UNIT 15

A.C Circuit

After studying this chapter the students will be able to

- describe the terms time period, frequency, instantaneous peak value and root mean square value of an alternating current and voltage.
- represent a sinusoidally alternating current or voltage by an equation of the form $x = x_o \sin \omega t$.
- describe the phase of A.C and how phase lags and leads in A.C Circuits.
- identify inductors as important components of A.C circuits termed as chokes (devices which present a high resistance to alternating current).
- explain the flow of A.C through resistors, capacitors and inductors.
- apply the knowledge to calculate the reactances of capacitors and inductors.
- describe impedance as vector summation of resistances and reactances.
- construct phasor diagrams and carry out calculations on circuits including resistive and reactive components in series.
- solve the problems using the formulae of A.C Power.
- explain resonance in an A.C circuit and carry out calculations using the resonant frequency formulae.
- describe that maximum power is transferred when the impedances of source and load match to each other.
- describe the qualitative treatment of Maxwell's equations and production of electromagnetic waves.
- become familiar with electromagnetic spectrum (ranging from radiowaves to γ-rays).

- identify that light is a part of a continuous spectrum of electromagnetic waves all of which travel in vacuum with same speed.
- describe that the information can be transmitted by radiowaves.
- identify that the microwaves of a certain frequency cause heating when absorbed by water and cause burns when absorbed by body tissues.
- describe that ultra violet radiation can be produced by special lamps and that prolonged exposure to the Sun may cause skin cancer from ultra violet radiation.

The electricity produced by most generators is in the form of alternating current. In general AC generators, motors and other electrical equipment's are simpler, cheaper and more reliable than their DC counterparts.

Investigation of the behaviour of resistance, inductance and capacitance in AC circuits prepares us to look into the many diverse uses of these circuit elements and AC sources. A theory describing relation between accelerating charges, circuits and electric and magnetic fields was given by James Clerk Maxwell in 1864 called electromagnetic theory. The theory predicts that accelerating electric charges radiate electromagnetic waves, which propagate at the speed of light. When the acceleration is in the form of a continuous oscillation, the frequency of the electromagnetic waves is equal to the frequency of oscillation of the charges.

Maxwell formulated four equations that are regarded as the basis of all electrical and magnetic phenomena.

The consequences of Maxwell's equations are very far reaching. Max well predicted the existence of electromagnetic waves and that light is a form of electromagnetic radiation. Thus Maxwell unified the subjects of optics and electromagnetism.

For your information

Metal detectors are used at air ports and other sensitive areas for security purposes. Metal objects cause changes in an electromagnetic field when they pass through the door way. A circuit detects the changes and sets off an alarm.

15.1 Alternating Voltage and Current

The supply of current for electrical devices may come from a direct current source (DC), or an alternating current source (AC). In direct current electricity, electrons flow continuously in one direction from the source of power through a conductor to a load and back to the source of power. The voltage in direct current remains constant. DC power sources include batteries and DC generators. In alternating current an AC generator is used to make electrons flow first in one direction then in another. A source which produces potential difference of changing polarity with time is called as alternating source. *A voltage which changes its polarity at regular interval of time is called an alternating voltage.*

When an alternating voltage is applied in a circuit, the current flows first in one direction and then in the opposite direction; the direction of current at any instant depends upon the polarity of the voltages. Fig. shows an alternating voltage source connected to a resistor *R*. In Fig the upper terminal of alternating voltage source is positive and lower terminal negative so that current flows in the circuit as shown in Fig.15.1 (i).

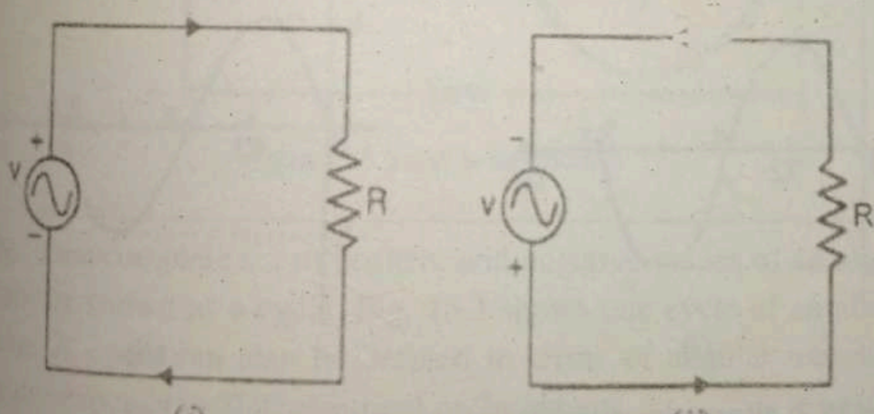


Figure 15.1: AC circuit

After some time, the polarities of the voltage source are reversed, so that current now flows in the opposite direction. This is called alternating current because the current flows in alternate directions in the circuit.

15.2 Sinusoidal Alternating Voltage and Current

We have studied in chapter 14 that sinusoidal alternating voltage can be produced by rotating a coil with a constant angular velocity (say ω rad/s) in a uniform magnetic field. AC voltage switches polarity over time. When, graphed over time, the "wave" traced by this voltage of alternating polarity from an alternator takes on a distinct shape, known as a sine wave. The sinusoidal alternating voltage can be expressed by the equation:

$$V = V_m \sin\omega t \qquad \text{...(15.1)}$$

Where V is Instantaneous value of alternating voltage, V_m is Max. value of alternating voltage and ω is angular velocity of the coil. Fig.15.2 (i)shows, the waveform of sinusoidal voltage whereas Fig.15.2 (ii) shows the waveform of sinusoidal current. Fig15.2: shows that sinusoidal voltage or current not only changes direction at regular intervals but the magnitude is also changing continuously. The change from one polarity to the other is a smooth one, the voltage level changing most rapidly at the zero ("crossover") point and most slowly at its peak.

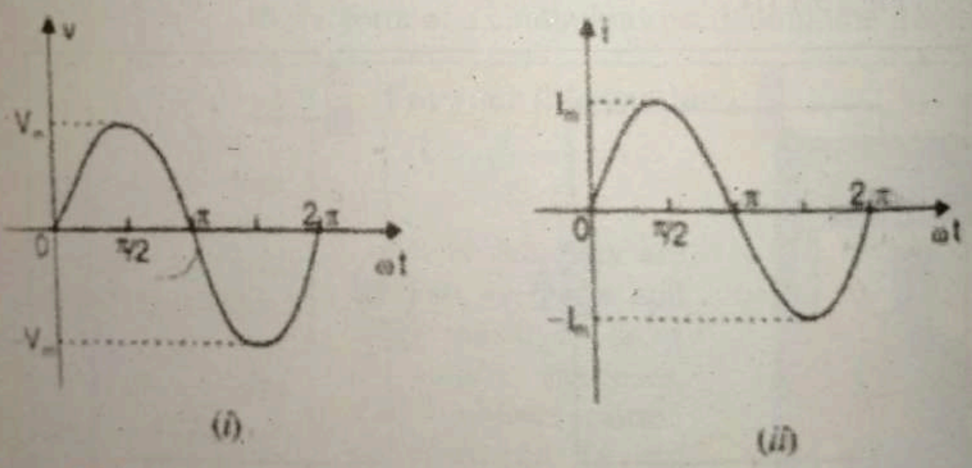


Figure 15.2: alternating current and voltage

15.3 A.C. Terminologies

Alternating voltage or current changes continuously in magnitude and alternates in direction at regular intervals of time.

It rises from zero to maximum positive value, falls to zero, increases to a maximum in the reverse direction and falls back to zero again (as shown in Fig15.3). The important A.C. terminology is defined below:

1. **Instantaneous value.** The value of an alternating quantity at any instant is called instantaneous value. The instantaneous values of alternating voltage and current are represented by V and I respectively. As an example, the instantaneous values of voltage at 0°, 90° and 270° are 0, + V_m, - V_m respectively as shown in Fig 15.3.

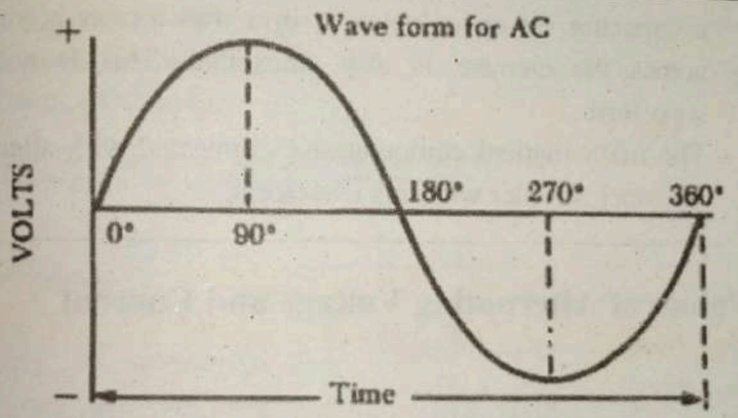


Figure 15.3: AC waveform

2. **Cycle.** One complete set of positive and negative values of an alternating quantity is known as a cycle. Fig. 15.3 shows one cycle of an alternating voltage. A cycle can also be defined in terms of angular measure. One cycle corresponds to 360° electrical or 2π radians.
3. **Time period.** The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.
4. **Frequency.** The number of cycle that occurs in one second is called the frequency (f) of the alternating quantity. It is measured in cycle /s (C/s) or Hertz (Hz). One hertz is equal to 1C/s.

and

The frequency of power system in Pakistan is 50 C/s or 50 Hz. It means that alternating voltage or current completes 50 cycles in one second. The 50 Hz frequency is the most popular because it gives the best results when used for operating both lights and machinery.

For your Information

Importance of Sine Waveform

Alternating voltages and currents can be produced in variety of waveforms (e.g. square waves, triangular waves, rectangular waves etc), but the engineers still choose to adopt sine waveform. It has following advantages:

1. The sine waveform produces the least disturbance in the electrical circuit and is the smoothest and efficient waveform. For example, when current in a capacitor, in an inductor or in a transformer is sinusoidal, the voltage across the element is also sinusoidal. This is not true of any other waveform.
2. The mathematical computations, connected with alternating current work, are much simpler with this waveform.

15.4 Values of Alternating Voltage and Current

The average value of an AC waveform is not the same value as that for a DC waveforms average value. This is because the AC waveform is constantly changing with time. It can be expressed as, Peak value, Average value or mean value, and R.M.S. value or effective value.

Peak Values

The maximum value reached by an AC waveform is called its peak value. The peak value of a sine wave occurs twice each cycle, once at the positive maximum value and once at the negative maximum value. The peak value of a waveform is sometimes also called its amplitude, but the term "peak value" is more descriptive. The knowledge of peak value is important in case of testing materials. However, peak value is not used to specify the magnitude of alternating voltage or current. The peak or maximum value of an alternating voltage or current is represented by V_m or I_m.

Average Value

The average value of a waveform is the average of all its values over a period of time. Finding an average value over time means adding all the values that occur in a specifying time interval and dividing the sum by that time. In performing such a computation, we regard the area above the time axis as positive area and area below the time axis as negative area. The algebraic signs of the areas must be taken into account when computing the total (net) area.

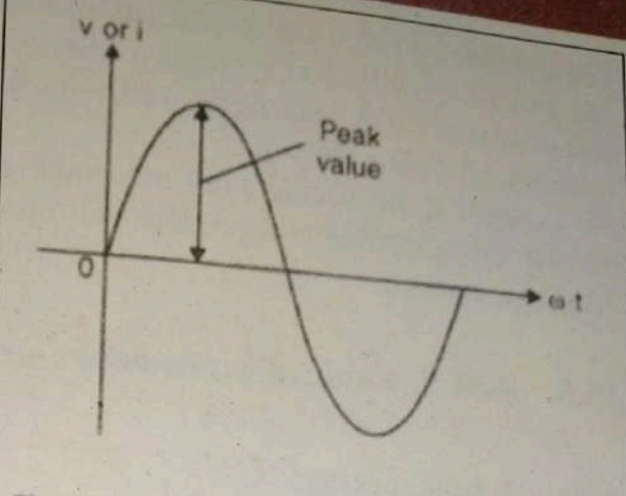


Figure 15.4: peak value

$$\text{Average} = \frac{\text{Total (net)area under curve for time } T}{\text{Time } T}$$

R.M.S. or Effective Value

In order to specify a sinusoidal voltage or current we do not use average value, because its value over one cycle is zero and cannot be used for power calculation. Therefore, we use another suitable method to measure the effectiveness of an alternating current. The equivalent average value for an alternating current system that provides the same power to the load as a DC equivalent circuit is called the "effective value".

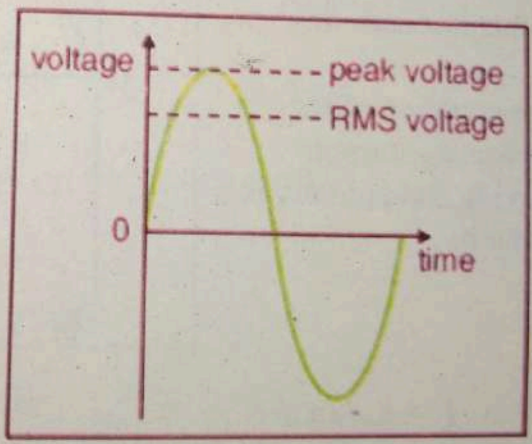


Figure 15.5(a): peak and RMS value of voltage

This effective power in an alternating current system is therefore equal to: (I^2R Average). As power is proportional to current squared, the effective current, I will be equal to $\sqrt{I^2}$ Ave.

Therefore, the effective current in an AC system is called the Root Mean Squared or R.M.S. value.

The effective or r.m.s. value of an alternating current is that steady current (d.c.)which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.

For example if the effective or r.m.s. value of an alternating current is 7A, then the alternating current will produce the same heating effect as that produced by 7A direct current.

15.5 R.M.S. Value of Sinusoidal Current

Although peak, average and peak to peak values may be important in some engineering applications, but it is the r.m.s. or effective value which is used to express the magnitude of an alternating voltage or current.

