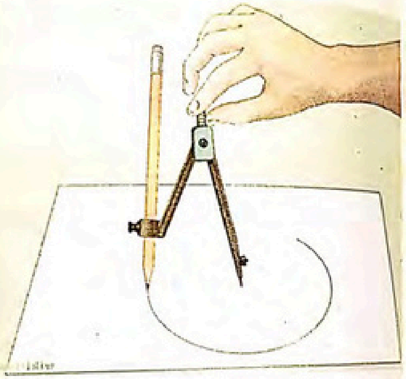
Unit 13

PRACTICAL GEOMETRY CIRCLE

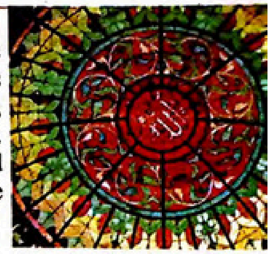
In this unit the students will be able to

- Locate the centre of a given circle.
- Draw a circle passing through three given non-collinear points.
- Complete the circle:
  - by finding the centre,
  - without finding the centre,

  When a part of its circumference is given.
- Circumscribe a circle about a given triangle.
- Inscribe a circle in a given triangle.
- Escribe a circle to a given triangle.
- Circumscribe an equilateral triangle about a given circle.
- Inscribe an equilateral triangle in a given circle.
- Circumscribe a square about a given circle.
- Inscribe a square in a given circle.
- Circumscribe a regular hexagon about a given circle.
- Inscribe a regular hexagon in a given circle..
- Draw a tangent to a given arc, without using the centre, through a given point P when P is
  - the middle point of the arc,
  - at the end of the arc,
  - outside the arc.
- Draw a tangent to a given circle from a point P when
  - on the circumference,
  - outside the circle.
- Draw two tangents to a circle meeting each other at a given angle.
- Draw direct common tangent or external tangent,
- Transverse common tangent or internal tangent to two equal circles.
- Draw
  - direct common tangent or external tangent,
  - transverse common tangent or internal tangent to two unequal circles.
- Draw a tangent to
  - two unequal touching circles,
  - two unequal intersecting circles
- Draw a circle which touches
  - both the arms of a given angle,
  - two converging lines and passes through a given point between them,
  - three converging lines

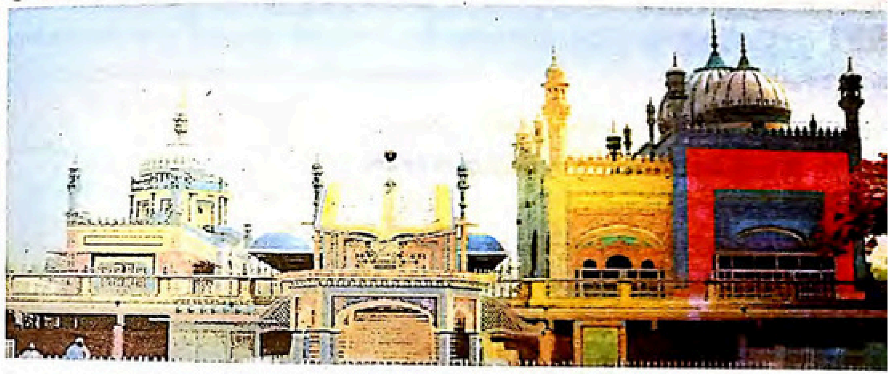


Why it's important



Greek geometers at the time of Euclid believed that circles have a special perfection. With the rediscovery of Euclid's Elements by English philosopher Adelard (twelfth century), this way of thinking made its way into the European world. The designs in many early buildings and churches were based on geometric principles learned from Euclid. Today the practical geometry of circles is at its best in masajids.

Practical geometry is a very important branch of mathematics. In this branch we mechanical methods of constructing various geometrical figures. It is the branch whi highly essential in all draftsmanship necessary in the work of engineers, architect, surv and others. The huge buildings, bridges and dams around us are all indebted to pra geometry.



Historical Bhong Masjid Rahim Yar Khan,Pakistan

13.1 Practical Geometry

13.1.1 Locate the centre of a given circle

Given: A circle without any mention of its centre.

Required: To locate the centre of the circle.

Steps of Construction:

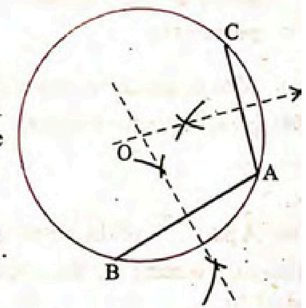
(i). Choose any three points A, B and C on the circumference of the given circle.

(ii). Join A with B and C.

(iii). Draw the perpendicular bisector of $\overline{AB}$ and $\overline{AC}$.

(iv). These bisectors intersect at O.

(v). O is the required center of the given circle.

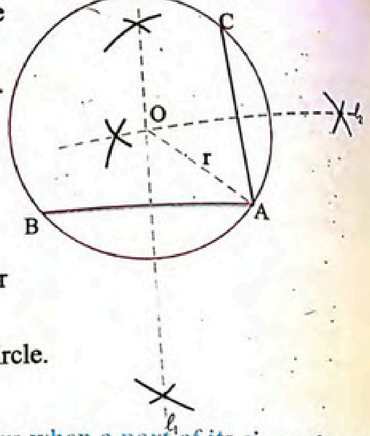


13.1.2 Draw a circle passing through three given non–collinear points

Given: Any three points A, B and C in a plane which are non–collinear.

Required: To draw a circle passing through these points.

Steps of Construction:



(i). Join A to B and C respectively.

(ii). Draw the perpendicular bisectors ℓ_1 and ℓ_2 of $\overline{AB}$ and $\overline{AC}$.

(iii). Point of intersection of these perpendicular bisectors is O.

(iv). Taking radius $= m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

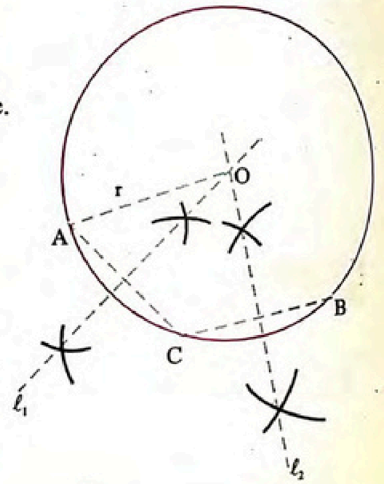
This is the required circle.

13.1.3 (a) Complete the circle, by finding the centre when a part of its circumference is given

Given: A part $\widehat{AB}$ of the circumference of a circle.

Required: To complete the circle by finding its centre.

Steps of Construction:



(i). Choose any point C in the given arc $\widehat{AB}$ at reasonable distance both from A and B with C as such we get $\overline{AC}$ and $\overline{BC}$.

(ii). Find the mid – points of $\overline{AC}$ and $\overline{BC}$ and draw their perpendicular bisectors ℓ_1 and ℓ_2.

(iii). The point of intersection of ℓ_1 and ℓ_2 is the centre of the required circle. Denote this point by O.

(iv). With O as centre draw a circle with radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$.

This gives the required circle.

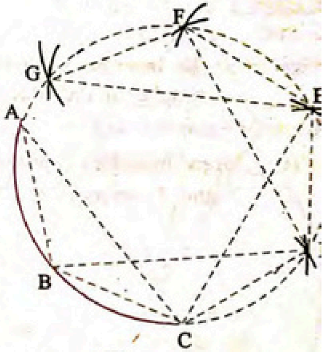
(b) Complete the circle without finding the center when a part of its circumference is given

Given: A part $\widehat{AC}$ of the circumference of a circle.

Required: To complete the circle without finding the centre.

Steps of Construction:

(i). Take any point B on the arc $\widehat{AC}$.



(ii). Join A to B, B to C and A to C.

(iii). With C as a centre and radius equal to $m\overline{AB}$, draw an arc. Again taking B as centre and radius radius equal to $m\overline{AC}$, draw another ar c, which cuts the first arc at point D.

(iv). Join B and C to D. With D as centre and radius equal to $m\overline{BC}$, draw an arc and with C as centre and radius equal to $m\overline{BD}$, draw another arc which cuts the first arc at point E.

(v). Join C and D to E. With E as centre and radius equal to $m\overline{CD}$, draw an arc and w D as centre and radius equal to $m\overline{CE}$, draw another arc which cuts the first arc a

(vi). Continue the same process which gives a sequence of points getting nearer a nearer to the point A. All the points D, E, F, G ······so obtained are the points lying the circumference of the required circle.

(vii). By free –hand drawing of arcs joining the above sequence of points.

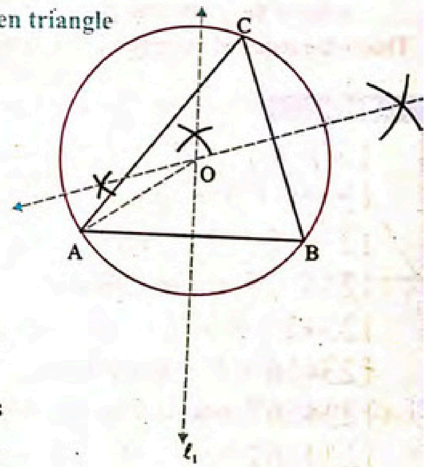
This gives the required circle.

13.2 Circles Attached to Polygons

13.2.1 Circumscribe a circle about a given triangle

Given: ΔABC

Required: To circumscribe a circle about the given ΔABC.



Steps of Construction:

(i). Draw ΔABC.

(ii). Draw perpendicular bisectors ℓ_1 and ℓ_2 of two sides $\overline{AB}$ and $\overline{BC}$, which intersect each other at O.

(iii). Draw a circle with centre O and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$..

(iv). This is the required circumscribed circle about the given triangle.

13.2.2 Inscribe a circle in a given triangle

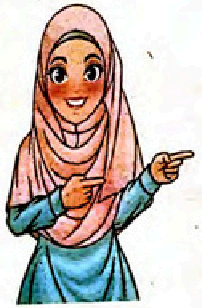
Given: ΔABC

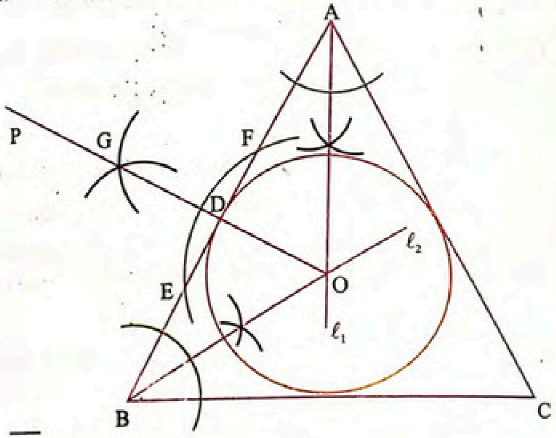
Required: To inscribe a circle in the given triangle i.e. to construct a circle which touches all the three sides of the given triangle.

Steps of Construction:

(i). Draw bisectors ℓ_1 and ℓ_2 of any two angles say $\angle A$ and $\angle B$ of the given ΔABC.

(ii). ℓ_1 and ℓ_2 intersect at O which is the centre of the required circle.





(iii). From O draw $\overline{OD} \perp \overline{AB}$.

(iv). With O as centre and $m\overline{OD}$ as radius draw a circle, which touches all the three sides of the given triangle.

This is the required inscribed circle in the given triangle.

Math Fun

$1 \times 8 + 1 = 9$

$12 \times 8 + 2 = 98$

$123 \times 8 + 3 = 987$

$1234 \times 8 + 4 = 9876$

$12345 \times 8 + 5 = 98765$

$123456 \times 8 + 6 = 987654$

$1234567 \times 8 + 7 = 9876543$

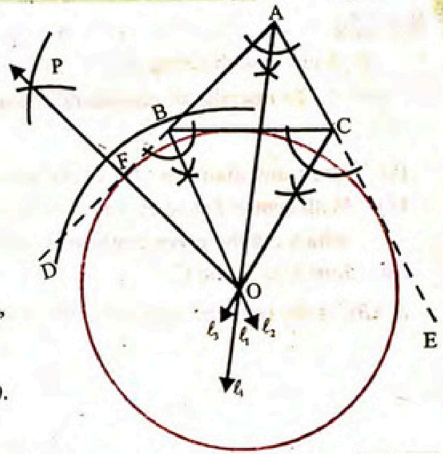
$12345678 \times 8 + 8 = 98765432$

$123456789 \times 8 + 9 = 987654321$

13.2.3 Escribe a circle to a given triangle

Given: ΔABC

Required: To draw an escribed circle opposite to vertex A of the ΔABC.



Steps of Construction:

(i). Produce $\overline{AB}$ and $\overline{AC}$ to form two exterior angles $\angle CBD$ and $\angle BCE$.

(ii). Draw bisectors ℓ_1, ℓ_2 and ℓ_3 of $\angle BAC$, $\angle CBD$ and $\angle BCE$ respectively.

(iii). All these angle bisectors intersect at O.

(iv). Draw $\overline{OF} \perp \overline{AB}$.

(v). With O as centre and $m\overline{OF}$ as the radius, draw a circle, which touches $\overline{BC}$, $\overline{BD}$ and $\overline{CE}$.

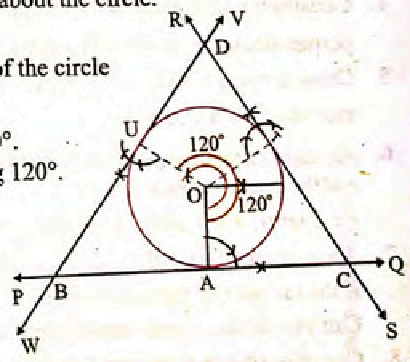
This circle is the required escribed circle. Similarly escribed circles opposite to the vertices B and C can be drawn.

13.2.4 Circumscribe an equilateral triangle about a given circle

Given: A circle with centre O.

Required: To circumscribe an equilateral triangle about the circle.

Steps of Construction:



(i). Take any point A on the circumference of the circle and join O to A. $\overline{OA}$ is radial segment.

(ii). Construct an angle $\angle AOT$ of measure 120°.

(iii). Construct another angle $\angle TOU$ measuring 120°.

(iv). The three points A, T, U which are lying on the circumference of the given circle are such that they divide the boundary of the circle into three equal arcs which are $\overset{\frown}{AT}$, $\overset{\frown}{TU}$ and $\overset{\frown}{UA}$.

(v). Draw perpendiculars to $\overline{OA}$, $\overline{OT}$ and $\overline{OU}$ at A, T and U respectively. These perpendicular lines are $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$ and $\overleftrightarrow{VW}$.

