6. If a varies jointly as b and the square root of c, and a = 21 when b = 5 and c = 36, find a when b = 12 and c = 225.
7. What number should be added to each of the numbers 3, 8, 11 and 20 to make them in proportion?
8. What number must be subtracted from each of 6, 8, 7 and 11 so that the remaining numbers are in proportion?
9. The ratio between two numbers is 8 : 3 and their difference is 20. Find the numbers.
10. Find three numbers in continued proportion such that their sum is 14 and sum of their squares is 84.
11. The mean proportional between two numbers is 6 and their sum is 13. Find the numbers.
12. Find the angles of a triangle which are in the ratio 3 : 4 : 5.
13. If $\frac{a}{b}=\frac{c}{d}$, then prove that $\frac{ac(a+c)}{bd(b+d)}=\frac{(a+c)^3}{(b+d)^3}$
14. If a, b, c are in continued proportion then prove that $\frac{a}{c}=\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a^2-b^2}{b^2-c^2}$
15. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then prove that $\frac{a^3+c^3+e^3}{b^3+d^3+f^3}=\frac{ace}{bdf}$

Summary

- A rate is a special type of ratio.
- A rate is a comparison of two quantities of different kinds.
- A ratio compares two quantities of the same type.
- A proportion is a statement expressing the equivalence of two rates or two ratios.
- Two quantities are in direct proportion when one quantity is doubled, the other quantity is also doubled; when one quantity increases x times, the other quantity also increases x times.
- Two quantities are in inverse proportion when one quantity is doubled, the other quantity is halved; when one quantity increases y times, the other quantity becomes 1/y of the original.
- If y varies directly to x, then y = kx, where k is a constant and k = 0 is direct variation.
- If y varies inversely to x, then xy = k, where k is a constant and k = 0 is inverse variation.

  If y varies directly to two or more quantities x and z, then y = kxz, where k is a constant and k = 0 is joint variation.
- Let: a : b : c : d be a proportion then [redacted] = [redacted] = k (say)

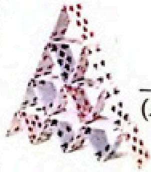
  Thus a = bk, c = dk

  These equation are used to evaluate certain expressions more easily. This method is called K-method.

Unit 4 PARTIAL FRACTIONS

In this unit the students will be able to

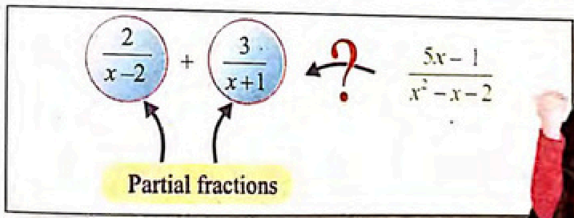
- Define proper, improper and rational fraction.
- Resolve an algebraic fraction into partial fractions when its denominator consists of
  - non – repeated linear factors,
  - repeated linear factors,
  - non-repeated quadratic factors,
  - repeated quadratic factors.



$\frac{4x-1}{(x+2)(x-1)}$

Why it's important

The partial fractions is simpler, which can help to solve more complicated fraction. For example, it is very useful in calculus, which is a branch of mathematics.



We can do this directly: $\frac{2}{x+1}+\frac{3}{x-2}\rightarrow\frac{5x-1}{x^2-x-2}$

Like this:

$$\frac{2}{x+1}+\frac{3}{x-2}=\frac{2(x-2)+3(x+1)}{(x+1)(x-2)}=\frac{2x-4+3x+3}{(x+1)(x-2)}=\frac{5x-1}{x^2-x-2}$$

... but how do we go in the opposite direction?

$$\frac{2}{x+1}+\frac{3}{x-2}\leftarrow\frac{5x-1}{x^2-x-2}$$

How to find the "parts" that make the single fraction (This is **"partial fractions"**).

Partial fractions

A procedure which does splitting up a fraction into two or more fractions with only one factor in the denominator is called partial fraction. In other words a set of fractions whose algebraic sum is a given fraction is called partial fraction.

