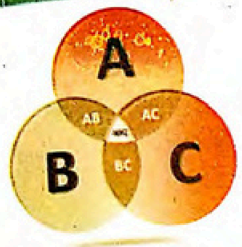
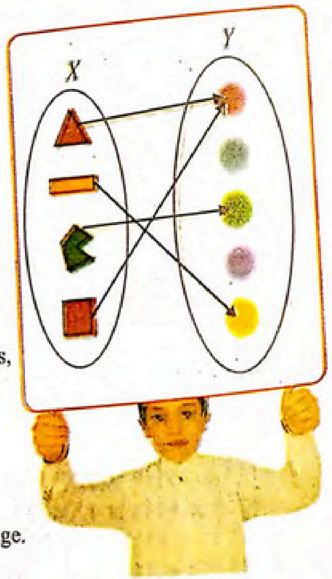
Unit 5

SETS AND FUNCTIONS



In this unit the students will be able to

- Recall the sets denoted by N, Z, W, E, O, P and Q.
- Recognize operation on sets ($\cup, \cap, \backslash, \ldots$)
- Perform operation on sets
  - union,
  - intersection,
  - difference,
  - complement.
- Give formal proofs of the following fundamental properties of union and intersection of two or three sets.
  - Commutative property of union,
  - Commutative property of intersection,
  - Associative property of union,
  - Associative property of intersection,
  - Distributive property of union over intersection,
  - Distributive property of intersection over union,
  - De Morgan's laws.
- Verify the fundamental properties for given sets.
- Use Venn diagram to represent
  - union and intersection of sets,
  - complement of a set.
- Use Venn diagram to verify
  - commutative law for union and intersection of sets,
  - De Morgan's Law,
  - associative laws,
  - distributive laws.
- Recognize ordered pairs and Cartesian product.
- Define binary relation and identify its domain and range.
- Define function and identify its domain, co-domain and range.
- Demonstrate the following:
  - into function,
  - one-one function,
  - into and one-one function (injective function),



  - onto function (surjective function),
  - one-one and onto functin (bijective function).
- Examine whether a given relation is a function or not.
- Differentiate between one-one correspondence and one-one function.
- Include sufficient exercise to clarify/differentiate between the above concepts.

Why it's important

Perhaps the single most important concept in mathematics is that of a function. At the heart of the function concept is the idea of a correspondence between two sets of objects. Correspondences between two sets of objects occur frequently in every day life. For example to each item in supermarket there corresponds a price (item $\rightarrow$ price).

5.1 Some important sets

N = Set of natural numbers $= \{1, 2, 3, \ldots\}$

W = Set of whole numbers $= \{0, 1, 2, 3, \ldots\}$

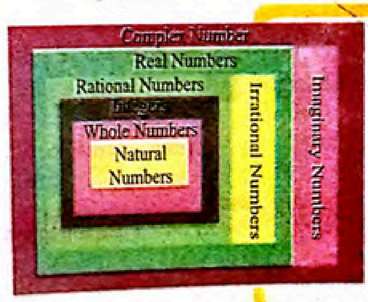
Z = Set of integers $= \{0, \pm 1, \pm 2, \ldots\}$

E = Set of even integers $= \{0, \pm 2, \pm 4, \ldots\}$

O = Set of odd integers $= \{\pm 1, \pm 3, \pm 5, \ldots\}$

P = Set of prime numbers $= \{2, 3, 5, 7, 11, \ldots\}$

Q = Set of rational numbers $= \left\{x \mid x = \dfrac{p}{q}, q \neq 0 \wedge p, q \in Z\right\}$



5.1.1 Operation on sets

(a) Union of two sets

If A and B two sets then union of set A and set B consists of all elements in set A or in set B or in both A and B, and it is denoted by A∪B. In set builder form,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Example 1 If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$

then $A \cup B = \{1, 2, 3\} \cup \{3, 4, 5, 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

(b) Intersection of two sets

If A and B are two sets then intersection of set A and set B consists of all those elements which are common to both A and B and it is denoted by A∩B symbolically.

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

(c) Disjoint sets

If A and B are disjoint sets then

$$A \cap B = \varphi$$

Example 2 Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$

$C = \{5, 11, 12\}$, $D = \{8, 9, 10\}$

Then

$A \cap B = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} = \{3, 4, 5\}$

and $B \cap C = \{3, 4, 5, 6, 7\} \cap \{5, 11, 12\} = \{5\}$

$A \cap D = \{1, 2, 3, 4, 5\} \cap \{8, 9, 10\} = \{\ \}$ or φ

Clearly here A and D are disjoint sets.

(d) Difference of two sets

If A and B are two sets then their difference consists of all those elements of A which are not in B and it is denoted by A\B or A−B. In set builder form

$$A \setminus B = \{x \mid x \in A \wedge x \notin B\}$$

Example 3

If $A = \{5, 6, 7, 8\}$, $B = \{7, 8, 9, 10\}$

Then $A \setminus B = \{5, 6, 7, 8\} \setminus \{7, 8, 9, 10\}$

$= \{5, 6\}$

and

$B \setminus A = \{7, 8, 9, 10\} \setminus \{5, 6, 7, 8\}$

$= \{9, 10\}$

(e) Complement of a set

If U is a universal set and A is subset of U then U \ A is called complement of the set A and is denoted by A' or A^c i.e. $A' = U \setminus A$

For your information

George Boole (1815-1864) introduce a symbolic approach to the study of logic.

This allowed him to clarify difficult logical problems in symbolic forms based on sets.

The algebra of sets having union and intersection as its basic operations is known as Boolean Algebra.

Today Boolean Algebra is used widely as a tool to aid sound reasoning.

Example 4 If $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{3, 4, 5\}$ and $B = \phi$, then find

(i) A' (ii) B'

Solution

$A' = U \setminus A = \{1, 2, 3, 4, 5, 6\} \setminus \{3, 4, 5\}$

$= \{1, 2, 6\}$

$B' = U \setminus B = \{1, 2, 3, 4, 5, 6\} \setminus \varphi$

$= \{1, 2, 3, 4, 5, 6\}$

Exercise 5.1

1. If $A = \{1, 2, 3\}$, $B = \{0, 1\}$ and $C = \{1, 3, 4\}$ then find

   (i). $A \cup B$ (ii). $A \cap B$ (iii). $A \cup C$
   (iv). $A \cap C$ (v). $B \cup C$ (vi). $A \cap A$

2. Find A \ B and B \ A when

   (i). $A = \{1, 3, 5, 7\}$, $B = \{3, 4, 5, 6, 7, 8\}$
   (ii). $A = \{0, \pm 1, \pm 2, \pm 3\}$, $B = \{-1, -2, -3\}$
   (iii). $A = \{1, 2, 3, 4, \ldots\}$, $B = \{1, 3, 5, 7, \ldots\}$

3. If $U = \{1, 2, 3, \ldots 20\}$, $A = \{2, 4, 6, \ldots 20\}$

   $B = \{1, 3, 5, \ldots 19\}$ and $C = \varphi$ then find

   (i). A' (ii). B' (iii). C' (iv). $A' \cup B'$
   (v). $A' \cap B'$ (vi). $A' \cap B$ (vii). $A' \cup C'$ (viii). $A \cap C'$
   (ix). $C' \cap C$ (x). $B' \cup C'$

4. If U = set of natural numbers up to 15

   and A = set of even numbers up to 15

   and B = set of odd numbers up to 15

   then find.

   (i). $A' \cup B'$ (ii). $A' \cap B'$ (iii). U' (iv). φ'
   (v). $B \cap A'$ (vi). $B \cup B'$ (vii). $A \cap A'$ (viii). $A \cup B'$

5.1.2 Properties of union and intersection

(a) Commutative property of union

If A and B are any two sets then

$$A \cup B = B \cup A$$

Proof Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

Hence $A \cup B \subseteq B \cup A$ (i)

(b) Commutative properly of intersection

Conversely

Let $x \in B \cup A$

$\Rightarrow$ $x \in B$ or $x \in A$

$\Rightarrow$ $x \in B$ or $x \in A$

$\Rightarrow$ $x \in A$ or $x \in B$

$\Rightarrow$ $x \in A \cup B$

Hence $B \cup A \subseteq A \cup B$ (ii)

From(i) and (ii), we have

$A \cup B = B \cup A$

(c) Associative property of union

If A, B and C are any three sets then

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Proof

Let $x \in A \cup (B \cup C)$

$\Rightarrow$ $x \in A$ or $x \in B \cup C$

$\Rightarrow$ $x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow$ $x \in A \cup B$ or $x \in C$

$\Rightarrow$ $x \in (A \cup B) \cup C$

Hence $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ (i)

Conversely,

Let $x \in (A \cup B) \cup C$

$\Rightarrow$ $x \in A \cup B$ or $x \in C$

$\Rightarrow$ $x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow$ $x \in A$ or $x \in B \cup C$

$\Rightarrow$ $x \in A \cup (B \cup C)$

Hence $(A \cup B) \cup C \subseteq A \cup (B \cup C)$ (ii)

