Based on National Curriculum 2022-23

Textbook of

Mathematics

GRADE 10

National Book Foundation

Ministry of Federal Education and Professional Training

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A Textbook of Mathematics for Grade 10
Based on National Curriculum of Pakistan (NCP) 2022-23

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PREFACE

Textbook of Mathematics for Grade 10 has been developed by NBF according to the National Curriculum of Pakistan 2022- 2023. The aim of this experimentation skills is to enhance learning abilities through inculcation of logical thinking in learners, and to develop higher order thinking processes by systematically building upon the foundation of learning from the previous grades. A key emphasis of the present experimentation skills is on creating real life linkages of the concepts and methods introduced. This approach was devised with the intent of enabling students to solve daily life problems as they go up the learning curve and for them to fully grasp the conceptual basis that will be built upon in subsequent grades.

After amalgamation of the efforts of experts and experienced author, this book was reviewed and finalized after extensive reviews by professional educationists. Efforts were made to make the contents student friendly and to develop the concepts in interesting ways.

The National Book Foundation is always striving for improvement in the quality of its books. The present book features an improved design, better illustration and interesting activities relating to real life to make it attractive for young learners. However, there is always room for improvement and the suggestions and feedback of students, teachers and the community are most welcome for further enriching the subsequent editions of this book.

May Allah guide and help us (Ameen).

Murad Ali Mohmand

Managing Director

Applications of Mathematics

Complex Numbers: Complex numbers play a critical role in fields such as engineering, electronics, and quantum physics. They are especially valuable for modeling oscillations in alternating current (AC) circuits, which are foundational to modern electrical systems, and for analyzing waveforms and resonances that occur naturally in many types of physical systems.

Quadratic Equations: Quadratic equations model a range of phenomena. In physics, they describe projectile motion; in finance, they are used for profit and cost optimization; and in architecture, they inform structural designs by calculating load distributions and parabolic shapes for stability.

Matrices and Determinants: Matrices are essential in solving complex systems of equations and executing transformations in areas like computer graphics and machine learning. Determinants, closely related to matrix invertibility, provide insights into properties of shapes and can be used to calculate area, volume, and to determine if linear systems have unique solutions.

Linear and Quadratic Inequalities: Linear and quadratic inequalities express ranges of values that satisfy given conditions, such as acceptable income levels, speed limits, or temperature thresholds. In real life, they are essential in budgeting, optimizing resources in industries, setting safety regulations, and environmental monitoring, where certain parameters must stay within safe or efficient boundaries.

Algebraic Fractions: Algebraic fractions simplify relationships found in finance (like calculating compound interest), physics (such as wave motion and light refraction), and data science (for probability ratios and odds in statistical models). They allow complex relationships to be expressed in simpler, comparable terms.

Functions and Graphs: Functions are mathematical tools that map inputs to outputs, visualized through graphs to represent relationships between variables, such as speed over time, cost versus quantity, or population growth over years. These graphical representations are fundamental in economics, biology, and physics, where understanding variable relationships is key to predicting and modeling real-world changes.

Vectors: Vectors combine magnitude and direction, making them indispensable in applications like navigation, physics (force and velocity calculations), and engineering (stress analysis in materials). In computer graphics, they help in rendering movements and 3D transformations, contributing to realistic visualizations.

Trigonometry: Trigonometry is crucial for solving right and non-right triangles. Its applications span navigation, construction, and astronomy, where precise distance and angle measurements are essential, from mapping locations to constructing stable structures.

Chords and Arcs of a Circle: Understanding chords and arcs forms the basis of circular geometry, used extensively in design, architecture, and mechanical engineering, where curves and circular components (like gears or wheels) must be accurately measured and constructed.

Tangents and Its Construction: A tangent touches a circle at one point, forming a right angle with the radius at that point. This principle has practical applications in road and race track design gear construction, and analyzing rotational motion, where precision at points of contact is crucial. Constructing tangents, arcs, and circular sections is fundamental in structural design, architecture, bridge engineering, and urban planning.

Statistics: Statistics, through measures like mean, median, mode, range, and standard deviation, help summarize and interpret data trends. Statistics play an essential role in business forecasting, healthcare analytics, social sciences, and environmental studies, where decisions must be made based on reliable data insights.

Table of Contents

01

Complex Numbers

After studying this unit students will be able to:

- Recognize complex numbers, conjugate and modulus of a complex numbers.
- Apply basic operations on complex numbers.
- Define commutative laws, associative laws and distributive laws for complex numbers.
- Recognize additive and multiplicative identity and find additive and multiplicative inverse of complex numbers.
- Find real and imaginary parts of $\left(\dfrac{x_1+iy}{x_2+iy}\right)^n$ for $n=\pm 1, \pm 2, \ldots$
- Find solution of equations having complex values.
- Apply the geometric interpretation of a complex numbers.
- Solve daily life problems involving complex numbers.

Until the sixteenth century, mathematicians were puzzled by square roots of a negative numbers. For example, the solutions of $x^2-2=0$ are $\sqrt{2}$, and $-\sqrt{2}$. But what are the solution of $x^2+2=0$? To find the solution of this equation, imaginary numbers were defined.

A French mathematician René Descartes was the first to emphasize the imaginary nature of numbers, positing that "one can imagine as many (numbers) as already mentioned in each equation, but sometimes, there is no quantity that matches what we imagine."

However, in 16th century an Italian mathematician Gerolamo Cardano, proved that having a negative term inside a square root can lead to the solution of an equation.

Later, in the 18th century, mathematician Carl Friedrich Gauss consolidated Cardano's premises, in addition to developing a treatise on complex numbers in a plane and thereby established the modern bases of the term.

René Descartes

Gerolamo Cardano

Carl Friedrich Gauss

Imaginary and Complex Numbers

Negative numbers do not have square roots in the real number system. However, a larger number system that contains the real number system is designed so that negative numbers do have square roots. This system is called the complex number system and it makes use of a number that is a square root of -1. We call this new number i.

Let us find the solution of:

$$x^2 + 1 = 0 \qquad \text{(i)}$$

From (i), $x^2 = -1$ which implies $x = \pm\sqrt{-1} = \pm i$

The number i is the solution of $x^2 + 1 = 0$, and is defined as:

$i = \sqrt{-1}$ where $i^2 = -1$

Since the square of a real number is not negative, therefore i is not a real number and is called the imaginary unit.

Using i we can define square root of any negative number.

Interesting Information

Girolamo Cardano began his career as a doctor and studied, taught and wrote mathematics as a side line. He held important positions at The University of Pavia and Bologna in Italy and wrote many works on arithmetic, astronomy, physics and medicine.

To express the roots of negative numbers in terms of i, we can use the fact that in the complex numbers, $\sqrt{-x} = \sqrt{-1}\sqrt{x} = i\sqrt{x}$. When x is a positive real number and i is the imaginary number, the number $i\sqrt{x}$ is called a pure imaginary number.

