	1.0	−0.9	0.81	1.0
15.3	12.3	1.1	−0.6	−0.66	1.21	0.36
13.6	13.2	−0.6	0.3	−0.18	0.36	0.09
15.5	11.6	1.3	−1.3	−1.69	1.69	1.69
14.0	13.1	−0.2	0.2	−0.04	0.04	0.04
14.3	12.3	0.1	−0.6	−0.06	0.01	0.36
12.8	13.9	−1.4	1.0	−1.4	1.96	1.0
14.8	13.1	0.6	0.2	0.12	0.36	0.04
		$\sum(x_i - \bar{x})$	$\sum(y_i - \bar{y})$	$\sum\{(x_i-\bar{x})\times(y_i-\bar{y})\}$ = −5.77	$\sum(x_i - \bar{x})^2$ =7.93	$\sum(y_i - \bar{y})^2$ = 5.38

Finally, we calculate the correlation coefficient (r):

$$r = \sum\{(x_i - \bar{x}) \times (y_i - \bar{y})\} / \sqrt{(\sum(x_i - \bar{x})^2 \times \sum(y_i - \bar{y})^2)}$$

$$r = (-5.77) / \sqrt{7.93 \times 5.38} \approx -0.88$$

The correlation coefficient (r) is approximately − 0.88, indicating a strong negative linear relationship between Age and Time. As Age increases, Time tends to decrease, and vice versa.

f) No, it is highly unreliable as it lies outside the range as there are no children with age less than 10 years.

Exercise 12.2

1. A mechanical engineer wants to analyze the relationship between the mass of a car and its fuel efficiency. Show the data by box and whisker plots.

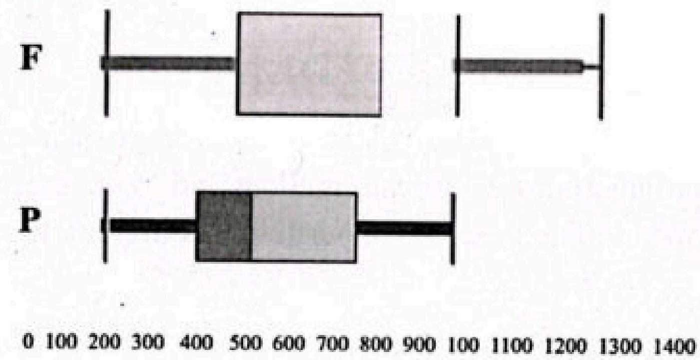
variables	Min/Max	Q1	median	Q3	Min/Max
Mass (kg)	1100	1200	1500	1700	2000
Fuel efficiency (Km/l)	24	20	14	12	10

a. Find IQR b. What is type of correlation?

c. What conclusion can be drawn from this?

2. Adjoining box & whisker plot shows wages for part time and full-time employees:

Hourly rate for Full and Part-time employees



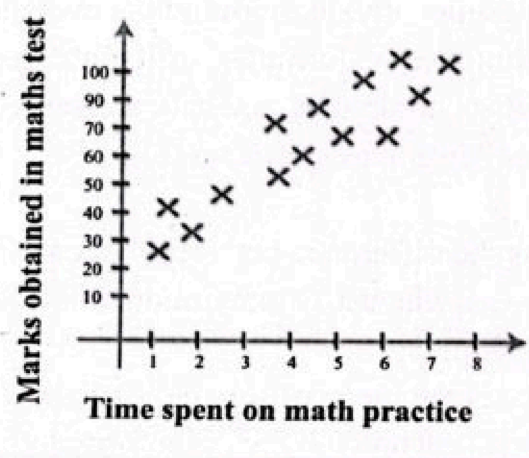
a) What is the amount of median wages paid to full time employees?
b) What are minimum and maximum wages paid to part time employees?
c) What is IQR for the dataset of fulltime employees?
d) What is IQR for the dataset of part time employees?
e) What type of correlation is observed in both datasets?

3. Maryam wants to investigate the relationship between the amount of rainfall and the number of flowers blooming in her garden. Draw the scatter diagram.

Rain (inch)	2	4	6	8	10
No. of flowers	10	22	55	70	80

a. Draw a line of best fit. b. Comment on type of correlation

4. Given scatter diagram is showing relation between practice time and marks obtained in math's test.
   a. Find type of correlation.
   b. Draw line of best fit.



5. Draw the scatter diagram for the data showing the relationship between amount of pollution (measured in Air Quality Index (AQI)) and number of sick days reported by a locality:

AQI (Pollution Level)	20	40	60	80	100	120	10	30	50	70
No. of Sick Days	5	18	25	40	60	80	3	15	20	27

Find correlation and what does this suggest?

6. A teacher wants to analyze the test scores of her students. Draw box and whisker plot to show a median score of 75, a first quartile of 60, and a third quartile of 85. Find:
   a. What percentage of students scored above 85?   b. IQR

Measures of Dispersion

The single value which can represent a data is known as average or measure of central tendency. In grade 9, different types of averages e.g. mean, median and mode are studied.
Let us find the arithmetic mean of scores of five participants of a math quiz:

V (Bilal's score) $= 5, 5, 5, 5, 5 \quad \Rightarrow \overline{V} = \frac{\Sigma V}{n} = \frac{25}{5} = 5$

W (Umer's score) $= 1, 3, 5, 8, 8 \quad \Rightarrow \overline{W} = \frac{\Sigma W}{n} = \frac{25}{5} = 5$

X (Usman's score) $= 2, 4, 5, 6, 8 \quad \Rightarrow \overline{X} = \frac{\Sigma X}{n} = \frac{25}{5} = 5$

Y (Hassan's score) $= -10, 0, 10, 12, 13 \quad \Rightarrow \overline{Y} = \frac{\Sigma Y}{n} = \frac{25}{5} = 5$

Z (Abdullah's score) $= -15, -5, 10, 15, 20 \quad \Rightarrow \overline{Z} = \frac{\Sigma Z}{n} = \frac{25}{5} = 5$

We have seen that all of the above five data have same measure of central tendency (arithmetic mean) but they vary in different ways. The values of above data are scattered away from their average in different ways. It is a confusing situation because we are unable to compare the data on the basis of their average. We can say that the average of a data is not more significant unless the scatterings of values around the average are measured.
The techniques or formulas which are used to measure, how the values of a data are scattered around its central value, are called measures of dispersion.

Types of Measures of Dispersion

(a) Range

Range is the difference between the maximum and the minimum values of the data. i.e.
$R = x_m - x_o$, where x_m = maximum value and x_o = Minimum value

(b) Variance

Variance is the mean of squared deviation taken from mean.
Usually it is denoted by S^2. The formula for variance of grouped data:

$$S^2 = \frac{\Sigma f(xi - \bar{x})^2}{\Sigma f} \qquad \text{(Proper mean formula)}$$

or $$S^2 = \frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2 \qquad \text{(Direct formula)}$$

(c) Standard Deviation

Standard deviation is a positive square root of variance. It is denoted by S and given by following formulae:

$$S = \sqrt{\frac{\Sigma f(xi - \bar{x})^2}{\Sigma f}} \quad \text{(Proper mean formula)}$$

or $$S = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} \quad \text{(Direct formula)}$$

Example: Data shows scores of Huffaz in a Hifz Quran test:

scores	41-50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
f	3	17	20	30	18	2

Find Range, variance and standard deviation of the data.

Solution: Range = $x_m - x_o$ = 100 – 41 = 59

For the variance and SD, we need to construct the following table:

scores	x	x^2	f	fx	fx^2
41-50	45.5	2070.25	3	136.5	6210.75
51-60	55.5	3080.25	17	943.5	52364.25
61-70	65.5	4290.25	20	1310	85805
71-80	75.5	5700.25	30	2265	171007.5
81-90	85.5	7310.25	18	1539	131584.5
91-100	95.5	9120.25	2	191	18240.5
			$\sum f = 90$	$\sum fx = 6385$	$\sum fx^2 = 465212.5$

Variance: $S^2 = \frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2 = \frac{465212.5}{90} - \left(\frac{6385}{90}\right)^2 = 135.9$ app.

Standard deviation: $S = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{465212.5}{90} - \left(\frac{6385}{90}\right)^2}$

$= \sqrt{135.9} = 11.6$

Coefficient of Variation (C.V.)

The percentage ratio between the standard deviation and mean of a data is called coefficient of variation i.e.

C.V. = $\frac{\text{SD}}{\text{Mean}} \times 100$

In above example, CV = $\frac{11.6}{70.5} \times 100 = 16.4$ app.

Key Fact:

C.V. is used to compare the dispersion of different data which differ in their means and units. It is also used as a criterion for consistent performance. The data having smaller C.V is more consistent than the data having larger C.V.

Comparison of 2 Data Sets using Mean and Standard Deviation

To compare two data sets using mean and standard deviation, follow these steps:

1. Calculate the arithmetic mean of each data set.
2. Calculate the standard deviation of each data set.
3. Compare the means which indicates a difference in the central tendency of the two data sets.
4. Compare the standard deviations which indicate a difference in the variability or spread of the two data sets.

Example: Following are scores of 2 girl sections orchid and teal in Math quiz.

Score chart for Teal:

scores	41-50	51-60	61-70	71-80	81-90	91-100
No. of students	3	8	6	7	4	2

Score chart for orchid:

scores	41-50	51-60	61-70	71-80	81-90	91-100
No. of students	3	6	4	9	1	7

i. Calculate the arithmetic mean of each data set.
ii. Calculate the standard deviation of each data set.
iii. Compare the means. What does this indicate?
iv. Compare the standard deviations. What does this indicate?
v. Calculate CV for both data sets.

Solution:

Teal section				
scores	x	f	fx	$f(x-\bar{x})^2$
41-49	45.0	4	180	2116
50-60	55.0	6	330	1014
61-70	65.5	6	393	37.5
71-80	75.5	8	604	450
81-90	85.5	4	342	1225
91-100	95.5	2	191	1512.5
		$\Sigma f = 30$	$\Sigma fx = 2040$	$\Sigma f(x-\bar{x})^2 = 6355$

Orchid section				
scores	y	f	fy	$f(y-\bar{y})^2$
41-49	45.0	3	135	2187
50-60	55.0	6	330	1734
61-70	65.5	4	262	169
71-80	75.5	9	679.5	110.25
81-90	85.5	1	85.5	182.25
91-100	95.5	7	668.5	3865.75
		$\Sigma f = 30$	$\Sigma fy = 2160.5$	$\Sigma f(y-\bar{y})^2 = 8248.25$

i) $\bar{x} = \dfrac{\Sigma fx}{\Sigma f} = \dfrac{2040}{30} = 68$

$\bar{y} = \dfrac{\Sigma fy}{\Sigma f} = \dfrac{2160.5}{30} = 72.0$

ii) SD teal :

$$S_{teal} = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{6355}{30}} = 14.6$$

SD orchid:

$$S_{orchid} = \sqrt{\frac{\Sigma f(y-\bar{y})^2}{\Sigma f}} = \sqrt{\frac{8248.25}{30}} = 16.6$$

Comparison:

iii. The mean score of Orchid section (72.0) is higher than the mean score of Teal section (68.0), indicating that the average performance of Orchid is better than Teal.

iv. The standard deviation of Orchid (16.6) is greater than the standard deviation of Teal (14.6), indicating that the performance of students in teal is more consistent and less spread out than in Orchid. Such performance is more reliable.

v. $CV_{teal} = \frac{14.6}{68} \times 100 = 21.47$ $\quad$ $CV_{orchid} = \frac{16.6}{72} \times 100 = 24.41$

Real Life Application of Variance and Standard Deviation

Variance and standard deviation have numerous real-life uses in various fields, e.g. Finance, Quality Control, Healthcare, Sports, Education, Environmental Science & Engineering.

Exercise 12.3

1. What is dispersion? Write names and formulae of 3 measures of dispersion.
2. The energy consumption for a month is given below for Rabee's home.

Units consumed	101-200	201-300	301-400	401-500	501-600	601-700
No. of months	2	1	3	1	3	2

Find Range and variance of the energy consumption.

3. Maria wants to analyze the water supply in her area, given in the data as shown:

Water supply				
Water supply (gallons)	501-700	701-900	901-1100	1101-1300
No. of hours	4	5	2	1

Find i. mean supply of water in the area. ii. Variance of the supply of water.
iii. SD of the supply of water. Iv. CV of the supply of water.

4. An organization wants to analyze salaries, grouped by department as shown below:

Department	Salary Range	Number of Employees
Elementary	40,000 - 60,000	10
Secondary	60,000 - 80,000	12
Higher secondary	80,000 - 100,000	8

Find the range, variance and standard deviation of the salaries across all departments. Also calculate CV of the data.

5. A teacher wants to analyze the scores of students in different sections of class 10 as shown:

Class	Scores Range	Number of Students
10 Mauve	70 - 80	15
10 teal	60 - 70	12
10 hazel	50 - 60	10

Find the variance and standard deviation of the grades of all sections .

6. Mutahhir wants to analyze the purchases made by customers, grouped by age as shown:

Age Group	Purchase worth (in Rs.)	Number of Customers
kids	0 - 500	20
juniors	500 - 1000	30
seniors	1000 - 2000	25

Find the range, variance and standard deviation of the purchases across all age groups.

7. Namra wants to analyze blood pressure of patients in OT & OPD grouped by age as shown:

Patients in OT						
BP	100-110	110-120	120-130	130-140	140-150	150-160
No. of patients	3	8	6	7	4	2

Patients in OPD						
BP	100-110	110-120	120-130	130-140	140-150	150-160
No. of patients	9	1	5	7	6	2

Find the Range, Mean, variance and standard deviation of the blood pressure of both datasets. Also find CV and comment on results.

8. Khola wants to forecast the temperature for the next week for Jhelum and Indiana. She has the history of a week for both of the cities as given:

Jhelum				
Temperature(F°)	96 - 100	101- 105	106- 110	111- 115
No. of days	2	3	1	1

Indiana				
Temperature(F°)	51 - 60	61- 70	71- 80	81- 90
No. of days	3	1	2	1

Help her to find:

i. Mean of both data sets. ii. Variance of both datasets

Also comment on the possible predictions for next week temperature for both cities. (Hint: In weather forecasting, variance is used to measure the spread or uncertainty in temperature predictions. A higher variance indicates a wider range of possible temperatures, making it more challenging to predict the exact temperature.)

Probability

Probability is a measure of the likelihood of an event occurring. It is a number between 0 and 1 both inclusive that represents the chance of an event happening. Here are some key concepts in probability:

Experiment:	An action or process that can produce a set of outcomes.
Outcome:	A specific result of an experiment.
Sample Space:	The set of all possible outcomes of an experiment.
Event:	A set of one or more outcomes of an experiment.
Probability of an Event:	A number between 0 and 1 both inclusive that represents the likelihood of an event.

Types of Probability

There are several types of probability such as:

- Theoretical Probability
  The number of favorable outcomes divided by the total number of possible outcomes.
- Experimental Probability
  sed on the results of repeated trials of an experiment.
- Conditional Probability
  The probability of an event occurring given that another event has occurred.
- Dependent Probability
  The probability of two or more events occurring dependently on each other.
- Mutually Exclusive Probability
  The probability of two or more events that cannot occur simultaneously.
- Joint Probability
  The probability of two or more events occurring together.