From (i) and (ii), it follows that

$$A \cup (B \cup C) = (A \cup B) \cup C$$

(d) Associative property of intersection

If A, B and C are any three sets then

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Proof

Let $x \in A \cap (B \cap C)$

$\Rightarrow$ $x \in A$ and $x \in B \cap C$

$\Rightarrow$ $x \in A$ and $x \in B$ and $x \in C$

$\Rightarrow$ $x \in A \cap B$ and $x \in C$

$\Rightarrow$ $x \in (A \cap B) \cap C$

Hence $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ (i)

Conversely,

Let $x \in (A \cap B) \cap C$

$\Rightarrow$ $x \in A \cap B$ and $x \in C$

$\Rightarrow$ $x \in A$ and $x \in B$ and $x \in C$

$\Rightarrow$ $x \in A$ and $x \in B \cap C$

$\Rightarrow$ $x \in A \cap (B \cap C)$

Hence $(A \cap B) \cap C \subseteq A \cap (B \cap C)$ (ii)

From (i) and (ii), it follows

$$A \cap (B \cap C) = (A \cap B) \cap C$$

(e) Distributive property of union over intersection

If A, B and C are any three sets then

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Proof

Let $x \in A \cup (B \cap C)$

$\Rightarrow$ $x \in A$ or $x \in B \cap C$

$\Rightarrow$ $x \in A$ or $x \in B$ and $x \in C$

$\Rightarrow$ $x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow$ $x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow$ $x \in (A \cup B) \cap (A \cup C)$

Hence $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ (i)

Conversely,

Let $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow$ $x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow$ $x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow$ $x \in A$ or $x \in B$ and $x \in C$

$\Rightarrow$ $x \in A$ or $x \in B \cap C$

$\Rightarrow$ $x \in A \cup (B \cap C)$

Hence $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ (ii)

From (i) and (ii), it follows

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(f) Distributive property of intersection over union

If A, B and C are any three sets then

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Proof

Let $x \in A \cap (B \cup C)$

$\Rightarrow$ $x \in A$ and $x \in B \cup C$

$\Rightarrow$ $x \in A$ and $x \in B$ or $x \in C$

$\Rightarrow \; x \in A \text{ and } x \in B \text{ or } x \in A \text{ and } x \in C$

$\Rightarrow \; x \in A \cap B \text{ or } x \in A \cap C$

$\Rightarrow \; x \in (A \cap B) \cup (A \cap C)$ (i)

Hence $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Conversely,

Let $x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow \; x \in A \cap B \text{ or } x \in A \cap C$

$\Rightarrow \; x \in A \text{ and } x \in B \text{ or } x \in A \text{ and } x \in C$

$\Rightarrow \; x \in A \text{ and } x \in B \text{ or } x \in C$

$\Rightarrow \; x \in A \text{ and } x \in B \cup C$

$\Rightarrow \; x \in A \cap (B \cup C)$ (ii)

Hence $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

From (i) and (ii),

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

De Morgan's Laws

(g) If U is universal set and A and B are any two subsets of U then;

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Proof

(i) Let $x \in (A \cup B)'$

$\Rightarrow \; x \notin A \cup B$

$\Rightarrow \; x \notin A \text{ and } x \notin B$

$\Rightarrow \; x \in A' \text{ and } x \in B'$

$\Rightarrow \; x \in A' \cap B'$ (i)

Hence $(A \cup B)' \subseteq A' \cap B'$

Conversely,

Let $x \in A' \cap B'$

$\Rightarrow \; x \in A' \text{ and } x \in B'$

$\Rightarrow \; x \notin A \text{ and } x \notin B$

$\Rightarrow \; x \notin A \cup B$

$\Rightarrow \; x \in (A \cup B)'$ (ii)

Hence $A' \cap B' \subseteq (A \cup B)'$

From (i) and (ii), it follows

$$(A \cup B)' = A' \cap B'$$

Proof (ii) Let $x \in (A \cap B)'$

$\Rightarrow \; x \notin A \cap B$

$\Rightarrow \; x \notin A \text{ and } x \notin B$

$\Rightarrow \; x \in A' \text{ or } x \in B'$

$\Rightarrow \; x \in A' \cup B'$

Hence $(A \cap B)' \subseteq A' \cup B'$

Conversely (i)

Let $x \in A' \cup B'$

$\Rightarrow \; x \in A' \text{ or } x \in B'$

$\Rightarrow \; x \notin A \text{ and } x \notin B$

$\Rightarrow \; x \notin A \cap B$

$\Rightarrow \; x \in (A \cap B)'$

Hence $A' \cup B' \subseteq (A \cap B)'$ (ii)

From (i) and (ii), it follows $(A \cap B)' = A' \cup B'$

5.1.3 Verification of fundamental properties of union and intersection

(a) Commutative property of union

For any two sets A and B, $A \cup B = B \cup A$

Example 5

Solution If $A = \{1,2,3\}$

$B = \{4,5,6\}$, then

$A \cup B = \{1,2,3\} \cup \{4,5,6\} = \{1,2,3,4,5,6\}$

and $B \cup A = \{4,5,6\} \cup \{1,2,3\} = \{1,2,3,4,5,6\}$

Hence $A \cup B = B \cup A$

(b) Commutative property of intersection

For any two sets A and B, $A \cap B = B \cap A$

Example 6

Solution If $A = \{a, b, c\}$

$B = \{b, c, d, e\}$, then

$A \cap B = \{a, b, c\} \cap \{b, c, d, e\}$

$= \{b, c\}$

and $B \cap A = \{b, c, d, e\} \cap \{a, b, c\}$

$= \{b, c\}$

Hence $A \cap B = B \cap A$

(c) Associative property of union

For any three sets A, B and C, $A\cup(B\cup C)=(A\cup B)\cup C$

Example 7

If $A=\{3,4,5\}$

$B=\{5,6,7\}$

and $C=\{8,9,10\}$, then prove that $A\cup(B\cup C)=(A\cup B)\cup C$

Solution

$$A\cup(B\cup C)=\{3,4,5\}\cup[\{5,6,7\}\cup\{8,9,10\}]$$
$$=\{3,4,5\}\cup\{5,6,7,8,9,10\}$$
$$=\{3,4,5,6,7,8,9,10\}$$

and $$(A\cup B)\cup C=[\{3,4,5\}\cup\{5,6,7\}]\cup\{8,9,10\}$$
$$=\{3,4,5,6,7\}\cup\{8,9,10\}$$
$$=\{3,4,5,6,7,8,9,10\}$$

Hence,

$$A\cup(B\cup C)=(A\cup B)\cup C$$

(d) Associative property of intersection

For any three sets A, B and C, $A\cap(B\cap C)=(A\cap B)\cap C$

Example 8

If $A=\{1,2,3\}$

$B=\{2,3,4\}$

and $C=\{3,4,5\}$, then prove that $A\cap(B\cap C)=(A\cap B)\cap C$

Solution

$$A\cap(B\cap C)=\{1,2,3\}\cap[\{2,3,4\}\cap\{3,4,5\}]$$
$$=\{1,2,3\}\cap\{3,4\}$$
$$=\{3\}$$

and $$(A\cap B)\cap C=[\{1,2,3\}\cap\{2,3,4\}]\cap\{3,4,5\}$$
$$=\{2,3\}\cap\{3,4,5\}$$
$$=\{3\}$$

Hence,

$$A\cap(B\cap C)=(A\cap B)\cap C.$$

(e) Distributive property of union over intersection

For any three sets A, B and C, $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Example 9 Let $A=\{1,2,3,4\}$

$B=\{5,6,7\}$

and $C=\{7,8,9\}$,

then prove that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

Solution

$$A\cup(B\cap C)=\{1,2,3,4\}\cup[\{5,6,7\}\cap\{7,8,9\}]$$
$$=\{1,2,3,4\}\cup\{7\}$$
$$=\{1,2,3,4,7\}$$

and $$(A\cup B)\cap(A\cup C)=[\{1,2,3,4\}\cup\{5,6,7\}]\cap[\{1,2,3,4\}\cup\{7,8,9\}]$$
$$=\{1,2,3,4,5,6,7\}\cap\{1,2,3,4,7,8,9\}$$
$$=\{1,2,3,4,7\}$$

Hence,

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C).$$

(f) Distributive property of intersection over union

For any three sets A, B and C, $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Example 10 Let $A=\{a,b,c\}$

$B=\{c,d,e\}$

and $C=\{e,f,g\}$,

then prove that $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$.

