TANGENTS OF A CIRCLE

Unit 26

• Weightage = 5%

Students Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Understand the following theorems along with their corollaries and apply them to solve allied problems.
  - If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.
  - The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.
  - The two tangents drawn to a circle from a point outside it are equal in length.
  - If two circles touch externally or internally, the distance between their centres is, respectively, equal to the sum or difference of their radii.

Introduction

The theorems and proofs related to **tangent(s)** to a circle are discussed here. The concept of tangent is also important in defining derivatives in a branch of Mathematics called **Calculus.** A straight line may or may not intersect a circle. If a line touches a circle at only one point, then it is a **tangent to the circle**. The common point between a tangent and the circle is the **point of contact** (or) **point of tangency**. A line which intersects the circle at two points is a **secant to the circle**. There are two points of contact between a secant and the circle. In Figure (i), lines l_1 and l_2 do not touch the circle. The lines m_1 and m_2 are tangents at P and Q, respectively. The lines n_1 and n_2 are secants to the circle at points A, B and C, D, respectively. A line segment from the point of tangency to any other point of the tangent is a **tangent segment**. In Figure (ii), $\overleftrightarrow{PQ}$ and $\overleftrightarrow{AB}$ are tangents to the circle, whereas $\overline{PQ}$ and $\overline{AB}$ are tangent segments, respectively. $m\overline{AB}$ is **length of tangent segment** $\overline{AB}$. For a point P lying outside the circle, only two tangents $\overrightarrow{PA}$ and $\overrightarrow{PB}$ can be drawn, as shown in Figure (iii).

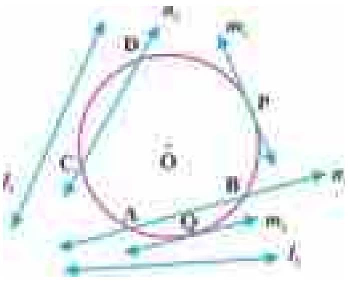


Figure (i).

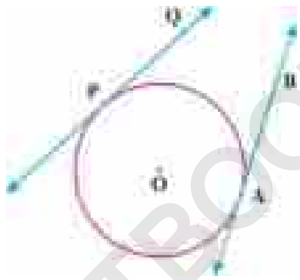


Figure (ii).

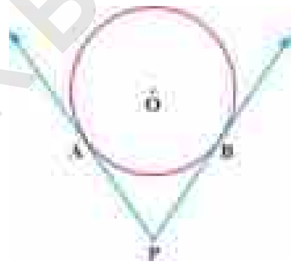


Figure (iii).

Circles touching each other internally or externally share the common point of tangency. The point *P* is the **common point of tangency** of circles with centres O, M and N in Figure (iv). Circles with centres O and M touch internally, whereas circles with centres O and N (also M and N) touch externally. Two circles not touching each other can have common tangents but different points of contact. A common tangent between two circles not touching each other is an **internal (or transverse) common tangent** if it intersects the segment joining their centres, otherwise an **external (or direct) common tangent**. In Figure (v), line *m* is an external (or direct) common tangent, whereas line *l* is an internal (or transverse) common tangent to the circles.

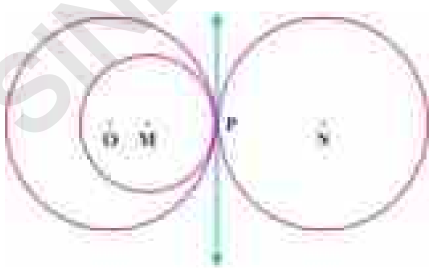


Figure (iv).

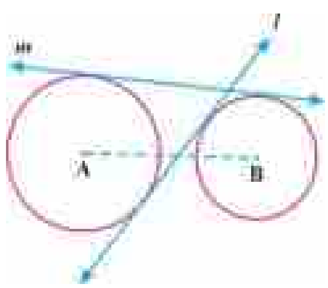
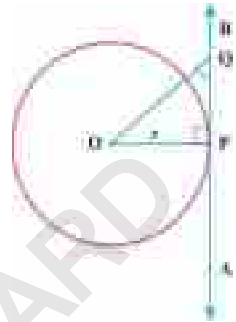


Figure (v).

26.1 Tangent(s) to a Circle:

Theorem 26.1: If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.



Given:

A circle with centre O and radius r. $\overleftrightarrow{AB}$ is drawn perpendicular on radial segment $\overline{OP}$, i.e. $\overleftrightarrow{AB} \perp \overline{OP}$ at its outer end point P.

