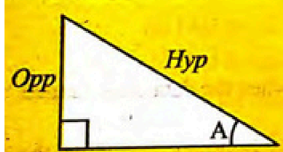
Unit 7

INTRODUCTION TO TRIGNOMETRY

In this unit the students will be able to

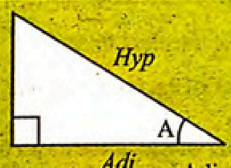
- Measure an angle in sexagesimal system (degree, minute and second).
- Convert an angle given in D°M′S″ form into a decimal form and vice versa.
- Define a radian (measure of an angle in circular system) and prove the relationship between radians and degrees.
- Establish the rule $\ell = r\theta$, where r is the radius of the circle, ℓ the length of circular arc and θ the central angle measured in radians.
- Prove that the area of a sector of a circle is $\frac{1}{2}r^2\theta$.
- Define and identify:
  - general angle (coterminal angles)
  - angle in standard position.
- Recognize quadrants and quadrantal angles.
- Define trigonometric ratios and their reciprocals with the help of a unit circle.
- Recall the values of trigonometric ratios for 45°,30°,60°.
- Recognize signs of trigonometric ratios in different quadrants.
- Find the values of remaining trigonometric ratios if one trigonometric ratio is given.
- Calculate the values of trigonometric ratios for 0°, 90°, 180°, 270°, 360°.
- Prove the trigonometric identities and apply them to show different trigonometric relations.
- Find angle of elevation and depression.
- Solve real life problems involving angle of elevation and depression.

Sine



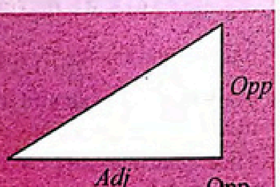
Sine of angle A = $\frac{\text{Opp}}{\text{Hyp}}$

Cosine



Cosine of angle A = $\frac{\text{Adj}}{\text{Hyp}}$

Tangent



Tangent of angle A = $\frac{\text{Opp}}{\text{Adj}}$

Why it's important

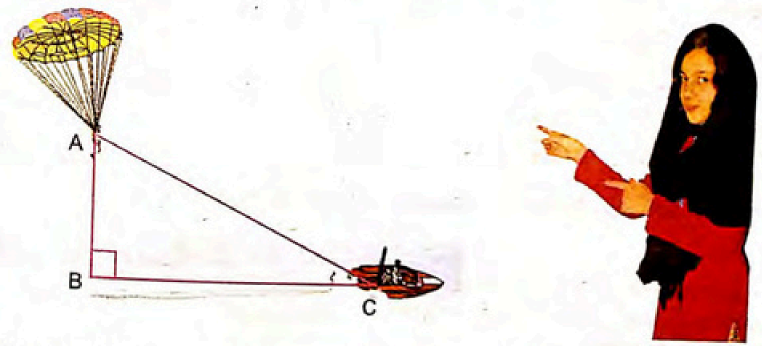
Mount Everest is Earth's highest mountain, peaking at an incredible 29,035 feet. The heights of mountains can be found using trigonometry.

Trigonometry is used in navigation, building, and engineering. For centuries, Muslims used trigonometry and the stars to navigate across the Arabian desert to Mecca, the birthplace of the prophet Muhammad ﷺ, the founder of Islam. The ancient Greeks used trigonometry to record the locations of thousands of stars and worked out the motion of the Moon relative to Earth. Today, trigonometry is used to study the structure of DNA, the master molecule that determines how we grow from a single cell to a complex, fully developed adult and much more.

Activity

How ratios in right triangles used in the real world?

In parasailing, a towrope is used to attach the parachute to the boat.



(a). What type of triangle do the towrope, water, and height of the person above the water form?

(b). What is the hypotenuse of the triangle.

(c). What type of angle do the towrope and the water form?

(d). Which side is opposite this angle?

(e). Other than the hypotenuse, what is the side adjacent to this angle.

7.1

7.1.1 Sexagesimal system (Degree, minute and second)

Sexagesimal (base 60) is a numeral system with sixty as its base. It originated with the ancient Sumerians in the 3rd millennium BC, was passed down to the ancient Babylonians, and is still used—in a modified form—for measuring time, angles and geographic coordinates.

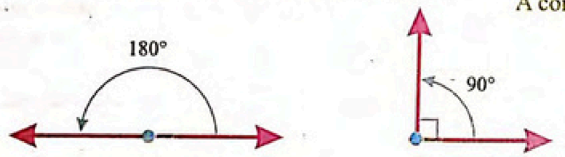
It is a system of measurement of angles, in which angles are measured in degrees, minutes and seconds. If the initial ray $\overrightarrow{OA}$ completes one rotation in anticlock wise direction, then the angle formed is said to be 360 degree or 360°.

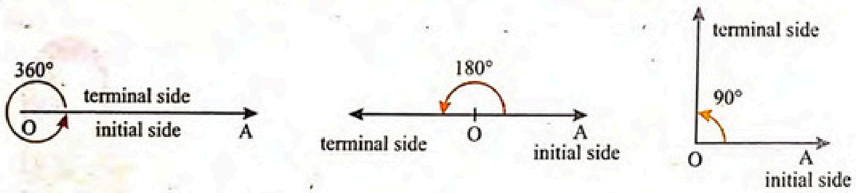
A complete 360° rotation

One complete rotation $= 360°$.

$\frac{1}{2}$ of complete rotation $= 180°$ is called straight angle.

$\frac{1}{4}$ of complete rotation $= 90°$ is called right angle.





Thus one degree is defined as the measure $\frac{1}{360}$th of a complete rotation and it is denoted by '1°'.

A degree is further divided into 60 equal parts called minutes and each minute is further subdivided into 60 equal parts called seconds. We denote minute by (′) and second by (″).

> **Try This**
> (i). Convert 5°42'30" to decimal degree notation.
> (ii). Convert 72.18° D°M'S" to decimal degree notation.

$$1° = 60 \text{ minutes} = 60'$$
$$1' = 60 \text{ seconds} = 60''$$
$$1° = 3600^{th} \text{ seconds} = 3600''$$

49 degree 25 minutes and 15 seconds are written in symbolic form as 49°25′15″

7.1.2 Conversion of D°M'S" form into decimal form and vice versa

Example 1 Convert 15°30′25″ to decimal form.

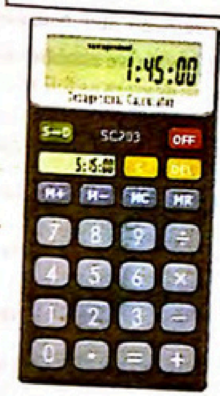
Solution since $1' = \left(\frac{1}{60}\right)^{\circ}$ and $1'' = \left(\frac{1}{3600}\right)^{\circ}$

therefore, $15°30'25'' = \left[15 + 30 \times \frac{1}{60} + 25 \times \frac{1}{3600}\right]^{\circ}$

$= [15 + 0.5 + 0.00694]°$

$= 15.50694°$

> **Activity**
> Use coolconversion.com to covert 142°34'12" to decimal degrees. And also 142.57° to D°M′S″ form.



Example 2 Convert 38.39° to D°M′S″ form.

Solution Since $1° = 60'$ and $1' = 60''$

therefore, $38.39° = 38° + (0.39)°$

$38.39° = 38° + (0.39 \times 60)'$

$= 38° + (23.4)'$

$= 38° + 23' + (0.4)'$

$= 38°23' + (0.4 \times 60)''$

$= 38°23'24''$

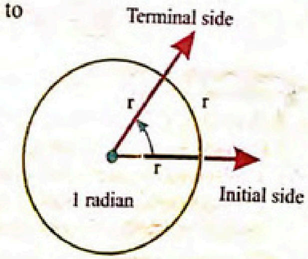
7.1.3 Circular system (Radians)

So far we have been using the measurement of 360 to denote the angle for one complete rotation. However, this value is arbitrary and in some branches of mathematics, angular measurement cannot be conveniently done in degrees. Thus a new unit called the radian, is introduced to describe the magnitude of an angle. This system of angular measurement, known as circular measure, is applied specially to those branches of mathematics which involve the differentiation and integration of trigonometric ratios.