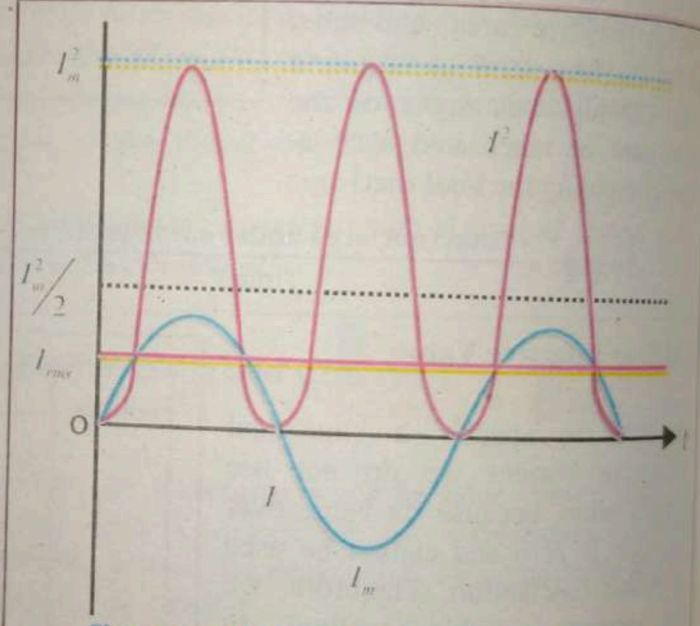


Figure 15.5(b): peak and RMS value of current

The equation of the alternating current varying sinusoidally is given by:

$$I = I_m \sin \omega t \qquad ...(15.2)$$

If this current is passed through a resistance R, then power delivered at any instant is

$$P = I^2R = (I_m \sin \omega t)^2 R$$

$$= I_m^2 R \sin^2 \omega t \qquad ...(15.3)$$

Because the current is squared, power is always positive. Since the value of $\sin^2 \omega t$ varies between 0 and 1, its average value is 1/2

$$\therefore \text{Average power delivered, } P = \frac{1}{2} I_m^2 R \qquad ...(15.4)$$

If $I_{r.m.s}$ is the r.m.s. (or effective) value of alternating current, then by definition,

Power delivered, $P = I^2_{r.m.s} R$ ...(15.5)

From Eqs. (15.4) and (15.5), we have,

$$I^2_{r.m.s}R = \frac{1}{2} I^2_m R$$

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = 0.7071\ I_m$$

$$I_{r.m.s} = 0.707\ I_m \quad ...(15.6)$$

An alternating current can also be represented as a cosine function of time. $I = I_m cos\omega t$. Similarly, alternating voltage can be represented as $V = V_m cos\omega t$.

15.6 Phase of A.C.

In electrical engineering, we are more concerned with relative phases or phase difference between different alternating quantities rather than with their absolute values. The word "phasor" is short for "phase vector." It is a way to represent a sine or cosine function graphically. Consider an alternating voltage wave of time period T second as shown in Fig.15.6.

The maximum positive value ($+ V_m$) occurs at $T/4$ second or $\pi/2$ radians. Therefore phase of maximum positive value is $T/4$ second or $\pi/2$ radians.

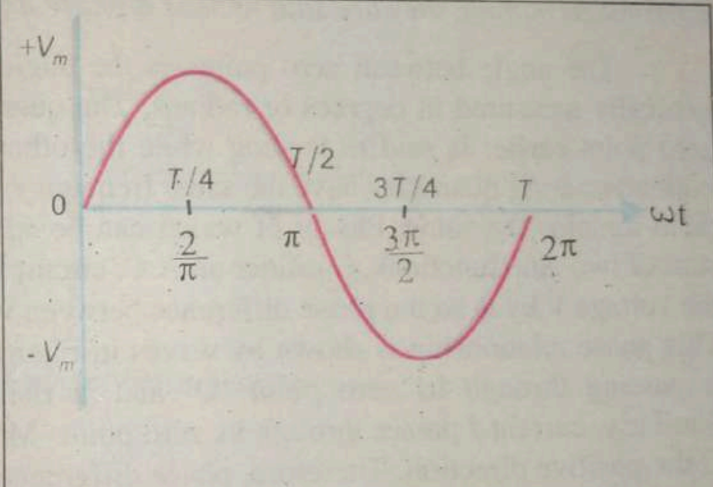


Figure 15.6: alternating voltage wave form

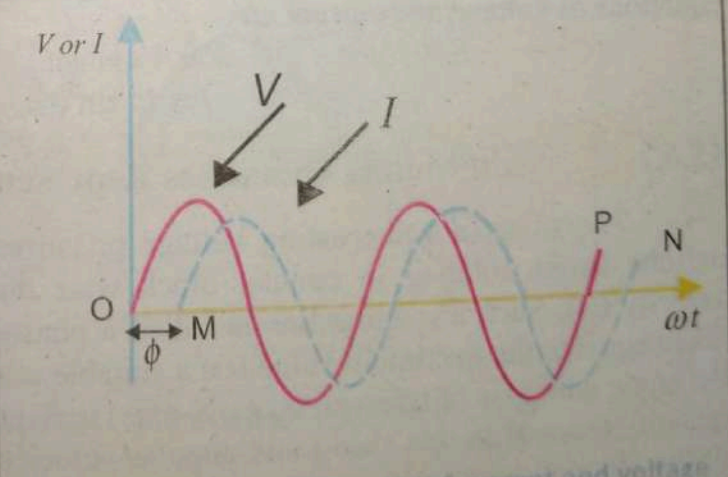


Figure 15.7: phase difference of current and voltage

Similarly, the phase of negative peak ($-V_m$) is $3T/4$ second or $3\pi/2$ radians.

Phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference.

Phase Difference

: In most of practical circuits, alternating voltage and current have different phases. Thus voltage may be passing through its zero point while the current has passed or it is yet to pass through its zero point in the same direction. We say that voltage and current have a phase difference.

Hence *when two alternating quantities of the same frequency have different zero point, they are said to have a phase difference.*

The angle between zero points is the angle of phase difference ϕ. It is generally measured in degrees or radians. The quantity which passes through its zero point earlier is said to leading while the other is said to be lagging. Since both alternating quantities have the same frequency, the phase difference between them remains the same. Phasor of waves can be added as vectors to produce the sum of two sine functions. Consider an A.C. circuit in which current I lags behind the voltage V by ϕ so the phase difference between voltage and current is ϕ. This phase relationship is shown by waves in Figure. Thus in Fig15.7, voltage V is passing through its zero point 'O' and is rising in the positive direction. Similarly, current I passes through its zero point 'M' as shown in fig and is rising in the positive direction. Therefore, phase difference between voltage and current is OM (= ϕ). Similarly, difference at other points P and N is PN (= ϕ). The equations of voltage and current are:

$$V = V_m \sin\omega t \qquad \text{(i)}$$

$$I = I_m \sin(\omega t - \phi) \qquad \text{(ii)}$$

15.6.1 Alternating Quantities Representation

The sinusoidal alternating voltage or current is represented by a line of definite length rotating in counter clock wise direction at a constant angular velocity (ω). Such a rotating line is called a phasor. The length of the phasor is taken equal to the maximum value (on a suitable scale) of the alternating quantity, the angle with axis of reference (i.e., X-axis) indicates the phase of the alternating quantity (current in this case) and angular velocity equal to the angular velocity of the alternating quantity. A phasor diagram permits addition and subtraction of alternation voltages or current with a fair degree of case.

In AC circuits, currents and voltages are all sinusoidal functions. The general mathematical form of such a function is: $I = I_m \sin \omega t$. Let line OA represents the maximum value I_m on the scale. Imagine the line OA (or Phasor, as it is called) to be rotating in anticlockwise direction at an angular velocity ω rad/s about the point O. Measuring the time from the instant when OA is horizontal, let OA rotate through an angle θ in the anticlockwise direction. The projection of OA on the Y-axis is OB.

$$OB = OA \sin\theta$$

$$I = I_m \sin \omega t$$

where I, is the value of current at that instant.

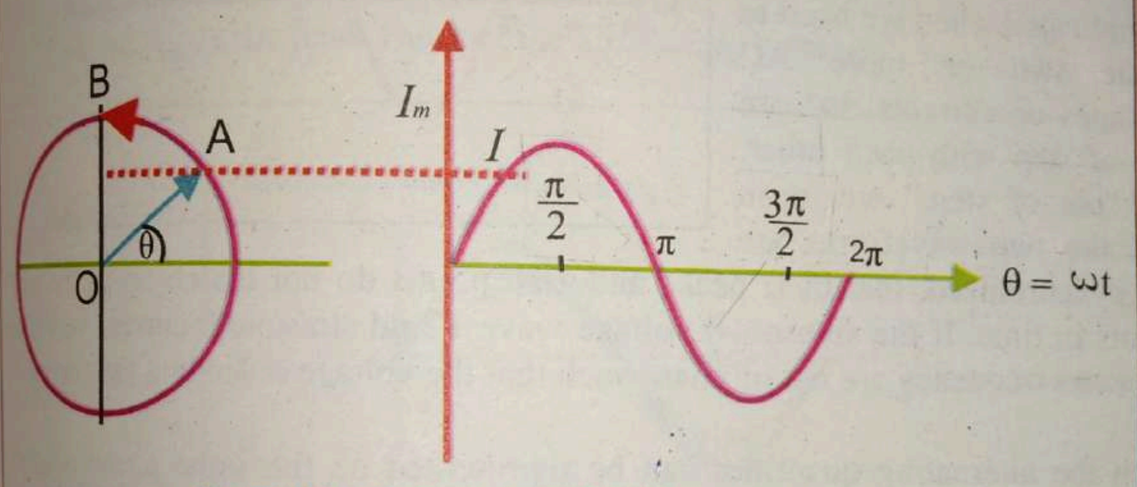


Figure 15.8:Phasor Representation

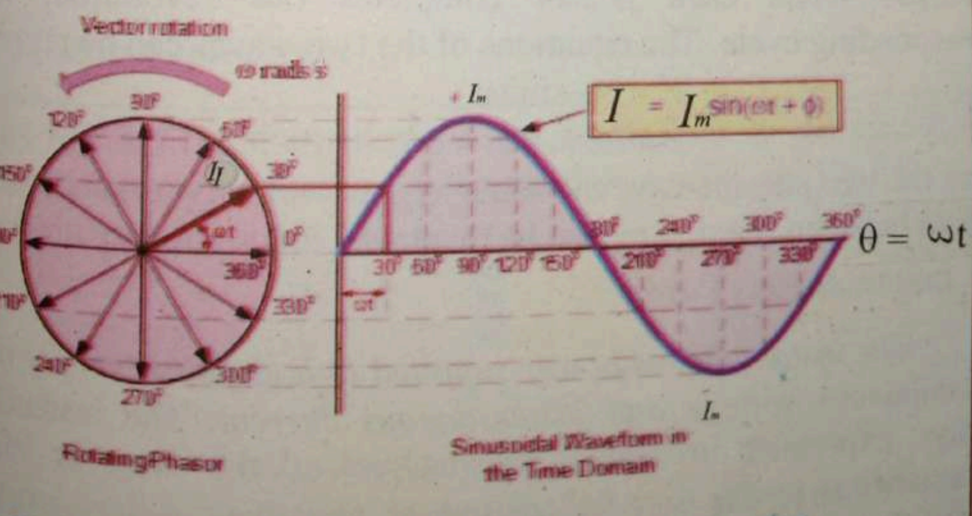


Figure 15.9: rotating vectors at different phase angles

Hence the projection of the phasor OA and the y-axis at any instant gives the value of current at that instant. Thus when θ= 90°, the projection on y-axis is OA (= I_m) itself. That the value of current at this instant (i.e. at θ or $\omega t = 90^o$) is I_m can be readily established if we put θ = 90° in the current equation. If we plot the projections of the phasor on the Y-axis versus its angular position point by point, a sinusoidal alternating current wave is generated as shown in Fig15.8. Thus the phase represents the sine wave for every instant of time. Things start to get complicated when we need to relate two or more AC voltages or currents that are out of step with each other. By "out of step," we mean that the two waveforms are not synchronized: that their peaks and zero points do not match up at the same points in time. If the sinusoidal voltage wave V and sinusoidal current wave I of the same frequency are out of phase such that the voltage is leading the current by ϕ^0.

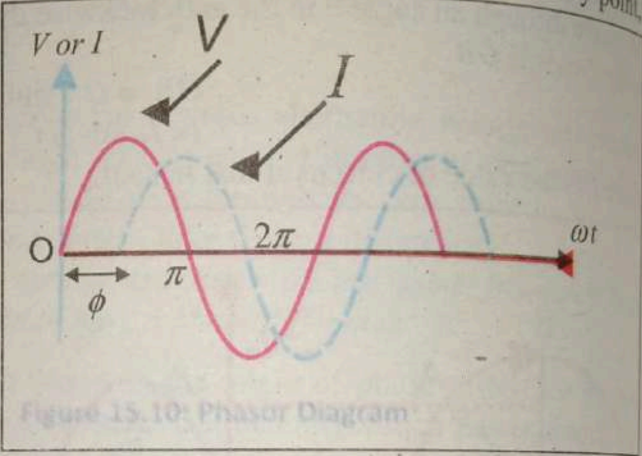


Figure 15.10: Phasor Diagram

Then the alternating quantities can be represented on the same phasor diagram because the phasors V_m and I_m rotate at the same angular velocity ω and hence phase difference ϕ between them remains the same at all times as shown in fig15.10: When each phasor completes one revolution, it generates the corresponding cycle. The equations of the two waves can be represented as:

$$V = V_m \sin \omega t$$

$$I = I_m \sin (\omega t - \phi) \qquad ...(15.7)$$

Since the two phasors have the same angular velocity (ω) and there is no relative motion between them, they can be displayed in a stationary diagram.

Instantaneous Power

The instantaneous power supplied to a circuit is simply the product of the instantaneous voltage and instantaneous current. The instantaneous power is always expressed in watts, irrespective of the type of circuit used. The instantaneous power may be positive or negative. A positive value means that power flows from the source to the load. Consequently, a negative value means that power flows from the load to the source.

15.7 A.C. Through Resistance

Consider a circuit containing a pure resistance of R connected across an alternating voltage source as shown in Fig15.11 (a), then free electrons flow in one direction for the first half-cycle of the supply and then flow in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit. The applied voltage and current pass through their zero values at the same instant and attain their positive and negative peaks at the same instant such that current is in phase with the applied voltage as shown in fig 15.11(b).The alternating voltage is be given by

$V = V_m \sin \omega t$ (i)

where, V_m is the peak value of the alternating voltage.

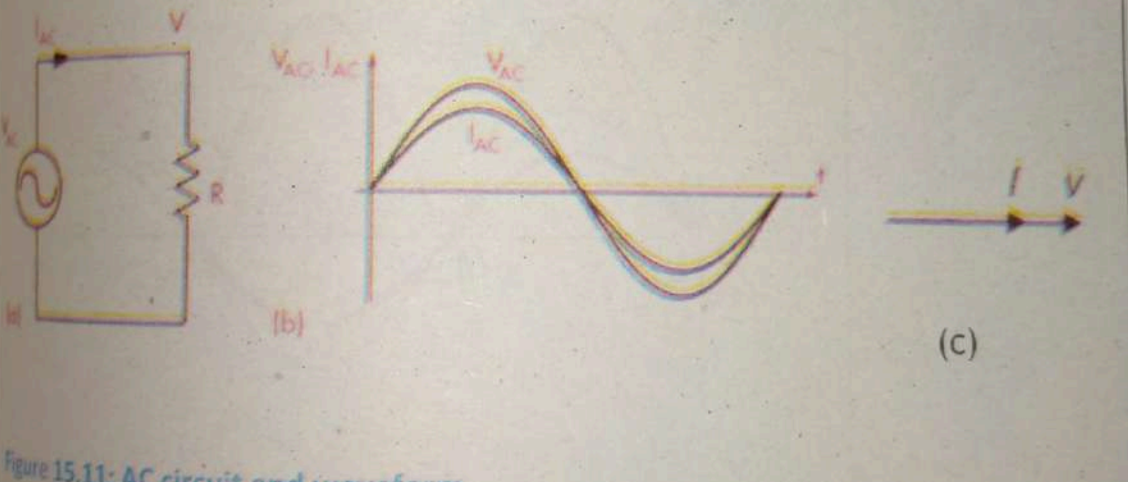


Figure 15.11: AC circuit and waveform

As a result of this voltage, an alternating current I will flow in the circuit. The applied voltage has to overcome the drop in the resistance only i.e., $V = IR$

Or $$I = \frac{V}{R} = \frac{V_m}{R} \sin \omega t$$

$\therefore$ $$I_m = \frac{V_m}{R}$$

$$I = I_m \sin \omega t \qquad \text{(ii)}$$

The value of I will be maximum (i.e. I_m) when $\sin \omega t = 1$. Eqs. (i) and (ii) shows that the applied voltage and the circuit current are in phase with each other. This is also indicated by the phasor diagram shown in Fig15.11(c).

