PRACTICAL GEOMETRY - CIRCLES

Unit 29

• Weightage = 6%

Students Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Locate the centre of a given circle
- Draw a circle passing through three given non-collinear points.
- Complete the circle:
  - By finding the centre,
  - Without finding the centre, when a part of its circumference is given.
- Circumscribe a circle about a given triangle.
- Inscribe a circle in a given triangle.
- Escribe a circle to a given triangle.
- Circumscribe an equilateral triangle about agiven circle.
- Inscribe an equilateral triangle in a given triangle.
- Inscribe an equilateral triangle in a given circle
- Circumscribe a square about a given circle.
- Circumscribe a regular hexagon about a givencircle.
- Inscribe a regular hexagon in a given circle.
- Draw a tangent to a given arc, without using thecentre, through a given point P, when P is :
  - The middle point of the arc,
  - At the end of the arc,
  - Outside the arc.
- Draw a tangent to a given circle from a point *P*, when *P* lies
  - On the circumference,
  - Outside the circle.
- Draw two tangents to a circle meeting each otherat a given angle.
- Draw
  - Direct common tangent or external tangent,
  - Transverse common tangent or internaltangent to two equal circles,
- Draw
  - Direct common tangent or external tangent,
  - Transverse common tangent or internaltangent to two unequal circles,
- Draw a tangent to
  - To unequal touching circles,
  - Two un equal intersecting circles,
- Draw a circle which touches
  - Both the arms of a given angle,
  - Two converging lines and passes through agiven point between them,
  - Three converging lines.

Introduction:

We know that Practical Geometry is an important branch of Geometry which is used in architecture, computer graphics, art etc. We have already learnt to construct angles, triangles, rectangle, pentagon etc in previous classes. Let us learn the construction of circles and related figures.

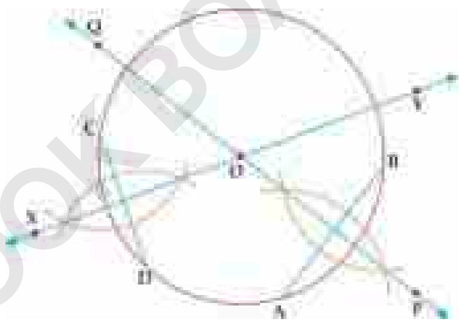
29.1 Construction of circles:

We know that a circle can easily be drawn by using compass when its centre and radius are given. In this section we will also learn to draw circles when centre and radius are not given

29.1(i) Locate the centre of a given circle:

In order to locate the centre of a given circle, we will use the fact that the right bisectors of two non-parallel chords of a circle always intersect each other at the centre of the circle. Method of locating the centre of a given circle is explained with the help of the following example.

Example: Locate the centre of given circle as shown in the figure.



Given: A circle

Required: To locate the centre of the given circle.

Steps of Construction:

(i) Draw two non-parallel chords $\overline{AB}$ and $\overline{CD}$ of the given circle.

(ii) Draw the right bisector $\overleftrightarrow{PQ}$ of the chord $\overline{AB}$.

(iii) Draw the right bisector $\overleftrightarrow{XY}$ of the chord $\overline{CD}$

(iv) Both the right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{XY}$ intersect each other at point O. The point O is the centre of the given circle.

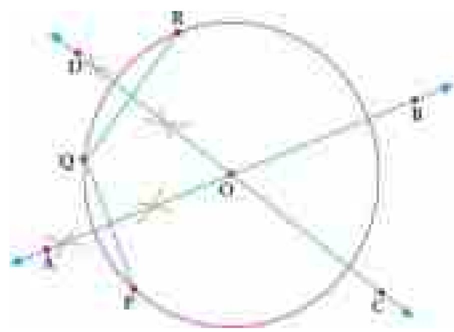
29.1(ii) Draw a circle passing through three given non-collinear points.

In order to draw a circle passing through three given non-collinear points, we will first locate the centre and then draw the circle as explained in the following example.

Example: Draw a circle passing through three non-collinear point P, Q and R

Given: Three non-collinear points P, Q and R.

Required: To draw a circle which passes through P, Q and R.



Steps of Construction:

(i) Take any three non-collinear points P, Q and R

(ii) Draw $\overline{PQ}$ and $\overline{QR}$

(iii) Draw the right bisector $\overleftrightarrow{AB}$ of $\overline{PQ}$

(iv) Draw the right bisector $\overleftrightarrow{CD}$ of $\overline{QR}$

(v) Both right bisectors $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect each other at point O which is the centre of the required circle.

(vi) With O as centre and radius equal to $m\overline{OP}$ or $m\overline{OQ}$ or $m\overline{OR}$, draw a circle. This circle is the required circle.

29.1(iii) Complete the Circle:

- **By finding the centre.**
- **Without finding the centre.**

When a part of its circumference is given.

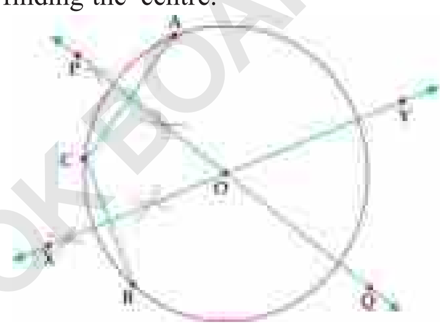
(a) Completion of circle by finding the centre when a part of its circumference is given.

We can complete a circle when a part of its circumference or an arc is given by finding its centre with the help of its any three non-collinear points. The method is explained in the following example.

Example: Complete a circle whose arc AB is given by finding the centre.

Given: A part of circumference or an arc AB is given.

Required: To complete a circle whose arc is given.



Steps of Construction:

(i) Draw the arc AB

(ii) Take a point C on AB other than A and B.

(iii) Draw $\overline{AC}$ and $\overline{BC}$.

(iv) Draw the right bisector $\overleftrightarrow{PQ}$ of $\overline{AC}$

(v) Draw the right bisector $\overleftrightarrow{XY}$ of $\overline{BC}$

(vi) Both $\overleftrightarrow{PQ}$ and $\overleftrightarrow{XY}$ interest each other at point O which is the centre of the circle.

(vii) With point O as centre and radius equal to $m\overline{AO}$ or $m\overline{OB}$ or $m\overline{OC}$, draw the remaining part of the circle.

Thus we have a complete circle of the given arc $\widehat{AB}$.

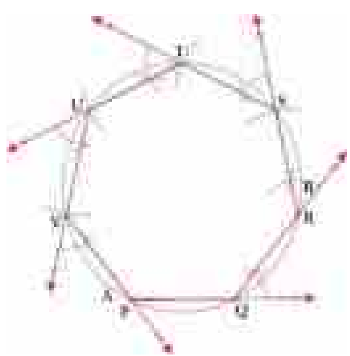
(b) Completion of a circle without finding the centre when a part of its circumference is given.

We can complete a circle without finding the centre when an arc is given with the help of regular polygon. We draw regular polygon with the help of either exterior angles or interior angles which are congruent. The method is explained with the help of the following example.

Example: Complete a circle whose arc AB is given without finding centre.

Given: An arc AB of a circle.

Required: To complete the circle of AB



Steps of Construction:

(i) Draw two congruent chords $\overline{PQ}$ and $\overline{QR}$ of given $\widehat{AB}$

(ii) Draw exterior $\angle R$ congruent to $\angle Q$ and draw $\overline{RS}$ congruent to $\overline{PQ}$

(iii) Draw exterior $\angle S$ congruent to $\angle R$ and draw $\overline{ST}$ congruent to $\overline{PQ}$

(iv) By the same way, we get point S, T, U and V

(v) So we get a regular heptagon PQRSTUV

(vi) Draw dotted arcs of $\overline{RS}$, $\overline{ST}$, $\overline{TU}$, $\overline{UV}$ and $\overline{VP}$

These dotted arcs complete the circle.

EXERCISE: 29.1

1. Draw a circle with the help of a circular object. Locate its centre.
2. Take three non-collinear points X, Y, Z and draw a circle which passes through these points.
3. Draw a minor arc AB by a circular object. Complete the circle by finding the centre.
4. Draw a major arc ABC by a circular object. Complete the circle by finding the centre.
5. Draw a minor arc AB by a circular object. Complete the circle without finding the centre.