A radian is the measure of the angle subtended at the centre of a circle by an arc ℓ equal in length to its radius r of the circle.

In general radians is the ratio of the length of an arc to the radius of the circle.

i.e. $\theta = \frac{\ell}{r}$



In the following figure, the radius of circle is r and length of an arc AB is also r then angle subtended at the centre is one radian i.e.

$$m\angle AOB = \frac{r}{r} = 1 \text{ radian.}$$

or one radian is an angle subtended at the centre by an arc of length equal to radius of circle.

7.1.4 Relation between radians and degrees

If r is radius of circle then number of radians in one complete rotation is given by

$$\frac{\text{circumference of circle}}{\text{radius of circle}}$$

Since circumference of circle $= 2\pi r$

Therefore number of radians in one complete rotation $= \frac{2\pi r}{r} = 2\pi$ radians but 2π radian $= 360°$ because the circumference of a circle subtends an angle of measure $360°$.

Hence we establish a relation between angles measure in radians and the angles measure in degrees as follows.

$$2\pi \text{ radians} = 360°$$
$$\pi \text{ radians} = 180° \text{ .... (i)}$$
$$\frac{\pi}{2} \text{ radians} = 90°$$
$$\frac{\pi}{3} \text{ radians} = 60°$$
$$\frac{\pi}{6} \text{ radians} = 30° \quad \text{and so on.}$$

Hence, $\quad 1 \text{ radian} = \frac{180°}{\pi} \approx \frac{180°}{3.14159} \approx 57.296°$

$$\boxed{1\text{radian} = 57.3°}$$

Target

To find equivalents for radian and degree measure.

Similarly from (i) we have

$$180° = \pi \text{ radians}$$
$$\Rightarrow \quad 1° = \frac{\pi}{180} = \frac{3.14159}{180} \text{ radians}$$
$$1° \approx 0.0175\text{radians}$$
$$\boxed{1° = 0.0175}$$

Did You Know?

WHAT IS A 'GOOGOL' ?

Googol is one of the largest numbers that has ever been named. It has been defined as 1 followed by one hundred zeros;

10000000000000000000000000000000000000000000000000000000000000000
000000000000000000000000000000000000

However, a googol can be expressed using exponents, as 10^{100}

Example 3 Convert $\frac{4\pi}{7}$ radians to degrees.

Solution Since π radians $= 180°$

Therefore $\quad \frac{4\pi}{7} = \frac{4}{7}(180°) = 102.85714°$
$\quad = 102°51'26''$

Example 4 Convert $31°\,45'$ to radians.

Solution Since $180° = \pi$ radians

$$\Rightarrow \quad 1° = \frac{\pi}{180} \text{ radians}$$

As $\; 31°45' = 31.75°$

$$\because \quad 31°45' = 31.75 \times \frac{\pi}{180} \text{ radians}$$
$$= \frac{31.75 \times 3.14159}{180} \text{ radians}$$
$$= 0.5541 \text{ radians}$$

Exercise 7.1

1. Convert the following angles from D°M'S" forms to decimal forms.
   (i). $8°15'35''$ (ii). $39°48'55''$
   (iii). $84°19'10''$ (iv). $18°6'21''$
2. Convert the following angles from decimal forms to D°M'S".
   (i). $4225°$ (ii). $57.325°$
   (iii). $12.9956°$ (iv). $32.625°$
3. Convert the following radian measures of angles into the measures of degrees.
   (i). 2 radians (ii). $\frac{5\pi}{3}$ radians
   (iii). $\frac{\pi}{6}$ radians (iv). $\frac{-3\pi}{4}$ radians
4. Convert the following angles in terms of radians.
   (i). $45°$ (ii). $120°$
   (iii). $-210°$ (iv). $60°35'48''$

7.2 Sector of a circle

7.2.1 Length of an arc of circle

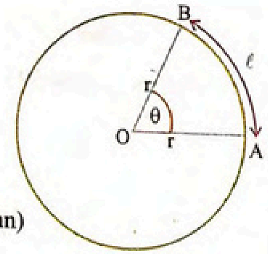
Consider a circle with centre 'O' and radius r, which subtends an angle θ radian at the centre O. Let $\widehat{AB}$ is the minor arc of the circle whose length is equal to ℓ as shown in figure.

By the definition of radian, we have

$$\text{Radian} = \frac{\text{length of an arc } \widehat{AB}}{\text{radius of cirlce}}$$

$$\theta = \frac{\ell}{r}$$

or $$\ell = r\,\theta$$



Which gives length of an arc in terms of angle (in radian) subtended at the center of a circle.

Example 5 Find the length of an arc of a circle of radius 5 cm which subtends an angle of $\frac{3\pi}{4}$ radians at the centre.

Solution

$$r = 5\text{cm}$$

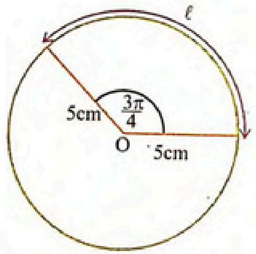
$$\theta = \frac{3\pi}{4} \text{ radians}$$

$$\ell = ?$$

we have

$$\ell = r\,\theta$$

$$\ell = 5 \times \frac{3\pi}{4} = \frac{15\pi}{4} = 11.78 \text{ cm}$$



Hence length of an arc = 11.78 cm.

Example 6

Find the distance travelled by a cyclist moving on a circle of radius 15m, if he makes 3.5 resolutions.

Solution

1 revolution = 2π radians

3.5 revolution = $2\pi \times 3.5$ m

Distance travelled = $l = r\theta$

$l = 15 \times 2\pi \times 3.5 = 105\pi$ m

Example 7 An arc of length 2.5 cm of a circle subtends an angle θ at the centre O of diameter 6 cm. Find the value of θ.

Solution Here

$$m\overline{AB} = 6\text{cm}$$

$$\therefore \quad \overline{OB} = \overline{OC} = r = 3\text{cm}$$

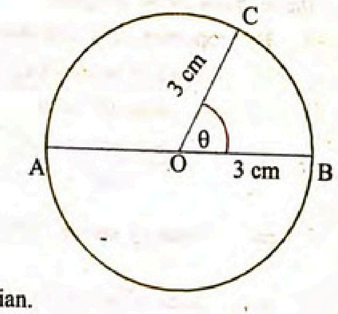
$$m\widehat{BC} = 2.5\text{cm}$$

$$\theta = ?$$

We know that $$\ell = r\,\theta$$

$$\Rightarrow \quad \theta = \frac{\ell}{r}$$

$$\theta = \frac{2.5}{3} \Rightarrow \theta = 0.833 \text{ radian.}$$



Example 8 If length of an arc of a circle is 5 cm which subtends an angle of measure 60°, find the radius of the circle.

Solution $\ell = 5\text{cm}$

$$\theta = 60° = \frac{60 \times \pi}{180} \text{ radians}$$

$$= 1.047 \text{radians}$$

$$r = ?$$

We have formula $$\ell = r\theta$$

$$\Rightarrow \quad r = \frac{\ell}{\theta} = \frac{5}{1.047}$$

$$r = 4.78\text{cm}$$

Hence radius of circle is 4.78cm.

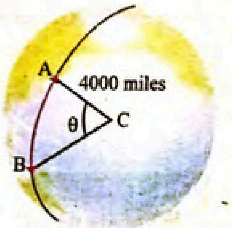
Tidbit

In general,
The length of an arc $= x/360 \times$ circumference
The length of an arc $= x/360 \times 2\pi r$
The area of a sector $= x/360 \times$ area of the circle $= x/360 \times \pi r^2$

Activity

How do we measure the distance between two points, A and B, on Earth?