4.1 Proper and improper rational fractions

(a) Rational fraction

A rational function can be written in the form:

$$f(x)=\frac{P(x)}{Q(x)}$$

Wherer P(x) and Q(x) are polynomials where $Q(x)\neq 0$

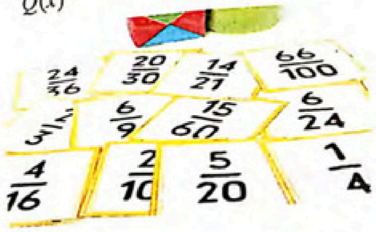
(b) Proper rational fraction

A rational fraction $\frac{P(x)}{Q(x)}$, $Q(x)\neq 0$ is proper fraction, if the degree of numerator $P(x)$ is less than the degree of denominator $Q(x)$.

For example,

$$\frac{1}{x+1},\ \frac{2x}{x^2+2},\ \frac{x^2+x-3}{x^3+x^2-x+1}$$

are proper rational fractions.



(c) Improper rational fractions

A rational fraction $\frac{P(x)}{Q(x)}$, $Q(x)\neq 0$ is as an improper fraction, if the degree of numerator $P(x)$ is greater than or equal to the degree of denominator $Q(x)$.

For example,

$$\frac{x^3+4}{(x+1)(x+2)},\ \frac{x}{2x+1},\ \frac{x^2+3x+2}{x^2+2x+3},\ \frac{x^3-x^2+x+1}{x^2+x-1}$$

Any improper rational fraction can be reduced into sum of polynomials and proper rational fraction by large division.

Tidbit

a fraction that has a larger number on the top than on the bottom

For example, consider $\frac{2x^2+1}{x-1}$, an improper rational fraction. Divide numerator by denominator as

$$\begin{array}{r|l} & 2x+2 \\ \hline x-1 & 2x^2+1 \\ & \underline{\cancel{2x^2}\ \mp 2x} \\ & 2\cancel{x}+1 \\ & \underline{-\cancel{2}x\mp 2} \\ & 3 \end{array}$$

Therefore we have $\frac{2x^2+1}{x-1}=2x+2+\frac{3}{x-1}$

4.2 Resolution of fraction into partial fractions

Resolution of rational fraction $\frac{P(x)}{Q(x)}$, where $Q(x)\neq 0$ into partial fractions depends upon the factors of denominator $Q(x)$. Thus there are four cases involved in resolution of fraction.

(i) When denominator $Q(x)$ consists of non-repeated linear factors.

(ii) When denominator $Q(x)$ consists of repeated linear factors

(iii) When denominator $Q(x)$ consists of non-repeated quadratic factors.

(iv) When denominator $Q(x)$ consists of repeated quadratic factors.

Case I When denominator consists of non-repeated linear factors

Let proper fraction $\frac{P(x)}{Q(x)}$ is given, factorize the polynomial Q(x) in the denominator if it is not already factorized. To every non-repeated linear factor $ax+b$ in the denominator corresponds a partial fraction of the form $\frac{A}{ax+b}$ where A is a constant. Thus if

$Q(x)=(a_1x+b_1)(a_2x+b_2)$ then

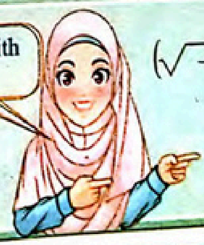
$$\frac{P(x)}{Q(x)}=\frac{A}{a_1x+b_1}+\frac{B}{a_2x+b_2}$$ where A and B are constants. Similarly if

$Q(x)=(a_1x+b_1)(a_2x+b_2)(a_3x+b_3)$ then

$$\frac{P(x)}{Q(x)}=\frac{A}{a_1x+b_1}+\frac{B}{a_2x+b_2}+\frac{C}{a_3x+b_3}$$

Where A,B and C are constants to be determined.

