UNIT 19

.... Atomic Spectra ......

At the end of this chapter the student will be able to:

- describe and explain the origin of different types of optical spectra.
- show an understanding of the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to spectral lines.
- explain how the uniqueness of the spectra of elements can be used to identify an element.
- analyse the significance of the hydrogen spectrum in the development of Bohr's model of the atom.
- explain hydrogen atom in terms of energy levels on the basis of Bohr Model.
- determine the ionization energy and various excitation energies of an atom using an energy level diagram.
- Solve problems and analyse information using $1/\lambda = R_H [1/p^2 - 1/n^2]$.
- understand that inner shell transitions in heavy elements result into emission of characteristic X-rays.
- explain the terms spontaneous emission, stimulated emission, meta stable states, population inversion and laser action.
- describe the structure and purpose of the main components of a He-Ne gas laser.

The beginning of the twentieth century saw the start of new branches of Physics – atomic structure and spectra which has a profound effect on revealing the inner mysteries of the structures of atoms.

The existence of line emission spectra from atomic gases is used to infer a structure of an atom in terms of discrete energy levels in atoms. J.J. Balmer in

1885 succeeded to devise an empirical formula which could explain the existence of the spectra of atomic hydrogen.
In this chapter we will study the line spectrum of hydrogen atom, the Bohr model of hydrogen atom, production of X-rays, working principle of CAT scanner and Laser.

For your Information

A CT scan stands for Computed Tomography scan. It is also known as a CAT (Computer Axial Tomography) scan. It is a medical imaging method that employs tomography. CT scanning is useful to get a very detailed 3-D image of certain parts of the body, such as soft tissues, the blood vessels, the lungs, the brain, abdomen, and bones.

19.1 Atomic Spectra

When a substance is heated, its atoms absorb energy and are excited, i.e. some of its electrons jump to higher energy states. The electron stays there for a short duration $(10^{-8}\,s)$ and fall back to its lower energy state. In this process it emits a radiation called photon which is supposed to be a discrete packet of light energy. A photon is a particle of light having wave characteristics, i.e. it has an frequency and wavelength.

The frequency of emitted radiation or photons is equal to the frequency with which the electron bounces back and forth between the higher and the lower energy state.

In a solid, a liquid or a dense gas, the atoms are closely packed and are, therefore, not free to emit radiation because of interaction.

Thus we do not get discrete radiation but instead obtain only a continuous spectrum. The atoms of gas are however free to emit a radiation when excited. The emission spectrum of a gas is, therefore, discrete, having line spectrum.

Suppose an evacuated glass tube is filled with a gas such as neon, helium, or argon. If a potential difference between electrodes in the tube produces an electric current in the gas, the tube will emit light whose color is characteristic of the gas. If the emitted light is analyzed by passing it through a narrow slit and then through a spectroscope, a series of discrete lines is observed, each line corresponding to a different wavelength or color. We refer to such a series of lines as a line spectrum. The wavelengths contained in a given line spectrum are characteristic of the elements emitting light. Because no two elements emit the same line spectrum, this phenomenon represents a practical technique for identifying elements in chemical substance. The first such spectral series was found by J.J Balmer in 1885 in the course of a study of the visible part of the hydrogen spectrum.

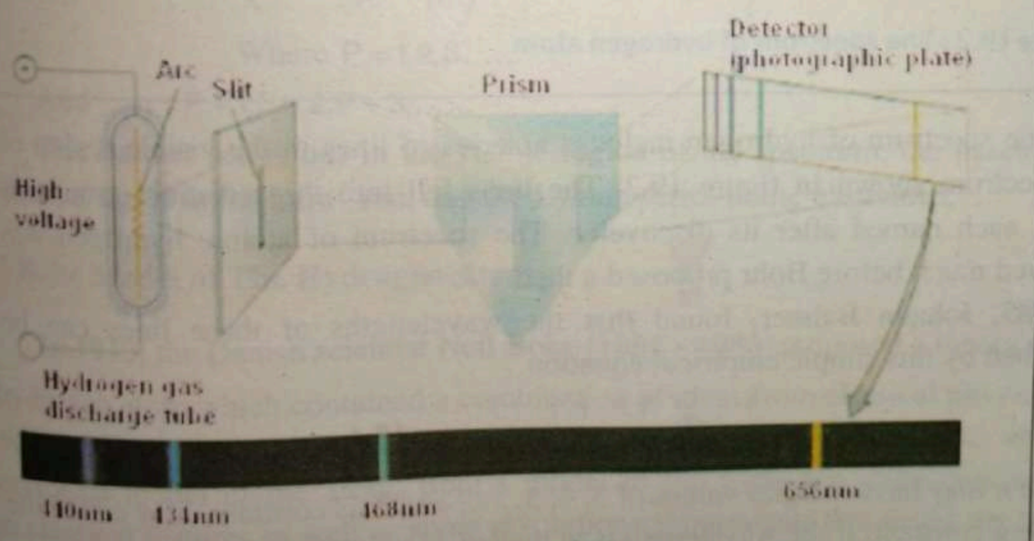


Figure 19.1 : line spectrum

19. 2 The Spectrum of Hydrogen Atom

When hydrogen gas is placed in a discharge tube, and a discharge is caused in it by means of high voltage across the tube, the gas becomes luminous and gives off a bluish –red light, Fig 19.2. This light can be analyzed by passing it through a dispersing device such as a prism or a grating. The spectrum of hydrogen atoms consists of a series of lines. Each line represents a wavelength of light given off by the light source.

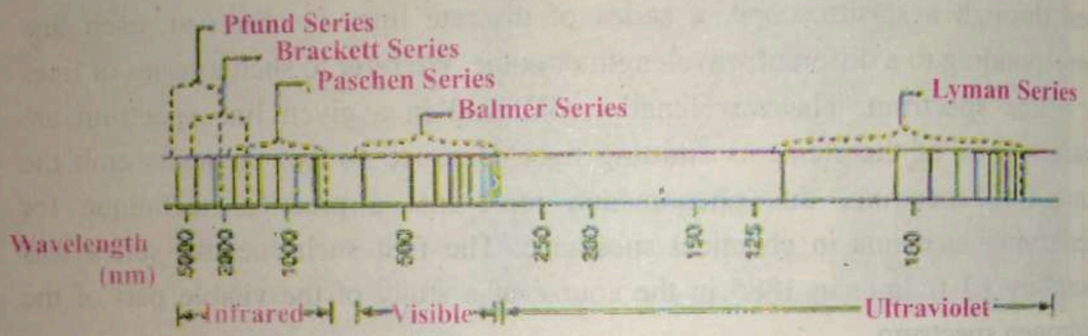


Figure 19.2 : line spectrum of hydrogen atom

The line spectrum of hydrogen includes a series of lines in the visible region of the spectrum shown in figure 19.2. The lines fall into three distinct groups or series, each named after its discoverer. The spectrum of atomic hydrogen was observed much before Bohr proposed a theory for it.

In 1885, Johann Balmer, found that the wavelengths of these lines can be described by this simple empirical equation.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad ......\ 19.1$$

Where n may have integral values of 3, 4, 5…… and R is constant, now called the Rydberg constant. If the wavelength is in meters, R has the value.

$$R = 1.0973732 \times 10^7 m^{-1}$$

Balmer predicted that other series of lines might exist outside the visible region which would obey the equations given below.

Several years later, other were discovered. These spectra are called the Lyman, Paschen and Brackett series after their discoverers. The wave lengths of the lines in these series can be calculated by the following empirical formulas.

Lyman series $\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$ ......(19.2)

Where $n=1,2,3,45,......$

paschen series $\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$ ......(19.3)

Where $n=4,5,6,7,.......$

Brachett series $\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$ ......(19.4)

Where $n=5,6,7...$

All the above formulas can be written in a general mathematical form as:

$$\frac{1}{\lambda_n}=R\left(\frac{1}{P^2}-\frac{1}{n^2}\right)......(19.5)$$

Where $P=1,2,3......$

And $n=P+1,P+2,P+3,......$

The Balmer series lies in the visible region of the spectrum, the paschen and Brakett series in the infra –red, and the Lyman series in the ultraviolet.

19. 3 Bohr Model of The Hydrogen Atom

In 1913, the Danish scientist Neil Bohr (1885 – 1963) proposed a theory of the hydrogen atom which contained a combination of ideas from classical physics, Plank's original quantum theory, Einstein's photon theory of light, and Rutherford's model of the atom. Bohr's model of the hydrogen atom contains some classical features as well as some revolutionary postulates that could not be justified within the frame work of classical physics. The Bohr model can be applied quite successfully to such hydrogen –like ions as single ionized helium and doubly ionized lithium. However, the theory does not properly describe the spectra of more complex atoms and ions.

The basic postulates of the Bohr model of the hydrogen atom are as follows.

1. The electron move in circular orbits about the nucleus under the influence of the coulomb force of attraction between the electron and the positively charged nucleus.

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \quad ....(19.6)$$

Where $\frac{mv^2}{r}$ and $\frac{ke^2}{r^2}$ are centripetal and coulomb forces respectively.

2. Only those stationary orbits are allowed for which orbital angular momentum is equal to an integral multiple of $\frac{h}{2\pi}$.

$$mvr = n\frac{h}{2\pi} \quad ....(19.7)$$

Where "h" is Plank's constant and its value is $h = 6.6256\times10^{-34}$ j s.