In terms of r.m.s. value,

$$\frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} \times R \qquad \ldots(15.8)$$

Or

$$V_{rms} = I_{rms} R \qquad \ldots(15.9)$$

15.7.1 Power Loss in an Resistor

The power curve for a pure resistive circuit is obtained from the product of the corresponding instantaneous values of voltage and current. Fig15.12 shows that power is always positive except at points L,M and N at which it drops to zero for a moment.

This means that the voltage source is constantly delivering power to the circuit which is consumed by the circuit.

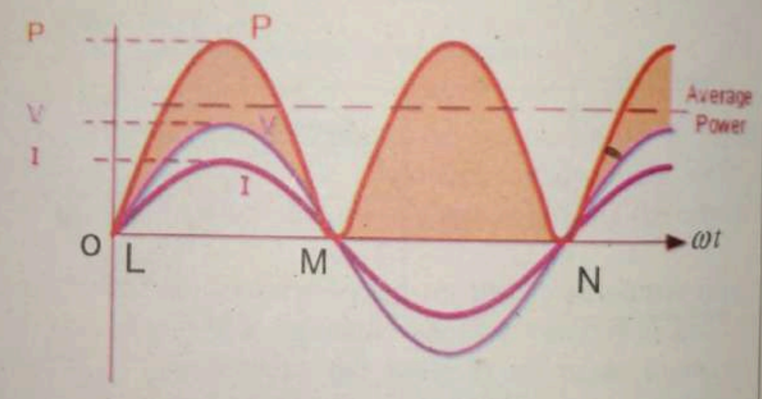


Figure 15.12: power curve

The average power dissipated in resistor R over one complete cycle of the applied is:

$$P = \langle VI \rangle = \langle V_m \sin \omega t \times I_m \sin \omega t \rangle \qquad \ldots(15.10)$$

$$= V_m I_m \langle \sin^2 \omega t \rangle$$

$$= \frac{V_m I_m}{2} \qquad \because \langle \sin^2 \omega t \rangle = \frac{1}{2}$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms} \qquad \ldots(15.11)$$

Example 15.1

An A.C. circuit consists of a pure resistance of 20Ω and is connected across A.C. supply of 220V, 50Hz. Calculate (a) current (b) power consumed and (c) equation for voltage and current.

Solution.

Resistance $R = 20\ \Omega$
Voltage $V = 220$ V
Frequency $f = 50$ Hz

Maximum value of an alternating voltage is $V_m = \sqrt{2}\,V = \sqrt{2} \times 220 = 311.1$V

(a) Current, $I = V/R = 220/20 = 11$A

(b) Average power P dissipated in the resistor is

$$P = VI = 220 \times 11 = 2420 \text{ w}$$

Maximum value of an alternating current $I_m = \sqrt{2}\,I = \sqrt{2} \times 11 = 15.55$ A

$$\omega = 2\pi f = 2\pi \times 50 = 314 \text{ rad s}^{-1}$$

(c) Equations for voltage and current is

$V = V_m \sin(\omega t)$ & $I = I_m \sin(\omega t)$

putting values

$V = 311.1\sin(314t)$ & $I = 15.55\sin(314t)$

15.8 A.C. Through Pure Inductance

An inductor is a two-terminal electrical component which resists changes in electric current passing through it. It consists of a conductor such as a wire, usually wound into a coil. Consider an alternating voltage applied to a pure inductance of L as shown in Fig15.13 when a sinusoidal current I flow in time t then a back e.m.f. ($=L\,\Delta I/\Delta t$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. As there is no drop in potential, so the applied voltage has to overcome the back. emf

∴ Applied alternating voltage = Back e.m.f.

So the energy which is required in building up current in inductance L, is returned back during the decay of the current.

Let the equation for alternating current is :

$$I = I_m \sin \omega t \qquad \ldots(i)$$

The changing current sets up a back e.m.f in the coil.
The magnitude of back e.m.f is

$$\mathcal{E} = L\frac{\Delta I}{\Delta t}$$

To maintain a constant current the applied e.m.f must be constantly applied.
The magnitude of applied voltage is

$$V = L\frac{\Delta I}{\Delta t} = L\frac{\Delta(I_m \sin \omega t)}{\Delta t}$$

$$= LI_m \frac{\Delta(\sin \omega t)}{\Delta t} \qquad (ii)$$

Using the result of simple calculus:

$$\frac{\Delta(\sin \omega t)}{\Delta t} = \omega \cos \omega t \qquad (iii)$$

putting the values of Eq (iii) in (ii) we get

$$V = \omega L I_m \cos \omega t$$

$$\therefore (\omega L I_m = V_m)$$

or $V = V_m \cos \omega t$

$$V = V_m \sin(\omega t + \frac{\pi}{2}) \qquad (iv)$$

From Eqs (i) and (iv) it is clear that current lags behind the voltage by $\pi/2$ radians or 90°. Hence in a pure inductance, current lags behind the voltage by 90°.
Fig. 15.13(b) also shows that current lags the voltage in an inductive coil. Inductance opposes the change in current and serves to delay the increase or decrease of current in the circuit.

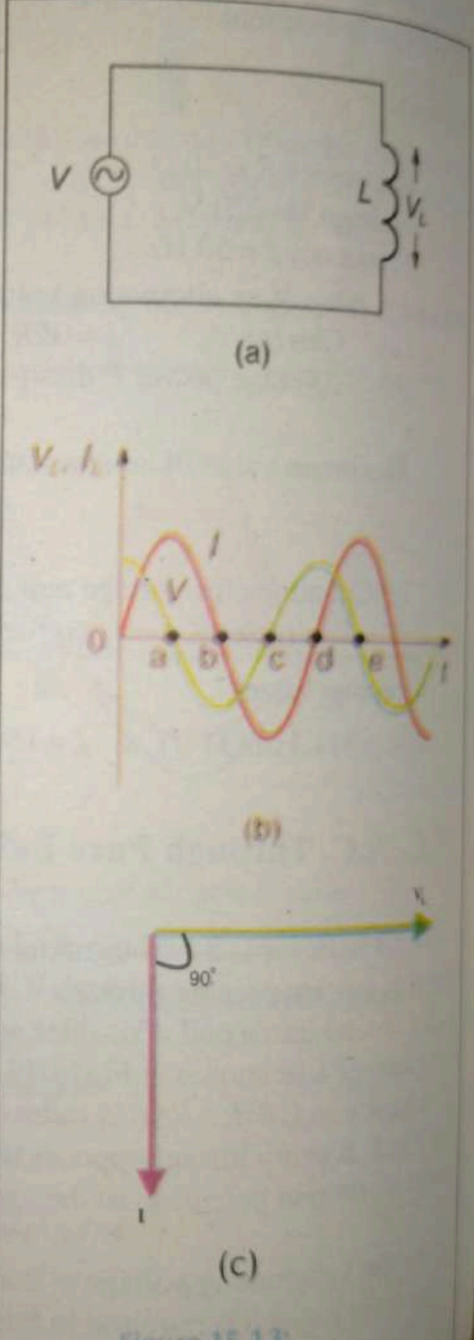


Figure 15.13:

This causes the current to lag behind the applied voltage which is indicated by the phasor diagram shown in Fig.15.13(c).

Inductance opposes the flow of current in the circuit.

So *the opposition offered by an inductor to the flow of A.C. is called inductive reactive reactance* X_L.

Therefore in analogy to ohm law we can write:

$$V_m = I_m X_L$$

Since inductive reactance is ratio of voltage to current. So

$$\text{or } \frac{V_m}{I_m} = X_L$$

$$X_L = \frac{V_m}{I_m}$$

$$X_L = \frac{I_m \omega L}{I_m} = \omega L$$

$$or \;\; X_L = \omega L \qquad \text{...(15.12)}$$

$$X_L = 2\pi f L \qquad \text{...(15.13)}$$

The reactance of coil depends upon frequency of A.C. In case of D.C. inductive reactance X_L is zero.

15.8.1 Power loss in an inductor

Fig15.14: shows the power curve for a pure inductive circuit. During the first 90° of the cycle, the voltage is positive and the current is negative, therefore, the

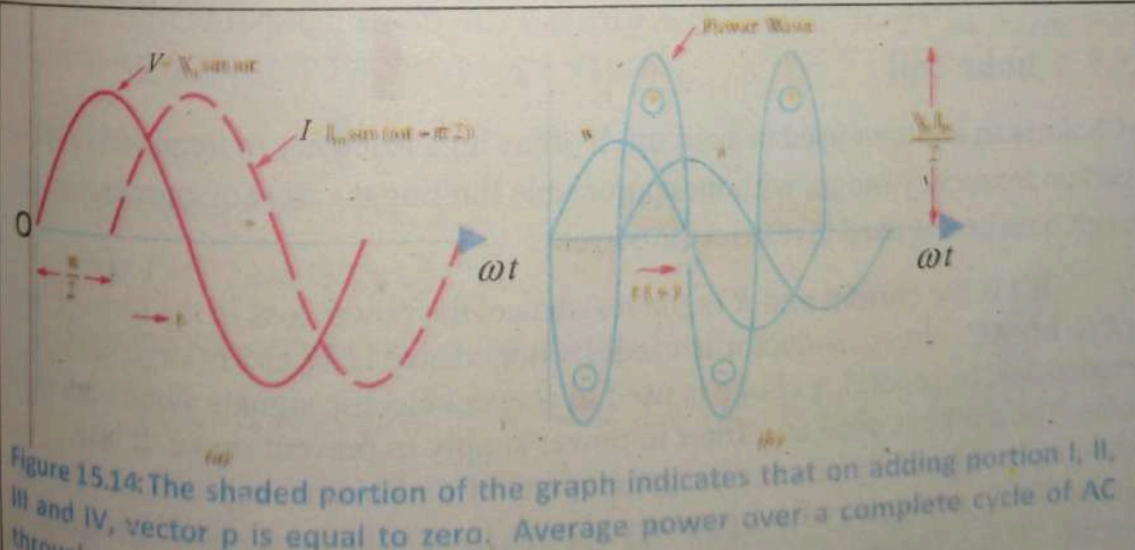


Figure 15.14: The shaded portion of the graph indicates that on adding portion I, II, III and IV, vector p is equal to zero. Average power over a complete cycle of AC through an ideal inductor is zero.

power supplied is negative. This means the power is flowing from the coil to the source. During the next 90° of the cycle, both voltage and current are positive and the power supplied is positive. Therefore, power flows from the source to the coil. Similarly, for the next 90° of the cycle, power flows from the coil to the source and during the last 90° of the cycle, power flow from the source to the coil. The power curve over one cycle shows that positive power is equal to the negative power. Hence the resultant power over one cycle is zero i.e. a pure inductance consumes no power. The electric power merely flows from the source to the coil and back again.

In any circuit, electric power consumed at any instant is the product of voltage and current at that instant. The average power loss in an inductive circuit is,

$$P = \langle VI \rangle = \langle V_m \cos \omega t \times I_m \sin \omega t \rangle$$

$$= V_m I_m \langle \sin \omega t \rangle \langle \cos \omega t \rangle$$

$$= 0 \qquad \because \langle \sin \omega t \rangle \langle \cos \omega t \rangle = 0$$

For your Information

The purpose of passing current through a circuit is to transfer power from the source to the circuit. *The power which is actually consumed in the circuit is called the true power or active power.*

We know that current and voltage are in phase in a resistance whereas they are 90° out of phase in L or C. Therefore, we come to the conclusion that current in phase with voltage produces true or active power whereas current 90° out of phase with voltage contributes to reactive power *i.e.*

True Power = Voltage × Current in phase with voltage

15.9 Choke coil

A Choke is an inductor used in a circuit. It offers high reactance to frequencies above a certain frequency range, without appreciable limiting the flow of current. In a DC circuit, a resistor is used to restrict the current.

If I is the current and R is the resistance, the power loss in the form of heat is I^2R. In AC circuit, inductor is used. Its impedance is X_L and is large at high frequencies. In general, a choke is used to prevent electric signals along undesired paths. The choke is used as a filter in power supply to prevent ripple. It also prevents unwanted signals to enter other parts of the circuits, e.g. radio frequency choke (RFC) prevents radio frequency signals from entering audio frequency circuits. Thus, undesired signals and noise can be attenuated.

Example: 15.2

A pure inductive coil allows a current of 20A to flow from a 220 V, 50Hz supply. Find (1) inductive reactance (2) inductance of the coil (3) power absorbed. Write down the equation for voltage and current.

Solution:

Current $I = 20$ A, Voltage $V = 220$ V

Frequency $f = 50$ Hz

1. Circuit current, $I = V/X_L$

∴ Inductive reactance, $X_L = V/I = 220/20 = 11\ \Omega$

2. Now, $X_L = 2\pi f L$

$$\therefore L = \frac{X_L}{2\pi f} = \frac{11}{2\pi \times 50} = 0.035\text{H}$$

3. Power absorbed = Zero

$V_m = 220 \times \sqrt{2} = 311.1\text{V}$; $I_m = 20 \times \sqrt{2} = 28.28$ A ;

$\omega = 2\pi \times 50 = 314$ rad s^{-1}

Since in a pure inductive circuit, current lags behind the voltage by $\pi/2$ radians, the equations are:

$V = 311.1 \sin(314t)$V $I = 28.28 \sin(314t - \pi/2)$ A

Example:15·3

The current through an 60 mH inductor is 0.2 sin $(377t-25^\circ)$A. Write the mathematical expression for the voltage across it.

Solution:

Inductance $L = 60$ m H ,

current $I = 0.2 \sin(377t - 25^\circ)$A

Mathematical expression for the voltage $V = ?$

Inductive reactance, $X_L = 2\pi f L = 377 \times 60 \times 10^{-3} = 22.62\Omega$

maximum value of an alternating $V_m = I_m X_L = 0.2 \times 22.62 = 4.5$ V

Since the voltage leads the current by 90^0 therefore 90^0 is added to the phase angle of voltage.

$V = V_m \sin(377t - 25^0 + 90^0)$ $V = 4.5 \sin(377t + 65^0)$ V

15.10 A.C. Through Capacitance

Consider an alternating voltage applied to a capacitor of capacitance C as shown in Fig. When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electron move to and fro around the circuit, connecting the plates, thus constituting alternating current. The basic relation between the charge q on the capacitor and voltage V across its plates i.e. $q=CV$ holds at every instant. Let the applied alternating voltage is

$$V = V_m \sin \omega t \qquad \text{(i)}$$

Then, at any instant I be the current and q be the charge on the plates.