Did You Know?



$$(\sqrt{-1})(\sqrt{-1})=\sqrt{(-1)(-1)}$$
$$(\sqrt{-1})^2=\sqrt{1}$$
$$-1=1$$

Example 1 Resolve $\dfrac{1}{(x+1)(x+2)}$ into partial fractions.

Solution

Let $\quad \dfrac{1}{(x+1)(x+2)}=\dfrac{A}{x+1}+\dfrac{B}{x+2} \qquad$ (i)

$\Rightarrow \quad \dfrac{1}{(x+1)(x+2)}=\dfrac{A(x+2)+B(x+1)}{(x+1)(x+2)}$

Multiplying both sides by $(x+1)(x+2)$ we get

$1=A(x+2)+B(x+1) \qquad$ (ii)

Putting $\quad x=-1$ in Eq. (2), we get

$1=A(-1+2)+B(-1+1)$

$\Rightarrow \quad 1=A(1)+B(0)$

$1=A$

or $\quad \boxed{A=1}$

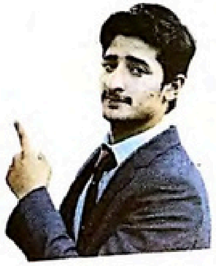
putting $x=-2$ in Eq. (ii), we get

$1=A(-2+2)+B(-2+1)$

$\Rightarrow \quad 1=A(0)+B(-1)$

$\Rightarrow \quad 1=0-B$

or $\quad \boxed{B=-1}$



Now putting these values of A and B in equation (i) we have

$$\frac{1}{(x+1)(x+2)}=\frac{1}{x+2}+\frac{-1}{x+2}$$

$$\Rightarrow \frac{1}{(x+1)(x+2)}=\frac{1}{x+1}-\frac{1}{x+2}$$

These are required partial fractions.

Math Fun

111,111,111 × 111,111,111

= 12,345,678,987,654,321.

Example 2 Find partial fractions of

$$\frac{3x+2}{x^2-x-2}$$

Solution

Since $\quad x^2-x-2=(x+1)(x-2)$

Therefore, $\quad \dfrac{3x+2}{x^2-x-2}=\dfrac{3x+2}{(x+1)(x-2)}$

Let $\quad \dfrac{3x+2}{(x+1)(x-2)}=\dfrac{A}{x+1}+\dfrac{B}{x-2} \qquad$ (i)

$\Rightarrow \quad \dfrac{3x+2}{(x+1)(x-2)}=\dfrac{A(x-2)+B(x+1)}{(x+1)(x-2)}$

Multiply both sides by $(x+1)(x-2)$ we have.

$3x+2=A(x-2)+B(x+1) \qquad$ (ii)

Putting $x=-1$ in Eq. (ii) we get.

$3(-1)+2=A(-1-2)+B(-1+1)$

$\Rightarrow \quad -3+2=A(-3)+B(0)$

$\Rightarrow \quad -1=-3A$

$\Rightarrow \quad \boxed{A=\dfrac{1}{3}}$

Putting $x=2$ in Eq. (ii), we get

$3(2)+2=A(2-2)+B(2+1)$

$\Rightarrow \quad 6+2=A(0)+B(3)$

$\Rightarrow \quad 8=-3B$

$\Rightarrow \quad \boxed{A=\dfrac{8}{3}}$

Putting these values of A and B in Eq. (i), we have

$$\frac{3x+2}{(x+1)(x-2)}=\frac{\frac{1}{3}}{x+1}+\frac{\frac{8}{3}}{x-2}$$

$$=\frac{1}{3(x+1)}+\frac{8}{3(x-2)}$$

or $\quad \dfrac{3x+2}{x^2-x-2}=\dfrac{1}{3(x+1)}+\dfrac{8}{3(x-2)} \quad$ These are required partial fractions.

ase II When denominator consists of repeated linear factors

et, in the proper fraction $\frac{P(x)}{Q(x)}$ the denominator $Q(x)$ contains a repeated linear factor $x+b)^2$, corresponds to two partial fractions of the form $\frac{P(x)}{Q(x)}=\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$, where A nd B are constant to be determined.

the denominator $Q(x)$ contains the repeated factor $(ax+b)^3$, then the corresponding three rtial fraction are $\frac{P(x)}{Q(x)}=\frac{A}{ax+b}+\frac{B}{(ax+b)^2}+\frac{C}{(ax+b)^3}$ where A,B and C are constants to be etermined.

