

Practical Geometry of Circles

After studying this unit students will be able to:

- Locate the centre of a given circle.
- Draw a circle passing through three given non-collinear points.
- Complete the circle (i) by finding its centre, (ii) without finding its centre, when a part of its circumference is given by
- Draw a tangent to a given arc, without using the centre, through a given point P when:
(i) P is middle point of the arc (ii) P is at the end of arc (iii) P is outside the arc.
- Draw a tangent to a given circle from a point P when:
(i) P is on the circumference (ii) P is outside the circle.
- Draw two tangents to a circle meeting each other at a given angle.
- Draw (i) direct common tangents or external tangents (ii) transverse common tangents or internal tangents, to two equal/unequal circles.
- Draw a tangent to two unequal (i) touching circles (ii) intersecting circles.
- Apply concepts of practical geometry of circles to real life word problems such as athletic tracks, recreational parks, Ferris wheels and mechanical machines.

A circle is symmetric about its centre and any of its diameters. Architects often use the circle's symmetrical properties when designing athletic tracks, recreational parks, buildings, roundabouts, Ferris-wheels, etc. Artists and painters find the circle almost indispensable in their work. Circular cylinders are used to print newspapers. Engineers exploit the circle's symmetrical properties as seen by the use of the circle in making watches, clocks, bicycles, cars, trains, ships, aero-planes, radios, telephones, trolleys, wheel-barrows, air-conditioners, rockets etc. Civilizations have progressed dramatically because of the invention of the circular wheel.



Construction of Circles

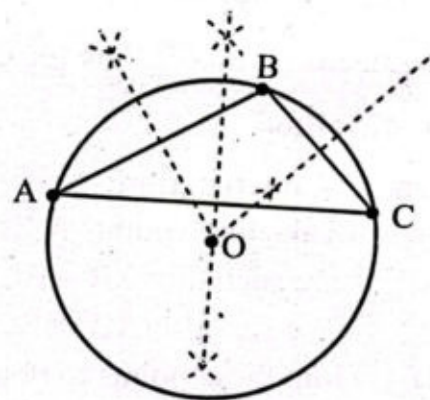
To Locate the Centre of a Circle

Given: A circle without centre.

Required: To locate the center of circle.

Steps of Construction:

- (i) Take three points A, B and C on a circle.
 - (ii) Join these points to obtain a triangle ABC.
 - (iii) Draw right bisectors of sides AB, BC and AC which meet each other at point O.
- O is required center of circle.



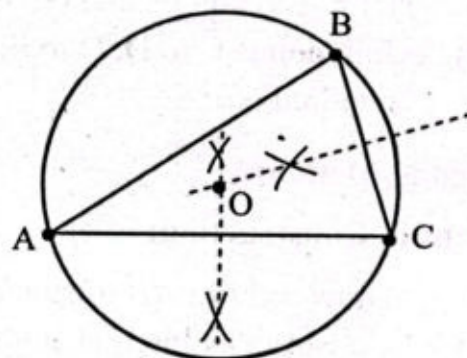
To Draw a Circle Passing Through Three Non-Collinear Points

Given: Three non-collinear points A, B and C.

Required: To draw a circle passing through these points.

Steps of Construction:

- (i) Take three non-collinear points A, B and C on a paper.
- (ii) Join these points to obtain a triangle ABC.
- (iii) Draw right bisectors of sides BC and AC which meet each other at point O.
- (iv) With center O, construct a circle of radius OA or OB.



To Complete the Circle when a Part of its Circumference is Given:

(a) By finding the Centre (b) Without finding the Centre

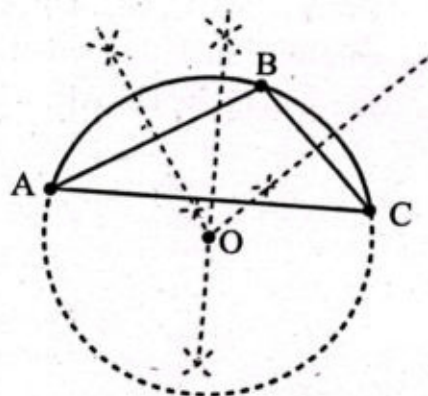
(a) To Complete a Circle by Finding its Centre when Part of Circumference is Given

Given: A part of circle i.e. an arc.

Required: To complete the circle by finding its center.

Steps of Construction:

- (i) Take three points A, B and C on the given arc.
- (ii) Join these points to obtain a triangle ABC.
- (iii) Draw right bisectors of sides AB, BC and AC which meet each other at point O.
- (iv) With center O, construct a circle of radius OA or OB of which the given arc is a part.