We measure along a circle with a center C, at the center of Earth. The radius of the circle is equal to the distance from C to the surface. Use the fact that Earth is a sphere of radius equal to approximately 4000 miles to solve the following questions.



(a). If two points A and B, are 8000 miles apart, express angle θ in radians and in degrees.
(b). If two points A and B, are 10,000 miles apart, express angle θ in radians and in degrees.
(c). If θ = 30°, find the distance between A and B to the nearest mile.
(d). If θ = 10°, find the distance between A and B to the nearest mile.

7.2.2 Area of sector

Consider a circle of radius r with centre O, $\overset{\frown}{PQ}$ is an arc which subtends an angle θ radians at the centre as shown in figure.

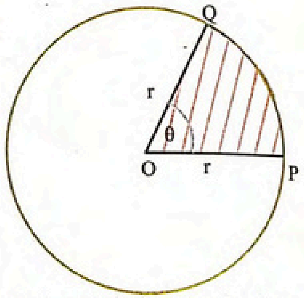
By proportion, it is clear that

$$\frac{\text{Area of sector POQ}}{\text{Area of circle}} = \frac{\text{Central angle of sector}}{2\pi}$$

$$\Rightarrow \quad \frac{\text{Area of sector POQ}}{\pi r^2} = \frac{\theta}{2\pi}$$

$$\Rightarrow \quad \text{Area of sector POQ} = \frac{\theta}{2\pi} \times \pi r^2$$

$$\Rightarrow \quad \text{Area of sector POQ} = \frac{1}{2} r^2 \theta$$



Hence, Area of sector of a circle with radius r, whose central angle is θ radian is given by

$$\boxed{A = \frac{1}{2} r^2 \theta}$$

Example 9

Find the area of sector with central angle of 60° in a circular region whose radius is 5 cm.

Solution

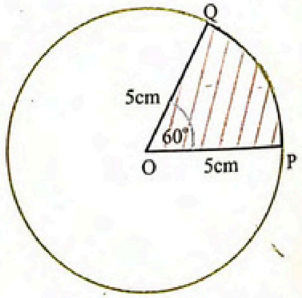
$r = 5\text{cm}$

$\theta = 60° = 60 \times \frac{\pi}{180}$ radian $= 1.047$ radian

Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} (5)^2 (1.047)$

$= 13.08\text{cm}^2$

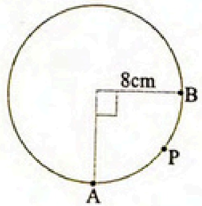


Did You Know?

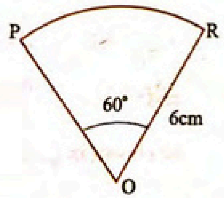
What are digits?
Digits are actually the alphabets of numbers. Just as we use the twenty-six letters of the alphabet to build words, we use the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to build numerals.

Exercise 7.2

1. Find ℓ when
   (i). $\theta = \frac{\pi}{6}$ radian, $r = 2\text{cm}$ (ii). $\theta = 30°$, $r = 6\text{cm}$
   (iii). $\theta = \frac{4\pi}{6}$ radian, $r = 6\text{cm}$
2. Find θ when
   (i). $\ell = 5$ cm, $r = 2$ cm (ii). $\ell = 30$ cm, $r = 6$ cm
   (iii). $\ell = 6$ cm, $r = 2.87$ cm
3. Fined r when
   (i). $\theta = \frac{\pi}{6}$ radians, $\ell = 2\text{cm}$ (ii). $\theta = 3\frac{1}{2}$ radians, $\ell = \frac{4}{7}\text{m}$
   (iii). $\theta = \frac{3\pi}{4}$ radians, $\ell = 15\text{cm}$
4. Find the area of sector of a circle whose radius is 4 m, with central angle 12 radian.
5. The arc of a circle subtends an angle of 30° at the centre. The radius of circle is 5 cm. find;
   (i). Length of the arc (ii). Area of sector formed.
6. An arc of a circle subtends an angle of 2 radian at the centre. If the area of sector formed is 64 cm^2, find the radius of the circle.
7. In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolutions. (3.5 revolutions $= 7\pi$).
8. What is the circular measure of the angle between the hands of the watch at 3 o'clock.
9. What is the length of the arc APB?



10. Find the area of the sector OPR.



7.3 Trigonometric Ratios

Trigonometry is a powerful tool for making indirect measurements of distance or height. It plays an important role in the field of surveying, navigation, engineering and many other branches of physical science.

7.3.1 (a) The general angle (Coterminal angles)

Angles having the same initial and terminal sides are called coterminal angles, and they differ by a multiple of 2π radians or $360°$. They are also called general angles.

If let $\theta = 30°$ is a terminal angle, then it coterminal angles are $\theta + 360°$ and $\theta - 360°$ i.e. $390°$ and $-330°$ as shown in the following figures.

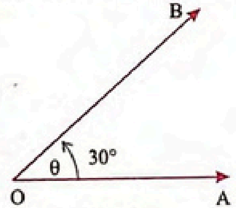


Fig.(a)

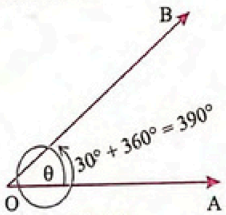


Fig.(b)

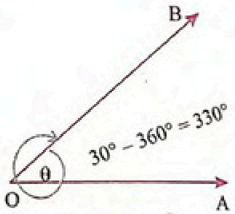
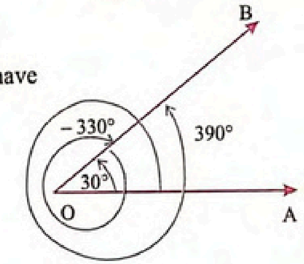


Fig.(b)

Combining figures a, b, c we have



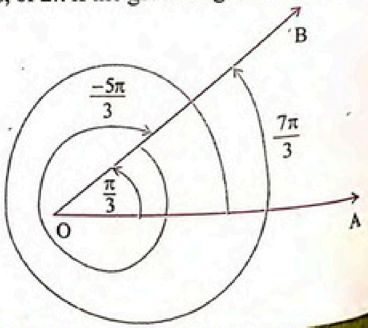
To find a positive and negative general angles (coterminal) with a given angle, add and subtract $360°$, if the given angle is measured in degrees, or 2π if the given angle is measured in radians. For example

coterminal angles of $\frac{\pi}{3}$ are

(i) $\frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

and (ii) $\frac{\pi}{3} - 2\pi = \frac{-5\pi}{3}$

as shown in figure.



Example 10 Find coterminal angle of $60°$ and $-60°$

Solution Since $60° + 360° = 420°$

and $60° - 360° = -300°$

therefore $420°$ and $-300°$ are coterminal angles of $60°$.

Now $-60° + 360° = 300°$

and $-60° - 360° = -420°$

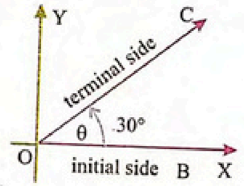
therefore $300°$ and $-420°$ are coterminal angles of $-60°$.

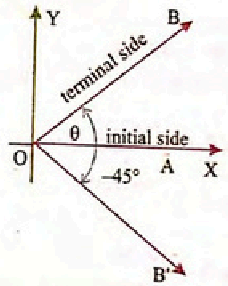
(b) Angle in standard position

In XY-plane, an angle is in standard position if

- its vertex is at the origin of a rectangular coordinate system and
- its initial side lies along the positive x-axis.

An angle is said to be positive if it is measured in anticlock wise direction from positive x-axis and is negative, if it is measured clockwise from +ve x-axis. In the following figure terminal side is in anti-clock wise direction which makes an angle of measure $45°$ and in clock wise direction makes an angle of measure $-45°$.