Example 3 Find partial fractions of $\frac{x}{(x+1)^2}$

Solution

Let $\frac{x}{(x+1)^2}=\frac{A}{x+1}+\frac{B}{(x+1)^2}$ (i)

$\Rightarrow \frac{x}{(x+1)^2}=\frac{A(x+1)+B}{(x+1)^2}$

Multiplying both sides by $(x+1)^2$, we get

$x=A(x+1)+B$ (ii)

$\Rightarrow x=Ax+A+B$ (iii)

Putting $x=-1$ in Eq. (ii), we get

$-1=A(-1-1)+B$

$\Rightarrow -1=A(0)+B$

$\Rightarrow -1=B$

$\Rightarrow \boxed{B=-1}$

To find, the value of A compare the co-efficient of x in Eq. (iii), we get

$\boxed{1=A}$

putting these values of A and B in Eq. (i), we get

$$\frac{x}{(x+1)^2}=\frac{1}{x+1}+\frac{-1}{(x+1)^2}$$

$$=\frac{1}{x+1}-\frac{1}{(x+1)^2}$$

These are required partial fractions.

Example 4 Find partial fractions of $\frac{2x^2+1}{(x-2)^2(x+3)}$.

Solution

Let $\frac{2x^2+1}{(x-2)^2(x+3)}=\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{x+3}$ (i)

$\Rightarrow \frac{2x^2+1}{(x-2)^2(x+3)}=\frac{A(x-2)(x+3)+B(x+3)+C(x-2)^2}{(x-2)^2(x+3)}$

Multiplying both sides by $(x-2)^2(x+3)$, we get

$2x^2+1=A(x-2)(x+3)+B(x+3)+C(x-2)^2$ (ii)

$2x^2+1=A(x^2+x-6)+B(x+3)+C(x^2-4x+4)$

$2x^2+1=(A+C)x^2+(A+B-4C)x+(-6A+3B+4C)$ (iii)

Putting $x=2$ in Eq. (ii), we get

$2(2)^2+1=A(2-2)(2+3)+B(2+3)+C(2-2)^2$

$\Rightarrow 8+1=A(0)(5)+B(5)+C(0)^2$

$\Rightarrow 9=0+5B+0 \Rightarrow 9=5B \Rightarrow \boxed{B=\frac{9}{5}}$

Putting $x=-3$ in Eq. (ii) we get

$2(-3)^2+1=A(-3-2)(-3+3)+B(-3+3)+C(-3-2)^2$

$\Rightarrow 2\times 9+1=A(-5)(0)+B(0)+C(-5)^2$

$\Rightarrow 19=0+0+C(25) \Rightarrow \boxed{C=\frac{19}{25}}$

To find value of A, compare co-efficient of x^2 from Eq. (iii), we get

$2=A+C$

Putting value of C, we get

$2=A+\frac{19}{25} \Rightarrow 2-\frac{19}{25}=A \Rightarrow \boxed{A=\frac{31}{25}}$

Putting values of A, B and C in Eq. (i), we get

$$\frac{2x^2+1}{(x-2)^2(x+2)}=\frac{\frac{31}{25}}{x-2}+\frac{\frac{9}{5}}{(x-2)^2}+\frac{\frac{19}{25}}{x+3}$$

$$\frac{2x^2+1}{(x-2)^2(x+2)}=\frac{31}{25(x-2)}+\frac{9}{5(x-2)^2}+\frac{19}{25(x+3)}$$

These are required partial fractions.

Exercise 4.1

Resolve the following fractions into partial fractions.