(b) To Complete a Circle without Finding its Centre when Part of Circumference is Given

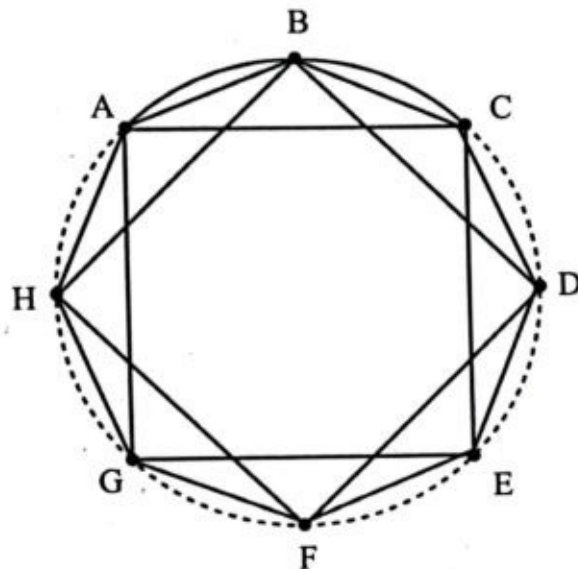
Given: A part of circle i.e. an arc.

Required: To complete the circle without finding its center.

First Method

Steps of Construction:

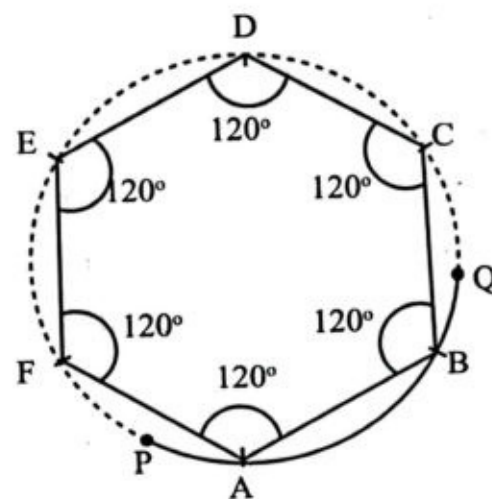
- (i) Take three points A, B and C on the given arc such that $AB = BC$.
(We can also $AB \neq BC$.)
- (ii) Join these points to obtain a triangle ABC.
- (iii) Draw $BD = AC$ and $CD = BC$ to obtain a triangle BCD.
- (iv) Again draw $CE = BD$ and $DE = CD$ to obtain a triangle CDE.
- (v) Continue in this way and construct triangles DEF, EFG, FGH and GHA.
- (vi) Join points C to D, D to E, E to F, F to G, G to H and H to A in circular path to obtain required circle.



Second Method

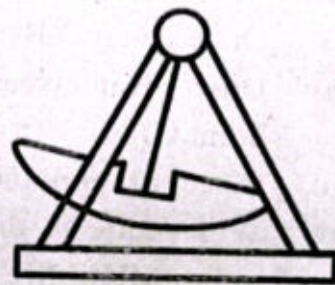
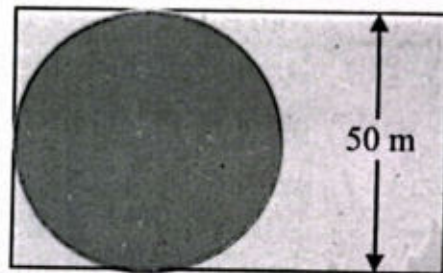
Steps of Construction:

- (i) Draw a chord AB of suitable length on an arc PQ.
- (ii) Construct an internal angle of measure 120° at point B and draw BC equal to AB.
- (iii) Again, construct an internal angle of measure 120° at point C and draw CD equal to BC.
- (iv) Continue this procedure to obtain points E and F.
- (v) Draw arcs BC, CD, DE, EF and FA carefully such that length of these arcs is equal to the arc AB.



Exercise 11.1

1. Given a circle, locate its centre.
2. Three points X, Y and Z are located on an arc. Find the centre of that arc.
3. Three non-collinear points A, B and C are such that $AB = BC = 3\text{cm}$ and $\angle B = 120^\circ$. Construct a circle passing through A, B and C.
4. PQ and QR are perpendicular segments in a plane such that $PQ = 4\text{cm}$ and $QR = 3\text{cm}$. Construct a circle passing through P, Q and R. What is the radius of that circle?
5. Draw a circle with the help of any round object, then find its centre.
6. Find the centre of an arc DEF, such that chord $DE = 3.5\text{cm}$, chord $EF = 4.2\text{cm}$ and angle between both segments is 60° .
7. Draw an arc of any length. Complete the circle without finding its centre of which the given arc is the part.
8. An engineer is constructing a circular path in a recreational park as shown below. He wants to plant a pine tree in the centre of the circular path.
 - (i) How will he locate the centre of the path?
 - (ii) What is the radius of the circular path?
9. A factory produces arc shaped tiles of different colors each of inner radius 7 inches. Arc length of each tile is 2 inches. Hamza wants to purchase these tiles for making a circular design on the wall. How many tiles are required?
10. In the figure a boat swing is shown. Trace the circular path of swing during a complete round.