(vi). These lines intersect each other at B, C and D forming a triangle BCD.

The ΔBCD is the required equilateral triangle circumscribed about the given circle.

13.2.5 Inscribe an equilateral triangle in a given circle

Given: A circle with centre *O*.

Required: To inscribe an equilateral triangle in the circle.

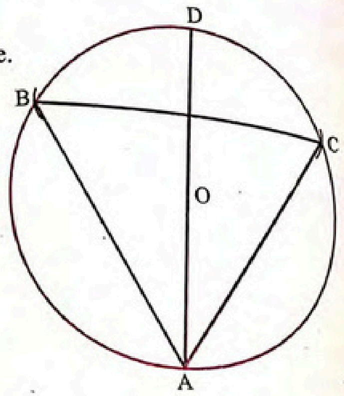
Steps of Construction:

(i). Draw any diameter $\overline{AD}$ of the given circle.

(ii). With centre *D* and radius *m OD*, draw two arcs which cut the given circle at B and C.

(iii). Join A to B and C.

ΔABC is the required equilateral triangle.



Exercise 13.1

1. Construct a triangle with sides 2 cm, 2.5 cm and 3 cm. Also draw its circumcircle.
2. Construct a triangle ABC such that $m\overline{AB} = 3"$ $m\overline{AC} = 4"$ and $m\angle A = 60°$. Draw circumcircle to this triangle.
3. Suppose we have a triangle whose sides are 3 cm, 4 cm and 6 cm respectively. Draw its inscribed circle.
4. Construct a triangle ABC with sides $m\overline{AB} = 5cm$, $m\overline{BC} = 6cm$ and $m\overline{CA} = 8cm$. Draw perpendicular bisectors of its sides and then circumscribe a circle.
5. Draw a triangle ABC with $m\angle A = 60°$ and $m\angle B = 45°$. Draw three angle bisectors and then inscribe a circle in it.
6. An equilateral triangle is inscribed in a circle. Find the altitude of the triangle if the radius *r* of the circle varies as under.
   $r = 3$ *units*, $r = 4$ *units*, $r = 6$*units*, $r = 12$ *units*. Can you deduce some result from this?
7. An equilateral triangle is circumscribed about a circle. Find the altitude of the triangle if the radius *r* of the circle varies as $r = 2$ units, $r = 5$ units $r = 10$ units. Can you deduce some result from this?
8. Circumscribe an equilateral triangle about a circle of radius 2", 3" and 1".
9. Draw a triangle with sides 2.5 cm, 3.5 cm and 4.5 cm long. Draw an escribed circle to the triangle touching the longest side of the triangle.
10. For the problem in Q,9 draw an escribed circle to the triangle touching the smallest side.

13.2.6 Circumscribe a square about a given circle

Given: A circle centre at O.

Required: To circumscribe a square about the given circle.

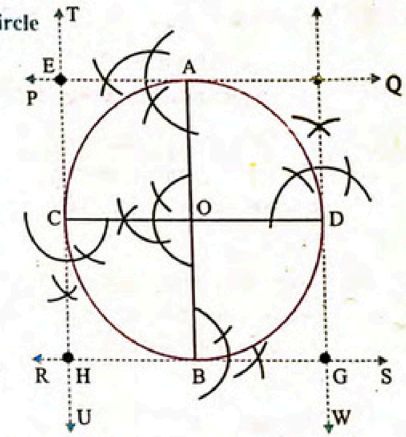
Steps of construction:

(i). Draw any diameter $\overline{AB}$ of the circle.

(ii). Draw another diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$.

(iii). Draw perpendicular $\overleftrightarrow{PQ}$, $\overleftrightarrow{RS}$, $\overleftrightarrow{TU}$ and $\overleftrightarrow{VW}$ at the extremities A, B, C, D of the diameters $\overline{AB}$ and $\overline{CD}$.

(iv). These lines cut each other at the points E, F, G and H.



(v). EFGH is the required circumscribed square about the given circle.

13.2.7 Inscribe a square in a given circle

Given: A circle with centre O.

Required: To inscribe a square in the given circle.

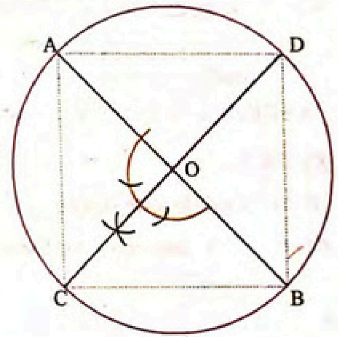
Steps of construction:

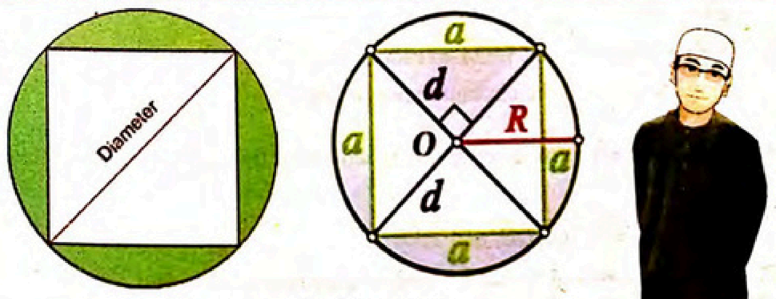
(i). Draw any diameter, say $\overline{AB}$, of the circle.

(ii). Draw another diameter $\overline{CD}$ of the circle which is perpendicular to $\overline{AB}$.

(iii). Draw $\overline{AC}$, $\overline{CB}$, $\overline{BD}$ and $\overline{DA}$.

ACBD is the required inscribed square.





Square inscribed in a circle.

13.2.8 Circumscribe a regular hexagon about a given circle

Given: A circle with centre O.

Required: To circumscribe a regular hexagon about the given circle.

Steps of construction:

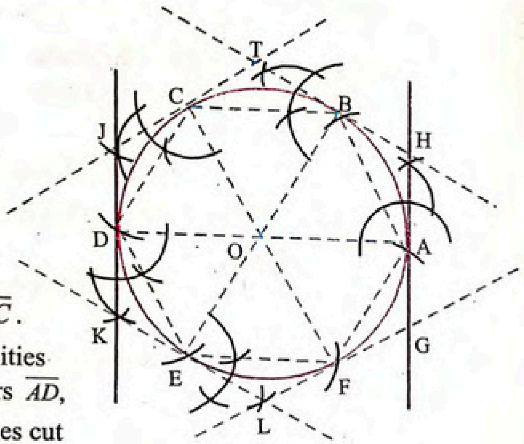
(i). Take any point A on the circumference of the given circle.

(ii). With A as centre and $m\overline{OA}$ as radius, draw two arcs which cut the circumference of the circle at B and F.

(iii). Through O, draw $\overline{AD}$, $\overline{BE}$ and $\overline{FC}$.

(iv). Draw perpendiculars at the extremities A, B, C, D, E and F of the diameters $\overline{AD}$, $\overline{EB}$ and $\overline{FC}$ of the circle. These lines cut each other at points G, H, I, J, K and L.

GHIJKL is the required circumscribed hexagon.



13.2.9 Inscribe a regular hexagon in given circle

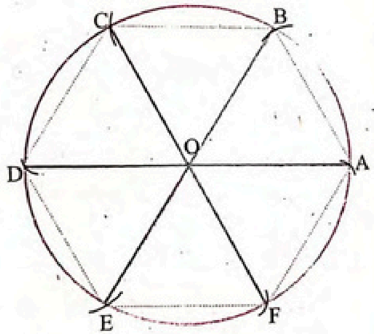
Given: A circle with centre O.

Required: To inscribe a regular hexagon in the given circle.

Steps of construction:

(i). Take any point A on the circumference of the given circle.

(ii). With A as centre and $m\overline{OA}$ as radius, draw an arc which cuts the circumference of the circle at B. Similarly draw successive arcs which cut the circumference of the circle at D, and F.



(iii). Draw $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}$ and $\overline{EF}$.

$ABCDEF$ is the required hexagon which is inscribed in the given circle.

Exercise 13.2

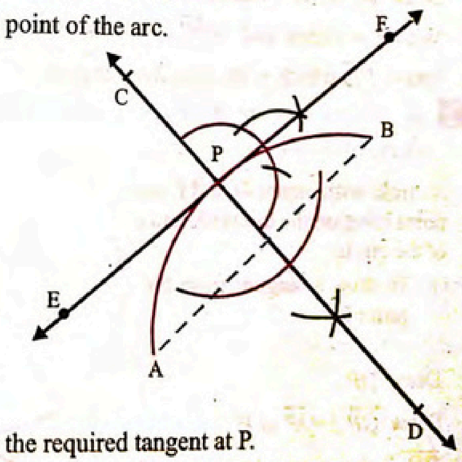
1. Circumscribe a square about a circle of radius 5cm.
2. Inscribe a square in a circle of radius 6 cm.
3. Draw a square of side 6 cm. Circumscribe a circle about that square and then inscribe a circle in the same square. Measure the radii of these two circles.
4. First draw a circle of suitable radius, so that the square circumscribed about that circle has sides of length 8 units.
5. Inscribe a square of side 10 cm in a circle. What will be the size of the radius?
6. Inscribe a regular hexagon in a circle of radius 4 cm.
7. Construct a circle of radius 4cm and draw a regular hexagon about the circle.
8. Draw a circle of radius 8 cm. Circumscribe a regular hexagon about that circle and also inscribe a regular hexagon in the same circle. Find the areas of these geometrical figures. Comment on the values of these areas.
9. Draw two regular hexagons of perimeters 6 cm and 30 cm respectively. Determine their centres. From their centers draw perpendicular to any of their sides respectively. What is the relation of these two perpendiculars?
10. Can you construct a square whose area equals the areas of a given circle? Discuss in detail.

13.3 Tangent to the circle

13.3.1 (i) Draw a tangent to a given arc, without using the centre, through a given point *P* when *P* is the middle point of the arc

Given: An arc such that P is the mid point of the arc.

Required: To draw a tangent at P.



Steps of construction:

(i). Draw $\overline{AB}$.

(ii). From P, draw $\overleftrightarrow{CD} \perp \overline{AB}$.

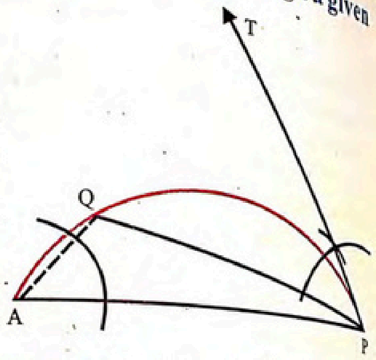
(iii). At P, draw $\overleftrightarrow{EF} \perp \overleftrightarrow{CD}$ which is the required tangent at P.

(ii) Draw a tangent to a given arc, without using the centre, through a given point P when P is at the end of the arc

Given: An arc $\widehat{AP}$ of a circle.

Required: To draw a tangent at the end point P.

Steps of Construction:



(i). Take any point Q on the arc $\widehat{AP}$ and draw the chords $\overline{AP}$ and $\overline{PQ}$.

(ii). Join A and Q.

(iii). Now construct $\angle QPT \cong \angle PAQ$.

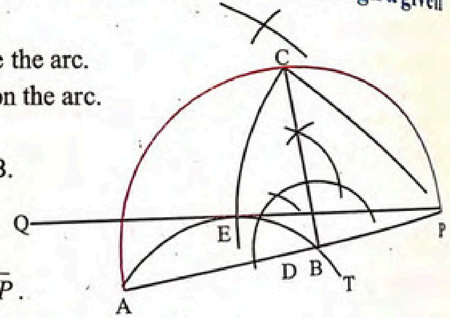
(iv). $\overline{PT}$ is the required tangent at P.

(iii) Draw a tangent to a given arc, without using the centre, through a given point P when P is outside the arc

Given: $\widehat{AT}$ is an arc and P is a point outside the arc.

Required: To draw a tangent from point P on the arc.

Steps of Construction:



(i). Join A and P. Which cuts the arc at B.

(ii). Bisect $\overline{AP}$ at D.

(iii). With D as centre and radius equal to $m\overline{DP}$, draw a semi - circle on $\overline{AP}$.

(iv). Draw $\overline{BC} \perp \overline{AP}$, which intersects the semi-circle at C.

(v). With P as centre and $m\overline{PC}$ as radius draw an arc to cut the given arc at E.

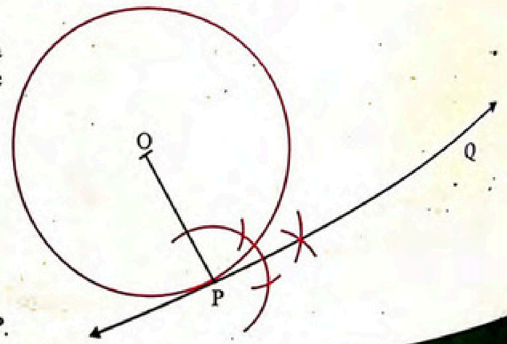
(vi). Draw $\overline{PE}$ which is the required tangent.

13.3.2 (i) Draw a tangent to a given circle from a point P when P lies on the circumference

Given: A circle with centre O and P is a point lying on the circumference of the circle.

Required: To draw a tangent from the point P.

Steps of construction:



(i). Draw $\overline{OP}$.

(ii). Draw $\overline{QP} \perp \overline{OP}$ at P.

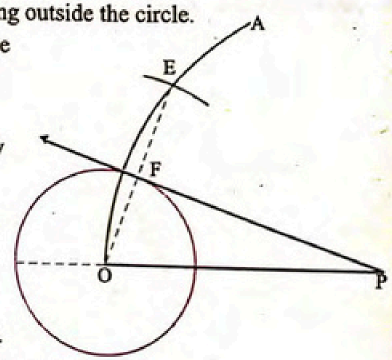
(iii). $\overline{PQ}$ is the required tangent at P.

(ii) Draw a tangent to a given circle from a point P when P lies outside the circle

Given: A circle with centre at O. P is a point lying outside the circle.

Required: To draw a tangent to the circle from the given point P.

Steps of Construction:



(i). With P as centre and $m\overline{OP}$ as radius, draw an arc $\widehat{OA}$.

(ii). With O as center and the diameter of the given circle as radius, draw another arc which intersects arc $\widehat{OA}$ at E.

(iii). Draw $\overline{OE}$ which intersects the circle at F.