29.2 Circles attached to polygons:

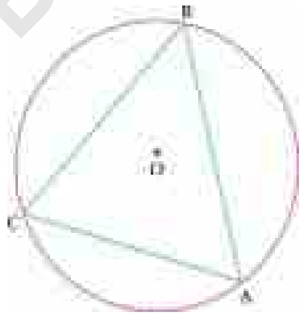
Recall that polygon is a three or more sided closed figure for example, triangle, quadrilateral, pentagon, hexagon etc.

The circles attached to polygons are those circles which either pass through all the vertices of the polygon or touch the sides of the polygon.

29.2 (i) Circumscribe a circle about a given triangle:

Circumcircle of a triangle:

A circle which passes through all the three vertices of a triangle is called circumcircle of the triangle. In the given figure, there is circumcircle of ΔABC with centre O which is called circumcentre.

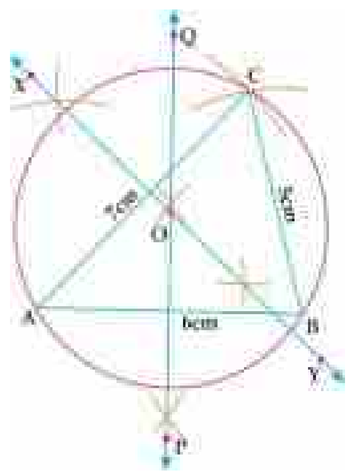


Example: Draw the circumcircle of ΔABC in which $m\overline{AB} = 6cm$, $m\overline{BC} = 5cm$ and $m\overline{AC} = 7cm$.

Given: A triangle ABC in which $m\overline{AB} = 6cm$, $m\overline{BC} = 5cm$ and $m\overline{AC} = 7cm$.

Required: To draw a circumcircle of ΔABC.

Steps of Construction:



(i) Construct a ΔABC with the help of given data.

(ii) Draw the right bisector $\overleftrightarrow{PQ}$ of $\overline{AB}$

(iii) Draw the right bisector $\overleftrightarrow{XY}$ of $\overline{AC}$

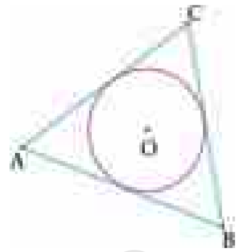
(iv) Both the right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{XY}$ intersect each other at point O.

(v) With O as centre and radius equal to $m\overline{OA}$ or $m\overline{OB}$ or $m\overline{OC}$, draw a circle which passes through A, B and C.

This is the required circumcircle of ΔABC.

29.2(ii) Inscribe a circle in a given triangle:

Inscribed circle or Incircle of a triangle:



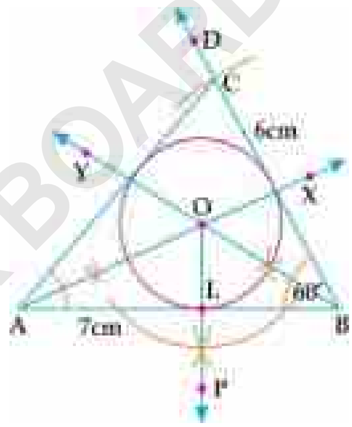
A circle which touches all the sides of a triangle is called incircle or inscribed circle of the triangle. In the adjacent figure, there is an incircle of ΔABC with centre O which is called the incentre

Example: Draw an inscribed circle of ΔABC in which $m\overline{AB} = 7cm$, $m\angle B = 60°$ and $m\overline{BC} = 6$ cm

Given: A triangle ABC with $m\overline{AB} = 7cm$, $m\angle B = 60°$ and $m\overline{BC} = 6cm$

Required: To draw an inscribed circle of ΔABC

Steps of Construction:



(i) Construct a ΔABC with the help of given data.

(ii) Draw the internal bisector $\overrightarrow{AX}$ of $\angle A$

(iii) Draw the internal bisector $\overrightarrow{BY}$ of $\angle B$

(iv) Both the internal bisectors $\overrightarrow{AX}$ and $\overrightarrow{BY}$ intersect each other at point O.

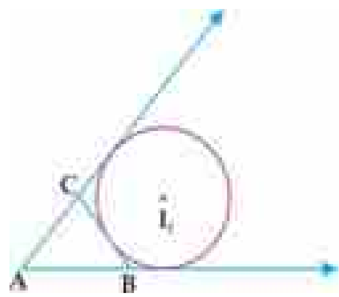
(v) Draw perpendicular OP on $\overline{AB}$ from point O. OP intersects $\overline{AB}$ at point L

(vi) With point O as centre and radius equal to $m\overline{OL}$, draw a circle which touches all the three sides of ΔABC.

This circle is the required inscribed circle.

29.2(iii) Escribe a circle to a given triangle

Escribed Circle or Excircle.



A circle which touches one side of a triangle externally and the two produced sides internally is called an escribed circle or excircle of the triangle.

In the given figure, there is an escribed circle of ΔABC opposite to vertex A with centre I_1, which is called excentre.

Similarly I_2 and I_3 are excenters of excircles of ΔABC opposite to vertices B and C respectively.

Example: Draw an escribed circle opposite to vertex A of ΔABC where $m\overline{AB} = 5cm$, $m\angle A = 30°$ and $m\angle B = 60°$

Given: A triangle ABC in which $m\overline{AB} = 5cm$, $m\angle A = 30°$ and $m\angle B = 60°$

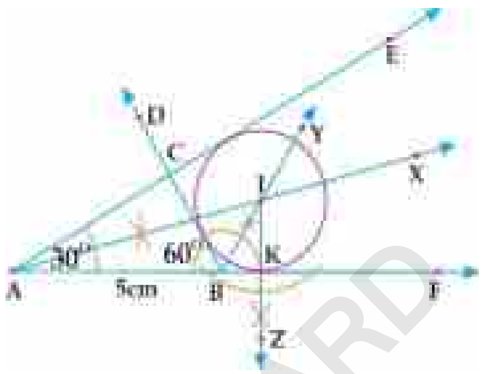
Required: To draw an escribed circle opposite to vertex A of ΔABC.

Steps of Construction.

(i) Construct a ΔABC with the help of given data.

(ii) Produce $\overline{AC}$ beyond C and produce $\overline{AB}$ beyond B making two exterior angles BCE and CBF respectively.

(iii) Draw the internal bisector $\overrightarrow{AX}$ of $\angle A$.
(iv) Draw the internal bisector $\overrightarrow{BY}$ of $\angle CBF$.
(v) Both bisectors $\overrightarrow{AX}$ and $\overrightarrow{BY}$ intersect each other at point I
(vi) Draw a perpendicular $\overrightarrow{IZ}$ on $\overrightarrow{AF}$ from I which cuts $\overrightarrow{AF}$ at point K.
(vii) With I as centre and radius equal to $m\overline{IK}$, draw a circle which touches $\overline{BC}$ externally and $\overrightarrow{AE}$ and $\overrightarrow{AF}$ internally.



This is the required escribed circle of ΔABC opposite to vertex A.

29.2(iv) Circumscribe an equilateral triangle about a given circle.

We will draw an equilateral triangle which circumscribes about a given circle by using the fact that if a circle is divided into three congruent arcs and tangents drawn at the points of division will be the sides of the equilateral triangle circumscribing about the circle. The method is explained in the following example.

Example: Draw a circle of radius 3cm with centre at point O and circumscribe an equilateral triangle about this circle.

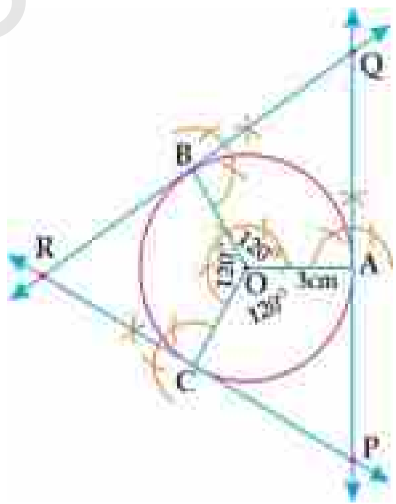
Given: A circle of radius 3cm with centre at point O.