7.3.2 (a) Quadrants

The Cartesian plane is divided into four quadrants and the angle θ is said to be in the quadrant where OP lies. In the diagram, θ is in the first quadrant.

As shown in figure (b).

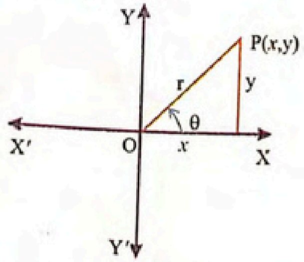


Fig.(a)

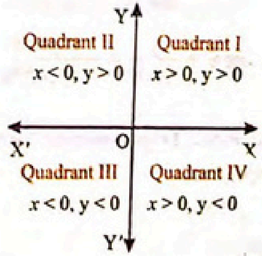
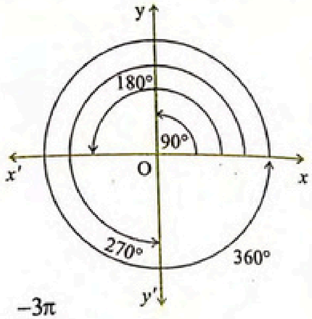


Fig.(b)

(b) Quadrantal angles

Quadrantal angles are those angles whose terminal sides coincide with co-ordinate axes i.e. x-axis or y-axis. The Quadrantal measures of angles are 0°, 90°, 180°, 270° and 360° or 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ and 2π radians as shown in figure.



Exercise 7.3

1. Find coterminal angles of the following angles.
   (i). 55° (ii). −45° (iii). $\frac{\pi}{6}$ (iv). $\frac{-3\pi}{4}$
2. State the quadrant in which the following angles lie?
   (i). $\frac{8\pi}{5}$ (ii). 75° (iii). −818° (iv). $\frac{-5\pi}{4}$ (v). 103°

7.3.3 Trigonometric ratios

A trigonometric ratio is a ratio of lengths of two sides in a right triangle. Trigonometric ratios are used to find the measure of a side or an acute angle in a right triangle. The trigonometric ratios for any acute angle θ of a right triangle ABC are

$$\sin\theta = \frac{\text{opposite side of }\theta}{\text{hypotenuse}}$$

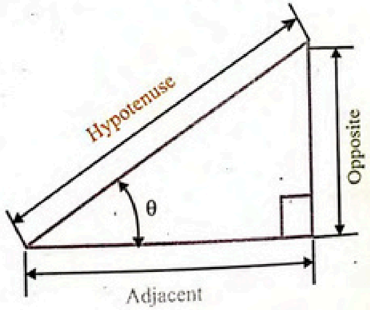
$$\cos\theta = \frac{\text{adjacent side of }\theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side of }\theta}{\text{adjacent side of }\theta}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite site of }\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}}$$



A trick to remember Trigonometric formulas

some people have
curly brown hair
through proper brushing

7.3.3.1 Trigonometric ratios with the help a unit circle

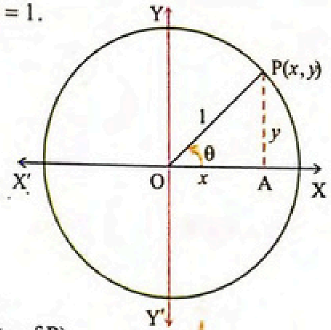
Consider a circle with centre 0 and radius 1 (unit circle). P (x, y) is any point on the circle. Radius $\overline{OP}$ makes an angle θ with the positive x-axis. θ can be measured either in degrees or in radians. Draw a perpendicular $\overline{PA}$ on x-axis, so that OAP is a right triangle in which $m\angle A = 90°$ as shown in the figure.

In ΔOAP opposite side of θ is $\overline{AP} = y$, adjacent side of θ is $\overline{OA} = x$ and hypotenuse is $\overline{OP} = 1$.
Therefore by definition

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\overline{AP}}{\overline{OP}} = \frac{y}{1} = y$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\overline{OA}}{\overline{OP}} = \frac{x}{1} = x$$

and $$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\overline{AP}}{\overline{OP}} = \frac{y}{x}$$



Hence trigonometric ratios of a unit circle are

$\sin\theta = y$ (the y co-ordinate of P)

$\cos\theta = x$ (the x co-ordinate of P)

$\tan\theta = \frac{y}{x}$ $\left(\frac{y \text{ co-ordinate of P}}{x \text{ co-ordinate of P}}\right)$

Reciprocal trigonometric ratios

$$\operatorname{cosec}\theta = \frac{\overline{OP}}{\overline{AP}} = \frac{1}{y} = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{\overline{OP}}{\overline{OA}} = \frac{1}{x} = \frac{1}{\cos\theta}$$

and $$\cot\theta = \frac{\overline{OA}}{\overline{AP}} = \frac{x}{y} = \frac{\cos\theta}{\sin\theta}$$

Note

(Important points to be noted)

1. If θ lies in 1st Quadrant, then we can write as $0 < \theta < \frac{\pi}{2}$ or $0 < \theta < 90°$.
2. If θ lies in 2nd Quadrant, then we can write as $\frac{\pi}{2} < \theta < \pi$ or $90° < \theta < 180°$.
3. If θ lies in 3rd Quadrant, then we can write as $\pi < \theta < \frac{3\pi}{2}$ or $180° < \theta < 270°$.
4. If θ lies in 4th Quadrant, then we can write as $\frac{\pi}{2} < \theta < 2\pi$ or $270° < \theta < 360°$.

7.3.4 Values of trigonometric ratios for 45°, 30° and 60°

The values of trigonometric ratios for 45°, 30° and 60° are given in the following table.

T-ratio\ θ	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\operatorname{cosec}\theta$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$
$\sec\theta$	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2
$\cot\theta$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

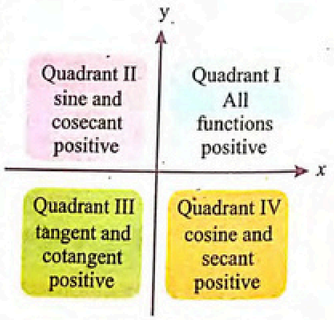
7.3.5 Recognizing signs of trigonometric ratios in different Quadrants

Signs of trigonometric ratios are different in different quadrants depending upon the terminal side of the angle θ.

(i). If θ lies in first quadrant, then $x > 0, y > 0$ therefore $\sin\theta$, $\cos\theta$, $\tan\theta$, $\operatorname{cosec}\theta$, $\sec\theta$, $\tan\theta$ are all positive.

(ii). If θ lies in 2nd quadrant, then $x < 0, y > 0$ therefore $\sin\theta$ and $\operatorname{cosec}\theta$ are positive and the remaining four trigonometric ratios are negative.

(iii). If θ lies in 3rd quadrant, then $x < 0, y < 0$ therefore $\tan\theta$ and $\cot\theta$ are positive and the remaining four trigonometric ratios are negative.



(iv). If θ lies in 4th quadrant, then $x > 0, y < 0$ therefore $\cos\theta$ and $\sec\theta$ are positive and the remaining trigonometric ratios are negative.

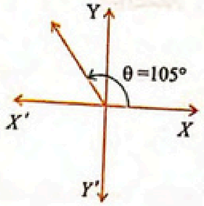
These four results are summarized in the figure at the right.

Example 11 Find the signs of the following trigonometric ratios and tell in which quadrant they lie?

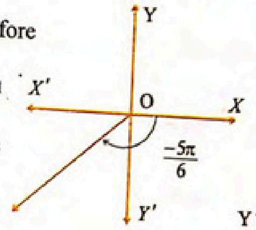
(i) $\sin 105°$ (ii) $\tan \frac{-5\pi}{6}$ (iii) $\sec 1030°$ (iv) $\cot 710°$

Solution

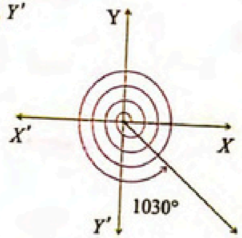
(i) $\theta = 105°$ lies in 2nd quadrant, therefore $\sin 105°$ also lies in 2nd Quadrant and hence sign of $\sin 105°$ is positive.