Probability of Combined Events

The probability of combined events depends on whether the events are independent, dependent, or mutually exclusive.

Probability of Independent Events

The probability of two or more independent events occurring is the product of their individual probabilities.

i.e. $P(A \text{ and } B) = P(A) \times P(B)$

Probability of Dependent Events

The probability of two or more dependent events occurring is the product of their individual probabilities, considering the conditional probability of each event.

i.e. $P(A \text{ and } B) = P(A) \times P(B|A)$

Probability of Mutually Exclusive Events

The probability of two or more mutually exclusive events is the sum of their individual probabilities.

i.e. P(A or B) = P(A) + P(B)

Conditional Probability

Probability of an event occurring given that another event has occurred.

i.e. P(A|B) = P(A and B) / P(B)

For example:

1. Probability of drawing two Kings (with replacement):
   P(King) × P(King) = 1/13 × 1/13= 1/169 ≈ 0.006 (or 0.6%)
   Probability of drawing two Kings (without replacement):
   P(King) × P(King) = 4/52 × 3/51 = 3/663 ≈ 0.0045 (or 0.45%)
2. Probability of rolling two 6s: P(6) × P(6) = 1/6 × 1/6 = 1/36
3. Probability of drawing a King and then a Queen:
   P(King) × P(Queen | King) = (4/52) × (4/51) = 16/2652

Remember to consider the type of events and their relationships when calculating the probability of combined events.

Probability of Combined Events using Sample Space Diagrams

A sample space diagram is a visual representation of all possible outcomes of an experiment. It can be used to calculate the probability of combined events by identifying the favorable outcomes and dividing them by the total number of possible outcomes.

To calculate the probability of combined events using a sample space diagram:

- Draw a sample space diagram showing all possible outcomes of the experiment.
- Identify the favorable outcomes that meet the conditions of the combined event.
- Count the number of favorable outcomes.
- Divide the number of favorable outcomes by the total number of possible outcomes in the sample space.

Example:

What is the probability that both coin A and coin B land heads up if both coins are flipped simultaneously?

Solution:

Favorable Outcomes: hH (both coins land heads up)

Number of Favorable Outcomes: 1

Total Number of Possible Outcomes: 4

Probability: 1/4 or 0.25

This means that the probability of both coins landing heads up is 1/4 or 25%.

Sample space diagram		Coin 1	
		H	T
Coin2	h	hH	hT
	t	tH	tT

Probability of Combined Events using Diagram

There are several types of sample space diagram, including:

Possibility Diagrams Tree Diagrams Venn Diagrams

By using an appropriate sample space diagram, we can easily visualize and calculate the probability of combined events. This method is especially helpful while dealing with complicated experiments and multiple events.

(a) Probability of Combined Events using Possibility Diagram

Possibility diagram is graphical representation of all possible outcomes of an experiment.

Example:

Raabi and Jerry are hoping on their play circles of four sectors each painted in red, green, yellow and blue colours.

Find the probability that they will jump on:

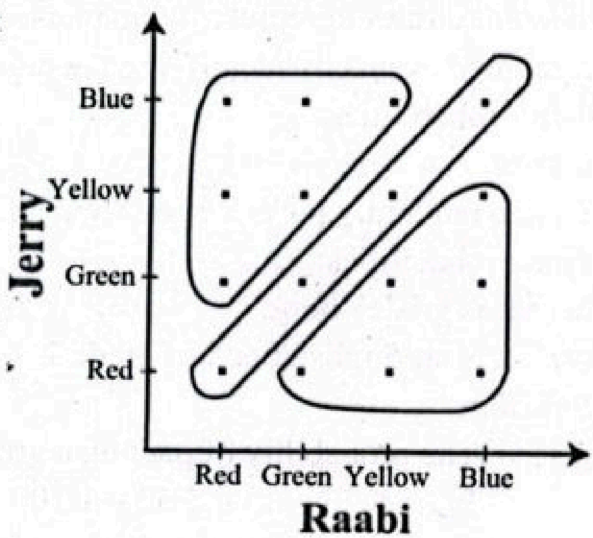
(a) At the same colour (b) At the different colours

Solution: We can associate the ordered pair (x, y), where x represents hops of Raabi and y represents hops of Jerry to show an outcome in the sample space. Here (blue, green) means Raabi lands on blue while Jerry lands on green.

So the sample space is:

S = {(red, red), (red, green), (red, yellow), (red, blue), (green, red), (green, green), (green, yellow), (green, blue), (yellow, red), (yellow, green), (yellow, yellow), (yellow, blue), (blue, red), (blue, green), (blue, yellow), (blue, blue)}

Which seems complicated and time taking method of writing sample space for bigger problems. We can express the same in more systematic and efficient way through possibility diagram.



There are 16 dots representing individual outcome, i.e. $n(S) = 16$

Let E_1 be the event that both girls jump on same colour

And E_2 be the event that they jump on different colours.

The loop on the diagonal enclosing four dots is associated With possible outcomes of E_1.

$\therefore n(E_1) = 4$ and $P(E_1) = 4/16 = 1/4$

The triangles enclosing 6+6 dots not on the diagonal are Possible outcomes of E_2.

$\therefore n(E_2) = 12$ and $P(E_2) = 12/16 = 3/4$

By using a possibility diagram, we can easily visualize and calculate the probability of combined events. This method is especially helpful when dealing with sequential experiments and conditional probability.

(b) Probability of Combined Events using Tree Diagram

Tree diagram is a graphical representation of a sequence of events or decisions, showing the possible outcomes and their probabilities in the form of:

Root Node (starting point) **Branches** (representing possible outcomes or decisions)
Leaf Nodes (representing the final outcomes)

Tree diagrams are used to visualize probability problems, identify possible outcomes, calculate probabilities, analyze decision-making processes and show conditional probability.

Example:

Maria and Haleema toss 2 coins Haleema after Maria.

(a) Draw tree diagram of the experiment. Using tree diagram, find probabilities:

(b) Both tails up (c) Maria's coin head up and Haleema's coin tail up

Solution:

(a) Tree diagram

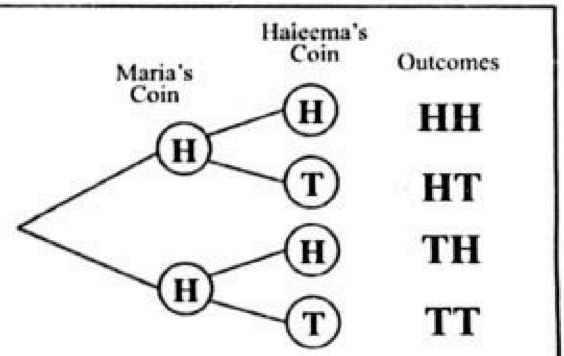
(b) Let E_1 be the probability of both heads up, then obviously there is only one such outcome among total of 4 outcomes. i.e. $E_1 = \{HH\}$

$\therefore P(E_1) = 1/4$

(c) Let E_2 be the probability of Maria's coin head up and Haleema's coin tail up, then obviously there is only one such outcome among total of 4 outcomes.

i.e. $E_2 = \{HT\}$

$\therefore P(E_2) = 1/4$



Tree Diagram

(c) Probability of Combined Events using Venn Diagram

A Venn diagram is a pictorial representation of sets and their relationships, using overlapping /disjoint circles to represent the intersection of sets. It can be used to calculate the probability of combined events by identifying the regions that represent the favorable outcomes and calculating their probabilities.

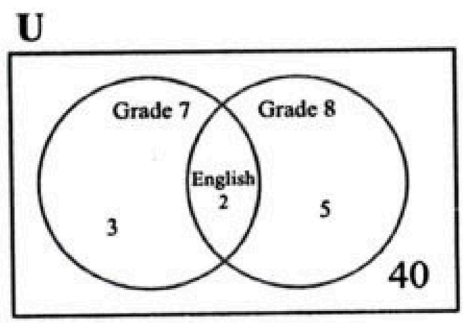
Example:

Zainab randomly picks a book from a library shelf containing 50 books, as shown in fig:

Find probabilities:

(a) It is grade7 book (b) It is an English book

(c) It is an English book of grade 7



Solution:

(a) The probability of picking a grade 7 book:
P(grade7 book) = 5/50 = 1/10

(b) The probability of picking an English book (given that it's a grade 7 book):
P(Englis book from grade 7 books) = 2/5

(c) To find the probability of picking a book that is both a grade 7 book and an English book, multiply the above probabilities:

$$\therefore P(E) = (1/10) \times (2/5) = 1/25$$

So, the probability of picking a book randomly from the 50 books that is a grade 7 English book is 1/25 or 4%.

By using a Venn diagram, we can easily visualize and calculate the probability of combined events. This method is especially helpful when dealing with multiple events and their intersections.

Probability Rules

Addition Rule

The probability of two mutually exclusive events is the sum of their individual probabilities.

Multiplication Rule

The probability of two or more events occurring independently is the product of their individual probabilities.

Complement Rule

The probability of an event not occurring is 1 minus the probability of the event occurring.

Addition Rule of Probability for Mutually Exclusive Events

The addition rule of probability states that the probability of two or more mutually exclusive events occurring is the sum of their individual probabilities.

P(A or B) = P(A) + P(B), where A and B are mutually exclusive events.

Applying addition rule of probability to problems of rolling even and odd number on a dice

The addition rule of probability can be applied to solve problems involving even and odd numbers rolled on a dice by treating the outcomes as mutually exclusive events

Example:

Hajrah rolled a fair cubical dice once. Find the probability of rolling an even or an odd number.

Solution:

On a fair cubical dice, there are 3 even numbers: 2, 4, 6 and 3 odd numbers: 1, 3, 5

Since the dice has 6 faces, the total number of possible outcomes is 6.

The probability of rolling an even number or an odd number is:

P(Even) = 3/6 = 1/2 and P(Odd) = 3/6 = 1/2

Since the events are mutually exclusive (a roll can't be both even and odd), we can add the probabilities:

P(Even or Odd) = P(Even) + P(Odd) = 1/2 + 1/2 = 1

Applying addition rule of probability to problems involving left and right hand turns

The addition rule of probability can be applied to solve problems involving taking left or right turns, by treating the outcomes as mutually exclusive events.

Example:

Mr. Bilal can make either a left turn (L) or a right turn (R) at an intersection on his journey to Tiri Mengal. The probabilities of making a left turn and a right turn are:

P(L) = 0.3 (30%) & P(R) = 0.7 (70%). Find the probability of making either a left or a right turn.

Solution:

Since making a left turn and making a right turn are mutually exclusive events (he can't make both at the same time), so

we can apply the addition rule to find the probability of making either a left turn or a right turn:

P(L or R) = P(L) + P(R) = 0.3 + 0.7 = 1

Applying addition rule of probability to problems involving tossing a coin

The addition rule of probability can be applied to solve problems involving tossing a coin by treating the outcomes as mutually exclusive events.

Example:

Sunayna tossed a fair coin with two outcomes: Heads (H) and Tails (T). The probabilities are:

P(H) = 0.5 (50% chance of getting Heads) and P(T) = 0.5 (50% chance of getting Tails)

find the probability of getting either Head or Tail.

Solution:

Since getting Head and getting Tail are mutually exclusive events (the coin can't land on both at the same time), so

we can apply the addition rule to find the probability of getting either Heads or Tails:

P(H or T) = P(H) + P(T) = 0.5 + 0.5 = 1

Applying addition rule of probability to problems involving winning or losing

The addition rule of probability can be applied to solve problems involving winning or losing by treating the outcomes as mutually exclusive events.

Example:

Affan played a game with two possible outcomes L and W such that P(W) = 0.4 (40%) and P(L) = 0.6 (60%). Find the probability of either winning or losing the game.

Solution:

Since winning and losing are mutually exclusive events (he can't both win and lose at the same time), we can apply the addition rule to find the probability of either winning or losing:

P(W or L) = P(W) + P(L) = 0.4 + 0.6 = 1

Multiplication Rule of Probability for Independent and Dependent Events

The multiplication rule of probability states that the probability of two or more events occurring is the product of their individual probabilities.

(i) For independent events:

P(A and B) = P(A) × P(B)

(ii) For dependent events:

P(A and B) = P(A) × P(B|A); where P(B|A) is the conditional probability of B given A.

Example: (Independent Events)

Haneen flipped a fair coin twice. What is the probability of getting heads both times?

Solution:

P(Heads on first flip) = ½ and P(Heads on second flip) = 1/2

Since the coin flips are independent, we can apply the multiplication rule:

∴ P (Heads on both flips) = P(Heads on first flip) × P(Heads on second flip) = 1/2 × 1/2 = 1/4

Example: (Dependent Events)

Zeemal drew 2 cards from a standard deck of cards. What is the probability of drawing two spades in a row?

Solution:

P(First card is spade) = 13/52 = 1/4

P(Second card is spade | First card is spade) = 12/51

Since the events are dependent, we can apply the multiplication rule with conditional probability

P(Two spades in a row) = P(First card is spade) × P(Second card is spade | First card is spade)

= 1/4 × 12/51 = 3/51

Multiplication Rule of Probability Involving Trade

The multiplication rule of probability can be applied to solve problems involving trade by treating the outcomes as independent events.

Example:

Mahmil made 2 trades, as given:

Trade 1: Buying Mangoes (BM) and Trade 2: Selling Berries (SB)

With the probabilities:

P(BM) = 0.6 and P(SB) = 0.7

Find the probabi'ity of both trades being successful.

Solution:

Since the trades are independent (the outcome of one trade doesn't affect the other), we can apply the multiplication rule to find the probability of both trades being successful:

P(BM and SB) = P(BM) × P(SB) = 0.6 × 0.7 = 0.42

This means there is a 42% chance of both trades being successful.

Multiplication Rule of Probability Involving Flipping a Coin

The multiplication rule of probability can be applied to solve problems involving flipping a coin by treating the outcomes as independent events.

Example:

Azkaa flipped a biased coin twice with following outcomes:

Flip 1: Heads (H) or Tails (T) and Flip 2: Heads (H) or Tails (T)

With the probabilities:

P(H) = 0.7 nad P(T) = 0.3 Find the probability of:

i. Both heads
ii. First head then tail
iii. First tail then head
iv. Both tails

Solution:

Since the coin flips are independent (the outcome of one flip doesn't affect the other), we can apply the multiplication rule to find the probability of specific outcomes:

i. P(HH) = P(H) × P(H) = 0.7 × 0.7 = 0.49 (49% chance of getting Heads both times)
ii. P(HT) = P(H) × P(T) = 0.7 × 0.3 = 0.21 (21% chance of getting Heads then Tails)
iii. P(TH) = P(T) × P(H) = 0.3 × 0.7 = 0.21 (21% chance of getting Tails then Heads)
iv. P(TT) = P(T) × P(T) = 0.3 × 0.3 = 0.09 (9 % chance of getting Tails both times)

Multiplication Rule of Probability for Drawing Cards with Replacement

The multiplication rule of probability can be applied to solve problems involving drawing 2 cards with replacement by treating the outcomes as independent events.