Example: Express each number in terms of i.

a. $\sqrt{-5}$ b. $\sqrt{-9}$ c. $\sqrt{-17}$

d. $-\sqrt{-64}$ e. $3i \times 5i$ f. $\sqrt{-5} \times \sqrt{-20}$

Solution:

a. $\sqrt{-5} = \sqrt{-1 \times 5} = \sqrt{-1} \times \sqrt{5} = i\sqrt{5} = \sqrt{5}i$

b. $\sqrt{-9} = \sqrt{-1 \times 9} = \sqrt{-1} \times \sqrt{9} = i \times 3 = 3i$

We prefer to write ia as ai, where a is rational number.

c. $-\sqrt{-17} = -\sqrt{-1 \times 17} = -\sqrt{-1} \times \sqrt{17} = -i\sqrt{17} = -\sqrt{17}i$

d. $-\sqrt{-64} = -\sqrt{-1 \times 64} = -\sqrt{-1} \times \sqrt{64} = -i\sqrt{64} = -i8 = -8i$

e. $3i \times 5i = 15i^2 = 15 \times (-1) = -15$ where $(i^2 = -1)$

f. $\sqrt{-5} \times \sqrt{-20} = \sqrt{-1 \times 5} \times \sqrt{-1 \times 20} = (\sqrt{-1} \times \sqrt{5}) \times (\sqrt{-1} \times \sqrt{20})$

$= i\sqrt{5} \times i\sqrt{20} = i^2\sqrt{5 \times 20} = -1\sqrt{100} = -10$

Imaginary Number

An imaginary number is a number that can be written $a + bi$, where a and b are real numbers, $b \neq 0$.

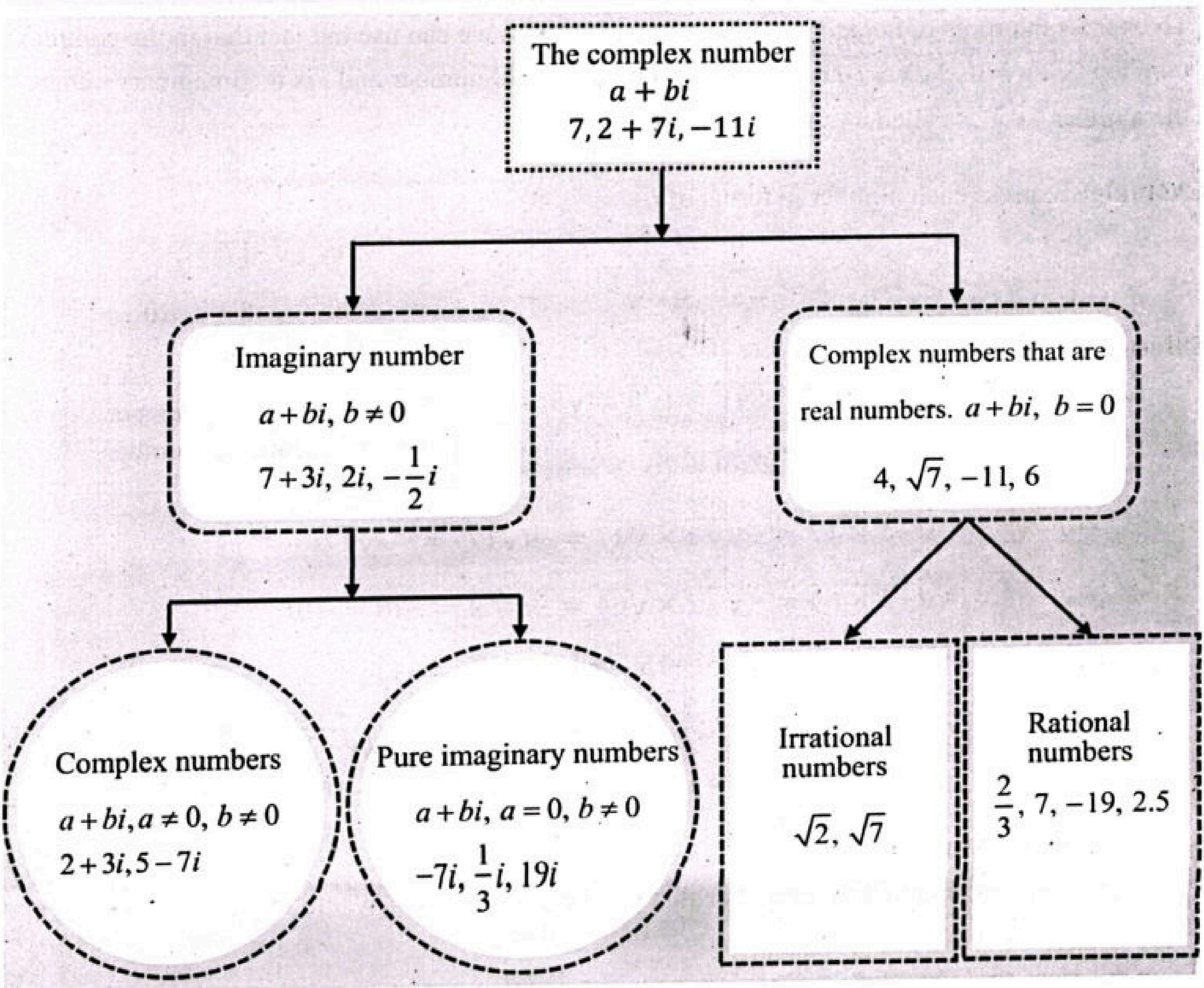
Complex Numbers

When a and b are real numbers and b is allowed to be 0, the number $a+bi$ is said to be complex.

A complex number is any number that can be written in the form of $a+bi$, where a and b are any real numbers and both a and b can be zero.

Key Fact:

- The numbers $2+5i$, $7-11i$, $18i$ are imaginary as $b \neq 0$ in these numbers. But they are also complex numbers.
- Complex number like $5i$ in which $a=0$ and $b \neq 0$ are imaginary number with no real part. Such numbers are called pure imaginary numbers.
- As 6 can be written as: $6+0i$. Therefore, every real number is a complex number with 0 as imaginary part.
- The set of complex numbers is denoted by C and R ⊆ C.



Addition and Subtraction of Complex Numbers

The complex numbers obey the commutative, associative and distributive law. Thus, we can add and subtract them as we do binomials. Moreover, the sum and difference of two complex numbers is also a complex number. If $z_1 = a + bi$ and $z_2 = c + di$ are two complex numbers then their sum and difference is defined as:

Sum: Add real part in real part and imaginary part in imaginary part.

$$z_1 + z_2 = (a + bi) + (c + di) = (a + c) + (b + d)i$$

Difference: Subtract real part in real part and imaginary part in imaginary part.

$$z_1 - z_2 = (a + bi) - (c + di) = (a - c) + (b - d)i$$

Example: Simplify:

a. $(7+5i)+(4+3i)$

b. $(5+6i)-(7-4i)$

Solution:

a. $(7+5i)+(4+3i)$
$=7+5i+4+3i$
$=7+4+5i+3i$ (combine real and imiginary parts)
$=11+8i$

b. $(5+6i)-(7-4i)$
$=5+6i-7+4i$
$=5-7+6i+4i$ (combining real and imiginary parts)
$=-2+10i$

Multiplication of Complex Numbers

If $a + bi$ and $c + di$ are two complex numbers then their product can be found by using distributive law.

$$(a + bi)(c + di) = a(c + di) + bi(c + di) = ac + adi + bci + bdi^2$$

$$= ac + adi + bci - bd \quad = (ac - bd) + (ad + bc)i \quad \text{..... (as } i^2 = -1)$$

Therefore, the product of two complex numbers is also a complex number.

For complex numbers, the property $\sqrt{a}\sqrt{b} = \sqrt{ab}$ does not hold in general, but it does when a is negative and b is a non-negative number.

To express square root of negative numbers, we first express them in terms of i. For example:
$\sqrt{-3}\times\sqrt{-7} = \sqrt{-1}\sqrt{3}\times\sqrt{-1}\sqrt{7} = i\sqrt{3}\times i\sqrt{7} = i^2\sqrt{21} = -\sqrt{21}$: correct.

But, $\sqrt{-2}\times\sqrt{-5} = \sqrt{-(2)(-5)} = \sqrt{10}$ is wrong.