Required: To circumscribe an equilateral triangle about the given circle.

Steps of Construction:

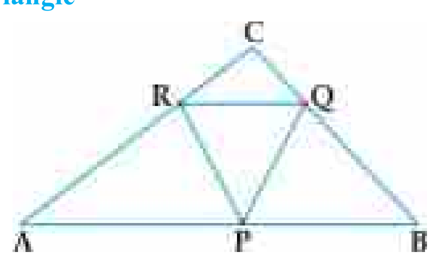
(i) Draw a circle of radius 3cm with centre at point O.
(ii) Divide the circle into three congruent arcs $\overset{\frown}{AB}$, $\overset{\frown}{BC}$, and $\overset{\frown}{AC}$ by dividing central angle into three congruent angles.
(iii) Draw tangents at points A,B and C by making right angles at these points.
(iv) These tangents intersect each other at points P, Q and R.

Now PQR is the required equilateral triangle.



29.2(v) Inscribe an equilateral triangle in a given triangle

We know that an equilateral triangle is said to be inscribed in a given triangle if all the vertices of the equilateral triangle lie on different sides of the given triangle. In the adjacent figure, ΔPQR is the equilateral triangle inscribed in the given ΔABC.



Example: Draw an equilateral triangle inscribed in a ΔABC where $m\overline{AB} = 4cm$, $m\overline{BC} = 5cm$ and $m\overline{AC} = 7cm$.

Given: A triangle ABC in which $m\overline{AB} = 4cm$, $m\overline{BC} = 5cm$ and $m\overline{AC} = 7cm$.

Required: To draw an equilateral triangle inscribed in ΔABC.

Steps of Construction:

(i) Construct ΔABC with given data.

(ii) Draw the internal bisector BD of $\angle B$ which is the suitable angle (usually the largest angle)

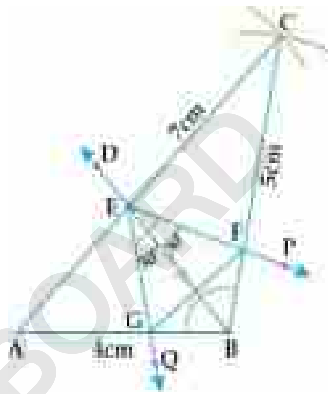
(iii) The internal bisector $\overrightarrow{BD}$ intersects $\overline{AC}$ at point E

(iv) At point E, draw two angles BEP and BEQ each of measure $30°$

(v) EP and EQ intersect $\overline{BC}$ and $\overline{AB}$ at points F and G respectively.

(vi) Draw $\overline{FG}$

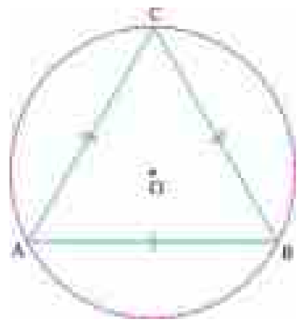
Now EFG is the required inscribed equilateral triangle of ΔABC



29.2(vi) Inscribe an equilateral triangle in a given circle

Inscribed equilateral triangle in a circle

An equilateral triangle is called inscribed equilateral triangle in a circle if the circle passes through its all the vertices. In the adjacent figure ABC is an inscribed equilateral triangle of circle with centre O.



Example: Draw an inscribed equilateral triangle of a circle with centre at O and radius of 3cm

Given: A circle of radius 3cm and with centre at point O.

Required: To draw an inscribed equilateral triangle of the given circle.

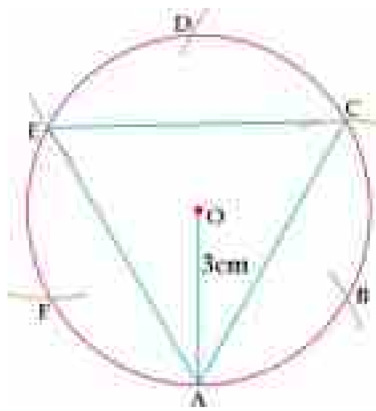
Steps of Construction:

(i) Draw a circle of radius 3cm with centre at point O.

(ii) Take a point A on the circle. Starting with A as centre and radius of 3cm divide the circle in six congruent parts at point B, C, D, E and F

(iii) Draw $\overline{AC}, \overline{AE}$ and $\overline{CE}$

Now AEC is the required inscribed equilateral triangle of the given circle.



EXERCISE: 29.2

1. Construct the ΔABC and draw its circumcircle in each case.
   (i) $m\overline{AB} = 5.5cm$, $m\overline{AC} = 6cm$ and $m\angle A = 50°$
   (ii) $m\overline{AB} = 6cm$, $m\overline{BC} = 4.5cm$ and $m\overline{AC} = 5cm$
2. Construct the ΔPQR and draw its incircle in each case
   (i) $m\overline{PQ} = 5cm$, $m\overline{QR} = 6.5cm$ and $m\overline{RP} = 5.5cm$
   (ii) $m\overline{PQ} = 6cm$, $m\angle P = 60$ and $m\angle Q = 50°$
3. Construct the ΔXYZ and draw its escribed circle opposite to $\angle Y$ in each case.
   (i) $m\overline{XY} = 4.5cm$, $m\overline{YZ} = 5cm$ and $m\angle Y = 30°$
   (ii) $m\overline{XY} = 5.5cm$, $m\overline{YZ} = 6cm$ and $m\overline{XZ} = 2.5cm$
4. Draw a circle of radius 3.5cm with centre at point O and circumscribe an equilateral triangle about this circle.
5. Draw an equilateral triangle inscribed in ΔPQR where
   $m\overline{PQ} = 4.5cm$, $m\overline{QR} = 5.5cm$ and $m\overline{PR} = 8cm$.
6. Draw an inscribed equilateral triangle of a circle with radius 3.8cm with centre O.

29.2(vii): Circumscribe a square about a given circle.

Before going to circumscribe or inscribe a square or hexagon or any other polygon about or in the given circle respectively. We must know the following theorem regarding regular polygons attached to the circles.

If the circumference of a circle be divided into n equal arcs then

(i) the points of division are the vertices of a regular n-gon inscribed in the circle.
(ii) the tangents drawn to the circle at these points will be the sides of a regular n-gon circumscribing about the circle.

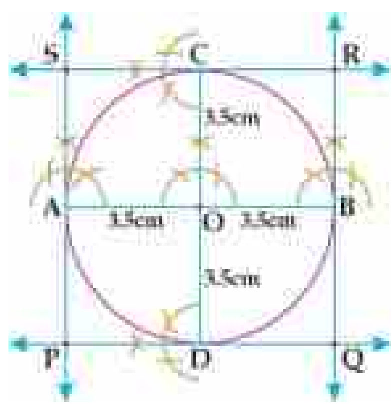
It means if we have to circumscribe or inscribe a square, regular pentagon, regular hexagon etc with a given circle then we have to divide the given circle into four, five, six etc equal parts respectively.

Let us learn how to circumscribe a square about a given circle with the help of the following example.

Example: Circumscribe a square about a circle whose radius is 3.5cm and centre at point O.

Given: A circle with radius 3.5cm and centre at point O.

Required: To construct a square which circumscribes the given circle.



Step of construction:

(i) With point O as centre, draw a circle of radius 3.5cm.
(ii) Draw a diameter $\overline{AB}$.
(iii) Draw another diameter $\overline{CD}$ which is perpendicular to $\overline{AB}$.

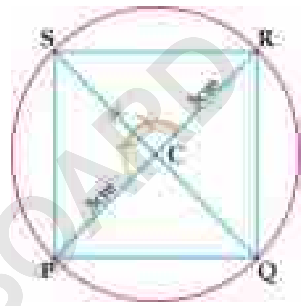
(iv) Draw tangents at points A,B,C and D.
(v) These tangents intersect each other at point P,Q,R and S.
Now PQRS is the required square which circumscribes about the given circle.