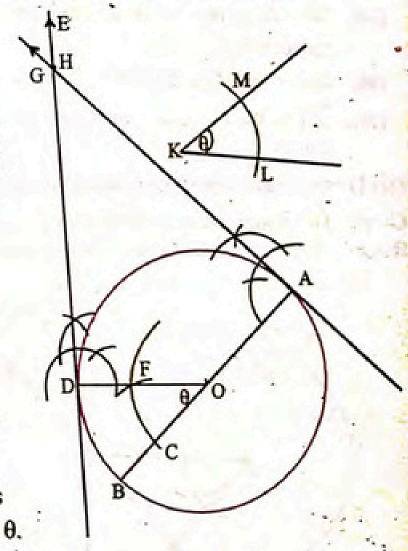
(iv). Through F draw $\overline{PF}$ which is the required tangent.

13.3.3 Draw two tangents to a circle meeting each other at a given angle

Given: A circle with centre O and an angle equal to angle θ. The vertex of the given angle is k.

Required: To draw two tangents to the given circle such that the tangents are inclined to each other at the angle θ.

Steps of Construction:



(i). Draw any diameter $\overline{AB}$.

(ii). At O, construct $\angle BOD \cong \angle\theta$.

(iii). At D draw $\overline{DE} \perp \overline{OD}$.

(iv). At A draw $\overline{AG} \perp \overline{OA}$.

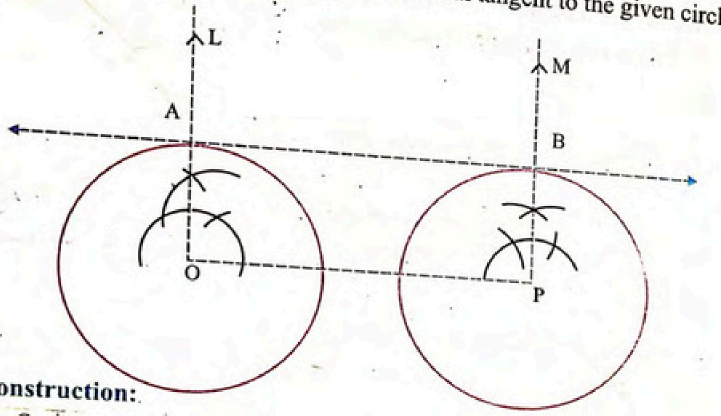
(v). These two perpendiculars intersect each other at H.

(vi). $\overline{DH}$ and $\overline{AH}$ are the required tangents inclined to each other at angle the angle θ.

13.3.4 **(i) Draw direct common tangent or external tangent to two equal circles**

Given: Two equal circles with centres at O and P respectively.

Required: To draw direct common tangent or external tangent to the given circles.



Steps of Construction:

(i). Join O to P.

(ii). Now draw two perpendiculars on $\overline{OP}$ at O and P which cut the circles at A and B respectively.

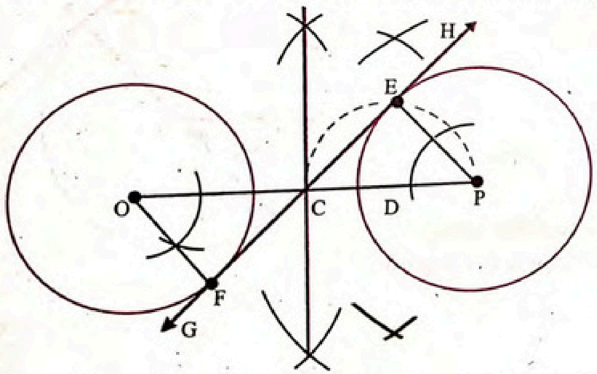
(iii). Join A and B and produce it towards both ends.

(iv). $\overleftrightarrow{AB}$ is the required direct common tangent or external tangent to two given equal circles.

(ii) Draw transverse common tangent or internal tangent to two equal circles

Given: Two equal circles with centres at O and P respectively.

Required: To draw transverse common tangent or internal tangent to the two given circles.



Steps of construction:

(i). Join O and P, the centers of the given circles.

(ii). Bisect $\overline{OP}$ at C.

(iii). Bisect $\overline{CP}$ at D.

(iv). Draw a semi−circle on $\overline{CP}$ diameter which cuts the given circle at E.

(v). Join E and P.

(vi). Draw $\overline{OF} \parallel \overline{EP}$.

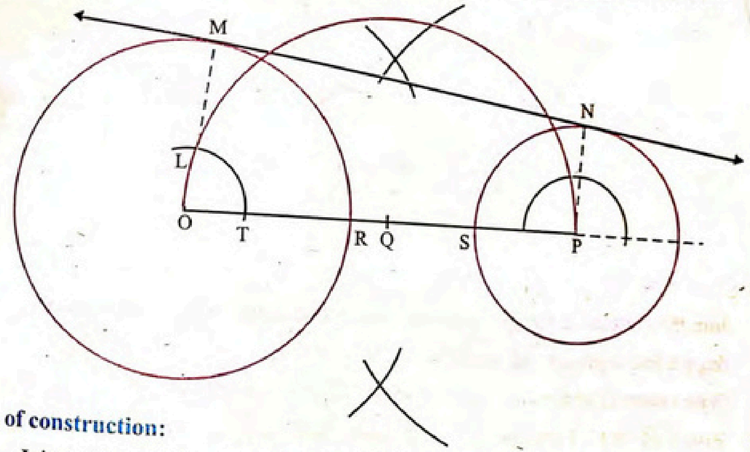
(vii). Join E and F and extend the line to both directions.

(viii). $\overleftrightarrow{GH}$ is the required tangent to both the circles.

13.3.5 **(i) Draw direct common tangent or external tangent to two un −equal circles**

Given: Two unequal circles with centres at O and P respectively.

Required: To draw a direct common tangent or external tangent to the given circles.



Steps of construction:

(i). Join centres of the circles O and P. $\overline{OP}$ cuts the circles at *R* and *S* respectively.

(ii). Bisect $\overline{OP}$ at Q. With Q as centre and $m\overline{QP}$ as radius, draw a semi circle.

(iii). Take a point T on $\overline{OP}$ in the bigger circle such that $m\overline{RT} = m\overline{SP}$ = length of the radius of the smaller circle.

(iv). With center O and $m\overline{OT}$ as radius, describe an arc cutting the semicircle at *L*.

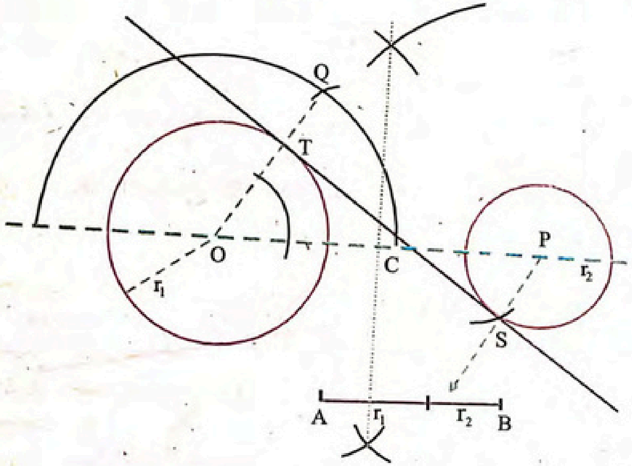
(v). Draw $\overrightarrow{OL}$ to meet the bigger circle at *M*.

(vi). Draw a $\overline{PN} \parallel \overline{OM}$. Join M and N and produce it to both sides to get the common tangent or external tangent $\overline{MN}$ to the two given circles.

(ii) Draw transverse common tangent or internal tangent to two unequal circles

Given: Two unequal circles with centres O and P having radii r_1 and r_2, where $r_1 > r_2$

Required: To draw a transverse common tangent or internal tangent to the given circles.



Steps of Construction:

(i). Join the centres of the given circles and produce the line to both directions. Also draw a line segment $\overline{AB}$ such that $m\overline{AB} = r_1 + r_2$.

(ii). With centre O and radius equal to $m\overline{AB}$, draw a semicircle.

(iii). Bisect $\overline{OP}$ at C. Then choosing C as centre and $m\overline{OC}$ as radius, draw an arc which cuts the semicircle at Q.

(iv). Draw $\overline{OQ}$ cutting the bigger circle at T. Now from P, draw $\overrightarrow{PS} \parallel \overline{OQ}$ in the opposite sense cutting the smaller circle at S.

(v). Join $\overline{OQ}$ cutting the given circle at T. Now draw a line parallel to $\overline{OQ}$ but passing through P which is the centre of the smaller circle. This line cuts the smaller circle at S.

(vi). Join T and S and produce it in both directions. The line $\overleftrightarrow{TS}$ so obtained is the required transverse tangent or internal tangent to the given unequal circles.

13.3.6 (a) Draw a tangent to two unequal touching circles

Given: Two unequal circles with centers O and P touching each other at T.

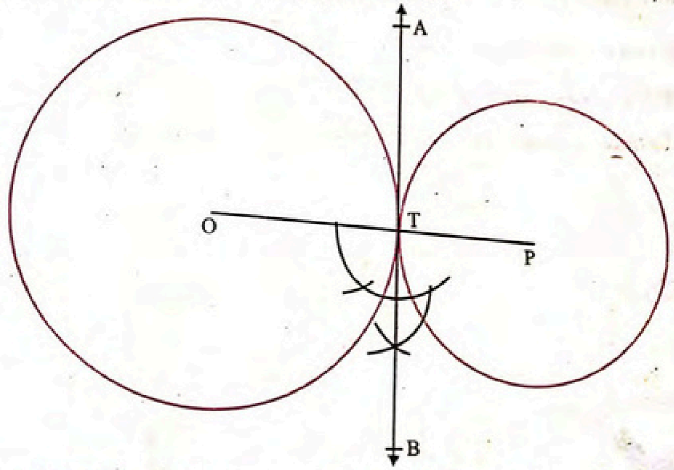
Required: To draw a tangent to the circles.

Steps of Construction:

(i). Join the centers O and P of the given circles. $\overline{OP}$ passes through T, the point contact.

(ii). At T draw a line $\overleftrightarrow{ATB} \perp \overline{OP}$.

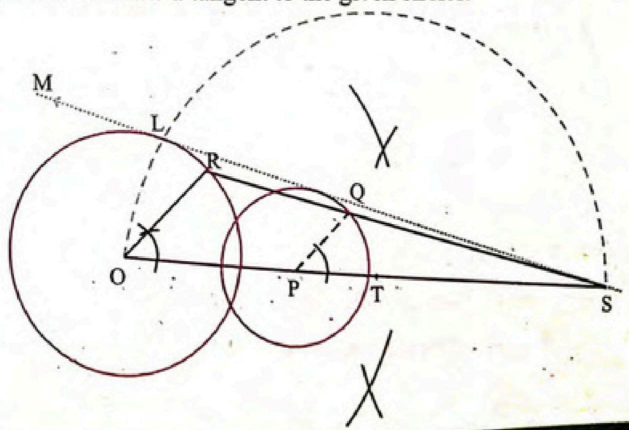
(iii). $\overleftrightarrow{ATB}$ is the required tangent to the given circles.



(b) Draw a tangent to two unequal intersecting circles

Given: Two unequal circles with centers O and P and intersecting each other.

Required: To draw a tangent to the given circles.



Steps of Construction:

(i). Join the centers of the given circles and produce it to the right.

(ii). Draw a line segment $\overline{PQ}$ as a radius of the smaller circle such that $\overline{PQ}$ is not along $\overline{OP}$. Also draw $\overline{OR}$, the radius of the bigger circle such that $\overline{OR}$ is parallel to $\overline{PQ}$.

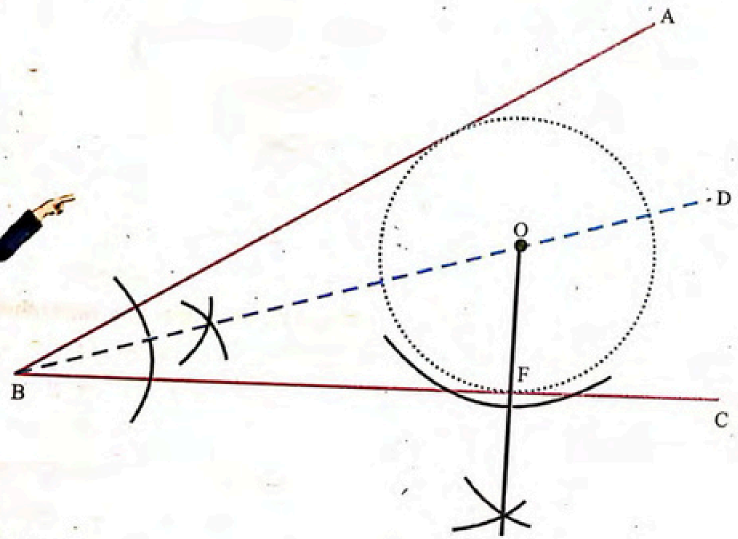
(iii). Join R and Q and produce it to meet the line $\overline{OP}$ produced at S. Bisect the line segment $\overline{OS}$ at T and draw a semicircle on $\overline{OS}$ which cuts the bigger circle at L.

(iv). Finally join S and L and produce it to M. $\overline{SM}$ is the required tangent.

13.3.7 (i) Draw a circle which touches both the arms of a given angle

Given: An angle $\angle ABC$ whose vertex is at B. $\overline{AB}$ and $\overline{BC}$ are its arms.

Required: To draw a circle which touches both the arms $\overline{AB}$ and $\overline{BC}$ of the given angle.



Steps of construction:

(i). Draw $\overline{BD}$, the bisector of $\angle ABC$.

(ii). Take any point O on $\overline{BD}$ and draw a perpendicular $\overline{OF}$ on $\overline{BC}$.

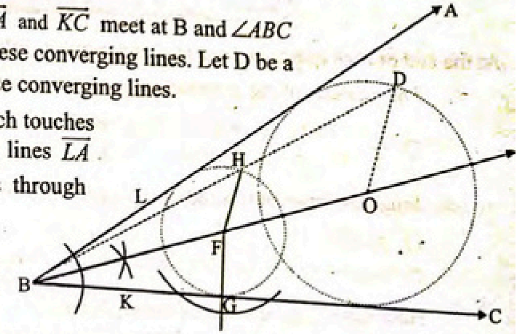
(iii). Taking O as centre and $m\overline{OF}$ as radius draw a circle.

(iv). This is the required circle.

(ii) Draw a circle which touches two converging lines and passes through given point between them

Given: Two converging lines $\overline{LA}$ and $\overline{KC}$ meet at B and $\angle ABC$ is an angle formed by these converging lines. Let D be a point lying between these converging lines.

Required: To draw a circle which touches the two converging lines $\overline{LA}$ and $\overline{KC}$ and passes through given point D.