Example: Multiply and simplify:

a. $\sqrt{-2}\sqrt{-11}$

b. $-2i.5i$

c. $-3i(2-8i)$

d. $\sqrt{-9}\sqrt{-36}$

e. $(3-2i)(1+5i)$

f. $(5-i)(9-3i)$

Solution:

a. $\sqrt{-2}\sqrt{-11} = \sqrt{-1}\sqrt{2}\times\sqrt{-1}\sqrt{11} = i\sqrt{2}\times i\sqrt{11} = i^2\sqrt{2\times 11} = -\sqrt{22}$

b. $-2i.5i = -10(i^2) = -10(-1) = 10$ (as $i^2 = -1$)

c. $-3i(2-8i) = -6i + 24(i^2) = -6i + 24(-1) = -6i - 24 = -24 - 6i$

d. $\sqrt{-9}\sqrt{-36} = \sqrt{-1\times 9}\sqrt{-1\times 36} = \sqrt{-1}\sqrt{9}\times\sqrt{-1}\sqrt{36} = i3\times i6 = 18i^2 = -18$

e. $(3-2i)(1+5i) = 3 + 15i - 2i - 10i^2 = 3 + 13i - 10(-1) = 13 + 13i$

f. $(5-i)(9-3i) = 45 - 15i - 9i + 3i^2 = 45 - 24i + 3(-1) = 42 - 24i$

Powers of *i*

We now want to simplify certain expressions involving higher powers of *i*. simplifying powers of *i* can then be done by using the fact that $i^2 = -1$ and expressing the given power of *i* in terms of i^2. Consider:

$i^3 = i^2.i = (-1).i = -i$
$i^5 = i^4.i = (i^2)^2.i = (-1)^2.i = i$
$i^7 = i^6.i = (i^2)^3.i = (-1)^3.i = -i$

$i^4 = (i^2)^2 = (-1)^2 = 1$
$i^6 = (i^2)^3 = (-1)^3 = -1$
$i^8 = (i^2)^4 = (-1)^4 = 1$

The powers of *i* cycle themselves through the values of *i*, –1, –*i* and 1.

Example: Simplify:

a. i^{11} b. i^{22} c. i^{61} d. i^{100}

Solution:

a. $i^{11} = (i^2)^5.i = (-1)^5.i = (-1).i = -i$

b. $i^{22} = (i^2)^{11} = (-1)^{11} = -1$

c. $i^{61} = (i^2)^{30}.i = (-1)^{30}.i = (1).i = i$

d. $i^{100} = (i^2)^{50} = (-1)^{50} = 1$

Key Fact:

Imaginary numbers appear in the field of engineering and physical sciences.

Example: Write the expressions in the form of $a+ib$.

a. $2+i^3$ b. $19+7i^2$ c. $i^{12}+i^{15}$ d. $4i^3+8i^6$

Solution:

a. $2+i^3 = 2+i^2.i = 2+(-1).i = 2-i$

b. $19+7i^2 = 19+7(-1) = 19-7 = 12+0i$

c. $i^{12}+i^{15} = (i^2)^6+(i^2)^7.i = (-1)^6+(-1)^7.i$
$= 1+(-1).i = 1-i$

d. $4i^3+8i^6 = 4i^2.i+8(i^2)^3 = 4(-1).i+8(-1)^3$
$= -4i+8(-1) = -4i-8 = -8-4i$

Conjugates of Complex Numbers

Conjugate of complex number is a complex number obtained by changing the sign of the imaginary part. In simple words, conjugate of a complex number is a number that has the same real part as the original complex number, and the imaginary part has the same magnitude but opposite sign.

The conjugate of a complex number $a+ib$ is $a-ib$.

For example, the conjugate of $5+6i$ is $5-6i$ and that of $3-11i$ is $3+11i$.

Commonly, the complex number is denoted by z and its conjugate by $\overline{z}$.

For example, if $z = 11+7i$ then $\overline{z} = \overline{11+7i} = 11-7i$

Example: Find the conjugate:

a. $5+9i$ b. $13-5i$ c. $-5-7i$ d. $11i$

Solution:

a. $5+9i$ the conjugate is $5-9i$

b. $13-5i$ the conjugate is $13+5i$

c. $-5-7i$ the conjugate is $-5+7i$

d. $11i$ the conjugate is $-11i$

Division of Complex Numbers

Dividing complex numbers is a little more complicated than addition, subtraction, and multiplication of complex numbers because it is difficult to divide a number by an imaginary number.

For dividing complex numbers, we need to find a term by which we can multiply the numerator and the denominator that will eliminate the imaginary part of the denominator so that we end up with a real number in the denominator.

If $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$ are the two complex numbers, then division of z_1 by z_2 is written as:

$$\frac{z_1}{z_2}=\frac{x_1+iy_1}{x_2+iy_2} \quad \ldots\ldots \text{(i)}$$

To eliminate the imaginary part in the denominator, we multiply the numerator and denominator of (i) by the conjugate of x_2+iy_2 i.e. by x_2-iy_2

$$\frac{z_1}{z_2}=\frac{x_1+iy_1}{x_2+iy_2}\times\frac{x_2-iy_2}{x_2-iy_2} \quad \ldots\ldots \text{(ii)}$$

After that, we simplify the expression (ii).

Steps for Dividing Complex Numbers

To divide the two complex numbers, follow these steps:

- First, calculate the conjugate of the complex number that is at the denominator of the fraction.
- Multiply the conjugate with the numerator and the denominator of the complex fraction.
- Apply the algebraic identity $(a+b)(a-b)=a^2-b^2$ in the denominator and substitute $i^2=-1$.
- Apply the distributive property in the numerator and simplify.
- Separate the real part and the imaginary part of the resultant complex number.

Example: Divide and simplify to the form $a+ib$.

a. $\dfrac{-7+3i}{2-5i}$ b. $\dfrac{11+5i}{10i}$ c. $\dfrac{1-7i}{3+4i}$

Solution:

a. $\dfrac{-7+3i}{2-5i}$

$=\dfrac{-7+3i}{2-5i}\times\dfrac{2+5i}{2+5i}$ multiplying and dividing by conjugate of denominator

$$=\frac{(-7+3i)(2+5i)}{(2-5i)(2+5i)}=\frac{-14-35i+6i+15i^2}{(2)^2-(5i)^2}$$

$$=\frac{-14-35i+6i+15(-1)}{4-25i^2}=\frac{-14-29i-15}{4-25(-1)}$$

$$=\frac{-29-29i}{4+25}=\frac{-29-29i}{29}=\frac{29(-1-i)}{29}$$

$$=-1-i$$

b. $\frac{11+5i}{10i}$

$\frac{11+5i}{10i}\times\frac{-10i}{-10i}$ multiplying and dividing by conjugate of denominator $(10i=0+10i)$

$$=\frac{(11+5i)(-10i)}{(10i)(-10i)}=\frac{-110i-50i^2}{-100i^2}=\frac{-110i-50(-1)}{-100(-1)}$$

$$=\frac{-110i+50}{100}=\frac{50-110i}{100}=\frac{50}{100}-\frac{110}{100}i$$

$$=\frac{1}{2}-\frac{11}{10}i$$

c. $\frac{1-7i}{3+4i}$

$=\frac{1-7i}{3+4i}\times\frac{3-4i}{3-4i}$ multiplying and dividing by conjugate of denominator

$$=\frac{(1-7i)(3-4i)}{(3+4i)(3-4i)}=\frac{3-4i-21i+28i^2}{(3)^2-(4i)^2}=\frac{3-25i+28(-1)}{9-16i^2}$$