Solution

$$A\cap(B\cup C)=\{a,b,c\}\cap[\{c,d,e\}\cup\{e,f,g\}]$$
$$=\{a,b,c\}\cap\{c,d,e,f,g\}$$
$$=\{c\}$$

and $$(A\cap B)\cup(A\cap C)=[\{a,b,c\}\cap\{c,d,e\}]\cup[\{a,b,c\}\cap\{e,f,g\}]$$
$$=\{c\}\cup\{\ \}$$
$$=\{c\}$$

Hence,

$$A\cap(B\cup C)=(A\cap B)\cup(A\cap C).$$

(g) De. Morgan's laws

For any two sets A and B which are subsets of U

(i) $(A\cup B)' = A'\cap B'$ and (ii) $(A\cap B)' = A'\cup B'$

Example 11

Solution If $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 3\}$

$B = \{3, 4, 5\}$

(i) $A\cup B = \{2, 3\}\cup\{3, 4, 5\}$

$= \{2, 3, 4, 5\}$

$(A\cup B)' = U\setminus(A\cup B)$

$(A\cup B)' = \{1, 2, 3, 4, 5, 6\}\setminus\{2, 3, 4, 5\}$

$(A\cup B)' = \{1, 6\}$ (i)

also $A' = U - A = \{1, 2, 3, 4, 5, 6\}\setminus\{2, 3\}$

$= \{1, 4, 5, 6\}$

and $B' = U - B = \{1, 2, 3, 4, 5, 6\}\setminus\{3, 4, 5\}$

$= \{1, 2, 6\}$

$\therefore$ $A'\cap B' = \{1, 4, 5, 6\}\cap\{1, 2, 6\}$

$A'\cap B' = \{1, 6\}$ (ii)

From (i) and (ii) it follows;

$(A\cup B)' = A'\cap B'$

(ii) Now

$A\cap B = \{2, 3\}\cap\{3, 4, 5\}$

$= \{3\}$

$(A\cap B)' = U - (A\cap B)$

$(A\cap B)' = \{1, 2, 3, 4, 5, 6\}\setminus\{3\}$

$= \{1, 2, 4, 5, 6\}$ (i)

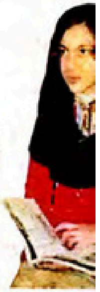
also $A'\cup B' = \{1, 4, 5, 6\}\cup\{1, 2, 6\}$

$= \{1, 2, 4, 5, 6\}$ (ii)

From (i) and (ii), it follows; $(A\cup B)' = A'\cap B'$

Exercise 5.2

1. Verify commutative property of union and intersection for the following sets.
   - (i). $A = \{1, 2, 3, ...12\}$, $B = \{2, 4, 5, 8, 10, 12\}$
   - (ii). $A = N$, $B = \{x \mid x \in N \wedge x$ is an even integer$\}$
   - (iii). A = Set of first ten prime numbers.
     B = Set of first ten composite numbers.
2. Verify associative properties of union and intersection for the following sets.
   - (i). $A = \{a, b, c, ...z\}$
     $B = \{a, e, i, o, u\}$
     $C = \{a, d, i, l, m, n, o\}$
   - (ii). $A = \{1, 2, ... 100\}$
     $B = \{2, 4, 6, ... 100\}$
     $C = \{1, 3, 5, ... 99\}$
3. Verify distributive properties of union over intersection and intersection over u
   - (i). $A = \{0, 1, 2\}$
     $B = \{0\}$
     $C = \varphi$
   - (ii). $A = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$
     $B = \{-1, -2, -3, -4, -5\}$
     $C = \{-1, -2, +3, +4\}$



4. Verify De. Morgan's laws for the following sets.
   - (i). $U = \{x \mid x \in N \wedge 1 \le x \le 20\}$
     $A = \{2, 3, 5, 7, 11, 12, 13, 17\}$
     $B = \{1, 4, 6, 8, 10, 14, 17, 18\}$
   - (ii). If $U = \{1, 2, 3, ... 10\}$
     $A = \{2, 4, 6, 8, 10\}$
     $B = \{1, 3, 5, 7, 9\}$

5.1.4 Venn diagrams

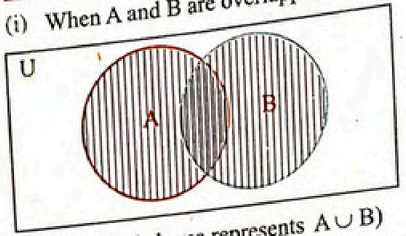
A Venn diagram is a visual way to show the relationships among or between sets th something in common. Usually, the Venn diagram consists of two or more ove circles, with each circle representing a set of elements, or members and univers represented by a rectangle. If two circles overlap, the members in the overlap belon sets; if three circles overlap, the members in the overlap belong to all three sets.

Example 12 Draw Venn diagrams for $A \cup B$, $A \cap B$ A/B and B/A when;

(i) A and B are overlapping sets. (ii) A and B are disjoint sets. (iii) $A \subseteq B$.

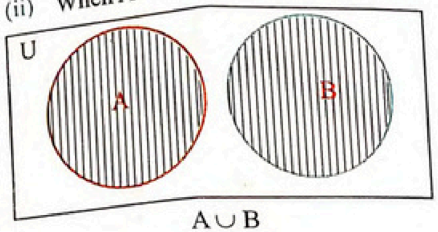
Solution Venn diagrams of union.

(i) When A and B are overlapping sets.



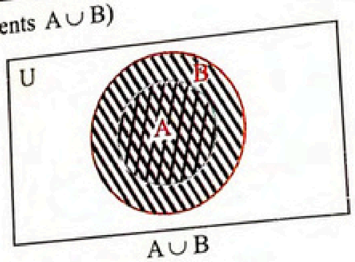
$A \cup B$ (Shaded area represents $A \cup B$)

(ii) When A and B are disjoint sets.



$A \cup B$

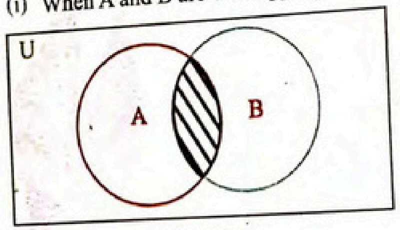
(iii) When $A \subseteq B$



$A \cup B$

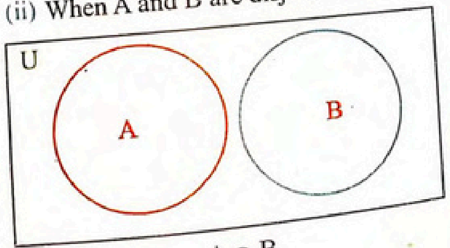
(a). **Venn diagrams of intersection**

(i) When A and B are overlapping sets.



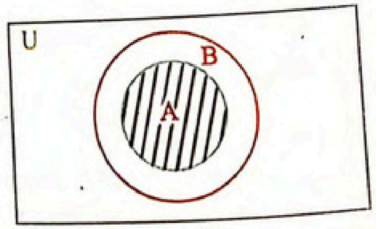
$A \cap B$

(ii) When A and B are disjoint sets.



$A \cap B$

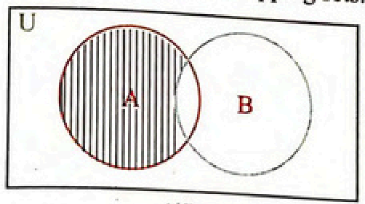
(iii) When $A \subseteq B$



$A \cap B$

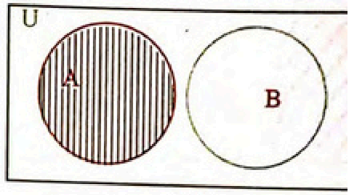
(b). **Venn diagrams of A\B**

When A and B are overlapping sets.



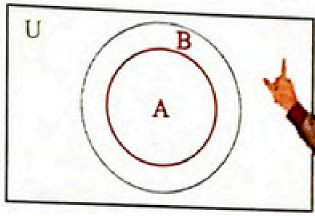
A\B

When A and B are disjoints sets.



A\B

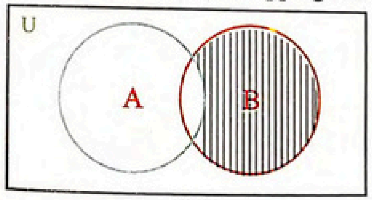
When $A \subseteq B$.



A\B

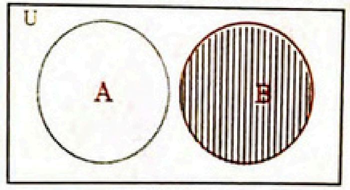
(c). **Venn diagrams of B\A**

When A and B are overlapping sets.



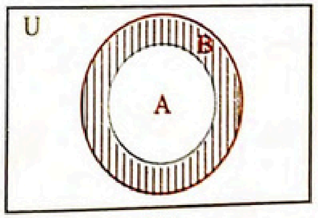
B\A

When A and B are disjoints sets.



B\A

When $A \subseteq B$.