3. The electron in stable orbit does not radiate energy as in the classical theory.

4. The atom radiates energy only when the electron jumps from one allowed stationary orbit to another. The frequency of the radiation obeys the condition.

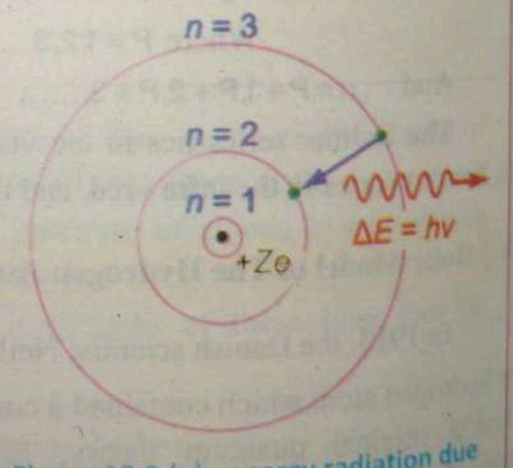


Figure 19.3 (a) : energy radiation due to jumping of electron from higher to lower orbit

$$hf = E_n - E_p \quad ...(19.8)$$

Where E_n and E_p are higher and lower energy states respectively.

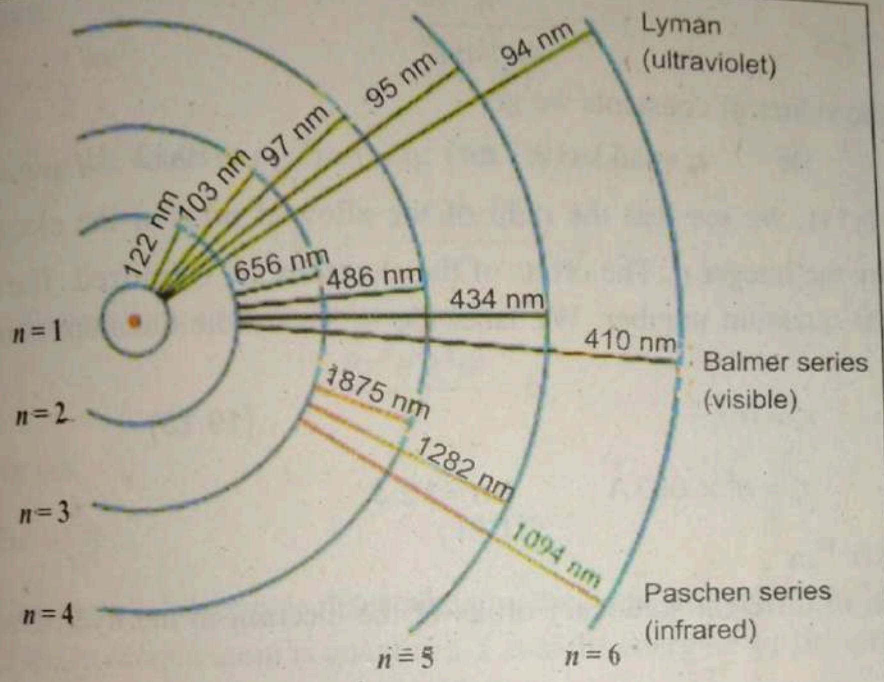


Figure 19.3 (b) : Lyman, Balmer, and paschen series.

The Radii of The Quantized Orbit

From Eq:(19.7)

$$v = \frac{nh}{2\pi m r_n} \qquad \ldots(19.9)$$

For electrons to stay in a circular orbit, the centripetal force is provided by the coulomb's force. Thus,

$$\frac{mv^2}{r_n} = \frac{ke^2}{r^2_n} \qquad \ldots\ldots(19.10)$$

$$k = \frac{1}{4\pi\varepsilon_0}$$

From Eq: 19.9 put the value in Eq: 19.10. After simplification we get.

$$r_n = \frac{n^2 h^2}{4\pi^2 kme^2} = n^2 r_1 \qquad \cdots 19 \cdot 11$$

Where $$r_1 = \frac{h^2}{4\pi^2 kme^2}$$

By putting the values of constants we get

Or $$r_1 = 0.53\times10^{-10}\,\text{m} \qquad \cdots 19.12$$

From Eq (19.11), we see that the radii of the allowed orbit of the electron are determined by the integer n. The orbits of the electrons are quantized. The integer n is called the quantum number. We label the orbits by the quantum number n. Thus,

$$r_n = n^2 r_1 \qquad \ldots(19.13)$$

$$r_n = n^2 \times .053\,\text{Å} \qquad : n = 1,2,3$$

Where $1\,\text{Å} = 10^{-10}\,\text{m}$

Thus the radii of different stationary orbits of the electron in the hydrogen atom are given by

$$r_n = r_1, 4r_1, 9r_1, 16r_1 \ldots\ldots$$

Energy of Electron in Quantized Orbit

Let us now compute the energy of the hydrogen system for a given n. The energy consists of the electron's kinetic energy and the electrostatic potential energy of the two charges. From Eq (19.10).

$$K\cdot E = \frac{1}{2}mv^2 = \frac{1}{2}\frac{ke^2}{r_n} \qquad \ldots(19.14)$$

And its electrostatic energy is

$$P\cdot E = -\frac{ke^2}{r_n} \qquad \ldots(19.15)$$

So the total energy will be

$$E_n = K\cdot E + P\cdot E$$

$$E_n = \frac{ke^2}{2r_n} - \frac{ke^2}{r_n}$$

Hence

$$E_n = -\frac{1}{2}\frac{ke^2}{r_n} \quad \ldots(19.16)$$

By substituting the value of r_n from Eq: (19.11), we have

$$E_n = -\frac{2\pi^2 e^4 k^2 m}{h^2} \times \frac{1}{n^2}$$

Write the constant factor in the above equation as

$$\frac{2\pi^2 e^4 k^2 m}{h^2} = E_o$$

We get

$$E = -\frac{E_o}{n^2} \quad \ldots(19.17)$$

Thus the total energy is determined by the quantum number *n*. The total energy of the hydrogen atom is quantized. Label the energy *E* by the quantum number *n*, we have

$$E_n = \frac{-E_o}{n^2} \quad \ldots\ldots(19.18)$$

Where *n*=1,2,3..

The minus sign shows that the electron is bound to the nucleus and cannot escape from it. Substituting the value of various constants, we find that

$$E_0 = 2.17 \times 10^{-18}\,j = +13.6ev$$

Thus

$$E_n = \frac{-13.6\,ev}{n^2} \quad \ldots\ldots(19.19)$$

The lowest stationary energy state, or ground state, corresponds to $n = 1$ and has energy $E_1 = -13 \cdot 6$ ev. The next state, corresponding to $n = 2$, has an energy $E_2 = -\frac{E_1}{4} = -3 \cdot 4$ ev, and so on.

Hydrogen Emission Spectrum

The result derived above for the energy levels along with postulate 4 can be used to derive the expression for the wavelength of the hydrogen spectrum. Suppose that the electron in hydrogen atom is in the excited state "*n*" with energy

E_n and makes a transition to a lower state "P" with energy E_p, where $E_n > E_p$, then $hf = E_n - E_P$.

Where $$E_n = \frac{-E_o}{n^2} \text{ and } E_P = -\frac{E_0}{P^2}$$

Hence $$hf = -E_0\left[\frac{1}{n^2} - \frac{1}{P^2}\right]$$

Substituting for $f = \frac{c}{\lambda}$ we have,

$$\frac{1}{\lambda} = \frac{E_0}{hc}\left(\frac{1}{P^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{P^2} - \frac{1}{n^2}\right) \quad \text{....(19.20)}$$

Where R_H is the Rydberg constant given by the equation.

$$R_H = \frac{E_0}{hc} = 1.0974 \times 10^7 \text{ m}^{-1} \quad \text{...(19.21)}$$

This value of R_H in Bohr model is in agreement with the value of the Rydberg's constant determined empirically by Balmer. Thus the Balmer empirical formula $\frac{1}{\lambda} = R\left(\frac{1}{P^2} - \frac{1}{n^2}\right)$ and that derived from Bohr's theory $\frac{1}{\lambda} = R_H\left(\frac{1}{P^2} - \frac{1}{n^2}\right)$ are actually the same. Thus, similar to the Balmer empirical formula, Bohr's theory can be used to compute the energies or wavelength of the transitions involved in the various emission series.

19.4 Energy –Level Diagram

According to Bohr's theory the total energy and the radii of the electron orbits in hydrogen atom are respectively given by the following relations.

$$E_n = \frac{-E_0}{n^2} = \frac{-13.6 \text{ ev}}{n^2} \text{ and } r_n = r_1 n^2$$

When n = 1, the electron is in the first orbit; the energy is minimum and has the value $E_1 = -E_0 = -13.6 \text{ ev}$. When $n \to \infty$, then $r_n \to \infty$, $E_n \to 0$.

The electron become free from the nucleus. The atom is then said to ionized. It is convenient to represent the energy of the quantized states of the atom on an energy level diagram as shown in Fig 19.4.

The energy levels of the atom E_n are represented by a series of horizontal lines. Transition between the levels are represented by vertical arrows. When the electron is free from the atom and is at rest, both its kinetic and potential energies are zero at $n = \infty$ level.