Charge on capacitor,

$$q = CV = C\, V_m \sin \omega t$$

$$I = \frac{\Delta q}{\Delta t} = \frac{\Delta(CV_m \sin(\omega t))}{\Delta t}$$

by using maths formulae

$$\therefore \Delta \sin(\omega t) = \omega \cos(\omega t)$$

$$= CV_m \omega \cos(\omega t)$$

$$= CV_m \omega \sin(\omega t + \frac{\pi}{2})$$

$$\therefore\ CV_m \omega = I_m \qquad ...(15.14)$$

$$I = I_m \sin(\omega t + \frac{\pi}{2}) \qquad ...(ii)$$

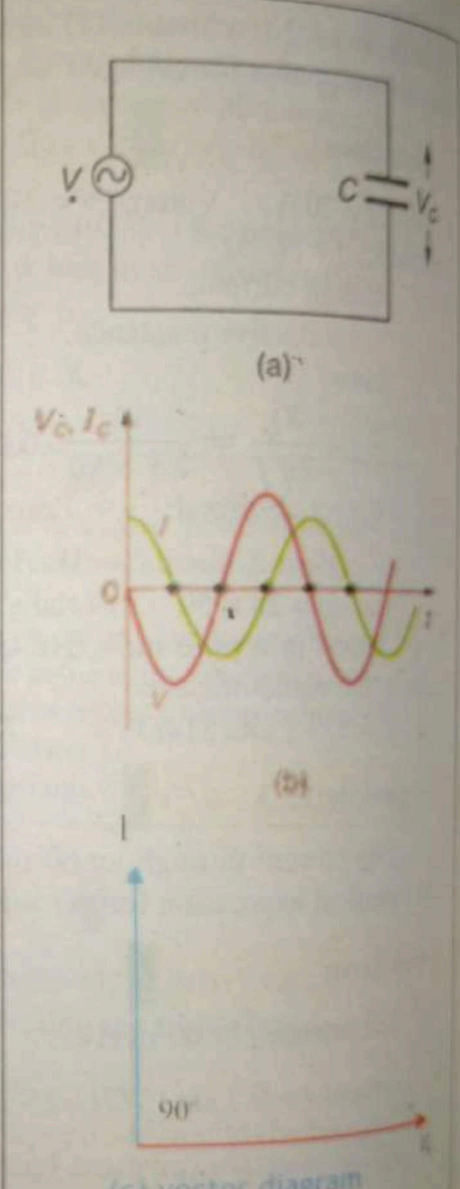


Figure 15.16 Capacitor in AC circuit

Eqs. (i) and (ii) shows that current leads the voltage by $\pi/2$ radians or 90°. Hence in a pure capacitance, current leads the voltage by 90°. Capacitance opposes the change in voltage and serves to delay the increase or decrease of voltage across the capacitor.

This causes the voltage to lag behind the current. This is also illustrated in the phasor diagram shown in Fig:15.16(b): Like inductance which opposes the flow of A.C., capacitance also opposes the flow of AC current in the circuit. From the above:

$$I_m = \omega C V_m \quad \text{...(15.15)}$$

Or $$\frac{V_m}{I_m} = \frac{1}{C\omega}$$

Just like ohm law the ratio of V/I is the measure of opposition offered by a resistor to the flow of current. In case of capacitor this opposition is capacitive reactance which opposes the flow of current.

$$\frac{V_m}{I_m} = \frac{V_C}{I} = \frac{1}{C\omega} \quad \text{...(15.16)}$$

Clearly, the opposition offered by capacitance to current flow is $I/\omega C$. This quantity $I/\omega C$ is called the capacitive reactance X_c of the capacitor. It has the same dimensions as resistance and is, therefore, measured in Ω.

$$I = V_c/X_c$$

Where capacitive reactance is

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi fC} \quad \text{...(15.17)}$$

The capacitive reactance depends upon frequency of A.C. In case of D.C., X_c has infinite value.

15.10.1 Power loss in a capacitive circuit

In pure capacitive circuit the current leads the voltage by 90^0 in phase therefore; the power curve for capacitor and inductor is same. The average power loss in capacitive circuit is,

$$P = \langle VI \rangle = \langle V_m \sin \omega t \times I_m \cos \omega t \rangle$$

$$= V_m I_m < \sin \omega t >< \cos \omega t >$$

$$= 0 \qquad \because < \sin \omega t >< \cos \omega t > = 0$$

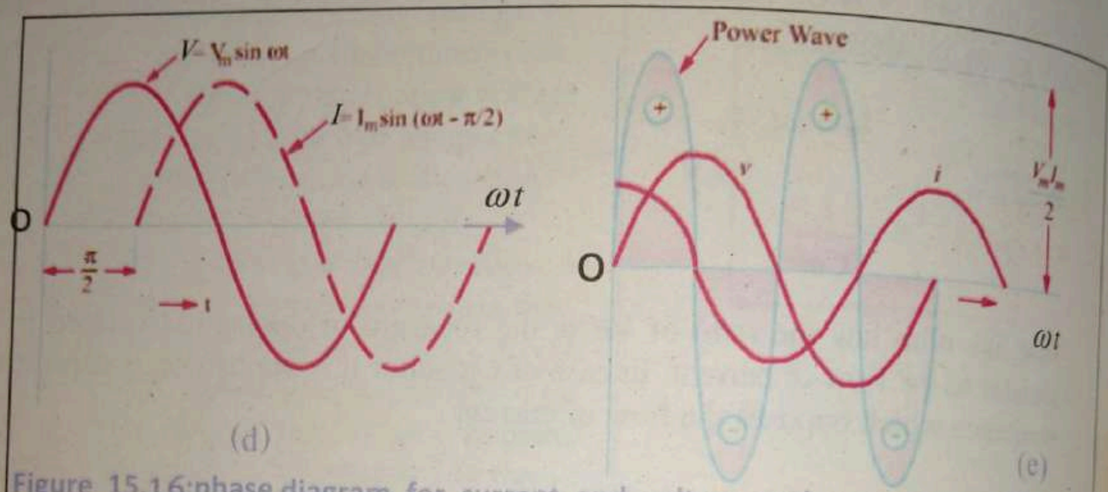


Figure 15.16:phase diagram for current and voltage and power curve for pure capacitive circuit

This fact is also illustrated in the wave diagram shown in Fig:15.16: Which shows that in one cycle the positive power is equal to negative power so power absorbed by capacitor in one cycle is zero.

Example 15.4

A318μF capacitor is connected across a 220V, 50Hz system. Determine (a) the capacitive reactance (b) RMS value of current and (c) equations for voltage and current.

Solution:

Capacitance, C= 318μF= 318×10^{-6} F

Voltage V=220 V

Frequency f = 50 Hz

(a) Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times50\times318\times10^{-6}} = 10.26\Omega$

(b) RMS value of current, $I = V / Xc = 220/10.26 = 21.44\text{A}$

(c) maximum value of an alternating voltage & current is

$V_m = 220\times\sqrt{2} = 311.1\text{V}$,

$I_m = I\times\sqrt{2} = \sqrt{2}\times21.44 = 30.32\text{A}$,

Frequency $\omega = 2\pi\times50 = 314\text{Hz}$

$\therefore$ Equations for voltage and current are:

$$V = V_m \sin(\omega t)$$

& $$I = I_m \sin(\omega t + \pi / 2)$$

putting values

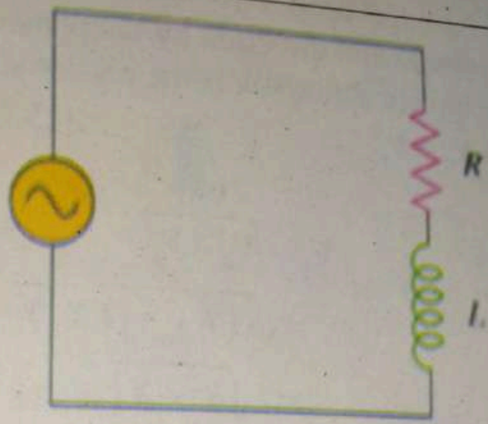
$$V = 311.1 \sin 314t,$$

$$I = 30.32 \sin(314t + \pi / 2),$$

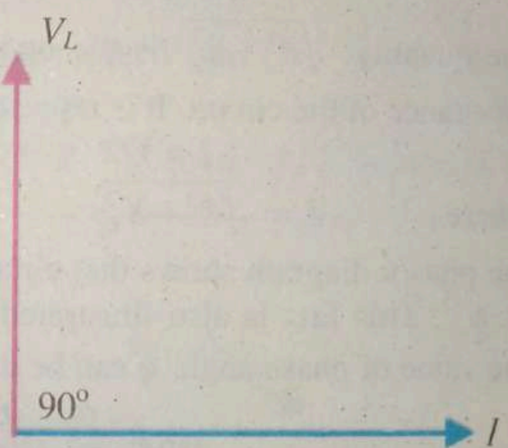
15.11 R.L Series A.C. Circuit

In an AC circuit that contains inductance, L and resistance, R the voltage, V will be the phasor sum of the two component voltages, V_R and V_L. This means that the current flowing through the coil will still lag the voltage, but by an amount less than 90^o depending upon the values of V_R and V_L. The new phase angle between the voltage and the current is known as the phase angle ϕ of the circuit. V is the rms value of the applied voltage, I is the r.m.s. value of the circuit current and $V_L = I X_L$. Fig15.17 (a): shows a pure resistance R connected in series with a coil of pure inductance L.

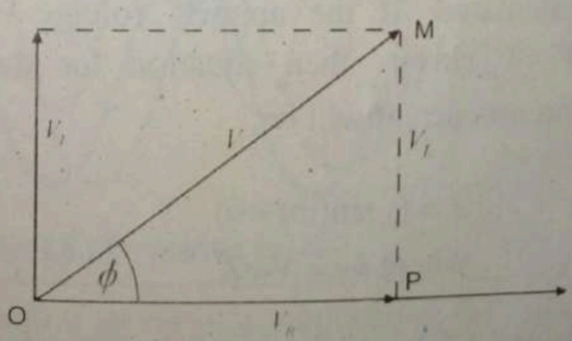
Taking current as the reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig.15.17(c).



(a) : RL series circuit



(b) phasor diagram



(c):phasor diagram of RL series circuit

Figure 15.17

The voltage drop V_R ($= I\,R$) is in phase with current and is represented in magnitude and direction by the phasor OP.
The voltage drop V_L ($= I\,X_L$) leads the current by 90^0 and is represented in magnitude and direction by the phasor *PM*.
The applied voltage *V* is the phasor sum of these two drops *i.e.*

$$V^2 = V_R^2 + V_L^2 \qquad \text{...(15.18)}$$

or

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(I\,R)^2 + (I\,X_L)^2}$$

$$= I\sqrt{R^2 + X_L^2} \qquad \text{...(15.19)}$$

$$or \quad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ is the opposition offered to current flow and is called impedance of the circuit. It is represented by Z and is measured in ohms (Ω)

$$I = V/Z$$

where $\quad Z = \sqrt{R^2 + X_L^2} \qquad \text{...(15.20)}$

The phasor diagram shows that circuit current *I* lags behind the applied voltage *V* by ϕ^{O}. This fact is also illustrated in the wave diagram shown in Fig.15.17(b). The value of phase angle φ can be determined from the phasor diagram.

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \qquad \text{...(15.21)}$$

Since X_L and *R* are known, φ can be calculated. If the applied voltage is $V = V_m \sin\omega t$, then equation for the circuit current will be:

$$I = I_m \sin(\omega t - \phi)$$

where $I_m = V_m/Z$

Fig15.17(d): shows that in an inductive circuit current lags behind the applied

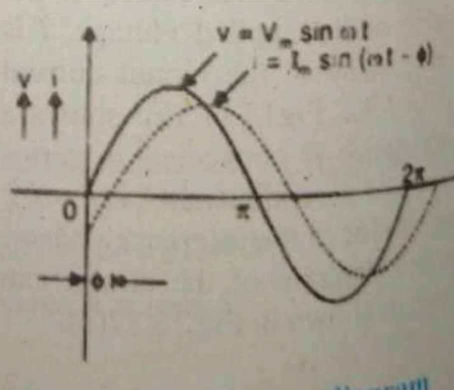


Figure 15.17 (d) :wave diagram

voltage. The angle (i.e. ϕ) of lagging is greater than $0°$ but less than $90°$. It is determined by the ratio of inductive reactance to resistance ($\tan\phi = X_L / R$) in the circuit.

The greater the value of this ratio, the greater will be the phase angle ϕ.

15.11.1 Power in RL circuit

$$\therefore \quad Average\ Power, P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} \times \cos\phi$$

or

$$P = V\,i\cos\phi \qquad \ldots(15.22)$$

Where V and I are the r.m.s. values of voltage and current. The term $\cos\phi$ is called power factor of the circuit and its value is given by (from phasor diagram):

Power factor,

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \qquad \ldots(15.23)$$

Or Power factor = $\cos\phi$ cosin of angle between V and I.

Or

$$P = VI\cos\phi = (IZ)I(R/Z) = I^2R \quad [\because \cos\phi = R/Z\ and\ V = IZ]$$

In a resistor, the current and voltage are in phase i.e. $\phi = 0°$.

Therefore, power factor of a pure resistive circuit is $\cos 0° = 1$. Similarly, phase difference between voltage and current in a pure : inductance or capacitance is $90°$. Hence power factor of pure L or C is zero.

This is the reason that power consumed by pure L or C is zero. For a circuit having R, L and C in varying proportions, the value of power factor will lie between 0 and 1.

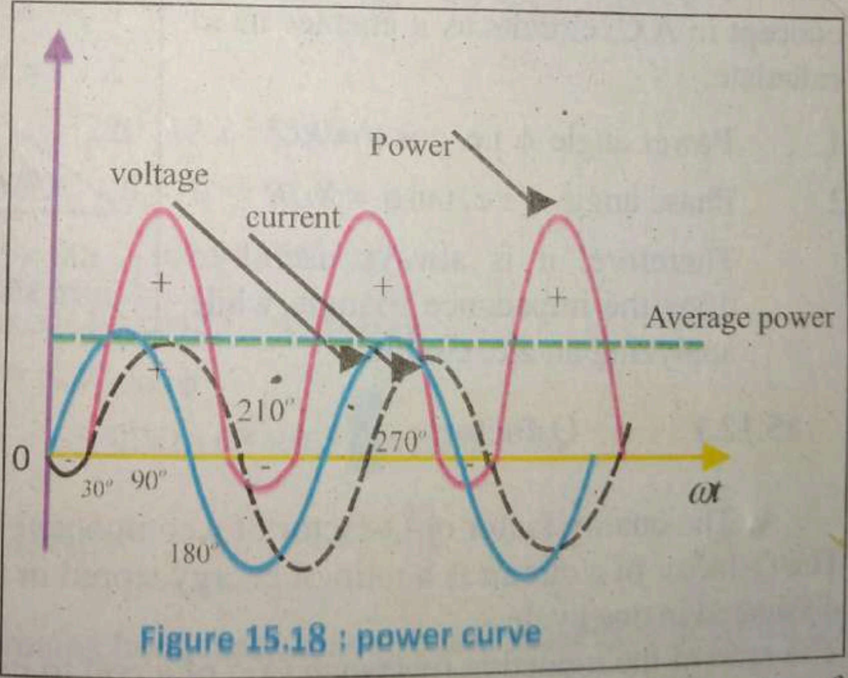


Figure 15.18 : power curve

Fig:15.18 shows that power is negative between 0^0 and 30^0 and between 180^0 and 210^0. The negative area means that the inductance of the circuit returns the power to the source. Conversely power is positive between 30^0 and 180^0 and so on. But as the area of positive curve is greater than negative area of curve. So net power over one cycle is positive. This shows that power is consumed in R-L series circuit.