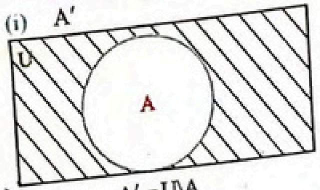


B\A

(d). Venn diagrams of complement sets A', B', $(A \cup B)'$ and $(A \cap B)'$

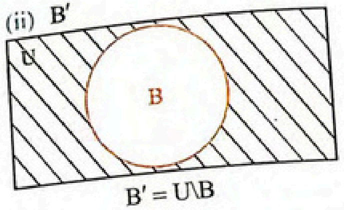
When A and B are overlapping sets

(i) A'



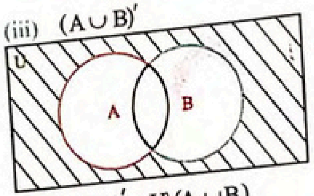
$A' = U \backslash A$

(ii) B'



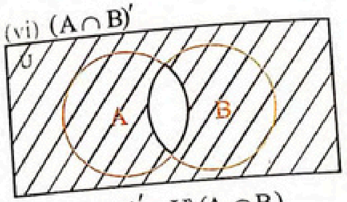
$B' = U \backslash B$

(iii) $(A \cup B)'$



$(A \cup B)' = U \backslash (A \cup B)$

(vi) $(A \cap B)'$



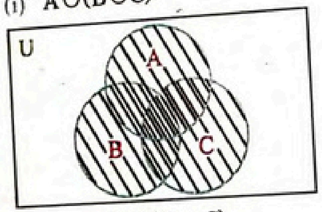
$(A \cap B)' = U \backslash (A \cap B)$

(e). Venn diagrams of union and intersection of three sets

Example 13 If A, B and C are any three sets then draw Venn diagrams of $A \cup (B \cup C)$, $(A \cap B) \cap C$, $A \cup (B \cap C)$ and $A \cap (B \cup C)$

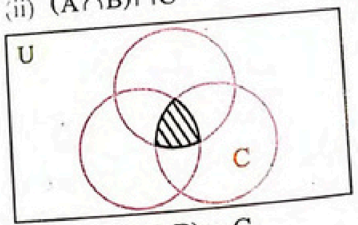
Solution

(i) $A \cup (B \cup C)$



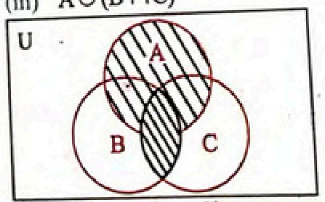
$A \cup (B \cup C)$

(ii) $(A \cap B) \cap C$



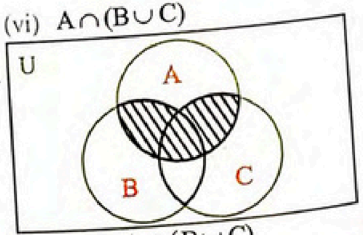
$(A \cap B) \cap C$

(iii) $A \cup (B \cap C)$



$A \cup (B \cap C)$

(vi) $A \cap (B \cup C)$



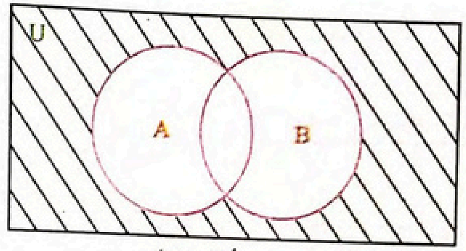
$A \cap (B \cup C)$

(f). Venn diagrams of De. Morgan's Laws

If A and B are any two sets then

(i) $(A \cup B)' = A' \cap B'$, (ii) $(A \cap B)' = A' \cup B'$

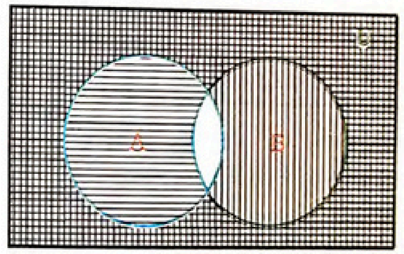
Venn diagrams of $(A \cup B)'$



$(A \cup B)'$ (Fig. 1)

Where U is universal set and A and B are overlapping sets of U. Here $A \cup B$ is white part and $(A \cup B)'$ is shaded part.

Venn diagrams of $A' \cap B'$



$A' \cap B'$ (Fig. 2)

In Fig. 2,

$A' = U \backslash A$ is represent by vertical lines.

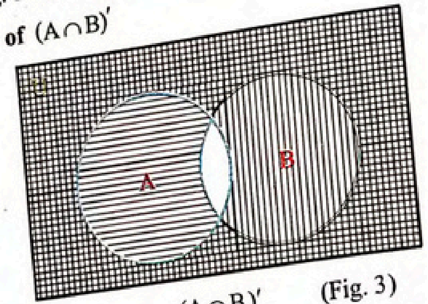
$B' = U \backslash B$ is represent by horizontal lines.

Therefore, $A' \cap B'$ is represented by lines i.e. horizontal and vertical lines cross each other as shown in figure.

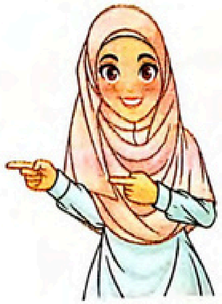
$(A \cup B)' = A' \cap B'$

Hence from Fig. 1 and Fig. 2, we see that

Venn diagram of $(A \cap B)'$

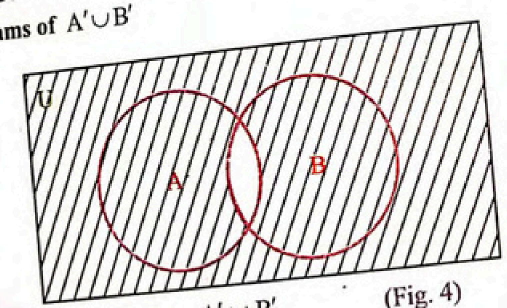


$(A \cap B)'$ (Fig. 3)



In this figure shaded area represents $(A \cap B)'$

Venn diagrams of $A' \cup B'$



$A' \cup B'$ (Fig. 4)

In Fig. 4,

A' is represented by vertical lines

B' is represented by horizontal lines

While $A' \cup B'$ is represented by either lines.

Hence From fig 3 and fig 4, we see that

$$(A \cap B)' = A' \cup B'$$

Verification of fundamental properties with the help of Venn diagrams

Example 14 If $A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6\}$
$C = \{3, 4, 7, 8\}$

Then verify the following with the help of Venn diagrams.

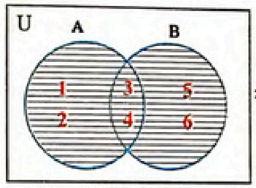
(i) $A \cup B = B \cup A$
(ii) $A \cap B = B \cap A$
(iii) $A \cup (B \cup C) = (A \cup B) \cup C$
(iv) $A \cap (B \cap C) = (A \cap B) \cap C$
(v) $A \cup (B \cap C) = (A \cap B) \cap (A \cup C)$
(vi) $A \cap (B \cup C) = (A \cap B) \cap (A \cap C)$

Solution

(i) $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

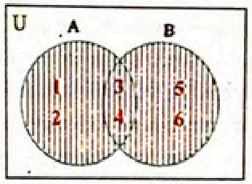
and $B \cup A = \{3, 4, 5, 6\} \cup \{1, 2, 3, 4\}$
$= \{1, 2, 3, 4, 5, 6\}$

Venn diagrams are



$A \cup B$ (Fig. a)

=



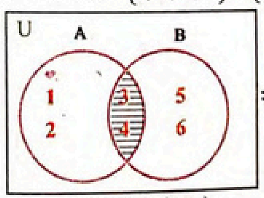
$B \cup A$ (Fig. b)

It is clear from Fig. (*a*) and Fig. (b), that both regions representing $A \cup B$ and $B \cup A$ are are identical therefore

$$A \cup B = B \cup A$$

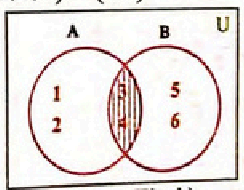
(ii) $A \cap B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} = \{3, 4\}$

and $B \cap A = \{3, 4, 5, 6\} \cap \{1, 2, 3, 4\} = \{3, 4\}$



$A \cap B$ (Fig. a)

=



$B \cap A$ (Fig. b)

It is clear from Fig. (*a*) and Fig. (b) that both regions representing $A \cap B$ and $B \cap A$ are are identical. Therefore,

$$A \cap B = B \cap A$$

(iii) $A\cup(B\cup C)=\{1,2,3,4\}\cup[\{3,4,5,6\}\cup\{3,4,7,8\}]$

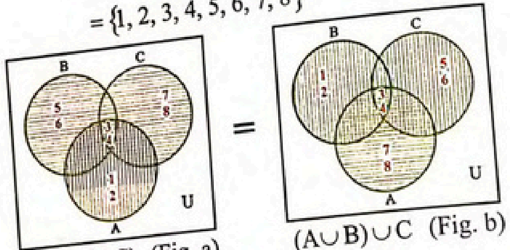
$=\{1,2,3,4\}\cup\{3,4,5,6,7,8\}$

$=\{1,2,3,4,5,6,7,8\}$

and $(A\cup B)\cup C=[\{1,2,3,4\}\cup\{3,4,5,6\}]\cup\{3,4,7,8\}$

$=\{1,2,3,4,5,6\}\cup\{3,4,7,8\}$

$=\{1,2,3,4,5,6,7,8\}$



$A\cup(B\cup C)$ (Fig. a) $\qquad$ $(A\cup B)\cup C$ (Fig. b)