To prove:

$\overleftrightarrow{AB}$ is tangent to given circle at P only.

Construction:

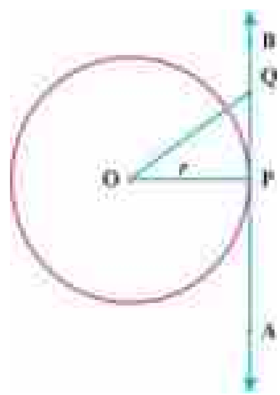
Draw $\overline{OQ}$, where Q is any other point on AB except P.

Proof:

Statements	Reasons
In right ΔOPQ, $m\angle OPQ = 90^\circ$	$\overleftrightarrow{AB} \perp \overline{OP}$ at P (given)
But, $m\angle OQP < 90^\circ$	Except right angle, other angles are acute in right Δ.
So, $m\overline{OQ} > m\overline{OP}$	Greater angle has greater side opposite.
Hence, Q lies outside the circle.	$m\overline{OQ} > r$
All points on $\overleftrightarrow{AB}$ except P lie outside the circle.	Q is any other point except P (construction),
Only point of contact of circle and $\overleftrightarrow{AB}$ is P. So, $\overleftrightarrow{AB}$ is tangent to circle at P.	By definition of tangent.

Q.E.D.

Theorem 26.2: The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.



Given:

A circle with centre O, radius r and a radial segment $\overline{OP}$.

A tangent $\overleftrightarrow{AB}$ at P.

To prove:

$\overline{OP} \perp \overleftrightarrow{AB}$

Construction:

Draw $\overline{OQ}$, where Q is any point on $\overleftrightarrow{AB}$ other than P.

Proof:

Statements	Reasons
Q lies outside the circle.	Q is any other point on $\overleftrightarrow{AB}$ except P.
So, $m\overline{OQ} > m\overline{OP} = r$ (i)	$\overline{OP}$ is radial segment.
Distance from the centre of all points on $\overleftrightarrow{AB}$ except P exceed r. So, all points except P lie outside the circle.	$m\overline{OP} = r$, and Q is any point on $\overleftrightarrow{AB}$ except P. (construction).
The shortest line segment from O to any point on $\overleftrightarrow{AB}$ is $\overline{OP}$.	From (i) for any Q on $\overleftrightarrow{AB}$ except P.
So, $m\angle OPQ = 90°$ and, $\overline{OP} \perp \overleftrightarrow{AB}$	All other angles are acute.

Q.E.D.

Note 1:

Theorems 26.1 and 26.2 highlight relationship between a tangent to a circle and the corresponding radial segment at the point of tangency.

Note 2:

Theorem 26.1 is a converse of Theorem 26.2, and vice-versa.

Corollary 1:

The line drawn at right angle to a tangent of a circle at its point of tangency passes through the centre of the circle.

Corollary 2:

The triangle formed by centre of the circle, point of tangency and any other point on the tangent is a right-angled triangle.

Corollary 3:

Only one tangent can be drawn to a circle at a given point on its boundary.

Example 1:

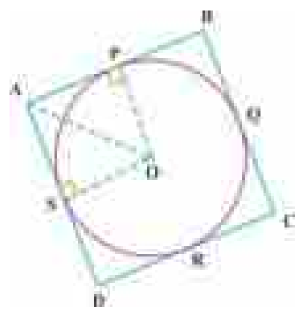
Show that the parallelogram circumscribing a circle is a rhombus.

Given:

$||^m$ ABCD circumscribing a circle so that: $m\overline{AB} = m\overline{CD}$ and $m\overline{AD} = m\overline{BC}$.

To prove:

$||^m$ $ABCD$ is a rhombus, i.e. $m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{AD}$.



Construction:

$\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{AD}$ are tangent to circle at P, Q, R and S, respectively. Draw $\overline{OS}, \overline{OA}$ and $\overline{OP}$.

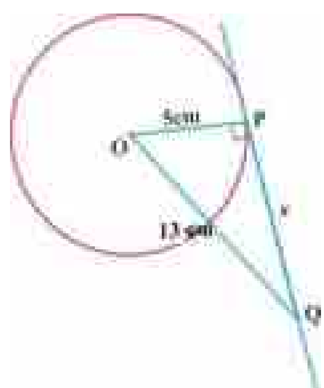
Proof:

Statements	Reasons
In $\Delta OSA \leftrightarrow OPA$	
$\overline{OS} \cong \overline{OP}$	Radial segments of same circle.
$m\angle OSA = 90^\circ = m\angle OPA$	Tangent perpendicular to radial segment
$\overline{OA} \cong \overline{OA}$	Common side of triangles.
$\Rightarrow \Delta OSA \cong \Delta OPA$	$H.S \cong H.S$
and $m\overline{AS} = m\overline{AP}$ (*i*)	Corresponding sides of congruent Δs.
$\Delta OPB \cong \Delta OQB$	By same process
Similarly, $m\overline{BQ} = m\overline{BP}$ (*ii*)	Corresponding sides of congruent Δs.
$\Delta OQC \cong \Delta ORC$	By same process
$m\overline{QC} = m\overline{RC}$ (*iii*)	Corresponding sides of congruent Δs.
$\Delta ORD \cong \Delta OSD$	By same process
$m\overline{DS} = m\overline{DR}$ (iv)	Corresponding sides of congruent Δs.
Now, $m\overline{AS} + m\overline{BQ} + m\overline{QC} + m\overline{DS}$ $= m\overline{AP} + m\overline{BP} + m\overline{RC} + m\overline{DR}$	Adding (*i*)-(*iv*)
or $\left(m\overline{AS} + m\overline{DS}\right) + \left(m\overline{BQ} + m\overline{QC}\right)$ $= \left(m\overline{AP} + m\overline{BP}\right) + \left(m\overline{RC} + m\overline{DR}\right)$	Re-arranging
or $m\overline{AD} + m\overline{BC} = m\overline{AB} + m\overline{CD}$ (v)	From figure
$2m\overline{BC} = 2m\overline{AB} \Rightarrow m\overline{AB} = m\overline{BC}$ (vi)	By definition of parallelogram (given)
$m\overline{AB} = m\overline{BC} = m\overline{CD} = m\overline{AD}$	Using (vi) and the given.
So, $\|^m$ ABCD is a rhombus.	

Q.E.D.

Example 2: Find the length of the tangent segment to a circle of radius 5cm from a point 13cm away from the centre of the circle.

Solution: Let centre of the circle is O. The point of tangency is P. Let Q be the point on tangent at a distance of 13cm from the centre as shown in the figure. From Figure, we have:



$m\overline{OP} = r = 5cm,\ m\overline{OQ} = 13cm$ and $m\overline{PQ} = x = ?$

Using Pythagoras' theorem:

$\left(m\overline{OQ}\right)^2 = \left(m\overline{OP}\right)^2 + \left(m\overline{PQ}\right)^2$

$(13)^2 = (5)^2 + (x)^2$

$169 = 25 + x^2 \quad or \quad x^2 = 169 - 25 = 144$

$m\overline{PQ} = x = \sqrt{144} = 12cm.$

Example 3:

Two concentric circles have radii 5cm and 3cm, respectively. Find the length of the tangent segment to smaller circle from the points on larger circle. Also find the length of chord of the larger circle which touches smaller circle.

Solution:

Two concentric circle of given radii are shown in the figure with common centre O. $m\overline{OA} = 3\text{cm}$ and $m\overline{OP} = 5\text{cm}$.

We first need to find $m\overline{AP}$ and $m\overline{AQ}$, i.e. length of tangent segment to smaller circle from points P and Q of larger circle.

Using Pythagoras' theorem, we have:

$$\left(m\overline{OP}\right)^2 = \left(m\overline{OA}\right)^2 + \left(m\overline{AP}\right)^2$$

$$25 = 9 + \left(m\overline{AP}\right)^2$$

$$\Rightarrow \quad \left(m\overline{AP}\right)^2 = 16 \quad or \quad m\overline{AP} = 4\text{cm}$$

But, the radial segment $\overline{OA}$ bisects the chord $\overline{PQ}$, so:

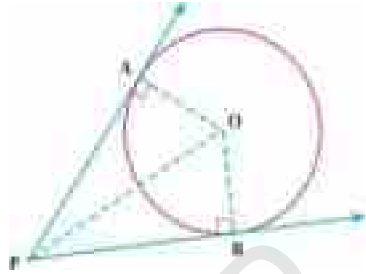
$$m\overline{AP} = m\overline{AQ} \quad \Rightarrow \quad m\overline{AQ} = 4cm$$

Finally, we need length of chord $\overline{PQ}$ of larger circle which touches smaller circle, which is: $m\overline{PQ} = m\overline{AP} + m\overline{AQ} = 8cm.$

EXERCISE 26.1

1. Show that a triangle circumscribing a circle is an equilateral triangle.
2. Show that a rectangle circumscribing a circle must be a square.
3. The diameters of two concentric circles are 10cm and 5cm, respectively. Find length of the tangent segment to the smaller circle to a point on it touching outer circle. Also find length of chord of outer circle which touches inner circle.
4. The length of tangent segment from a point at a distance of 5cm from centre of the circle is 4cm. Find diameter, circumference and area of the circle?
5. Find length of the tangent segment to a circle of radius 7cm from a point at a distance of 25cm from the centre of circle.
6. How far from centre of the circle of radius 3cm, a tangent segment of length 10cm can be drawn?