Example:

Romaisa draws 2 cards from standard deck with replacement as given :

First draw → Any card (52 possibilities)

Second draw → Any card (52 possibilities, since the first card is replaced)

Find the probabilities of:

i. Both black cards ii. Both hearts

iii. First spade then heart iv. First Queen of Spades then King of Hearts

Solution:

Since the events are independent (the outcome of the second draw does not depend on the first draw), we can apply the multiplication rule to find the probability of specific outcomes:

i. P(Both black cards) = P(black) × P(black) = 26/52 × 26/52 = 1/4

ii. P(Both hearts) = P(heart) × P(heart) = 13/52 × 13/52 = 1/16

iii. P(spade then heart) = P(spade) × P(heart) = 13/52 × 13/52 = 1/16

iv. P(Queen of Spades then King of Hearts)

= P(Queen of Spades) × P(King of Hearts)

= 1/52 × 1/52 = 1/ 1704

Multiplication Rule of Probability for Drawing Cards without Replacement

The multiplication rule of probability can be applied to solve problems involving drawing 2 cards without replacement by treating the outcomes as dependent events.

Example:

Sundas draws 2 cards one after the other from a standard pack of playing cards, such that:

First draw → Any card (52 possibilities)

Second draw → Any card (51 possibilities, since one card has already been drawn)

then find the probabilities of:

i. Both black cards ii. Both hearts

iii. First spade then heart iv. First Queen of Spades then King of Hearts

Solution:

Since the events are dependent we can apply the multiplication rule to find the probability :

i. P(Both black cards) = P(black) × P(black/black) = 26/52 × 25/51 = 25/102

ii. P(Both hearts) = P(heart) × P(heart/heart) = 13/52 ×12/51 = 3/51

iii. P(spade then heart) = P(spade) × P(heart/spade) = 13/52 × 13/51 = 13/204

iv. P(first Queen of Spades then King of Hearts)

= P(Queen of Spades) × P(King of Hearts/ Queen of Spades) = 1/52 × 1/51 = 1/ 2652

Exercise 12.4

1. Musaab picked a card randomly from a box containing cards bearing numbers 1,2 and 3. He then replaced and picked another after well shuffling. He found product of two numbers he got in 2 trials. Draw the possibility diagram of this experiment and find probability of getting an even product.

2. Ashas flipped a coin thrice. Draw tree diagram to find the probability of getting three Tails?
3. A survey of 100 students shows that 40 like pizza, 45 like burgers. Draw venn diagram and find probability of students like either pizza or burgers, assuming none like both?
4. What is the probability of a patient testing positive for a disease with either Test A or Test B, given that the probability of testing positive with Test A is 0.3 and the probability of testing positive with Test B is 0.4, and the tests are independent?
5. What is the probability of getting either Job A or Job B, given that the probability of getting Job A is 0.5 and the probability of getting Job B is 0.4 and the offers are independent?
6. What is the probability of taking either Volvo bus or Bedford bus to work, given that the probability of taking Volvo bus is 1/2 and the probability of taking Bedford bus is 1/3, and the buses are to be chosen independently?
7. What is the probability of either Team Sudaisee or Team Huzaifee winning the championship, given that the probability of Team Sudaisee winning is 12/27 and the probability of Huzaifee team winning is 13/27, and the teams play independently?
8. HJ produces party abayaas. In the products, probability of colour damage is 0.1 and the probability of bad stitching is 0.2. Find the probability of a product having either colour damage or bad stitching.
9. Fakeha is tested for a disease with Test A and Test B and the probability of testing positive with A is 0.9 and the probability of testing positive with B is 0.7. Find the probability of Fakehaa for testing positive with both Test A and Test B, when the tests are independent?

10. Cocobakes produces festive cakes. The quality control department performs 2 quality checks A and B on an item. Find the probability of a product passing both Quality Control Test A and Quality Control Test B, given that the probability of passing Test A is 0.9 and the probability of passing Test B is 0.8, and the tests are independent?

I have Learnt

- A table consisting of values of a data along with their frequencies is called a frequency table.
- In a group data, the table consisting of class intervals and their respective frequencies is called frequency table or a frequency distribution.
- The representation of data by a polygon in which class marks are plotted along x-axis and frequencies along y-axis is called a frequency polygon.
- In a group data, number of values less than or within the limits of a class is called cumulative frequency of the respective class.
- A polygon made by joining the points whose x-coordinates are class marks and y-coordinates are their cumulative frequencies is called a cumulative frequency polygon.

- An ogive is a smooth curve, while a cumulative frequency polygon is a polygon-shaped graph made by straight segments.
- The three values which divide the whole data into four equal parts are called quartiles.
- Deciles divide a dataset or a distribution into 10 equal parts, each representing 10% of the data.
- Percentiles divide a dataset or a distribution into 100 equal parts, each representing 1% of the data.
- IQR = Q3 – Q1, where IQR represents the spread of the data within the middle half of the distribution.
- A box-and-whisker plot is a graphical representation of a dataset that displays the distribution of values through 5 data points: min, max, Q_1, Q_2, Q_3
- A single value which can represent the whole data is called measure of central tendency, measure of location or an average.
- The middle most value of an arranged data is called median average or simply median.
- The measures which tell us how the values of a data are scattered around their average, are called measures of dispersion.
- Range is the difference of largest and smallest values of the data.
- Variance is the ratio between sum of squares of deviations from mean and the number of values.
- Standard deviation is the positive square root of variance.
- Probability is a measure of the likelihood of an event occurring.
- The percentage ratio between the standard deviation and mean of a data is called coefficient of variation i.e. $\text{C.V.} = \frac{\text{SD}}{\text{Mean}} \times 100$

MISCELLANEOUS EXERCISE-12

1. Encircle the correct option in the following.

i. Which of the following is not measure of dispersion?
(a) variance (b) standard deviation (c) range (d) arithematic mean

ii. What is median of the data 4, 3, 0, 2, 1?
(a) 0 (b) 2 (c) 3 (d) 4

iii. Which of the following is measure of dispersion?
(a) arithmetic mean (b) range (c) quartile (d) median

iv. Which of the following is used to compare the consistency of two data?
(a) arithmetic mean (b) standard deviation (c) C.V. (d) G.M.

v. What is sum of deviations taken from arithmetic mean?
(a) $\sum f$ (b) $\sum x$ (c) n (d) 0

vi. What is variance of five values 4, 4, 4, 4, 4?
(a) does not exist (b) 4 (c) 5 (d) 0

vii. Which of the following divides the data into four equal parts?
(a) decile (b) quartile (c) percentile (d) median

viii. Which of the following divides the data into ten equal parts?
(a) decile (b) quartile (c) percentile (d) median

ix. Which of the following divides the data into two equal parts?
(a) decile (b) quartile (c) percentile (d) median

x. Which of the following divides the data into hundred equal parts?
(a) decile (b) quartile (c) percentile (d) median

xi. line of best fit is given by the equation:
(a) $y = x^2$ (b) $y = mx^2 + c$ (c) $y = mx + c$ (d) $x = y^2$

xii. CV is given by the formula:
(a) $\frac{SD}{100} \times$ mean (b) $\frac{SD}{range} \times 100$ (c) $\frac{mean}{SD} \times 100$ (d) $\frac{100}{mean} \times SD$

xiii. Probability of drawing a king in one draw is -------- than probability of drawing 2 kings in 2 draws.
(a) greater (b) smaller (c) equal (d) no relation

xiv. Probability of drawing 2 aces in 2 draws with replacement is -------- than probability of drawing 2 jacks in 2 draws without replacement.
(a) greater (b) smaller (c) equal (d) no relation

xv. Probability of rolling a standard cubical dice for an even number in 2 attempts is -------- than probability of rolling an odd numbers in 2 attempts.
(a) greater (b) smaller (c) equal (d) no relation

2. For the following data, draw cumulative frequency polygon, then estimate 3 quartiles graphically. Also find variance and stand deviation of the data.

x	5	10	15	20	25	30	35	40
f	2	4	6	8	10	7	5	3

3. Aatika wants to examine the relationship between the time of exercise of patients and their blood pressure. Show the data by box and whisker plots .

variables	Min/Max	Q1	median	Q3	Min/Max
Time (min)	0	15	30	45	60
Blood Pressure	140/90	130/85	120/80	120/75	115/70

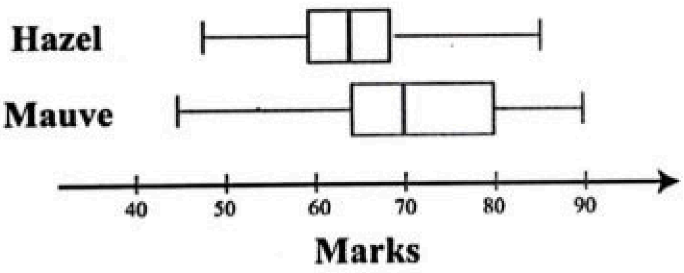
a. Find IQR of both data sets
b. What is type of correlation?
b. What conclusion can be drawn from this?

4. A sports coach wants to analyze the timed performance of athletes in an event. Draw the box and whisker plot to show a median time of 20 seconds, Q1 of 10 seconds and Q3 of 30 seconds. Find what percentage of athletes could perform in first 10 seconds? Also find IQR?

5. Given box and whisker plots show the marks of 4 boys of mauve section and 4 boys of hazel section in math test.

a) Find Median scores of both sections

b) Find IQR of scores of both sections



6. Bareera and Hadiya draw cookie packs, Hadiya after Bareera from a bag of 2 choco cookies and 3 lemon cookies. Find the probability of drawing both lemon cookies

7. A company has two machines, A & B, which produce 60% & 40% of the total products, respectively. Machine A produces 20% defective products & Machine B produces 15% defective products. What is the probability that a randomly selected product is defective?

> **Maths Play Ground:**
>
> Help students to make colour wheel/ number wheel and let them perform trials of moving the wheel to record the outcomes. Then ask them different related questions of probability.

ANSWERS

UNIT 1

Exercise 1.1

1. (i) $\sqrt{3}i$ (ii) $12i$ (iii) $\frac{2}{3}i$ (iv) $-2\sqrt{5}i$ (v) $4-2\sqrt{15}\,i$ (vi) -4
2. (i) 9+4i (ii) 2 (iii) −9+5i (iv) − 1 (v) −6+8i (vi) −10−11i
(vii) −16−16i (viii) 5+12i (ix) 45 (x) 16+30i
(xi) $1+\sqrt{6}+(\sqrt{2}-\sqrt{3})i$ (xii) 3
3. (i) i (ii) i (iii) 1 (iv) $-i$ (v) −27i (vii) 16
4. (i) 8+0=8 (ii) $-17+i$ (iii) 1 −13i (iv) $22i$ (v) 2i (vi) −2
5. (i) $\frac{12}{17}+\frac{3}{17}i$ (ii) $-\frac{15}{16}+\frac{18}{61}i$(iii) $\frac{2}{7}-\frac{3\sqrt{5}}{7}i$ (iv) $\frac{7}{5}-\frac{2}{5}i$ (v) $-\frac{9}{41}+\frac{40}{41}i$
(vi) $\frac{8}{5}+\frac{1}{5}i$ (vii) $\frac{7}{5}-\frac{9}{5}i$ (viii) $\frac{a^2-b^2}{a^2+b^2}+\frac{2ab}{a^2+b^2}i$ (ix) $-\frac{1}{2}+\frac{1}{2}i$ (x) 4

Exercise 1.2

1. (a) $4-5i$ (b) $3+3i$ (c) $-5+5i$ (d) $-4i$
3. (a)$\frac{1}{2}-\frac{1}{2}i$ (b) $\frac{7}{58}+\frac{3}{58}i$ (c) $\frac{5}{122}+\frac{3}{61}i$ (d) $\frac{5}{2}-\frac{1}{2}i$ (e) 3+2i (f) $\frac{a}{a^2+b^2}+\frac{b}{a^2+b^2}i$
4. (a) 16 (b) 2 (c) 49 (d) 40 (e) 181 (f) 137
6. Do yourself 5. b. (i) $\sqrt{10}$ (ii) $\sqrt{34}$ (iii) 1
7. (a) Re=−1 , Im = $-2\sqrt{6}$ (b) Re=1 , Im =$2\sqrt{2}$ (c) Re=$\frac{41}{10}$, Im=$-\frac{3}{10}$ (d) Re=$-\frac{1}{2}$, Im= $\frac{\sqrt{3}}{2}$
(e) Re=−1 , Im =1 (f) Re = $\frac{1}{2}$, Im = 0 (g) Re= $\frac{-8}{25}$, Im = $-\frac{6}{25}$

Exercise 1.3

1.$\{\pm\sqrt{7}i\}$ 2. $\{\pm 3i\}$ 3. $\{\pm 10i\}$ 4. *yes* 5. *yes* 6. No 7. Yes 8.$(x-4i)(x+4i)$
9. $(a-bi)(a+bi)$ 10. $(x-5yi)(x+5yi)$ 11.W=1−i , Z=4−i
12. W=3−2i , Z=1 13. (a) (i) $I=9+\frac{37}{4}J$ (ii) $I=\frac{-37}{5}+\frac{2}{5}J$ (b) (i) Z= $-\frac{5}{2}-10J$
(ii) $Z=-\frac{770}{73}-\frac{380}{73}J$ (c) $\frac{225}{41}+\frac{200}{41}i$

Miscellaneous Exercise-1

1. (i) (c) (ii) (b) (iii) (a) (iv) (c) (v) (b) (vi) (c)
(vii) (d) (viii) (b) (ix) (c) (x) (d)
2. a $-10+9i$ (b) $1-3i$ 3. (a) $2x-3y+(3x+2y)i$ (b) $-45i$
4. (a) $3\sqrt{2}-\sqrt{7}i$ (b) $\sqrt{2}i$ 5. (a) $\frac{5}{4}$ (b) 245 6. (a) $\frac{-4}{5}-\frac{-3}{5}i$ (b) $\frac{3\sqrt{5}}{5}-\frac{\sqrt{5}i}{5}$
7. a. $2(x-3i)(x+3i)$ b. $-(x-5i)(x+5i)$
8. a. $3(x-\sqrt{5}i)(x+\sqrt{5}i)$ b. $6(x-\sqrt{6}i)(x+\sqrt{6}i)$

UNIT 2

Exercise 2.1

1. (i) $x^2-x-11=0$ (ii) $3x^2+26x-2=0$ (iii) $x^2-2x=0$
2. (i) $\{1,4\}$ (ii) $\{1\}$ (iii) $\{-1,8\}$ (iv) $\{3,5\}$ (v) $\{3,-10\}$
3. (i) $\{4,-8\}$ (ii) $\{0,-8\}$ (iii) $\{-3\pm 3\sqrt{2}\}$ (iv) $\left\{\frac{-2\sqrt{3}\pm 2}{\sqrt{3}}\right\}$ (v) $\left\{\frac{-1\pm\sqrt{-3}}{2}\right\}$ (vi) $\left\{\frac{5}{2},-\frac{1}{2}\right\}$

4. (i) $\{3,-3\}$ (ii) $\left\{\frac{-5\pm\sqrt{17}}{4}\right\}$ (iii) $\{-1, 24\}$ (iv) $\{0,2\}$ (v) $\left\{1,-\frac{3}{2}\right\}$ (vi) $\left\{\frac{-11\pm\sqrt{165}}{2}\right\}$

5. $\{-3, -3\}$ 6. $-1\pm\sqrt{3}i$ 8. Only two solutions, degree is 2

9. The width of the strips added is approximately 16.85 meters.
The new dimensions of the playground are approximately 76.85 meters by 46.85 meters.