The energy level diagram can be used to illustrate the origin of various spectral series observed in the emission spectrum of hydrogen. The transition from various energy level to the lowest level $(n = 1)$ gives rise to lyman series. Balmer series occurs for transition ending at second energy level $(n = 2)$. The paschen, bracket and pfund series occurs for transitions from various energy levels to the $r = 3, 4, 5$ energy levels, respectively.

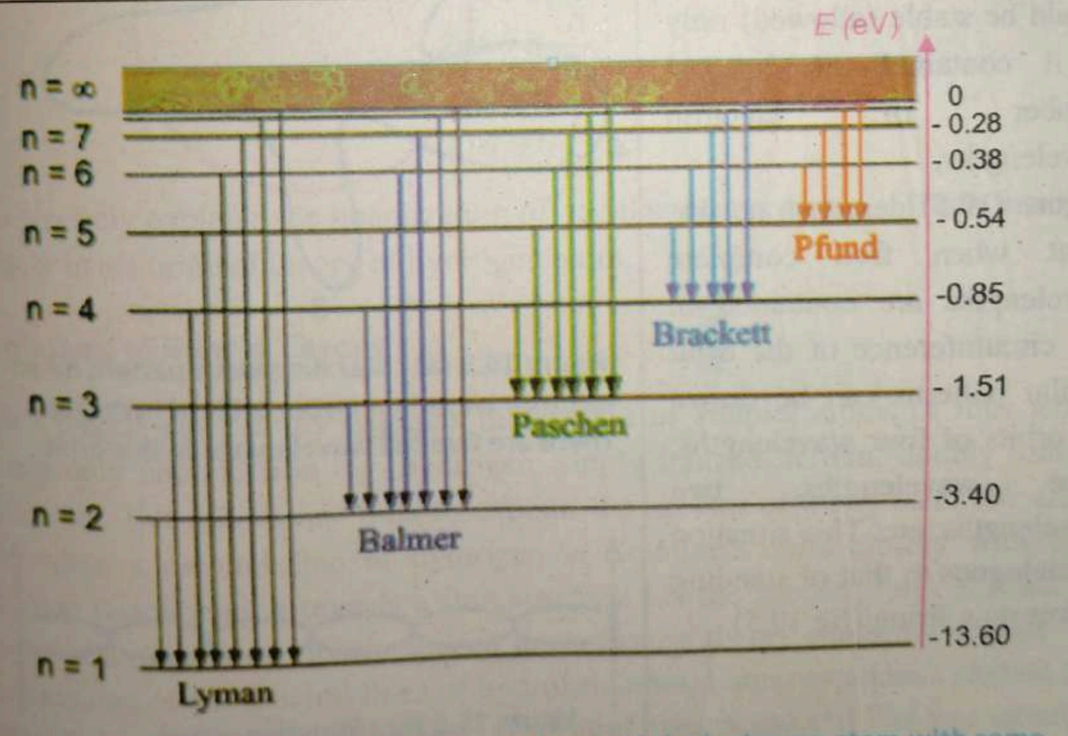


Figure 19.4 : energy level diagram Energy levels of the hydrogen atom with some of the transitions between them to give rise to the spectral lines

19.5 De –Broglie Waves And The Hydrogen Atom

One of the postulates made by Bohr in his theory of the hydrogen atom was that angular momentum of the electron is quantized in units of

$$\frac{h}{2\pi}, \quad \text{or} \quad mvr = n\frac{h}{2\pi}$$

For more than a decade following Bohr's publication, no one was able to explain why the angular momentum of the electron was restricted to these discrete values. Finally, de Broglie recognized a connection between his theory of the wave character of material properties and the quantization condition given above. De–Broglie assumed that an electron orbit would be stable (allowed) only if it contained an integral number of electron wavelengths.

Figure (19.5) demonstrate this point when five complete wavelengths are contained in one circumference of the orbit similar patterns can be drawn for orbits of four wavelengths, three wavelengths, two wavelengths, etc. This situation is analogous to that of standing waves on a string (fig 19.5).

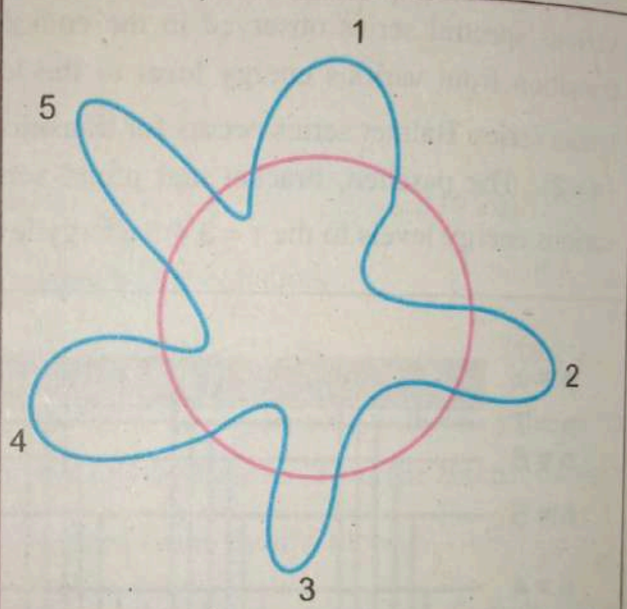


Figure 19.5 (a) : Standing wave pattern for an electron wave in a stable orbit of hydrogen. There are five full wavelengths in this orbit.

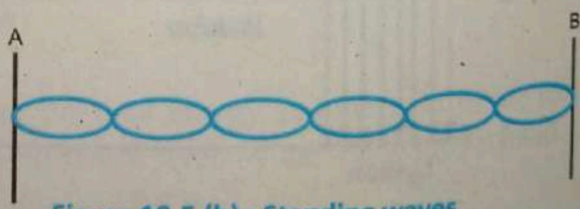


Figure 19.5 (b) : Standing waves pattern for a vibrating stretched string

Now imagine that the vibrating string is removed from its support at A and B and bent into a circular shape such that A and B are brought together. The end result is a pattern similar to that shown in (fig 19.5). Standing waves pattern for a vibrating stretched string fixed at its ends. This pattern has three full wavelengths.

In general, the condition for de –Broglie standing wave in an electron orbit is that the circumference must contain an integral multiple of electron wavelengths. We can express this condition as

$$2\pi r = n\lambda \quad \cdots(19.21)$$

$$(n = 1, 2, 3 \ldots\ldots)$$

De –Broglie's equation for the wavelength of an electron in terms of its momentum.

$$\lambda = \frac{h}{mv} \quad \cdots 19.22$$

Substituting λ in Eq:(19.21), we have,

$$2\pi r = n\frac{h}{mv}$$

$$mvr = n\frac{h}{2\pi}$$

This precisely explains the quantization of angular momentum condition imposed by Bohr in his original theory of hydrogen atom.

Limitations of Bohr's Theory

Bohr's theory successfully explains the spectra of simpler atoms or ions which contain only one electron e.g. hydrogen, singly ionized helium, doubly ionized lithium etc. But this theory fails to explain the spectra of many electrons atom. Also when a spectral line of hydrogen is examined more closely with high precision instruments it reveals a fine structure i.e. the spectral lines is found to consist of a number of closely spaced lines. Bohr's theory could not explain the fine structure of the spectral lines of hydrogen atom. Later researchers studied the effect of electric and magnetic fields on spectral lines.A spectral line was found to split into a number of lines under the influence of magnetic field (Zeeman Effect) and electric field (Stark effect). Bohr's theory cannot explain these two effects.

Check Points

What postulate of Bohr's model is justified by de –Broglie?

Example 19.1

The electron in the hydrogen atom makes a transition from n = 2 energy state to the ground state n = 1. Find the wavelength of the emitted photon.

Solution:

We can use the equation

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda = \frac{4}{3R}$$

$$\lambda = \frac{4}{3(1.097\times10^7)}$$

$$\lambda = 1.215\times10^{-7}\,\text{m} = 121.5\text{ nm}$$

Example 19.2

The Balmer series for hydrogen atom corresponds to electronic transitions that terminate in the state of quantum number n = 2. Find the longest wavelength of photon emitted.

Solution:

The longest –wavelength in the Balmer series result from the transition. From $n=3$ to $n=2$.

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}R$$

$$\lambda_{max} = \frac{36}{5R}$$

$$\lambda_{max} = \frac{36}{5(1.097\times10^7)} = 656.3\text{ nm}$$

Example 19.3

Find the shortest wavelength photon in the Balmer series.

Solution:

The shortest –wavelength photon in the Balmer series is emitted when the electron makes transition from $n = \infty$ to $n = 2$. Therefore,

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R}{4}$$

$$\lambda_{min} = \frac{4}{R} = \frac{4}{1.097\times10^7} = 364\cdot6 \text{ nm}$$

19.6 Excitation and Ionization Potential

The Bohar model as well as the current quantum mechanical model of an atom predicts that the total energy of an electron in an atom is quantized. The allowed energies are given by a relation of the form.

$$E_n = \frac{-E_0}{n^2}; \qquad n = 1,2,3......$$

The state $n = 1$ is called ground state, while states with $n = 2,3,4,......$ are called exited states. When energy is supplied to the atom, then an electron in the atom reaches one of its exited states. The atom in an exited state cannot stay for a long time. The electron in an exited atom soon returns to lower energy levels by emitting photons.