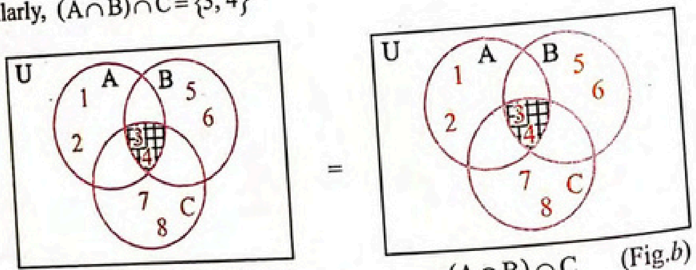
It Fig. (*a*) A is represented by vertical lines and $B\cup C$ represented by horizontal lines. The region which is lined either in one or both ways represented $A\cup(B\cup C)$. In Fig. (b) $A\cup B$ is represented by vertical lines and C is represented by horizontal lines. The region which is lined either in one or both ways represents $A\cup(B\cup C)$. It is clear from Fig. (a) and Fig. (b) both the regions representing $A\cup(B\cup C)$ and $(A\cup B)\cup C$ are identical. Therefore,

$A\cup(B\cup C)=(A\cup B)\cup C$

(iv) $A\cap(B\cap C)=\{1,2,3,4\}\cap[\{3,4,5,6\}\cap\{3,4,7,8\}]$

$=\{1,2,3,4\}\cap\{3,4\}=\{3,4\}$

Similarly, $(A\cap B)\cap C=\{3,4\}$



$A\cap(B\cap C)$ (Fig.*a*) $\qquad$ $(A\cap B)\cap C$ (Fig.*b*)

It is clear from fig.(*a*) and fig.(b) that both regions representing $A\cap(B\cap C)$ and $(A\cap B)\cap C$ are identical.

Therefore, $A\cap(B\cap C)=(A\cap B)\cap C$.

(v) $A\cup(B\cap C)=\{1,2,3,4\}\cup[\{3,4,5,6\}\cap\{3,4,7,8\}]$

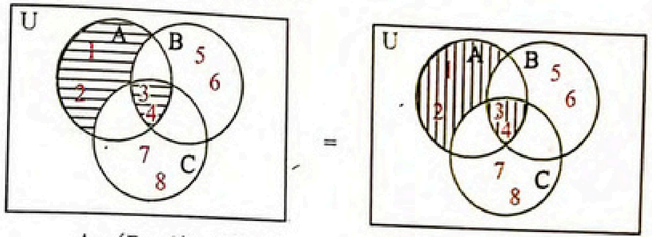
$=\{1,2,3,4\}\cup\{3,4\}$

$=\{1,2,3,4\}$

$(A\cup B)\cap(A\cup C)=[\{1,2,3,4\}\cup\{3,4,5,6\}][\{1,2,3,4\}\cup\{3,4,7,8\}]$

$=\{1,2,3,4,5,6\}\cap\{1,2,3,4,7,8\}$

$=\{1,2,3,4\}$



$A\cup(B\cap C)$ (Fig.*a*) $\qquad$ $(A\cup B)\cap(A\cup C)$ (Fig.*b*)

It is clear from Fig. (a) and Fig. (b) that both regions representing $A\cup(B\cap C)$ an $(A\cup B)\cap(A\cup C)$ are identical.

(vi) $A\cap(B\cup C)=\{1,2,3,4\}\cap[\{3,4,5,6\}\cup\{3,4,7,8\}]$

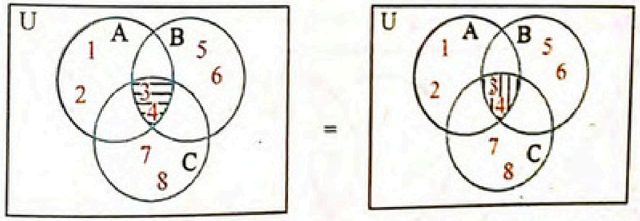
$=\{1,2,3,4\}\cap\{3,4,5,6,7,8\}$

$=\{3,4\}$

$(A\cap B)\cup(A\cap C)=[\{1,2,3,4\}\cap\{3,4,5,6\}]\cup[\{1,2,3,4\}\cap\{3,4,7,8\}]$

$=\{3,4\}\cup\{3,4\}$

$=\{3,4\}$



$A\cap(B\cup C)$ (Fig.*a*) $\qquad$ $(A\cap B)\cup(A\cap C)$ (Fig.*b*)

Example 15 If $U = \{1, 2, 3, 4, 5, 6, 7\}$
$A = \{2, 5, 6\}$ and $B = \{1, 2, 3\}$

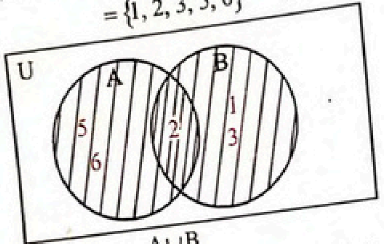
then draw Venn diagrams for

(i) $A \cup B$ (ii) $A \cap B$ (iii) A' (iv) B' (v) $(A \cup B)'$
(vi) $A' \cap B'$ (vii) $(A \cap B)'$ (viii) $A' \cup B'$

Hence verify De Morgan's laws.

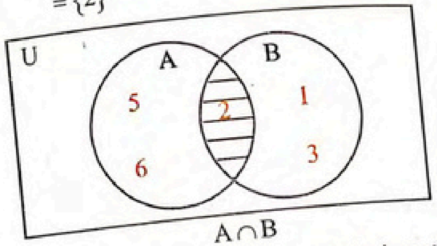
Solution

(i) $A \cup B = \{2, 5, 6\} \cup \{1, 2, 3\}$
$= \{1, 2, 3, 5, 6\}$



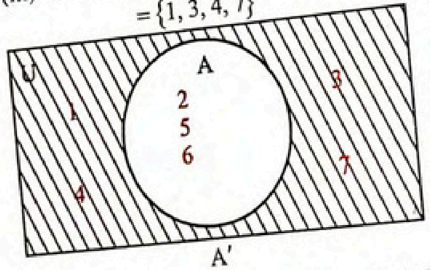
$A \cup B$

(ii) $A \cap B = \{2, 5, 6\} \cap \{1, 2, 3\}$
$= \{2\}$



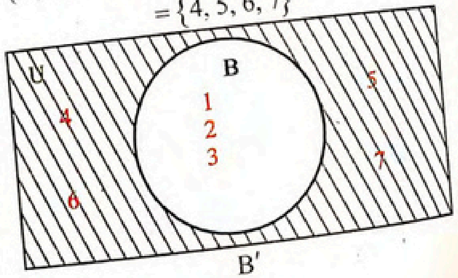
$A \cap B$

(iii) $A' = U \setminus A = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2, 5, 6\}$
$= \{1, 3, 4, 7\}$



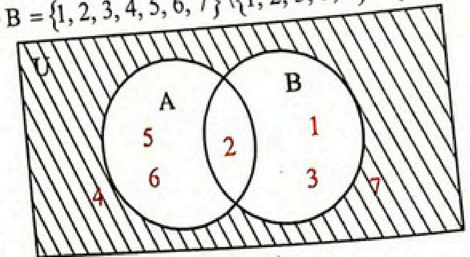
A'

(iv) $B' = U \setminus B = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3\}$
$= \{4, 5, 6, 7\}$



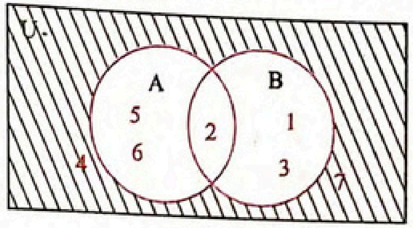
B'

(v) $(A \cup B)' = U \setminus A \cup B = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{1, 2, 3, 5, 6\} = \{4, 7\}$



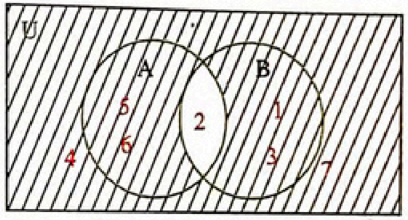
$(A \cup B)'$

(vi) $A' \cap B' = \{1, 2, 3, 4, 7\} \cap \{4, 5, 6, 7\} = \{4, 7\}$



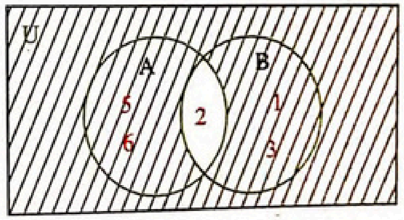
$A' \cap B'$

(vii) $(A \cap B)' = U \setminus A \cap B = \{1, 2, 3, 4, 5, 6, 7\} \setminus \{2\} = \{1, 3, 4, 5, 6, 7\}$