10. It takes approximately 5.65 seconds for the car to travel 145 meters.

Exercise 2.2

1. (i) $ay^2+by+c=0$ (ii) $9y^2-3y+7=0$ (iii) $y^2+1=0$ (iv) $z^2-4z+3=0$
(v) $3y^2+8y+5=0$ (vi) $3z^2+7z+11=0$ (vii) $az^2+bz+c=0$
(viii) $8y^2-7y-1=0$ (ix) $y^2-16y-42=0$ (x) $4z^2+19z+13=0$

2. (i) $\{\pm2,\pm4\}$ (ii) $\{\pm3,\pm5i\}$ (iii) $\{-32,-243\}$ (iv) $\left\{\pm1,\pm\frac{2}{\sqrt{3}}\right\}$ (v) $\left\{0,-\frac{2}{5}\right\}$
(vi) $\left\{\pm\sqrt{\frac{14}{5}},\pm\sqrt{\frac{-2}{5}}\right\}$ (vii) $\left\{-1,2,\frac{1}{2}\right\}$ (viii) $\left\{3,-4,\frac{1}{3},-\frac{1}{4}\right\}$
(ix) $\left\{\frac{1\pm\sqrt{5}}{2},\frac{-3\pm\sqrt{73}}{8}\right\}$ (x) $\{1,5\}$ (xi) $\{1,2\}$ (xii) $\{2,3\}$
(xiii) $\left\{1,-8,\frac{-7\pm\sqrt{97}}{2}\right\}$ (xiv) $\{2,-4,-1\pm2i\}$ (xv) $\{-1\pm\sqrt{17},-1\pm\sqrt{41}\}$

3. (i) $\left\{\pm1,\pm\frac{1}{\sqrt{2}}\right\}$ (ii) $\left\{1,-\frac{1}{2}\right\}$ (iii) $\{\pm1\}$ (iv) $\{-1\}$

4. do yourself 5. Take $z=y-4$, The solution $y=10, y=5$ 6. $\left\{5, \frac{-5+5\sqrt{3}i}{2}, \frac{-5-5\sqrt{3}i}{2}\right\}$

Exercise 2.3

1. i) 144 ii) 49 iii) −28 iv) 841
2. i) Rational and unequal ii) Rational and unequal
iii) Rational and unequal iv) Imaginary
v) Irrational and unequal
3. k = ±24 4. k = ±6
6. i) m = 9 ii) m = -1/4 iii) m = -1, -11/3

Exercise 2.4

1. i) S= 5, P = 2 ii) S = -3/2, P = 1/2 iii) S = 2/5, P = 2/5
iv) S = -2, P = 9/4 v) S = 17/16, P = -3/4 vi) S = 25.67, P =6.0
2. i) $3x^2+5x-8=0$ ii) $x^2-3\sqrt{3}\,x+6=0$ iii) $x^2-4x+1=0$
iv) $x^2+25=0$ v) $x^2-14x+53=0$
3. i) -5/4 ii) -20/9 iii) 16/3 iv) 1/2
v) -5/3 vi) 8/9 vii) -80/27 viii) 25/3
4. i) $49x^2-2x+49=0$ ii) $7x^2+10x+7=0$ iii) $49x^2-2x+49=0$
iv) $49x^2+96=0$ v) $7x^2+6x+15=0$ vi) $49x^2-2x+49=0$
vii) $49x^2+143x+100=2$ viii) $49x^2+100x+100=0$ ix) $x^2+68x/49+359/343=0$

5. $x^2-60x+864=0$ 6. $2x^2+38x-7=0$ 7. $k=\pm\sqrt{5}$ 8. k= 4

Exercise 2.5

1. S.S. = {(-1, 3), (9/5, -13/5)} 2. S.S. = {(1, 1), (-5, -8)} 3. S.S. = {(2, -9), (-3, 6)}
4. S.S. = {(1/2, 7/2)} 5. S.S. = {$(9/\sqrt{2}, 3/\sqrt{2}), (-9/\sqrt{2}, -3/\sqrt{2}), (3\sqrt{5/2}, 3\sqrt{5/2}), (-3\sqrt{5/2}, -3\sqrt{5/2})$}

6. S.S. $=\left\{(\pm\sqrt{6},\pm\sqrt{6}),\left(\sqrt{\frac{5}{2}},-6\sqrt{\frac{5}{2}}\right),\left(-\sqrt{\frac{5}{2}},6\sqrt{\frac{5}{2}}\right)\right\}$ 7. S.S.= (-1/2, -4), (2, 7/2)

8. S.S. = {(-1, -1), (2, -4)} 9. S.S = {$\pm 1/\sqrt{2}, \pm 3/\sqrt{2}$} 10. S.S = {$(\pm 3, \pm\sqrt{2})$}

Exercise 2.6

1. 10, 12 2. 20 3. 6, 5 4. (3,6) (6,3) 5. 5, 6 6. 3,5 7. Son's age = 13 , Father's age = 37 8. 83
9. (24, -4)(15, 5) 10. 4, 6 11. 625, 400 12. 8, 9 13. Appro 4.1 seconds 14. 20.82 Km/h
15. 11 cm long and 10 cm wide

Miscellaneous Exercise-2

1. (i) (c) (ii) (d) (iii) (b) (iv) (b) (v) (a) (vi) (a)
(vii) (d) (viii) (b) (ix) (c) (x) (b) (xi) (c) (xii) (d) (xiii) (a)

2. $\left\{1,-\frac{1}{2},\frac{-1\pm\sqrt{-3}}{2},\frac{-1\pm\sqrt{-3}}{4}\right\}$ 3. m = 3/2, x = -3 4. $\frac{c+3(b+3a)}{a}$ 5. $a=\pm 2\sqrt{2}$, b = -2 6. $L = x + 1$, then $W = x + 3$

Exercise 3.1

1. (a) 1, 2, 1-by-2 (b) 2, 2, 2-by-2 (c) 1, 1, 1-by-1 (d) 1, 1, 1-by-1
2. (a) not equal (b) not equal (c) not equal (d) equal 3. $R=\begin{bmatrix}7 & 11 & 10\end{bmatrix}, C=\begin{bmatrix}7\\11\\10\end{bmatrix}$

4. (a) scalar matrix (b) square matrix (c) unit matrix (d) unit matrix (e) scalar matrix
5. (a) symmetric matrix (b) skew symmetric matrix (c) skew symmetric matrix
(d) skew symmetric matrix (e) symmetric matrix

Exercise 3.2

1. (a) $x = 2, y = 9$ (b) impossible, corresponding elements are not equal . (c) impossible; orders are not equal
(d) $x=-4, y=-2, z=-8$ (e) $x=3, y=3, z=6$ (f) impossible; orders are not equal .
(g) $x=6, y=5$ (h) $x=2, y=5, z=10$ (i) $x=4, y=2$ (j) $x=-10, y=-6, z=-4$

2. $-R=\begin{bmatrix}-5 & 0 & -3\\-7 & 9 & 1\\8 & -5 & -6\end{bmatrix}, -S=\begin{bmatrix}5 & -2\\-3 & 6\\9 & -4\end{bmatrix}, -T=\begin{bmatrix}-5 & 6 & -1\end{bmatrix}$ 3. (i) $\begin{bmatrix}20 & 12\\-8 & 11\end{bmatrix}$ (ii) $\begin{bmatrix}0 & -1\\2 & 3\end{bmatrix}$

4. (i) yes (ii) no (iii) no (iv) no 8. (i) $Z=\begin{bmatrix}1\\7\end{bmatrix}$ (ii) $Z=\begin{bmatrix}5\\-2\end{bmatrix}$

9. (a) (i) impossible (ii) 1-by-2 (iii) 2-by-1 (iv) impossible (v) impossible (vi) impossible
(vii) 2-by-1 (viii) impossible (ix) 1-by-2 (x) impossible (xi) 1-by-1 (xii) 1-by-1
(xiii) 2-by-2 (xiv) 2-by-2 (xv) 2-by-1

(b) (ii) $\begin{bmatrix}8 & 0\end{bmatrix}$ (iii) $\begin{bmatrix}8\\0\end{bmatrix}$ (vii) $\begin{bmatrix}5\\15\end{bmatrix}$ (ix) $\begin{bmatrix}45 & 15\end{bmatrix}$ (xi) $\begin{bmatrix}50\end{bmatrix}$ (xii) $\begin{bmatrix}20\end{bmatrix}$ (xiii) $\begin{bmatrix}10 & 20\\15 & 30\end{bmatrix}$ (xiv) $\begin{bmatrix}2 & 5\\4 & 3\end{bmatrix}$ (xv) $\begin{bmatrix}19\\-2\end{bmatrix}$

10. (i) $AB=\begin{bmatrix}2 & 1\\4 & -7\end{bmatrix}, BA=\begin{bmatrix}4 & 3\\-6 & -9\end{bmatrix}, AB\neq BA$ (ii) $AC=CA=\begin{bmatrix}3 & 0\\0 & 3\end{bmatrix}$

(vi) $A^2=\begin{bmatrix}1 & 0\\8 & 9\end{bmatrix}, B^2=\begin{bmatrix}4 & -1\\0 & 9\end{bmatrix}, A+B=\begin{bmatrix}3 & 1\\2 & 0\end{bmatrix}, A-B=\begin{bmatrix}-1 & -1\\2 & 6\end{bmatrix},$

$(A+B)(A-B)=\begin{bmatrix}-1 & 3\\-2 & -2\end{bmatrix}, A^2-B^2=\begin{bmatrix}-3 & 1\\8 & 0\end{bmatrix}$ (vii) no (viii) no (ix) no (x) yes

Exercise 3.3

1. (a) A is non singular, B is non singular, C is singular, D is non singular and E is singular. (b) $k = 1$ (c) $x = \frac{7}{3}$

2. (a) $R^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 3 & 2 \end{bmatrix}$, but S^{-1}, T^{-1} & U^{-1} are not possible.

4. Do yourself. 5. (i) $\begin{matrix} 4x + 2y = 6 \\ 3x + y = 4 \end{matrix}$ (ii) $\begin{matrix} 5x = 10 \\ 4y = 20 \end{matrix}$ (iii) Not possible (iv) $5x + 3y = 2$

6. (a)
(i) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 1 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 90 \\ 10 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(iv) $\begin{bmatrix} \frac{5}{2} & -3 \\ -4 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (b) $K = 8$ 7. (a) (i) $\{(-3, 2)\}$ (ii) $\left\{\left(0, \frac{3}{4}\right)\right\}$ (iii) impossible

(iv) impossible (b) (i) $\{(4, -1)\}$ (ii) $\left\{\left(\frac{5}{3}, \frac{-1}{2}\right)\right\}$ (iii) $\{(20, 10)\}$ (iv) impossible

Exercise 3.4

1. $50^\circ, 40^\circ$ 2. literature = 5, science=10 3. Note book =Rs.100, Book=Rs.150
4. Sundas =15 years, Zenab =7 years 5. The first truck (with a capacity of 10 tons) made 7 round trips. The second truck (with a capacity of 12 tons) made 13 round trips. 6. 8 kg of the metal with 55% and 12 kg of the metal with 80% aluminum content. 7. 150 pounds of soybean and 200 pounds of cornmeal
8. Length=76 meters Width=19 meters

Miscellaneous Exercise-3

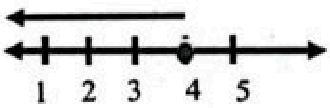
1. (i) (b) (ii) (d) (iii) (b) (iv) (d) (v) (d) (vi) (d)
(vii) (a) (viii) (a) (ix) (d) (x) (c) (xi) (c) (xii) (b)
(xiii) (d) (xiv) (c) (xv) (b) (xvi) (d)

3. impossible, matrix of coefficient is singular. 4. impossible, matrix of coefficient is singular.
5. impossible, matrix of coefficient is singular.

UNIT 4

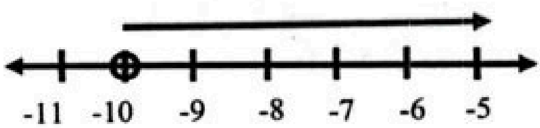
Exercise 4.1

1. $x \leq 4$

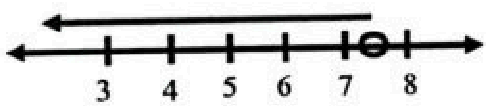


2. No solution

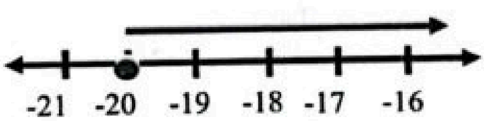
3. $x > -10$



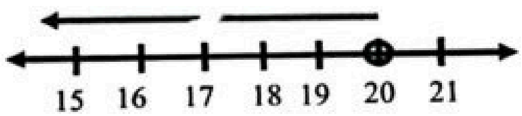
4. $x < \frac{55}{7}$



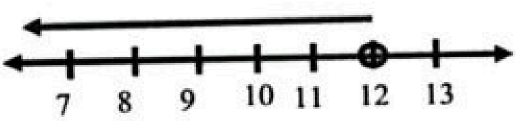
5. $x \geq -20$



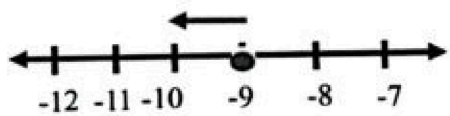
6. $x < 20$



7. $3x + 4 < 40, x < 12$



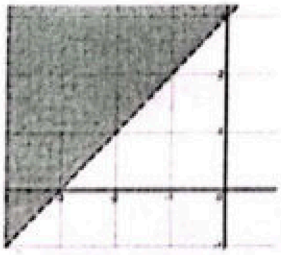
8. $2(x + 8) \leq -2, x \leq -9$

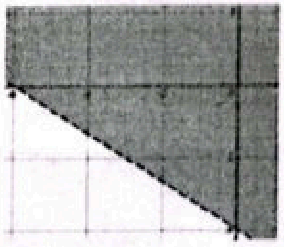


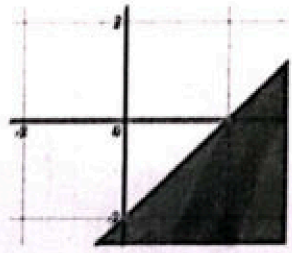
9. $x > 7$ 10. $x \leq \frac{9}{2}$ 11. $x < -1$ or $x > 3$ 12. $-1 < x < 9$
13. $x < -9$ or $x > 10$ 14. $-7 < x < 3$ 15. $x \neq 4$ 16. $4 - 15 < x < 15 + 4$
17 $10 \leq W \leq 24$ 18 There are no solutions except -1 19. $0 \leq x \leq 24.54$ 20. $L: x \leq 10m, W: 2x \leq 20m$

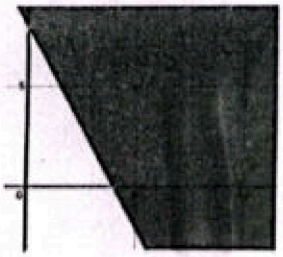
Exercise 4.2

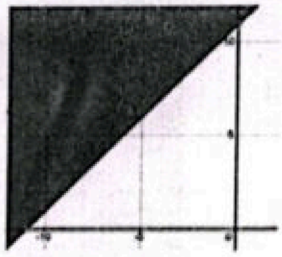
1. No 2. Yes 3. Yes 4. No 5. Yes 6. No

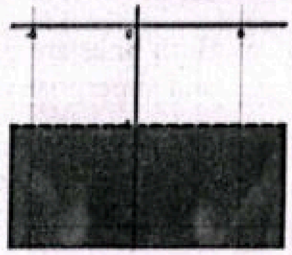
7. 