$$=\frac{3-25i-28}{9-16(-1)}=\frac{-25-25i}{9+16}=\frac{-25-25i}{25}=\frac{25(-1-i)}{25}$$

$$=-1-i.$$

Exercise 1.1

1. Simplify and express in terms of iy.

   i. $\sqrt{-3}$ ii. $6\sqrt{-4}$ iii. $\sqrt{-\frac{4}{9}}$

   iv. $-\sqrt{-20}$ v. $4-\sqrt{-60}$ vi. $\sqrt{-8}\sqrt{-2}$

2. Simplify:

   i. $(4-i)+(5+5i)$ ii. $(7-6i)-(5-6i)$ iii. $(-2+8i)-(7+3i)$

   iv. $(4-2i)-(5-2i)$ v. $(2+4i)(1+2i)$ vi. $(1-4i)(2-3i)$

   vii. $-8i(2-2i)$ viii. $(3+2i)^2$ ix. $(3-6i)(3+6i)$

x. $(-5-3i)^2$ xi. $(1+\sqrt{2}i)(1-\sqrt{3}i)$ xii. $(\sqrt{2}+i)(\sqrt{2}-i)$

3. Simplify:

i. i^9	ii. i^{13}	iii. i^{28}
iv. $(-i)^{21}$	v. $(3i)^3$	vi. $(-2i)^4$

4. Simplify in the form of $a+bi$.

i. $9+i^6$	ii. $-17+i^5$	iii. i^4-13i
iv. i^5+21i	v. i^5+i^7	vi. $i^{74}-i^{100}$

5. Divide and simplify in the form of $a+bi$.

i. $\dfrac{3}{4-i}$	ii. $\dfrac{3i}{6+5i}$	iii. $\dfrac{3-i\sqrt{5}}{3+i\sqrt{5}}$	iv. $\dfrac{2+7i}{5i}$
v. $\dfrac{4+5i}{4-5i}$	vi. $\dfrac{3+2i}{2+i}$	vii. $\dfrac{5+i}{1+2i}$	viii. $\dfrac{a+ib}{a-ib}$
ix. $\dfrac{1+i}{(1-i)^2}$	x. $\dfrac{(2+2i)^2}{(1+i)^2}$		

Developing skilled knowledge

- ✓ What is an imaginary number?
- ✓ Is an imaginary number real number?
- ✓ Provide examples of numbers that are pure imaginary numbers.
- ✓ What is the pattern formed by the powers of i?
- ✓ Can we say product of $(2-3i)(2-3i)$ is imaginary?
- ✓ If conjugate of z is $\overline{z}$, then is $z\overline{z}$ a real number?

Properties of Complex Numbers

Let $z=a+bi$, $w=c+di$ and $v=e+fi$ be any three complex numbers, then the following properties hold.

- $z+w=w+z$ (commutative law of addition)
- $z\times w == w\times z$ (commutative law of multiplication)
- $z+0=0+z=z$ (existence of additive identity where $0=0+0i$)
- $z\times 1=1\times z=z$ (existence of multiplicative identity where $1=1+0i$)
- $(z+w)+v=z+(w+v)$ (associative law of addition)
- $(z\times w)\times v=z\times(w\times v)$ (associative law of multiplication)
- $z(w+v)=zw+zv$ (distributive law of multiplication over addition)
- $z(w-v)=zw-zv$

Challenge

Prove the above properties by taking, $z=1+2i$, $w=3-i$ and $v=2+2i$

Additive Inverse of Complex Numbers

If $z = a + bi$ is a complex number then its additive inverse is $-z = -a - bi$.

In the same way, additive inverse of $-z = -a - bi$ is $z = a + bi$.

Sum of a complex number and its additive inverse is zero.

$z + (-z) = (a + bi) + (-a - bi) = a + bi - a - bi = 0 = 0 + 0i$

Example:

Find the additive inverse of $3-2i$.

Solution:

Let $a+bi$ be the additive inverse of $3-2i$.

Then by additive inverse rule:

$$(3 - 2i) + (a + bi) = 0 + 0i$$
$$(3 + a) + (-2i + bi) = 0 + 0i$$
$$(3 + a) + (-2 + b)i = 0 + 0i$$

Comparing real and imaginary parts of both sides, we get:

$3 + a = 0; -2 + b = 0 \quad \Rightarrow \quad a = -3; b = 2$

Thus, additive inverse of $3 - 2i$ is $-3 + 2i$.

Equality of Complex Numbers

If two complex numbers $a + bi$ and $c + di$ are equal then:

$a = c$ and $b = d$

Direct Method:

We can apply the rule that additive inverse is the opposite sign of $3-2i$ that is $-3+2i$. Similarly the additive inverse of $-7+11i$ is $7-11i$.

Multiplicative Inverse of Complex Numbers

The multiplicative inverse of $a+bi$ is $\dfrac{1}{a+bi}$.

Example: Find the multiplicative inverse of $7-5i$.

Solution: The multiplicative inverse of $7-5i$ is $\dfrac{1}{7-5i}$.

Now, simplifying the expression $\dfrac{1}{7-5i} = \dfrac{1}{7-5i} \times \dfrac{7+5i}{7+5i}$ multiplying by conjugate

$$= \frac{7 + 5i}{(7)^2 - (5i)^2} \Rightarrow \frac{7 + 5i}{49 - 25i^2} \Rightarrow \frac{7 + 5i}{49 - 25(-1)}$$

$$= \frac{7 + 5i}{49 + 25} = \frac{7 + 5i}{84} = \frac{7}{84} + \frac{5}{84}i$$

$$= \frac{1}{12} + \frac{5}{84}i$$

Verification: $\quad (7 - 5i) \times \left(\dfrac{1}{12} + \dfrac{5}{84}i\right) = (7 - 5i)\left(\dfrac{7 + 5i}{84}\right)$

$$= \frac{(7)^2 - (5i)^2}{84} = \frac{49 - 25i^2}{84} = \frac{49 - 25(-1)}{84}$$

$$= \frac{49 + 25}{84} = \frac{84}{84} = 1$$

Key Fact:

The product of a complex number and its multiplicative inverse is 1.

Properties of Conjugate of Complex Number

Key Fact:

The complex number $a+ib$ can be written as (a,b), a is a real part and b is imaginary part.

a. $z\bar{z}$ is a real number.

Consider $z=7-6i$ then $\bar{z}=\overline{7-6i}=7+6i$

$z\bar{z}=(7-6i)(7+6i)=(7)^2-(6i)^2=49-36i^2$

$=49-36(-1)=49+36=85$

b. $\overline{z_1 z_2}=\bar{z}_1.\bar{z}_2$

Consider $z_1=1-2i$ and $z_2=4+3i$, then:

$\bar{z}_1=1+2i$ and $\bar{z}_2=4-3i$

Now, LHS $= z_1.z_2=(1-2i)(4+3i)=4+3i-8i-6i^2$

$=4-5i-6(-1)=4-5i+6=10-5i$

$\overline{z_1.z_2}=\overline{10-5i}=10+5i$

RHS $=\bar{z}_1.\bar{z}_2=(1+2i)(4-3i)=4-3i+8i-6i^2$

$=4+5i-6(-1)=4+5i+6=10+5i$

Hence, $\overline{z_1.z_2}=\bar{z}_1.\bar{z}_2$

c. $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2}$

Consider $z_1=-1+5i$ and $z_2=2+i$

Then, $\bar{z}_1=-1-5i$, $\bar{z}_2=2-i$

Taking $\frac{z_1}{z_2}=\frac{-1+5i}{2+i}=\frac{-1+5i}{2+i}\times\frac{2-i}{2-i}=\frac{(-1+5i)(2-i)}{(2+i)(2-i)}$