15.12 R-L Series Impedance Triangle

In a DC circuit, the ratio of voltage to current is called resistance. However, in an AC circuit this ratio is known as Impedance, Z. Impedance is the total resistance to current flow in an "AC circuit" containing both resistance and inductive reactance. In R.L. series circuit,

$$Z = \sqrt{R^2 + X_L^2} \quad \text{where } X_L = 2\pi f L$$

The magnitude of impedance in R.L series circuit depends upon the values of R.L and the supply frequency *f*. The R-L series circuit is shown in Fig15.17(a). The phasor diagram is a triangle whose sides represent R, X_L and Z. This triangle is called an "Impedance Triangle".

Impendence triangle is a useful concept in A.C. circuits as it enables us to calculate:

1. Power angle ϕ i.e. $\cos\phi = R/Z$
2. Phase angle ϕ i.e. $\tan\phi = X_L/R$

Therefore, it is always useful to draw the impedance triangle while analyzing an a.c. circuit.

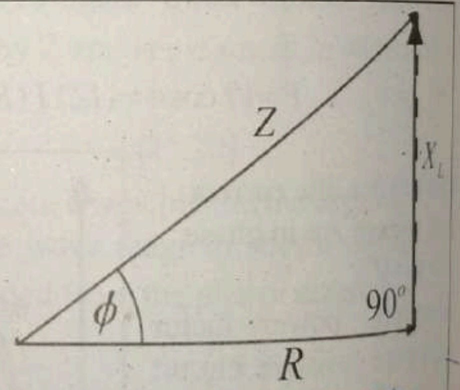


Figure 15.19: impedance triangle

15.12.1 Q-factor

The quality factor or Q-factor of a component is its energy storing ability. The Q-factor of a circuit is a ratio of energy stored in the circuit to the energy dissipated in one cycle.

The ratio of the inductive reactance (X_L) of a coil to its resistance (R) at a given frequency is known as Q-factor of the coil at that frequency i.e.,

$$Q - \text{factor} = \frac{X_L}{R} = \frac{\omega L}{R} \qquad \ldots(15.24)$$

Also,

$$Q-\text{factor}=2\pi\times\frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}}$$

The Q-factor is used to describe the quality or effectiveness of a coil. A coil is usually designed to have high value of L compared to its resistance R. The greater the value of Q-factor of a coil, the greater is its inductance (L) as compared to its resistance (R).

Example:15.5

A coil having a resistance of 7Ω and an inductance of 31.8 mH is connected to 220V, 50Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor and (d) power consumed.

Solution.

Inductance, $L = 31.8$ m H= 31.8×10^{-3}H
Voltage V=220 V
coil resistance R=7Ω
Frequency f = 50 Hz

(a) Inductive reactance, $X_L = 2\pi f L = 2\pi\times50\times31.8\times10^{-3} = 10\Omega$

Coil impedance, $Z=\sqrt{R^2+X_L^2}=\sqrt{7^2+10^2}=12.2\Omega$

Circuit current, $I = V/Z$

$=220/12.2=18.03\text{A}$

(b) $\tan\phi = X_L/R = 10/7$

∴ Phase angle, $\phi=\tan^{-1}(10/7)=55^0$ lag

(c) Power factor $=\cos\phi=\cos 55^0=0.573$ lag

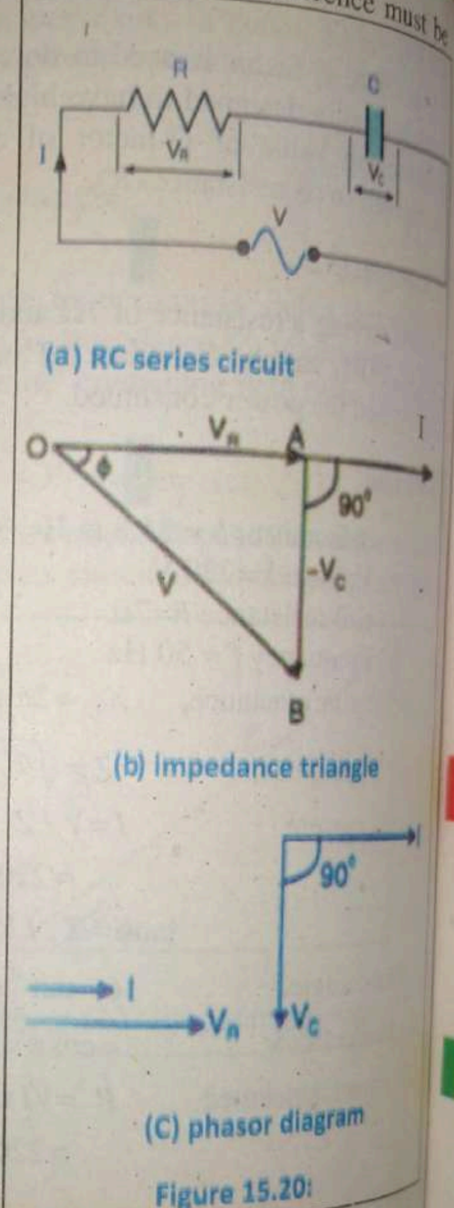
(d) Power consumed, $P = VI\cos\phi$

$=220\times18.03\times0.573=2272.8$ W

15.13 R.C Series A.C. Circuit

Consider AC circuit that contains both capacitance, C and resistance, R which are connected in series with each other as shown in fig. 15.20(a). The voltage, V across the combination is equal to the phasor sum of two component voltages, V_R and V_C. Where $V_R = IR$ and, $V_C = IX_C$.

In order to draw a vector diagram for a capacitance a reference must be found. In a series AC circuit the current is common and can therefore be used as the reference source because the same current flows through the resistance and capacitance. The individual vector diagrams for a pure resistance and a pure capacitance is shown in fig15.20.



(a) RC series circuit

(b) impedance triangle

(C) phasor diagram

Figure 15.20:

The voltage drop V_R $(= IR)$ is in phase with current and is represented by the phasor OA. The voltage drop V_C $(= IX_C)$ lags behind the current by 90^0 and is represented in magnitude and direction by the phasor AB. The applied voltage V is the phasor sum of these two drop i.e.

$$V^2 = V_R^2 + V_C^2 \qquad (i)$$

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} \qquad ...(15.25)$$

The quantity $\sqrt{R^2 + X_C^2}$ offer opposition to current flow and is called impedance of the circuit.

$$I = V/Z$$

where

$$Z = \sqrt{R^2 + X_C^2} \qquad ...(15.26)$$

The phasor diagram shows that circuit current I leads the applied voltage V by ϕ. this fact is also illustrated in the wave diagram and impedance triangle (as shown in Fig15.21(b)) of the circuit.

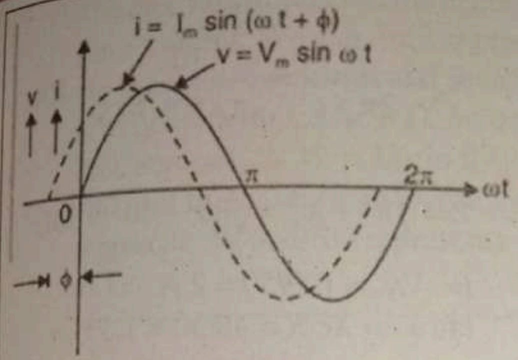
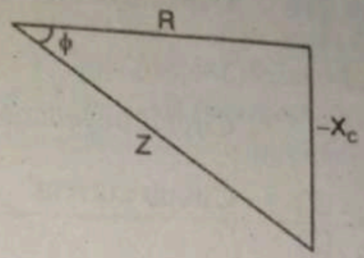


Figure 15.21(a): phasor diagram



(b) impedance triangle

The value of the phase can be determined as under:

$$\tan\phi = -\frac{V_C}{V_R}$$

$$= -\frac{IX_C}{IR} = -\frac{X_C}{R} \qquad \ldots(15.27)$$

15.14 Power in R.C. circuit

The equation for voltage and current are:

$$V = V_m \sin\omega t \quad ; \quad I_m \sin(\omega t + \phi)$$

$\therefore$ Average power, $\langle P \rangle = \langle V \rangle \langle I \rangle$

$$= VI\cos\phi$$

Example 15.6

A 100V, 50Hz a.c. supply is applied to a capacitor of capacitance $79.5\,\mu$ F connected in series with a non-inductive resistance of $30\,\Omega$. Find (1) impedance (2) current (3) phase angle and (4) equation for the instantaneous value of current.

Solution:

Voltage $V = 100$ V

resistance $R = 30\Omega$

Frequency $f = 50$ Hz

, capacitance C = 79.5μF

(1) Capacitive reactance, $X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 79.5} = 40\Omega$

Circuit impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{30^2 + 40^2} = 50\Omega$

(2) Circuit current, $I = V/Z = 100/50 = 2$ A

(3) $\tan\phi = Xc/R = 40/30 = 1.33$

∴Phase angle, $\phi = \tan^{-1} 1.33 = 53^\circ$ *lead*

$$I_m = I \times \sqrt{2} \Rightarrow I_m = 2 \times \sqrt{2} = 2.828\text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314\text{ rad s}^{-1}$$

(4) equation for current is $I = 2.828 \sin(314t + 53^\circ)$

15.15 R-L-C Series A.C Circuit

Many AC circuits are very useful for us, which include resistance, inductive reactance and capacitive reactance. In this section, we will look at some implications of connecting a resistor (*R*), an inductor (*L*), and a capacitor (*C*) together in what is called a series RLC circuit. The simplest and most important AC circuit we can analyze is the series LRC circuit, illustrated in Fig15.22 (a).

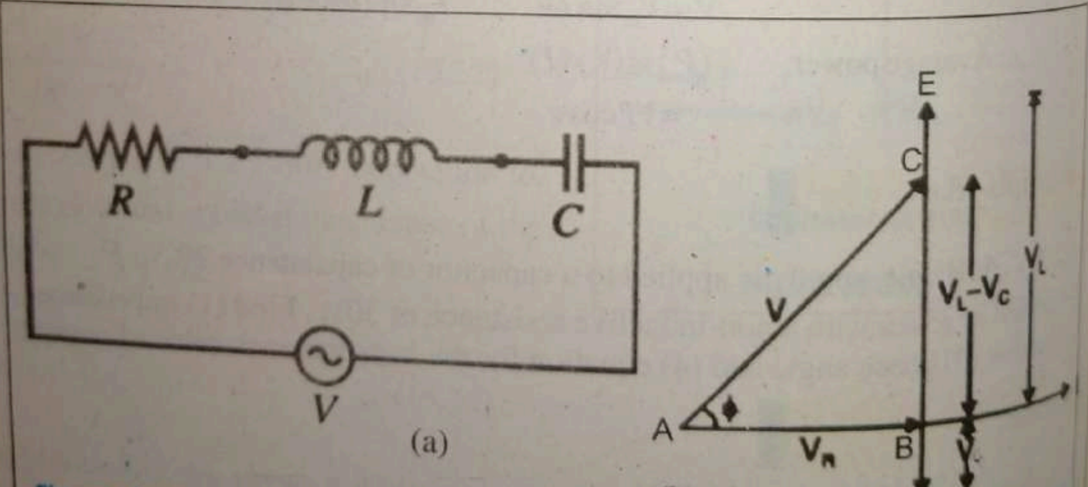


Figure 15.22(a) AC circuit of *R*, *L* and *C* connected in series across a supply voltage *V* (*r.m.s.*). The resulting circuit current is *I* (*r.m.s.*). (b) phasor sum

The anal... since all the circuit elements share the same current. We can draw a phasor diagram for the current and voltages across the inductor, capacitor, and resistor. The P.D. across R, is $V_R = IR$ in this case V_R is in phase with I. The P.D. across L, is $V_L = IX_L$ in this case V_L leads I by 90^0. The P.D. across C, is $V_C = IX_C$ in this case where V_C lags I by 90^0. V_L and V_C are thus 180^0 out of phase. In phasor diagram (Fig15.22 (b)), AB represents V_R, BE represents V_L and BD represents V_C. It may be seen that V_L is in phase opposition to V_C.

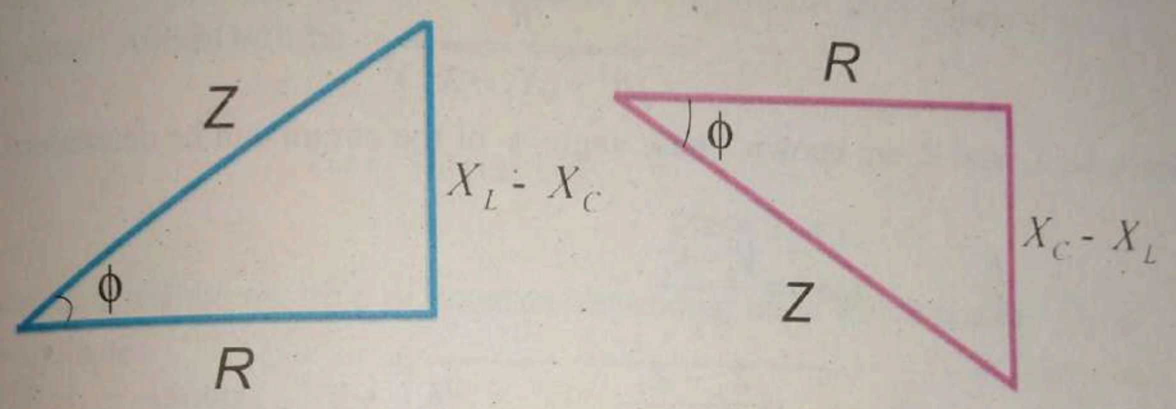


Figure 15.22 (c), (d) : impedance triangle

It follows that the circuit can either be effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. If $V_L > V_C$ then the net voltage drop across L-C combination is $V_L - V_C$ and their resultant is in the direction V_L represented by BC. Therefore, the applied voltage V is the phasor sum of V_R and $V_L - V_C$ and represented by AC.

$$V^2 = V_R^2 + (V_L - V_C)^2 \qquad ...(15.28)$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{(R)^2 + (X_L - X_C)^2}$$

$$= IZ$$

Where $Z = \sqrt{(R)^2 + (X_L - X_C)^2} \qquad ...(15.29)$

The quantity $(X_L - X_C)$ is called the reactance of the circuit, denoted by X

$$X^2 = (X_L - X_C)^2$$

Finally we can write

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X^2}$ is the opposition offered to current flow and is called impedance of the circuit.

$$\text{Circuit power factor, } \cos\phi = \frac{R}{Z}$$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad (15.30)$$

Since X_L, X_C and R, are known, phase angle ϕ of the circuit can be determined.

