PYTHAGORAS THEOREM

Unit 23

- Weightage = 7%

Students Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Understand the following theorem along with itscorollaries and apply them to solve allied problems.
  - In a right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides. (Pythagoras'Theorem).
  - If the square of one side of a triangle is equal to thesum of the squares of the other two sides then the triangle is a right angled triangle, (converse to Pythagoras' Theorem).

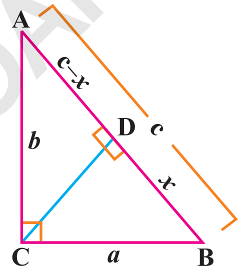
Introduction:

Pythagoras, a Greek philosopher and mathematician was born around 570BC. He discovered a very important relationship between the sides of right angled triangle. The Pythagorean theorem or Pythagoras theorem is a fundamental relation in Ecludian geometry among the three sides of a right angled triangle. He developed this relationship in the form of theorem called Pythagoras theorem after his name. The theorem can be proved by various methods. Here we shall prove it by using the concept of similar triangles. We shall state and prove its converse also and then apply them to solve different problem of daily life.

23.1.1 Pythagoras Theorem:

Theorem 23.1.

In a right angled triangle, the square of the length of hypotenuse is equal to sum of the squares of the length of the other two sides.



Given: ΔABC is a right angled triangle, having right angle at C. The measures of sides $\overline{AB}, \overline{AC}$ and $\overline{BC}$ are c, b and a respectively.

To prove: $c^2 = a^2 + b^2$

Construction:

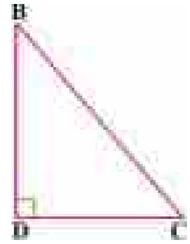
Draw an altitude of vertex C to side $\overline{AB}$. say $\overline{BD} = x$

Proof:

Statements	Reason
In $\Delta ABC \leftrightarrow \Delta CBD$	
$\angle ACB \cong \angle CDB$	$\angle CDB$ is right angle (Construction)
$\angle B \cong \angle B$	Common angle
and $\angle BAC \cong \angle BCD$	Complements of $\angle B$
$\therefore$ $\Delta ACB \quad \Delta CBD$	By definition of similar Δs.
Hence $\frac{c}{a} = \frac{a}{x}$	Corresponding sides of similar triangles
$cx = a^2$ ...(i)	
Again in $\Delta ACB \leftrightarrow \Delta ADC$	
$\angle ACB \cong \angle ADC$	$\angle ADC$ is right angle (Construction)
$\angle A \cong \angle A$	Common angles
and $\angle CBA \cong \angle DCA$	Complements of $\angle A$
$\therefore$ $\Delta ACB \quad \Delta ADC$	
Hence $\frac{c}{b} = \frac{b}{c-x}$	Corresponding sides of similar triangles
$c(c-x) = b^2$	
or $c^2 - cx = b^2$...(ii)	
Adding equation (i) and (ii) we get,	
$cx + c^2 - cx = a^2 + b^2$	
$c^2 = a^2 + b^2$	

Q.E.D

Corollary: In a right angled triangle *ABC*, right angle at *B*.



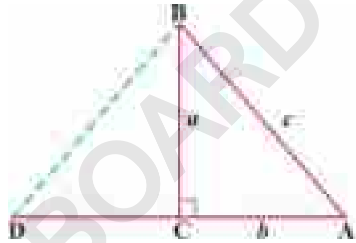
i. $\left(m\overline{BC}\right)^2 = \left(mAC\right)^2 - \left(mAB\right)^2$

ii. $\left(m\overline{AB}\right)^2 = \left(mAC\right)^2 - \left(mBC\right)^2$

Theorem 23.2

(Converse of Pythagoras theorem)

If the square of one side of triangle is equal to the sum of the squares of the other two sides, then the triangle is a right angled triangle.



Given:

In a $\Delta ABC, m\overline{BC} = a, m\overline{AC} = b$ and $m\overline{AB} = c$

such that $a^2 + b^2 = c^2$

To prove:

ΔABC is a right angled triangle.

Construction:

Draw $\overline{CD}$ perpendicular to $\overline{BC}$ such that $\overline{CD} \cong \overline{CA}$. Join the points *B* and *D*.