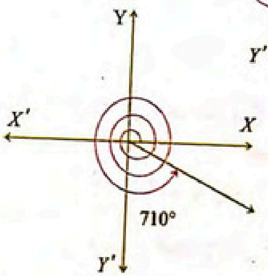
(ii) Since $\frac{-5\pi}{6}$ lies in the 3rd quadrant, therefore $\tan\left(\frac{-5\pi}{6}\right) = -\tan\frac{5\pi}{6}$ also lies in 3rd quadrant and it has positive sign. As shown in figure.



(iii) Since $\theta = 1030° = 720° + 310°$, therefore it lies in 4th quadrant and $\sec 1030°$ in 4th quadrant has positive sign.



(iv) Since $\theta = 710° = 360° + 350°$, therefore it lies in 4th quadrant and $\cot 710°$ in 4th quadrant is negative.



Example 12 If $\tan\theta < 0$ and $\cos\theta > 0$, name the quadrant in which angle θ lies.

Solution

When $\tan\theta < 0$, θ lies in quadrant II or IV. When $\cos\theta > 0$, θ lies in quadrant I or IV. When both conditions are met ($\tan\theta < 0$ and $\cos\theta > 0$), θ must lie in quadrant IV.

Try This

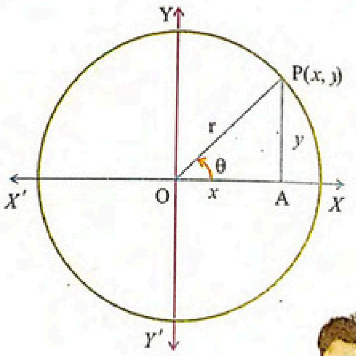
If $\sin\theta < 0$ and $\cos\theta < 0$, name the quadrant in which angle θ lies.

7.3.6 Finding the values of remaining trigonometric ratios if one trigonometric ratio is given

Example 13 If $\tan\theta = 1$, find the other trigonometric ratios, when θ lies in first quadrant.

Solution $\tan\theta = 1 = \dfrac{y}{x}$

$\Rightarrow \quad y = 1 \; and \; x = 1$



Hence by Pythagoras theorem;

$$r^2 = x^2 + y^2$$
$$r^2 = (1)^2 + (1)^2 = 1 + 1$$
$$r^2 = 2$$
$$r = \sqrt{2}$$

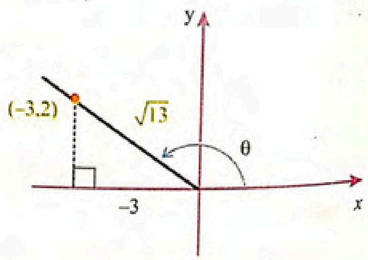
Therefore $\quad \sin\theta = \dfrac{y}{r} = \dfrac{1}{\sqrt{2}}$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}}$$
$$\cot\theta = \frac{x}{y} = \frac{1}{1} = 1$$
$$\text{cosec}\theta = \frac{r}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

Example 14 Given that $\tan\theta = -\dfrac{2}{3}$ and θ is in the second quadrant, find the other function values.

Solution $\tan\theta = \dfrac{y}{x} = -\dfrac{2}{3} = \dfrac{2}{-3}$,

$\therefore y = 2$ and $x = -3$



Expressing $-\dfrac{2}{3}$ as $\dfrac{2}{-3}$ sine θ is in quadrant II

Hence by Pythagoras theorem;

$$r^2 = x^2 + y^2$$
$$r^2 = (-3)^2 + (2)^2 = 9 + 4 = 13$$
$$r^2 = 13$$
$$r = \sqrt{13}$$

Therefore,

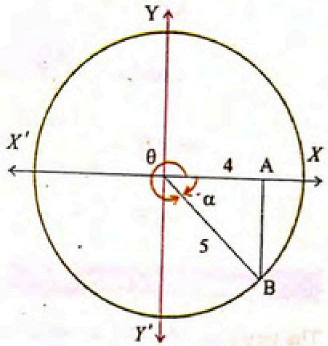
$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}} = -\frac{3}{\sqrt{13}}$$
$$\cot\theta = \frac{x}{y} = \frac{-3}{2} = -\frac{3}{2}$$
$$\text{cosec}\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$$
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$$

Example 15 If $\cos\theta = \dfrac{4}{5}$ where θ lies in 4^{th} quadrant, find the values of other trigonometric ratios.

Solution Given

$$\cos\theta = \frac{4}{5} = \frac{x}{r}$$

$\Rightarrow \; x = 4 \; and \; r = 5$



Hence by Pythagoras theorem;

$$r^2 = x^2 + y^2$$
$$y^2 = r^2 - x^2 = (5)^2 + (4)^2 = 25 - 16 = 9$$
$$y^2 = 9 \Rightarrow y = \pm 3$$
$$y = -3 \; (\because \theta \text{ lies in } 4^{th} \text{ quadrant.})$$

Therefore,

$$\sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{4} = -\frac{3}{4}$$
$$\cot\theta = \frac{x}{y} = \frac{4}{-3} = -\frac{4}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{4}$$
$$\text{cosec}\theta = \frac{r}{y} = \frac{5}{-3} = -\frac{5}{3}$$

Try This

If $\sin\theta = \dfrac{1}{2}$ and θ lies in 2^{nd} Quadrant, find the other trigonometric ratios.

7.3.7 Calculating the values of trigonometric ratios of 0°, 90°, 180°, 270° and 360°

Trigonometric ratio of 0°

In XY-plane, the terminal side $\overrightarrow{OP}$ of the angle θ° coincides with $\overrightarrow{OX}$.

Therefore radius $\overline{OP} = r = 1$

or $x = 1$ and $y = 0$

so $\sin 0° = \frac{y}{r} = \frac{0}{1} = 0$

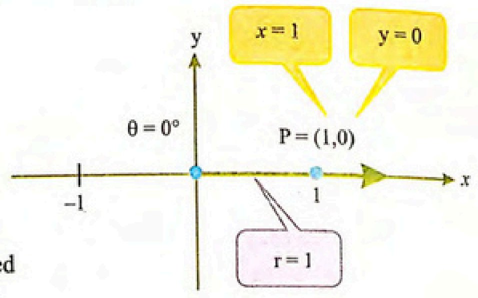
$\cos 0° = \frac{x}{r} = \frac{1}{1} = 1$

$\tan 0° = \frac{y}{x} = \frac{0}{1} = 0$

$\cot 0° = \frac{x}{y} = \frac{1}{0} =$ undefined

$\operatorname{cosec} 0° = \frac{r}{y} = \frac{1}{0} =$ undefined

$\sec 0° = \frac{r}{x} = \frac{1}{1} = 1$



Trigonometric ratio of 90°

The terminal side $\overrightarrow{OP}$ of angle 90° coincides with $\overrightarrow{OY}$.