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$= \frac{X_L - X_C}{R}$$

$$= \frac{X}{R} \qquad (15.31)$$

So if the current is represented by a cosine function, $I = I_m \cos \omega t$

The source voltage leads the current by an angle and its equation is

$$I = I_m \cos(\omega t + \phi)$$

Power consumed, $P = VI\cos\phi$

We have seen that the impedance of a R-L-C series circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

(i) When $X_L - X_C$ is positive (*i.e.* $X_L > X_C$), phase angle ϕ is positive and the circuit will be inductive. In other words, in such a case, the circuit current I will lag behind the applied voltage V by ϕ.

(ii) When $X_L - X_C$ is negative (*i.e.* $X_C > X_L$), phase angle ϕ is negative and the circuit is capacitive. That is to say the circuit current I leads the applied voltage V by ϕ ; the value of ϕ being given by Eq.(15.31) above.

(iii) When $X_L - X_C = 0$ (i.e. $X_L = X_C$), the circuit is purely resistive. In other words, circuit current I and applied voltage V will be in phase i.e. $\phi = 0^o$ the circuit will then have unity power factor.

If the equation for the applied voltage is $V = V_m \sin \omega t$, then equation for the circuit current will be

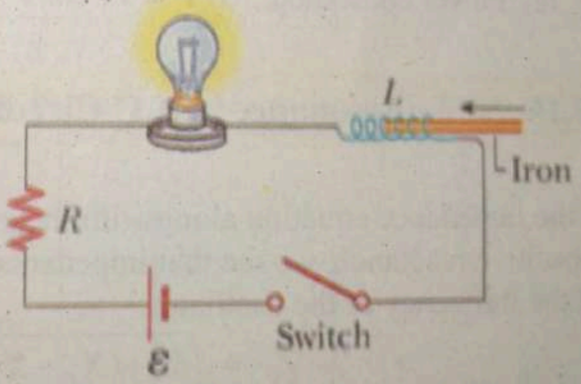
$$I = I_m \text{Sin}(\omega t \pm \phi)$$

where $I_m = V_m / Z$

The value of ϕ will be positive or negative depending upon which reactant (X_L or X_C) predominates.

Fig.15.22(*c*) shows the impedance triangle of the circuit for the case when $X_L > X_C$ whereas impendence triangle in Fig.(*d*) is for the case when $X_C > X_L$.

Quiz?



The switch in the circuit shown in Figure is closed and the light bulb glows steadily. The inductor is a simple air-core solenoid. As an iron rod is being inserted into the interior of the solenoid, the brightness of the light bulb (a) increases, (b) decreases, or (c) remains the same.

Example15.7

A 220V, 50Hz A.C. supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance connected in series with a 6.8 μ F capacitor. Calculate

(a) impedance (b) current (c) phase angle between current and voltage (d) power factor and (e) power consumed.

Solution

$$X_L = 2\pi f L = 2\pi \times 50 \times .06 = 18.85\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{10^6}{2\pi \times 50 \times 6.8} = 468\Omega$$

(a) Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(2.5)^2 + (18.85 - 468)^2} = 449.2\Omega$$

(b) Circuit Current, $I = V/Z = 220/449.2 = 0.4897\text{A}$

(c) $\tan\phi = \frac{X_L - X_C}{R} = \frac{18.85 - 468}{2.5} = -179.66$

∴ Phase angle, $\phi = \tan^{-1}(-179.66) = -89.7^\circ$

The negative sign with ϕ shows that current is leading the voltage

(d) Power factor, $\cos\phi = \frac{R}{Z} = \frac{2.5}{449.2} = 0.00557$

(e) Power consumed, $P = VI\cos\phi = 220 \times 0.4897 \times 0.00557 = 0.60007\text{w}$

15.16 Resonance in A.C Circuits

In the impedance equation along with the equations for the inductive and capacitive reactance, we see that impedance has a rather complicated dependence on the frequency of the oscillator.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L \text{ and } X_C = \frac{1}{\omega C}$$

When the frequency is very small, the capacitive reactance is large and $X_C \approx Z$. When the frequency is very large, the inductive reactance is large and $X_L \approx Z$. Z is a minimum when $X_L = X_C$, and Z is a minimum, the current in the circuit is a maximum. When this happens, the resistance provides the only impedance in the circuit, $Z=R$. This condition is called resonance and is electrical analog to resonance in harmonic oscillators such as a swinging pendulum or a mass on the end of a spring.

Resonance means to be in step with. When applied voltage and circuit current in an A.C. circuit are in step with (i.e. phase angle is zero or power factor is unity), the circuit is said to be in electrical resonance. If this condition exists in a series a.c. circuit, it is called series resonance. The frequency at which resonance occurs is called resonant frequency (f_r).

An A.C. circuit containing reactive elements (L and C) is said to be in resonance when the circuit power factor is unity.

15.16.1 Resonance in R-L-C Series Circuits

R-L-C series circuit is said to be in resonance when the circuit power factor is unity i.e. $X_L = X_C$. The frequency f_r at which it occurs is called resonant frequency. The resonance (i.e. $X_L = X_C$) in an R-L-C series circuit can be achieved by changing the supply frequency because X_L and X_C are frequency dependent. At a certain frequency f_r, X_L becomes equal to X_C and resonance takes place.

At series resonance, $$X_L = X_C \quad \text{...(15.32)}$$

Or $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

∴ Resonant frequency, $$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \text{...(15.33)}$$

From Eq. (15.33), it is clear that on increasing either the inductance or the capacitance causes the resonant frequency to decrease. For a given value of inductance and capacitance, there is only one resonant frequency.

There are an infinite number of inductor and capacitor combinations for any specified resonant frequency.

Resonance Curve: The curve between current and frequency is known as resonance curve of a typical *R-L-C* series circuit. Current reaches its maximum value at the resonant frequency (f_r), falling off rapidly on either side at that point.

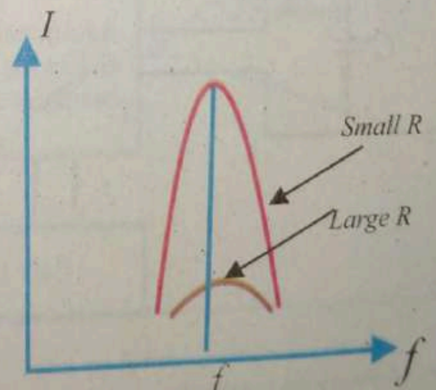


Figure 15.23: resonance curve

It is because if the frequency is below f_r, $X_C > X_L$ and the net reactance is no longer zero. If the frequency is above f_r, then $X_L > X_C$ and the net reactance is again not zero. In both cases, the circuit impedance will be more than the impedance ($Z_r = R$) at resonance. The result is that the magnitude of circuit current decreases rapidly as the frequency changes from the resonant frequency.

The Q-factor of a series circuit circle indicates how many times the P.D. across L or C is greater than the applied voltage at resonance. For example, when R-L-C series circuit is connected to a 220V source having a Q-factor of the coil as 20, then voltage across the coil or capacitor will be

$$V_C = V_L = QV_R = 20 \times 220 = 4400 \text{ V at resonance.}$$

15.17 Principle of metal detectors

A coil and capacitor are electrical components, which together can produce oscillations of current. An L-C circuit behaves just like an oscillating mass-spring system. In this case energy oscillates between a capacitor and an inductor. The circuit is called an electrical oscillator. Two such oscillators A and B are used for the operation of common type of metal detector (Fig 15.24). In the absence of any nearby metal object, the inductances L_A and L_B are the same and hence the resonance frequency of the two circuits is also same.

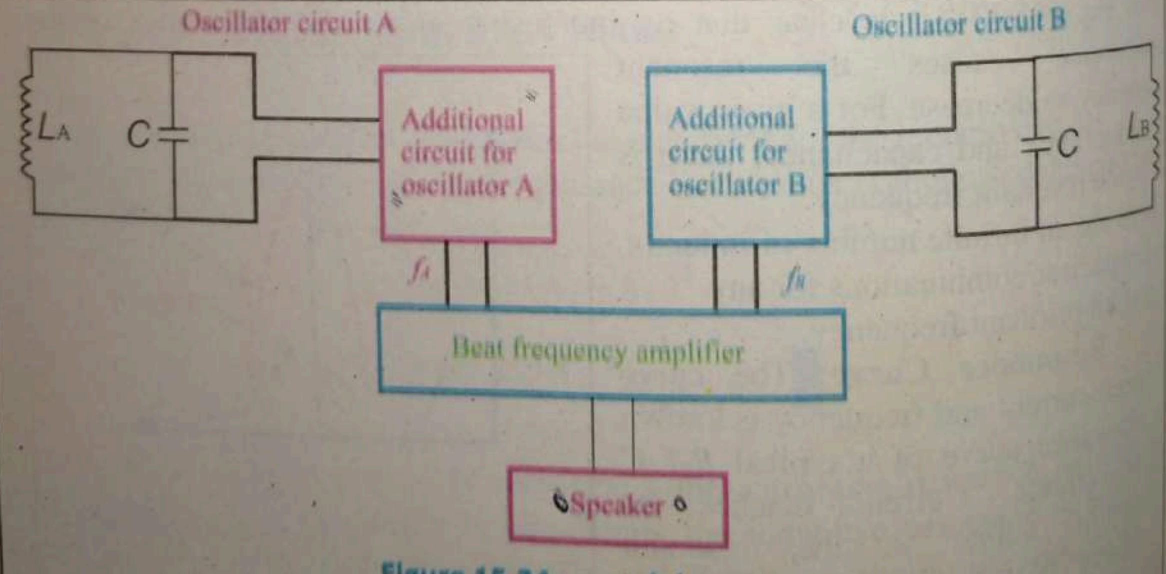


Figure 15.24 : metal detectors

When inductor B, called the search coil comes near a metal object the inductance L_B decreases and corresponding oscillator frequency increases and thus a beat note is heard in the attached speaker. Such detectors are extensively used not only for various security checks but also to locate buried metal objects.

15.18 Maximum power transfer

The maximum power-transfer theorem says that to transfer the maximum amount of power from a source to a load, the load impedance should match the source impedance.

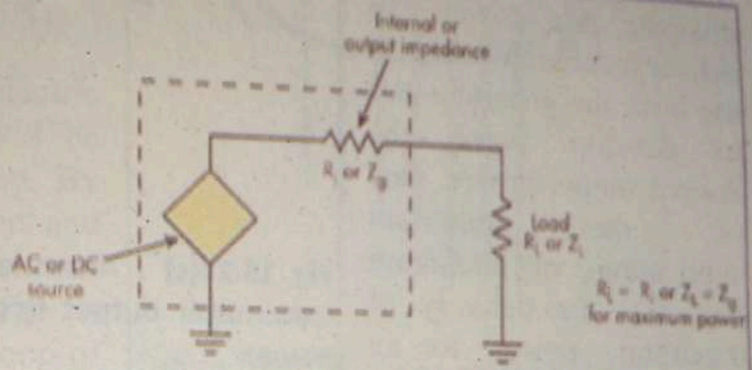


Figure 15.25(a): Maximum power is transferred from a source to a load when the load resistance equals the internal resistance of the source.

In the basic circuit, a source may be AC or DC, and its internal resistance (R_i), or generator output impedance (Z_g) drives a load resistance (R_L) or impedance (Z_L) (Fig15.25(a):

$R_L = R_i$

Or

$Z_L = Z_g$ ...(15.34)

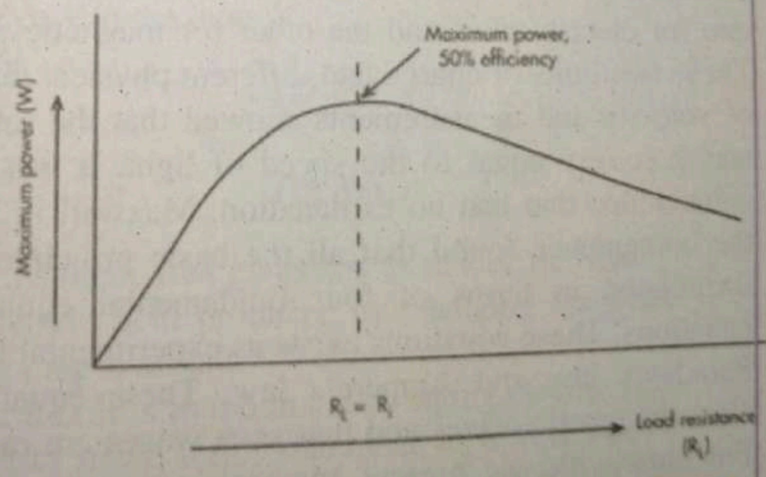


Fig 15.25(b). Varying the load resistance on a source shows that maximum power to the load is achieved by matching load and source impedances. At this time, efficiency is 50%.

A plot of load power versus load resistance reveals that matching load and source impedances will achieve maximum power (fig15.25(b)).

A key factor of this theorem is that when the load matches the source, the amount of power delivered to the load is the same as the power dissipated in the source. Therefore, transfer of maximum power is only 50% efficient.

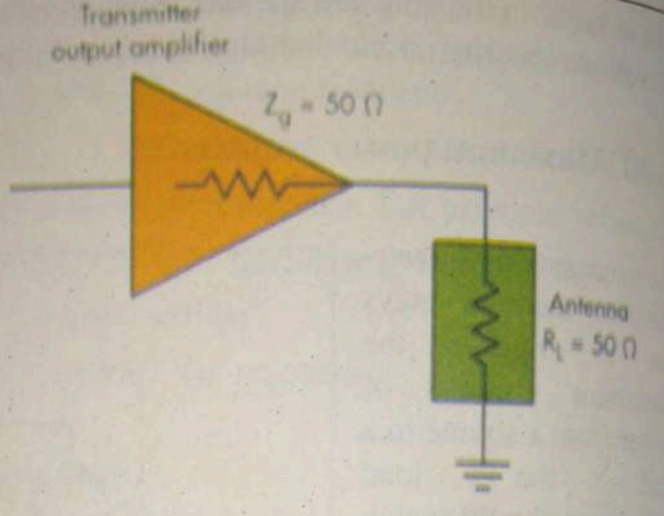


Fig 15.25(c) . Antenna impedance must equal the transmitter output impedance to receive maximum power.

The source must be able to dissipate this power. To deliver maximum power to the load, the generator has to develop twice the desired output power. One of the important applications of maximum power is the delivery of maximum power to an antenna (fig15.25(c)).