Excitation Energy

The energy required to move electron from its ground state to an exited state is known as excitation energy. For example the first and second excitation energies of hydrogen atom are calculated to be.

$$\frac{-E_0}{2^2} - (-E_0) = \frac{3}{4}E_0 = \frac{3}{4}(13.6\,\text{eV}) = 10.2\,\text{eV}_0$$

$$\frac{-E_0}{3^2} - (-E_0) = \frac{8}{[illegible]}E_{[illegible]} = \frac{8}{[illegible]}(13.6\text{ eV}) = 12.1\text{ eV}_0$$

Excitation Potential

The potential difference V in volts applied to an electron in its ground state to get an amount of energy equal to the excitation energy of the electron in the atom is called excitation potential of the atom. For example, the first and second excitation potential of H –atom are respectively 10.2 V and 12.1 V.

Ionization Energy

If an atom absorbs sufficient amount of energy, an electron may be raised to a level $n = \infty$. The electron then becomes free from the attractive force of the nucleus, i.e., the electron is removed from the atom. An atom which has lost one or more electrons is said to be ionized. The minimum energy required to remove an electron from its ground state is called ionization energy of the atom. But the energy of the electron in the initial (ground) state is E_o, and its energy in the final (ionized) state is zero. Thus the ionization energy of the atom is $\{0-(-E_0)\} = E_0$. This means that the ionization energy of the atom is numerically equal to the ground state energy of the atom. For example, the ionization energy of H –atom is 13.6 eV.

Ionization Potential

The potential difference applied to an electron to provide it the requisite amount of ionization energy is called ionization potential.

Example 19 .4

When a hydrogen atom is bombarded, the atom may be raise into a higher energy state. As the excited electron fall back to the lower energy levels, light is emitted. What are the three longest wavelength spectral lines emitted by the hydrogen atom as it returns to the $n = 1$ state from higher energy states?

Solution:

$$n=2 \rightarrow n=1: \quad \Delta E_{2,1} = -3.4-(-13.6) = 10.2\,eV$$

$$n=3 \rightarrow n=1: \quad \Delta E_{3,1} = -1.5-(-13.6) = 12.1\,eV$$

$$n=4 \rightarrow n=1: \quad \Delta E_{4,1} = -0.85-(-13.6) = 12.8\,eV$$

To find the corresponding wavelengths we can use $\Delta E = hf = \frac{hc}{\lambda}$.

For $n=2$ to $n=1$ transition

$$\lambda = \frac{hc}{\Delta E_{2,1}}$$

$$\lambda = \frac{6.63\times10^{-34}\,\text{j}\cdot\text{s}\left(3\times10^{8}\,\text{ms}^{-1}\right)}{(10.2\text{eV})\left(1.60\times10^{-19}\,\text{JeV}^{-1}\right)} = 121\text{ nm}$$

For $n=3$ to $n=1$ transition

$$\lambda = \frac{hc}{\Delta E_{3,1}}$$

$$\lambda = \frac{\left(6.63\times10^{-34}\,\text{js}\right)(3\times10^{8}\,\text{ms}^{-1})}{(12.1\text{eV})\left(1.60\times10^{-19}\,\text{JeV}^{-1}\right)} = 102\text{ nm}$$

For $n=4$, to $n=1$ transition

$$\lambda = \frac{hc}{\Delta E_{4,1}}$$

$$\lambda = \frac{\left(6.63\times10^{-34}\,\text{j}\cdot\text{s}\right)\left(3\times10^{8}\,\text{ms}^{-1}\right)}{(12\cdot8\,\text{eV})\left(1.60\times10^{-19}\right)\text{JeV}^{-1}} = 96\cdot9\text{nm}$$

These are the first three lines of the Lyman series.

19 .7 Inner shell Transition and Characteristic X–Rays

The transitions of electron in hydrogen atom results in the emission of spectral lines in the infrared, visible or ultraviolet region of electromagnetic spectrum due to small energy difference in the transition levels.

In heavy atoms, the electrons are assumed to be arranged in concentric shells labeled as K, L, M, N, O, etc. The K shell being closest to the nucleus, the L shell next, and so on. The inner shell electrons are tightly bound and large amount of energy is required for their displacement from their normal energy levels. When a heavy target material is bombarded with a beam of electrons, that has been accelerated by several k eV. Some of these electrons will collide with inner–shell electrons of the target and knock them out of their respective atoms.

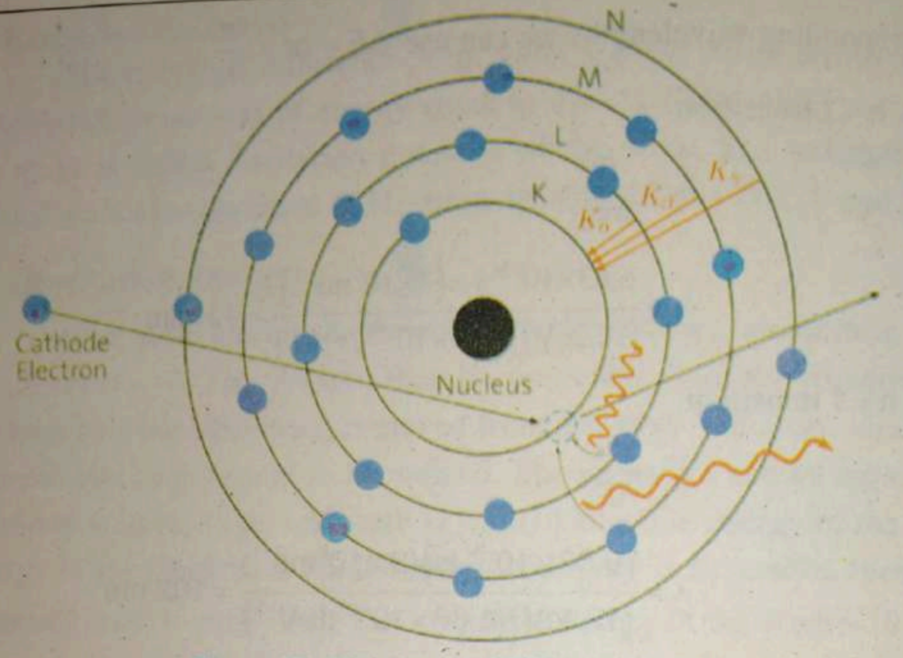


Figure 19.6 (a) :Inner shell Transition

Let a K–shell electron is knocked out from an atom creating a vacancy in K–shell.

Then an electron from either, L, M, or N –shell will quickly jump down to fill the vacancy in the K–shell emitting the excess energy as x –rays photon.

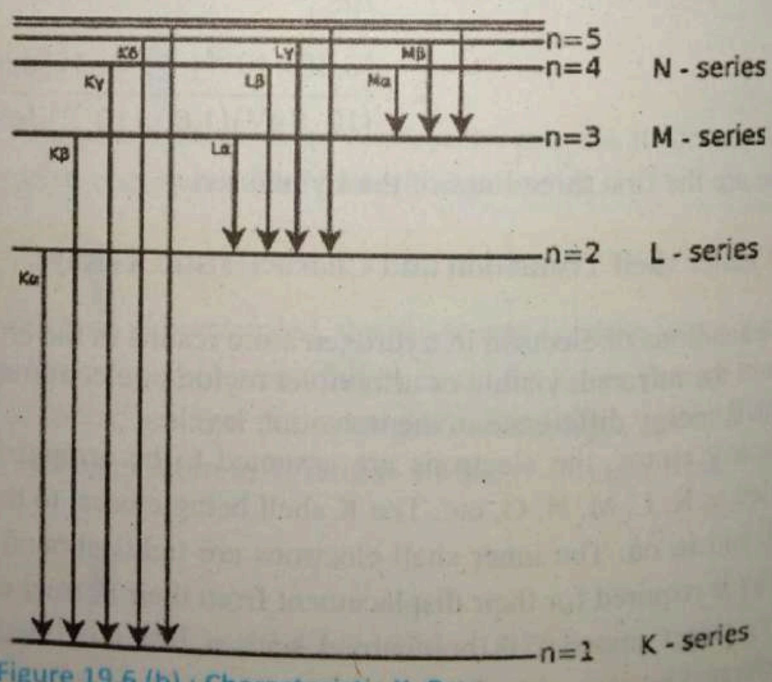


Figure 19.6 (b) : Characteristic X–Rays

These x–rays consists of series of specific wavelengths or frequencies and hence are called characteristic x–rays. An x–ray photon due to transition from L–shell to the vacancy in the K–shell is called K_α characteristic x –rays. The transition from M -and N –shell to the K –shell gives rise to K_β and K_γ characteristic x –rays respectively. The study of characteristic x –rays spectra has played a very important role in the study of atomic structure and the periodic table of elements.

19.7.1 Continuous x –rays

Another process that can rise the emission of x –rays, is illustrated in (fig 19 .7). Consider an electron traveling towards a target nucleus in the x –rays tube. The incident electron has coulomb interaction with orbital electrons as well as the positive nucleus. Because of the concentrated positive charge, the interaction with the nucleus is very strong. The force of attraction accelerates the electron. According to the classical theory of electromagnetism, an accelerated charge emits radiation called Bremsstrahlung, a Germen word meaning braking radiation. This Bremsstrahlung is called continues x –rays.