Proof:

Statements	Reason
ΔDCB is a right angled triangle	Construction
$\therefore \quad \left(m\overline{BD}\right)^2 = a^2 + \left(m\overline{DC}\right)^2$	Pythagoras theorem
$\Rightarrow \quad \left(m\overline{BD}\right)^2 = a^2 + b^2$	
But $\quad a^2 + b^2 = c^2$	Given
$\therefore \quad \left(m\overline{BD}\right)^2 = c^2$	Transitive property of equality
$\therefore \quad m\overline{BD} = c$	By taking square root of both sides
Now in	
$\Delta DCB \leftrightarrow \Delta ACB$	
$\overline{CD} \cong \overline{CA}$	Construction
$\overline{BC} \cong \overline{BC}$	Common
$\overline{BD} \cong \overline{AB}$	Each side equal to C (Proved above).
$\therefore \quad \Delta DCB \cong \Delta ACB$	S.S.S ≅ S.S.S
$m\angle DCB = m\angle ACB$	Corresponding angles of congruent triangles
But $\quad m\angle DCB = 90^\circ$	Construction
$\therefore \quad m\angle ACB = 90^\circ$	Transitive property of equality.
Hence ABC is a right angled triangle.	$\therefore \quad m\angle ACB = 90^\circ$

Q.E.D

Corollary:

Let a, b and c be the sides of a triangle. Such that the side c be the longest side. Then,

i. If $a^2 + b^2 = c^2$ then the triangle is right angled triangle.

ii. If $a^2 + b^2 > c^2$ then the triangle is acute angled triangle.

iii. If $a^2 + b^2 < c^2$ then the triangle is obtuse angled triangle.

Example 1:

Measures of the sides of a triangle are given decide which of these represent right triangles.

i. $a = 5\text{cm},\ b = 12\text{cm}$ and $c = 13\text{cm}$

ii. $a = 6\text{cm},\ b = 7\text{cm}$ and $c = 8\text{cm}$

iii. $a = 9\text{cm},\ b = 12\text{cm}$ and $c = 15\text{cm}$

Solution:

i. Since $(13)^2 = (5)^2 + (12)^2$

$$169 = 25 + 144$$
$$169 = 169$$

By the converse of Pythagoras theorem, the a,b and c are the sides of right triangle.

ii. Since $(8)^2 \neq (6)^2 + (7)^2$

i.e. $64 \neq 36 + 49$

or $64 \neq 85$

By the converse of Pythagoras theorem, the a,b and c are not the sides of right triangle.

iii. Since $(15)^2 = (12)^2 + (9)^2$

$$225 = 144 + 81$$
$$225 = 225$$

By the converse of Pythagoras theorem, the a,b and c are the sides of right triangle.

Example 2:

An anchor line for a tower needs to be placed. The tower is 96fts. The anchor line is 105fts long. How far from the tower can it be placed?

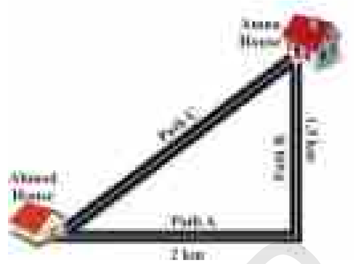
Solution:

Let the anchor line be placed at x ft from the tower. By Pythagoras theorem.

$$(105)^2 = (96)^2 + (x)^2$$
$$\Rightarrow \quad 11025 = 9216 + x^2$$
$$\Rightarrow \quad x^2 = 1809$$
$$x = \sqrt{1809} = 42.53\text{ft}$$

Hence the required distance is 42.53 ft approx.

Example 3:



There are two streets that one can choose to go from Amna's house to Ahmed's house. One way is to take C path, and the other way requires to take A path of 2 km and then B path of 1.5 km. How much shorter is the direct path along C path as shown in the figure.

Solution:

Let x be the length of path C.

By Pythagoras theorem.

$$x^2 = (2)^2 + (1.5)^2$$

$$\Rightarrow \quad x = \sqrt{(2)^2 + (1.5)^2} = \sqrt{6.25} = 2.5\text{km}$$

By using the alternative way, he has to cover the distance $= 2 + 1.5 = 3.5\text{km}$

The difference between these two paths $= 3.5 - 2.5 = 1\text{km}$

EXERCISE 23.1

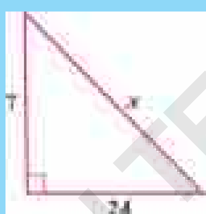
1. Following are the length of sides of triangle, verify that the triangle is right triangle.

i. $a = 16 \;\; b = 30 \;\; c = 34$ ii. $a = 6 \;\; b = 8 \;\; c = 10$

iii. $a = 15 \;\; b = 20 \;\; c = 25$ iv. $a = 13 \;\; b = \sqrt{56} \;\; c = 15$

2. Find unknown values in each of the following figures.

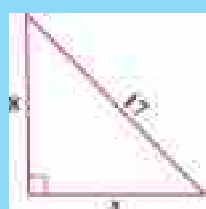
i.



ii.

iii.

iv.