Let us learn how to inscribe a square in a given circle with the help of the following example.

Example: Construct a square inscribed in the circle whose radius is 3cm and centre at point C.

Given: A circle with radius 3cm and centre at C.

Required: To construct a square inscribed in the given circle.



Step of construction:

(i) With C as centre , draw a circle of radius 3cm

(ii) Draw a diameter $\overline{PR}$

(iii) Draw a diameter $\overline{QS}$ which is perpendicular to $\overline{PR}$

(iv) Given circle has been divided into four equal parts at point P,Q,R and S

(v) Draw $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SP}$. Now PQRS is the required inscribed square in the given circle.

29.2(viii) circumscribe a regular hexagon about a given circle:

The method of circumscribing a regular hexagon about a given circle is explained with the help of the following example.

Example: Circumscribe a regular hexagon about a circle of radius 4cm with centre at point O.

Given: A circle of radius 4cm with centre at point O

Required: To construct a regular hexagon which circumscribes the given circle.

Steps of construction:

(i) With O as centre draw a circle of radius 4cm

(ii) Take any point A on the circle.

(iii) Starting with A and radius equal to $m\overline{OA}$ draw arcs to divide given circle in six equal parts with points of division are A,B,C,D,E and F

(iv) Draw tangents at these points of division

(v) These tangents intersect each other at points P,Q,R,S,T and U
Now PQRSTU is the required regular hexagon circumscribing the given circle.

29.2 (ix) Inscribe a regular hexagon in a given circle:

The method of inscribing a regular hexagon is explained with the help of the following example.

Example: Construct a regular hexagon inscribed in a circle of radius 4.5 cm and centre at point O.

Given: A circle of radius 4.5cm and with centre at point O.

Required: To construct a regular hexagon which inscribes the given circle.

Steps of construction:

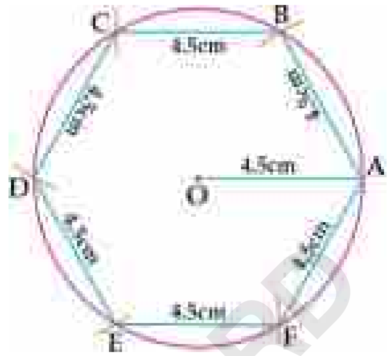
(i) With centre at point O, draw a circle of radius 4.5cm

(ii) Take any point A on the circle.

(iii) Starting with point A and radius equal to $m\overline{AO}$ or 4.5cm draw arcs to divide the given circle in six equal parts at points A ,B, C, D, E and F

(iv) Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$ and $\overline{AF}$

Now ABCDEF is the required regular hexagon which is inscribed in the given circle.



EXERCISE 29.3

1. Circumscribe a square about a circle of radius 4.2 cm and with centre at point C.
2. Inscribe a square in a circle of radius 3.6 cm with centre at point O.
3. Circumscribe a regular hexagon about a circle of radius 4.8 cm with centre at point O.
4. Inscribe a regular hexagon in a circle of radius 4.4 cm with centre at point C
5. Inscribe a regular pentagon in a circle of radius 4.3 cm and centre O
   (Hint: Draw five congruent angles at centre)

29.3 Tangents to a circle

Recall that tangent to a circle is a line which touches the circle at a single point. Tangent and radial segment of a circle are always perpendicular to each other at the point of contact

29.3 (i) Draw a tangent to a given arc without using the centre through a given point P when P is

- The middle point of the arc
- At the end of the arc.
- Outside the arc

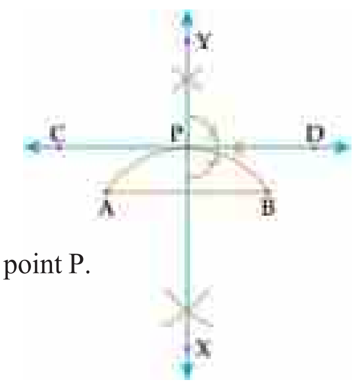
Case1. When P is the middle point of the arc

The method of drawing tangent to a given arc without using the centre through a point P which is middle point of the arc , is explained with the help of the following example.

Example: Take an arc AB. Draw tangent to the arc AB through the middle point P of AB without using centre.

Given: Point P is the mid-point of $\overset{\frown}{AB}$

Required: To draw a tangent through P without using centre



Step of construction:

(i) Take an arc $\overset{\frown}{AB}$

(ii) Draw the chord AB

(iii) Draw the right bisector $\overleftrightarrow{XY}$ of $\overline{AB}$ which cuts the arc at point P.

(iv) At point P, draw a perpendicular $\overleftrightarrow{CD}$ on $\overleftrightarrow{XY}$

Thus $\overleftrightarrow{CD}$ is the required tangent

Case2. When P is at the end of the arc

The method of drawing tangent to an arc without using centre through an end point P of the arc is explained with the help of the following example

Example: Take an arc $\overset{\frown}{PAB}$

Draw a tangent to the $\overset{\frown}{PAB}$ through its end-point P without using centre.

Given: Point P is the end point of $\overset{\frown}{PAB}$

Required: To draw a tangent to $\overset{\frown}{PAB}$ at the end point P without using centre.

Steps of Construction:

(i) Draw a $\overset{\frown}{PAB}$ with radius and centre of your choice

(ii) Draw chord $\overline{AP}$

(iii) Draw a perpendicular $\overrightarrow{AX}$ on $\overline{PA}$ at point A which cuts the given arc at point Q.

(iv) Draw $\overline{PQ}$ which is the diameter of the arc

(v) At point P draw a perpendicular $\overrightarrow{CD}$ on $\overline{PQ}$

Now $\overrightarrow{CD}$ is the required tangent.

Case 3. When P is outside the arc

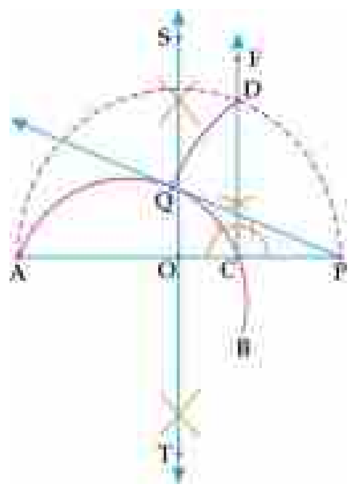
The method of drawing tangent to an arc through point P which is outside the arc without using centre is explained with the help of the following example.

Example: Take an arc $\overset{\frown}{AB}$

Draw a tangent to the arc $\overset{\frown}{AB}$ through point P which is outside the arc without using centre

Given: A point P outside an arc $\overleftrightarrow{AB}$

Required: To draw a tangent to the $\overset{\frown}{AB}$ from the point P without using centre



Steps of Construction:

(i) Draw an arc AB of your choice

(ii) Take a point P outside the AB

(iii) Draw $\overline{AP}$ which cuts the given arc at point C.

(iv) Draw the right bisector ST of $\overline{AP}$ which cuts $\overline{AP}$ at point O.

(v) With O as centre and radius equal to $m\overline{AO}$ or $m\overline{OP}$, draw a semi-circle

(vi) Draw a perpendicular $\overleftrightarrow{CF}$ on $\overline{AP}$ at point C which cuts the semi-circle at point D.

(vii) With P as centre and radius equal to $m\overline{PD}$, draw an arc which cuts $\overset{\frown}{AB}$ at point Q.

(viii) Draw $\overrightarrow{PQ}$

Now $\overrightarrow{PQ}$ is the required tangent.

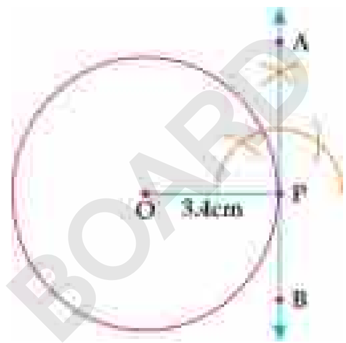
29.3 (ii) Draw a tangent to a given circle from a point P, when P lies

- On the circumference
- Outside the circle

Case1. When point P is on the circumference

The method of drawing a tangent to a given circle from a point P when P lies on the circumference is explained with the help of the following example.