15.19 MAXWELLS EQUATIONS

In the early days of 19th century, two different units of electric charge were used, one for electrostatics and the other for magnetic phenomena involving currents. These two units of charge had different physical dimensions. Their ratio has units of velocity and measurements showed that the ratio had a numerical value that was precisely equal to the speed of light. It was regarded as an extraordinary coincidence that had no explanation. Maxwell in a search for an explanation of the coincidence found that all the basic principles of electromagnetism can be formulated in terms of four fundamental equations, now called Maxwell's equations. These equations exists as experimental laws in the form of Gauss' law, Faraday's law and Ampere's law. These equations predict the existence of electromagnetic waves and that such waves are radiated by accelerating charges. For simplicity, we present Maxwell's equations as applied to free space. We know that changing magnetic flux density B through a certain region of space produces an induced emf in the region.

So an induced current will flow in a closed loop of wire in the region as shown in Fig(15.26(a)

According to Faraday's law the induced emf or the induced potential difference V is given by

$$\mathcal{E} = \frac{\Delta\Phi}{\Delta t}$$

$$= \frac{\Delta}{\Delta t}(B.A) \qquad ...(15.34)$$

As potential difference is due to an electric field, it means an electric field will be generated at each point of the loop. By symmetry E is circular in direction and constant in magnitude E at each point of the loop. If a unit positive charge is circulated once round the circular loop of radius r, the work done will be

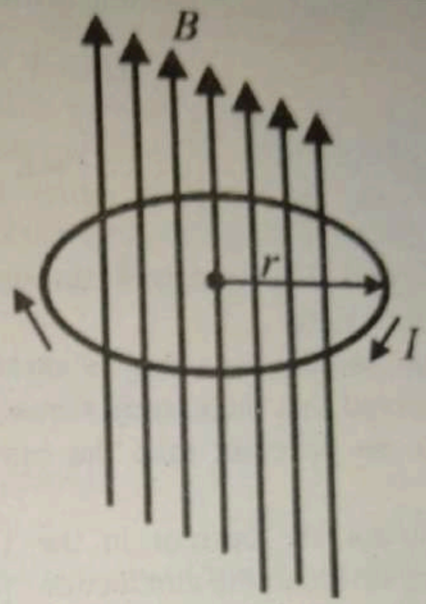


Figure 15.26 :changing magnetic field

$$W = 2\pi r F_e = 2\pi r(qE)$$

$$\mathcal{E} = \frac{W}{q} = 2\pi r E \qquad ...(15.35)$$

By definition W will equal to the emf or V in the loop:

$$\mathcal{E} = \frac{\Delta\Phi}{\Delta t} = 2\pi r E \qquad ...(15.36)$$

$$E = \frac{1}{2\pi r}\frac{\Delta\Phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t} \qquad ...(15.37)$$

This equation shows that a changing magnetic flux gives rise to an electric field. Experiments have shown that the electric field produced by changing magnetic field is present even if the conducting loop is absent.

(b): Analogues to changing magnetic flux it is found that a changing electric flux gives rise to a magnetic field. In order to arrive at this statement, let a capacitor be connected to a battery as arranged in Fig. (15.27). Current starts growing in the circuit but very quickly decreases to zero when the capacitor is fully charged. An electric field is established between the plates of the capacitor.

The charge Q on an air filled capacitor of capacitance C, plate area A, plate separation d and potential difference V is given by

$$Q = CV = \frac{\varepsilon_o A}{d} V = \varepsilon_o AE \quad ...(15.38)$$

$$\therefore \frac{V}{d} = E$$

In Fig (15.27) the current through the dielectric of the capacitor is due to changing electric field.

Suppose the capacitor is now connected to an alternating emf source. It is observed that the current flows continuously in the circuit. It does not stop. Why is to so different than the case of the battery! The reason is that outside the capacitor the current in the wires is due to the conduction electrons but what is the entity that drives that current in the dielectric between the plates of the capacitor. We note that in the present case the E-Field between the plates of the capacitor changes with time. There exists $\Delta E/\Delta t$. It was first conceived by Maxwell that the change in the electric field is the cause of current in the capacitor. From Eq. (15.38'), we have

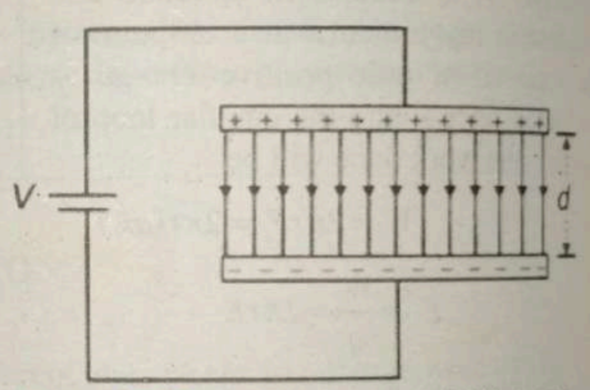


Figure 15.27 **Uniform electric field between a pair of charged plate.**

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta(\varepsilon_o AE)}{\Delta t} = \frac{\varepsilon_o \Delta(AE)}{\Delta t} \quad ...(15.39)$$

$$I = \frac{\varepsilon_o \Delta(\Phi)}{\Delta t} \quad ...(15.40)$$

Where I is the current and Φ is the electric flux through the area A. Eq. (15.40) shows that a changing electric flux is equivalent to a current. This type of current, which is due to changing electric flux is called displacement current.

We must extend our concept about current. A current can arise due to the flow of charges and also due to changing electric flux. The former is called conduction current and the later is called displacement current. According to Ampere's law, each type of current produces magnetic field around itself. We have thus shown that a changing E –field creates a B-field.

(c). A changing E-field creates a B-field, which in turn creates an E-field, an electromagnetic disturbance or waves are generated. The fundamental requirement for generation of electromagnetic waves is an electric charge with changing velocity (acceleration) since it will create changing electric flux. The velocity of an oscillating charge as it moves to and fro along a wire is always changing.

(d). Light is a type of electromagnetic waves. Maxwell predicted theoretically that the velocity of electromagnetic waves in free space is given by

$$c=\frac{1}{\sqrt{\varepsilon_o\mu_o}} \qquad ...(15.41)$$

Where $\varepsilon_o = 8.85\times10^{-12}C^2\ N^{-1}m^{-2}$ is the permitivity of free space and

$\mu_o = 4\pi\times10^{-7}$ Wb $A^{-1}m^{-1}$ is the permeability of free space.

Put these values in the above relation we find that the velocity of lights is given by

$$c=\frac{1}{\sqrt{8.85\times10^{-12}\times4\pi\times10^{-7}}}=3\times10^{8}\,m\,s^{-1}$$

Faraday's law shows that a changing magnetic field gives rise to an electric field. Ampere-Maxwell law shows that a changing electric field gives rise to a magnetic field. It follows that when either electric or magnetic field varies with time, the other field is induced in space. The net effect is that an electromagnetic disturbance is generated due to changing electric and magnetic fields. The disturbance propagates in the form of an electromagnetic wave.

15.20 ELECTROMAGNETIC WAVE

Electromagnetic radiations such as infrared ,ultravoilet,etc are different from each other due to their properties. But they have some features in common such as electric field and magnetic field. Therefore it can be described in terms of electric and magnetic fields, and they all travel through vacuum with the same speed (the speed of light).

Fundamentally these radiations are different only in wavelength or frequency. The names given to them in Fig. 15.28 shows various regions of the spectrum along with given names. There are no gaps in the spectrum, nor their sharp boundaries between the various categories. (Certain regions of the spectrum are assigned by law for commercial or other uses, such as TV. AM. or FM broadcasting).

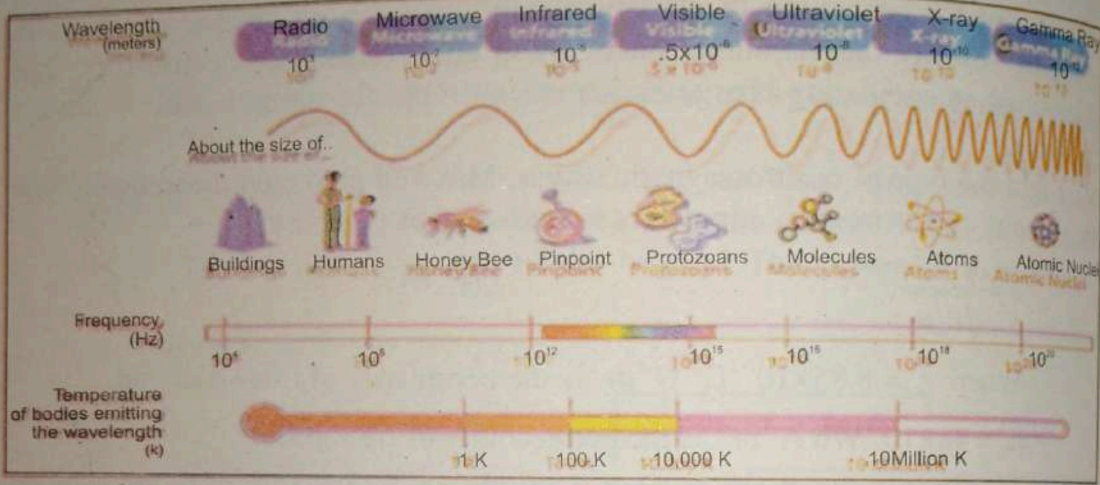


Figure 15.28(a) : electromagnetic spectrum

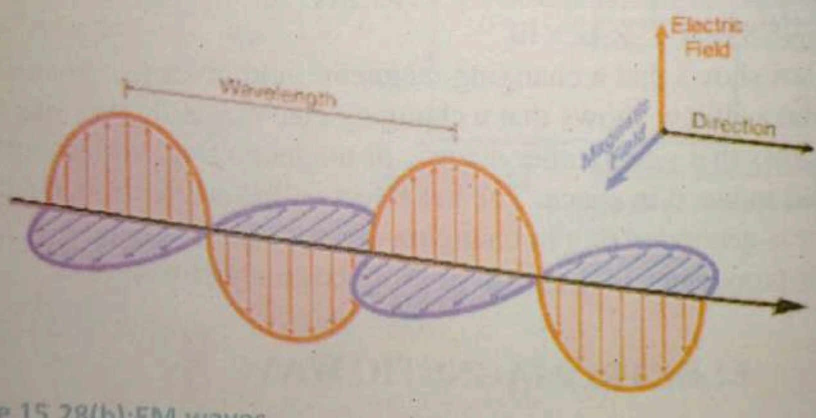


Figure 15.28(b):EM waves

Let us consider some of these types of electromagnetic radiation in more details.

1. **Light.** The visible region of the spectrum, most familiar to us, is the electromagnetic radiation emitted by the Sun. The wavelength of the visible region ranges from about 400 nm (violet) to about 700nm (red).

Light is often emitted when the outer (or valence) electrons in atoms change their state of motion: for this reason, such transitions in the state of the electron are called optical transitions. The color of the light tells us something about the atoms or the object from which it was emitted. The study of the light emitted from the Sun and from distant stars gives infomation about their composition.

2. Infrared. Infrared radiation, which has wavelength stonger than the visible (from 0.7 μm to about 1 mm), is commonly emitted by atoms or molecules when they change their rotational or vibrational motion. Infrared radiation is an important means of heat transfer and is sometimes called heat radiation. The warmth you feel when you place your hand near a glowing light bulb is primarily a result of the infrared radiation emitted from the bulb.

All objects emit electromagnetic radiation (called "thermal radiation;") because of their temperature. Objects of temperatures ranges from , 3 K to 3000 K emit their most intense thermal radiation in the infrared region of the spectrum.

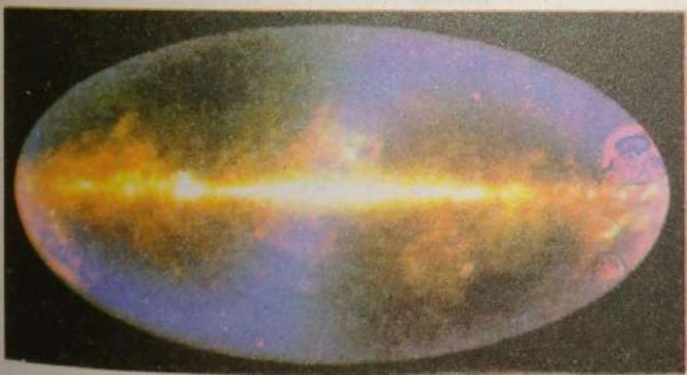


Figure 15.29: infrared image of Milky Way galaxy

Figure 15.30: Microwaves relay station

A **remote control** is a component of an electronics device, most commonly a television set, DVD player and home theater systems originally used for operating the device wirelessly from a short line-of-sight distance. The main technology used in home remote controls is infrared light. The signal between a remote control handset and the device it controls consists of pulses of infrared light, which is invisible to the human eye. Infrared radiation is also used for cooking the surface of food (the interior is then heated by convection and conduction).

3. Microwaves. Microwaves can be regarded as short radio waves, with typical wavelengths in the range 1 mm to 1 m. They are commonly produced by electromagnetic oscillators in electric circuits, as in the case of microwave ovens. Microwaves are often used to transmit telephone conversations; Fig. 15.31 shows a Microwaves station that serves to relay telephone calls. Microwaves also reach us from extraterrestrial sources.

Figure 15.31: remote

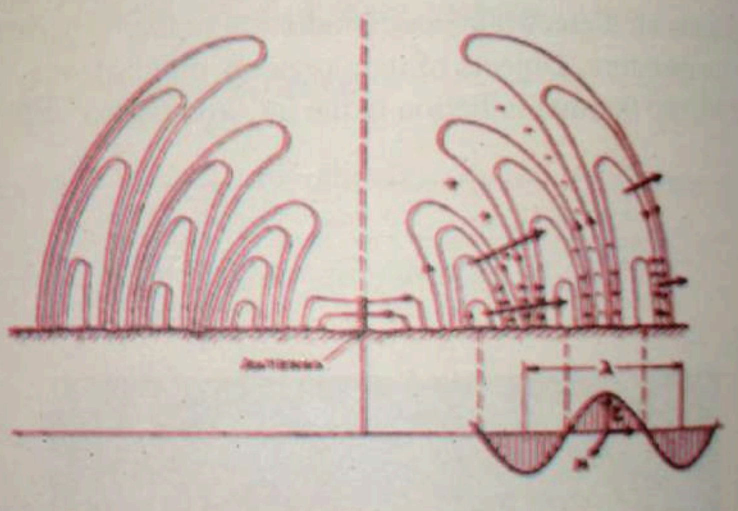


Figure 15.32: radio waves propagation

Neutral hydrogen atoms, which populate the regions between the stars in our galaxy, are common extraterrestrial source of Microwaves emitting radiation with a wavelength of 21 cm.

4. Radio waves. Radio waves have wavelengths longer than 1 m. They are produced from terrestrial sources through electrons oscillating in wires of electric circuits. By carefully choosing the geometry of these circuits, as in an antenna, we can control the distribution in space of the emitted radiation (if the antenna acts as a transmitter) or the sensitivity of the detector (if the antenna acts as a receiver. Traveling outward at the speed of light, the expanding of TV signals transmitted on Earth.

Radio waves reach us from extraterrestrial sources, the sun being a major source that often interferes with radio or TV reception on Earth. Mapping the radio emissions from exraterrestrial sources, known as radio astronomy, has provided infomation about the universe that is often not obtainable using optical telescopes.