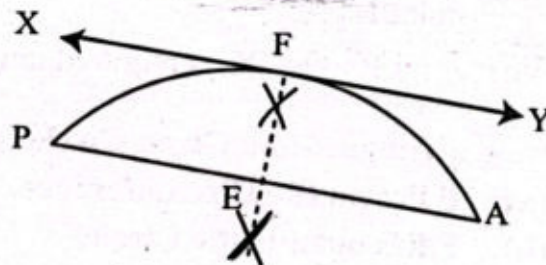


Tangent to the Circle

To draw a Tangent to a Given Arc without using the Centre, through a Given Point P when P is:

- (a) The middle point of the arc.
- (b) At the end of the arc.
- (c) Outside the arc.

- (a) **Given:** An arc PA without centre.
Required: To draw tangent at mid-point of arc.



Steps of Construction:

- (i) Draw an arc PA.
- (ii) Joint P to A.
- (iii) Draw right bisector EF of PA where E is mid-point of $\overline{PA}$ and F is a point on given arc.
- (iv) Draw a line XY passing through F parallel to $\overline{PA}$ using set square or by any other method.

$\overline{XY}$ is required tangent.

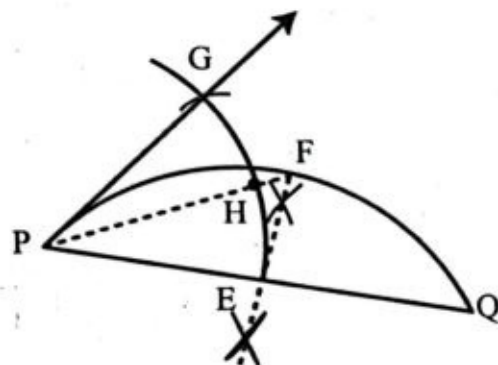
- (b) **Given:** An arc PQ without centre.

Required: To draw a tangent at end point P of the arc.

Steps of Construction:

- (i) Draw an arc PQ.
- (ii) Join P to Q.
- (iii) Draw right bisector EF of PQ where E is mid-point of $\overline{PQ}$ and F is a point on given arc.
- (iv) Join point P to F.
- (v) With centre P, draw an arc of radius PE intersecting PF at point H.
- (vi) Take point G on this arc such that EH = HG. Now join P to G.

$\overline{PG}$ is required tangent.

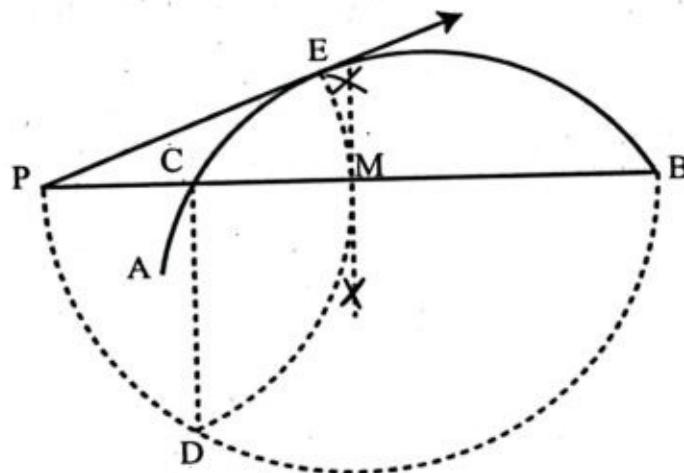


- (c) **Given:** An arc AB without knowing its centre.

Required: To draw a tangent from a point P where P is outside the arc.

Steps of Construction:

- (i) Draw an arc AB.
- (ii) Draw PB intersecting the arc AB at point C.
- (iii) Bisect PB at point M.
- (iv) With centre M, draw a semi-circle of radius MP or MB.
- (v) Draw CD perpendicular to PB intersecting semicircle at point D.
- (vi) With centre P, draw another arc of radius PD intersecting given arc AB at point E.
- (vii) Joint P to E. $\overline{PE}$ is required tangent.