Theorem 26.3: The two tangents, drawn to a circle from a point outside it, are equal in length.



Given: $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are tangents to the circle at A and B from a point P outside it.

To prove: $\overline{PA} \cong \overline{PB}$.

Construction: Draw $\overline{OA}, \overline{OB}$ and $\overline{OP}$.

Proof:

Statements	Reasons
$\overline{OA} \perp \overrightarrow{PA}$ and $m\angle OAP = 90^\circ$	Tangent is perpendicular to radial segment
Similarly, $\overline{OB} \perp \overrightarrow{PB}$ and $m\angle OBP = 90^\circ$	Tangent is perpendicular to radial segment
In right triangles $\Delta OAP \leftrightarrow \Delta OBP$	
$\overline{OP} \cong \overline{OP}$	Common side
$\overline{OA} \cong \overline{OB}$	Radial segments of same circles
$\therefore$ $\Delta OAP \cong \Delta OBP$	$H.S \cong H.S$
$\therefore$ $\overline{PA} \cong \overline{PB}$	Corresponding sides of congruent Δs.

Q.E.D.

Note 1:

Theorem 26.3 demonstrates that the two tangents to a circle intersecting at a point outside the circle must be equal in length.

Corollary 1:

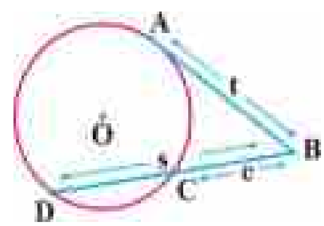
The two tangents drawn to a circle from an external point subtend congruent angles at the centre.

Corollary 2:

Two parallel tangents to a circle do not intersect at the same point outside it.

Note 2:

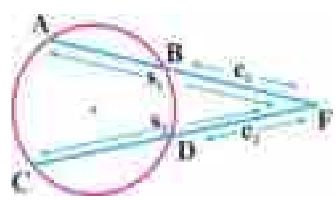
If a tangent and a secant to a circle intersect outside a circle as shown in adjacent figure, then the square of the length of the tangent segment equals the product of the lengths of the secant segment and its external portion, i.e.



$$\left(m\overline{AB}\right)^2 = \left(m\overline{DB}\right)\cdot\left(m\overline{BC}\right) \quad \text{or} \quad t^2 = s \cdot e$$

Note 3:

If two secants intersect outside a circle as shown in the adjacent figure, then product of the lengths of one secant segment and its external portion equals the product of the lengths of other secant segment and its external portion, i.e.

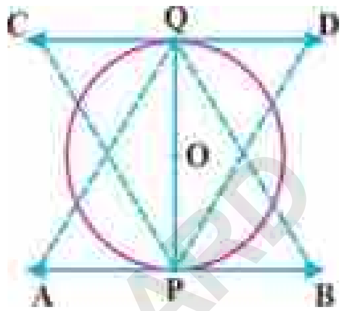


$$\left(m\overline{AF}\right)\times\left(m\overline{BF}\right) = \left(m\overline{CF}\right)\times\left(m\overline{DF}\right) \quad \text{or} \quad s_1e_1 = s_2e_2$$

Example 1:

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Given:



A circle centred at O. AB and CD are tangent at end points P and Q of the diameter, respectively.

To prove:

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Construction:

Draw $\overline{CP}, \overline{DP}, \overline{AQ}$ and $\overline{BQ}$.

Proof:

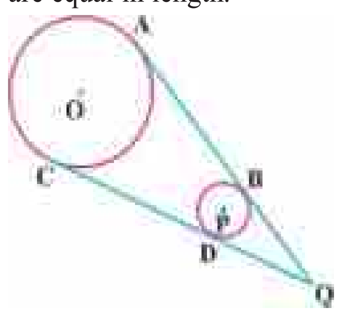
Statements	Reasons
$m\angle APO = 90^\circ = m\angle BPO$	$\overleftrightarrow{AB} \perp \overline{OP}$
$m\angle CQO = 90^\circ = m\angle DQO$	$\overleftrightarrow{CD} \perp \overline{OQ}$
$m\angle CPQ = m\angle BQP$ (i)	Alternating angles
and $m\angle AQP = m\angle QPD$ (ii)	Alternating angles
$\therefore$ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Same interior alternating angles, (i), (ii)

Q.E.D.