$=\frac{-2+i+10i-5i^2}{(2)^2-(i)^2}=\frac{-2+11i-5(-1)}{4-(-1)}=\frac{-2+11i+5}{4+1}=\frac{3+11i}{5}=\frac{3}{5}+\frac{11}{5}i$

LHS $=\overline{\left(\frac{z_1}{z_2}\right)}=\frac{3}{5}-\frac{11}{5}i$

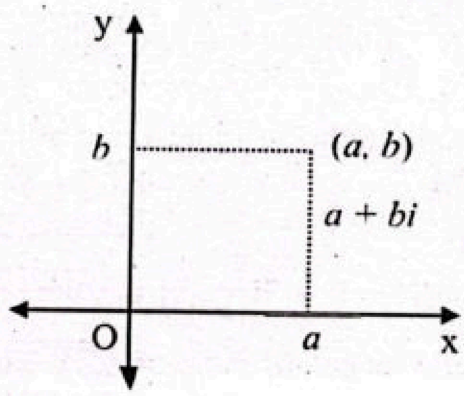
RHS $=\frac{\bar{z}_1}{\bar{z}_2}=\frac{-1-5i}{2-i}=\frac{-1-5i}{2-i}\times\frac{2+i}{2+i}=\frac{(-1-5i)(2+i)}{(2-i)(2+i)}$

$=\frac{-2-i-10i-5i^2}{(2)^2-(i)^2}=\frac{-2-11i-5(-1)}{4-(-1)}=\frac{-2-11i+5}{4+1}=\frac{3-11i}{5}=\frac{3}{5}-\frac{11}{5}i$

Hence, $\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2}$

Graphical Representation of Complex Number

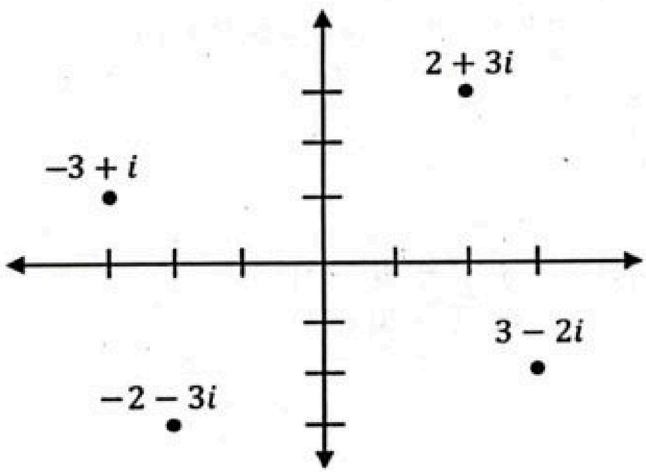
In complex number $z=a+ib$, there are two parts of z, a is a real part and b is an imaginary part associated with i. On the graphical representation of complex number z, the real part of z is taken along x-axis and imaginary part is taken along y-axis and considered as to real axis and imaginary axis respectively. The coordinate plane itself is called the complex plane or z-plane.



Example: Represent the numbers in the complex plane.

$2+3i,\ 3-2i,\ -3+i,\ -2-3i$

Solution:



Key Fact:

The complex plane is also called "The Argand Diagram" after the French-Swiss mathematician Jean Robert Argand (1768-1822) who was the first to represent a complex number geometrically as point in the plane.

We observe that the complex numbers appear in all the four quadrants due to the negative and positive signs of their real and imaginary parts.

Key Fact:

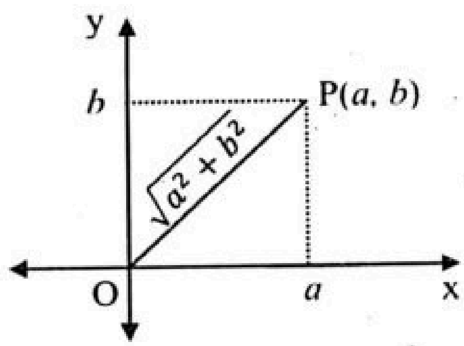
If $x=0$, then $x+yi=yi$ is an imaginary number located on vertical axis.
If $y=0$, then $x+yi=x$ is a real number located on horizontal axis.

Absolute Value or Modulus of a Complex Number

Let $z=(a,b)=a+ib$ be a complex number.
The absolute value or modulus of z, denoted by $|z|$ is defined as:

$$|z|=\sqrt{a^2+b^2}$$

which is a real number. From the diagram, it is clear that the modulus of a complex number is the distance OP of complex number from the origin.



Key Fact:

If, $z=a+ib$, then $|z|=\sqrt{(\text{real part})^2+(\text{img part})^2}=$ Real Number

Example: Find the absolute value of the complex numbers:

a. $5i$ b. 2 c. $4-7i$ d. $-3+2i$

Solution:

a. $z=5i=0+5i=|z|=\sqrt{0^2+5^2}=\sqrt{25}=5$

b. $z=2=2+0i=|z|=\sqrt{2^2+0^2}=\sqrt{4}=2$

c. $z = 4 - 7i = |z| = \sqrt{(4)^2 + (-7)^2} = \sqrt{16 + 49} = \sqrt{65}$

d. $z = -3 + 2i = |z| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9 + 4} = \sqrt{13}$

Properties of Modulus

If z is a complex number, then following properties holds:

a. $|z| = |-z| = |\overline{z}| = |-\overline{z}|$ *b.* $\overline{z}.z = |z|^2$

a. Consider a complex number $z = 2 - 5i$

$|z| = \sqrt{(2)^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29}$...... (i)

$-z = -(2 - 5\,i) = -2 + 5i$

$|-z| = \sqrt{(-2)^2 + (5)^2} = \sqrt{4 + 25} = \sqrt{29}$...... (ii)

$-\overline{z} = -2 - 5i$

$|\overline{z}| = \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$ ... ... (iii)

$\overline{z} = \overline{2 - 5i} = 2 + 5i$

$|-\overline{z}| = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29}$... (iv)

From (i), (ii), (iii), and (iv): $|z| = |-z| = |\overline{z}| = |-\overline{z}|$

b. Consider $z = 2 - 5i$

$\overline{z} = 2 + 5i$ then,

$\overline{z}.z = (2 + 5i)(2 - 5i)$

$= (2)^2 - (5i)^2 = 4 - 25i^2$

$= 4 - 25(-1) = 4 + 25$

$= 29$ ..... (i)

$|z| = \sqrt{(2)^2 + (-5)^2}$

$= \sqrt{4 + 25} = \sqrt{29}$

$|z|^2 = (\sqrt{29})^2 = 29$ ... ... (ii)

From (i) and (ii),

$\overline{z}.z = |z|^2$

Example: Find the real and imaginary parts of the complex numbers:

a. $(2-3i)^2$ b. $(1+2i)^{-2}$ c. $\dfrac{(\sqrt{2}+i\sqrt{3})^2}{1+i}$ d. $\dfrac{(1-i)^2}{(1+i)^2}$

Solution:

a. $(2-3i)^2 = (2)^2 + (3\text{i})^2 - 12i = 4 + 9i^2 - 12i = 4 + 9(-1) - 12\text{i}$