To draw a Tangent to a Given Circle from a Point P when:

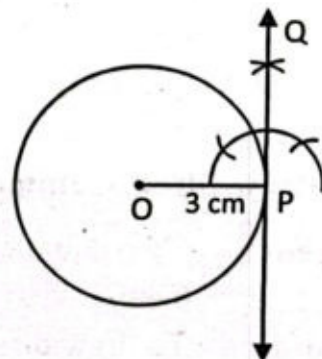
- (a) P lies on the Circumference.
- (b) P lies outside the Circle.

(a) **Given:** A circle of radius 3 cm.

Required: To draw a tangent from a point P lying on the circle.

Steps of Construction:

- (i) Construct a circle of radius 3 cm by taking O as centre.
- (ii) Take a point P on the circle and join point O to point P.
- (iii) Construct an angle of 90° at point P and draw a line PQ. The line PQ is required tangent.

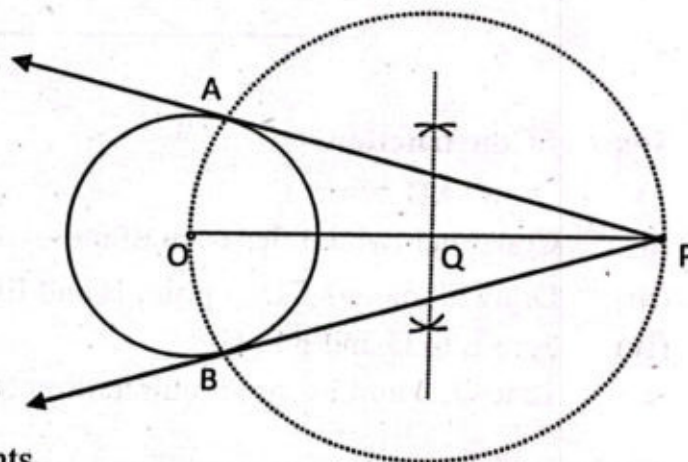


(b) **Given:** A circle of radius 3 cm and a point P 8 cm away from the centre of circle.

Required: To draw a tangent from a point P at a distance of 8 cm from the center of the circle.

Steps of Construction:

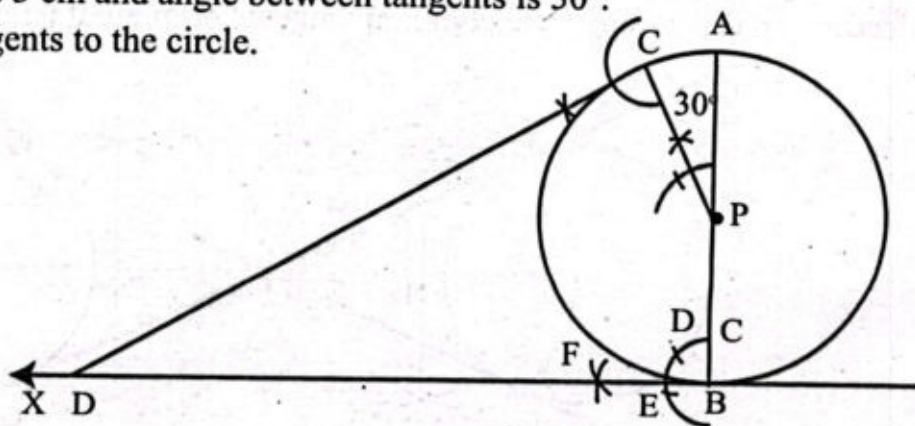
- (i) Construct a circle of radius 3 cm by taking O as centre.
- (ii) Take a point P at a distance of 8 cm from the centre.
- (iii) Bisect $\overline{OP}$ at point Q.
- (iv) Draw a dotted circle of radius OQ or PQ.
- (v) The two circles intersect each other at points A and B.
- (vi) Draw $\overrightarrow{PA}$ and $\overrightarrow{PB}$ which are required tangents.



To Draw two Tangents to a Circle Meeting each other at a Given Angle

Given: A circle of radius 3 cm and angle between tangents is 30° .

Required: To draw two tangents to the circle.



Steps of Construction:

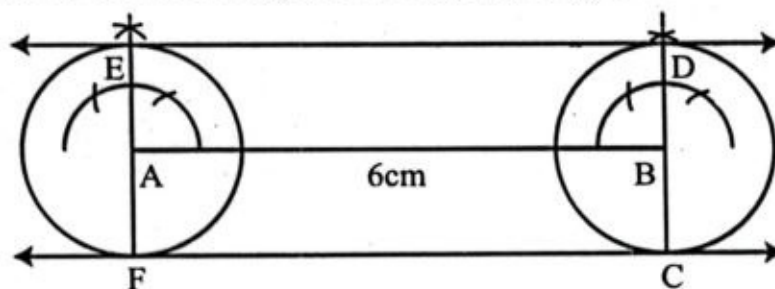
- (i) Construct a circle of radius 3 cm at point P.
- (ii) Draw a diameter AB.
- (iii) Draw a tangent BX to the circle at point B.