Example 2:

Prove that the two direct common tangents to two circles are equal in length.

Given:



$\overline{AB}$ and $\overline{CD}$ are directcommon tangents to circles with centres O and P.

To prove:

$m\overline{AB} = m\overline{CD}$

Construction:

Extend $\overline{AB}$ and $\overline{CD}$ to intersect at a point, say Q.

Proof:

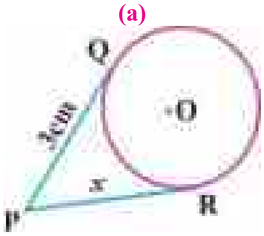
Statements	Reasons
$m\overline{AQ} = m\overline{CQ}$ (i)	Tangents drawn from point Q outside O.
$m\overline{BQ} = m\overline{DQ}$ (ii)	Tangents drawn from point Q outside P.
$m\overline{AQ} - m\overline{BQ} = m\overline{CQ} - m\overline{DQ}$	Subtracting (ii) from (i).
$\therefore$ $m\overline{AB} = m\overline{CD}$	Using figure.

Q.E.D.

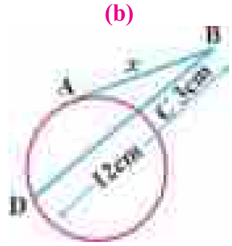
Example 3:

Find unknown length x in the following figures:

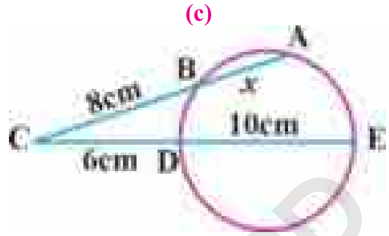
(a)



(b)



(c)



Solution:

(a). Tangents $\overline{PQ}$ and $\overline{PR}$ meet at point P outside the circle, so they must be equal in length, i.e. $m\overline{PQ} = m\overline{PR}$. Therefor, $x = 3\text{cm}$.

(b). The tangent $\overline{AB}$ meets the secant $\overline{DC}$ at point B outside the circle, so we have:
$\left(m\overline{AB}\right)^2 = \left(m\overline{DB}\right)\cdot\left(m\overline{BC}\right)$. (or) $x^2 = 12\times 3 \Rightarrow x^2 = 36 \Rightarrow x = 6cm.$

(c). Two secants $\overline{AB}$ and $\overline{ED}$ of circle intersect at C outside it, so we have:
$\left(m\overline{AC}\right)\times\left(m\overline{BC}\right) = \left(m\overline{EC}\right)\times\left(m\overline{CD}\right)$ (or) $(x+8)\times 8 = (10+6)\times 6 \Rightarrow 8x + 64 = 96$

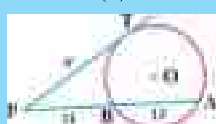
Finally, $8x = 96 - 64 \Rightarrow 8x = 32 \Rightarrow x = 4\text{cm}.$

EXERCISE 26.2

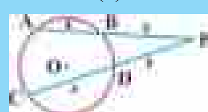
1. Show that the tangents drawn at the ends of a chord in a circle make equal angles with the chord.
2. Show that if two tangents to a circle are parallel, then the points of tangency are the end points of a diameter of the circle.
3. Show that if two tangents to a circle are not parallel, then the points of tangency are end points of a chord of the circle.
4. Find unknown x in the following.

(a)

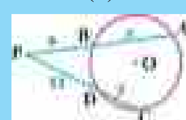
(b)



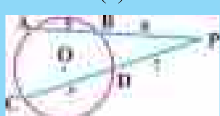
(c)



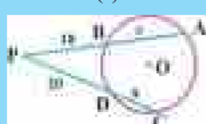
(d)



(e)



(f)

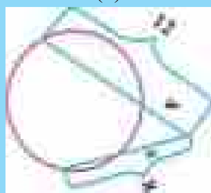


(g)

(h)

5. Find unknown x in the following.

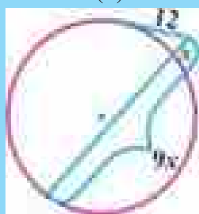
(a)



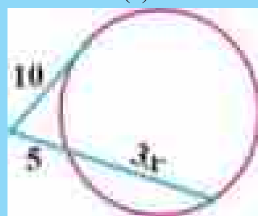
(b)

(c)

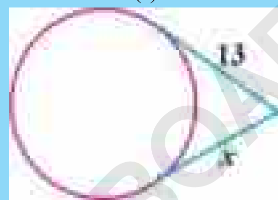
(d)



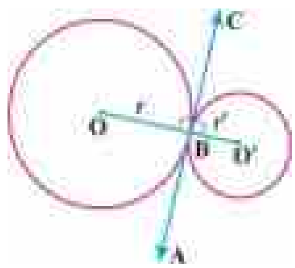
(e)



(f)



Theorem 26.4: Case (A): If two circles touch externally, the distance between their centres is equal to the sum of their radii.