(1). $\dfrac{3x-2}{2x^2-x}$ (2). $\dfrac{x-1}{x^2+6x+5}$ (3). $\dfrac{1}{x^2-1}$

(4). $\dfrac{x}{x^2+4x-5}$ (5). $\dfrac{4x+2}{(x+2)(2x-1)}$ (6). $\dfrac{x^2+5x+3}{(x^2-1)(x+1)}$

(7). $\dfrac{x^2+2}{(x+2)(x^2+5x+6)}$ (8). $\dfrac{2x-1}{x(x-3)^2}$ (9). $\dfrac{x^2}{x^2+2x+1}$

(10). $\dfrac{x^2}{(x-1)^2(x+1)}$

Case III When denominator consists of non-repeated quadratic factors

Let the proper fraction $\dfrac{P(x)}{Q(x)}$ is given then to every non-repeated quadratic factor ax^2+bx+c in Q(x) which is not factorizable corresponds the partial fraction of the form

$\dfrac{P(x)}{Q(x)}=\dfrac{Ax+B}{ax^2+bx+c}$ where A and B are constants to be determined.

Example 5 Find partial fractions of $\dfrac{1}{(x+1)(x^2+2)}$.

Solution Let $\dfrac{1}{(x+1)(x^2+2)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+2}$ (i)

$$\Rightarrow \quad \frac{1}{(x+1)(x^2+2)}=\frac{A(x^2+2)+(Bx+C)(x+1)}{(x+1)(x^2+2)}$$

Multiplying both sides by $(x+1)(x^2+2)$, we get

$$1=A(x^2+2)+(Bx+C)(x+1) \qquad \text{(ii)}$$
$$1=A(x^2+2)+Bx^2+Bx+Cx+C$$
$$1=(A+B)x^2+(B+C)x+2A+C \qquad \text{(iii)}$$

Putting $x=-1$ in Eq. (ii), we get

$$1=A\left[(-1)^2+2\right]+\left[B(-1)+C\right](-1+1)$$
$$\Rightarrow \quad 1=A(1+2)+(-B+C)(0)$$
$$\Rightarrow \quad 1=A(3)+0$$

$$1=3A \Rightarrow \boxed{A=\frac{1}{3}}$$

To find values of B and C comparing co-efficient of x^2 and x in Eq. (iii)

We get, $0=A+B$ (iv)

and $0=B+C$ (v)

using value of A in Eq. (iv), we get

$$0=\frac{1}{3}+B$$

$\Rightarrow \boxed{B=\dfrac{-1}{3}}$ putting $B=-\dfrac{1}{3}$ in Eq. (v), $-\dfrac{1}{3}+C=0\Rightarrow C=\dfrac{1}{3}$

Putting these values of A, B and C in Eq. (i), we get

$$\frac{1}{(x+1)(x^2+2)}=\frac{\frac{1}{3}}{x+1}+\frac{\frac{-1}{3}x+\frac{1}{3}}{x^2+2}$$
$$\frac{1}{(x+1)(x^2+2)}=\frac{1}{3(x+1)}+\frac{-x+1}{3(x^2+2)}$$
$$\frac{1}{(x+1)(x^2+2)}=\frac{1}{3(x+1)}+\frac{1-x}{3(x^2+2)}$$

These are required partial fractions.

Example 6 $\dfrac{4x^2-28}{x^4-x^2-6}=\dfrac{4x^2-28}{(x^2+3)(x^2-2)}=\dfrac{Ax+B}{x^2+3}+\dfrac{Cx+D}{x^2-2}$

Solution $4x^2-28=(Ax+B)(x^2-2)+(Cx+D)(x^2+3)$

$$=(Ax^3+Bx^2-2Ax-2B)+(Cx^3+Dx^2+3Cx+3D)$$
$$=(A+C)x^3+(B+D)x^2+(3C-2A)x-2B+3D$$

Equating coefficients of like powers of x,

$$A+C=0, B+D=4, 3C-2A=0, -2B+3D=-28$$

Solving simultaneously, $A=0, B=8, C=0, D=-4$.