Steps of construction:

(i). Draw the bisector $\overline{BE}$ of the angle $\angle ABC$.

(ii). Take a point F on $\overline{BE}$ and draw $\overline{FG} \perp \overline{BC}$. With F as centre and $m\overline{FG}$ as radius draw a circle. This circle touches the given converging lines tangentially.

(iii). Join D and B $\overline{DB}$ cuts the circle at H. Join F to H and draw $\overline{DO} \parallel \overline{FH}$.

(vi). With O as centre and $m\overline{DO}$ as radius, draw a circle C which as required.

Exercise 13.3

1. Draw an arc of length 7 cm. Without using the center draw a tangent through a given poin P when P is
   (i). The middle point of the arc.
   (ii). End point of the arc.
   (iii). Outside the arc.
2. Draw a circle passing through a point D and touching a given line $\overleftrightarrow{BC}$ at point D.
3. Describe a circle of radius 4cm, passing through a given point C and touching a give straight line $\overleftrightarrow{AB}$.
4. Radius of a circle is 2.5 cm. A point Q is at a distance of 5cm from the centre. Draw tanger to the circle from the point Q.
5. Radii of two circles are 2 cm and 3cm and their centres are 8 cm apart. Draw direc common tangents to the circles.
6. Two congruent circles are of radius 4 cm each. Their centres are 10 cm apart. Draw transverse common tangents to these circles.
7. Radii of two circles are 2cm and 2.5 cm respectively. Distance between their centres is 5. cm. Draw transverse common tangents to the circles.
8. Draw $\angle ABC$ of measure $60°$. Construct a circle having radius 2.5 cm and touching th arms of the angle.

Review Exercise 13

1. At the end of each question, four circles are given. Fill in the correct circle only.

(i). The measure of the external angle of a regular hexagon is

○ $\frac{\pi}{3}$ ○ $\frac{\pi}{4}$ ○ $\frac{\pi}{6}$ ○ none of these

(ii). Tangents drawn at the end points of the diameter of circle are

○ parallel ○ perpendicular
○ intersecting ○ none of these

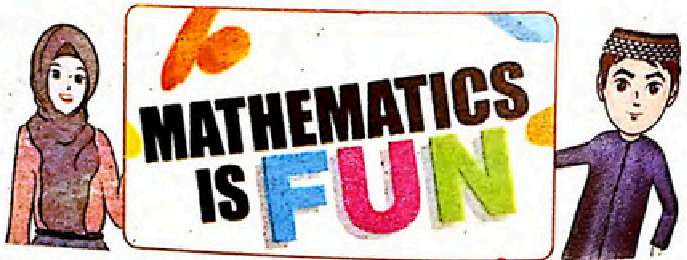
(iii). How many tangents can be drawn from a point outside the circle?

○ 1 ○ 2 ○ 3 ○ 4

(iv). If the distance between the centers of two circles is equal to the sum of their radii, then the circles will

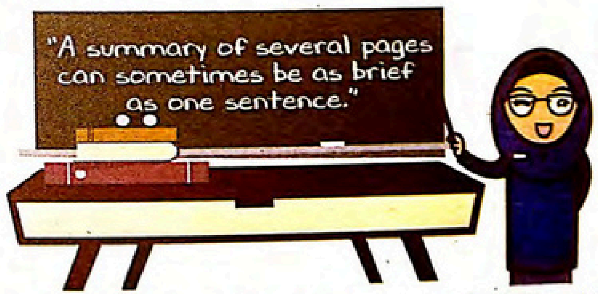
○ intersect ○ do not intersect
○ touch each other externally ○ touch each other internally

2. Practically find the centre of an arc ABC.
3. Escribe a circle opposite to vertex A of a triangle ABC with sides $|AB| = 5$cm, $|BC| = 4$cm, $|CA| = 3$cm. Find its radius also.
4. Circumscribe a circle about an equilateral triangle ABC with each side of length 5cm.
5. Circumscribe a regular hexagon about a circle of radius 4cm.
6. Construct a circle of radius 3 cm. Draw two tangents making an angle of 60° with each other.
7. Draw two equal circles each of radius 3.5cm. If the distance between their centres is 7cm, then draw their transverse tangents.
8. Draw two comon tangents to two intersecting circles of radii 2.5cm and 3.5cm.
9. Draw two common tangents to two touching circle of radii 3 cm and 4cm.



Summary

- The perpendicular bisectors of two non-parallel chords of a circle intersect at a point which is known as centre of the circle.
- A circle of any radius can be traced by rotating a compass about fixed point.
- A circle can be drawn through given three non-collinear points.
- If a triangle, the circumscribed circle, inscribed circle and escribed circle opposite to each vertex can be constructed.
- When a part of circumference of a circle is given, the circle can be completed.
- If a circle is given, then the circumscribed and inscribed equilateral triangles can be constructed.
- For a given circle, the circumscribed an inscribed squares can be drawn.
- For a given circle, the circumscribed and inscribed regular hexagon can be constructed.
- Tangents can be drawn to a given circle, when a point is an its circumference and from a point outside the circle.
- We can draw tangents to a given arc as its mid point, its any end point, and a point not on the arc.
- Tangents to two unequal touching circles can be traced.
- We can construct a circle touching the arms of a given angle.
- Direct or transverse common tangents of two equal circles or two unequal circles can be drawn.
- A circle passing through a given point between two converging lines and touching each of them, can be traced.



ANSWERS

Exercise 1.1

1.

i. $\{-1, -4\}$

ii. $\{1, 5\}$

iii. $\{-5, 2\}$

iv. $\left\{\frac{1}{2}, \frac{5}{3}\right\}$

v. $\left\{-1, \frac{7}{2}\right\}$

vi. $\{3, -7\}$

2.

i. $\{-10, 4\}$

ii. $\{5+\sqrt{14}, 5-\sqrt{14}\}$

iii. $\{0, -2\}$

iv. $\{-12, 14\}$

v. $\left\{\frac{1}{9}, \frac{5}{9}\right\}$

vi. $\{-2, 5\}$

3.

i. $\{3, 5\}$

ii. $\{1+\sqrt{5}, 1-\sqrt{5}\}$

iii. $\left\{0, \frac{-3}{4}\right\}$

iv. $\left\{1+\frac{\sqrt{6}}{3}, 1-\frac{\sqrt{6}}{3}\right\}$

v. $\left\{\frac{3}{2}, \frac{4}{3}\right\}$

vi. $\left\{\frac{1}{2}, -\frac{1}{4}\right\}$

4.

i. $\{1, 7\}$

ii. $\{12, -6\}$

iii. $\{-2+\sqrt{3}, -2-\sqrt{3}\}$

iv. $\{-10\}$

5. $\{-13\}$ **6.** $\left\{1, \frac{3}{2}\right\}$ **7.** $\{\sqrt{85}\}$

Exercise 1.2

1.

(i) $\{\pm 1, \pm 2\}$ (ii) $\{\pm 2, \pm\sqrt{3}\}$ (iii) $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{\frac{5}{3}}\right\}$ (iv) $\left\{0, \frac{-5}{2}\right\}$

(v) $\{1\pm\sqrt{5}\}$ (vi) $\left\{10, \frac{-2}{5}\right\}$ (vii) $\left\{1, 3, \frac{1}{3}\right\}$

(viii) $\{1, 4\pm\sqrt{15}\}$ (ix) $\left\{\frac{-1\pm\sqrt{5}}{2}, 1\pm\sqrt{2}\right\}$ (x) $\{2, 0\}$

(xi) $\{0, -2\}$ (xii) $\{-1, 2\}$ (xiii) $\{-1\pm\sqrt{71}, 2, -4\}$

(xiv) $\left\{\frac{-5\pm\sqrt{5}}{2}\right\}$ (xv) $\{-4\pm\sqrt{5}\}$

2. $\{\pm 1, 1\pm\sqrt{2}\}$

Exercise 1.3

1.

(i) $\{3\}$ (ii) $\{5\}$ (iii) $\{5\}$ (iv) $\{1\}$

(v) $\{2\}$ (vi) $\{0\}$ (vii) $\{4\}$ (viii) $\{6\}$

(ix) $\{0, -2\}$ (x) $\left\{-\frac{1}{2}, -1\right\}$

2. -13

Review Exercise 1

1.

i. $x = -1, 5$

ii. $\frac{1\pm\sqrt{5}}{2}$

iii. cannot be simplified

iv. $a = 2, b = -1, c = -3$

v. $x = 4, -1$

vi. $x = \frac{-2\pm\sqrt{22}}{2}$

vii. $x = \pm\frac{1}{2}$

viii. 2 or -9

ix. $x = 3$ or $x = 5$

2. $\left\{\pm\sqrt{2}, \pm\frac{1}{\sqrt{2}}\right\}$ **3.** $a = -2, b = 0$ **4.** $x = -2$

5. $\left\{6, \frac{22}{49}\right\}$

Exercise 2.1

1. (i) -36 (ii) 9 (iii) -3

2. (i) Real (irrational) and unequal (ii) Real (rational) and unequal
(iii) Complex and unequal

3. (i) ± 6 (ii) ± 12 (iii) $\frac{25}{4}$

4. (i) $\left\{\frac{-5\pm\sqrt{5}}{2}\right\}$ (ii) $\left\{\frac{-3}{2},\frac{-3}{2}\right\}$ (iii) $\left\{\frac{1}{2},\frac{-2}{3}\right\}$

5. (i) The roots are real (rational) and unequal: $3, \frac{1}{3}$
(ii) The roots are real (irrational) and unequal $: 3+\sqrt{5}, 3-\sqrt{5}$
(iii) The roots are real (irrational) and unequal $: \sqrt{3}, -\sqrt{3}$

6. (i) (a) $k \le \frac{9}{8}$ (b) $k > \frac{9}{8}$ (ii) (a) $k \le 1$ (b) $k > 1$
(iii) (a) $k \le \frac{25}{4}$ (b) $k > \frac{25}{4}$

Exercise 2.2

1. (i) $-1, -\omega, -\omega^2$ (ii) $2, 2\omega, 2\omega^2$ (iii) $-3, -3\omega, -3\omega^2$

2. (i) 0 (ii) $-128\omega^2$ $6(\omega-1)$

Exercise 2.3

1. (i) Sum of the roots = 1, Product of the roots $=\frac{-3}{4}$
(ii) Sum of the roots $=\frac{-5}{2}$, Product of the roots = 3
(iii) Sum of the roots $=\frac{-2}{3}$, Product of the roots $=\frac{-5}{3}$

2. $k=-6$ 3. $k=\pm 1$
4. $k=6$ 5. $k=16$
6. $k=6$ 7. $m=5, n=-3$

Exercise 2.4

1. (i) $\frac{c\left(b^2-2ac\right)}{a^3}$ (ii) $\frac{b^2-4ac}{a^2}$

2. (i) $2x^2-3x+1=0$ (ii) $x^2-x-12=0$ (iii) $x^2-6x+7=0$
(iv) $x^2+ax-2a^2=0$

3. $a^2x^2-\left(b^2-2ac\right)x+c^2=0$

4. (i) $\frac{5}{2}$ (ii) 5 (iii) $-\frac{9}{4}$

5. $15x^2+26x+15=0$ 6. $2x^2-12x+17=0$

Exercise 2.5

1. (i) $Q(x)=3x^2-7x+20, R=-61$
(ii) $Q(x)=2x^2-x+9, R=0$
(iii) $Q(x)=2x^3-4x^2+5x-5, R=3$

2. k = 8 3. $p=-4, q=1$ 4. $a=-2, b=-1$

5. $-1, -2$ 6. $2\pm\sqrt{3}$

Exercise 2.6

1.
(i) $(1,-1), \left(\frac{7}{5}, -\frac{1}{5}\right)$ (ii) $(-4, 2), (4, -2)$ (iii) $(2, 12), (-4, 0)$
(iv) $(1, 2), \left(\frac{13}{5}, -\frac{6}{5}\right)$ (v) $(1, 0), (-1, 0)$

vi. $(3, 6), (-3, 6), (3, -6), (-3, -6)$

vii. $\left(\frac{1}{\sqrt{2}}, 1\right), \left(-\frac{1}{\sqrt{2}}, 1\right), \left(\frac{1}{\sqrt{2}}, -1\right), \left(-\frac{1}{\sqrt{2}}, -1\right)$

2. i. $(18, -9)$ ii. $(-2, 2), (-1, 3)$

Exercise 2.7

1. 8, 9
2. 3, 4, 5 or −3, −4, −5,
3. 9 m, 4m
4. 4, 7
5. 6 and 8, −6 and −8
6. 21m, 12m
7. 9 cm, 6 cm
8. 9 cm, 12 cm
9. 3 cm, 5 cm, 4 cm
10. 15 goats.