Example: Draw a circle of radius 3.4cm with centre at O. Draw tangent to the circle at a point P of its circumference.



Given: A circle of radius 3.4cm with centre at point O and a point P on it circumference

Required: To draw a tangent to the circle at point P.

Steps of Construction:

(i) With point O as centre, draw a circle of radius 3.4cm

(ii) Take a point P on its circumference

(iii) Draw $\overline{OP}$

(iv) At point P, draw a perpendicular $\overleftrightarrow{AB}$ on $\overline{OP}$

Now $\overleftrightarrow{AB}$ is the required tangent.

Case2. When point P is outside the circle

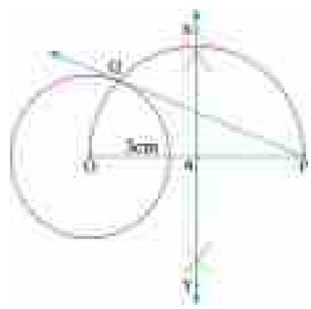
The method of drawing a tangent to a given circle from a point P which lies outside the circle is explained with the help of the following example.

Example: Draw a circle of radius 3cm with centre O. Draw a tangent to the circle from a point P outside the circle.

Given: A circle with centre O and radius of 3cm. Also a point P outside the circle

Required: To draw a tangent to the circle from point P outside it

Steps of Construction:



(i) With O as centre, draw a circle of radius 3cm

(ii) Take point P outside the circle

(iii) Draw $\overline{OP}$

(iv) Draw the right bisector $\overleftrightarrow{XY}$ of $\overline{OP}$ which cuts it at point A.

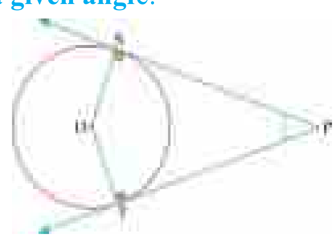
(v) With A as centre and radius equal to $m\overline{AO}$, draw a semi-circle which cuts the given circle at point Q

(vi) Draw $\overrightarrow{PQ}$

Now $\overrightarrow{PQ}$ is the required tangent

29.3 (iii) Draw two tangents to a circle meeting each other at a given angle:

Consider a circle with centre O. $\overrightarrow{PS}$ and $\overrightarrow{PT}$ are two tangents to the circle where $\angle P$ is the angle between the tangents.



Here $m\angle PSO = 90°$ ($\overline{OS} \perp \overrightarrow{PS}$)

and $m\angle PTO = 90°$ ($\overline{OT} \perp \overrightarrow{PT}$)

Now, in quadrilateral PSOT

$$m\angle P + m\angle PSO + m\angle O + m\angle PTO = 360^\circ$$

$$m\angle P + 90^\circ + m\angle O + 90^\circ = 360^\circ \qquad \text{i.e.,}$$

$$\Rightarrow \; m\angle P + m\angle O = 180^\circ$$

$$\Rightarrow \; \boxed{m\angle O = 180^\circ - m\angle P}$$

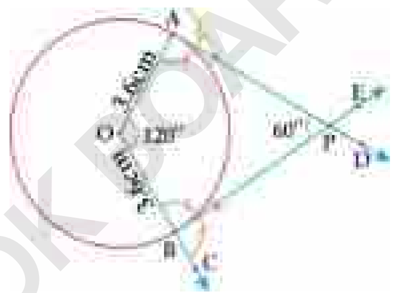
i.e., the central angle of circle joining the points of contact is supplement of the angle between the tangents

Using this fact, we will draw two tangents to a circle meeting each other at a given angle. The method is explained with the help of the following example.

Example: Draw two tangents to a circle of radius 3.6cm and centre at point O meeting each other at angle of measure 60°

Given: A circle with centre O and radius of 3.6cm whereas angle between the tangents is 60°

Required: To draw two tangents to the given circle meeting each other at angle of 60°



Steps of Construction:

(i) With centre O , draw a circle of radius 3.6cm

(ii) Take a point A on the circle and join it with O

(iii) Draw $\angle AOC$ of 120° (i.e $180^\circ - 60^\circ = 120^\circ$) such that $\overrightarrow{OC}$ cuts the given circle at point B

(iv) Draw perpendicular $\overrightarrow{AD}$ on $\overline{OA}$ at point A.

(v) Draw perpendicular $\overrightarrow{BE}$ on $\overline{OB}$ at point B.

(vi) Both $\overrightarrow{AD}$ and $\overrightarrow{BE}$ intersect each other at point P

Thus $\overrightarrow{AP}$ and $\overrightarrow{BP}$ are the required tangents such that $m\angle P = 60^\circ$

EXERCISE 29.4

1. Take a minor arc $\overparen{PQ}$. Draw a tangent to $\overparen{PQ}$ through its midpoint A without using centre.
2. Take a major arc $\overparen{PQR}$. Draw a tangent to $\overparen{PQR}$ through its endpoint Q without using centre.
3. Take a minor arc $\overparen{XY}$. Draw a tangent to $\overparen{XY}$ through point Z which is outside arc without using centre
4. Draw a circle of radius 2.9cm with centre C. Draw a tangent to the circle from a point P of the circle.
5. Draw a circle of radius 3.2cm with centre P. Draw tangent to the circle from a point Q which is at a distance of 8cm from P.
6. Draw two tangents to a circle of radius 3.3cm with centre C meeting each other at an angle of measure (i) 50° (ii) 63°

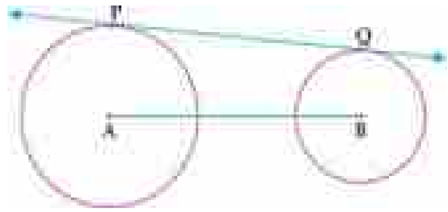
29.3(iv) Draw

- **Direct common tangents or external tangents.**
- **Transverse common tangents or internal tangents to two equal circles.**

(a) Drawing of direct common tangents or external tangents to two equal circles

Direct Common Tangent:

A common tangent is called direct common tangent or external tangent to two circles if it does not intersect the line segment joining the centres of the two given circles.



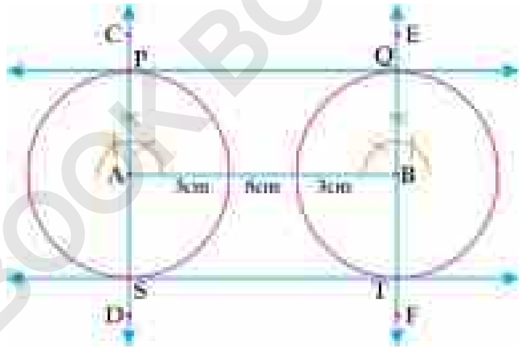
In the figure, $\overleftrightarrow{PQ}$ is a direct common tangent to two circles with centres at A and B.

Note that $\overleftrightarrow{PQ}$ does not intersect $\overline{AB}$. The method of drawing direct common tangents to two equal circles is explained by the following example.

Example: Draw two direct common tangents to two equal circles with centres at points A and B and having radius 3cm each such that $m\overline{AB} = 8cm$

Given: Two equal circles of radius 3cm each and centres at points A and B such that $m\overline{AB} = 8cm$

Required: To draw two direct common tangents to the given circles



Steps of Construction:

(i) Draw $\overline{AB}$ of 8cm

(ii) Draw two equal circles, each of radius 3cm at points A and B.

(iii) Draw perpendicular $\overleftrightarrow{CD}$ on $\overline{AB}$ at point A which cuts the circle at points P and S

(iv) Draw perpendicular EF on $\overline{AB}$ at point B which cuts the other circle at points Q and T

(v) Draw $\overleftrightarrow{PQ}$ and $\overleftrightarrow{ST}$

Now $\overleftrightarrow{PQ}$ and $\overleftrightarrow{ST}$ are the required direct common tangents to the given circles.