$= 4 - 9 - 12i = -5 - 12i$. Real part $= -5$, Imiginary part $= -12$

b. $$(1+2i)^{-2} = \frac{1}{(1+2i)^2} = \frac{1}{(1)^2 + (2\text{i})^2 + 4i} = \frac{1}{1 + 4i^2 + 4i} = \frac{1}{1 + 4(-1) + 4\text{i}}$$

$$= \frac{1}{1 - 4 + 4i} = \frac{1}{-3 + 4i} = \frac{1}{-3 + 4i} \times \frac{-3 - 4i}{-3 - 4i}$$

$$= \frac{-3 - 4i}{(-3)^2 - (4i)^2} = \frac{-3 - 4i}{9 - 16i^2} = \frac{-3 - 4i}{9 - 16(-1)}$$

$$= \frac{-3 - 4i}{9 + 16} = \frac{-3 - 4i}{25} = \frac{-3}{25} - \frac{4}{25}i.$$ Real part $= \dfrac{-3}{25}$, Imiginary part $= \dfrac{-4}{25}$

c. $\dfrac{(\sqrt{2}+i\sqrt{3})^2}{1+i}=\dfrac{(\sqrt{2})^2+(i\sqrt{3})^2+2i\sqrt{2}\sqrt{3}}{1+i}=\dfrac{2+3i^2+2i\sqrt{3\times2}}{1+i}$

$=\dfrac{2+3(-1)+2i\sqrt{6}}{1+i}=\dfrac{2-3+2i\sqrt{6}}{1+i}=\dfrac{-1+2i\sqrt{6}}{1+i}$

$=\dfrac{-1+2i\sqrt{6}}{1+i}\times\dfrac{1-i}{1-i}=\dfrac{(-1+2i\sqrt{6})(1-i)}{(1)^2-(i)^2}=\dfrac{-1+2i\sqrt{6}+i-2i^2\sqrt{6}}{1-i^2}$

$=\dfrac{-1+(2\sqrt{6}+1)i+2\sqrt{6}}{1-(-1)}=\dfrac{-1+2\sqrt{6}+(2\sqrt{6}+1)i}{2}=\dfrac{-1+2\sqrt{6}}{2}+\dfrac{(2\sqrt{6}+1)}{2}i$

Real part $=\dfrac{-1+2\sqrt{6}}{2}$, Imiginary part $=\dfrac{(2\sqrt{6}+1)}{2}$

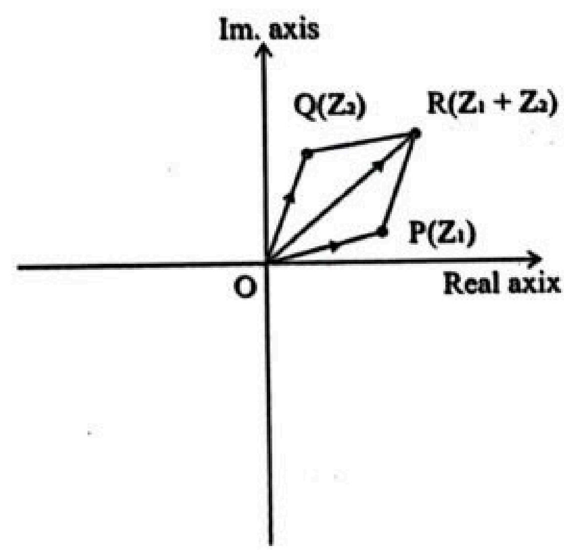
d. $\dfrac{(1-i)^2}{(1+i)^2}=\dfrac{(1)^2+(i)^2-2i}{(1)^2+(i)^2+2i}=\dfrac{1-1-2i}{1-1+2i}=\dfrac{-2i}{2i}=-1$

Real part $=-1$, Imiginary part $=0$

Geometrical Representation of Algebraic Operations

The graphical representation of algebraic operations in the complex number system makes it easier to understand the concepts.

(i) Geometrical Representation of Addition



Geometrically, addition of two complex numbers z_1 and z_2 can be visualized as addition of the vectors by using the *parallelogram law*. The vector sum $z_1 + z_2$ is represented by the diagonal of the parallelogram formed by the two original vectors.

If two points P and Q represent complex numbers z_1 and z_2 respectively in the Argand Plane, then the sum $z_1 + z_2$ is represented by the point R of the diagonal OR of parallelogram OPRQ having OP and OQ as two adjacent sides.

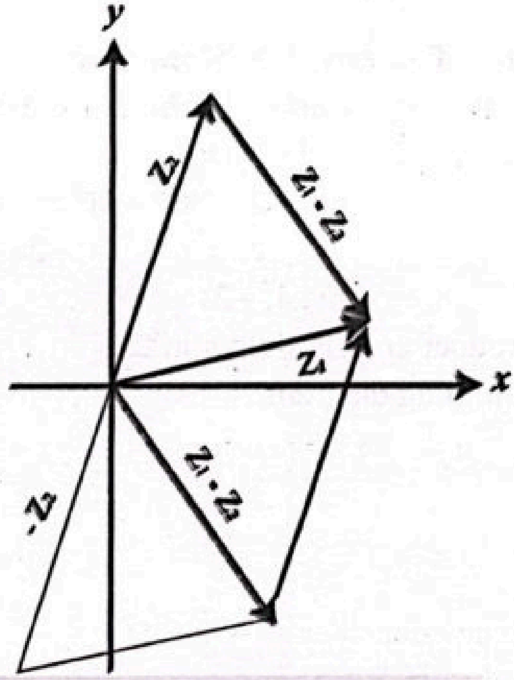
(ii) Geometrical Representation of Subtraction

The representation of the difference of two complex numbers, is slightly complicated than the addition of the complex numbers. The easiest way of presenting the subtraction is to think of addition of a negative vector.

If we want to represent $z_1 - z_2$, then the easiest way of representing it would be to think of adding a negative vector $z_1 + (-z_2)$.

The negative vector is the same as the positive one, the only difference being that the negative vector points in the opposite direction.

The difference vector $z_1 - z_2$ is represented in the adjoining figure.



Key Fact:

It is important to note here that the vector representing the difference of the vectors $z_1 - z_2$ may also be drawn joining the end point of z_2 to the tip of z_1 instead of the origin. This kind of representation does not alter the meaning or interpretation of the difference operator.

Check Point:

Can you think about a geometric interpretation of the addition of three or more complex numbers?

Example:

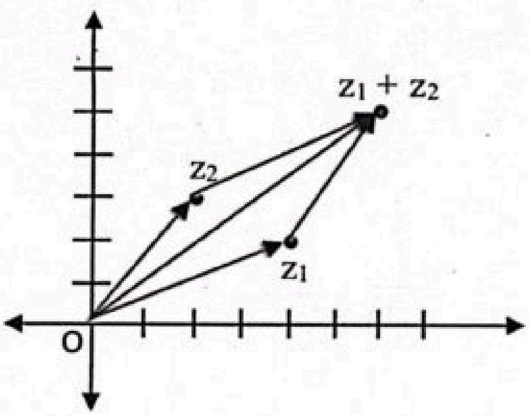
Represent sum and difference of $z_1 = 4 + 2i$ and $z_2 = 2 + 3i$ graphically.

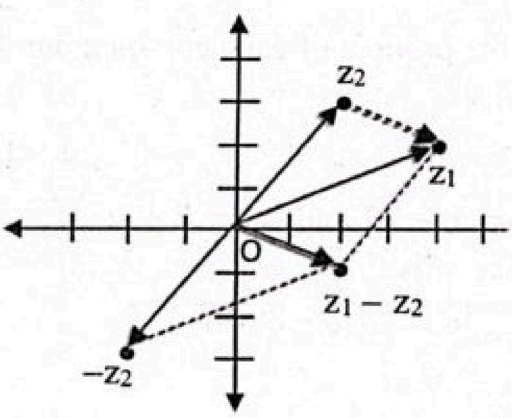
Solution:

Given, $z_1 = 4 + 2i = (4, 2)$ and $z_2 = 2 + 3i = (2, 3)$

Sum $= z_1 + z_2 = (4 + 2i) + (2 + 3i) = 6 + 5i = (6, 5)$

Difference $= z_1 - z_2 = (4 + 2i) - (2 + 3i) = 2 - i = (2, -1)$





(iii) Geometrical Representation of Multiplication and Division

Consider, $z_1 = 4 + 3i$ and $z_2 = 2 - 2i$.