Review Exercise 2

1. i. 2 ii. $x^2 - 2x + 15 = 0$ iii. none of these
   iv. $\frac{3}{7}$
2. $k = \frac{25}{12}$
3. -128
4. (i) Sum of roots $= 0$, Product of roots $= -\frac{1}{4}$
   (ii) Sum of roots $= -\frac{4}{3}$, Product of roots $= 0$
5. $k = \frac{4}{3}$
6. $k = 1$
7. $6x^2 - 17x + 12 = 0$
8. The other root is 1, $k = -6$
9. $-1, -2$
10. (i) $(-1, 4), (4, -1)$
    (ii) $(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, \sqrt{2}), (\sqrt{2}, -\sqrt{2}), (-\sqrt{2}, -\sqrt{2})$
11. 8cm, 6cm

Exercise 3.1

1. 151:208
2. $x = 63$
3. First Partner = Rs. 8,000, Second Partner = Rs. 12,800, Third Partner = Rs. 19,200
4. 14.5
5. (i). $y = 81$ (ii). $x = 10, -10$
6. (i) $y = 2$ (ii). $x = 1$
7. (i). $r = 54$ (ii). $p = 18$
8.

x	4	6	7	15
y	2	3	3.5	7.5

Exercise 3.2

1. (i) Continued Proportion
   (ii) Not in continued proportion (iii) Continued Proportion
2. ± 6
3. 45
4. $\frac{4}{7}$
5. $\pm(a + b)$
7. (i) $x = -9$ (ii) $-\frac{1}{8}, 5.5$ (iii) $\sqrt{\frac{5}{3}}a$

Exercise 3.3

1. $y = \frac{968}{9}$
2. $f = 405$
3. $a = 2$
4. $p = 2400$
5. $a = \frac{21}{2}$
6. $k = \frac{3}{2}$
7. $y = kx^2z$, 32
8. $p = \frac{12}{5}, \frac{qr^2}{st^2}, -\frac{128}{5}$

Exercise 3.5

1. (i) 18cm (ii). 3
2. 119N/cm^2
3. 100N/cm^2
4. 8 days
5. 400N/m^2
6. 6cm.
7. $V = 2816\ cm^3$

Review Exercise 3

1.
   - i. $a \propto b$
   - ii. $mn = k$
   - iii. 3 to 2
   - iv. $\frac{a}{c} = \frac{b}{d}$
   - v. $x = 21$
   - vi. $\frac{y^2}{x}$
   - vii. $x \propto z$
   - viii. $a = \frac{11}{2}$
   - ix. $\frac{\ell a + mc + ne}{\ell b + md + nf}$
   - x. $x = \frac{7}{16}y$
2. $k = \frac{1}{20}$
3. $x = 2$
4. 300 units
5. 60amp
6. $a = 126$
7. 7
8. 5
9. 32, 12
10. 8, 4, 2
11. 4, 9
12. 45°, 60°, 75°

Exercise 4.1

1. $\frac{2}{x} - \frac{1}{2x-1}$
2. $\frac{3}{2(x+5)} - \frac{1}{2(x+1)}$
3. $-\frac{1}{2(x+1)} + \frac{1}{2(x-1)}$
4. $\frac{5}{6(x+5)} + \frac{1}{6(x-1)}$
5. $\frac{6}{5(x+2)} + \frac{8}{5(2x-1)}$
6. $\frac{9}{4(x-1)} - \frac{5}{4(x+2)} + \frac{1}{2(x+1)^2}$
7. $\frac{11}{x+3} - \frac{10}{x+2} + \frac{6}{(x+2)^2}$
8. $-\frac{1}{9x} + \frac{1}{9(x-3)} + \frac{5}{3(x-3)^2}$
9. $1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$
10. $\frac{1}{4(x+1)} + \frac{1}{2(x-1)^2} + \frac{3}{4(x-1)}$

Exercise 4.2

1. $\frac{1}{x} - \frac{x}{x^2+1}$
2. $\frac{5}{4(x-1)} - \frac{x-11}{4(x^2+3)}$
3. $\frac{-3x+1}{x^2+1} + \frac{3}{2(x-1)}$
4. $-\frac{3}{5x^2} + \frac{3}{5(x^2+5)}$
5. $-\frac{2}{7(x+4)} + \frac{1}{7}\frac{6x-3}{3x^2+1}$
6. $\frac{-5}{x+1} + \frac{5x-5}{x^2-2} - \frac{5x-10}{(x^2-2)^2}$
7. $\frac{4}{5(x-2)} - \frac{4x+8}{5(x^2+1)} + \frac{x-2}{(x^2+1)^2}$
8. $\frac{4x-5}{(x^2+4)^2}$
9. $-\frac{4}{(x^2+1)^2} + \frac{2}{x^2+1} + \frac{1}{x+1} - \frac{1}{x-1}$
10. $\frac{-1-x}{(x^2+1)^2} + \frac{1}{2}\frac{-3x-3}{x^2+1} + \frac{3}{2(x-1)}$

Review Exercise 4

1. (i) $\frac{1}{2(x-1)}-\frac{1}{2(x+1)}$ (ii) Rational fraction (iii) Improper fraction

(iv) x^2-3x+1

2. i. $2+\frac{1}{x-1}-\frac{1}{x+1}$ ii. $2x+3+\frac{30}{x-2}-\frac{16}{x-1}$

iii. $-\frac{1}{x}+\frac{1}{x-1}+\frac{2}{(x-1)^2}$ iv. $\frac{1}{x-1}+\frac{2}{(x-1)^2}$

v. $\frac{1}{x^2-2}+\frac{1}{x^2+2}$ vi. $\frac{2}{x-1}-\frac{1}{2(x+1)}-\frac{3x-1}{2(x^2+1)}$

vii. $\frac{x+3}{x^2+1}-\frac{x+2}{(x^2+1)^2}$ viii. $2-\frac{1}{x^2}-\frac{3}{x+1}+\frac{1}{x}$

ix. $\frac{x-1}{x^2+2x+4}+\frac{3}{x-2}$

3. $\frac{x}{(x^2+1)^2}+\frac{1}{x+1}$

Exercise 5.1

1. (i) {0, 1, 2, 3} (ii) { 1 } (iii) {1, 2, 3, 4} (iv) {1, 3}
(v) {0, 1, 3, 4} (vi) {1, 2, 3}

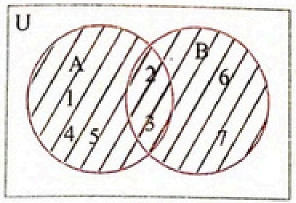
2. (i) {1}, {4, 6, 8} (ii) {0, 1, 2, 3}, { } (iii) {2, 4, 6, … }, { }

3. (i) {1, 3, 5, 7 , … 19} (ii) {2, 4, 6, … 20}
(iii) {1, 2, 3, … 20} (iv) {1, 2, 3, … 20}
(v) { } (vi) {1, 3, 5, … 19}
(vii) {1, 2, 3, … 20} (viii) {2, 4, 6, … 20}
(ix) { } (x) {1, 2, 3, … 20}

4. (i) {1, 2, 3, … 15} (ii) { }
(iii) { } (iv) {1, 2, 3, … 15}
(v) {1, 3, … 15} (vi) {1, 2, 3, … 15}
(vii) { } (viii) {2, 4, 6, … 14}

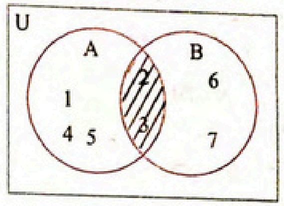
Exercise 5.3

1. (i)



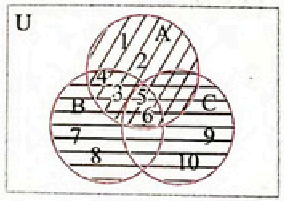
A∪B

(ii)



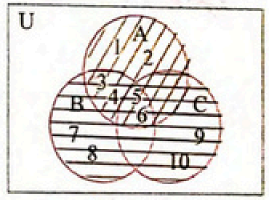
A∩B

2. (i)



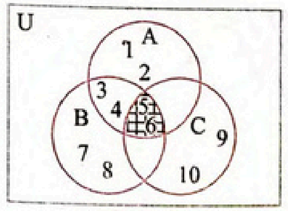
A∪(B∪C)

=



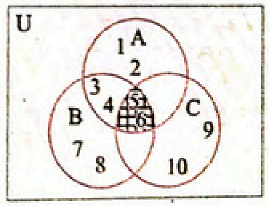
(A∪B)∪C

(ii)



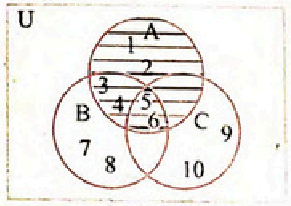
A∩(B∩C)

=



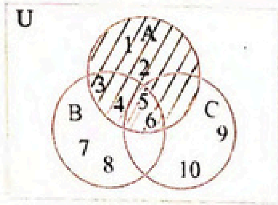
(A∩B)∩C

(iii)



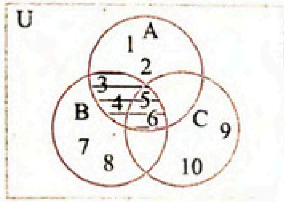
A∪(B∩C)

=



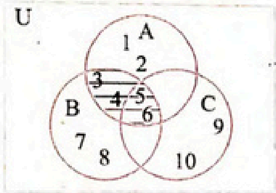
(A∪B)∩(A∪C)

(iv)



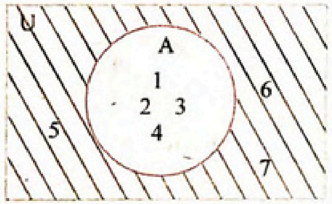
A∩(B∪C)

=

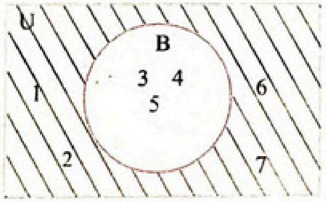


(A∩B)∪(A∩C)

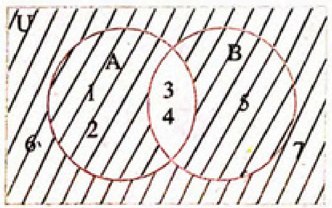
3.



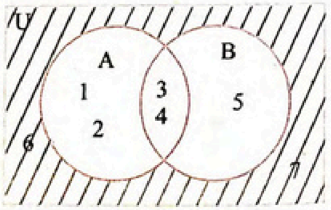
A′



B′

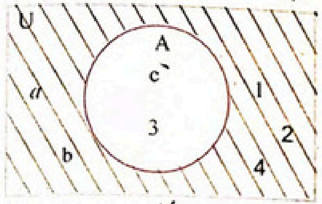


A′∪B′

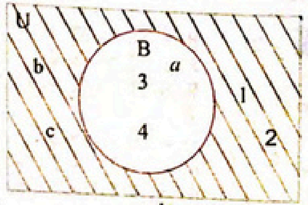


A′∩B′

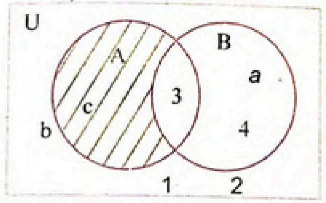
4.



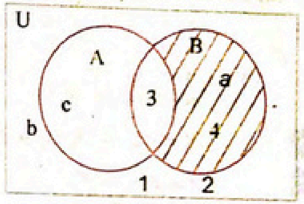
A′



B′

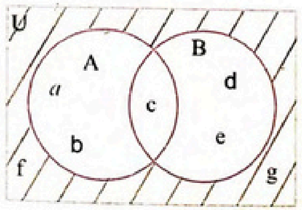


A\B



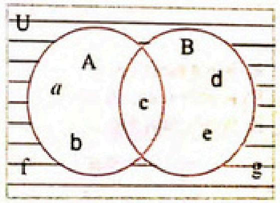
B\A

5.

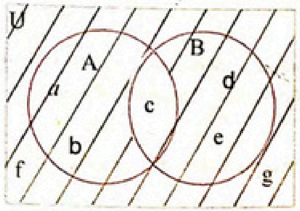


(A∪B)′

=

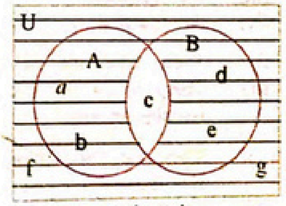


A′∩B′



(A∩B)′

=



A′∪B′

Exercise 5.4

1. (i) $R_1 = \{(1, 4), (2, 4)\}$
$R_2 = \{(2, 5), (3, 4)\}$
$R_3 = \{(1, 4), (2, 5), (3, 5)\}$
(ii) $R_1 = \{(4, 1)\}$
$R_2 = \{(5, 1), (5, 2)\}$
$R_3 = \{(4, 1), (4, 2), (4, 3)\}$
$R_4 = \{(4, 2), (4, 3), (5, 1), (5, 2)\}$
(iii) $R_1 = \{(1, 1), (1, 2)\}$
$R_2 = \{(1, 3), (2, 2)\}$
$R_3 = \{(1, 1), (2, 2), (3, 3)\}$
$R_4 = \{(3, 1), (3, 2)\}$
(iv) $R_1 = \{(4, 4)\}$
$R_2 = \{(4, 4), (4, 5)\}$

2. $R = \{(2, 1), (3, 1), (4, 1), (4, 3)\}$

3. Range = $\{0, 8, 16\}$

4. Range = $\{1, 7, 17, 31\}$

Exercise 5.5

1. R_1 is not function.
R_2 is onto function.
R_3 is into function.

2. (i) into function
(ii) not a function, because both conditions of being a function are not satisfied.
(iii) not a function
(iv) into function

3. (i) There exists one–one correspondence between set A and set B.
(ii) There is not one-one correspondence because $P \in B$ remains unpaired.

4. (i) There does not exist one-one correspondence between set A and set B. It is one-one function.
(ii) There does not exist one-one correspondence between set A and set B. It is into function.

5. (i) $\{(1, 5), (2, 6), (3, 7), (4, 8)\}$
(ii) $\{(1, 6), (2, 5), (3, 8), (4, 7)\}$
(iii) $\{(1, 6), (2, 5), (3, 7), (4, 8)\}$
(iv) $\{(5, 1), (6, 2), (7, 3), (8, 4)\}$
(v) $\{(5, 4), (6, 3), (7, 2), (8, 1)\}$
(vi) $\{(1, 5), (2, 6), (3, 5), (4, 6)\}$

6. (i) It is a function.
Range = $\{1, 2, 3, 5\} \neq A$, not onto
(ii) It is a function.
Range = $\{1, 2, 3, 4\} \neq A$, not onto
(iii) It is a function.
Range = $\{1, 2, 4, 5\} \neq A$, not onto

Review Exercise 5

1. (i) An onto function from A to B (ii) 2×3 (iii) $\{0,4,8,12\}$ and $\{6,10,14,18\}$
(iv) 13 (v) Range $f = B$ (vi) $\{0,8,10,14\}$

2. (i) $\{1, 2, 3, \ldots 100\}$ (ii) $\{\ \}$ (iii) $\{2, 4, 6, .. 100\}$
(iv) $\{1, 3, 5, \ldots 99\}$

7. (i) Function (ii) Bijective function
(iii) Bijective function (iv) Not a function

8. (i) Function, Range = $\{1, 2, 3, 5\}$, not onto
(ii) Function, Range = $\{1, 2, 3, 4\}$, not onto
(iii) Function, Range = $\{1, 2, 3, 5\}$, not onto
(iv) Not a Function

9. (i) $\{(-5, 1), (-6, 2), (-4, 3), (-3, 4)\}$
(ii) $\{(-5, 2), (-6, 1), (-4, 4), (-3, 3)\}$
(iii) $\{(-5, 4), (-6, 3), (-4, 2), (-3, 1)\}$
(iv) $\{(-5, 2), (-6, 2), (-4, 1), (-3, 3)\}$

Note: Several answers are possible.