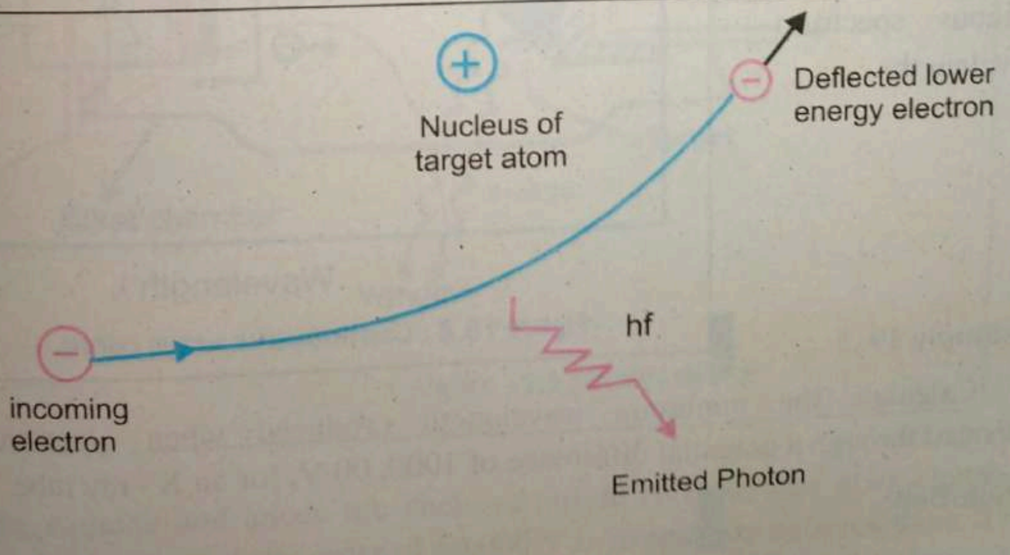


Figure 19.7 : electron moving towards nucleus

According to quantum theory, this radiation must appears in the form of photon. Since the radiated photon (shown in Fig:19.7) carries energy, the electron must lose kinetic energy because of its encounter with the target nucleus. Let us consider an extreme example in which the electron losses all of its energy in a single collision. In this case, the initial energy of the electron (eV) is transformed completely into the energy of the photon (hf_{max}). In equation form we have

$$eV = hf_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV} \qquad \cdots(19.23)$$

Where eV is the energy of the electron after it has been accelerated through a potential difference of V volt and e is the charge on electron.

All radiation produced does not have the wavelength given in Eq:19.23 because many of the electrons are not stopped in a single collision.

This results in the production of the continuous spectrum of wavelengths.

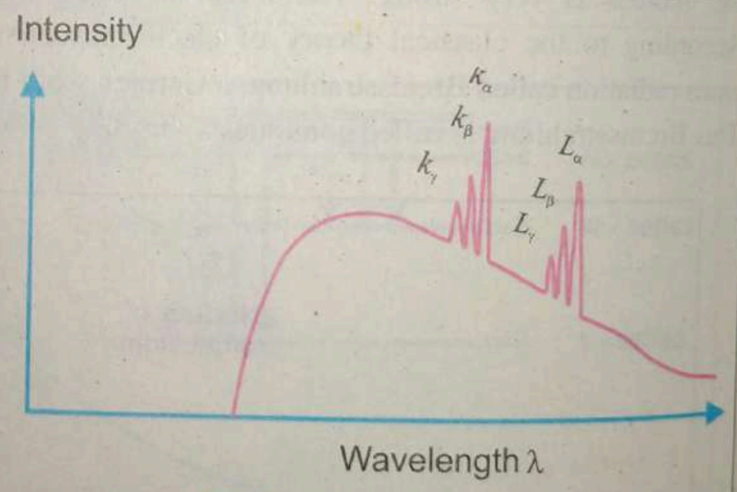


Figure 19.8 : Continuous x –rays curve.

Example 19 .5

Calculate the minimum wavelength produced when electrons are accelerated through a potential difference of 1000, 00 V, for an X –ray tube.

Solution:

Using (Eq 19.23)

$$\lambda_{min} = \frac{hc}{eV}$$

$$\lambda_{min} = \frac{(6.63\times10^{-34} j\cdot s)(3\times10^{8}\ m/_{sec})}{(1.60\times10^{-19} c)(10^{5} v)}$$

$$\lambda_{min} = 1.24\times10^{-11} m$$

19.7.2 Production of X –Rays

Fig (19.9) shows an arrangement of producing X –rays. It consists a filaments F, heated by the current supplied from a battery B, emits electrons. This serves as a hot cathode. The anode is made of a solid copper bar "c". A high melting –point metal like platinum or tungsten is embedded at end of the copper rod and it serve as a target T.

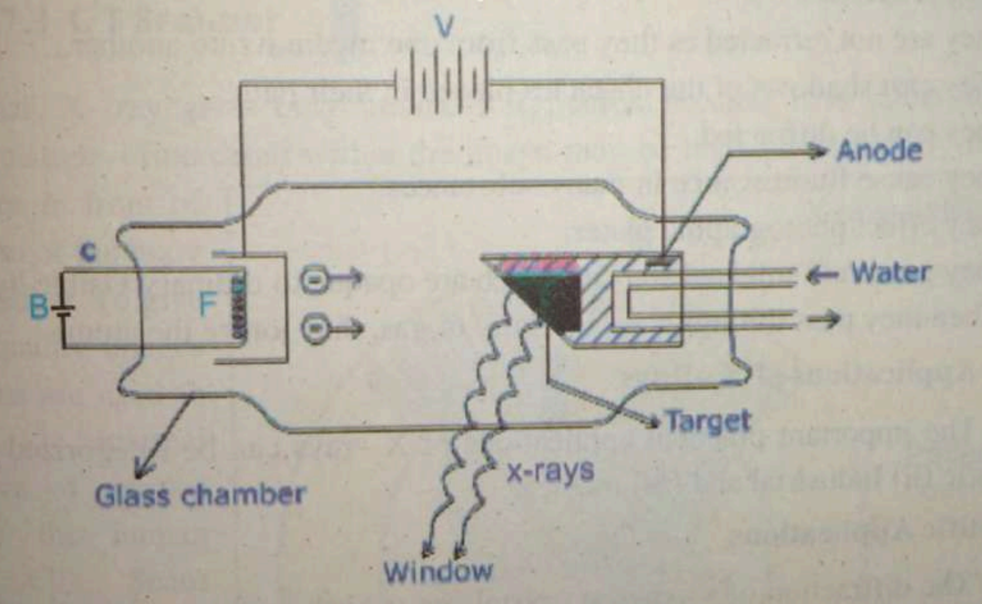


Figure 19.9 : X-rays device

The cathode and anode are enclosed inside an evacuated glass chamber and a high DC voltage of the order of 50,000 V is maintained between them. The electrons emitted from the cathode are accelerated by the high potential difference. The energetic electrons strike the target T and X–rays are produced.

It may be mentioned that a small part of the kinetic energy of the incident electrons is converted into X –rays, the rest is converted into heat. The target T becomes very hot and must, therefore, have a high melting point. The heat generated in target T is dissipated through the copper rod. Sometime the anode is cooled by water flowing behind the anode.

When such highly energetic electrons are suddenly stopped by target T, an intense beam of X –rays produced. These X –rays have large penetrating capacity and are called hard X –rays, while those with small penetrating power are called soft X–rays.

Properties of X–rays

Preliminary experimental investigations revealed that X –rays have the following properties.

1. They are not refracted as they pass from one medium into another.
2. They cast shadows of the obstacles placed in their path.
3. They can be diffracted.
4. They cause fluorescence in many substances.
5. They effect photographic plates.
6. They penetrate solid substances which are opaque to ordinary visible light.
7. When they pass through a solid, liquid or gas, they ionize the atoms.

Applications of X –Rays

The important practical applications of X –rays can be categorized as (i) Scientific (ii) Industrial and (iii) medical.

Scientific Applications

The diffraction of x –rays at crystals gave birth to x –ray crystallography. The Laue diffraction pattern can be used to determine the internal structure of the crystals. The spacing and dispositions of the atom of a crystal can be precisely determined.

Industrial Applications

Since X –rays penetrate the materials on which they are incident, they are used in industry to detect defects in metallic structures in big machines, railway tracks and bridges.

X -rays are used to analyse the compositions of alloys such as bronze, steel and artificial pearls. The structure of rubber and plastics can be analysed and controlled by X -rays studies.