- (iv) Construct $\angle APC = 30^\circ$ and draw another tangent $\overline{CD}$ at point C intersecting the first tangent at point D.
- (v) Measure $\angle BDC$. It will be 30° .

To draw Direct Common Tangents (External Tangents) to two Equal Circles

Given: Two equal circles each of radius 2.5 cm and distance between centers of circles is 6 cm.

Required: To draw direct common tangents or external tangents.



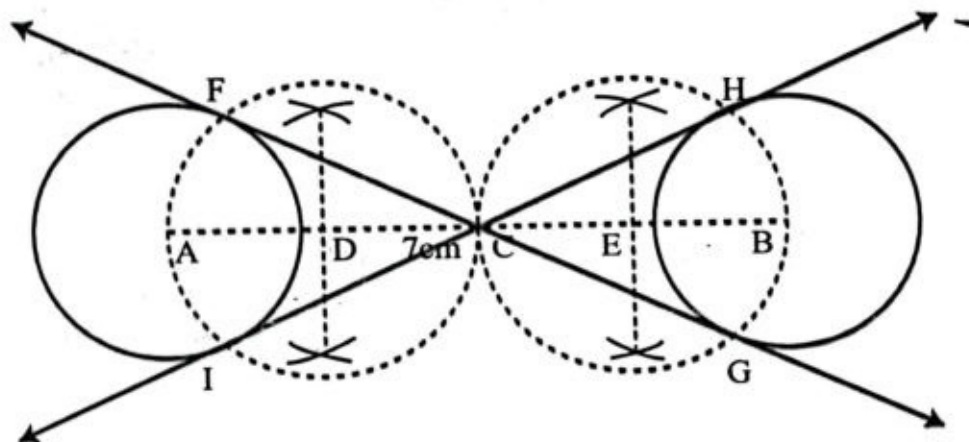
Steps of Construction:

- (i) Draw $\overline{AB} = 6$ cm.
 - (ii) Construct two circles each of radius 2.5 cm at points A and B.
 - (iii) Draw diameters CD at point B and EF at point A, perpendicular to $\overline{AB}$.
 - (iv) Join E to D and F to C.
- Lines ED and FC are required direct common tangents.

To draw Transverse Common Tangents (Internal Tangents) to two Equal Circles

Given: Two equal circles each of radius 3 cm and distance between centers of circles is 7 cm.

Required: To draw transverse common tangents or internal tangents.



Steps of Construction:

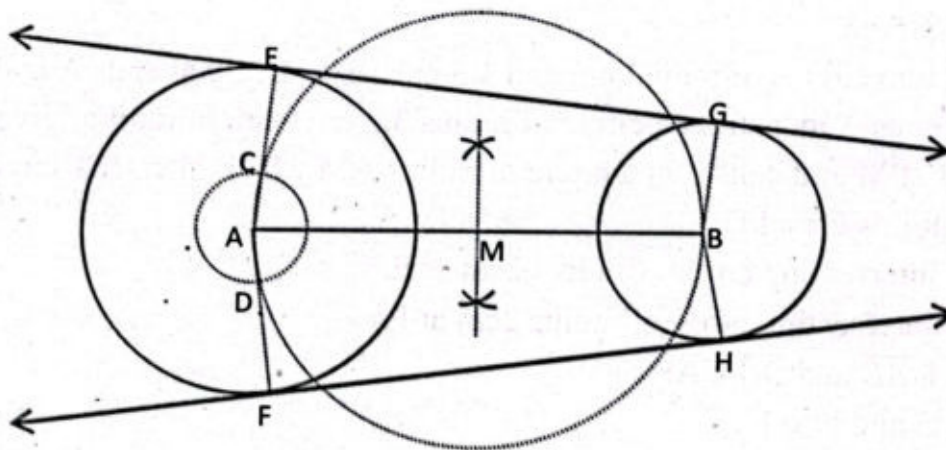
- (i) Draw $\overline{AB} = 7$ cm.
- (ii) Construct two circles each of radius 3 cm at points A and B.
- (iii) Bisect $\overline{AB}$ at point C.