Exercise 6.1

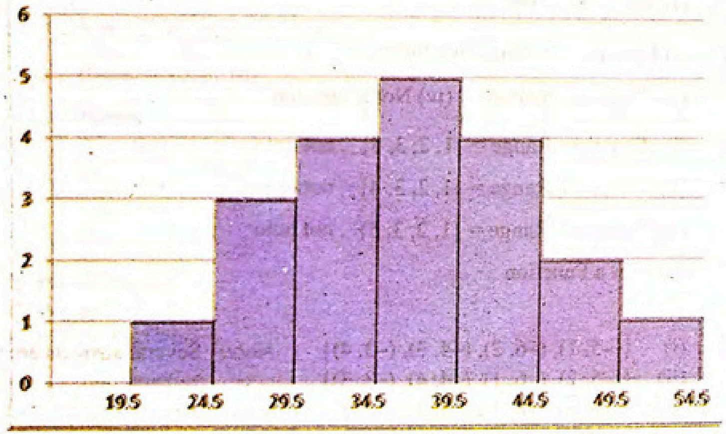
1.

Class-Limits	Tally Marks	Frequency
35 — 45	II	2
46 — 55	IIII	4
56 — 65	~~IIII~~ I	6
66 — 75	~~IIII~~ IIII	9
76 — 85	~~IIII~~ I	6
86 — 90	III	3

2.

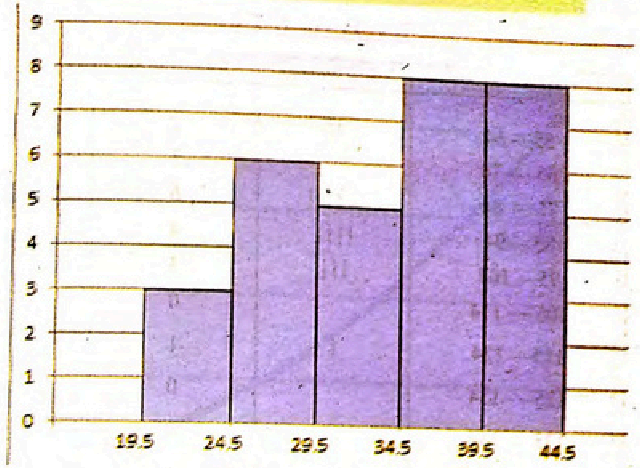
S. No	Class interval	Frequency
1	1—3	01
2	4—6	04
3	7—9	04
4	10—12	03
5	13—15	04
6	16—18	04
7	19—21	03
8	22—24	02
		Σf = 25

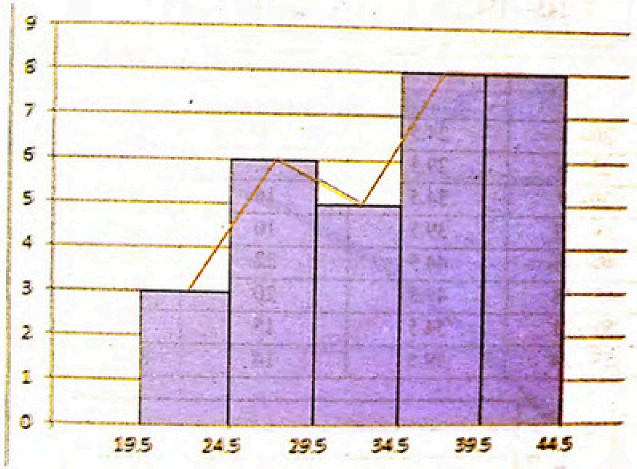
3.



4.

Class-Limits	Tally Marks	Frequency
20 — 24	|||	3
25 — 29	𝍸		6
30 — 34	𝍸	5
35 — 39	𝍸 |||	8
40 — 44	𝍸 |||	8





5.

Class interval	Frequency
1—2	15
3—4	11
5—6	4
	Σf =30

Exercise 6.2

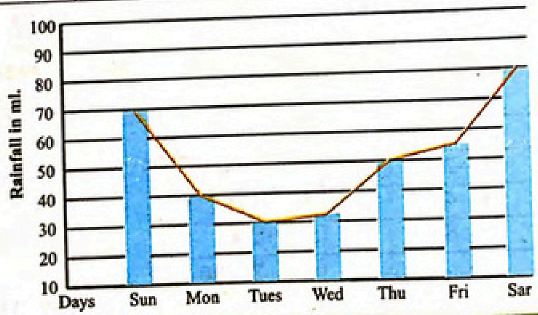
1.

Class-Limits	Tally Marks	Frequency
55— 64	II	2
65 — 74	III	3
75 — 84	~~IIII~~ I	6
85 —94	IIII	4
95— 104	III	3
105 — 114		0
115— 124	I	1
125 — 134		0
135 — 144		0
145— 154	I	1

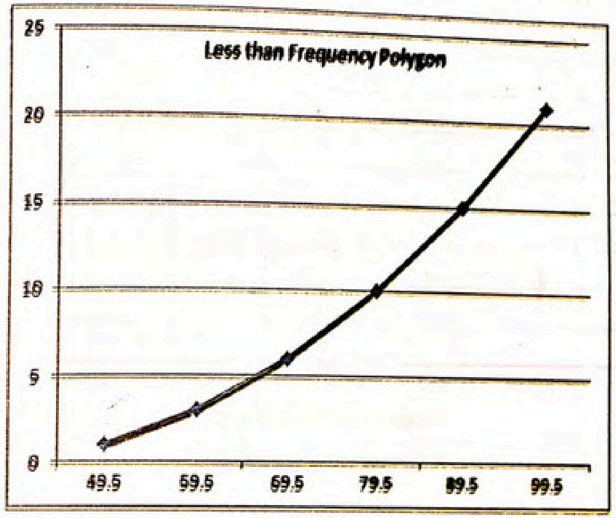
2.

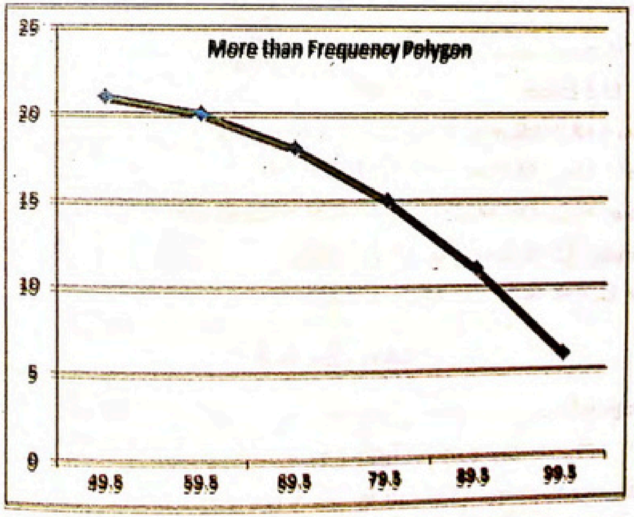
Age in years	Upper class Boundary	Frequency	Cumulative frequency
20—3	24.5	1	1
25—6	29.5	2	1+2=3
30—9	34.5	16	3+16=19
35—12	39.5	10	19+10=29
40—15	44.5	22	29+22=51
45—18	49.5	20	51+20=71
50—21	54.5	15	71+15=86
55—24	59.5	14	86+14=100

3.



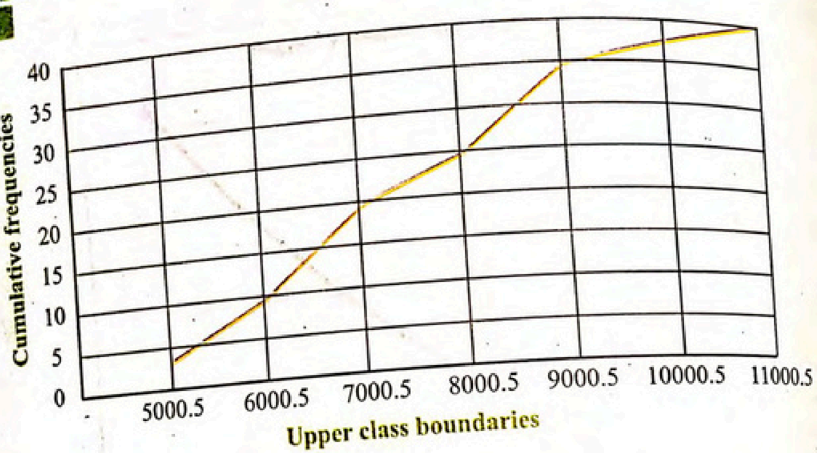
4.





5. (i). 20 (ii). 6 (iii). 5 (iv). 10 (v). 79.5

6.



Exercise 6.3

1. 37.8
2. 37.8
3. 1247(for assumed mean of 1244)
4. 53.5 marks
5. (i) 65.5 inches (i) 7500
6. Mean = 66, Median = 67, Mode = 71, GM = 64.44
7. Mean = 154.3 Median = 156.5, Mode = 160
8. Median = 121.125, Mode = 119.62, G.M = 122.49, HM = 119.44
9. Median = 12.5th item Q_1 = 4.25th item
   Q_3 = 12.75th item Mode = 26.166

Exercise 6.4

1. Range = 12
2. (i) 7.72 (ii) 8.01 (iii) 1.71
3. Range = 65, Variance = 10.38 and S.D = 3.22
4. (a) AM: Section A = 7, Section B = 7
   (b) Variance: Section A = 2.4, Section B = 6.4

5. S.D: Maths = 9.96, Physics = 3.15, in Physics students are more consistent.
6. Variance = 2.88, S.D = 1.7

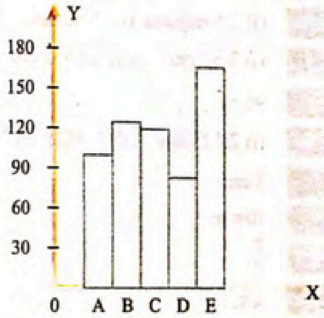
Review Exercise 6

1. (i) Class interval (ii) Ogive (iii) Frequency
   (iv) 1 (v) 1, 2, 3, 3, 2, 1, 2 (vi) Geometric mean
   (vii) 66 (viii) 51 (ix) 7
   (x) 0 (xi) Rs. 600/ (xii) None of these
   (xiii) Range (xiv) Arithmetic Mean (xv) 0.5
   (xvi) Variance (xvii) Mode

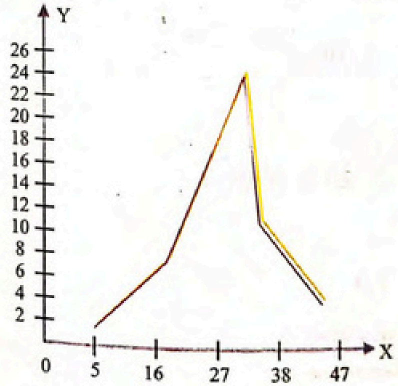
2.

Classes	f
15 – 16	12
17 – 18	12
19 – 20	3
Total	27

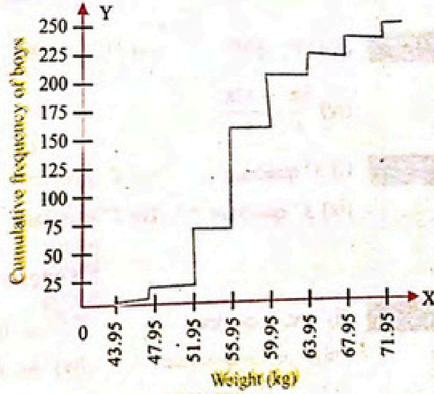
3.



4.



5.



Exercise 7.1

1. (i) 8.2597° (ii) 39.8152° (iii) 84.3194° (iv) 18.1058°
2. (i) 42°15′ (ii) 57°19′30″ (iii) 12°59′44″ (iv) 32°37′30″
3. (i) 114.6° (ii) 300° (iii) 30° (iv) −135°
4. (i) $\frac{\pi}{4}$ radians (ii) $\frac{2\pi}{3}$ radians (iii) $\frac{-7\pi}{6}$ radians (iv) 1.0576 radians

Exercise 7.2

1. (i) 1.047 cm (ii) 3.1415 cm (iii) 12.57 cm
2. (i) 2.5 radians (ii) 5 radians (iii) 2.09 radians
3. (i) 3.82 cm (ii) 0.1632 cm (iii) 6.366 cm
4. 96 m^2
5. (i) 2.625 cm (ii) 6.5625 cm^2
6. 8 cm
7. 70π m
8. $\frac{\pi}{2}$
9. 4πcm
10. $6\pi cm^2$

Exercise 7.3

1. (i) 415°, −305° (ii) 315°, −405° (iii) $\frac{13\pi}{6}, -\frac{11\pi}{6}$ (iv) $\frac{5\pi}{4}, \frac{-11\pi}{4}$
2. (i) 3rd quadrant (ii) 1st quadrant (iii) 3rd quadrant (iv) 3rd quadrant (v) 2nd quadrant

Exercise 7.4

1. (i) +ve, II quadrant (ii) +ve, II-quadrant (iii) +ve, III-quadrant (iv) −ve, II-quadrant (v) −ve, III-quadrant (vi) +ve, II-quadrant

2. (i) $\sin(-180°)=0$, $\cos(-180°)=-1$, $\tan(180°)=0$, $\operatorname{cosec}(-180°)=$ undefined, $\sec(-180°)=-1$, $\cot(-180°)=$ undefined

(ii) $\sin(-270°)=1$, $\cos(-270°)=0$, $\tan(-270°)=$ undefined, $\operatorname{cosec}(-270°)=1$, $\sec(-270°)=$ undefined, $\cot(-270°)=0$

(iii) $\sin 720°=0$, $\cos 720°=1$, $\tan 720°=0$, $\operatorname{cosec}720°=$ *undefined*, $\sec 720°=1$, $\cot 720°=$ *undefined*

(iv) $\sin(1470°)=\frac{1}{2}$, $\cos(1470°)=\frac{\sqrt{3}}{2}$, $\tan(1470°)=\frac{1}{\sqrt{3}}$, $\operatorname{cosec}(1470°)=2$, $\sec(1470°)=\frac{2}{\sqrt{3}}$, $\cot(1470°)=\sqrt{3}$

3. $\cos\theta=\frac{1}{2}$, $\sin\theta=-\frac{\sqrt{3}}{2}$, $\tan\theta=-\sqrt{3}$, $\operatorname{cosec}\theta=-\frac{2}{\sqrt{3}}$, $\cot\theta=-\frac{1}{\sqrt{3}}$
4. $\operatorname{cosec}\theta=\frac{5}{4}$, $\cos\theta=-\frac{3}{5}$, $\tan\theta=-\frac{4}{3}$, $\sec\theta=-\frac{5}{3}$, $\cot\theta=\frac{3}{4}$
5. (i) 1 (ii) $\frac{1}{\sqrt{3}}$ (iii) $\frac{1}{1+\sqrt{2}}$ (iv) 2 (v) 0
6. (i) 1st quadrant (ii) 3rd quadrant (iii) 2nd quadrant (iv) 4th quadrant (v) 4th quadrant (vi) 2nd quadrant
7. (i) 25.56 (ii) 26.16 (iii) 7.79
8. 333.92yd

Exercise 7.6

1. 40°
2. $50\sqrt{3}\,m$
3. 26.56°
4. 160m
5. 260 feet
6. $300(\sqrt{3}+1)m$
7. $\frac{210}{\sqrt{3}-1}m$

Review Exercise 7

1. (i) angle of depression (ii) $\frac{\cos\theta}{\sin\theta}$ (iii) $\sec^2\theta$ (iv) $-\frac{1}{2}$
(v) 1^{st} quadrant (vi) $\frac{\pi}{4}$ (vii) 2.6496 (viii) 0.4
(ix) $2\sqrt{2}$ (x) $45°$
2. $45.4916°$ 3. $216°40'12''$
4. (i) $\frac{3\pi}{2}$ *radian* (ii) 2π *radian*
5. (i) $550°, -170°$ (ii) $110°, -610°$
6. (i) $\sin 390° = \frac{1}{2}, \cos 390° = \frac{\sqrt{3}}{2}, \tan 390° = \frac{1}{\sqrt{3}}$
$\operatorname{cosec} 390° = 2, \sec 390° = \frac{2}{\sqrt{3}}, \cot 390° = \sqrt{3}$
(ii) $\sin(-240°) = \frac{\sqrt{3}}{2}, \cos(-240°) = \frac{-1}{2}, \tan(-240°) = -\sqrt{3}$
$\operatorname{cosec}(-240°) = \frac{2}{\sqrt{3}}, \sec(-240°) = -2, \cot(-240°) = \frac{-1}{\sqrt{3}}$
8. $\cos\theta = \frac{1}{2}, \sin\theta = \frac{-\sqrt{3}}{2}, \tan\theta = -\sqrt{3}, \operatorname{cosec}\theta = \frac{-2}{\sqrt{3}}, \cot\theta = -\frac{1}{\sqrt{3}}$
9. 44 m (approximately). 10. 194 feet (approximately).