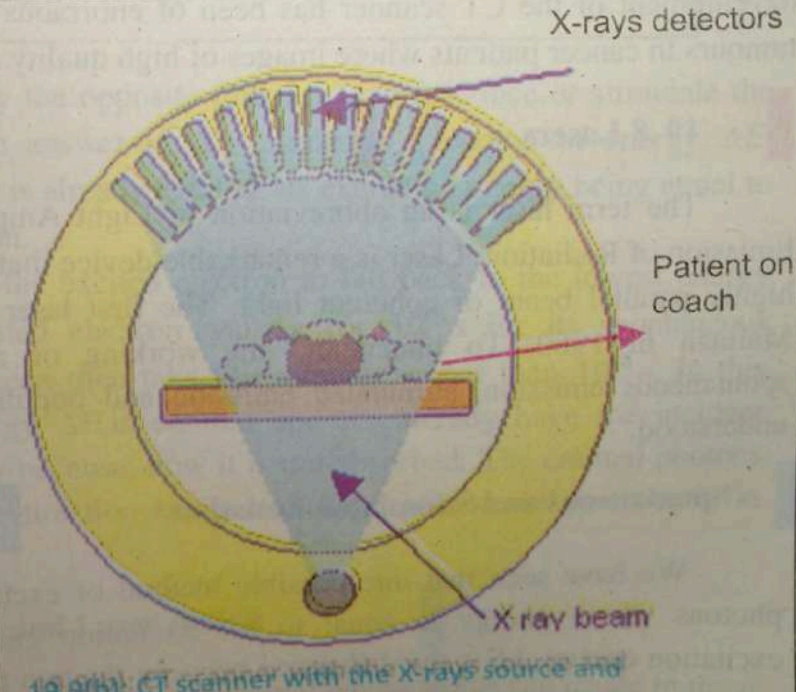
Medical Applications

Almost immediately after their discovery by Roentgen, X -rays were used in hospitals in Vienna for surgical operations. Since bone is more opaque to x -rays than flesh, if x -rays are allowed to pass through a human body, the bones cast their shadow on the photographic plate. The X -ray photographs reveal fractures of bones or the presence of foreign bodies. X -rays can also be used for curing malignant tissues of the body. X -ray therapy has also been used for the treatment of cancer.

19.7.3 C T Scanner

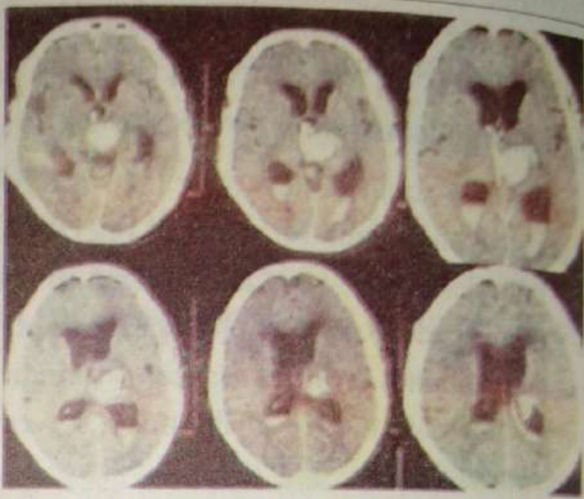
A 'normal' X- ray gives only limited information because it is rather like a shadow picture – fine detail within the image may be invisible especially if one organ lies in front of the region of the body being studied. To give a high quality images CT Scans are used to identify internal structures of various part of the human body. CT Scans Machine is 3D machine with computer model

In the CT scanner there is one X-ray source but a large number of detectors.



19.9(b): CT scanner with the X-rays source and detectors

The source and the detectors are mounted in a large ring-shaped machine and the patient is placed inside this on a couch as shown in figure. Each detector records an image and the source and detectors are then rotated around the patient to give views from a variety of direction. The image is called a tomogram. The couch and patient are then moved along the axis of the machine and another set of images is taken.


19.9(c): CT scan images.

This large number of images (many hundreds) are then combined by a computer to give a composite detailed 3D image of the organs under investigation. The development of the CT scanner has been of enormous help in the study of the tumours in cancer patients where images of high quality are essential.

19 .8 Lasers

The term laser is an abbreviation of Light Amplification by Stimulated Emission of Radiation. Laser is a remarkable device that produces an intense and highly parallel beam of coherent light. The first laser was fabricated by T.H. Maiman in 1960. To understand the working of a laser, terms such as spontaneous emission, stimulated emission and population inversion must be understood.

Spontaneous and Stimulated Emission

We have seen that one possible method of exciting an atom is to send photons whose energy is equal to the excitation energy of the atom. The excitation energy ΔE is the difference between the two possible energy states of an atom. The excited atom wait for a brief period of about 10^{-8} s and then spontaneously drops back to its lowest energy state, emitting light or photon of energy exactly equal to ΔE.

The only role of the passing photon is to give up its entire energy in exciting the electron to a higher energy state. This is a form of resonance in which a photon induce an upward transition.

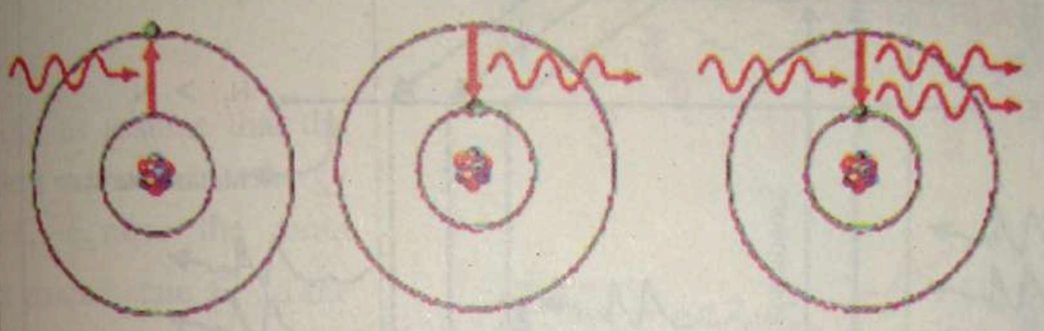


Figure 19. 10 : Electron falls back to its lower energy state E_1 and emitting a photon, thus duplicating the stimulated photon.

Can the photon play the opposite role, i. e. can it induce or stimulate the downward transition? The answer is, Yes. Imagine a photon of energy ΔE incident on an atom which is already excited, its excitation energy being equal to the energy ΔE of the photon.

The photon can stimulate the excited electron to fall back to the lowest energy state, instead of the excited electron waiting for 10^{-8} s for its spontaneous transition. This transition can then take place much sooner than 10^{-8} s. In this process a photon of energy ΔE is emitted and we already have the incident photon of the same energy because, now it is not absorbed. The emitted photons travel in exactly the same direction as the stimulated photon and are exactly in phase. (Fig 19. 10).

Population Inversion and Laser Action

Let us consider a simple case of a material whose atoms can reside in three different states as shown in fig 19 .11. State E_1 which is ground state, the excited state E_3, in which the atoms can reside only for 10^{-8} s and the metastable state E_2, in which the atoms can reside for 10^{-3} s, much longer than 10^{-8} s.

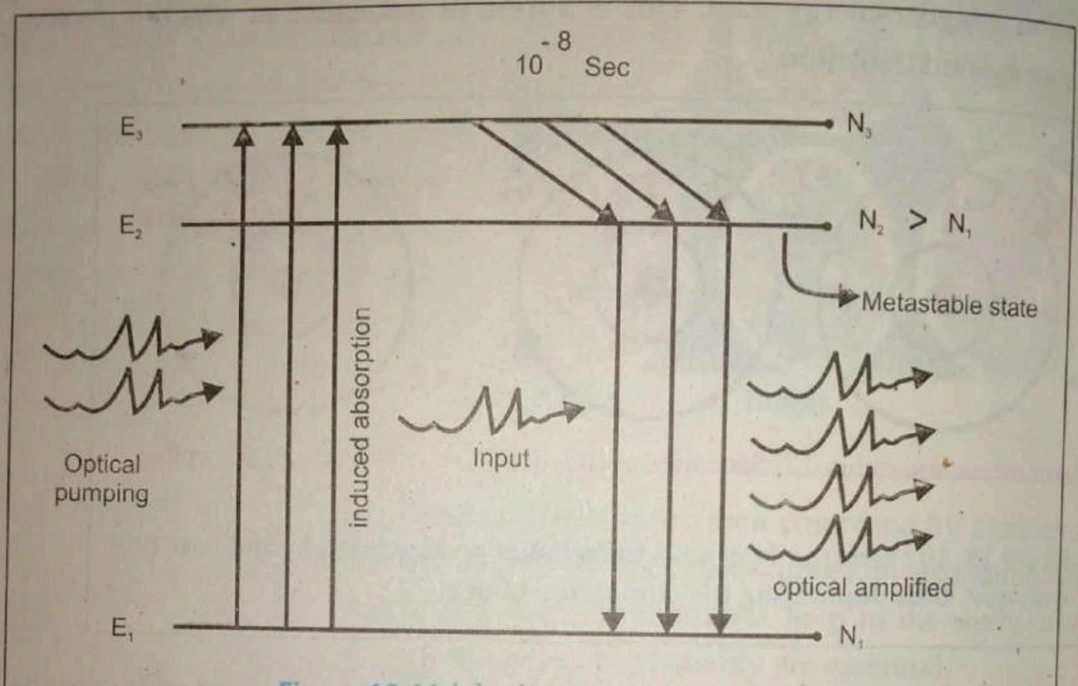


Figure 19.11 (a): electrons transition

A metastable state is an excited state in which an excited electron in unusually stable and from which the electron spontaneously falls to lower state only after relatively longer time.