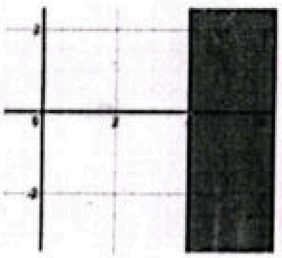
8. 

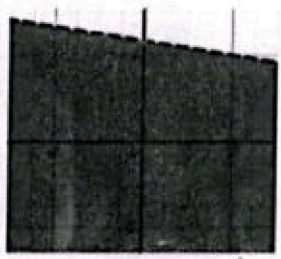
9. 

10. 

11. 

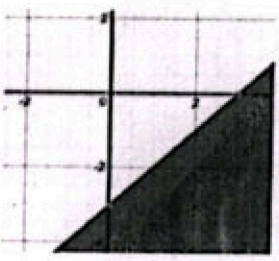
12. 

13. 

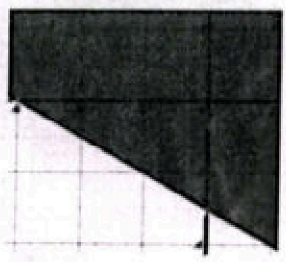
14. 

15. Do Yourself.

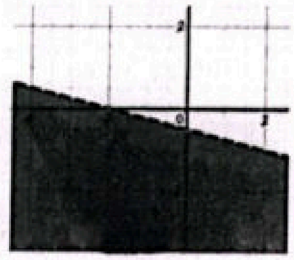
16. $x - 3 \geq y$



17. $-2y \leq x + 6$



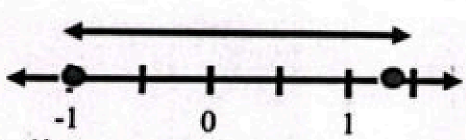
18. $x + 4y < -2$



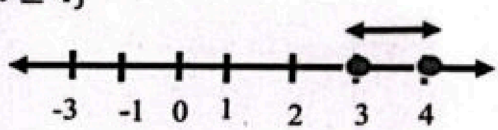
Exercise 4.3

1. $\{-8, 14\}$ 2. $\{-\frac{39}{2}, \frac{21}{2}\}$ 3. No solution 4. $\{4\}$ 5. $\{1\}$ 6. $\{-\frac{38}{5}, \frac{46}{5}\}$ 7. Absolute must be positive 8. Absolute Deviation= ±0.5 psi, Minimum Pressure=8 psi−0.5 psi=7.5 psi Maximum Pressure=8 psi+ 0.5 psi=8.5 psi

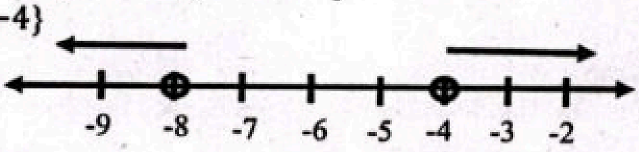
9. $\{-1 \leq x \leq \frac{11}{9}\}$



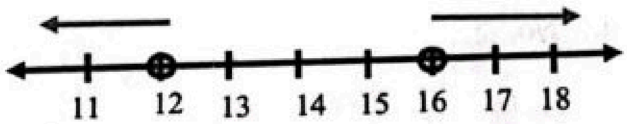
10. $\{3 \leq x \leq 4\}$



11. $\{x < -8 \text{ or } x > -4\}$

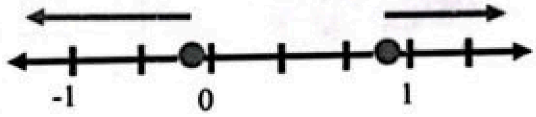


12. $\{x < 12 \text{ or } x > 16\}$,



13. $\{-\frac{8}{3} \leq x \leq 4\}$

14. $\{ x \leq -\frac{2}{15} \text{ or } x \geq \frac{14}{15}\}$

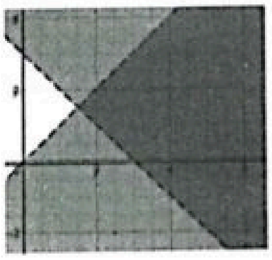


15. $\{470 \leq W \leq 530 \}$

Exercise 4.4

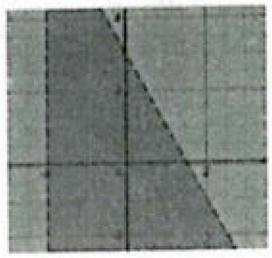
2. *a.* (0,0) and (−3,1) are solutions   *b.* (3,2) is solution

3. a.

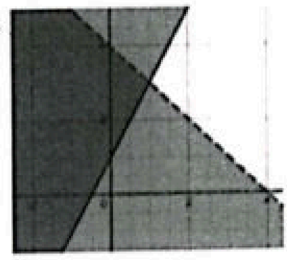


c. Both lines are parallel and superpose each other

e.



g.



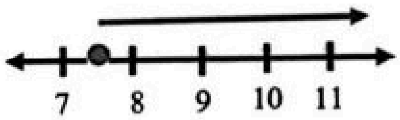
b. Do Yourself.   d. Do Yourself.   f. Do Yourself.   h. Do Yourself.

4. No solution

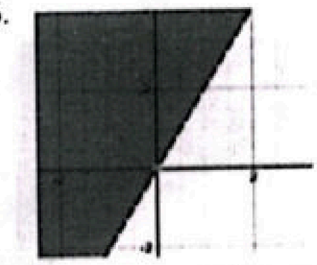
Miscellaneous Exercise-4

1. (i) (d) (ii) (c) (iii) (b) (iv) (b) (v) (c) (vi) (d)
(vii) (b) (viii) (c)

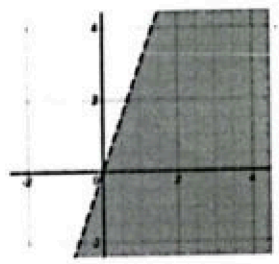
2. $x \geq \frac{15}{2}$



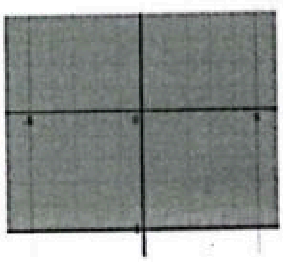
3. $x \geq 3$ 4. No solution 5. $x \geq -1$ 6.



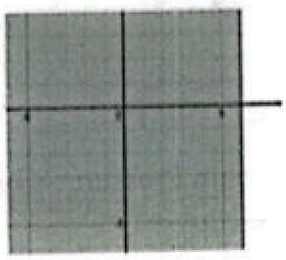
7.



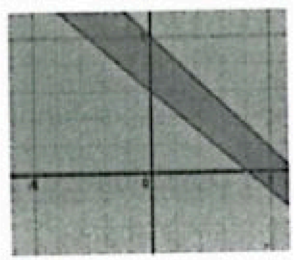
8.



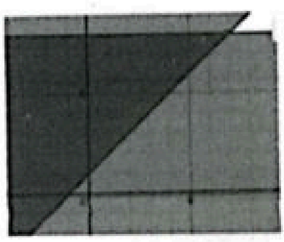
9.



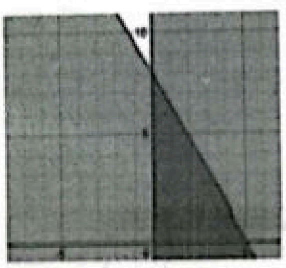
10.



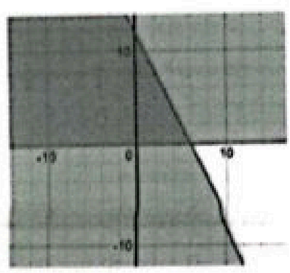
11.



12.



13.



14 $2 \le x \le 4$, $1 \le y \le 4$, 15 $x \ge 3$, $y \ge \frac{2}{3}x - 2$, $y \le \frac{4}{3}x - 2$

UNIT 5

Exercise 5.1

1. (i) $\frac{3x^2}{5ay^4}$ (ii) $\frac{2p^2m^2}{3k}$ (iii) $\frac{nq}{mp^3}$ (iv) $\frac{1}{5ab}$ (v) $\frac{2lmn}{3}$ (vi) -1

(vii) $x-9$ (viii) $\frac{r+3}{r-4}$

2. (i) 9 (ii) 294 (iii) 96.6 (iv) −7.5 (v) 1.002 (vi) $-3\frac{17}{56}$

3. 5050 5. $-11\frac{2}{3}$ 6. 21 cm

Exercise 5.2

1. (i) $\frac{7x}{10}$ (ii) $\frac{11x+2}{18}$ (iii) $\frac{81x+24}{28}$ (iv) $\frac{3x+2}{x+5}$ (v) $\frac{7x+3}{x-3}$

2. (i) $\frac{x+12}{5}$ (ii) $\frac{17x-101}{21}$ (iii) $\frac{-2x^3+2x^2+1}{x}$

3. (i) x^2+2x-3 (ii) $x^2-2xy+y^2$ (iii) $\frac{6(x-2)}{x}$ (iv) $\frac{x}{x-5}$

4. (i) $\frac{47x}{60}$ (ii) $\frac{29-2x}{6}$ (iii) $\frac{-4}{x^2-1}$ (iv) $\frac{3x^2-100}{15}$ (v) $\frac{9b^2cz^2}{4x^3y}$ (vi) $\frac{p^2q^2y}{x^2}$ (vii) $\frac{x+1}{x-3}$

(viii) $\frac{4}{3}$ (ix) $\frac{12a^2-4a+7}{3(4a^2-9)}$ (x) $\frac{23x}{(1+2x)(2+x)(5-9x)}$ (xi) $\frac{1}{8m}$ (xii) $\frac{1-q}{p}$ (xiii) $\frac{1}{1+x}$

Exercise 5.3

1. {2} 2. {2, 3} 3. No solution 4. only 4 (-1 is not solution) 5. {−1, −5}
6. {−6, 8}
7. No solution 8. approximately $2\frac{2}{9}$ or about 2 hours and 13 minutes
9. add 12 pints of yellow paint to the mixture to achieve a paint mixture that is 80% yellow.
10. Wasi takes 36 hours and Waqar takes 45

Miscellaneous Exercise-5

1.(i) (b) (ii) (a) (iii) (d) (iv) (c) (v) (c)
(vi) (d) (vii) (b) (viii) (d) (ix) (a) (x) (c)
(xi) (d) (xii) (d) (xiii) (c)

2. 78.571 cm 3. (i) 24.64 square inches (ii) 1.75 feet 4. (i), (ii) and (iv)

5. 20 6. $3x-9$ 7. $x-2$ 8. (i) $\frac{7x^2+30x+16}{8-2x^2}$ (ii) $\frac{1}{3}$

9. (i) {−2, −6} (ii) {5, −6} 3 is not solution 10. Essential Step: Ensure that solutions do not make any denominator zero, as such values are not valid for rational equations.

UNIT 6

Exercise 6.1

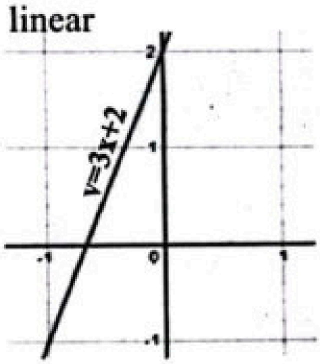
1. i. Yes ii. No iii. No iv. Yes 2. i. Yes, Bijective ii. No iii. Yes, Into
3. i. 6 ii. 9/2 iii. 28/9 iv. $\frac{1}{2}(2t^2+3t+8)$
4. Injective. 5. a = t = 1 6. Yes

Exercise 6.2

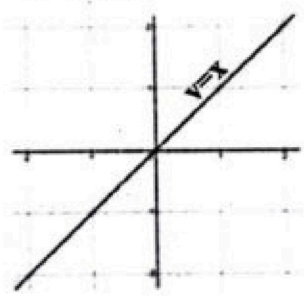
1. i. x^2+2x-3 ii. $-x^2+6x-5$ iii. $4(x-1)^3$ iv. $\frac{4}{x-1}$
2. i. $4(x+1)$ ii. $4x+1$ iii. $16x$ iv. $x+2$
3. i. $f\circ g(x)=-2x+1$, $g\circ f(x)=-2x+4$ ii. $f\circ g(x)=\frac{x-1}{x}$, $g\circ f(x)=\frac{4}{2-x}$ iii. $f\circ g(x)=\frac{6}{\sqrt{x-1}}$, $g\circ f(x)=\frac{2}{\sqrt{3x-1}}$ iv. $f\circ g(x)=x-2$, $g\circ f(x)=\sqrt{x^2-2}$
4. i. $x=0,-2$ ii. $x=1\pm\sqrt{2}$ iii. $x=\pm 2$
5. i. $f^{-1}(x)=\frac{1+x}{2}$ ii. $f^{-1}(x)=\frac{2+3x}{x}$ iii. $f^{-1}(x)=x^2-5$ iv. $3\pm\sqrt{x}$
6. i. $f^{-1}(1)=8$, i. $g^{-1}\left(\frac{1}{2}\right)=-1$

Exercise 6.3

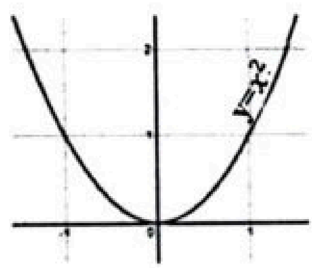
1. (i), (iii), (iv) and (iv) are graphs of functions.
2. i. linear



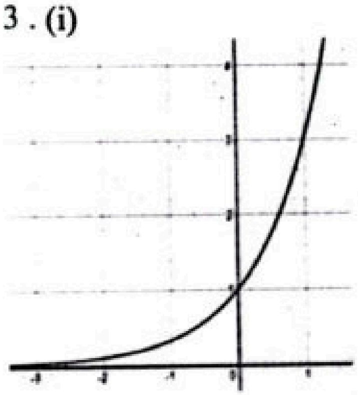
i. linear



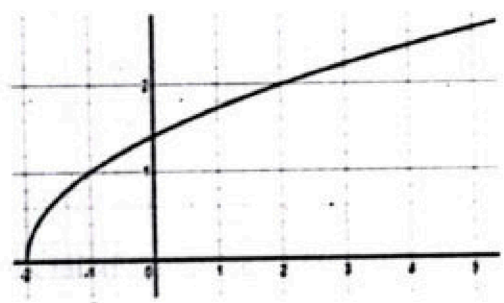
iii. not linear



3 . (i)

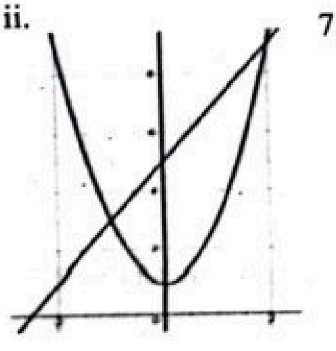


(iv)

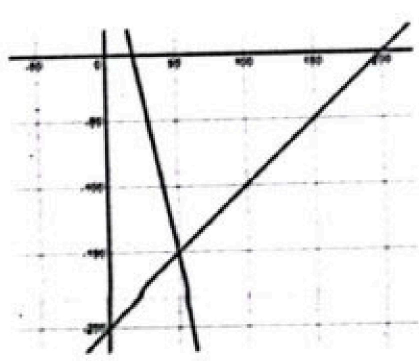


(ii), (iii),(v) draw yourself

4. i. upwards, x-intercept: 1 , y-intercept: 1
   ii. downwards, x-intercepts: $\pm\sqrt{2}$, y-intercept: 2
   iii. downwards, x-intercept: $2\pm\sqrt{3}$, y-intercept: 2
   iv. downwards, x-intercepts: -1, 3, y-intercept: 3
5. h=4/9, k = 2
6. i. draw yourself ii.