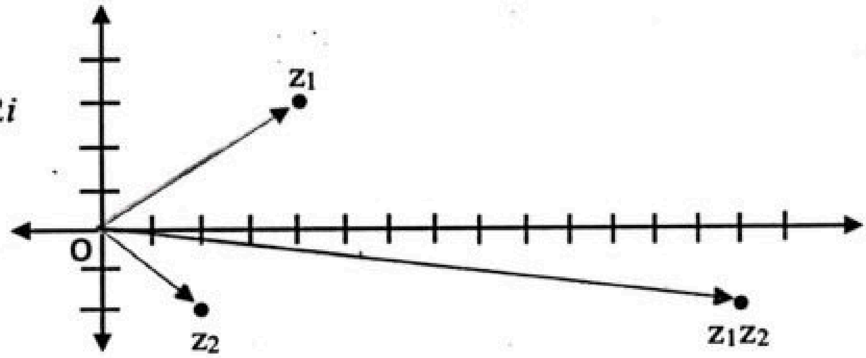
Now, $z_1 z_2 = (4 + 3i)(2 - 2i)$

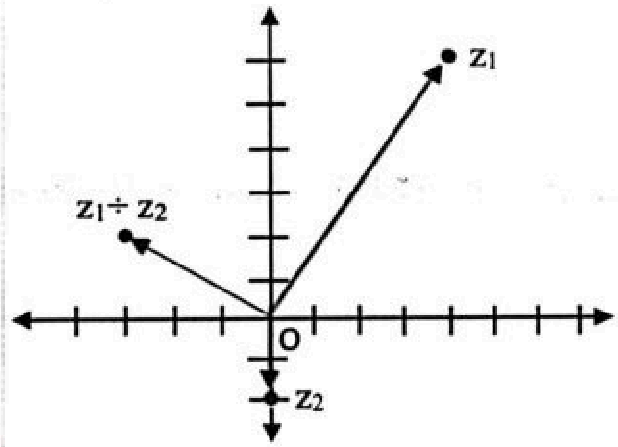
$= 8 - 8i + 6i - 6i^2$

$= 8 - 2i + 6 = 14 - 2i$

$= (14, -2)$

Product $z_1 z_2$ is shown in the adjoining diagram.





Again, consider $z_1 = 4 + 6i$ and $z_2 = -2i$ then:

$z_1 \div z_2 = (4 + 6i) \div (-2i)$

$= -3 + 2i$

The result is shown in the diagram.

Exercise 1.2

1. Find the additive inverse of each complex number.

   a. $-4+5i$ b. $-3-3i$ c. $5-5i$ d. $4i$

2. Show that each pair of complex numbers are multiplicative inverse of each other.

   a. $2+3i, \dfrac{2-3i}{13}$ b. $5-4i, \dfrac{5+4i}{41}$ c. $6+8i, \dfrac{3-4i}{50}$

3. Find the multiplicative inverse of each complex number.

   a. $1+i$ b. $7-3i$ c. $10-12i$

   d. $\dfrac{2}{5-i}$ e. $\dfrac{-i}{2-3i}$ f. $a-bi$

4. Find the product of each complex number and its conjugate.

   a. 4 b. $1-i$ c. $7i$

   d. $6-2i$ e. $10+9i$ f. $-4-11i$

5. If $z_1 = 1 - 2i$ and $z_2 = 2 + i$:

   a. Show that:

      i. $\overline{z_1 . z_2} = \overline{z_1} . \overline{z_2}$ ii. $\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}}$

iii. $|z_1| = |-z_1| = |\overline{z_1}| = |-\overline{z_1}|$ iv. $z_2.\overline{z_2} = |z_2|^2$

b. Find:

i. $|z_1 + z_2|$ ii. $|z_1 z_2|$ iii. $\left|\frac{z_1}{z_2}\right|$

6. Represent the numbers in the complex plane.

a. $-1-3i$ b. $2+4i$ c. $-3+2i$ d. $2-3i$

e. $2i$ f. $-3i$ g. 2

7. Separate into real and imaginary parts of each complex number.

a. $(\sqrt{2}-\sqrt{3}\,i)^2$ b. $(\sqrt{2}+\mathrm{i})^2$ c. $\dfrac{(2+3\mathrm{i})^2}{1-3i}$ d. $\left[\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right]^2$

e. $\dfrac{1-i}{(i)^2}$ f. $\dfrac{1}{i(1-i)^2}$ g. $\dfrac{(1+i)^2}{(1-2i)^2}$

8. Taking any complex number and show that:

a. $z.\overline{z}$ is a real number. b. $z^2 + (\overline{z})^2$ is a real number.

c. $(z - \overline{z})^2$ is a real number. d. $|z|$ and $|\overline{z}|$ are real numbers.

e. $z^2 - (\overline{z})^2$ is an imaginary number.

9. Represent sum and difference of complex numbers graphically.

(i) $z_1 = 5 + 3i$ and $z_2 = 2 - 3i$ (ii) $z_1 = -3 + 2i$ and $z_2 = 4 + 3i$

10. Represent product of complex numbers graphically.

(i) $z_1 = 4 + 2i$ and $z_2 = -2 + 3i$ (ii) $z_1 = -2 + 4i$ and $z_2 = 3 - i$

11. Represent $z_1 \div z_2$ graphically when:

(i) $z_1 = 6 - 4i$ and $z_2 = 3$ (ii) $z_1 = -4 - 6i$ and $z_2 = 1 + i$

Applications of Complex Numbers

Complex numbers are used in many fields in real life. Major Fields where complex numbers are used are:

Electronics	Electromagnetism	Computer science engineering
Civil engineering	Mechanical engineering	Control systems

In modelling and problem solving, we find many equations whose roots are complex numbers. Dealing such kind of problems, we must have conceptual understanding of complex numbers. Practically, negative numbers do not have square roots in the number system. However, a large number system that contains the real number system is designed so that negative numbers do have square roots.

Example: Solve $x^2+5=0$

Solution: $x^2+5=0$

$x^2=-5$

$x=\pm\sqrt{-5}=\pm\sqrt{-1\times 5}=\pm\sqrt{-1}\times\sqrt{5}$

$=\pm\sqrt{5}i$

The solution $+\sqrt{5}i$ and $-\sqrt{5}i$ are both pure imiginary numbers.

Check Point:

$x^2+5=0$

Solution is $\pm\sqrt{5}i$.

Verify the solution.

Sometimes we need factorization of the equation, but in real number system in some expressions making factors are not possible. By using the complex numbers properties, we can make factorization possible.