- (iv) Further bisect $\overline{AC}$ at point D and $\overline{BC}$ at point E.
- (v) With centre D, construct a dotted circle of radius AD, intersecting the circle with centre A at points F and I.
- (vi) With centre E, construct another dotted circle of radius BE, intersecting the circle with centre B at points H and G.
- (vii) Join H to I and F to G.
 $\overleftrightarrow{FG}$ and $\overleftrightarrow{HI}$ are the required transverse common tangents.

To draw Direct Common Tangents to two Unequal Circles

Given: Two circles of radii 2 cm and 1.2 cm respectively and distance between centers of circles is 5 cm.

Required: To draw direct common tangents or external tangents.



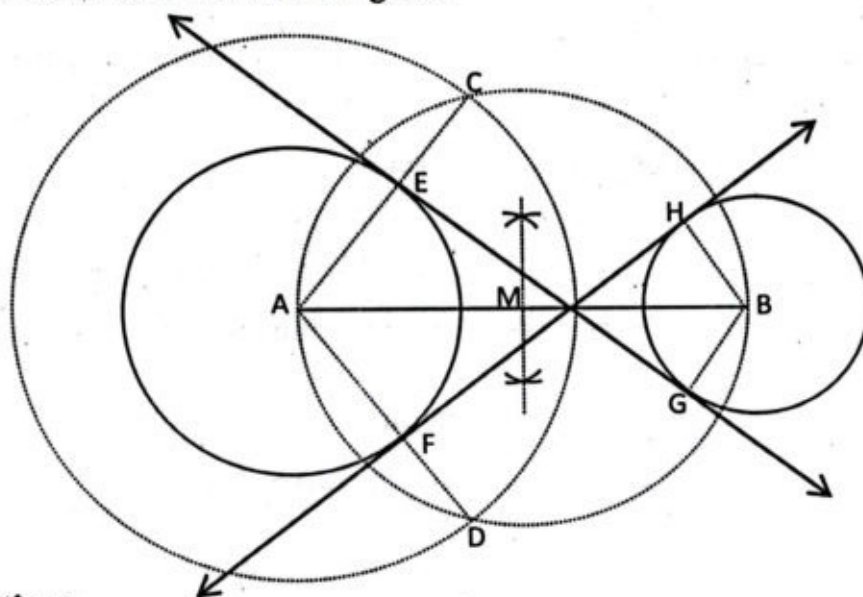
Steps of Construction:

- (i) Draw $AB = 5\text{cm}$.
- (ii) Construct two circles of radii 2 cm and 1.2 cm respectively at ends A and B.
- (iii) With center as A, construct a circle of radius 0.8 cm (difference of radii of given circles).
- (iv) Bisect $\overline{AB}$ at M and construct a circle of radius AM which intersects circle of radius 0.8cm at points C and D.
- (v) Produce $\overline{AC}$ to point E and $\overline{AD}$ to point F.
- (vi) Draw $\overline{BG} \parallel \overline{AE}$ and $\overline{BH} \parallel \overline{AF}$.
- (vii) Join E to G and F to H.
 $\overleftrightarrow{EG}$ and $\overleftrightarrow{FH}$ are required direct common tangents.

To draw Transverse Common Tangents to two Unequal Circles

Given: Two circles of radii 2 cm and 1.3 cm respectively and distance between centers of circles is 5 cm.

Required: To draw transverse common tangents



Steps of Construction:

- (i) Draw $AB = 5\text{cm}$.
 - (ii) Construct two circles of radii 2 cm and 1.3 cm respectively at ends A and B .
 - (iii) With center as A , construct a circle of radius 3.3 cm (sum of radii of given circles).
 - (iv) Bisect $\overline{AB}$ at M and construct a circle of radius AM which intersects circle of radius 3.3cm at points C and D .
 - (v) Draw $\overline{AC}$ intersecting circle of radius 2cm at E .
 - (vi) Draw $\overline{AD}$ intersecting circle of radius 2cm at F .
 - (vii) Draw $\overline{BG} \parallel \overline{AE}$ and $\overline{BH} \parallel \overline{AF}$.
 - (viii) Join E to G and F to H .
- $\overline{EG}$ and $\overline{FH}$ are required transverse common tangents.

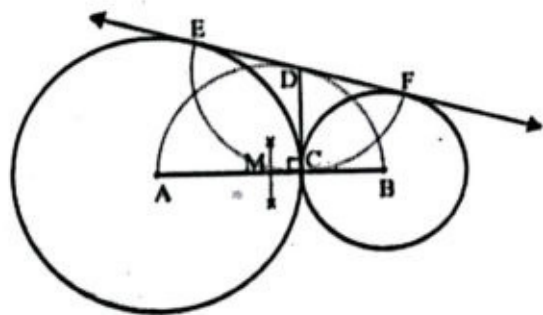
To draw a Tangent to two Unequal Touching Circles

Given: Two touching circles of radius 2.5 cm and 2 cm .