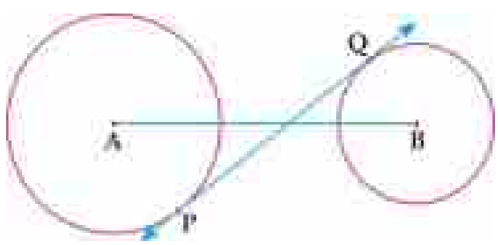
(b) Drawing of transverse common tangents or internal tangents to two equal circles

Transverse Common Tangent:

A common tangent is called transverse common tangent or internal tangent to two circles if it intersects the line segment joining the centres of the two given circles.

In the figure, $\overleftrightarrow{PQ}$ is a transverse common tangent to two given circles with centres A and B.

Note that $\overleftrightarrow{PQ}$ intersects $\overline{AB}$. The method of drawing transverse common tangents to two equal circles is explained with the help of the following example.



Example: Draw two transverse common tangents to two equal circles each of radius 2.8cm with centres at points A and B whereas $m\overline{AB} = 7.8$cm

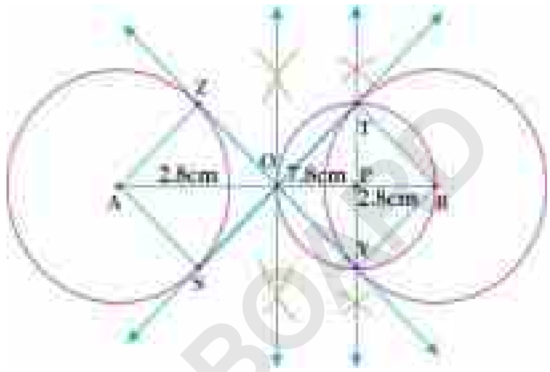
Method 1

Given:

Two circles each of radius 2.8cm with centres at A and B where $m\overline{AB}$=7.8cm

Required:

To draw two transverse common tangents to the given circles



Steps of Construction:

(i) Draw $\overline{AB}$ of 7.8cm

(ii) At points A and B , draw two circles each of radius 2.8cm

(iii) Draw the right bisector of $\overline{AB}$ which meets it at point O.

(iv) Draw a right bisector of $\overline{OB}$ meeting it at point P

(v) With P as centre and radius equal to $m\overline{OP}$, draw a circle which cuts the circle of centre B at points T and Y.

(vi) Draw $\overline{BT}$ and $\overline{BY}$

(vii) Draw $\overline{AZ}$ $\overline{BY}$ and $\overline{AS}$ $\overline{BT}$ using set squares.

(viii) Draw $\overleftrightarrow{ST}$ and $\overleftrightarrow{YZ}$.

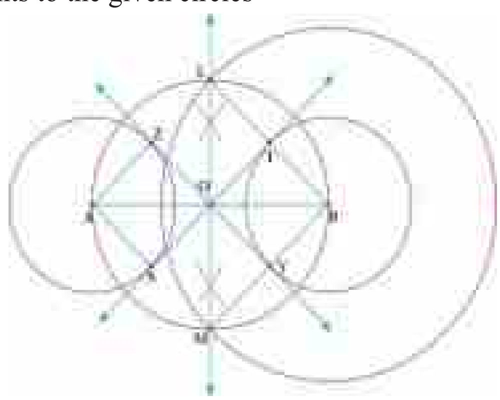
Now $\overleftrightarrow{ST}$ and $\overleftrightarrow{YZ}$ are the required transverse common tangents.

Method-2

Given: Two circles each of radius 2.8cm with centres at A and B such that $m\overline{AB} = 7.8$cm

Required: To draw two transverse common tangents to the given circles

Steps of Construction:



(i) Draw $\overline{AB}$ of 7.8cm.

(ii) At points A and B, draw two circles each of radius 2.8cm.

(iii) Bisect $\overline{AB}$ at O and draw a circle of radius equal to $m\overline{AO}$ with centre O.

(iv) With B as centre and radius equal to 5.6cm (double of 2.8cm), draw a circle which cuts the previous circle of centre O at points L and M.

(v) Draw $\overline{BL}$ and $\overline{BM}$ which cut the smaller circle with centre B at points T and Y respectively.

(vi) Draw $\overline{AS}$ $\overline{BT}$ and $\overline{AZ}$ $\overline{BY}$ by using set squares.

(vii) Draw $\overleftrightarrow{ST}$ and $\overleftrightarrow{YZ}$.

Now $\overleftrightarrow{ST}$ and $\overleftrightarrow{YZ}$ are the required transverse common tangents.

29.3(v) Draw

- **Direct common tangents or external tangents.**
- **Transverse common tangents or internal tangents to two unequal circles.**

(a) Drawing of direct common tangents or external tangents to two unequal circles.

Following example is suitable for learning the method of drawing direct common tangents to two unequal circles

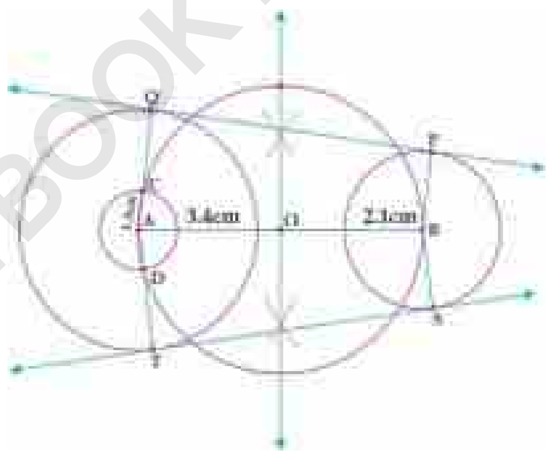
Example: Draw direct common tangents to the given circles of radius 3.4cm and 2.1cm, where distance between their centres is 7.5 cm.

Method 1

Given: Two circles with centres A and B having radii 3.4 cm and 2.1 cm respectively such that $m\overline{AB} = 7.5cm$

Required: To draw direct common tangents to the given circles.

Steps of Construction:



(i) Draw $\overline{AB}$ of 7.5cm

(ii) With centres A and B , draw two circles of radii 3.4 cm and 2.1cm respectively.

(iii) Bisect $\overline{AB}$ at point O and draw a circle of radius equal to $m\overline{AO}$ with centre O.

(iv) With centre A of larger circle, draw a circle of radius 1.3cm $(\text{i.e. } 3.4 - 2.1 = 1.3)$ which cuts the previous circle at points C and D.

(v) Draw $\overline{AC}$ and produce it to intersect the given concentric circle at point Q

(vi) Draw $\overline{AD}$ and produce it to intersect the given concentric circle at point T

(vii) Draw $\overline{BP} \parallel \overline{AQ}$ and $\overline{BS} \parallel \overline{AT}$ by using set squares

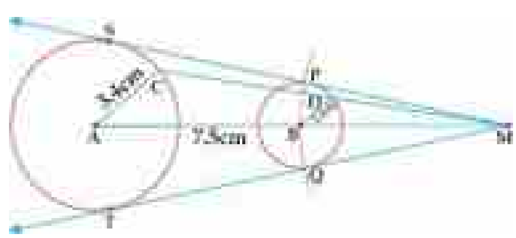
(viii) Draw $\overleftrightarrow{PQ}$ and $\overleftrightarrow{ST}$

Now $\overleftrightarrow{PQ}$ and $\overleftrightarrow{ST}$ are the required direct common tangents.

Method 2

Given: Two circles with centres, A and B and having radii 3.4cm and 2.1 cm respectively such that $m\overline{AB} = 7.5cm$

Required: To draw direct common tangents to the given circles.



Steps of Construction:

(i) Draw $\overline{AB}$ of 7.5cm.

(ii) Draw circles of radii 3.4cm and 2.1cm at points A and B respectively.

(iii) Take a point C on circle with centre A and draw $\overline{AC}$.

(iv) Draw $\overline{BD} \parallel \overline{AC}$ using set squares.

(v) Draw $\overline{CD}$ and produce it beyond D.

(vi) Produce $\overline{AB}$ to intersect CD at point M.

(vii) With M as centre and radius equal to $m\overline{MB}$, draw an arc which cuts the circle of centre B at points P and Q.