Example: Factorize:

a. z^2+9 b. $3z^2+5$

Solution:

a. z^2+9

$=(z)^2-9i^2$

$=(z)^2-(3i)^2$

$=(z-3i)(z+3i)$

The factors of z^2+9 are $(z-3i)(z+3i)$

b. $3z^2+5$

$=(\sqrt{3})^2z^2-i^2(\sqrt{5})^2$

$=(\sqrt{3}\,z)^2-(i\sqrt{5})^2$

$=(\sqrt{3}z-i\sqrt{5})(\sqrt{3}z+i\sqrt{5})$

The factors are $(\sqrt{3}z-i\sqrt{5})(\sqrt{3}z+i\sqrt{5})$

Example: Mr. Waqas is an electrical engineer designing the electrical circuits for a new office building. There are three basic things to be considered in an electrical circuit: The flow of the electric current I, the resistance to that flow Z, called impedance, and electromotive force E, called voltage. Their quantities are related in the formula $E=IZ$. The current of the circuit Waqas is designing is to be $(35-40J)$ amp. Electrical engineers use the letter J to represent the imaginary unit. Find the impedance of the circuit if the voltage is to be $(430-330J)$ volts.

Solution: We know that $E = IZ$ ...... (i)

We know it is given that $I=35-40J$, $E=430-330J$

Substituting the values in (i) $E=IZ$

and $Z=\dfrac{E}{I}$

$$Z=\frac{430-330J}{35-40J}$$

$$=\frac{430-330J}{35-40J}\times\frac{35+40J}{35+40J}$$

$$=\frac{15050+12200J-11550J-13200J^2}{1225-1600J^2},\qquad J^2=-1$$

$$= \frac{2825+5650J}{2825} = 1+2J$$

The impedance will be $1+2J$ ohms.

Example: If $z_1 = 5.5 + 5i, z_2 = 7.7 + 7i$, then calculate:

a. $z_1 + z_2$ b. $z_1 z_2$ c. $\frac{z_1}{z_2}$

Solution:

a. $z_1 + z_2 = 5.5 + 5i + 7.7 + 7i = 13.2 + 12i$

b. $z_1 z_2 = (5.5+5i)(7.7+7i) = 42.35 + 38.5i + 38.5i + 35i^2$

$= 7.35 + 77i$

c. $\frac{z_1}{z_2} = \frac{5.5+5i}{7.7+7i} = \frac{5.5+5i}{7.7+7i} \times \frac{7.7-7i}{7.7-7i} = \frac{(5.5+5i)(7.7-7i)}{(7.7)^2-(7i)^2}$

$= \frac{42.35-38.5i+38.5i-35i^2}{53.9-49i^2} = \frac{77.35}{102.9}$

Solution of System of Equations with Complex Coefficients

In this section, we shall find solution of different equations in complex variables either with real or complex coefficients. Consider the following equation:

$$pz + qw = r......(i)$$

Where p, q and r are complex numbers (every real number is a complex number). The equation (i) is called a linear equation in two complex variables z and w.

$$p_1 z + q_1 w = r_1$$

$$p_2 z + q_2 w = r_2$$

These two equations together form a system of linear equations in two variables z and w. The linear equations in two variables are also called simultaneous linear equations.

Example: Solve:

$$z + w = 3i$$

$$2z + 3w = 2$$

Solution: Given equations are:

$$z + w = 3i.........(i)$$

$$2z + 3w = 2......(ii)$$

Multiply equation (i) by 2 to equate the coefficient of z with equation (ii) and subtracting

$$\begin{array}{r} 2z+2w=6i \\ \pm 2z \pm 3w = \pm 2 \\ \hline -w=6i-2 \end{array}$$

$$w=2-6i \text{ (Substitute in (i) to find z)}$$

$$z+2-6i=3i$$
$$z=9i-2$$

Solution is: $z=9i-2,\ w=2-6i$

Exercise 1.3

Solve:

1. $x^2+7=0$
2. $x^2+9=0$
3. $x^2+100=0$

Determine whether the given complex number is a solution of the equation.

4. $1+2i,\ x^2-2x+5=0$
5. $1-2i,\ x^2-2x+5=0$
6. $1-i,\ x^2+2x+2=0$
7. $i,\ x^2+1=0$

Factorize the expressions:

8. x^2+16
9. a^2+b^2
10. x^2+25y^2

Solve the following system of linear equations.

11. $z-4w=3i$
    $2z+3w=11-5i$
12. $3z+(2+i)w=11-i$
    $(2-i)z-w=-1+i$

13. In an electrical circuit, the flow of the electric current I, the impedance Z and the voltage E, are related by the formula $E=IZ$.
    a. Find I, given the values:
       i. $E=(70+220J)$ volts, $Z=(16+8J)$ ohms
       ii. $E=(85+110J)$ volts, $Z=(3-4J)$ ohms
    b. Find Z given the value:
       i. $E=(-50+100J)$ volts, $I=(-6-2J)$ amp
       ii. $E=(100+10J)$ volts, $I=(-8+3J)$ amp
    c. Evaluate $\dfrac{1}{z-z^2}$ when $z=\dfrac{1-i}{10}$

I have Learnt

- Recognizing complex numbers, and conjugate and modulus of a complex numbers.
- Applying basic operations on complex numbers.
- Defining commutative laws, associative laws and distributive laws for complex numbers.
- Recognizing additive and multiplicative identity and finding additive and multiplicative inverse of complex numbers.
- Finding real and imaginary parts of $\left(\dfrac{x_1+iy_1}{x_2+iy_2}\right)^n$ for $n=\pm1, \pm2, \ldots$
- Finding solution of equations having complex values.
- Applying the geometric interpretation of a complex numbers.
- Solving daily life problems involving complex numbers.

MISCELLANEOUS EXERCISE-1

1. Tick the correct option.

i. $\sqrt{-1}$ is equal to:

a. 1 b. -1 c. i d. $-i$

ii. If $x<0$, then $\sqrt{x}$ is:

a. Real b. Complex c. Irrational d. Rational

iii. Conjugate of $\sqrt{x}-i\sqrt{y}$ is:

a. $\sqrt{x}+i\sqrt{y}$ b. $x-iy$ c. $x-y$ d. $x+iy$

iv. If $z=x+iy$, then $z\bar{z}$ is:

a. Imaginary b. Complex
c. Non-negative number d. Negative number

v. $\sqrt{-25}+\sqrt[3]{8}$ is equal to:

a. $-5+\sqrt{8}$ b. $2+5i$ c. $-5+2i$ d. $2\sqrt{2}+5i$

vi. $1+(-i)^9=?$

a. $1+i$ b. $1+\sqrt{-1}$ c. $1-i$ d. $-i$

vii. $\dfrac{2}{1-i}=?$

a. $\dfrac{1+i}{2}$ b. $\dfrac{(1+i)^2}{2}$ c. $1-i$ d. $1+i$

viii. $(-xi)^{19}=?$

a. $-x^{19}i$ b. $x^{19}i$ c. $-i^{19}$ d. $-x^{19}$

ix. If $z = 3+4i$, then $|z|^2$ is:

a. 5 b. $\sqrt{5}$ c. 25 d. 16

x. The solution of $x^2+4=0$ is:

a. $2i$ b. $-2i$ c. ± 2 d. $\pm 2i$

2. Simplify:

a. $(-2+4i)-(8-5i)$ b. $(-3+4i)+(-7i+4)$

3. Find the product:

a. $(x+iy)(2+3i)$ b. $(-3+6i)(-6+3i)$

4. Write in the form of $a+bi$, then find its conjugate:

a. $3\sqrt{2}-\sqrt{-7}$ b. $\sqrt{-2}$

5. Find $z.\bar{z}$:

a. $z=\dfrac{-1}{2}+i$ b. $z=14-7i$

6. Simplify:

a. $\dfrac{-3-i}{-3+i}$ b. $\dfrac{1+3i}{i\sqrt{5}}$

7. Factorize:

a. $2x^2+18$ b. $-x^2-25y^4$

8. Solve:

a. $3x^2+15=0$ b. $6y^2+36=0$