Exercise 8.1

1. 14 cm 2. 1.5 cm 3. 6.1
4. $\angle B$ and $\angle D$ are obtuse.

Exercise 8.2

1. $5\sqrt{7}\ cm, \sqrt{142}$ cm, $\sqrt{58}cm$
2. $\frac{\sqrt{79}}{2}$ cm
3. $m\overline{CD} = 6\ cm,\ m\overline{AD} = 6\sqrt{3}$ cm, $m\overline{AB} = 4\sqrt{13}\ cm$
4. $m\overline{AD} = \frac{36}{\sqrt{61}}$ cm, $m\overline{CD} = \frac{25}{\sqrt{61}}\ cm$

Review Exercise 8

1. (i) Greek (ii) Perga (iii) Mathematician
2. 1.53 units.
3. $m\overline{BC} = \sqrt{89},\ m\overline{AD} = \frac{40}{\sqrt{89}}$ cm, $m\overline{BD} = \frac{25}{\sqrt{89}}\ cm$
4. $m\overline{BE} = \sqrt{41}$ cm
5. $x = \frac{86}{19} = 4.53(approx.)$
$y = \frac{275}{19} = 14.47(approx.)$
$h = \frac{4\sqrt{1794}}{19} = 8.92(approx.)$

Exercise 9.1

1. $40\sqrt{2}$ cm 2. 60 cm 3. $\frac{3}{2} = 1.5$ units
4. $\sqrt{41}$ cm 5. $2\sqrt{39}$ cm 6. 24

Exercise 9.2

1. 1 cm or 7 cm 2. 8.39 cm
3. (a) QT>PR
(b) $\overline{PR}$ is farther from C than $\overline{QT}$.

Review Exercise 9

1. (i) congruent (ii) *CD* is closer to O (iii) 24 centimeters (iv) 15 units
(v) 14.4 units (vi) 6 cm (vii) 8
2. (i) AC = 12 m, AB = 24 m (ii) Radius = 10 cm, Diameter = 20 cm
(iii) 8 cm (iv) 48 Units (v) $5\sqrt{2}$ m
3. 17 cm

Exercise 10.1

1. $8\,cm$ 2. $\sqrt{17}cm$
4. (i) 2 (ii) 15.65 (iii) 10
5. 16 cm
6. (i) $x = 49, y = 14$ (ii) $x = 58, y = 15$ (iii) $x = 34, y = 14.8$ (iv) $x = 35, y = 55$
7. (i) 6 cm (ii) 3 cm (iii) $3\sqrt{5}$ cm (iv) $3\left(\sqrt{5}-1\right)$cm

Exercise 10.2

1. 11 cm 2. 12cm 3. 24 cm
4. 12 cm 5. 16cm 6. 10 cm
7. $2\sqrt{7}$ cm 8. 6 cm

Review Exercise 10

1. (i) an arc (ii) $\overline{OT} \perp \overleftrightarrow{PQ}$ (iii) equal
2. 39 ft
3. 24^0
4. (i) 26^0 (ii) 122^0
5. (a) $i = 8, j = 67.4$ (b) $k = 12.6, l = 50.0$

Exercise 11

1. (i) $x = 12,$ $y = 90°$ (ii) $x = 11,$ $y = 90°$ (iii) $x = 12,$ $y = 67.4°$ (iv) $x = 11.0,$ $y = 61.9°$ (v) $x = 16,$ $y = 53.1°$ (vi) $x = 6,$ $y = 50.2°$
2. $x = \frac{3}{2}$

Review Exercise 11

1. (i) $3\sqrt{2}$ (ii) $\overline{AB} \cong \overline{CD}$
3. $\angle ADB = 65°, \angle BDC = 25°, m\widehat{BC} = 50°$

Exercise 12

1. $x = 35^0, y = 145^0$
2. 10 units
3. (i) $c = 62,\ x = 10$ (ii) $a = 45, b = 30$

Review Exercise 12

1. (i) 160° (ii) 60° (iii) 110° (iv) 40° (v) $37\frac{1°}{2}, 37\frac{1°}{2}$ (vi) 50° (vii) 25° (viii) 125°
2. (i) $x = 98°, y = 60°$ (ii) $x = 38°, y = 25\frac{1°}{3}$
4. (i) $x = 80^0$ (ii) $x = 125^0$ (iii) $x = 50^0$ (iv) $x = 45^0$ (v) $y = 12^0$ (vi) $x = 70^0$

Review Exercise 13

1. (i) none of these (ii) none of these (iii) 4 (iv) touch each other internally

GLOSSARY

UNIT 1

Quadratic equation

An equation that can be written in the form

$$ax^2+bx+c=0$$

where *a*, *b*, and *c* are real numbers with $a \neq 0$, is a quadratic equation.

Standard form

A quadratic equation written in the form $ax^2+bx+c=0$ is in standard form.

Zero-factor property of real numbers

If a and b are real numbers, with $ab=0$, then either $a=0$ or $b=0$

Square root property

The solution set of $x^2=k$ is

$$\{\sqrt{k}, -\sqrt{k}\}$$

Quadratic formula

The solutions of the quadratic equation $ax^2+bx+c=0$, where $a \neq 0$, are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Radical equation

An equation in which the variable appears in one or more radicands.

UNIT 2

The discriminant

The expression b^2-4ac that appears under the radical sign in the quadratic formula is called the discriminant.

Cube roots of unity

$1, \omega, \omega^2$ are the cube roots of unity, where $\omega=\frac{-1+i\sqrt{3}}{2}$ and $\omega^2=\frac{-1-i\sqrt{3}}{2}$.

Properties of the cube roots of unity

1. The sum of the cube roots of unity is zero, *i.e.* $1+\omega+\omega^2=0$
2. The product of the cube roots of unity is 1, *i.e.* $1\times\omega\times\omega^2=\omega^3=1$.
3. Each complex cube root of unity is reciprocal of the other, i.e. $\omega=\frac{1}{\omega^2}$ and $\omega^2=\frac{1}{\omega}$.

Relation between the roots and the coefficients of a quadratic equation

If α, β are the roots of the quadratic equation $ax^2+bx+c=0$, then

Sum of the roots $=\frac{-b}{a}$

and product of the roots $=\frac{c}{a}$

Formation of quadratic equation

The quadratic equation whose roots are α, β is given by

x^2- (sum of the roots) x + product of the roots $=0$.

Simultaneous equations

More than one equation which are satisfied by the same values of t variables involved, are called simultaneous equations or a system of equations.

UNIT 3

Ratio

A relation between two quantities of the same kind is called ratio.

Proportion

A proportion is a statement, which is expressed as equivalence of two ratios.

If two ratios a : b and c : d are equal, then we can write a : b = c : d

Direct variation

If two quantities are related in such a way that when one change in any ratio so does the other is called direct variation.

Inverse variation

If two quantities are related in such a way that when one quantity increases, the othe decreases is called inverse variation.

Joint Variation

Joint variation is the same as direct variation with two or more quantities, i.e. Joir variation is a variation where a quantity varies directly as the product of two or mor other quantities. If x is jointly proportional to y and z, we can write $xkyz$=for som constant k. We can also write this relationship as

$$\frac{x}{yz}=k.$$

UNIT 4

Rational fraction

If $P(x)$ and $Q(x)$ are two polynomials and $Q(x)$ is non zero polymomials than the fraction $\frac{P(x)}{Q(x)}$ is called rational fraction.

Proper rational fraction

A rational fraction , $Q(x)\neq 0$ is a proper rational fraction, if the degree of numerator $P(x)$ is less than the degree of denominator $Q(x)$.

Improper rational fraction

A rational fraction $\frac{P(x)}{Q(x)}$, $Q(x)\neq 0$ is an improper rational fraction, if the degree of numerator $P(x)$ is equal to or greater than the degree of denominator $Q(x)$.

Partial fractions

Splitting up a single rational fraction into two or more rational fraction with single factor in denominator, such a procedure is called partial fractions

UNIT 5

A set is a "collection of well-defined distinct objects. Sets are represented by capital English alphabets, A, B, C .... Z and elements of sets are represented by small English alphabets, *a*, b, c, ... z.

Union of two sets.

If A and B are two sets, then the union of set A and set B consists of all elements in set A or in set B or in both A and B, and it is denoted by $A\cup B$. Symbolically.

$$A\cup B=\{x \mid x\in A \text{ or } x\in B\}$$

Intersection of two sets

If A and B are two sets, then the intersection of set A and set B consists of all those elements which are common to both A and B, and it is denoted by $A\cap B$, Symbolically,

$$A\cap B=\{x \mid x\in A \wedge x\in B\}$$

Disjoint sets

Two sets A and B are disjoint if $A\cap B=\varphi$.

Complement of a set

If U is universal set and A is a subset of U, then U \ A is called complement of set A, and is denoted by A' or A^c.

Difference of two sets

If A and B are two sets, then their difference consists of all those elements of A which are not in B, and it is denoted by A \ B or A – B symbolically

$$A\setminus B=\{x \mid x\in A \wedge x\notin B\}$$

Commutative property of union

For any two sets A and B

$$A\cup B=B\cup A$$

Commutative property of intersection

For any two sets A and B

$$A\cap B=B\cap A$$

Associative property of union

For any three sets A, B and C

$$A\cup(B\cup C)=(A\cup B)\cup C$$

Associative property of intersection

For any three sets

$$A\cap(B\cap C)=(A\cap B)\cap C$$

Distributive property of union over intersection

For any three sets A, B and C

$$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$$

Distributive property of intersection over union

For any three sets A, B and C

$$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$$

De. Morgan's laws

For any two sets A and B

(i) $(A\cup B)'=A'\cap B'$ (ii) $(A\cap B)'=A'\cup B'$

Venn diagrams

Concept of sets i.e. union, intersection, complement, difference of sets, can be explained easily with the help of Venn diagrams. In these diagrams, a set is usually represented by a circle and universal set is represented by a rectangle.

Ordered pairs

(*a*, b) is called an ordered pair of two elements *a* and b of a set or of different sets, where a is the first element and b is the second element.

$$(a, b)\neq(b, a)$$

Cartesian product

If A and B are two non- empty sets then A × B is called Cartesian product, which is set of all ordered pairs such that the first element of each ordered pair belongs to set A and second element of each ordered pair belongs to set B symbolically.

$$A\times B=\{(x, y) \mid x\in A \text{ and } y\in B\}$$

If $A\neq B$, then $A\times B\neq B\times A$

Binary relation

Any subset of A × B is called a binary relation, where A and B are two non-empty sets.

Domain and range of binary relation

If R is a binary relation from set A to set B, i.e.

$R=\{(x, y) \mid x\in A\wedge y\in B\}$, then domain of R is set of first elements of all ordered pairs in R and is denoted by "Dom R".

The range of R is set of second elements of all ordered pairs in R, and is denoted by "Range R".

Function

If A and B are two non-empty sets, then a binary relation f is said to be a function from A to B, if

(i) Dom f = A

(ii) There should be no repetition in the first elements of all ordered pairs contained in *f*.

Symbolically

$f : A \longrightarrow B$

Domain, co-domain and range of a function

Let $f : A \longrightarrow B$ be a function, then set A is called domain of f, set B is called co-domain of f and the set of second elements of all ordered pairs contained in f is called range of f. The range is subset of co-domain

i.e. Range $f \subseteq B$

Into function

Let f be a function from A to B, then f is into function, if

Range $f \neq B$

One – one or 1–1 function

Let f be a function from A to B, then f is one-one function, if for each $x \in A$ there exist unique $y \in B$, i.e. there is no repetition in the second element of all ordered pairs contained in f.

Into and one-one function (injective function)

Let f be a function from A to B, then f is injective (into and one - one) function if

(i) Range $f \neq B$

(ii) There is no repetition in the second elements of all ordered pairs contained in f.

Onto function (surjective function)

Let f be a function from A to B, then f is onto function if

Range f = B

One-one and onto function (bijective function)

Let f be function from A to B, then f is one-one and onto (bijective) function if it is both one-one and onto.

One-one correspondence

If A and B are two non-empty sets then one-one correspondence between A and B is a rule for which each element of set A is paired with one and only one element of B and each element of B is paired with one and only one element of A, and non of the members of any set remains unpaired. It is also known as one-to-one function. In one-one correspondence both sets A and B have same number of elements.

UNIT 6

Histogram

A graphical representation of data in the form of rectangles is called Histogram.

Frequency

A number showing the repetition of a value in a given set of data.

Frequency Polygons

A curve on the graph showing the frequency of values.

Median

A value lying in the middle of arranged data is called median.

Mode

A value repeated maximum times in a given set of data is called mode. It shows the trend of a data and hence is usually used to find public opinion.