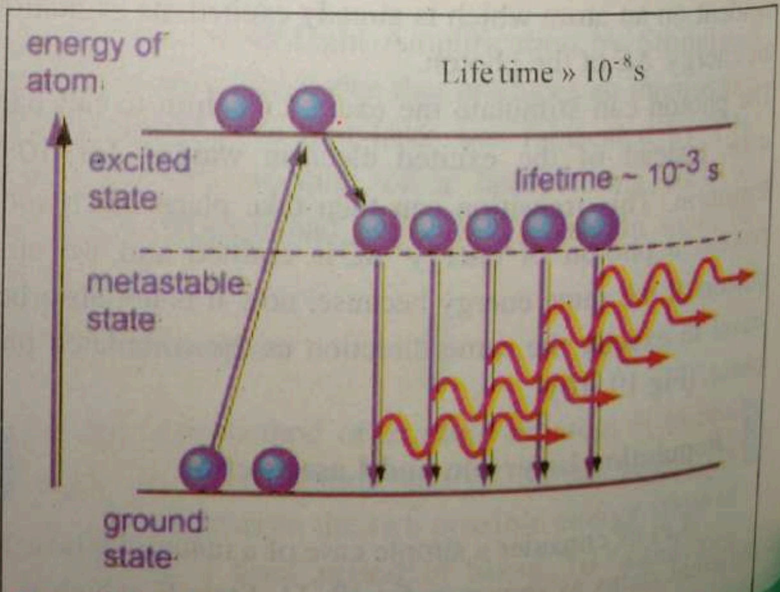


Figure 19.11 (b), metastable states

The transition from or to this state are difficult as compared to other excited states.

Hence, instead of direct excitation to this state, the electrons are excited to higher level for spontaneous fall to metastable state.

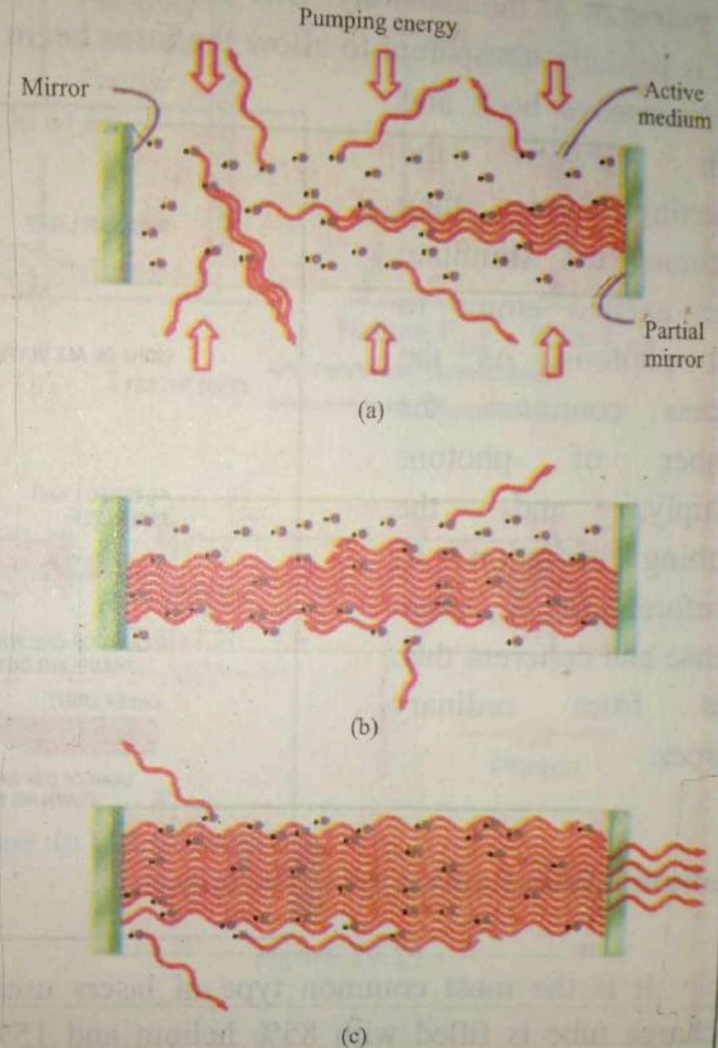


Figure 19.12 : photons move back and forth inside reflecting mirrors.

Also let us assume that the incident photons energy $hf = E_1 - E_3$ raise the atom from ground state E_1 to the excited state E_3, but the excited atoms do not decay back to E_1. Thus the only alternative for the atoms in the excited state E_3 is to decay spontaneously to state E_2. This eventually leads to the situation that the state E_2 contains more atoms than state, E_3. This situation is known as population inversion. Once the population inversion has been reached, the lasing action of a laser is simple to achieve.

The atoms in the metastable state E_2 are bombarded by photons of energy $hf = E_2 - E_1$, resulting in an induced emission, giving an intense, coherent beam in the direction of the incident photon.

The emitted photons must be confined in the assembly long enough to stimulate further emission from other excited atoms. This is achieved by using mirrors at the two ends of the assembly. One end is made totally reflecting, and the other end is partially transparent to allow the laser beam to escape (Fig 19.12). As the photons move back and forth between the reflecting mirrors they continue to stimulate other excited atoms to emit photons. As the process continues the number of photons multiply, and the resulting radiation is, therefore, much more intense and coherent than light from ordinary sources.

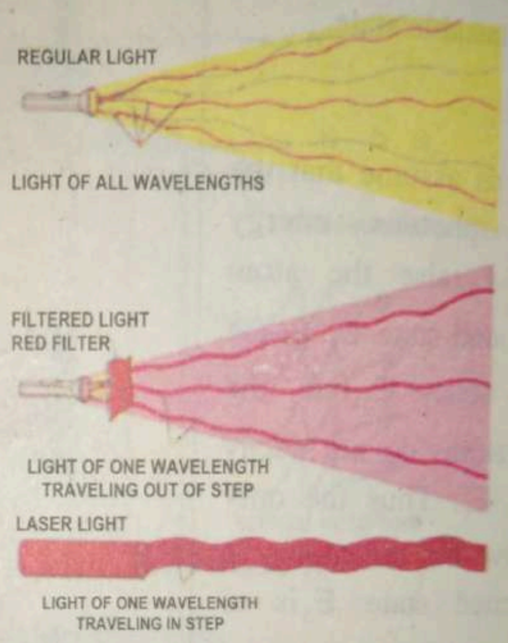


Figure 19.12 (d): comparison of light sources with laser.

Helium –Neon Laser

It is the most common type of lasers used in physics laboratories. Its discharge tube is filled with 85% helium and 15% neon gas. The neon is the lasing or active medium in this tube. By chance, helium and neon have nearly identical metastable states, respectively located 20.61 eV and 20.66 eV level. The high voltage electric discharge excites the electrons in some of the helium atoms to the 20.61 eV state. In this laser, population inversion in neon is achieved by direct collisions with same energy electrons of helium atoms.

Thus excited helium atoms collide with neon atoms, each transferring its own 20.61 eV of energy to an electron in the neon atom along with 0.05eV of K.E from the moving atom. As a result, the electrons in neon atoms are raised to the 20.66 eV state.

In this way, a population inversion is sustained in the neon gas relative to an energy level of 18.70 eV. Spontaneous emission from neon atoms initiate laser action and stimulated emission causes electrons in the neon to drop from 20.66 eV to the 18.70 eV level and red laser light of wavelength 632.8 nm corresponding to 1.96 eV energy is generated (Fig 19.13).

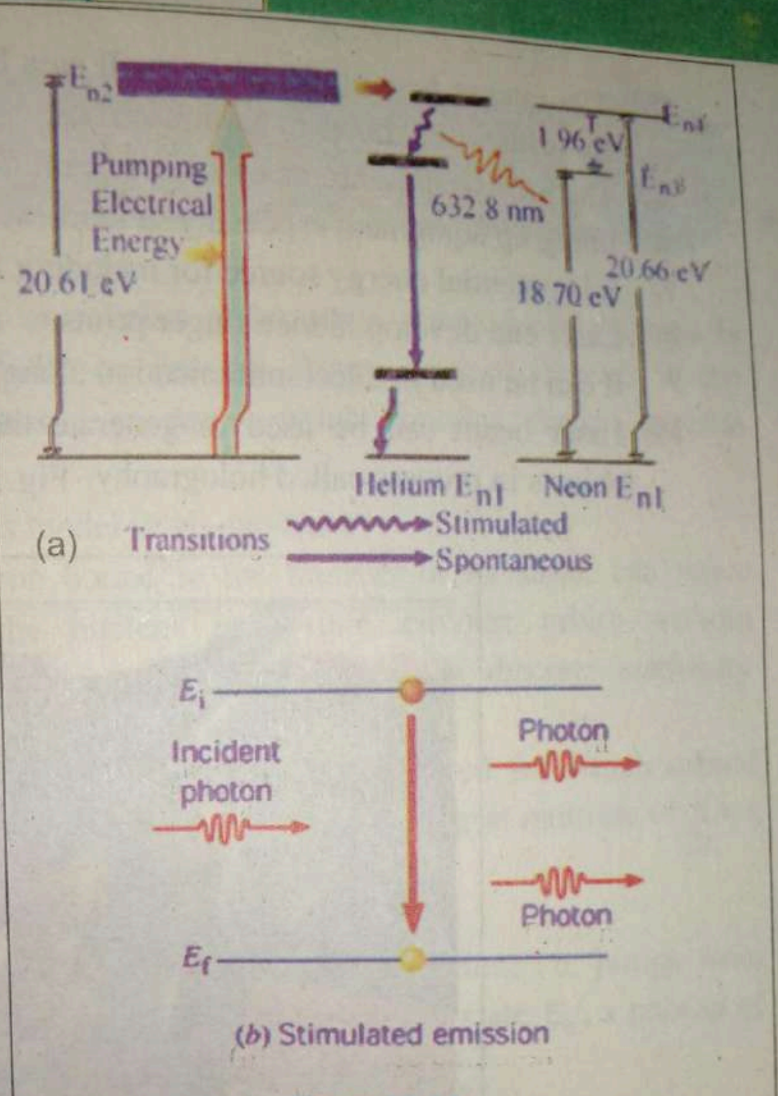


Figure 19.13 :

Uses of Laser

1. Laser beams are used as surgical tool for "welding" detached retains.
2. The narrow intense beam of laser can be used to destroy tissue in a localized area. Tiny organelles with a living cell have been destroyed by using laser to study how the absence of that organelle effects the behaviour of the cell.
3. Finely focused beam of laser has been used to destroy cancerous and pre –cancerous cells.
4. The heat of laser seals off capillaries and lymph vessels to prevent spread of the disease.