(viii) Draw $\overline{MP}$ and produce it to meet other circle at S.

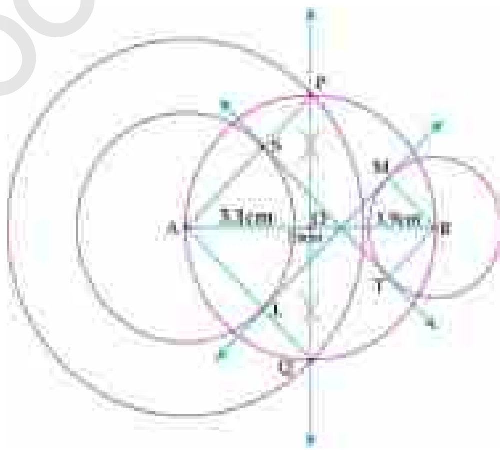
(ix) Draw $\overline{MQ}$ and produce it to meet the other circle at point T.

Now $\overleftrightarrow{SP}$ and $\overleftrightarrow{TQ}$ are the required direct common tangents to the given circles.

(b) Drawing of transverse common tangents or internal tangents to two unequal circles.

We explain the method of drawing transverse common tangents to two unequal circles by the following example.

Example: Draw the transverse common tangents to two circles with centres A and B, also having radii 3.1cm and 1.9cm such that the distance between their centres is 7.6cm



Given: Two circles with centres A and B having radii 3.1cm and 1.9cm respectively and $m\overline{AB}$ =7.6cm

Required: To draw transverse common tangents to two given circles.

Steps of Construction:

(i) Draw $\overline{AB}$ of 7.6cm.

(ii) Draw circles of radii 3.1cm and 1.9cm at points A and B respectively.

(iii) Bisect $\overline{AB}$ at point O and draw a circle of radius equal to $m\overline{OA}$ with centre O.

(iv) With centre A (centre of bigger circle) and radius of 5 cm (i.e,3.1cm+1.9cm=5cm), draw a circle which cuts the previous circle at points P and Q.

(v) Draw $\overline{AP}$ which cuts the given larger circle at point S.

(vi) Draw $\overline{AQ}$ which cuts the given larger circle at point L.

(vii) Draw $\overline{BM}$ $\overline{AQ}$ and $\overline{BT}$ $\overline{AP}$.

(viii) Draw $\overleftrightarrow{ST}$ and $\overleftrightarrow{LM}$.

Now $\overleftrightarrow{ST}$ and $\overleftrightarrow{LM}$ are the required transverse common tangents.

29.3(vi) Draw a tangent to

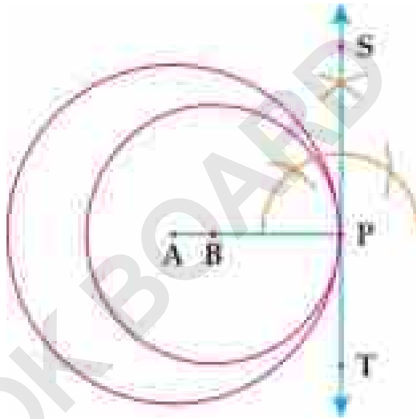
- Two unequal touching circles
- Two unequal intersecting circles

(a) Drawing of a tangent to two unequal touching circles

There are two cases

Case 1: when circles touch internally.

The method is explained by the following example



Example: Draw a tangent to two unequal circles of radii 3cm and 2cm with centres A and B respectively whereas circles touch internally.

Given: Two circles with centres A and B having radii 3cm and 2cm respectively which touch internally.

Required: To draw a tangent to these two circles

Steps of Construction:

(i) With centre at point A, draw larger circle of radius 3cm

(ii) Take any point P on the circle and draw $\overline{AP}$

(iii) Take a point B on $\overline{AP}$ at a distance of 2cm from point P

(iv) With B as centre and radius of 2cm draw a circle which touches the larger circle at point P

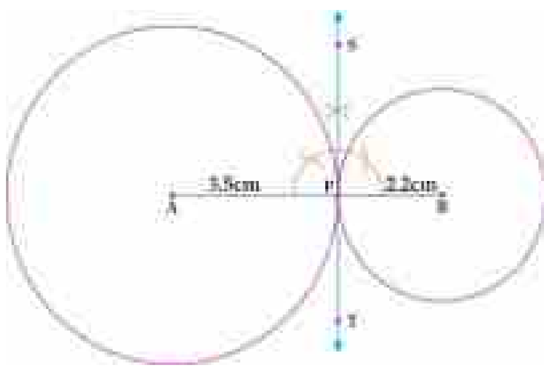
(v) Draw a perpendicular $\overleftrightarrow{ST}$ on $\overline{AP}$ at point of contact P

Now ST is the required tangent.

Case 2: When circles touch externally.

The method is explained in the following example

Example: Draw a tangent to two unequal circles of radii 3.5cm and 2.2cm with centres A and B respectively whereas both circles touch each other externally.



Given: Two circles of radii 3.5cm and 2.2cm with centres at A and B whereas circles touch each other externally.

Steps of Construction:

(i) With centre A, draw a circle of radius 3.5cm.

(ii) Take a point P on the circle.

(iii) Draw $\overline{AP}$ and produce it to point B such that $m\overline{PB}$=2.2cm

(iv) With P as centre and radius of 2.2cm, draw a circle which touches first circle at point P

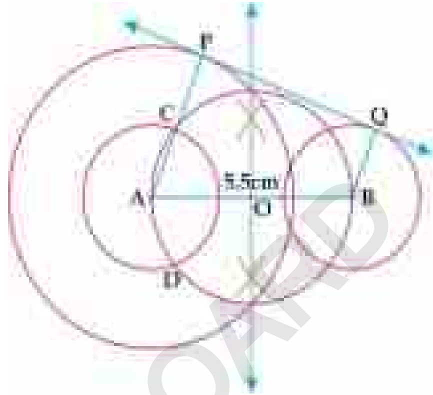
(v) Draw a perpendicular $\overleftrightarrow{ST}$ on $\overline{AP}$ at point of contact P

Now $\overleftrightarrow{ST}$ is the required tangent.

(b) Drawing of a tangent to two unequal intersecting circles.

The method is explained with the help of the following example

Example: Draw a tangent to two unequal interesting circles of radii 4cm and 2cm with centres A and B respectively such that $m\overline{AB}=5.5\text{cm}$



Given: Two unequal intersecting circles of radii 4cm and 2cm with centre A and B respectively such that $m\overline{AB}=5.5\text{cm}$

Required: To draw a tangent to both the circles

Steps of Construction:

(i) Draw $\overline{AB}$ of 5.5cm.

(ii) Draw circles of radii 4cm and 2cm with centres A and B respectively.

(iii) With centre A (centre of larger circle), draw a circle of radius $2\text{cm}(4\text{cm}-2\text{cm}=2\text{cm})$

(iv) Bisect $\overline{AB}$ at point O. With O as centre and radius equal to $m\overline{AO}$, draw a circle which cuts the smaller circle of centre A at point C and D.

(v) Draw $\overline{AC}$ and produce it to meet the larger concentric circle of centre A at point P.

(vi) Draw $\overline{BQ} \parallel \overline{AP}$ using set squares.

(vii) Draw $\overleftrightarrow{PQ}$

Now $\overleftrightarrow{PQ}$ is the required tangent.

29.3(vii) Draw a circle which touches

- **Both the arms of a given angle**
- **Two converging lines and passing through a given point between them**
- **Three converging lines**

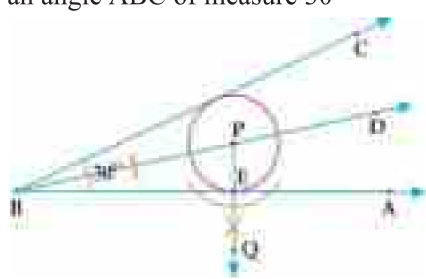
(a) Drawing a circle which touches both the arms of a given angle.