Hence, $\dfrac{4x^2-28}{x^4-x^2-6}=\dfrac{4x^2-28}{x^4+x^2-6}=\dfrac{8}{x^2+3}-\dfrac{4}{x^2-2}$

Case IV When denominator consists of repeated quadratic factors

Example 7 Resolve $\dfrac{1}{(x-1)(x^2+1)^2}$ into partial fractions.

Solution Let $\dfrac{1}{(x-1)(x^2+1)^2}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+1}+\dfrac{Dx+E}{(x^2+1)^2}$ (i)

$$\Rightarrow \frac{1}{(x-1)(x^2+1)^2}=\frac{A(x^2+1)^2+(Bx+C)(x-1)(x^2+1)+(Dx+E)(x-1)}{(x-1)(x^2+1)^2}$$

Multiplying both sides by $(x-1)(x^2+1)^2$ we get

$$1=A(x^2+1)^2+(Bx+C)(x-1)(x^2+1)+(Dx+E)(x-1) \quad \text{(ii)}$$

$$1=A(x^4+2x^2+1)+(Bx+C)(x^3-x^2+x-1)+Dx^2-Dx+Ex-E$$

$$1=A(x^4+2x^2+1)+Bx^4-Bx^3+Bx^2-Bx+Cx^3-Cx^2+Cx-C+Dx^2-Dx+Ex-E$$

$$1=(A+B)x^4+(-B+C)x^3+(2A+B-C+D)x^2+(-B+C-D+E)x+(A-C-E)$$

Putting $x=1$ in Eq. (ii), we get

$$1=A\left[(1)^2+1\right]^2+\left[B(1)+C\right](1-1)\left[(1)^2+1\right]+\left[D(1)+E\right](1-1)$$

$$1=A[1+1]^2+(B+C)(0)(1+1)+(D+E)(0)$$

$$1=A(2)^2+0+0$$

$$1=A(4)$$

$$\Rightarrow \boxed{A=\frac{1}{4}}$$

Comparing co-efficients of x^4, x^3, x^2, x and constant respectively.

$0=A+B$ (iii)

$0=-B+C$ (iv)

$0=2A+B-C+D$ (v)

$0=-B+C-D+E$ (vi)

$1=A-C-E$ (vii)

Using value of A in (iii) we get

$$0=\frac{1}{4}+B$$

$$\Rightarrow \boxed{B=-\frac{1}{4}}$$

Using value of B in (4) we get

$$0=-\left(-\frac{1}{4}\right)+C$$

$$0=\frac{1}{4}+C$$

$$\Rightarrow \boxed{C=-\frac{1}{4}}$$

Using values of A, B and C in Eq. (v), we get

$$0=2\left(\frac{1}{4}\right)+\left(-\frac{1}{4}\right)-\left(-\frac{1}{4}\right)+D$$

$$0=\frac{1}{2}-\cancel{\frac{1}{4}}+\cancel{\frac{1}{4}}+D$$

$$0=\frac{1}{2}+D$$

$$\boxed{D=\frac{-1}{2}}$$

Using values of A and C in Eq. (vii), we get

$$1=\frac{1}{4}-\left(-\frac{1}{4}\right)-E$$

$$1=\frac{1}{4}+\frac{1}{4}-E$$

$$1=\frac{1}{2}-E$$

$$E=\frac{1}{2}-1=-\frac{1}{2}$$

$$\boxed{E=-\frac{1}{2}}$$

Putting values of A, B, C, D, E in Eq. (i), we get

$$\frac{1}{(x-1)(x^2+1)^2}=\frac{\frac{1}{4}}{x-1}+\frac{\frac{-1}{4}x-\frac{1}{4}}{x^2+1}+\frac{\frac{-1}{2}x-\frac{1}{2}}{(x^2+1)^2}$$

$$=\frac{1}{4(x-1)}+\frac{-x-1}{4(x^2+1)}+\frac{-x-1}{2(x^2+1)^2}$$

$$\frac{1}{(x-1)(x^2+1)^2}=\frac{1}{4(x-1)}-\frac{x+1}{4(x^2+1)}-\frac{x+1}{2(x^2+1)^2}$$

Exercise 4.2

Resolve the following fractions into partial fractions.