5. Ultraviolet. The radiations of wavelengths shorter than the visible begin with the ultraviolet (1 nm to 400nm), which can be produced in atomic transitions of the outer- electrons as well as in radiation from thermal sources such as the Sun. Because our atmosphere absorbs strongly at ultraviolet wavelengths, little of this radiation from the Sun reaches the ground. However, the principal agent of this absorption is atmospheric ozone, which has been depleted in recent years as a result of chemical reactions with fluorocarbons released from aerosol sprays, refrigeration equipment, and other sources. Brief exposure to ultraviolet radiation causes common sun burn but long-term exposure can lead to more serious effects, including skin cancer.

Ultraviolet Lamp,(UV Light).A lamp producing Ultraviolet (UV) radiation is emitted through clear, pre-filtered, particle free water. This UV light is extremely effective in killing and eliminating bacteria, yeast's, viruses, molds and other harmful organisms known to man. Used in industry and hospitals to treat water. Many times used as a post disinfecting method for residential water treatment.

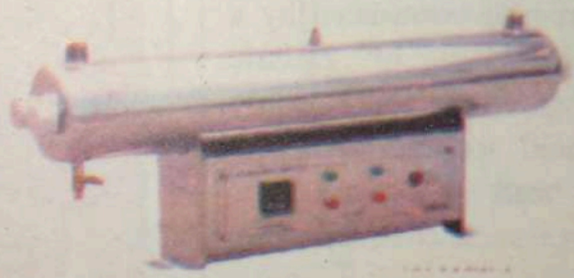


Figure 15.33: ultraviolet lamp

6. X-rays. X rays (typical wavelengths 0.01 nm to 10 nm) can be produced with discrete wavelengths in individual transitions among the inner (most tightly bound) electrons of an atom, and they can also be produced when charged particles (such as electrons) are decelerated. X rays can easily penetrate soft tissue but are stopped by bone and other solid matter: for this reason they have found wide use in medical diagnosis.

7. Gamma rays. Gamma rays are electromagnetic radiations with the shortest wavelengths (less than 10 pm).

They are the most penetrating of electromagnetic radiations, and exposure to intense gamma radiation can have a harmful effect on the human body. These radiations can be emitted in transitions of an atomic nucleus from one state to another and can also occur in the decays of certain elementary particles: for example, a neutral pion can decay into two gamma rays according to

$\pi^{\circ} \rightarrow \gamma + \gamma$

15.21 Electrocardiogram (E.C.G)

The electrocardiogram or ECG is worldwide used for diagnosing heart conditions. An electrocardiogram is a recording of the small electric waves being generated during heart activity.

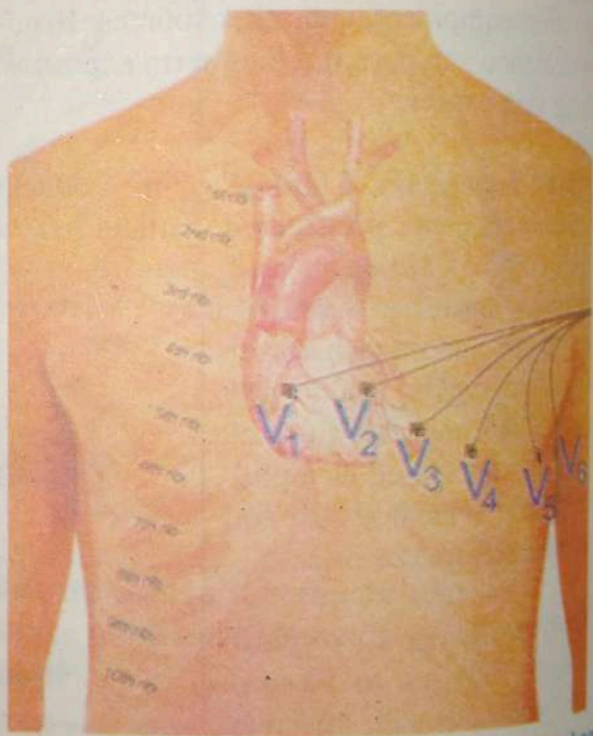


Figure 15.34(a) : Sensitive electrodes are placed on certain parts of the body.

The electric activity starts at the top of the heart and spreads down. A normal heart beat is initiated by a small pulse of electric current. This tiny electric "shock" spreads rapidly in the heart and makes the heart muscle contract.

If the whole heart muscle contracted at the same time, there would be no pumping effect.

Therefore the electric activity starts at the top of the heart and spreads down, and then up again, causing the heart muscle to contract in an optimal way for pumping blood. The electric waves in the heart are recorded in millivolts by the electrocardiograph.

Our heart produces time-varying voltages as it beats. These heart voltages produce small voltage differences between points on your skin that can be measured and used to diagnose the condition of your heart. The waves are registered by electrodes placed on certain parts of the body. Which are then printed on paper in form of a curve as shown in fig:15.34(b).

Some of the characteristics of ECG curve are shown in fig15.34(b): When the curve falls below the base line it shows a negative deflection and when it rises above the base line it shows a positive deflection.

A negative deflection indicates that the recorded wave has traveled away from the electrode and a positive deflection means it has traveled towards it.

Fig 15.34(b):

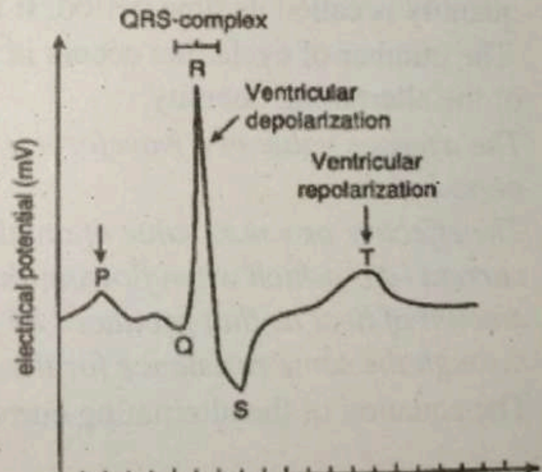


Fig 15.34(c) : An ECG curve reflects the perspective of the electrode recording it.

The tiny rise and fall in the voltage between two electrodes placed either side of the heart which is displayed as a wavy line either on a screen or on paper.

A typical plot of voltage difference between two points on the human body vs time is shown in fig 15.34(b). The P deflection corresponds to the contraction of the atria at the start of the heartbeat. The QRS group corresponds to the contraction of the ventricles. The T deflection corresponds a re-polarization or recovery of the heart cells in preparation for the next beat.

Key points

- *A voltage which changes its polarity at regular interval of time is called an alternating voltage.*
- The sinusoidal alternating voltage can be expressed by the equation:

$$V = V_m \sin\omega t$$

- The shape of the curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abscissa is called its waveform or wave shape.
- One complete set of positive and negative values of an alternating quantity is known as a cycle.
- The time taken in seconds to complete one cycle of an alternating quantity is called its time period. It is generally represented by T.
- The number of cycle that occurs in one second is called the frequency (f) of the alternating quantity.
- *The average value of a waveform is the average of all its values over a period of time.*
- *The effective or r.m.s. value of an alternating current is that steady current (d.c.)which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.*
- The equation of the alternating current varying sinusoidally is given by:

$$I = I_m \sin \omega t$$

- When two alternating quantities of the same frequency have different zero point, they are said to have a phase difference.
- Sinusoidal alternating voltage or current is represented by a line of definite length rotating in counter clock wise direction at a constant angular velocity (ω). Such a rotating line is called a phasor.
- The phasor representation enables us to quickly obtain the numerical value and at the same time as the events taking place in the circuit.

- The applied voltage and current across resistor are in phase with each other. As they pass through their zero values at the same instant and attain their positive and negative peaks at the same instant.

- When an alternating current flows through a pure inductive coil then the current lags behind the voltage by $\pi/2$ radians or 90^o. *The opposition offered by an inductor to the flow of charges is called inductive reactance X_L.*
- When an alternating voltage is applied across the plates of a capacitor then the current is leading the voltage by $\pi/2$ radians or 90^o.

Exercise

Multiple choice questions:

Each of the following questions is followed by four answers. Select the correct answer in each case.

1. The AC system is preferred to DC system because.
   (a) AC voltage can be easily changed in magnitude
   (b) DC motor angular velocity is effected badly
   (c) High voltage AC transmission is less efficient
   (d) Domestic appliances require AC voltage for their operation.
2. A capacitor is perfectly insulator for
   (a) Direct current (b) Alternating current
   (c) Direct as well as alternating current (d) none of these
3. The peak value of alternating current is $5\sqrt{2}$ A. The mean square value of current will be
   (a) 5A, (b) 2.5A, (c) $5\sqrt{2}$ A (d) 5^2 A
4. In choke coil the reactance X_L and resistance R are
   (a) $X_L = R$ (b) $X_L \ll R$
   (c) $X_L \gg R$ (d) $X_L = \infty$
5. In an LRC circuit, the capacitance is made one-fourth, when in resonance. Then what should be change in inductance, so that the circuit remains in resonance?
   (a) 4 times (b) 1/4 times
   (c) 8 times (d) 2 times
6. In AC system we generate sine wave form because
   (a) It can be easily draw
   (b) It produces least disturbance in electrical circuits
   (c) It is nature standard
   (d) Other waves cannot be produced easily.

7. The phase difference between the current and voltage at resonance is

(a) 0 (b) π (c) $-\pi$ (d) $\pi/2$

8. An alternating voltage is given by 20 sin 157 t. The frequency of alternating voltage is

(a) 50Hz (b) 25Hz (c) 100Hz (d) 75Hz

9. In LR current which one of the following statements is correct?

(a) *L* and *R* opposes each other (b) *R* value increases with frequency

(c) The inductive reactance increases with frequency

(d) The inductive reactance decreases with frequency.

10. An alternating quantity (voltage or current) is completely known if we know it's

(*a*) maximum value

(*b*) frequency and phase.

(*c*) effective value

(d) both (a) & (b)

11. For electromagnetic waves, Maxwell generalized

(a) gausses law for magnetism

(b) gausses law for electricity

(c) faradays law

(d) amperes law

12. An electromagnetic wave goes from air to glass which of the following does not change?

(a) Radio waves (b) X-rays

(c) Ultra violet radiation (d) Ultra sound waves

13. The circuit in which current and voltage are in phase, the power factor is

(a) Zero (b) 1 (c) −1 (d) 2

Conceptual questions

1. (a) Sketch a graph of e.m.f. induced in an inductive coil against rate of change of current. What is the significance of the gradient?

(b) Explain why it is difficult to measure the rate of change of current?

(c) How do graphs of e.m.f. against time and current against time make it possible to measure self-inductance?

2. (a) Current and voltage provided by an AC generator are sometimes negative and sometimes positive. Explain why for, an AC generator connected to a resistor, power can never be negative?

   (b) Explain, using sketch graphs, why the frequency of variation of power in an AC generator is twice as that of the current and voltage.

3. What determines the gradient of a graph of inductive reactance against frequency?

4. How does doubling the frequency affect the reactance of (a) an inductor (b) a capacitor?

5. If the peak value of a sine wave is 1,000 volts, what is the effective (E_{eff}) value?

6. Show that reactance is measured in ohms for both inductors and capacitors.

7. Describe the principle of ECG.

Comprehensive questions

1. Explain with diagrams sinusoidal alternating voltage and sinusoidal alternating current.

2. Define mean, peak and rms value of sinusoidal current and sinusoidal voltage. Obtain mathematical expression for the rms value of current.

3. A sinusoidal alternating voltage of angular frequency ω is connected across a resistor R. Find mathematical expression for instantaneous voltage instantaneous current and the average power dissipated per cycle of the applied voltage.

4. A sinusoidal alternating voltage of angular frequency ω is connected across a capacitor C. Find mathematical expression for instantaneous voltage instantaneous current and the average power dissipated per cycle of the applied voltage.

5. A sinusoidal alternating voltage of angular frequency ω is connected across an inductor of inductance L and resistance R. Find mathematical expression for instantaneous voltage, instantaneous current, average power dissipated per cycle of the applied voltage and draw power curve.

6. Explain the term impedance of an AC circuit. Find expression for the of the RLC series circuit.

7. Describe that maximum power is transferred when the impedances of source and load match to each other.

8. In an RL series circuit will the current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.

9. In an RC series circuit will the current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.

10. What do you mean by the term phasor diagram? Why we use it for a sinusoidal current and voltage?

11. Explain the resonance of a series RLC circuit. Show that resonance occurs at a frequency determined by: $f_r = \frac{1}{2\pi\sqrt{LC}}$

12. Describe that maximum power is transferred when the impedances of source and load match to each other.

13. Describe the statement of four Maxwell's Equations. Use Maxwell's theory to show that $E = \frac{1}{2\pi r}\frac{\Delta\Phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t}$ and $I = \frac{\varepsilon_o \Delta(\Phi)}{\Delta t}$

14. What is the principle of ECG? Sketch the wave curve of heart beats and explain the terms positive deflection and negative deflection.

15. Describe the principle of metal detectors with suitable diagram.

16. What is Ecg and its principle, by sketching its curve identify the terms positive deflection, negative deflection ,P deflection, QRS and T deflection?

Numerical Problems

1. The peak voltage of an ac supply is 300 V. What is the rms voltage?
(212.1V)

2. The rms value of current in an ac circuit is 10 A. What is the peak current?
(14.1 A)

3. The a.c. voltage across a 0.5 μF capacitor is 16 sin $(2\times10^3 t)$V. Find (a) the capacitive reactance (b) the peak value of current through the capacitor.
(1000Ω,16mA)

4. The voltage across a 0.01μF capacitor is 240 sin $(1.25 \times 10^4 t - 30^o)$V. Write the mathematical expression for the current through it.
(I=.03sin$(1.25\times10^4 t+60^o)$A)

5. An inductor with an inductance of 100μH passes a current of 10mA when its terminal voltage is 6.3V. Calculate the frequency of A.C supply.
(10^6 Hz)

6. (a) Calculate the inductive reactance of a 3.00 mH inductor, when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120 V?
((a) 1.13 Ω, 188 Ω (b) 106 A, 0.637 A)

7. For the same *RLC* series circuit having a 40.0 Ω resistor, a 3.00 mH inductor, and a 5.00 μF capacitor: (a) Find the resonant frequency. (b) Calculate I_{rms} at resonance if V_{rms} is 120 V.
((a) 1.30 kHz. , (b) 3.00 A)

8. A coil of pure inductance 318mH is connected in series with a pure resistance of 75 Ω. The voltage across resistor is 150V and the frequency of power supply is 50Hz. Calculate the voltage of power supply and the phase angle.

(250V, 53.06° lag)

9. A resistor of resistance 30 Ω is connected in series with a capacitor of capacitance 79.5μF across a power supply of 50Hz and 100V. Find (a) impedance (b) current (c) phase angle and(c) equation for the instantaneous value of current.

(a. 50 Ω , b. 2A, c. 53° lead, d. 2.828 sin (314t + 53°)

10. A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor.(d) power consumed.

(18.85, 55° lag, 0.573 lag, 2484.24W)