7.

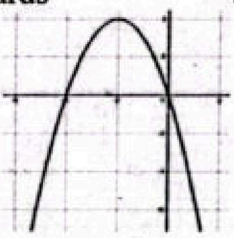


8. 63814 (apporx.)

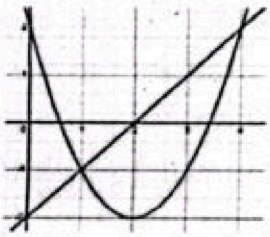
Supply will become equal to demand at x = 50. 9. draw yourself 10. 175 dollars

Miscellaneous Exercise-6

1. i. b ii. c iii. c iv. a v. c vi. c vii. b viii. c ix. d x. b
2. $f^{-1}(x) = \frac{x}{2x-3}$, $f^{-1}(x)$ is undefined at $x = \frac{3}{2}$, $f^{-1}(-1) = \frac{1}{5}$
3. downwards 4. $y = -10x^2 + 10$ 5. $x = -2, 3$



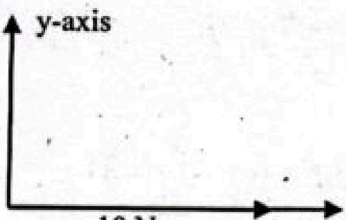
6. $F = \frac{9}{5}C + 32$, $77^{\circ}F$
7. $(1, -1), (4, 2)$ 8. 17 hrs.(approx.) 9. 2 units



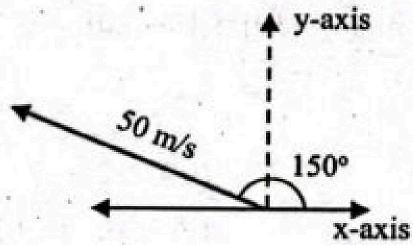
UNIT 7

Exercise 7.1

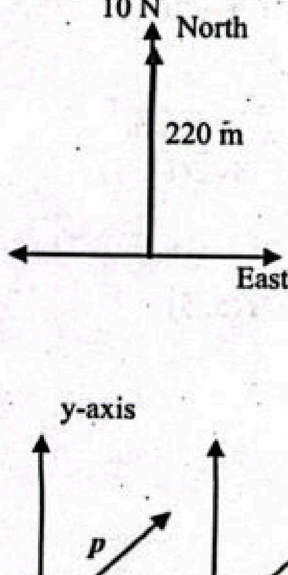
1. (i)



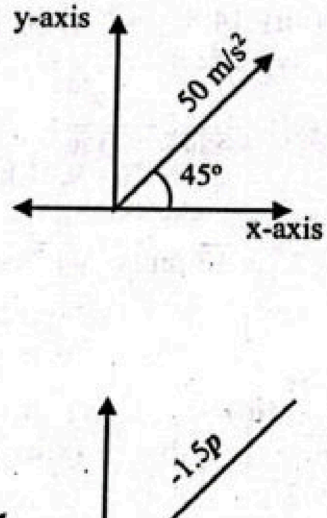
(ii)



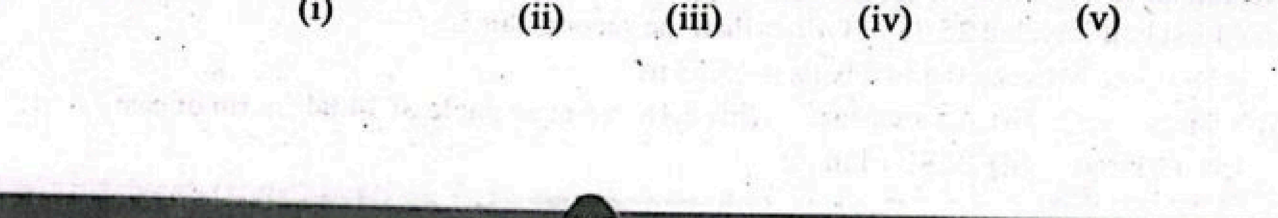
(iii)



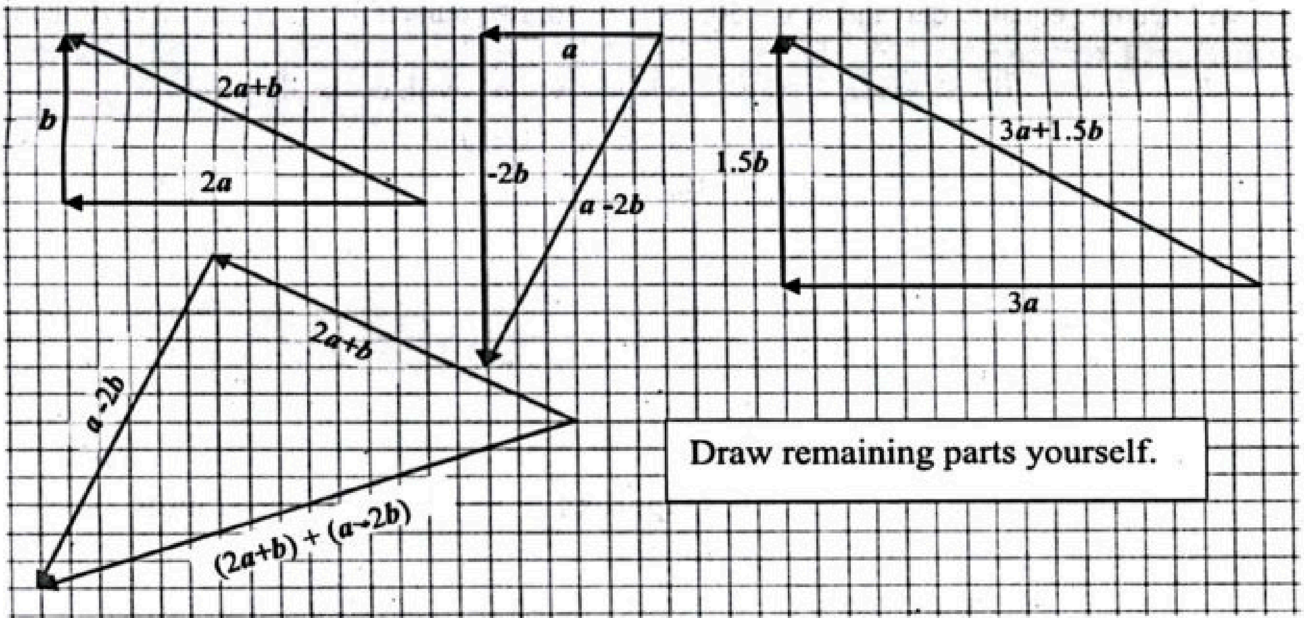
(iv)



2.



3.



4. $\vec{b} = -0.5\vec{a}$; $\vec{c} = 2\vec{a}$; $\vec{d} = -\vec{a}$; $\vec{e} = 3\vec{a}$; $\vec{f} = 0.5\vec{a}$; $\vec{g} = -2\vec{a}$

5. $(5, -10)$ 6. $\overrightarrow{[-1, 1]}$

7. (i) $A'(1,6), B'(4,4)\ C'(-1,1)$ (ii) $A'(-7,2), B'(-4,0), C'(-9,-3)$ 8. $[6, -5]$

9. (i) $\vec{a}$ (ii) $-b$(iii) $\vec{a} + b$ (iv) $-\vec{a} + b$ (v) $\frac{1}{2}(\vec{a} + b)$ (vi) $\frac{1}{2}(-\vec{a} + \vec{b})$ (vii) $2\vec{a} + \vec{b}$

10. (i) $-2\vec{q}$ (ii) $-\vec{q}$ (iii) $\vec{q}$ (iv) $\vec{p}$

11. (i) $[3, 3] = 3\hat{\imath} + 3\hat{\jmath}$ (ii) $[4, -10] = 4\hat{\imath} - 10\hat{\jmath}$ (iii) $[-5, -2] = -5\hat{\imath} - 2\hat{\jmath}$

12. (i) $\frac{3}{5}\hat{\imath} + \frac{4}{5}\hat{\jmath}$ (ii) $\frac{5}{13}\hat{\imath} - \frac{12}{13}\hat{\jmath}$ (iii) $\frac{2}{5}\hat{\imath} - \hat{\jmath}$

13. $x = 2, y = -2$

14. (i) $-6\hat{\imath} - 3\hat{\jmath}$ (ii) $-3\hat{\imath} - 2\hat{\jmath}$ (iii) $12\hat{\imath} - 7\hat{\jmath}$ (iv) $-\frac{3}{2}\hat{\imath} + 16\hat{\jmath}$ (v) $-5\hat{\imath} + 4\hat{\jmath}$

15. (i) $7\hat{\imath} - 3\hat{\jmath}$ (ii) $22\hat{\jmath}$ (iii) $40\hat{\imath} + 30\hat{\jmath}$ (iv) $\frac{1}{2}\hat{\imath} + \frac{9}{2}\hat{\jmath}$ (v) $\sqrt{26}$
(vi) $\sqrt{26}$ (vii) 14.8 (viii) 1.4 16. $p = 2$

17. (i) $\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath}$ (ii) $\frac{3}{\sqrt{130}}\hat{\imath} - \frac{11}{\sqrt{130}}\hat{\jmath}$ (iii) $\frac{1}{\sqrt{5}}\hat{\imath} - \frac{2}{\sqrt{5}}\hat{\jmath}$ (iv) $\frac{3}{\sqrt{10}}\hat{\imath} - \frac{1}{\sqrt{10}}\hat{\jmath}$
(v) $\hat{\imath}$ (vi) $\frac{7}{\sqrt{130}}\hat{\imath} - \frac{9}{\sqrt{130}}\hat{\jmath}$ 18. (i) $-4\hat{\jmath}$ (ii) $-3\sqrt{2}(\hat{\imath} + \hat{\jmath})$

19. 383.28 mph 20. 9.01 km away, 82.5°

Exercise 7.2

1. Parallelogram 2. $\sqrt{13}$ units, $\sqrt{17}$ units 4. $\frac{11}{5}\hat{\imath} - \frac{2}{5}\hat{\jmath}$ 5. Y(5, 5)

6. $\left(\frac{11}{3}, \frac{7}{3}\right)$ 7. 1 8. D(−4, 7) 9. B(1, 3)

Miscellaneous Exercise-7

1. (i) b (ii) a (iii) c (iv) d (v) c (vi) d (vii) a (viii) b (ix) d
(x) c (xi) a (xii) b (xiii) a (xiv) c

2. $\vec{p} \neq \vec{q}$ 3. $x = 2$

4. (i) $\frac{1}{2}\vec{a}$ (ii) $-\frac{1}{2}\vec{a}$ (iii) $\vec{b} - \frac{1}{2}\vec{a}$ (iv) $\vec{a}$

5. Resultant force = 533.82 N, direction = 44.05° with horizontal

6. (i) First ball traveled 25.73 m farther than the second ball.
(ii) Distance between the two balls = 43.68 m

7. (i) 7.5 m/s west (ii) 4.5 m/s east (iii) 6.18 m/s at an angle of 14.04° north of east

8. (i) 158.11 km/h (ii) 1,581.1 km

Exercise 8.1

1. (i) 55° (ii) 42° (iii) 69° (iv) 38°

2. 144° 3. 93° 4. $\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$, $\tan 120^\circ = -\sqrt{3}$

5. $\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$, $\tan\frac{3\pi}{4} = -1$ 6. (i) -0.559
(ii) 0.83 (iii) -1.485 (iv) -0.674 (v) -1.789 (vi) 1.204

Exercise 8.2

1. (i) $\beta = 50^\circ, a = 9.64m, b = 11.49m$ (ii) $\alpha = 54.5^\circ, c = 86.1m, a = 70.1m$
(iii) $c = 19.5cm, \alpha = 50.20^\circ, \beta = 39.8$

2. (i) $\alpha = 55^\circ, H = 14.64cm, P = 8.4cm$ (ii) $\beta = 64.41^\circ$, $a = 14.54cm$, $c = 33.5cm$
(iii) $c \cong 56cm, \alpha \cong 63.43^\circ, \beta \cong 26.57^\circ$ (vi) $b \cong 26.46cm, \alpha \cong 41.84^\circ, \beta \cong 48.16^\circ$
(v) $c \cong 40.5cm, a \cong 32.6cm, \beta \cong 53.75^\circ$ (vi) $c \cong 38.7cm, b \cong 37.32cm, \beta \cong 14.12^\circ$

3. $42.89m$ 4. $116{,}5m$ 5. 5.18 ft 6. 26.59 7. 48.6° 8. 24 ft 9. 61° 10. $32.47m$
11. $814.3m$ 12. $4.7m$ 13. $23.2\ m$ 14. $6.66\ m$ 15. $9.33m$

Exercise 8.3

1. (i) $\alpha = 36.1^\circ, \beta = 85.4^\circ, c = 10.2$ (ii) $\gamma = 48.3^\circ, \beta = 95.7^\circ, c = 11.8$
(iii) $\alpha = 86^\circ, \gamma = 79^\circ, b = 38.9$ (iv) $\alpha = 71.57^\circ, \gamma = 63.43^\circ, b = 30$
(v) $\alpha = 37.7^\circ, \beta = 58.3^\circ, \gamma = 88.0^\circ$ (vi) $\alpha = 33.33^\circ, \beta = 33.33^\circ$, $\gamma = 112.89^\circ$
(vii) $\alpha = 36.3^\circ, \beta = 62.7^\circ, \gamma = 81.0^\circ$ (viii) $\alpha = 60^\circ, \beta = 60^\circ, \gamma = 60^\circ$
(ix) $\alpha = 22.62^\circ, \beta = 67.38^\circ, \gamma = 90^\circ$

3. (i) $\gamma = 80^\circ, b \cong 13.5, c \cong 15.3$ (ii) $\gamma = 60^\circ, a \cong 16.3, c \cong 18.4$
(iii) $\gamma = 60^\circ, a \cong 9.8, c \cong 13.4$ (iv) $\alpha \cong 19.42^\circ, a \cong 9.1, c \cong 23.8$
(v) $\gamma \cong 69.33^\circ, b \cong 34.2, c \cong 32.1$ (vi) $\beta \cong 52.28^\circ, a \cong 35.2, b \cong 41.2$

5. (i) $\alpha = 48.6^\circ, \gamma \cong 51.4^\circ, c \cong 10.4$ (ii) $\alpha = 71.8^\circ, \gamma \cong 39.2^\circ, a \cong 20.1$
(iii) $\alpha = 90^\circ, \beta = 26^\circ, a \cong 13.8$ (iv) $\gamma = 101.3^\circ, \beta = 23.6^\circ, c = 43$
(v) $\alpha \cong 57.4^\circ, \beta \cong 35.1^\circ, \gamma \cong 87.5^\circ$ (vi) $\alpha \cong 83.6^\circ, b \cong 36.2, c \cong 88$

6. Distance $= 362.7.5km$ 7. Perimeter $= 1425\ m$, Area $\cong 84852.8m^2$
8. Largest Angle $\cong 83.41^\circ$, Smallest Angle $\cong 44.64^\circ$ 9. 70.75 cm, 98.97 cm
10. (i) BC = 10.15 inch (ii) $\angle BDC = 19.49^\circ$ 11. (i) BC = 11.49 m (ii) $\angle EDG = 56.9^\circ$
12. 284.51 ft 13. 9.51 miles 14. 25.98 cm 15. 36.71°

Exercise 8.4

1. (i) 52.27 square units (ii) 269.74 square units (iii) 63.3 square units (iv) 814.8 square units
(v) 5.13 square units (vi) Not possible (not triangle) (vii) 87.56 square units (viii) 254.73 square units
(ix) 136.04 square units (x) 154.55 square units (xi) 61.5 square units (xii) 295.1 square units
(xiii) 168 square units (xiv) 252 square units (xv) 0.033 square units (xvi) 74.53 square units
(xvii) 1.69 square units

2. 127.28 square units 3. 387.4 square units

4. (a) $\cos\theta = \frac{1}{3}$ (b) $\sin\theta = \frac{2\sqrt{2}}{3}$ (c) $5.66\ sq.units$ (d) $6\ sq.units$

5. $A \cong 233.75$ sq units 6. $125\sqrt{3}$ sq units 7. Do yourself.