Given: Circles with centres O and O’ having radii r and r', respectively, touching each other externally at point B.

To prove: $m\overline{OO'} = r + r'$

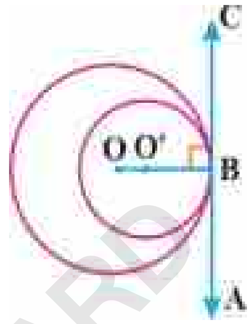
Construction: Draw a common tangent $\overleftrightarrow{AC}$ through B of both circles.

Proof:

Statements	Reasons
$\overline{OB} \perp \overleftrightarrow{AC}$ so $m\angle OBC = 90°$ (i)	Tangent is perpendicular to radial segment.
$\overline{O'B} \perp \overleftrightarrow{AC}$ so $m\angle O'BC = 90°$ (ii)	Tangent is perpendicular to radial segment.
$m\angle OBC + m\angle O'BC = 180°$ (iii)	Adding (i) and (ii).
O,B and O′ are collinear, and B lies between O and O′.	From (i), (ii) and (iii)
$\therefore$ $m\overline{OB} + m\overline{O'B} = m\overline{OO'}$	By definition.
or $m\overline{OO'} = r + r'$	$m\overline{OB} = r$ and $m\overline{O'B} = r'$.

Q.E.D.

Theorem 26.4: Case (B): If two circles touch internally, the distance between their centres is equal to the difference of their radii.



Given:

Circles with centres O and O' with radii r and r', respectively, touching each other internally at B.

To prove:

$m\overline{OO'} = r - r'$

Construction:

Draw a common tangent $\overleftrightarrow{AC}$ through B of both circles.

Proof:

Statements	Reasons
$\overline{OB} \perp \overleftrightarrow{AC}$ so $m\angle OBC = 90^\circ$ (i)	Tangent is perpendicular to radial segment
$\overline{O'B} \perp \overleftrightarrow{AC}$ so $m\angle O'BC = 90^\circ$ (ii)	Tangent is perpendicular to radial segment
$m\angle OBC = m\angle O'BC = 90^\circ$ (iii)	From (i) and (ii).
O, O' and B are collinear, and O' lies between O and B,	From (i), (ii) and (iii).
$\therefore$ $m\overline{OO'} + m\overline{O'B} = m\overline{OB}$ (iv)	By definition.
or $m\overline{OO'} = m\overline{OB} - m\overline{O'B} = r - r'$	From (iv), and $m\overline{OB} = r$, $m\overline{O'B} = r'$.

Q.E.D.

Corollary 1:

If distance between centres of two circles is sum (difference) of their radii, then circles touch externally (internally).

Corollary 2:

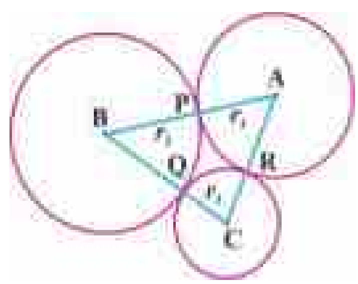
If the distance between centres of two circles is not equal to the sum or difference of their radii, then circles do not touch each other.

Example 1.

If three circles touch in pair externally, then the perimeter of a triangle formed by joining their centres is equal to twice the sum of their radii.

Given:

Three circles with centres A, B, C and radii r_1, r_2, r_3, respectively, touch in pairs externally at points P, Q and R.



To prove:

Perimeter of $\Delta ABC = 2(r_1 + r_2 + r_3)$.

Construction:

Construct $\overline{AB}, \overline{BC}$ and $\overline{AC}$ through P, Q and R, respectively, to form ΔABC.