Therefore $\overrightarrow{OP} = r = 1$

or $x = 0$ and $y = 1$

Hence $\sin 90° = \frac{y}{r} = \frac{1}{1} = 1$

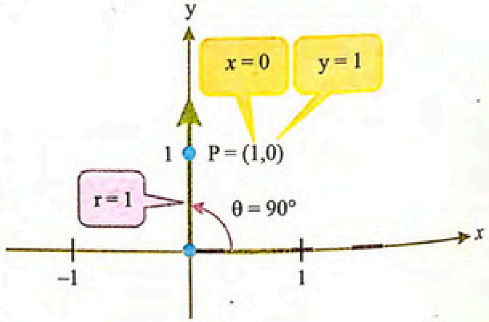
$\cos 90° = \frac{x}{r} = \frac{0}{1} = 0$

$\tan 90° = \frac{y}{x} = \frac{1}{0} =$ undefined

$\operatorname{cosec} 90° = \frac{r}{y} = \frac{1}{1} = 1$

$\sec 90° = \frac{r}{x} = \frac{1}{0} =$ undefined

$\cot 90° = \frac{x}{y} = \frac{0}{1} = 0$



Trigonometric ratio of 180°

In this case, the terminal side $\overrightarrow{OP}$ of angle 180° coincides with $\overrightarrow{OX'}$

Therefore $\overrightarrow{OP} = r = 1$

or $x = -1$ *and* $y = 0$

$\sin 180° = \frac{y}{r} = \frac{0}{r} = 0$

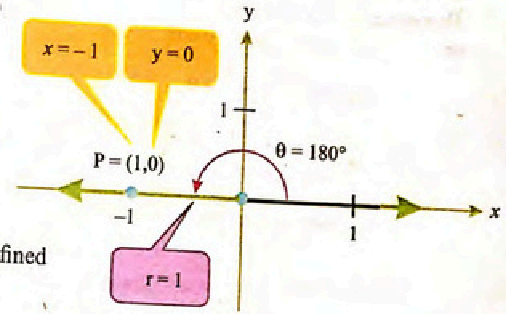
$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$\tan 180° = \frac{y}{x} = \frac{0}{-1} = 0$

$\operatorname{cosec} 180° = \frac{r}{y} = \frac{1}{0} =$ undefined

$\sec 180° = \frac{r}{x} = \frac{1}{-1} = -1$

$\cot 180° = \frac{x}{r} = \frac{-1}{0} =$ undefined



Trigonometric ratio of 270°

In this case, the terminal side $\overrightarrow{OP}$ of angle 270° coincides with $\overrightarrow{OY'}$

Therefore $\overrightarrow{OP} = r = 1$

or $x = 0$ *and* $y = -1$

$\sin 270° = \frac{y}{r} = \frac{-1}{1} = -1$

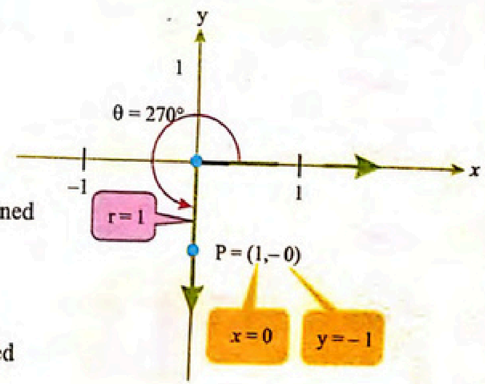
$\cos 270° = \frac{x}{r} = \frac{0}{1} = 0$

$\tan 270° = \frac{y}{x} = \frac{-1}{0} =$ undefined

$\operatorname{cosec} 270° = \frac{r}{y} = \frac{1}{-1} = -1$

$\sec 270° = \frac{r}{x} = \frac{1}{0} =$ undefined

$\cot 270° = \frac{x}{r} = \frac{0}{-r} = 0$

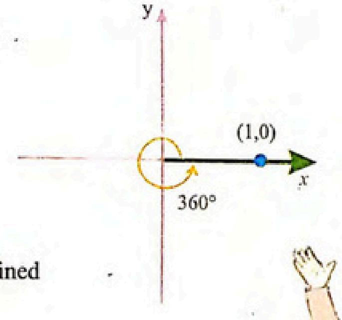


Trigonometric ratio of 360°

In this case, the terminal side $\overrightarrow{OP}$ of angle 360° coincides with $\overrightarrow{OX}$.

Therefore $\overrightarrow{OP} = r = 1$

or $x = 1$ *and* $y = 0$



$$\sin 360° = \frac{y}{r} = \frac{0}{1} = 0$$

$$\cos 360° = \frac{x}{r} = \frac{1}{1} = 1$$

$$\tan 360° = \frac{y}{x} = \frac{0}{1} = 0$$

$$\text{cosec}360° = \frac{r}{y} = \frac{1}{0} = \text{undefined}$$

$$\sec 360° = \frac{r}{x} = \frac{1}{1} = 1$$

$$\cot 360° = \frac{x}{r} = \frac{1}{0} = \text{undefined}$$

Summarizing all the results, we have a table

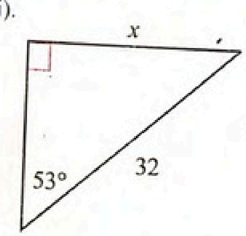
The trigonometric table

θ	0°	30°	30°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cot θ	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cosec θ	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

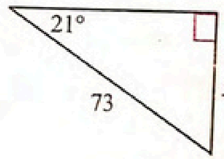
Exercise 7.4

1. Find the signs of the following trigonometric ratios and tell in which quadrant they lie?
   (i). sin 98° (ii). sin 160° (iii). tan 200°
   (iv). sec 120° (v). cosec 198° (vi). sin 460°
2. Find the trigonometric ratios of the following angles.
   (i). −180° (ii). −270° (iii). 720°
   (iv). 1470°
3. If $\sec\theta = 2$ where θ lies in 4th quadrant, find the other values of trigonometric ratios.
4. If $\sin\ \theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$ then find other trigonometric ratios.
5. Find the values of
   (i). $2\sin 45° \cos 45°$ (ii). $\dfrac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°}$ (iii). $\dfrac{\cos 45°}{\sin 45° + \tan 45°}$
   (iv). $\tan 30° \tan 60° + \tan 45°$ (v). $\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6}$
6. In which quadrant θ lies?
   (i). $\sin\theta > 0,\ \tan\theta > 0$ (ii). $\sin\theta < 0,\ \cot\theta > 0$ (iii). $\sin\theta > 0,\ \cos\theta < 0$
   (iv). $\cos\theta > 0,\ \text{cosec}\theta < 0$ (v). $\tan\theta < 0,\ \sec\theta > 0$ (vi). $\cos\theta < 0,\ \tan\theta < 0$
7. For each triangle, find each missing measure to two decimal places.

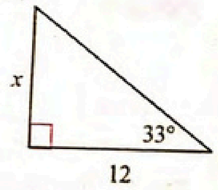
(i).



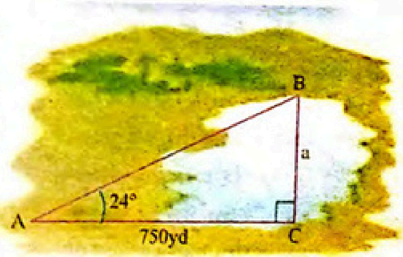
(ii).



(iii).



8. The irregular blue shape in the diagram represents a lake. The distance across the lake, a, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?



7.4 Trigonometric identities

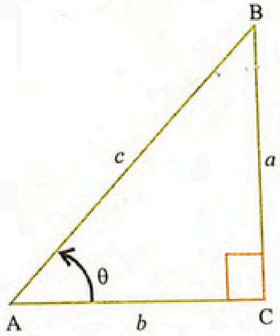
For any real number θ, we have following fundamental trigonometric identities.