8. Length of side = 22.3 m, Third Angle = 75° 9. $27\sqrt{3}\ \text{cm}^2$

10. (a) Sine of angle B $= \frac{2}{3}$ (b) $\angle B = 41.8^\circ$ 11. (a) Area $= (ac)\sin\theta$ (b) $\theta = 90^\circ$

Exercise 8.5

1. (i) $r \cong 3.24, R \cong 8.53, r_1 \cong 6.48, r_2 \cong 9.26, r_3 \cong 21.6$
(ii) $r \cong 6.87, R \cong 15.18, r_1 \cong 16.31, r_2 \cong 18.64, r_3 \cong 32.61$
(iii) $r = 1, R = 2.5, r_1 = 2, r_2 = 3, r_3 = 3$
(iv) $r \cong 16.33, R \cong 35.75, r_1 \cong 36.74, r_2 \cong 48.99, r_3 \cong 73.48$

10. $r \cong 2.43$ cm, $R \cong 7.64$ cm
11. (i) $2\pi r_1 \cong 20.16\,cm$ (ii) $2\pi r_3 \cong 70.56\,cm$ **12.** $A \cong 2.72\ cm^2$, $C \cong 5.85$ cm

Exercise 8.6

1. 16 cm **2.** 78.86° **3.** $6\sqrt{2}$ cm, $6\sqrt{2}$ cm **4.** 13.9° **5.** (i) 20cm (ii) 25 cm (iii) 26.9°
6. 46.7° **7.** (i) 10.82m (ii) 12.44m (iii) 53.07° (iv) 64.52°
8. (i) 29 km (ii) 29 km (iii) 56 km

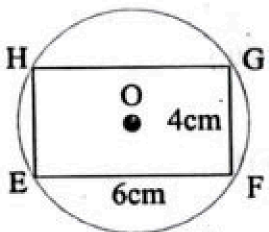
Miscellaneous Exercise-8

1. (i) a (ii) c (iii) b (iv) d (v) c (vi) d
(vii) b (viii) a (ix) c (x) d (xi) b (xii) a
2. (i) $\gamma = 145°, b = 10.6, c = 17.7$ (ii) $\beta = 32.4°, \gamma = 32.6°, c = 63.5$
(iii) $\gamma = 56.9°, \alpha = 24.9°, a = 7.37$ (iv) $b \approx 114.74, a \approx 146.99, \gamma = 20°$
(v) $b \cong 66.86, \alpha = 28.88°, \gamma = 43.12°$ (vi) $\alpha = 53.13°, \beta = 36.87°, \gamma = 90°$
3. 14.48 cm **4.** a. 1.8 km b. 4.25 km **5.** a. 33.73 inches b. 57.5° **6.** 1717.62 m^2
7. a. 8.84 m b. 32.18 m **8.** 137.3 meters and 121.6 meters
9. Yes, it is possible for Aamir to draw a parallelogram with one side of 12 cm, one diagonal of 10 cm, and one angle of 120°. The calculated length of the other diagonal is approximately 20.8 cm, which is consistent with the geometry of a parallelogram.
10. Approximately 73.8feet (higher observer) and 45.3 feet (lower observer). The distance from the pipe to the apartment complex (horizontal distance) is approximately 42.3 feet. **11.** 5.22 miles **12.** Both are 21.9 cm
16. 1702.3 meters **17.** 24.96° **18.** 21°

UNIT 9

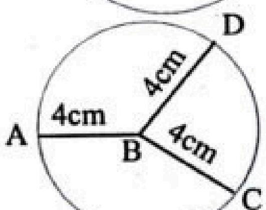
Exercise 9.1

2. (i) $x = 4cm$ (ii) $y = 6cm$ 3. $CD = 26cm, CE = 1cm$ 4. $CD = 24cm$
5. $AB = 24cm$
6.

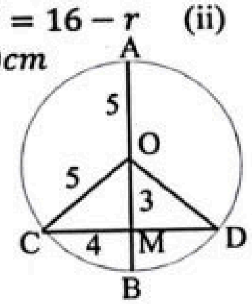


As the circle has only one centre O, therefore one and only one circle can pass through vertices of rectangle.
As the circle has only one centre B, therefore one and only one circle can pass through points A, C and D.

7.



8. (i) $OU = 16 - r$ (ii) $r^2 = 5^2 + (16 - r)^2$; $r = 8.8cm$
9. $r = 10cm$
10.



Sides of triangle OCM satisfy Pythagorean triplet (3, 4, 5). Therefore, $\angle OMC = 90°$ proving the result.

11. Prove $\triangle ACD \cong \triangle BCD$, with the help of SAS postulate, then $AC = BC$.

Exercise 9.2

1. $\angle OCF = 15°, \angle CFD = 75°$ 2. $2.24cm$, Yes, they are equal. 3. $7cm$
4. (i) $a = 9cm, y = 53°$ (ii) $a = 6cm, x = 45°, y = 8.49cm$
5. $16.53cm$ 6. $6cm, 4cm$ 7. $31cm$ 8. $\sqrt{10}cm = 3.16cm$ 9. $5.15cm$

Exercise 9.3

1. (i) yes (ii) yes (iii) chord AB < chord AD
4. (i) no (ii) both are proportional 5. (i) yes (ii) CD = 4 cm (iii) 2.24 cm
6. 120° 8. 6 cm 11. (a) 130° (b) 85° (c) 75° (d) 70°
13. $\angle AOB = \angle COD = 100^\circ$; Equal chords of a circle subtend equal angles at the centre.
14. (i) All the four chords make central angle of 90°. (ii) $\angle PCB = 45^\circ, \angle ACB = 90^\circ$
    (iii) $\angle CBD, \angle BDA, \angle DAC$ (iv) square (v) one and only one
15. (i) $\angle PTQ = 45^\circ$, $\angle RTS = 135^\circ$ (ii) 1 : 3

Exercise 9.4

1. (i) Length of minor arc = 12.21 cm, length of major arc = 31.75 cm (ii) 43.96 cm; yes
2. (i) Area of minor sector = 33.51 cm^2, area of major sector = 167.55 cm^2
   (ii) 201.06 cm^2; yes 3. 26.2 cm
4. (i) 22cm (ii) 10.5cm (iii) 11.5cm (iv) 231cm^2 (v) 52.4cm^2 (vi) 178.6cm^2
5. (i) 19.8cm (ii) 22cm (iii) 41.8cm (iv) 98cm^2 (v) 154cm^2 (vi) 56cm^2
6. 12.57cm 7. 4 cm 8. A = 12.32 ft^2; P = 14.4 ft 9. 35.9 m 10. 28 m
11. P = 12.48 ft; A = 1.85 ft^2 12. 29.3 ft 13. (i) 12228.1 m^2 (ii) 65.1 m

Miscellaneous Exercise-9

1. (i) b (ii) d (iii) c (iv) c (v) b (vi) a (vii) d (viii) a (ix) d
   (x) c (xi) b (xii) c (xiii) b (xiv) d (xv) b (xvi) a
   (xvii) b (xviii) d

UNIT 10

Exercise 10.1

1. (i) $r = 5cm$ (ii) $d = 4\sqrt{7}cm$ (iii) $x = 8cm,\ y = 4.62cm$
   (iv) $x = 25^\circ, y = 65^\circ$ (v) $a = 30^\circ, b = 10\sqrt{3}cm$ (vi) $a = b = 60^\circ$
   (vii) $a = 15^\circ$ (viii) $x = 63^\circ, y = 60^\circ, z = 57^\circ$
2. (i) $\angle OBA = 90^\circ$ (ii) $\angle OCB = 30^\circ$ (iii) $\angle BAC = 60^\circ$ (iv) $\angle ABC = 60^\circ$
4. 4 ft 5. 2cm, 6cm, 10cm 6. 5.06 cm 7. $\angle PGH = 35^\circ$ 8. $x = 48^\circ, y = 68^\circ$

Exercise 10.2

1. $\angle Q = 30^\circ$, $\angle ROS = 60^\circ$ 2. $\angle ABO = 30^\circ$, $\angle AOB = 120^\circ$, $\angle ACB = 60^\circ$
3. $\angle ABC = 90^\circ$, $\angle BCA = 50^\circ$, $\angle CAD = 30^\circ$ 4. $\angle O = 60^\circ$, $\angle C = 30^\circ$
5. $\angle AEB = 110^\circ$, $\angle B = 40^\circ$, $\angle C = 40^\circ$, $\angle D = 30^\circ$
6. (i) $4cm$ (ii) $\angle ABE = 60^\circ$ (iii) $\angle ABC = 90^\circ$ (iv) $\angle EBD = 30^\circ$
7. (i) 2 (ii) $4cm$ (iii) $\angle BQC = \angle BPO = 90^\circ$ (iv) $5\sqrt{5}\ cm$ (v) $15\ cm$
8. (i) $\angle QSR = 120^\circ$ (ii) $\angle PRS = 30^\circ$ (iii) $\angle PQS = 30^\circ$ (iv) $\angle QTR = 60^\circ$
   (v) Angle in a segment greater than semicircle is acute angle.
9. (i) $\angle BAD = 65^\circ$ (ii) $\angle BCD = 115^\circ$ (iii) $\angle ABC + \angle ADC = 180^\circ$
10. (i) $\angle QRS = 85^\circ$ (ii) $\angle PQR = 95^\circ$ (iii) $\angle PSR = 85^\circ$
11. (i) $\angle BAC = 30^\circ$ (ii) $\angle CAD = 30^\circ$
12. (i) $\angle ECD = 65^\circ$ (ii) $\angle CDE = 95^\circ$ (iii) $\angle CED = 20^\circ$
13. (i) 2.2 cm (ii) 1.2 cm 14. 40 cm, 125.6 cm

Miscellaneous Exercise-10

1. (i) a (ii) c (iii) b (iv) b (v) d (vi) a (vii) d (viii) a (ix) b
   (x) c (xi) d (xii) c (xiii) d (xiv) b (xv) d (xvi) a
   (xvii) c (xviii) c (xix) b (xx) d
2. $6\sqrt{2}\ cm$ 3. $8cm$ 4. $4.75cm$

UNIT 11

Exercise 11.1

8. (i) He will take three points on circular path and join them to obtain a triangle. Point of intersection of right bisectors of sides of the triangle is the required point.
(ii) 25 cm 9. 22 tiles

Exercise 11.2

11. 122 inches 12. 12451.51 km 13. 61.68 cm 14. 5 cm

Miscellaneous Exercise-11

3. (i) d (ii) a (iii) b (iv) c (v) a (vi) c (vii) d (viii) b (ix) a
(x) d (xi) c (xii) d (xiii) b (xiv)

UNIT 12

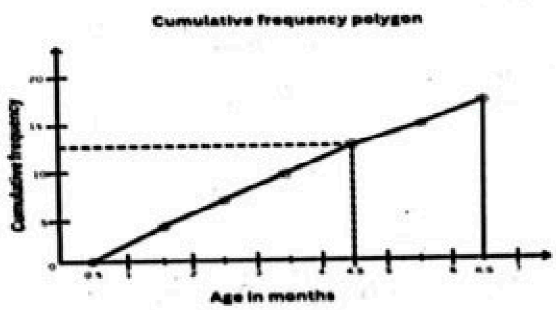
Exercise 12.1

1.

Class Interval	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60
Frequency	3	4	7	9	5	2
Cumulative freq	3	7	14	23	28	30

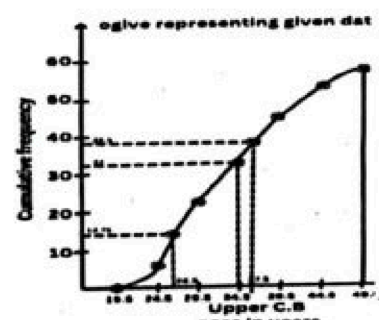
i. 30 ii. 21-30 iii. 7 iv. 31-40 v. 50.5

2.



From the graph, mass of the baby at the age of $4\frac{1}{2}$ months is 12kg app.

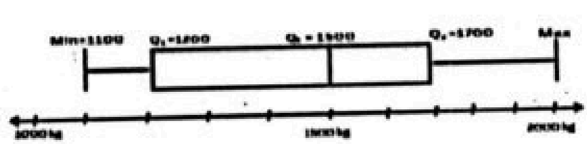
3. 33 a. 26.5 app b. 37app c. 34.5app

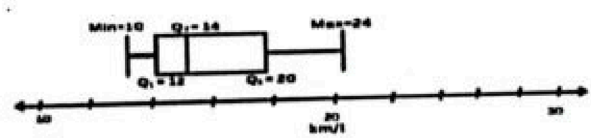


4. Draw ogive yourself. a.. 90 b. 72.5 c. 45 d. 9 e. 27 f. 1 5. do yourself
6. a. 75 app b. 45% app c. 3 : 1 app d. 25 e. 55m app f. 80m app
7. Range = Rs.125,000 & IQR = Rs. 40,000 8. She performed above 75% of of her class
9. Between Rs. 25,000,000 & Rs. 30,000,000
10. i. 38app ii. 30app iii. 50app iv. 20app v. 48app vi. 32app

Exercise 12.2

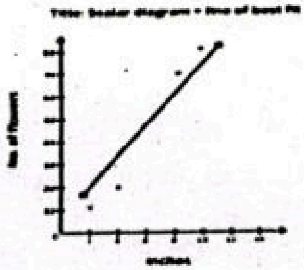
1.





a. $IQR_1 = 500$, $IQR_2 = 8$, b. negative c. lighter vehicles are more fuel efficient
2. a. Rs. 500 b. 0 &700 c. Rs. 400 d. Rs. 300 e. positive

3.