$(A \cap B)'$

(viii) $A' \cup B' = \{1, 3, 4, 7\} \cup \{4, 5, 6, 7\} = \{1, 3, 4, 5, 6, 7\}$



$A' \cup B'$

From (v) and (vi), it is proved that

$$(A \cup B)' = A' \cap B'$$

and from (vi) and (viii), it is proved that

$$(A \cap B)' = A' \cup B'$$

Exercise 5.3

1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 6, 7\}$, then draw Venn diagrams for the following.
   (i). $A \cup B$ (ii). $A \cap B$
2. If $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7, 8\}$ and $C = \{5, 6, 9, 10\}$, then verify with the help of Venn diagrams.
   (i). $A\cup(B\cup C)=(A\cup B)\cup C$ (ii). $A\cap(B\cap C)=(A\cap B)\cap C$
   (iii). $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ (iv). $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$
3. If $U = \{1, 2, 3, 4, 5, 6, 7,\}$, $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$
   Draw Venn diagrams for A', B', $A'\cup B'$, $A'\cap B'$ and also verify that
   (i). $(A\cup B)' = A'\cap B'$ (ii) $(A\cap B)' = A'\cup B'$
4. If $U = \{a, b, c, 1, 2, 3, 4\}$, $A = \{c, 3\}$ and $B = \{a, 3, 4\}$ then draw Venn diagrams for A', B', $A\backslash B$ and $B\backslash A$.
5. If $U = \{a, b, c, d, e, f, g\}$, $A = \{a, b, c\}$ and $B = \{c, d, e\}$ then verify De Morgan's laws with the help of Venn diagrams.

5.1.5 Ordered pairs and Cartesian product

Ordered Pairs

An ordered pair (a, b) consists of two elements "a" and "b" in which "a" is the first element and "b" is the second element.
The ordered pairs (a, b) and (c, d) are equal if, and only if, a = c and b = d.
Note that (a, b) and (b, a) are not equal unless a = b.

Example 16 Find x and y given $(2x, x + y) = (6, 2)$

Solution Two ordered pairs are equal if and only if the corresponding components are equal. Hence, we obtain the equations:

$$2x = 6 \quad\ldots\ldots\ldots\ldots\ldots\text{(i)}$$
$$\text{and } x + y = 2 \quad\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Solving equation (i) we get $x = 3$ and when substituted in (ii) we get $y = -1$.

Cartesian Product Of Two Sets

The Cartesian product of A and B, denoted $A \times B$ (read "A cross B") is the set of all ordered pairs (a, b), where a is in A and b is in B.

Symbolically: $A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$

Remarks:

(i) $A \times B \neq B \times A$ for non-empty and unequal sets A and B (ii) $A \times \phi = \phi \times A = \phi$

5.2 Binary Relation:

Let A and B be sets. A (binary) relation R from A to B is a subset of $A \times B$.
When $(a, b) \in R$, we say a is related to b by R, written a R b.
Otherwise if $(a, b) \notin R$, we write $a \not R b$.

Example 17 If $A = \{a, b\}$ and $B = \{1, 2\}$ then
$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

Solution Number of elements in $A \times B = 2 \times 2 = 4$
Number of all possible subsets of
$A \times B = 2^4 = 16$

Since every subset of $A \times B$ is a binary relation from A to B. Therefore, here we have 16 binary relations which are given below.

$R_1 = \varphi$	$R_2 = \{(a, 1)\}$
$R_3 = \{(a, 2)\}$	$R_4 = \{(b, 1)\}$
$R_5 = \{(b, 2)\}$	$R_6 = \{(a, 1), (a, 2)\}$
$R_7 = \{(a, 1), (b, 1)\}$	$R_8 = \{(a, 1)(b, 2)\}$
$R_9 = \{(a, 2)(b, 1)\}$	$R_{10} = \{(a, 2), (b, 2)\}$
$R_{11} = \{(b, 1), (b, 2)\}$	$R_{12} = \{(a, 1), (a, 2), (b, 1)\}$
$R_{13} = \{(a, 1), (b, 1), (b, 2)\}$	$R_{14} = \{(a, 1), (a, 2), (b, 2)\}$
$R_{15} = \{(a, 2), (b, 1), (b, 2)\}$	$R_{16} = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

Similarly, total number of binary relation in $B \times A = 2^4 = 16$

Example 18 Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$
Then $A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$
Write any five relations from A to B.

Solution

Let

$R_1 = \{(1,1), (1, 3), (2, 2)\}$
$R_2 = \{(1, 2), (2, 1), (2, 2), (2, 3)\}$
$R_3 = \{(1, 1)\}$
$R_4 = A \times B$
$R_5 = \varnothing$

All being subsets of $A \times B$ are relations from A to B.

Real Life Application

Relationships between elements of sets occur in many contexts. Every day we deal with relationships such as those between a business and its telephone number, an employee and his or her salary, a person and a relative, and so on. In mathematics we study relationships such as those between a positive integer and one that it divides, a real number and one that is larger than it, a real number x and the value $f(x)$ where f is a function, and so on.

Note

If set A has m elements and set B has n elements then A B has m n elements.

Domain Of A Relation

The domain of a relation R from A to B is the set of all first elements of the ordered pairs which belong to R denoted Dom(R).

Symbolically: Dom (R) = $\{a \in A | (a,b) \in R\}$

Range Of A Relation

The range of a relation R from A to B is the set of all second elements of the ordered pairs which belong to R denoted Ran(R).

Symbolically: Range(R) = $\{b \in B | (a,b) \in R\}$

Example 19 Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$.
Define a binary relation R from A to B as follows:
$R = \{(a, b) \in A \times B \mid a < b\}$

Then
(i) Find the ordered pairs in R.
(ii) Find the Domain and Range of R.
(iii) Is 1R3, 2R2?

Solution Given $A = \{1, 2\}$, $B = \{1, 2, 3\}$.
$A \times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$

(i) $R = \{(a, b) \in A \times B \mid a < b\}$
$R = \{(1,2), (1,3), (2,3)\}$

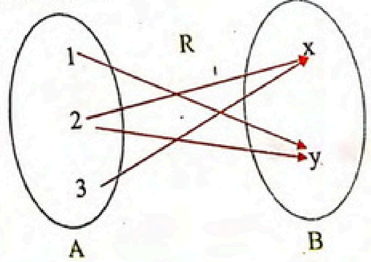
(ii) $Dom(R) = \{1,2\}$ and $Range(R) = \{2, 3\}$

(iii) Since $(1,3) \in R$ so 1R3
But $(2, 2) \notin R$ so 2 is not related with 3.

Arrow Diagram Of A Relation

Let
$A = \{1, 2, 3\}$, $B = \{x, y\}$ and
$R = \{(1,y), (2,x), (2,y), (3,x)\}$
be a relation from A to B.

The arrow diagram of R is:



Exercise 5.4

1. If $A = \{1, 2, 3\}$, $B = \{4, 5\}$ then
   (i). Write three binary relations from A to B.
   (ii). Write four binary relations from B to A.
   (iii). Write four binary relations on A.
   (iv). Write two binary relations on B.
2. If $A = \{1, 2, 3, 4\}$, $B = \{1, 3, 5\}$ and $R = \{(x, y) \mid y < x\}$ is a binary relation from A to B, then write it in tabular form.
3. Domain of a binary relation $R = \{(x, y) \mid y = 2x\}$ is the set $\{0, 4, 8\}$, find Range of R.
4. Domain of a binary relation $R = \{(x, y) \mid y + 1 = 2x^2\}$ is set N. Find its range.

5.3 Functions

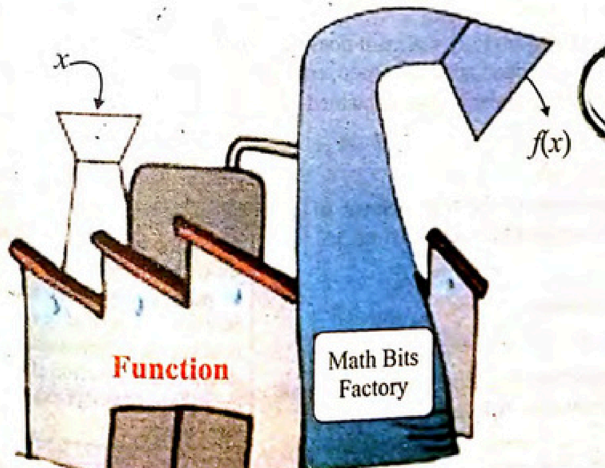
Let A and B are two non-empty sets, then a binary relation f is said to be a function from A to B if

- Dom $f = A$
- There should be no repetition in the first element of all ordered pairs contained in f.

Symbolically, we write it as

$$f: A \longrightarrow B$$

and say f is function from A to B.



Example 20

If $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$ then which of the following are functions?