Range

The difference between maximum value and minimum value in given set of data.

UNIT 7

An angle is a union of two rays which have a common point (vertex) one of the ray is called "initial side" and other ray is called "terminal side".

Sexagesimal system (degrees, minutes, seconds)

It is the system of measurement of an angle in which one complete rotation is divided into 360 parts called degrees, written as 360°. One degree is divided into 60 parts called minutes, written as $60'$ and one minute is again divided into 60 parts called seconds, written as $60''$.

Circular system (radians)

It is another system of measurement of an angle. In this system unit of measure of angle is radian. One radian is an angle subtended at the centre of a circle an arc whose length is equal to radius of the circle.

Length of an arc is measured by the formula $\ell = r\theta$ where ℓ is length of an arc, θ is central angle of a circle measured in radians and r is radius of circle.

Area sector of circle

Area of sector of a circle is given by $A = \frac{1}{2}r^2\theta$. Where r is radius of circle, θ is central angle of sector measured in radians.

Coterminal angles

Angles having the same initial and terminal sides are called co-terminal angles and differ by a multiple of 2π radians or 360° they are also called general angles.

Angle in standard position

In XY-plane (co-ordinate plane), if the vertex of an angle lies at origin and initial side lies on positive x-axis, then such an angle is said to be in standard position.

Quadrants and quadrantal angles

XY-plane divided into four equal parts each part is called quadrant. Being in the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} quadrants means measure of angle from 0° to 90°, 90° to 180°, 180° to 270° and 270° to 360° respectively quadrant angles are 0°, 90°, 180°, 90, 360°.

Trigonometric ratios of unit circle

These are $\sin\theta = y$, $\cos\theta = x$, $\tan\theta = \frac{y}{x}$

and $\cos = \frac{1}{y}$, $\sec\theta = \frac{1}{x}$, $\cot\theta = \frac{x}{y}$

Trigonometric identities are

(i) $\cos^2\theta + \sin^2\theta = 1$

(ii) $1 + \tan^2\theta = \sec^2\theta$

(iii) $1 + \cot^2\theta = \cos^2\theta$

Angle of elevation

If an object is above the level of observer's sight then the angle between the horizontal line and observer's line of sight is called angle of elevation.

Angle of depression

If an object is below the level of observer's sight then the angle between the horizontal line and observer's line of sight is called angle of depression.

UNIT 8-13

Acute angle

An angle is acute if and only if it has a measure greater than 0° and less than 90°.

Acute triangle

A triangle is acute if and only if it has three acute angles.

Adjacent angles

Two coplanar angles are adjacent if and only if they have a common arm but no points in the interior of one angle are in the interior of the other.

Altitude of a triangle

An altitude of a triangle is the perpendicular segment joining a vertex of the triangle to the line that contains the opposite side.

Angle

An angle is the union of two non-collinear rays with the same end point.

Arc of a circle

Any portion of the circumference of a circle is called an arc of the circle.

Bisector of an angle

A ray that divides an angle into two equal adjacent angles is the bisector of the angle.

Central angle

A central angle of a given circle is an angle whose vertex is at the centre of the circle.

Chord

A chord is a segment whose end points are on the circle.

Circle

A circle is the set of all coplanar points equidistant from a given point.

Circular region

A circular region is the union of a circle and its interior

Circumference of a circle

The perimeter of a circle is called its circumference.

Collinear points

Points are collinear if and only if there is a line that contains all of them.

Complementary angles

Two angles are complementary if and only if the sum of their measures is 90°.

Concentric circles

Two or more circles (in the same plane) are concentric if they share the same centre point.

Concurrent lines

Two or more lines are concurrent if and only if there is a single point that lies on all of them.

Congruent angles

Angles of the same degree measure.

Congruent arcs

Two arcs of a circle are said to be congruent if and only if they have the same degree measures.

Congruent circles

Circles with congruent radii are called congruent circles.

Congruent figures

Geometric figures are congruent if and only if they have the same size and shape.

Congruent segments

Two or more line segments are congruent if and only if they have the same length.

Congruent triangles

Given a correspondence ABC « DEF between the vertices of two triangles, if the corresponding sides are congruent, and the corresponding angles are congruent then the correspondence ABC « DEF is called a congruence between the two triangles.

Coplaner points

points are coplaner if and only if there is a plane that contains all of them.

Corollary

A statement that can easily be proved by applying a theorem.

Diameter

A chord passing through the centre of a circle or in the same circle, two collinear radii form a diameter.

Externally tangent

Two or more circles are externally tangent if they intersect in exactly one point and if their interiors do not intersect.

Half – plane

Given a line and the plane containing it, the two sets separated by the line are called half – planes.

Internally tangent

Two or more circles are internally tangent if they intersect in exactly one point and if the interior of one contains the interior of the other.

Linear pair

Two angles form a linear pair if and only if they are adjacent angles and the noncommon sides are opposite rays.

Major and Minor arcs

The larger of the two arcs is called the major arc and the smaller one is called the minor arc.

Measure of an arc

The degree measure of a minor arc is the measure of the corresponding centre angle.

The degree measure of a major arc is 360° minus the degree measure of the corresponding minor arc.

The degree measure of a semi-circle is 180°. The degree measure of a circle is taken to be 360°.

Median of a triangle

A segment is a median of a triangle if and only if its end points are a vertex of the triangle and the midpoint of the opposite side.

Midpoint

A point B is called the midpoint of $\overline{AC}$ if and only if (i) B is between A and C and (ii) $m\overline{AB} = m\overline{BC}$.

Obtuse angle

An angles is obtuse if and only if it has a measure greater than 90° but less than 180°.

Obtuse triangle

A triangle is obtuse if and only if it has one obtuse angle.

Opposite rays

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called opposite rays if and only if A is between B and C.

Parallel lines

Two lines are parallel if they do not intersect.

Perpendicular bisector

In a given plane, the perpendicular bisector of a segment is the line that is perpendicular to the segment at its midpoint.

Radius of a circle

A radius of a circle is

(i) Any segment with one endpoint at the centre and the other end point on the circle and

(ii) The distance from the centre to the circle.

Ray

Ray $AB\left(\overrightarrow{AB}\right)$ is the figure that contains A and every point on the same side of A as B.

Right angle

An angle of measure 90°.

Secant

A secant is a line that intersects a circle in exactly two points.

Sector

If $\overset{\frown}{AB}$ is an arc of a circle with centre O and radius r, then the union of all segments $\overline{OP}$, where P is any point of $\overset{\frown}{AB}$, is a sector.

Segment of a circle

A segment of a circle is the region bounded by a chord and an arc of the circle.

Supplementary angles

Two angles are supplementary if and only if the sum of their measures is 180°.

Tangent

A line that intersects a circle at exactly one point is called a tangent, and the point of intersection is called the point of tangency (or point of contact).

Index

A

Alternando 58
Angle 192
Angle of depression 187
Angle of elevation 187
Angle in standard position 173
Apollonius theorem 198
Arc 226
Area of a circular sector 170
Arithmetic mean 133
Associative property 88

B

Basic Statistics 120
Bijective function 113
Binary relation 107
Biquadratic 4

C

Cartesian product 106
Central angle 206
Central tendency 133
Chord 202
Circle 202
Circum-angle 241
Circular system 165
Circumference 240
Circumscribe 257
Circumscribed circle 249
Circumscribed hexagon 258
Class boundaries 123
Class limits 123
Class width 123
Co-domain 111
Co-efficients 5,31
Common tangent 264
Commutative property 88
Complement of a set 86
Completing square 5
Complex cube roots 28
Componendo 58,59
Componendo-dividendo 58
Concentric circles 212
Convergent lines 267
Congruent circles 227
continued proportion 57
Correspondence 113
Corresponding chords 207
Coterminal angles 172
Cube roots of unity 28
Cumulative frequency 130
Cumulative frequency polygon 130
Cyclic quadrilateral 249

D

Data 121
De Morgan's Laws 94
Degree 166
Denominator 72
Diameter 202
Disjoint set 86
Difference of sets 86
Direct method 134
Direct proportion 67
Direct variation 53
Discriminant 23
Dispersion 153
Distributive property 89
Dividendo 58
Domain 108,111

E

Equidistant 203
Equilateral triangle 256
Exponential equations 13
Externally 219
Extraneous roots 16
Extremes 52

F

Factorization 3
Formation of quadratic equation 36
Fourth proportional 57
Frequency distribution 122
Frequency polygon 127
Function 109

G

General angle 172
Geometric mean 143
Grouped data 133

H

Half circle 226
Harmonic mean 145,146
Hexagon 247
Histograms 125

I

Imaginary 23
Improper fraction 72
Injective function 112
Inscribed angle 249
Inscribed polygon 249
Internal tangent 264
Intercepted arc 226,249
Internally 221
Intersecting circles 265
Intersection of sets 86
Into function 111
Inverse proportion 55
Inverse variation 55
Invertendo 58
Irrational 23

J.K.L

Joint variation 61
K-method 63
Linear equation 49
Linear factors 73

M

Major segment 241
Mean proportion 57
Mean 52,133,135
Median 133,139
Midpoint 123
Minor arc 226
Major arc 226
Minor segment 241
Mode 139,150

N

Nature of the roots 23,25
Non-collinear points 250
Non-repeated 73
Numerator 12

O

Obtuse angle 194
One-One function 112
Onto function 113
Ordered pairs 106

P

Partial fractions 71
Perfect square 5,7
Perpendicular bisectors 251
Polygon 253
Polynomial 72

Product of the roots 32
Projection 194,196
Proper fraction 72
Properties of cube roots of unity 29
proportions 52
Pythagorean theorem 184,205

Q

Quadrantal angle 174
Quadrants 173,176
Quadratic equation 2
Quadratic formula 6
Quadrilateral 245
Quartic equation 42
Quartiles 151
Quotient 41

R

Radian 166
Radical equation 16
Radius 202
Range 108,111
Ratio 57
Rational fraction 72
Real life problems 46,68
Reciprocal equations 11
Reducible to quadratic form 9
Remainder 40,41
Roots 2

S

Secant 212
Sector of a circle 168
Semi-circle 226
Sets 85
Sexagesimal 164
simultaneous equations 43
Solution 2
Solution set 2
Standard deviation 155
Standard form 5
Standard position 173
Sum of the roots 32
Supplementary 245
Surjective function 113
Symmetric functions 35
Synthetic division 39
System of equations 45

T

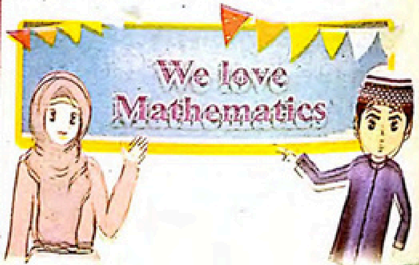
Tally marks 121
Tangent 212
Third proportional 57
Touching circles 265
Transverse 264
Trigonometric identities 184
Trigonometric ratios 172

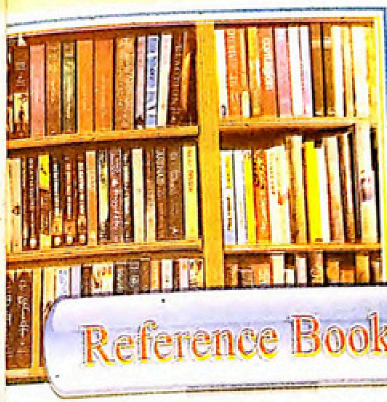
U

Unequal circles 265
Ungrouped data 133
Union of sets 86

V,Z

Variable 52
Variance 155
Variations 52
Venn diagrams 95
Weighted mean 147





Reference Books

1. Pre-Algebra, Student Edition-Glencoe by Carol Malloy, Ph.D., Jack Price, Ed.D. Published by McGraw-Hill (2002)
2. Algebra and Trigonometry, 3rd Ed. by Judith A. Beecher, Judith A. Penna, Marvin L. Bittinger Published by Addison Wesley; (2007)
3. Holt Geometry by Schults. Holt Rinehart & Winston Published by Houghton Mifflin Harcourt (2004)
4. Geometry 1,001 Practice Problems For Dummies by Allen Ma, Amber Kuang (2015)
5. Fundamentals of college geometry by Edwin M.Hemmerling
6. Geometry by Cindy J. Boyd, Jerry Cummins, Carol E. Malloy, John A. Carter, Alfinio Flores Published by Mcgraw Hill (2007)
7. Basic math and algebra-United States NAVY (1985)
8. Introduction to algebra (the art of problem solving) by Richard Rusczyk Published by AoPS Incorporated (2007)
9. Eureka-Math-Fun-from-Many-Angles by David Lewis Published by Perigee Trade
10. Maths-for-Fun-Creative-and-Innovative-Techniques by T. Satyanarayana Raju
11. Teach yourself algebra by Abbott P. Published by Hodder and Stoughton (1988)

About the Authors

Dr. Muhammad Shah

Dr. Muhammad Shah did his M.Sc from University of Peshawar in 1998. He did his PhD in 2012 from Quaid-i-Azam University Islamabad in computational Algebra. In 2010, he was awarded IRSIP scholarship of HEC for Birmingham University UK.

Dr. Shah has been teaching Mathematics on postgraduate level since 2002 in colleges of Khyber Pakhtunkhwa. He has been the Subject Specialist of Mathematics and Computer Science of Khyber Pakhtunkhwa Textbook Board since 2014 to 2017. He is the editor and reviewer of several textbooks of Mathematics and Computer Science.

Mr. Naseer Ahmad

Mr. Naseer Ahmad is a retired Asistant Professor of Mathematics in the Mathematics Department, University of Peshawar. He secured M.Sc from the University of Peshawar with Gold Medal and MS from Temple University USA. He has published a number of research papers.

Ms. Shoukat Qadeer

Ms. Shoukat Qadeer did M.Sc Mathematics from Punjab University Lahore. She is a Lecturer in Maths in Agricultural University Public School And College for Girls Peshawar.

Mr. Zulfiqar Ali Jamil

Mr. Zulifiqar Ali Jamil is Senior Teacher in Islamia Collegiat School Peshawar. He is an M.Phil in Education and has vast experience as a teacher.

Mr. Abdul Salam Kohsary

Mr. Abdul Salam Kohsary is a Senior Subject Teacher in G.H.S.S. Gujar Garhi (Mardan).

He joined several national and international level seminars and competition. He has a vast experience in writing of textbooks.

مومن تو آپس میں بھائی بھائی ہیں

تو اپنے دو بھائیوں میں صلح کرادیا کرو

اور اللہ سے ڈرتے رہو

تاکہ تم پر رحم کیا جائے۔

(سورۃ الحجرات : ۱۰)