(1). $\dfrac{1}{x(x^2+1)}$ (2). $\dfrac{x^2+3x+1}{(x-1)(x^2+3)}$ (3). $\dfrac{2x+1}{(x^2+1)(x-1)}$

(4). $\dfrac{-3}{x^2(x^2+5)}$ (5). $\dfrac{3x-2}{(x+4)(3x^2+1)}$ (6). $\dfrac{5x}{(x+1)(x^2-2)^2}$

(7). $\dfrac{5x^2-4x+8}{(x^2+1)^2(x-2)}$ (8). $\dfrac{4x-5}{(x^2+4)^2}$ (9). $\dfrac{8x^2}{(x^2+1)(1-x^4)}$

(10) $\dfrac{2x^2+4}{(x^2+1)^2(x-1)}$

Review Exercise 4

1. At the end of each question, four circles are given. Fill in the correct circle only.

(i). $\dfrac{1}{x^2-1}=$

○ $\dfrac{1}{x+1}-\dfrac{1}{x-1}$ ○ $\dfrac{1}{2(x+1)}-\dfrac{1}{2(x-1)}$

○ $\dfrac{1}{2(x-1)}-\dfrac{1}{2(x+1)}$ ○ $\dfrac{2}{x-1}-\dfrac{1}{2(x+1)}$

(ii). If $P(x)$ and $Q(x)$ are two polynomials then $\dfrac{P(x)}{Q(x)}$, $Q(x)\neq 0$ is

○ Rational fraction ○ Irrational fraction

○ Proper fraction ○ Improper fraction

(iii). $\dfrac{x^2+2}{x^2+2x+2}$ is

○ Proper fraction. ○ Improper fraction

○ Irrational fraction ○ None of these

(iv). What is the quotient when $x^3-8x^2+16x-5$ is divided by $x-5$?

○ x^2-x+5 ○ x^2-3x+2

○ x^2-3x+1 ○ $x^2+13x-49+\dfrac{240}{(x+5)}$

2. Resolve the following fractions into partial fractions.

(i). $\dfrac{2x^2}{(x+1)(x-1)}$ (ii). $\dfrac{2x^3-3x^2+9x+8}{x^2-3x+2}$ (iii). $\dfrac{3x-1}{x^3-2x^2+x}$

(iv). $\dfrac{x+1}{(x-1)^2}$ (v). $\dfrac{2x^2}{x^4-4}$ (vi). $\dfrac{3x^2+3x+2}{x^4-1}$

(vii). $\dfrac{x^3+3x^2+1}{(x^2+1)^2}$ (viii). $\dfrac{2x^3-1}{x^3+x^2}$ (ix). $\dfrac{4x^2+3x+14}{x^3-8}$

Challenge !

3. Resolve the following fraction into partial fractions. $\dfrac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}$

Summary

- If $P(x)$ and $Q(x)$ are two polynomials and $Q(x)$ is non zero polynomial than the fraction $\dfrac{P(x)}{Q(x)}$ is called a **rational fraction**.
- A rational fraction, $Q(x)\neq 0$ is a **proper rational fraction**, if the degree of numerator $P(x)$ is less than the degree of denominator $Q(x)$.
- A rational fraction $\dfrac{P(x)}{Q(x)}$, $Q(x)\neq 0$ is a **improper rational fraction**. if the degree of numerator $P(x)$ is equal to or greater than the degree of denominator $Q(x)$.
- Splitting up a single ration al fraction into two or more rational fraction with single factor in denominator, such a procedure is called **partial fractions**.

Math Fun

1 x 1 = 1
11 x 11 = 121
111 x 111 = 12321
1111 x 1111 = 1234321
11111 x 11111 = 123454321
111111 x 111111 = 12345654321
1111111 x 1111111 = 1234567654321
11111111 x 11111111 = 123456787654321
111111111 x 111111111 = 12345678987654321