Proof:

Statements	Reasons
$m\overline{AB} = m\overline{AP} + m\overline{BP}$ (i)	P lies between A and B.
$m\overline{BC} = m\overline{BQ} + m\overline{CQ}$ (ii)	Q lies between B and C.
$m\overline{AC} = m\overline{AR} + m\overline{CR}$ (iii)	R lies between A and C.
$m\overline{AB} + m\overline{BC} + m\overline{AC}$ $= m\overline{AP} + m\overline{BP} + m\overline{BQ} + m\overline{CQ} + m\overline{AR} + m\overline{CR}$	Adding (i), (ii) and (iii)
$m\overline{AB} + m\overline{BC} + m\overline{AC}$ $= \left(m\overline{AP} + m\overline{AR}\right) + \left(m\overline{BP} + m\overline{BQ}\right) + \left(m\overline{CQ} + m\overline{CR}\right)$	Re-arranging
$m\overline{AB} + m\overline{BC} + m\overline{AC} = \left(r_1 + r_1\right) + \left(r_2 + r_2\right) + \left(r_3 + r_3\right)$ $= 2r_1 + 2r_2 + 2r_3 = 2\left(r_1 + r_2 + r_3\right)$	$m\overline{AP} = m\overline{AR} = r_1$ $m\overline{BP} = m\overline{BQ} = r_2$ $m\overline{CQ} = m\overline{CR} = r_3$
So, Perimeter of $\Delta ABC = 2\left(r_1 + r_2 + r_3\right)$	

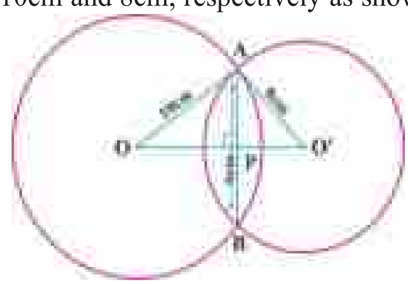
Q.E.D.

Example 2:

The radii of two intersecting circles are 10cm and 8cm. If the length of their common chord is 6cm, what is the distance between their centres?

Solution:

Consider circles with centres O and O' with radii 10cm and 8cm, respectively as shown in the figure. From figure, we have:



$m\overline{OA} = 10\text{cm},\ m\overline{O'A} = 8\text{cm},\ m\overline{AB} = 6\text{cm},\quad m\overline{OO'} = ?$

P bisects $\overline{AB}$, so $m\overline{AP} = \frac{1}{2} m\overline{AB} = \frac{6}{2} = 3\text{cm}.$

In ΔAPO, using Pythagoras' theorem we have:

$10^2 = 3^2 + \left(m\overline{OP}\right)^2 \quad \Rightarrow m\overline{OP} = \sqrt{100-9} = \sqrt{91}\text{cm}.$

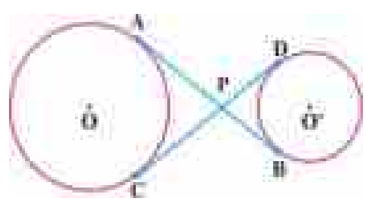
In $\Delta APO'$, using Pythagoras' theorem we have:

$8^2 = 3^2 + \left(m\overline{O'P}\right)^2 \quad \Rightarrow m\overline{O'P} = \sqrt{64-9} = \sqrt{55}\text{cm}$

Finally, $m\overline{OO'} = m\overline{OP} + m\overline{O'P} = \sqrt{91} + \sqrt{55} = 16.955\text{cm}$ (approximately).

Example 3:

The two transverse common tangents to the two circles are equal in length.



Given: $\overline{AB}$ and $\overline{CD}$ are two internal (transverse) common tangents to circles with centres O and O', respectively.

To prove: $m\overline{AB} = m\overline{CD}$

Construction: Tangents $\overline{AB}$ and $\overline{CD}$ intersect at point P.

Proof:

Statements	Reasons
$m\overline{AP} = m\overline{CP}$ (i)	Tangents from a point outside circle.
$m\overline{BP} = m\overline{DP}$ (ii)	Tangents from a point outside circle.
$m\overline{AP} + m\overline{BP} = m\overline{CP} + m\overline{DP}$ (iii)	Adding (i) and (ii)
$\Rightarrow m\overline{AB} = m\overline{CD}$	From (iii) and figure.

Q.E.D.

Example 4:

Two circles touch each other externally, and the length of segment joining their centres is 7cm. If radius of one is 3cm, what is area of other circle?