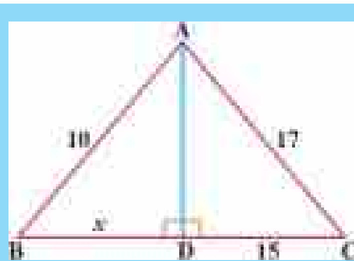


3. The three sides of a triangle are of measure 9.5cm, 7.5cm and $x$cm. For what value of x will the side represent right triangle?

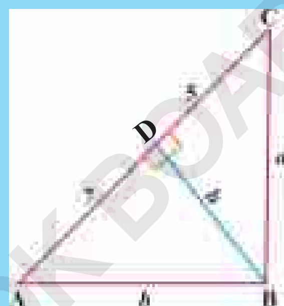
4. ABC is an isosceles triangle with $m\overline{AB} = m\overline{AC} = 13\text{cm}$ and $m\overline{BC} = 10\text{cm}$. Calculate the perpendicular distance from A to $\overline{BC}$.

5. The foot of a ladder is placed 6 feet from a wall. If the top of the ladder rests 8 feet up on the wall. How long is the ladder?

6. Find the value of x in the adjacent figure.



7. In the ΔABC, $m\angle B$ is a right angle and $\overline{BD}$ is a perpendicular on $\overline{AC}$ such that $m\overline{CD}=5$ units and $m\overline{AD}=7$ units as shown in the figure. Find the lengths of the unknowns a, h and b.



8. The sides of a rectangular swimming pool are 50m and 30m. What is the length between the opposite corners?
9. The length of each side of an equilateral triangle is 8 units. Find the length of any one altitude.
10. The sides of a triangle have lengths $x, x+4$ and 20. If the length of the longest side is 20. What values of x make the right triangle?
11. A manager goes 18m due east and then 24m due north. Find the distance of his current position from the starting point.
12. In the rectangle ABCD, $m\overline{BC}+m\overline{CD}=17\text{cm}$ and $m\overline{BD}+m\overline{AC}=26\text{cm}$ calculate the length and breadth of the rectangle.

REVIEW EXERCISE 23

1. Encircle the correct option

i. Diagonal of a rectangle measures 6.5cm. If its width is 2.5cm, its length is

(a) 9cm (b) 4cm
(c) 6cm (d) 3cm

ii. Which of the following are the sides of a right angled triangle?

(a) 3, 4, 5 (b) 2, 3, 4
(c) 5, 6, 7 (d) 4, 5, 6

iii. In a right angled triangle the greatest angle is

(a) 100^o (b) 90^o
(c) 80^o (d) 110^o

iv. In a right angled triangle hypotenuse is opposite side to

(a) Acute angle (b) Right angle

(c) Obtuse angle (d) None

v. If *a*, *b*, *c* are sides of right angled triangle, with *c* is the larger side, then

(a) $c^2 = a^2 + b^2$ (b) $b^2 = c^2 + a^2$

(c) $a^2 = b^2 + c^2$ (d) $c^2 = a^2 - b^2$

vi. If 5cm and 12cm are two sides of a right angled triangle. Then hypotenuse is

(a) 16 (b) 15

(c) 14 (d) 13

vii. If hypotenuse of an isosceles right-angled triangle is $3\sqrt{2}$cm, then each of other side is of length

(a) 2cm (b) 5cm

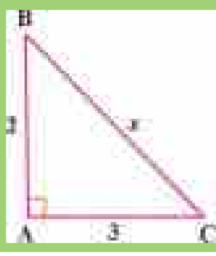
(c) 3cm (d) 1cm

2. Explain the Pythagoras theorem.

3. A ladder 25m long rests against a vertical wall. The foot of the ladder is 7m away from the base of the wall. Find the height of the wall given that ladder reaches at its top end.

4. Find the unknown values in each of the following figures

i)



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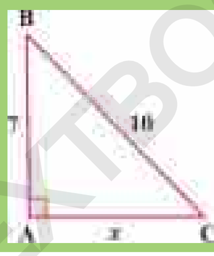
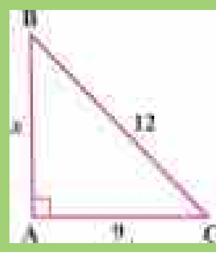


iii)



SUMMARY

- In a right angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
- If the square of the one side of a triangle is equal to the sum of the square of the other two sides then the triangle is a right angled triangle.