Required: To draw a tangent to the circles.

Steps of Construction:

- (i) Construct a circle of radius 2.5 cm at point A and a circle of radius 2 cm at point B in such a way that both circles touch each other externally at point C .
- (ii) Join A to B .
- (iii) Bisect $\overline{AB}$ at point M .
- (iv) With centre M , draw a dotted semi-circle of radius AM or BM .
- (v) Draw a perpendicular $\overline{CD}$ on $\overline{AB}$ where D is a point on the semi-circle.
- (vi) With centre D , draw an arc of radius CD intersecting the given circles at points E and F respectively.
- (vii) Join E to F .



Line EF is the required tangent.

Class Activity:

Draw a common tangent to two circles that are:

- (i) touching internally,
- (ii) touching externally,

such that tangent drawn is perpendicular to the radial segments of both circles.

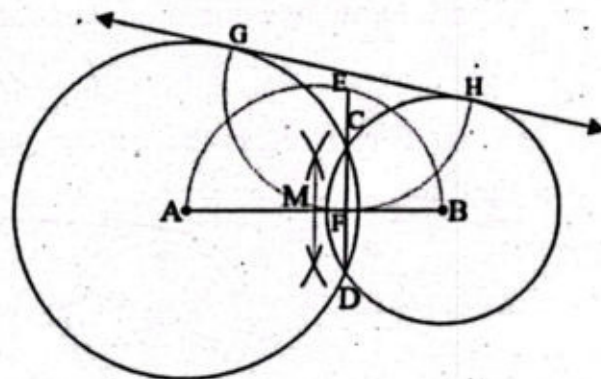
To draw a Tangent to two Unequal Intersecting Circles

Given: Two intersecting circles of radius 2.5 cm and 3 cm.

Required: To draw a tangent to the circles.

Steps of Construction:

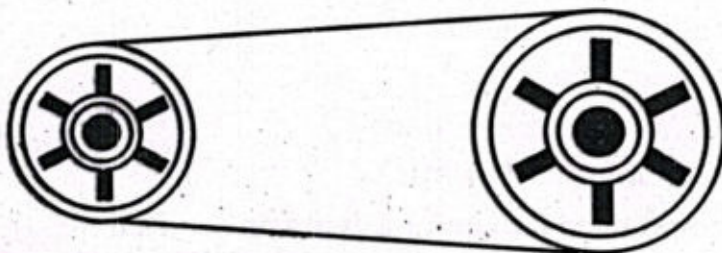
- (i) Construct a circle of radius 2.5 cm at point A. Construct another circle of radius 3 cm at point B in such a way that both circles intersect each other at points C and D.
- (ii) Join A to B.
- (iii) Bisect $\overline{AB}$ at point M. With centre M, draw a dotted semi-circle of radius AM or MB.
- (iv) Join point D to C and produce it to point E lying on semi-circle. $\overline{DE}$ intersects $\overline{AB}$ at point F.
- (v) With centre E, draw an arc of radius EF intersecting the given circles at points G and H respectively.
- (vi) Join G to H. Line GH is the required tangent.



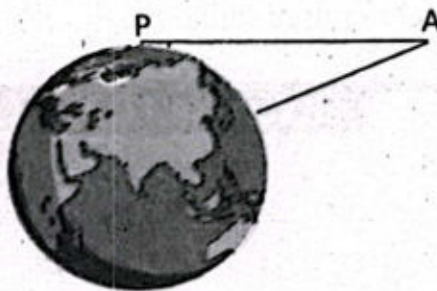
Exercise 11.2

1. Draw an arc and divide it into two equal parts.
2. Draw an arc of any length. Draw a tangent without using centre, through point A, when point A is: (i) in the middle of arc. (ii) at the right end of the arc.
3. Draw an arc ABC. Take a point X outside the arc and draw a tangent from X to the arc without using centre of the arc.
4. Construct a circle of radius 4cm. Mark a point P on the circumference and draw a tangent passing through P.
5. Construct a circle of radius 3cm. Draw two tangents from a point A, 5.8cm away from the centre of circle.
6. Construct a circle of radius 2.5cm. Draw two tangents making an angle of:
 - (i) 45° (ii) 90° with each other.