$f_1 = \{(1, a), (2, b)\}$

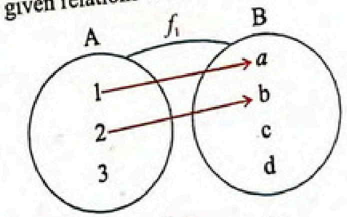
$f_2 = \{(1, a), (2, b), (3, c), (3, d)\}$

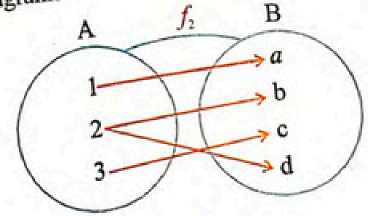
$f_3 = \{(1, a), (2, b), (3, c)\}$

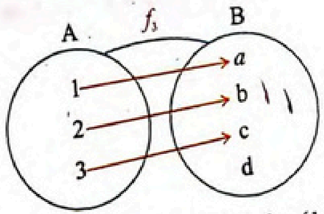
$f_4 = \{(1, a), (2, a), (3, d)\}$

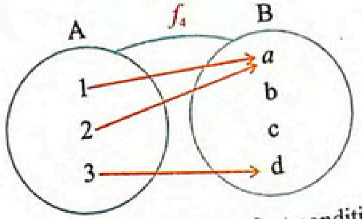
Solution

The given relations are shown in the following diagrams.









f_1 is not a function because Dom $f_1 = \{1, 2\} \neq A$ i.e. it does not satisfy the first condition of being a function. f_2 is also not a function because second condition of being a function is not satisfied i.e. first element $2 \in A$ of ordered pairs contained in f_2 is repeated. f_3 is a function because

(i) Dom $f_3 = \{1, 2, 3\} = A$

(ii) There is no repetition in the first element of all ordered pairs contained in f_3. It can be written as

$$f_3 : A \longrightarrow B$$

f_4 is a function because

(i) Dom $f_4 = \{1, 2, 3\} = A$

(ii) There is no repetition in the first element of all ordered pairs contained in f_4. It can be written as

$$f_4 : A \longrightarrow B$$

Note

A relation is any pairing of set of inputs with a set of outputs. Every function is relation, but not every relation is a function. A relation is a function if for every input there is exactly one output.

5.3.1 Domain, Co-domain and Range of a function

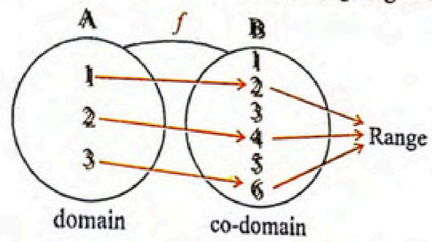
Let $f : A \longrightarrow B$ be a function, then the set A is called domain of "f" (or set of first elements of the ordered pairs contained in f).

The set B is co-domain of f and the set of second elements of all ordered pairs contained in f is called range of the function. Range is always a subset of co-domain. i.e. Range $f \subseteq B$.

Example:

Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

$f : A \longrightarrow B$ as shown in the following diagram.



In this diagram, the set A is domain, the set B is co-domain and that set $\{2, 4, 6\}$ which is a subset of B is range of the function f.

5.3.2 Kinds of a function

(a) Into function

Let f be a function from A to B, then f is into function if Range $f \neq B$

It can be written as

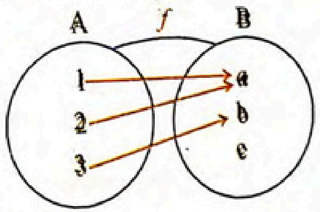
$$f : A \xrightarrow{\text{into}} B$$

Example:

Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ then a function f from A to B is defined by

$f = \{(1, a), (2, a), (3, b)\}$

The following figure illustrates the function f.



We see that f is into function because

Range $f = \{a, b\} \neq B$

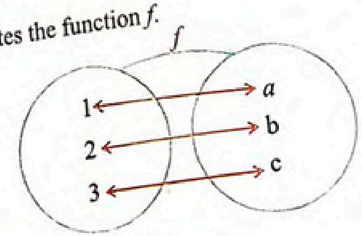
(b) One-one function

Let f be a function from A to B, then f is one-one function, if there is no repetition in the second element of all ordered pairs contained in f. This means that element of the domain set A is associated with one and only one element of the set B. It can be written as:

$$f: A \xrightarrow{\text{one-one}} B$$

Example:

If A = {1, 2, 3} and B = {a, b, c} then
f = {(1, a), (2, b), (3, c)}
The following figure illustrates the function f.



From figure it is clear that f is one – one function.

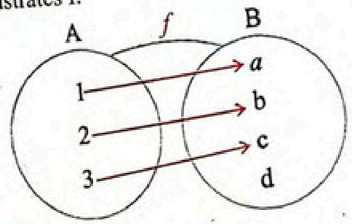
(c) Into and one-one function (Injective function)

Let f be a function from A to B, then f is into one-one or injective function, if
(i) Range $f \neq$ B
(ii) There is no repetition in the second element of all ordered pairs contained in f.
It can be written as

$$f: A \xrightarrow{\text{injective}} B$$

Example:

If A = {1, 2, 3} and B = {a, b, d} and f is a function from A to B defined by,
f= {(1, a), (2, b), (3, c)}
The following figure illustrates f.



Here we see that f is injective function because
(i) Range f= {a, b, c} $\neq$ B
(ii) There is no repetition in the second element of all ordered pairs contained in f.

(d) Onto function (surjective function)

Let f be a function from A to B, then f is onto function if
Range f =B
It can be written as

$$f: A \xrightarrow{\text{onto}} B$$

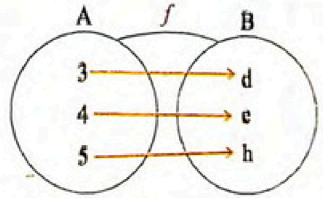
(e) One-one and onto function (bijective function)

Let f be a function from A to B, then f is one-one and onto (bijective) function if it is both one-one and onto.

i.e. $f: A \xrightarrow{\text{bijective}} B$

Example:

Let A = {3, 4, 5} and B = {d, e, h} then a function f from A to B is define by
f= {(3, d), (4, e), (5, h)}
The following figure illustrates f.



Here we see that f is onto function
Range f= {d, e, h} = B

f is one – one function because for each $x \in A$ there is unique $y \in B$.
For $3 \in A$ there is unique $d \in B$
For $4 \in A$ there is unique $e \in B$
For $5 \in A$ there is unique $h \in B$
Hence f is bijective function.

Difference between one – one correspondence and one – one function

If A and B are two non-empty sets then a rule for which each element of A is paired with one and only one element of B and each element of B is paired with one and only one element of A is called one-one correspondence.

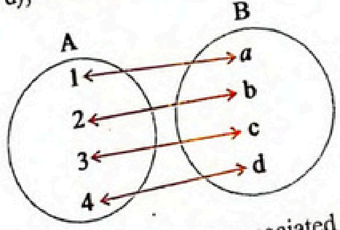
Remember

If the two sets are finite and if there exists a one-one correspondence between them, they have the same number of elements.

The difference is clear from the following two examples.

Example

Let A = {1, 2, 3, 4} and B = {a, b, c, d} then a one – one correspondence is given by {(1, a), (2, b), (3, c), (4, d)} as shown in the following figure.

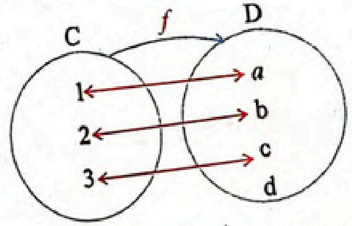


In one – one function every element of the set A is associated with one and only one element of set B. This means that Range f may not be equal to set B.

Example

If C = {1, 2, 3} and D = {a, b, c, d}

then f = {(1, a), (2, b), (3, c)} is one – one function, as shown in the following figure.



From figure, it is clear that there does not exist one – one correspondence between sets C and D because $d \in D$ is unpaired.

WARNING

(a) Note that $f(a+3) \neq f(a) + f(3)$. Function notation is not to be confused with the distributive law.

(b) Note that $f(x^2) \neq f \cdot x^2$. The use of parentheses in function notation does not imply multiplication.

Exercise 5.5

1. A = {1, 2, 3, 4} and B = {6, 7} and the following are the relations from A to B, then state whether these are functions or not?

   If these are functions then state which kind of functions are these?

   R_1 = {(1, 6), (2, 7), (3, 6)}

   R_2 = {(1, 6), (2, 6), (3, 7), (4, 7)}

   R_3 = {(1, 6), (2, 6), (3, 6), (4, 6)}

2. Which of the following relations on set {a, b, c, d} are functions? State the kind of functions as well

   (i). {(a, b), (c, d), (b, d), (d, b)}
   (ii). {(b, a), (c, b), (a, b), (d, d)}
   (iii). {(d, c), (c, b), (a, b), (d, d)}
   (iv). {(a, b), (b, c), (c, b), (d, a)}

3. If A = {0, 1, 2, 3} and B = {x, y, z, p} then state whether following relations show that there exist one – one correspondence between the elements of set A and B. If not, give reasons.