We explain the method of drawing a circle which touches both the arms of given angle in the following example

Example: Draw a circle which touches both the arms of an angle ABC of measure 30°

Given: An angle ABC of measure 30°

Required: To draw a circle which touches both the arms of the angle.



Steps of Construction:

(i) Draw an $\angle ABC$ of 30°

(ii) Draw the internal bisector $\overrightarrow{BD}$ of $\angle ABC$

(iii) Take any point P on $\overrightarrow{BD}$

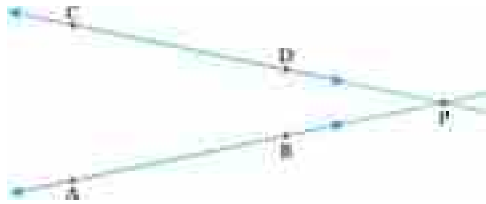
(iv) From point P, draw a prependiculer $\overrightarrow{PQ}$ on $\overrightarrow{BA}$ which cuts it at point E

(v) With P as centre and radius equal to $m\overline{PE}$, draw a circle which touches $\overrightarrow{BA}$ and $\overrightarrow{BC}$

This circle is the required circle.

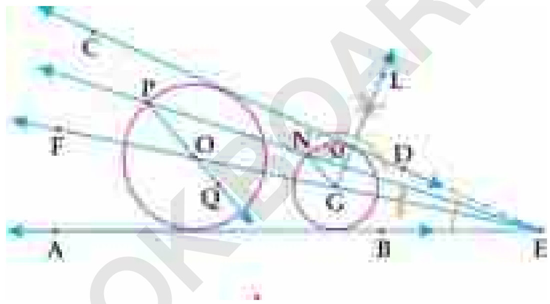
(b) Drawing a circle which touches two converging lines and passing through a given point between them.

Converging Lines:



Two or more lines which get closer and closer to each other and finally meet at a point, are said to be converging lines, In the adjacent figure, two lines AB and CD are converging lines and they finally meet at a point P.

Example: Draw a circle which passes through a point P between the converging lines AB and CD and touch both the lines.



Given: Two converging lines AB and CD. And a point P between them

Required: To draw a circle which passes through point P and touches both the given converging lines.

Steps of Construction:

(i) Draw two converging lines AB and CD which meet at point E.

(ii) Take any point P between these lines.

(iii) Draw the internal bisector $\overrightarrow{EF}$ of $\angle E$.

(iv) Take any point G on $\overrightarrow{EF}$.

(v) From point G, draw a perpendicular $\overrightarrow{GL}$ on $\overrightarrow{EC}$ meeting it at point M

(vi) With G as centre and radius equal to $m\overline{GM}$, draw a circle which touches both the converging lines.

(vii) Draw $\overrightarrow{EP}$ which cuts this circle at point N.

(viii) Draw $\overline{NG}$. Also draw $\overline{PQ} \parallel \overline{NG}$.

(ix) $\overrightarrow{PQ}$ cuts $\overrightarrow{EF}$ at point O.

(x) With O a centre and radius equal to $m\overline{OP}$, draw a circle which passes through P and touches the two converging lines.

This is the required circle.

(c)Drawing a circle which touches three converging lines

Drawing a circle which touches three converging lines in a plane is not possible. However, it is possible in space which is beyond the scope of this book.

EXERCISE 29.5

1. Draw two equal circles each of radius 3.3cm with centres at points A and B such that $m\overline{AB} = 7.8cm$.

(a) Draw direct common tangents to these circles.

(b) Draw transverse common tangents to these circles.

2. Draw two unequal circles of radii 3.3cm and 2.1cm with centres, A and B respectively such that $m\overline{AB}$=8cm.
   (a) Draw direct common tangents to these circles.
   (b) Draw transverse common tangents to these circles.
3. Draw a tangent to two unequal circles of radii 3.8cm and 2.2cm with centres A and B respectively whereas
   (i) circle touch internally
   (ii) circles touch externally
   (iii) circles intersect each other and $m\overline{AB}$=5.6cm.
4. Draw a circle which touches both the arms of an angle of measure
   (i) $35°$ (ii) $40°$
5. Draw a circle which passes through a point M which lies between two converging lines PQ and ST such that the circle also touches both the lines.

REVIEW EXERCISE 29

Tick the correct option

(i) Right bisectors of non-parallel___________ intersect at the centre of the circle.
(a) Radial segments (b) Chords (c) Tangents (d) none of these

(ii) ______________circles can pass through a single point.
(a) One (b) Two (c) Three (d) Infinite

(iii)_____________circles can pass through three non-collinear points.
(a) One (b) Two (c) Three (d) Infinite

(iv)_____________angles of regular polygon are equal in measure.
(a) Interior (b) Exterior (c) both a and b (d) none of these

(v) Each interior angle of regular hexagon is equal to______.
(a) $90°$ (b) $108°$ (c) $120°$ (d) $135°$

(vi)A circle which touches all the sides of triangle is called___________.
(a) circumcircle (b) incircle (c) excircle (d) tricircle

(vii) A circle which touches one side of triangle externally and two produced sides internally is called_____________.
(a) excircle (b) circumcircle (c) incircle (d) tricircle

(viii) The centre of inscribed circle is called________________
(a) excentre (b) incentre (c) centroid (d) orthocentre

(ix)Right bisectors of sides of a triangle intersect each other at___________
(a) incentre (b) excentre (c) centroid (d) circumcentre

(x) The point of intersection of internal bisectors of angles of a triangle is called
(a) excentre (b) circum centre (c) centroid (d) incentre

(xi)If a regular hexagon is inscribed in a circle then length of each side of hexagon __________ radius of the circle
(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

(xii) Angle between tangent and radial segment at the point of contact is ___________ angle
(a) right (b) obtuse (c) acute (d) reflex

(xiii) The central angle of circle joining the points of contact of tangents is ___________ of the angle between the tangents

(a) complement (b) supplement (c) square (d) cube

(xiv) ___________ common tangents do not intersect the line segment joining the centres of the circle

(a) internal (b) transverse (c) direct (d) vertical

(xv) Direct common tangents of two equal circles are ____________

(a) intersecting (b) coincident (c) equal (d) parallel

(xvi) _____________ common tangents of two equal circles intersect at the midpoint of the line segment joining the centres of the circles

(a) direct (b) transverse (c) external (d) parallel

(xvii)If two circles of radii 5cm and 2cm touch each other externally then the distance between their centers is _____________

(a) 5cm (b) 10cm (c) 3cm (d) 7cm

(xviii) If two circles of radii 5cm and 2cm touch each other internally then the distance between their centres is______________

(a) 5cm (b) 10cm (c) 3cm (d) 7cm

(xix) Two or more converging lines always intersect each other at ________

(a) single point (b) two points

(c) more than one point (d) none of these

(xx) Three or more sided closed figure is called ___________

(a) pentagon (b) hexagon (c) heptagon (d) polygon

SUMMARY

- Centre of a circle can be located by drawing the right bisectors of two non-parallel chords which meet each other at the centre.
- From three non-collinear points, a unique circle can be drawn.
- A circle can be completed without finding the centre when an arc is given with the help of regular polygon.
- Circumcircle always passes through all the vertices of a polygon.
- Incircle of a triangle always touches all the sides of the triangle.
- Excircle of a triangle touches one side of a triangle externally and two produced sides internally.
- Three excircles of a triangle can be constructed.
- Tangent is a line which touches circle at a single point
- Secant always cuts the circle at two points.
- Tangent to a circle is always perpendicular to the radial segment at the point of contact.
- Only one tangent can be drawn to a circle from a point of its circumference.
- Only two tangents to a circle can be drawn from a point outside the circle.
- The central angle of a circle joining the points of contact is supplement of the angle between the tangents.
- If a line is tangent to more than one circle, it is called common tangent.

- Direct common tangents do not intersect the line segment joining the centres of the two given circles.
- Transverse common tangents always intersect the line segment joining the centres of the two given circles.
- If two circles touch each other externally then the distance between their centers is the sum of their radii.
- If two circles touch each other internally then the distance between their centers is equal to the difference of their radii.
- Converging lines get closer and closer to each other and finally meet at a point.