(i) $\cos^2\theta+\sin^2\theta=1$ (ii) $1+\tan^2\theta=\sec^2\theta$ (iii) $1+\cot^2\theta=\text{cosec}^2\theta$

Proof Consider a right triangle ABC in which

$m\angle ACB=90^\circ,\ m\angle BAC=\theta$

$m\overline{AB}=c,\ m\overline{AC}=b$ and $m\overline{CB}=a$



Then we have $\sin\theta=\frac{a}{c},\ \cos\theta=\frac{b}{c}$ (i)

$\tan\theta=\frac{a}{b},\ \sec\theta=\frac{c}{b}$ (ii)

$\cot\theta=\frac{b}{a},\ \text{cosec}\theta=\frac{c}{a}$ (iii)

By Pythagoras theorem

$$c^2=b^2+a^2 \qquad \text{(iv)}$$

Dividing both sides by c^2, we have

$$\frac{c^2}{c^2}=\frac{b^2}{c^2}+\frac{a^2}{c^2}$$

$$\Rightarrow\quad 1=\left(\frac{b}{c}\right)^2+\left(\frac{a}{c}\right)^2$$

Putting the values of $\frac{b}{c}$ and $\frac{a}{c}$ from (i), we get

$$1=(\cos\theta)^2+(\sin\theta)^2$$

$$\Rightarrow\quad 1=\cos^2\theta+\sin^2\theta$$

$$\text{or}\quad \boxed{\cos^2\theta+\sin^2\theta=1} \qquad \text{(v)}$$

Now dividing equation (iv) by b^2, we have

$$\frac{c^2}{b^2}=\frac{b^2}{b^2}+\frac{a^2}{b^2}$$

$$\left(\frac{c}{b}\right)^2=1+\left(\frac{a}{b}\right)^2$$

Putting values from (ii), we get

$$(\sec\theta)^2=1+(\tan\theta)^2$$

$$\Rightarrow\quad \sec^2\theta=1+\tan^2\theta$$

$$\text{or}\quad \boxed{1+\tan^2\theta=\sec^2\theta} \qquad \text{(vi)}$$

Again dividing Eq. (iv) by a^2, we have

$$\frac{c^2}{a^2}=\frac{b^2}{a^2}+\frac{a^2}{a^2}$$

$$\left(\frac{c}{a}\right)^2=\left(\frac{b}{a}\right)^2+1$$

J

Putting values from (3), we get

$$(\text{cosec}\theta)^2=(\cot\theta)^2+1$$

$$\Rightarrow\quad \text{cosec}^2\theta=\cot^2\theta+1$$

$$\text{or}\quad \boxed{1+\cot^2\theta=\text{cosec}^2\theta} \qquad \text{(vii)}$$

Example 16 Show that $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$

?

Solution L.H.S $=(\sin\theta+\cos\theta)^2$

$=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$

$=1+2\sin\theta\cos\theta$ $[\because \sin^2\theta+\cos^2\theta=1]$

$=R.H.S$

Hence $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta$

Example 17 Prove that $\sin\theta=\sqrt{1-\cos^2\theta}$

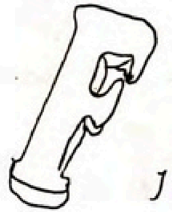
Solution As we know that

$\cos^2\theta+\sin^2\theta=1$

$\Rightarrow\quad \sin^2\theta=1-\cos^2\theta$

$\Rightarrow\quad \sin\theta=\sqrt{1-\cos^2\theta}$

Which is the required result.



Joker

Example 18 Prove that $\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}=\cot\theta$

Solution L.H.S $=\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

As we know that

$\cos^2\theta+\sin^2\theta=1$

$\Rightarrow\quad \cos^2\theta=1-\sin^2\theta$

Hence L.H.S $=\frac{\sqrt{\cos^2\theta}}{\sin\theta}=\frac{\cos\theta}{\sin\theta}=\cot\theta=R.H.S$

Hence, $\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}=\cot\theta$

Example 19 Prove that $\sec^2\theta + \tan^2\theta = \dfrac{1+\sin^2\theta}{1-\sin^2\theta}$

Solution

$$\text{L.H.S} = \sec^2\theta + \tan^2\theta$$
$$= \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \frac{1+\sin^2\theta}{\cos^2\theta} \quad \text{.............................. (i)}$$

As we know that,

$$\cos^2\theta + \sin^2\theta = 1$$
$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$

Putting value of $\cos^2\theta$ in (i), we get

$$\text{L.H.S} = \frac{1+\sin^2\theta}{1-\sin^2\theta} = \text{R.H.S}$$

Hence,

$$\sec^2\theta + \tan^2\theta = \frac{1+\sin^2\theta}{1-\sin^2\theta}$$

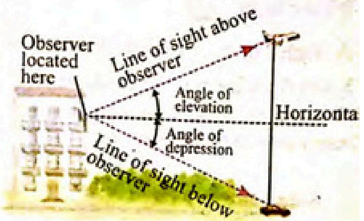
Exercise 7.5

Prove the following trigonometric identities.

1. $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$
2. $\tan\theta + \sec\theta = \dfrac{1+\sin\theta}{\cos\theta}$
3. $(\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta$
4. $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
5. $\tan\theta + \cot\theta = \sec\theta\,\mathrm{cosec}\,\theta$
6. $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$
7. $\sin\theta\sqrt{1+\tan^2\theta} = \tan\theta$
8. $\cos\theta = \sqrt{1-\sin^2\theta}$
9. $(1+\cos\theta)(1-\cos\theta) = \dfrac{1}{\mathrm{cosec}^2\theta}$
10. $\cos x - \cos x\sin^2 x = \cos^3 x$
11. $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = 2\,\mathrm{cosec}\,x$
12. $\dfrac{\sin x}{1+\cos x} = \dfrac{1-\cos x}{\sin x}$
13. $\dfrac{1}{1+\cos a} + \dfrac{1}{1-\cos a} = 2 + 2\cot^2 a$
14. $\cos^4 b - \sin^4 b = 1 - 2\sin^2 b$
15. $\dfrac{\sin y + \cos y}{\sin y} + \dfrac{\cos y - \sin y}{\cos y} = \sec y\ \mathrm{cosec}\ y$
16. $(\sec x - \tan x)^2 = \dfrac{1-\sin x}{1+\sin x}$
17. $\sin x\tan x + \cos x = \sec x$.

7.5 Angle of Elevation and Depression

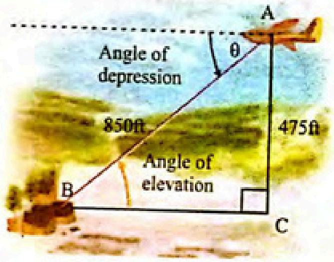
Many applications of right triangle trigonometry involve the angle made with an imaginary horizontal line. As shown in the figure , an angle formed by a horizontal line and the line of sight to an object that is above the horizontal line is called the angle of elevation. The angle formed by a horizontal line and the line of sight to an object that is below the horizontal line is called the angle of depression.



Example 20 An aerial photographer who photographs a farm house for a company has determined from experience that the best photo is taken at a height of approximately 475 ft and a distance of 850 ft from the farmhouse. What is the angle of depression from the plane to the house?

Solution

When parallel lines are cut by a transversal, alternate interior angles are equal. Thus the angle of depression from the plane to the house, θ, is equal to the angle of elevation from the house to the plane, so we can use the right triangle shown in the figure. Since we know the side opposite ∠B and the hypotenuse, we can find ∠B by using the sine function.

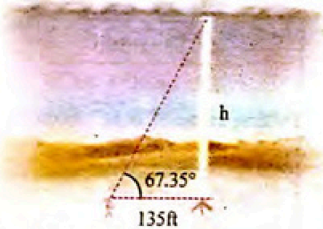


$$\sin\theta = \sin B = \frac{475\,ft}{850\,ft} \approx 0.5588$$

$$\theta \approx 34°$$

Thus the angle of depression is approximately 34°.

Example 21 To measure cloud height at night, a vertical beam of light is directed on a spot on the cloud. From a point 135 ft away from the light source, the angle of elevation to the spot is found to be 67.35°. Find the height of the cloud.



Solution From the figure, we have tan 67.35°

$$\tan 67.35° = \frac{h}{135}$$

$$h = 135\tan 67.35° \approx 324\,ft$$

The height of the cloud is about 324 ft.

Tidbit

Trigonometry helps to find the height of buildings and mountains. Clinometer is used to obtain the angle of elevation of the top of an object.

Example 22

A light house is 300 meter above the sea level. Angles of depression of two boats from the top of light house are 30° and 45° respectively. If line joining the boats passes through the foot of the light house, find the distance between the boats when they are on the same side of the light house.