Positive correlation indicates increase in number of flowers with increased rain quantity.

4. a. Positive correlation indicates higher score with more practice time. b. draw yourself

Exercise 12.3

1. Measure of spread of data ; Range , Variance , Standard deviation
2. range = 599, Variance = 28886 app 3i. 758.8 ii. 19097 iii. 138.19 iv. 18.2
4. range = Rs.60,000 , Variance = 246437000app , SD = 15698.3 CV = 22861.5
5. mean = 66.35 , Variance = 67.57 , SD = 8.22
6. range = 2,000 , Variance = 243806app, SD = 493.8
7. OT: range = 60, mean = 127.3, Variance = 204.7app , SD = 14.31 CV = 11.24
   OPD: range = 60, mean = 127, Variance = 292.4app , SD = 17.1 CV = 13.46
   OPD patients BP data have more variation.
8. Mean : 103, 66.9 Variance: 33.3, 147.6
   Next week average temp seems to be 103F app in Jhelum and 147.6F app in Indiana.

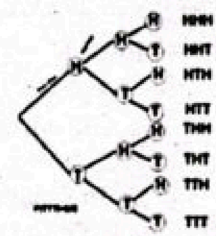
Exercise 12.4

1.

trials	First trial			
	×	1	2	3
Second trial	1	1	2	3
	2	2	4	6
	3	3	6	9

P(Even) = 5/9

2.



3. 0.85 4. 0.7 5. 0.9 6. 5/6 7. 25/27 8. 0.28 9. 0.63 10. 0.72

Miscellaneous Exercise-12

1. i. d ii. b iii. b iv. c v. d vi. d vii. b viii. a ix. d x. c
   xi. c xii. d xiii. a xiv. a xv. c
2. Draw the graph yourself , variance = 83.14 , SD = 9.12
3. a. $IQR_1 = 30$, $IQR_2 = 10/10$ b. negative c. more exercise time caused reduction in BP.
4. 25%, IQR = 20 5. Hazel: M = 65 app, IQR = 84-48 = 36,
   Mauve: M = 68 app, IQR = 86-45 = 41
6. 3/10 7. 0.18

GLOSSARY

Absolute value of a complex number: The absolute value of z, denoted by $|z|$ is defined as $|z| = \sqrt{a^2 + b^2}$ and is a real number.
Absolute valued function: The function defined by $f(x) = |x|$, is called the absolute valued function.
Additive identity in matrices: Additive identity is such a matrix which causes no change in any matrix A while 'adding to' or 'subtracting from' it.
Additive inverse of a matrix: Additive inverse of a matrix 'A' is such a matrix which when added to A, gives additive identity matrix of the same order.
Additive inverse of complex number: Complex number that is added to a given complex number in order to make the sum zero is known as the additive inverse.
Algebraic expressions: A statement in which variables or constants or both are connected by arithmetic operations.
Imaginary Number: An imaginary number is a number that can be written $a+bi$, where a and b are real numbers, $b \neq 0$
Angle in a segment: An angle whose vertex is a point on the arc of a circle and whose arms pass through end points of chord.
Arc of a circle: An arc is a part of a circle. An arc AB is denoted by $\widehat{AB}$.
Associative property of matrix addition: If A, B and C are three matrices of same order, then (A + B) + C = A + (B + C), is called associative property of matrix addition.
Associative property of matrix multiplication: If A, B, and C are matrices and products AB and BC are possible, then (AB) C = A (BC)
Bijective function: A function which is both onto and 1-1 is called a bijective function.
Binomial: A polynomial having two terms is called a binomial.
Box-and-whisker plot: It is a graphical representation of a data set that displays the distribution of values through 5 data points i.e. min, max, Q_1, Q_2, Q_3.
Central angle: An angle whose vertex is the centre of circle and whose arms pass through end points of arc is known as central angle of arc.
Chord of a circle: A line segment joining two points of a circle is called chord of that circle.
Circumcenter: It is the point of intersection of right bisectors of sides of a triangle.
Circumcircle: A circle passing through the vertices of any triangle (polygon) is called circumcircle.
Coefficient of variation: The percentage ratio between the standard deviation and mean of a data.
Column matrix: A matrix having only one column is called a column matrix.
Column: Vertical arrangement of elements is in a matrix called a column.
Commulative frequency: In a group data, number of values less than or within the limits of a class is called cumulative frequency of the respective class.
Commutative property of matrix addition: If A and B are two matrices of same order, then A + B = B + A, is called commutative property of matrix addition.
Complex number: A complex number is any number that can be written in the form of $a + bi$, where a and b are any real numbers and both a and b can be zero.
Composition of functions: The composition of $f(x)$ and $g(x)$ is a function denoted by $f_\circ g$ and is written as: $(fog)(x) = f(g(x))$
Conditional probability: The probability of an event occurring given that another event has occurred.
Congruent arcs: Two arcs of a circle are congruent if their central angles are congruent.
Conjugates of complex numbers: Conjugate of complex number is a complex number obtained by changing the sign of the imaginary part.
Constant function: A polynomial function of degree zero is called a constant function.
Constant polynomial: A polynomial having degree zero is called a constant polynomial.
Coterminal angles: Two angles having same terminal ray in standard position, are called coterminal angles.
Cubic function: A polynomial function of degree three is called a cubic function.
Cubic polynomial: A polynomial having degree three is called a cubic polynomial.

Cumulative frequency polygon: A polygon made by joining the points whose x-coordinates are class marks and y-coordinates are their cumulative frequencies is called a cumulative frequency polygon.
Cyclic quadrilateral: A quadrilateral inscribed in a circle is called cyclic quadrilateral.
Deciles: Deciles divide a data set into 10 equal parts, each representing 10% of the data.
Dependent probability: The probability of two or more events occurring dependently on each other.
Determinant: The determinant of its matrix $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ is denoted by $\begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc.$
Diagonal matrix: A square matrix in which every element except the primary (principal) diagonal elements is zero is called a diagonal matrix.
Domain of a function: A set containing first element of each ordered pair in the function is called its domain.
E-center: It is the point of intersection of one internal bisector of angle and external bisectors of remaining angles of a triangle.
Equal matrices: Two matrices are said to be equal if and only if their order is same and corresponding elements are same.
Equal vectors: Two vectors $\vec{u}$ and $\vec{v}$ are said to be equal if both have the same magnitude and direction.
Equality of complex numbers: If two complex numbers $a + bi$ and $c + di$ are equal then $a = c$ and $b = d$.
Escribed circle: A circle touching one of the sides of a triangle externally and the extensions of its two other sides internally, is called an escribed circle.
Evaluation of a function: It is a process of finding the value of dependent variable by substituting any specific value of the independent variable.
Evaluation: The process of getting a numerical answer by substituting numbers for each variable is called evaluation.
Experimental probability: Probability based on the results of repeated trials of an experiment.
Exponential function: The exponential function is of the form $f(x) = a^x$, where $a > 0$ and $a \neq 1$.
Frequency polygon: The representation of data by a polygon in which class marks are plotted along x-axis and frequencies along y-axis is called a frequency polygon.
Frequency table: A table consisting of values of a data along with their frequencies.
Function: A function f from set A to set B is a relation, rule or mapping which maps each element of set A to a unique element of set B.
Graph of inequality: On a number line, the graph of an inequality in one variable is the set of points that represents all solutions of the inequality.
Head to tail rule: The method of adding vectors in which tail of the second vector is joined with the head of the first vector.
Imaginary number: An imaginary number is the product of a real number and the imaginary unit i, which is defined by its property $i^2 = -1$.
Incentre: It is the point of intersection of angle bisectors of a triangle.
Incircle: A circle drawn inside and touching the sides of a triangle (polygon) is called incircle.
Injective function: A function which is both into and 1-1 is called an injective function.
Inter quartile range: The IQR represents the range of values within which the middle 50% of the data points fall.
Into function: A function $f : A \rightarrow B$ will be into if there is at least one element in set B which is not an image of any element of set A.
Inverse of a function: The inverse of any function $f(x)$ is a function denoted by $f^{-1}(x)$ which reverses the effect of $f(x)$ and it undoes what $f(x)$ does.
Irrational expression: An algebraic expression which cannot be expressed in the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$.
Joint probability: The probability of two or more events occurring together.
Linear function: A polynomial function of degree one is called a linear function.
Linear polynomial: A polynomial having degree one is called a linear polynomial.
Matrix: A matrix is a rectangular arrangement of numbers.
Measure of central tendency: A single value which can represent the whole data.

Measures of dispersion: The measures which tell us how the values of a data are scattered around their average, are called measures of dispersion.
Minor and major arcs: Minor arc is the arc included in a semicircle while the arc which includes a semicircle is called major arc.
Minor and major segments: The region between chord and minor arc is called minor segment and the region between chord and major arc is called major segment.
Monomial: A polynomial having one term is called a monomial. e.g., 3, x, 0 etc.
Multiplicative identity in matrices: Multiplicative identity is such a matrix which causes no change in any matrix A when multiplied with A.
Mutually exclusive probability: The probability of two or more events that cannot occur simultaneously.
Negative correlation: If the boxes and whiskers of both variables tend to move in opposite directions it may indicate a negative correlation.
Negative of a vector: A vector having the same magnitude but opposite in direction of a given vector is called the negative of that vector.
No correlation: If the boxes and whiskers of both variables appear unrelated or don't show a clear pattern, it may indicate no correlation.
Null matrix: A matrix of any order, having all the elements equal to zero, is called a null matrix.
Oblique triangle: An oblique triangle is any triangle that is not a right triangle.
One-to-one function: A function f is one-to-one if distinct elements of set A have distinct images in set B.
Onto function: A function f : A→B will be onto if every element of set B is an image of at least one element of set A.
Operations on functions: Operations on functions are the ways of combining functions to create new functions.
Order of matrix: If 'A' is a matrix with 'm' number of rows and 'n' number of columns, then order of the matrix is m-by-n.
Parallel vectors: Two non-zero vectors $\vec{u}$ and $\vec{v}$ are said to be parallel if $\vec{u} = \lambda\vec{v}$.
Percentiles: Percentiles divide a data set into 100 equal parts, each representing 1% of the data.
Polynomial function: A function f(x) = $a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + +a_{n-1}x^{n-1} + a_nx^n$, where $a_0, a_1, a_2, a_3 \ldots, a_{n-1}, a_n$ are all real numbers and n is a non-negative integer.
Polynomials: Algebraic expressions in which exponents of the variables involved are non-negative integers.
Position vector: The vector used to specify the position of a point P with respect to origin O is called position vector of P.
Positive correlation: If the boxes and whiskers of both variables tend to move in the same direction it may indicate a positive correlation.
Probability: Probability is a measure of the likelihood of an event occurring.
Product of matrices: Two matrices A and B are conformable for product AB, if:
Quadrantal angles: Angles in standard position whose terminal side lies on an axis.
Quadratic equation: It is an equation that can be written in the form of $ax^2 + bx + c = 0$, where $a \neq 0$.
Quadratic function: A polynomial function of degree two is called a quadratic function.
Quadratic inequality: $ax^2 + bx + c > 0$ or $ax^2 + bx + c < 0$is called quadratic inequality.
Quadratic polynomial: A polynomial having degree two is called a quadratic polynomial.
Quartiles: The three values which divide the whole data into four equal parts are called quartiles.
Range of a function: A set containing second element of each ordered pair in the function is called its range.
Range: Range is the difference of largest and smallest values of the data.
Rational equations: An equation that contains one or more rational expressions is called a rational equation.
Rational expression: Algebraic expression of the form $\frac{P(x)}{Q(x)}$, where $P(x)$and $Q(x)$ are polynomials and $Q(x) \neq 0$.
Reciprocal function: The function defined by $f(x) = \frac{1}{x}$, where $x \neq 0$ is called a reciprocal function.
Rectangular matrix: A matrix with unequal number of rows and columns is called a rectangular matrix.
Row matrix: A matrix having only one row is called a row matrix.

Row: Horizontal arrangement of elements in a matrix is called a row.
Scalar matrix: A diagonal matrix in which all the elements of primary diagonal are equal and non-zero is called a scalar matrix.
Scalar multiplication of a matrix: Scalar multiplication of a matrix means multiplication of a matrix with a constant.
Scalar multiplication: Multiplication of a vector by a scaler.
Scalar: A physical quantity that possesses magnitude only, is called a scalar quantity.
Segment of circle: A chord of a circle divides the circular region into two parts, called segment of the circle.
Semicircle: A diameter divides a circle into two equal parts. Each part is called semicircle.
Singular and non-singular matrices: A is singular matrix, if $|A| = 0$ otherwise non-singular.
Skew symmetric matrix: A is skew Symmetric if, $A = -A^t$.
Solution of a system of linear inequalities: It is an ordered pair that is a solution of each inequality in the system.
Solution of inequality: A solution of an inequality in two variables x and y is an ordered pair (x, y) that produces a true statement when the values of x and y are satisfied into the inequality.
Solution of the system: The set of all the ordered pairs (x, y) which satisfies the system of equations is called the solution of the system.
Square matrix: A matrix with equal number of rows and columns is called a square matrix.
Square root function: The function defined by $f(x) = \sqrt{x}$, where $x \geq 0$ is called a square root function.
Standard deviation: It is the positive square root of variance.
Substitution: The process of replacing the variables by numbers in an algebraic expression is called ' substitution'.
Subtraction of two vectors: To subtract a vector, form the other we find the negative vector of the vector to be subtracted and then add it to the other vector.
Symmetric matrix: A is symmetric if, $A = A^t$.
System of linear inequalities: A system of linear inequalities in two variables or simply a system of inequalities consists of two or more linear inequalities in the same variable
System of simultaneous equations: A system of equations having a common solution is called a system of simultaneous equations.
Tangent line: A line which touches the circle at only one point is called tangent line.
Theoretical probability: The number of favorable outcomes divided by the total number of possible outcomes.
Three-D Trigonometry: It is an application of the trigonometric skills developed for 3-dimensional shapes in order to find unknown sides and angles.
Transpose of a matrix: If rows (columns) of a matrix P are changed into columns (rows) then the resulting matrix is called transpose of the matrix P.
Trigonometric ratio: The ratio between any two sides of a right-angled triangle is called trigonometric ratio.
Trinomial: A polynomial having three terms is called a trinomial.
Unit matrix: A scalar matrix in which all the primary diagonal elements are equal to 1, is called a unit matrix.
Unit vector: A vector which is in the direction of non-zero vector $\vec{u}$ and has magnitude 1 is called a unit vector of $\vec{u}$.
Variance: Variance is the ratio between sum of squares of deviations from mean and the number of values.
Vector: A physical quantity that possesses both magnitude and direction, is called a vector quantity.
Zero or null vector: If the initial and terminal points of a vector coincide then the vector has zero length and is called zero vector.
Zero polynomial function: A polynomial function of no degree is called a zero polynomial function.
Zero polynomial: '0' is called a polynomial of no degree.
Zero product property: For any real number a and b, if $ab = 0$, then either $a = 0$ or $b = 0$.

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