5. The intense heat produced in small area by a laser beam is also used for drilling tiny holes in hard materials.
6. The precise straightness of a laser beam is also useful to surveyors for lining up equipment especially in inaccessible locations.
7. It is potential energy source for including fusion reactions.
8. Laser can develop hidden finger prints.
9. It can be used in telecommunication along optical fibers.
10. Laser beam can be used to generate three dimensional images of objects in process called holography. Fig.19.14

Figure 19.14 : hologram

Key points

- When an atom gas or vapours at less than atmospheric pressure is suitably exited, usually by passing electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only.
- Postulates of Bohr's model of H –atom are:
  - i. An electron bound to the nucleus in an atom, can move around the nucleus in certain circular orbits without radiating. These orbits are called the discrete stationary states of the atom.
  - ii. Only those stationary states are allowed for which orbital angular momentum is, equal to an integral multiple of $\frac{h}{2\pi}$ i.e,

    $$mvr = \frac{nh}{2\pi}$$
  - iii. Whenever an electron makes a transition, i.e, jumps from high energy state E_n to a lower energy state E_p, a photon of energy hf is emitted so that $hf = E_n - E_p$.
- The transition of electrons in the hydrogen or other light elements results in the emission of spectral lines in the infrared, visible or ultraviolet region of electromagnetic spectrum due to small energy difference in the transition levels.
- The X–rays emitted in inner shell transition are called characteristics X–rays, because their energy depends upon the type of target materials.
- The X–rays that are emitted in all directions and with a continuous range of frequencies are known as continuous X–rays.
- Laser is the acr[illegible]ht amplification by stimulated Emission of Radiation.

- The incident photon absorbed by an atom in the ground state E_1, there by leaving the atom in the exited state E_2 called stimulated or induces absorption.
- Spontaneous or induced emission is that in which the atom emits a photon of energy $hf = E_2 - E_1$ in any arbitrary direction.
- Stimulated or induced emission is that in which the incident photon of energy $hf = E_2 - E_1$ induce the atom to decay by emitting a photon that travels in the direction of the incident photon. For each incident photon, we have two photons going in the same direction giving rise to an amplified as well as unidirectional coherent beam.

Exercise ?

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. If 13.6eV energy is required to ionize the hydrogen atom, then the required energy to remove an electron from $n = 2$ is
   a. 10.2eV b. 0 eV c. 3.4 eV d. 6.8 eV

2. For an atom of hydrogen atom the radius of the first orbit is given by,
   a. $\frac{h}{me^2}$ b. $\frac{me}{4h^2}$
   c. $\frac{h^2}{4\pi^2 kme^2}$ d. h^2me^2

3. The Balmer series is obtained when all the transition of electrons terminate on
   a. 1^{st} orbit b. 2^{nd} orbit c. 3^{rd} orbit d. 4^{th} orbit

4. In accordance with Bohr's theory the K.E of the electron is equal to
   a. $\frac{1}{2}\frac{Ze^2}{r}$ b. $\frac{Ze^2}{r}$ c. $\frac{Ze^2}{r^2}$ d. $\frac{1}{2}\frac{Ze^2}{r^2}$

5. According to Bohr's theory the radius of quantized orbit is given by
   a. $\frac{4\pi^2 m}{n^2h^2Ze^2}$ b. $\frac{n^2h^2}{4\pi^2 mZe^2}$
   c. $\frac{4\pi^2 m Ze^2 k}{n^2 h^2}$ d. $\frac{n^2h^2 Ze^2}{4\pi^2 m}$

6. In the Bohr's model of the hydrogen atom, the lowest orbit corresponds to
   a. Infinite energy b. Maximum energy
   c. Minimum energy d. Zero energy

7. When an electron in an atom goes from a lower to higher orbit its
   a. K.E increases, P.E decreases
   b. K.E increases, P.E increases
   c. K.E decreases, P.E increases
   d. K.E decreases, P.E decreases

8. Frequency of X –rays depends upon
   a. Number of electrons striking target
   b. Accelerating potential
   c. Nature of the target
   d. Both b and c

9. Target material used in X –rays tube must have following properties.
   a. High atomic number and high melting point
   b. High atomic number and low melting point.
   c. Low atomic number and low melting point.
   d. High atomic number only

10. Laser is a device which can produce.
    a. Intense beam of light
    b. Coherent beam of light
    c Monochromatic beam of light
    d. All of the above

Conceptual Questions

1. Why does the spectrum of hydrogen consists of many lines even though a hydrogen atom has only a single electron?
2. Suppose that the electron in hydrogen atom obeyed classical mechanics rather quantum mechanics. Why would such a hypothetical atom emit a continuous spectrum rather than the observed line spectrum?
3. Can the electron in the ground state of hydrogen absorb a photon of energy (a) less than 13.6eV (b) greater than 13.6eV? Explain.
4. Why do solids give rise to continuous spectrum while hot gases give rise to line spectrum?
5. Explain the difference between laser light and light from an incandescent lamp.
6. Why Bohr extends quantum theory to the structure of the atom?
7. Why $_2He^4$ has larger ionization energy than H?
8. X –rays can emit electrons from metal surface and X –rays can be diffracted. Comment?
9. Why X-rays have different properties from light even though both originate from orbital transition of electrons in excited atoms?

10. What is meant by the statement that a laser beam is coherent, monochromatic and parallel?
11. What are laser knives?
12. Why we cannot see atom?
13. What meant by breaking radiation?
14. What is optical pumping?

Comprehensive Questions

1. Describe the spectrum of hydrogen atom in detail.
2. What are Bohr's postulates about hydrogen atom? Hence derive expression for the (a) radii of electron orbit (b) energy of the electron.
3. What do you understand by the terms normal state, Excited state, Excitation energy, ionization energy.
4. What are X–rays? Give an account of the properties, and uses of X–rays.
5. What is a laser? Explain the principle and operation of a laser. Describe some practical uses of lasers.

Numerical Problems

1. Find the shortest wavelength photon emitted in the lyman series of hydrogen. (91nm)
2. What is the wavelength of the second line of paschen series? [1281.43nm].
3. Calculates the longest wavelength of radiation for the paschen series [1875 nm].
4. The series limit wavelength of the Balmer series is emitted as the electron in the hydrogen atom falls from $n=\infty$ to the $n=2$ state. What is the wavelength of this line. Where $\Delta E=3.40eV$. [365nm].
5. A photon is emitted from a hydrogen atom, which undergoes a transition from that $n=3$ state to the $n=2$ state. Calculate (a) the energy (b) the wavelength, and (c) frequency of the emitted photon. [(a)1.89eV,(b)658 nm (c)4.56×10^{14}Hz]

6. Find the longest wavelength of light capable of ionizing a hydrogen atom. How much energy in needed to ionize a hydrogen atom? [91.2 nm, 13.6 eV].

7. Calculate the radius of the innermost orbital level of the hydrogen atom. [5.3×10^{-11}m].

8. (a) Determine the energy associated with the innermost orbit of the hydrogen atom (n=1). (b) Determine the energy associated with the second orbit of the hydrogen atom. (c) What energy does an incoming photon possess to raise an electron from first to the second allowed orbit of the hydrogen atom? [(a) = −13.6eV, (b) − 3.4eV, (c) − 10.2 eV].

9. An electron drops from the second energy level to the first energy level within an excited hydrogen atom (a) determine the energy of the photon emitted (b) calculate the frequency of the photon emitted (c) calculate the wavelength of the photon emitted.

[(a) 10.2 eV (b) 2.5×10^{15}Hz (c) 1.2×10^{-7}m]

10. An electron is in the first Bohr orbit of hydrogen. Find (a) the speed of the electron. (b) the time required for the electron to circle the nucleus. [(a) 2.19×10^{6}ms^{-1}, (b) 1.52×10^{-6} s]

11. Electrons in an x –ray tube are accelerated through a potential difference of 3000 V. if these electrons were slowed down in a target, what will be the minimum wavelength of x –rays produced. [4.14 Å]

12. Compute the potential difference through which an electron must be accelerate in order that the short –wave limit of the continuous x –ray spectrum shall be exactly 0.1nm. [12,400V].