   (i). {(0, x), (2, z), (3, y), (1, p)}
   (ii). {(0, x), (1, z), (2, y), (3, z)}

4. If A = {a, b, c} and B = {2, 3, 4, 5} then state, whether the following relations show that there exists one – one correspondence between the elements of sets A and B, if not what kind of the relations they are?

   (i). {(a, 2), (b, 3), (c, 4)}
   (ii). {(a, 3), (b, 4), (c, 3)}

5. If X = {1, 2, 3, 4} and Y = {5, 6, 7, 8} then write

   (i). a function from X to Y.

   (ii). a one- one function from X to Y.

   (iii). a relation which shows that there exist one – one correspondence between X and Y.

   (iv). a function which is onto from Y to X.

   (v). bijective function from Y to X.

   (vi). a function from X to Y which is neither one- one nor onto.

6. Let $A = \{1,2,3,4,5\}$. Check whether the following sets are functions on A. In case these are functions, indicate their ranges. Which function is onto.

   (i). {(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)}
   (ii). {(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)}
   (iii). {(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)}

Review Exercise 5

1. At the end of each question, four circles are given. Fill in the correct circle only.

(i). If A = {1, 2, 3}, B = {4, 5} and R = {(1, 4), (2, 5), (3, 4)} then R is ____________

○ A one-one function from A to B ○ A function from A to B

○ Not a function ○ An onto function from A to B

(ii). If A has two elements and B has three elements, then number of binary relations in A × B is ____________.

○ 2×3 ○ 2^3 ○ 2^6 ○ 2^2

(iii). Which one of the following is an example of disjoint sets?

○ {0,1,2,3} and {3,2,1,0} ○ {0,2,4,6} and {2,4,6,8}

○ {0,3,6,9} and {9,16,25,36} ○ {0,4,8,12} and {6,10,14,18}

(iv). If the universal set $U = \{x|x$ is a positive odd integer less than 30}, R={1, 5, 7}, and S={1, 3, 7, 11, 13}, how many elements are in $(R \cap S)'$?

○ 15 ○ 13 ○ 7 ○ 2

(v). If f is a function from A to B, then f is onto function if

○ Range f = B ○ Range $f \neq$ A ○ Dom f = A

○ second element of all ordered pairs contained in f is not repeated.

(vi). If R = {(0,0), (8, 2), (10, 3), (14, 12)}, then Dom R = ____________

○ {0,8,10,14} ○ {0,2,3,12} ○ {8,10,4} ○ {0,10}

2. If U = { Natural numbers up to 100}
A = {positive even numbers up to 100}
B = {positive odd numbers up to 100}

Then find.

(i). $A' \cup B'$ (ii). $A' \cap B'$ (iii). $A \cap B'$ (iv). $A' \cap B$

3. If A = {1, 2, 3, 5, 7}, B = {2, 4, 6}, C = {2, 5, 9} then verify.

(i). Associative property of union
(ii). Associative property of intersection
(iii). Distributive property of union over intersection.
(iv). Distributive property of union over union.

4. If $U = \{x \mid x \in N \wedge 1 \leq x \leq 40\}$
A = {1, 6, 11, 16, 21, 26, 31}
B = {2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32}
then verify De. Morgan's laws.

5. If U = {1, 2, 3, 5, 6, 7}
A = {2, 5, 6}
B = {1, 2, 3}

Then verify De-Morgan's laws with the help of Venn diagrams.

6. If U = {1, 2, … 10}
A = {1, 2, 3, 4}
B = {3, 4, 5, 6}
C = {3, 4, 7, 8}

Then verify distributive laws with the help of Venn diagrams.

7. Let A = {−2, −1, 0, 1, 2} B = {a, b, c, d, e}. Determine which sets of ordered pair represent a function. In case of a function, mention one-one function, onto function and bijective function.

(i). {(−2, a), (−1, a), (0, b), (1, c), (2, d)} (ii). {(−1, a), (1, e), (−2, d), (0, c), (2, b)}

(iii). {(2, d), (0, a), (−2, b), (−1, c), (1, e)} (iv). {(−2, b), (−1, b), (0, a), (1, d), (−2, e)}

8. Let A = {1, 2, 3, 4, 5}, check whether the following sets are functions on A. In case these are functions, indicate their ranges. Which function are onto.

(i). {(1, 5), (2, 3), (3, 3), (4, 2), (5, 1)} (ii). {(1, 1), (2, 4), (3, 2), (4, 1), (5, 3)}

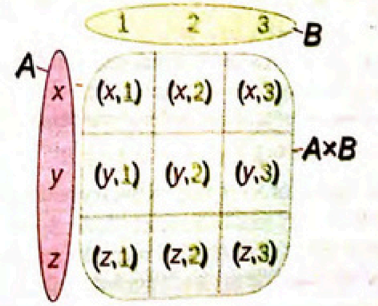
(iii). {(1, 2), (2, 1), (3, 1), (4, 4), (5, 5)} (iv). {(1, 2), (2, 3), (1, 4), (3, 5)}

9. If X = {−6, −5, −4, −3}
Y = {1, 2, 3, 4} then write

(i). A one – one function from X to Y. (ii). Onto function from X to Y.
(iii). A function which is one – one and onto from X to Y.
(iv). A function from X to Y which is neither one – one nor onto.

PROJECT

(−2, 5), (−1, 0), (0, 1), and (2, 5) is a function. If the x-values and y-values of this function are switched, the relation that results is not a function. Provide an example of a function that will still be a function if the x-values and y-values are switched.



Summary

- A set is a "collection of well-defined distinct objects, sets are represented by capital English alphabets, A, B, C, ... , Z and elements of sets are represented by small English alphabets, a, b, c, ... z.
- If A and B are two sets, then **union** of set A and set B consists of all elements in set A or in set B or in both A and B, and it is denoted by $A \cup B$. Symbolically.

  $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- If A and B are two sets, then **intersection** of set A and set B consists of all those elements which are common to both A and B, and it is denoted by $A \cap B$, Symbolically

  $A \cap B = \{x \mid x \in A \text{ or } x \in B\}$
- Two sets A and B are disjoint if $A \cap B = \varphi$.
- If U is universal set and A is subset of U, then U \ A is called **complement** of set A and is denoted by A' or A^c.
- If A and B are two sets, then their **difference** consists of all those elements of A which are not in B, and it is denoted by A \ B or A – B symbolically,

  $A \setminus B = \{x \mid x \in A \text{ or } x \notin B\}$
- For any two sets A and B,
  - $A \cup B = B \cup A$
  - $A \cup (B \cup C) = (A \cup B) \cup C$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  - $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
  - $A \cap B = B \cap A$
  - $A \cap (B \cap C) = (A \cap B) \cap C$
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- Concept of sets i.e. union, intersection, complement, difference of sets, can be explained easily with the help of **Venn diagrams**. In these diagrams, a set is usually represented by a circle and universal set is represented by a rectangle.
- (a, b) is called **ordered pair** of two elements a and b of a set or of different sets, where a is the first element and b is the second element.

  $(a, b) \neq (b, a)$
- If A and B are two non- empty sets then A × B is called **Cartesian product**, which is set of all ordered pairs such that the first element of each ordered pair belongs to set A and second element of each ordered pair belongs to set B symbolically.
  - $A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$
  - If $A \neq B$, then $A \times B \neq B \times A$
- Any subset of A × B is called **binary relation**, where A and B are two non-empty sets. If R is binary relation from set A to set B, i.e. $R = \{(x, y) \mid x \in A \wedge y \in B\}$, then domain of R is set of first elements of all ordered pairs in R and is denoted by "Dom R". Range of R is set of second elements of all ordered pairs in R, and is denoted by "Range R".

- If A and B are two non-empty sets, then a binary relation f is said to be a function from A to B, if
  - Dom f = A
  - There should be no repetition in the first elements of all ordered pairs contained in f. Symbolically $f : A \longrightarrow B$.
- Let $f : A \longrightarrow B$ be a function, then set A is called **domain** of f, set B is called **co-domain** of f and the set of second elements of all ordered pairs contained in f is called **range** of f. The range is subset of co-domain i.e. Range $f \subseteq B$.
- Let f be a function from A to B, then f is **into function**, if Range $f \neq B$.
- Let f be a function from A to B, then f is **one-one function**, if for each $x \in A$ there exist unique $y \in B$, i.e. there is no repetition in the second element of all ordered pairs contained in f.
- Let f be a function from A to B, then f is **injective** (into and non- one) function if
  (i). Range $f \neq B$
  (ii). There is no repetition in the second elements of all ordered pairs contained in f.
- Let f be a function from A to B, then f is **onto function** if Range f = B.
- Let f be function from A to B, then f is **one-one and onto (bijective) function** if it is both one-one and onto.
- If A and B are two non-empty sets then **one-one correspondence** between A and B is a rule for which each element of set A is paired with one and only one element of B and each element of B is paired with one and only one element of A, and none of the members of any set remains unpaired. It is also known as **one-to-one function**. In one-one correspondence both sets A and B have same number of elements.