Solution

Let boats are at point C and D respectively.

Height of light house $m\overline{AB} = 300m$

Distance between two boats $m\overline{CD} = ?$

Angles of depression are $m\angle EBD = 30°$ and $m\angle EBC = 45°$

But $\quad m\angle EBD = m\angle BDA$ *and* $m\angle EBC = m\angle BCA$

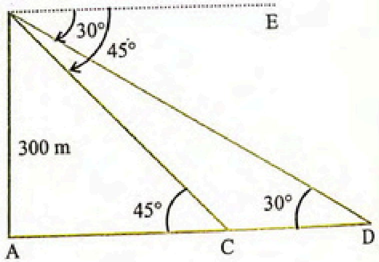
Therefore $\quad m\angle BDA = 30°$ *and* $m\angle BCA = 45°$

Consider $\quad \Delta ABC$

$$\tan 45° = \frac{m\overline{AB}}{m\overline{AC}}$$

$$1 = \frac{300\,m}{m\overline{AC}}$$

or $\quad m\overline{AC} = 300\,m \qquad$ (i)



Now

Consider ΔABD

$$\tan 30° = \frac{m\overline{AB}}{m\overline{AD}}$$

$$\frac{1}{\sqrt{3}} = \frac{300}{m\overline{AC} + m\overline{CD}} \qquad \because m\overline{AD} = m\overline{AC} + m\overline{CD}$$

Putting value of $m\overline{AC}$ from (i).

$$\frac{1}{\sqrt{3}} = \frac{300}{300 + \overline{CD}}$$

$\Rightarrow \quad 300 + m\overline{CD} = 300\sqrt{3}$

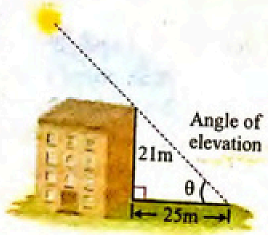
$\Rightarrow \quad m\overline{CD} = 300\sqrt{3} - 300$

$m\overline{CD} = 300\left(\sqrt{3} - 1\right)m = 219.6\,m$

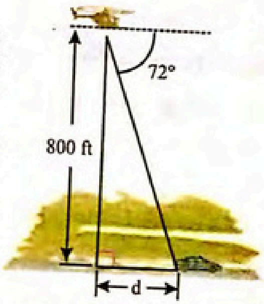
Hence distance between two boats is 219.6 m.

Exercise 7.6

1. A building that is 21 metres tall casts a shadow 25 metres long. Find the angle of elevation of the sun to the nearest degree.



2. A light house is 150 m above the sea level. Angle of depression of a boat from its top is 60°. Find the distance between the boat and the lighthouse.
3. A tree is 50 m high. Find the angle of elevation of its top to a point, on the ground 100 m away from the foot of tree.
4. From the top of hill 240 m high, measure of the angles of depression of the top and bottom of minaret are 30° and 60° respectively. Find the height of minaret.
5. A police helicopter is flying at 800 feet. A stolen car is sighted at an angle of depression of 72°. Find the distance of the stolen car, to the nearest foot, from a point directly below the helicopter.



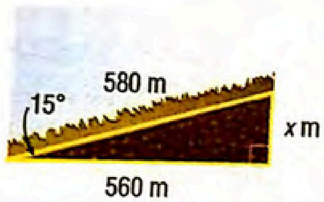
6. A light house is 300 m above the sea level. The angle of depression of two boats from the top of light house are 30° and 45° respectively. If the line of joining of the boats passes through the foot of light house, find the distance between two boats when they are on the opposite side of the light house.
7. The angle of elevation of the top of a cliff is 30°. Walking 210 metre from the point towards the cliff, the angle of elevation becomes 45°. Find the height of cliff.

Activity

Find the error.

Zia and Rabia are finding the height of the hill.

Zia	Rabia
$\sin 15° = \dfrac{x}{580}$	$\sin 15° = \dfrac{x}{560}$
$580(\sin 15°) = x$	$560(\sin 15°) = x$
$150 \approx x$	$145 \approx x$



Who is correct?

Review Exercise 7

1. At the end of each question, four circles are given. Fill in the correct circle only.

(i). If an object is above the level of observer then the angle formed between the horizontal line and observer's line of sight is called

○ angle of depression ○ angle of elevation
○ obtuse angle ○ none of the above

(ii). cot = ________.

○ $\frac{\sin\theta}{\cos\theta}$ ○ $\frac{1}{\cos\theta}$ ○ $\frac{\cos\theta}{\sin\theta}$ ○ $\frac{1}{\sin\theta}$

(iii). $1+\tan^2\theta=$ ______

○ $\sin^2\theta$ ○ $\cos^2\theta$ ○ $\text{cosec}^2\theta$ ○ $\sec^2\theta$

(iv). If $\tan\theta=1$ then $\sin\theta=$ ________ when θ lies in 3rd quadrant.

○ $\frac{1}{2}$ ○ $\frac{-1}{2}$ ○ $-\frac{1}{\sqrt{2}}$ ○ $\frac{1}{\sqrt{2}}$

(v). Sin (–350°) lies in ________.

○ 1st quadrant ○ 2nd quadrant ○ 3rd quadrant ○ 4th quadrant

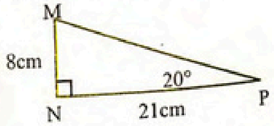
(vi). 45°=______ radian.

○ $\frac{\pi}{3}$ ○ $\frac{\pi}{4}$ ○ $\frac{\pi}{6}$ ○ $\frac{\pi}{2}$

(vii). If the measure of the hypotenuse of a right triangle is 5 feet and $m\angle B=58°$, what is the measure of the leg adjacent to ∠B?

○ 4.2402 ○ 8.0017 ○ 0.10060 ○ 2.6496

(viii). Find the value of tan P to the nearest tenth.



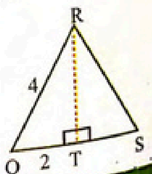
○ 2.6 ○ 0.5
○ 0.4 ○ 0.1

(ix). RT is equal to TS. What is ∠S?

○ $2\sqrt{6}$ ○ $2\sqrt{3}$ ○ $4\sqrt{3}$ ○ $2\sqrt{2}$

(x). RT is equal to TS. What is ∠S?

○ 25° ○ 30° ○ 45° ○ 60°



2. Convert 45°35′30″ into decimal form.

3. Convert 216.67° into D°M′S″ form.

4. Through how many radians does a minute of a clock turn through

(i). in 45 minutes (ii). in one hour.

5. Find coterminal angle of 190° and –250°.

6. Find trigonometric ratios of

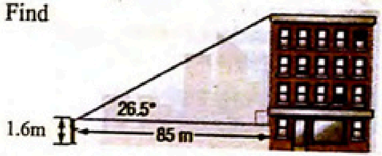
(i). 390° (ii). –240°

7. Prove that

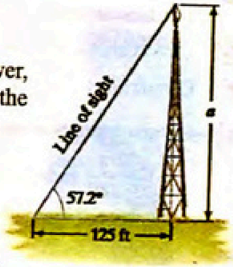
(i). $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\sec\theta-\tan\theta$ (ii). $2\cos\theta\sec\theta-\tan\theta\cot\theta=1$

8. If $\sec\theta=2$ and θ does not lie in first quadrant. Find remaining trigonometric ratios.

9. Refer to the diagram shown.
What is the height of the building?



10. From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2°. Approximate the height of the tower to the nearest foot.



The Amazing number 1,089

1089 • 1 = 1089 1089 • 9 = 9801
1089 • 2 = 2178 1089 • 8 = 8712
1089 • 3 = 3267 1089 • 7 = 7623
1089 • 4 = 4356 1089 • 6 = 6534

A unique property among two digit numbers.
$33^2 = 1{,}089 = 65^2 - 56^2$