Solution:

Two circles touch externally, so:

Length of segment joining centres $= r_1 + r_2$

$$7 = 3 + r_2 \qquad \Rightarrow \qquad r_2 = 7 - 3 = 4cm.$$

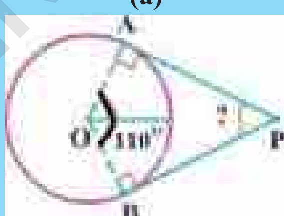
Now, area of second circle $= \pi r_2^2 = \pi \cdot (4)^2 = 16\pi \text{cm}^2$

$$= 16\left(\frac{22}{7}\right)\text{cm}^2 = 50.286\text{cm}^2 \text{ (approximately).}$$

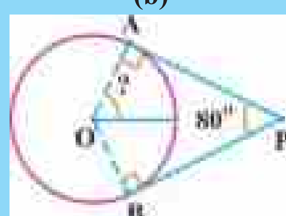
EXERCISE 26.3

1. If length of the segment joining centres of two circles is the sum of their radii, show that the circles touch externally.
2. If perimeter of the triangle with vertices at the centres of three circles is equal to the sum of their diameters, show that the three circles touch in pairs externally.
3. If length of segment joining two congruent circles touching externally is 12cm, find their radii and circumferences.
4. If $\overline{PA}$ and $\overline{PB}$ are tangents to the given circle from a point P outside as indicated in the figures. Find the unknown angle.

(a)



(b)



5. Show that if two circles touch externally, then the point of contact lies on the segment joining their centres.

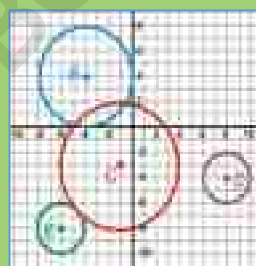
6. If two chords in a circle of radius 2.5cm are 3.9cm apart, and length of a chord is 1.4cm, then find the length of the other chord.
7. If three circles touch in pairs externally and radii of two circles are 4cm and 5cm, then find the radius of the third circle if the triangle formed through their centres has perimeter of 35cm.

Review Exercise 26

1. Tick the correct answer.

i. If a point is outside the circle then from this point we can draw ____ tangent(s) to the circle.
(a) one (b) two (c) three (d) none

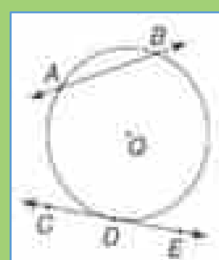
ii. Angle between the radial segment and tangent at its outer end point is ____ .
(a) 45^0 (b) 60^0 (c) 90^0 (d) 120^0



iii. In the adjacent figure, circles with centres E and C __ .
(a) touch internally (b) touch externally
(c) do not touch (d) are congruent

iv. In the adjacent figure, circles with centres B and C have ______ point(s) of contact.
(a) no (b) one
(c) two (d) none of these

v. In adjacent figure, circles with centres E and C have ______ point(s) of contact.
(a) no (b) one
(c) two (d) none of these



vi. In the adjacent figure, $\overleftrightarrow{AB}$ is ________ .
(a) tangent (b) secant
(c) chord (d) none of these

vii. In the adjacent figure, $\overleftrightarrow{CDE}$ is _______ .
(a) tangent (b) secant
(c) chord (d) none of these

viii. In the adjacent figure, the point of tangency is __ .
(a) A (b) B (c) C (d) D

ix. Tangents drawn at end points of a diameter of a circle are ______ to each other
(a) parallel (b) perpendicular (c) intersecting (d) both (b) and (c)

x. The maximum number of common tangents between two circles touching internally is ____ .
(a) 0 (b) 1 (c) 2 (d) 3

xi. The maximum number of common tangents between two circles touching externally is ____ .
(a) 0 (b) 1 (c) 2 (d) 3

SUMMARY

- A straight line touching a circle at only one point is a tangent to the circle, and the common point is called point of tangency or point of contact.
- A straight line intersecting a circle at two points is a secant to the circle. There are two points of contact between a secant and the circle.
- A tangent segment is a line segment from the point of tangency to any other point on the tangent.
- The triangle formed by joining the centre of circle, the point of tangency and another point on tangent line is always a right angled triangle.
- When two circles touch internally or externally, they share a common tangent between them and the common point of contact.
- Two circles not touching each other can also have common tangents: external (direct) and internal (transverse) common tangents.
- The lengths of two direct common tangents to two circles are equal.
- The lengths of two transverse common tangents to two circles are equal.
- Only one tangent can be drawn to a circle at a point on it.
- If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.
- The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.
- The two tangents drawn to a circle from a point outside it are equal in length.
- If two circles touch externally, the distance between their centres is equal to the sum of their radii.
- If two circles touch internally, the distance between their centres is equal to the difference of their radii.