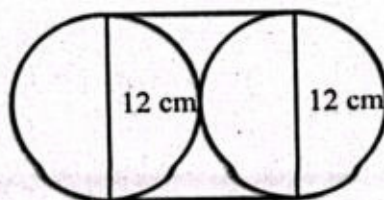
7. Construct two equal circles each of radius 3cm. Centres of the circles are 6cm apart. Draw their:
- direct common tangents.
 - transverse common tangents.
8. Construct two circles of radii 2cm and 3.5cm. Draw:
- direct common tangents
 - transverse common tangents to the circles, if the distances between centres of circles is 6.4cm.
9. Construct two touching circles of radii 2.6cm and 3.6 cm. Draw:
- a common tangent that passes through the point of contact of both circles.
 - a common tangent that does not pass through the point of contact of both circles.
 - two common tangents.
10. Construct two intersecting circles of radii 2.5cm and 3cm whose centres are 4.8cm apart. Draw two common tangents to the circles.
11. In the figure, two pulleys of radii 7 inches and 14 inches respectively are shown. Centres of pulleys are 28 inches apart. Find the approximate length of rope moving on pulleys.



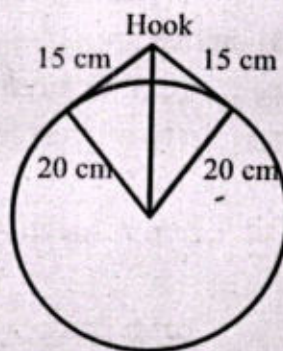
12. A satellite A is moving around Earth at an altitude of 7600 km from the surface. The radius of Earth is 6400 km. How far is the satellite from the city P?



13. Two cylindrical rods are bound with a strap. Each rod has diameter 12 cm. How long is the strap?



14. A circular mirror with radius 20 cm hangs by a wire from a hook. The wire is 30 cm long and is a tangent to the mirror in two places. How far above the top of the mirror is the hook?



I have Learnt

- Locating the centre of a given circle.
- Drawing a circle passing through three given non-collinear points.
- Completing the circle (i) by finding its centre, (ii) without finding its centre, when a part of its circumference is given by
- Drawing a tangent to a given arc, without using the centre, through a given point P when:
(i) P is middle point of the arc (ii) P is at the end of arc (iii) P is outside the arc.
- Drawing a tangent to a given circle from a point P when:
(i) P is on the circumference (ii) P is outside the arc.
- Drawing two tangents to a circle meeting each other at a given angle.
- Drawing (i) direct common tangents or external tangents (ii) transverse common tangents or internal tangents, to two equal/unequal circles.
- Drawing a tangent to two unequal (i) touching circles (ii) intersecting circles.
- Applying concepts of practical geometry of circles to real life word problems such as athletic tracks, recreational parks, Ferris wheels and mechanical machines.

MISCELLANEOUS EXERCISE-11

1. Encircle the correct option in the following.
 - i. Angle between tangent and radial segment of a circle is:
 (a) 30° (b) 45° (c) 60° (d) 90°
 - ii. Direct common tangents of equal circles are:
 (a) parallel (b) intersecting (c) converging (d) not parallel
 - iii. How many tangents can be drawn to two intersecting circles?
 (a) 1 (b) 2 (c) 3 (d) infinite
 - iv. Two circles having radii 3cm and 3.2cm respectively, touch externally. The distance between centres of the circles is:
 (a) 6cm (b) 0.2cm (c) 6.2cm (d) 5.3cm
 - v. Centres and point of contact of two touching circles are: _____
 (a) collinear (b) non-collinear (c) converging (d) coincident
 - vi. In which type of triangle, incentre and circumcenter are coincident?
 (a) scalene (b) isosceles (c) equilateral (d) right triangle
 - vii. Two circles with radii 4cm and 4.8cm respectively, touch internally. The distance between centres of the circles is:
 (a) 4cm (b) 8 cm (c) 8.8cm (d) 0.8cm
 - viii. Two tangents are drawn at the ends of diameter of a circle of radius 3.5cm. The distance between tangents is:
 (a) 3.5cm (b) 7cm (c) 5cm (d) 10.5cm
 - ix. Transverse common tangents intersect each other at:
 (a) 1 point (b) 2 points (c) 3 points (d) 4 points
 - x. A circle is inscribed in a square having length of side 6cm. What is the radius
 (a) 24 (b) 12 (c) 6 (d) 3
 - xi. A square is inscribed in a circle of radius 5cm. What is the length of diagonal of the square?
 (a) 2.5cm (b) 5cm (c) 10cm (d) 20cm
 - xii. Perpendicular bisectors of always pass through centre of circle.
 (a) tangents (b) secants (c) radial segments (d) chords
 - xiii. How many circles can be drawn through three non-collinear points?
 (a) infinite (b) 1 (c) 2 (d) 3
 - xiv. A circle have pair(s) of parallel tangents.
 (a) 1 (b) 2 (c) 3 (d) infinite