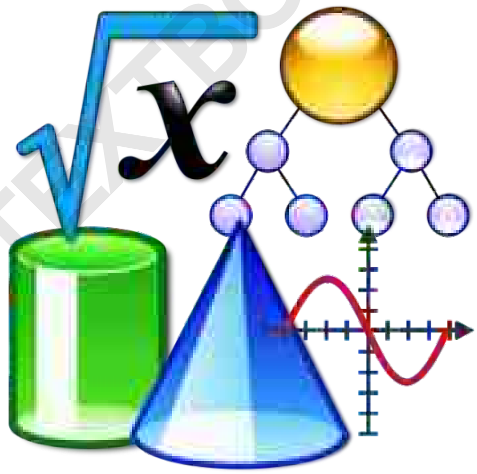
TEST EDITION

THE TEXTBOOK OF

MATHEMATICS

For Class X



Sindh Textbook Board, Jamshoro

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Cheif Supervisor
Yousuf Ahmed Shaikh
Sindh Textbook Board, Jamshoro

Supervisor
Daryush Kafi
Sindh Textbook Board, Jamshoro

Authors
- ☆ Mr. Abdul Saleem Memon
- ☆ Mr.Aijaz Ali Subehphoto
- ☆ Dr. Kashif Ali Abro
- ☆ Dr. Muhammad Mujtaba Shaikh
- ☆ Prof. Muhammad Farooq Khan
- ☆ Dr. Hafeezullah Mahar

Editors
- ☆ Dr. Zubair Ahmed Kalhoro
- ☆ Mr. Mir Sarfraz Khalil Saand

Reviewers
- ☆ Prof. Dr. Zain-ul-Abdin Khuhro
- ☆ Professor Muhammad Farooq khan
- ☆ Mr. Muhammad Saghir Shaikh
- ☆ Mr. Aijaz Ali Subehpoto
- ☆ Mr. Muhammad Waseem
- ☆ Mr. Zohaib Haseeb
- ☆ Mr. Riaz Hussain
- ☆ Mr. Aftab Ali
- ☆ Mr. Mir Sarfraz Khalil Saand

Consultants
- ☆ Mr. Kamran Latif Laghari ; A.S.S
- ☆ Mr. Mir Sarfraz Khalil Saand ; J.S.S

Composing
- ☆ Zia-ul-Haq Lodhi
- ☆ Muhammad Atif Ghouri

Designing & Illustration
- ☆ Muhammad Arslan Shafaat Gaddi

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PREFACE

The Sindh Textbook Board, is assigned with preparation and publication of the textbooks to equip our new generation with knowledge, skills and ability to face the challenges of new millennium in the fields of Science, Technology and Humanities. The textbooks are also aimed at inculcating the ingredients of universal brotherhood and to reflect the valiant deeds of our forebears and portray the illuminating patterns of our rich cultural heritage and tradition.

The textbook, in your hand, of Mathematics for class X has been developed according to provincial curriculum 2017 in the continuity of Mathematics for class IX. The curriculum contains 30 units, among them, first 16 units were included in Mathematics for class IX. Hence, remaining 14 units (from 17 to 30) are included in this textbook of Mathematics for class X.

The Sindh Textbook Board has taken great pains and incurred expenditure in publishing this book inspite of its limitations. A textbook is indeed not the last word and there is always room for improvement. While the authors have tried their level best to make the most suitable presentation, both in terms of concept and treatment, there may still have some deficiencies and omissions. Learned teachers and worthy students are, therefore, requested to be kind enough to point out the shortcomings of the text or diagrams and to communicate their suggestions and objections for the improvement of the next edition of this book.

In the end, I am thankful to our learned authors, editors and specialist of Board for their relentless service rendered for the cause of education.

Chairman
Sindh Textbook Board

CONTENTS

- Total weightage for examination SSC-II paper is 100%
- Unit-wise weightage for examination SSC-II paper is mentioned in each unit as per curriculum reviewed by DCAR, Sindh.

SETS AND FUNCTIONS

Unit 17

• Weightage = 11%

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- Recall the sets denoted by $\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{E}, \mathbb{O}, \mathbb{P}, \mathbb{C}, \mathbb{Q}, \mathbb{Q}'$ and $\mathbb{R}$
- Type of sets and representation of sets
- Operations on sets
    - Union,
    - Intersection,
    - Difference,
    - Complement.
- Symmetric difference of two sets
- Give formal proof of the following fundamentalproperties of union and intersection of two or three sets.
    - Commutative property of union,
    - Commutative property of intersection,
    - Associative property of union,
    - Associative property of intersection,
    - Distributive property of union over intersection,
    - Distributive property of intersection over union,
    - De-Morgan's laws,
- Verify the fundamental property of given sets.
- Use Venn diagrams to represent
    - Union and intersection of sets,
    - Complement of a set.
    - Symmetric difference of two sets.
- Use Venn diagram to verify
    - Commutative property of union overintersection of sets,
    - De-Morgan's laws,
    - Associative laws,
- Distributive laws.
- Recognize order pairs.
- To form Cartesian products.
- Define a binary relation and identify its domain and range.
- Define a function and identify its domain, co-domain and range.
- Demonstrate the following:
    - Into and one-one function (injective function),
    - Onto function (subjective function),
    - One-one and onto function (bijective function).
- Examine whether a given relation is a function or not.
- Differentiate between one-one correspondence and one-one function.

17.1 Operations on Sets:

17.1(i) Recall the sets denoted by $\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{E}, \mathbb{O}, \mathbb{P}, \mathbb{C}, \mathbb{Q}, \mathbb{Q}'$ and $\mathbb{R}$

As a matter of fact, set is one of the fundamental concepts of Mathematics which is useful in formulating and analyzing many mathematical notions. The concept of set, in detail, has already been discussed in previous classes. Let us recall some important sets.

Set of Natural Numbers: $\mathbb{N} = \{1,2,3,...\}$

Set of Whole Numbers: $\mathbb{W} = \{0,1,2,3,...\}$

Set of Integers: $\mathbb{Z} = \{0,\pm1,\pm2,\pm3,...\}$

Set of Even Integers: $\mathbb{E} = \{0,\pm2,\pm4,\pm6,...\}$

Set of Odd Integers: $\mathbb{O} = \{\pm1,\pm3,\pm5,...\}$

Set of Prime Numbers: $\mathbb{P} = \{2,3,5,7,...\}$

Set of Composite Numbers: $\mathbb{C} = \{4,6,8,9,10, ...\}$

Set of Rational Numbers: $\mathbb{Q} = \left\{x \mid x = \frac{p}{q}, p \wedge q \in Z,\ q \neq 0\right\}$

Set of Irrational Numbers: $\mathbb{Q}' = \left\{x \mid x \neq \frac{p}{q}, p \wedge q \in Z,\ q \neq 0\right\}$

Set of Real Numbers: $\mathbb{R} = \left\{x \mid x = \frac{p}{q} \vee x \neq \frac{p}{q}, p \wedge q \in Z,\ q \neq 0\right\}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$

Notes:
- The above sets can also be represented by N,W,Z,E,O,P,C,Q,Q′ and R.
- $\mathbb{R}^+$ and $\mathbb{R}^-$ denote the set of positive and negative real numbers respectively
- The set of all rational, irrational and real numbers cannot be written in tabular form

17.1 (ii) Types of sets and representation of sets

Firstly, we discuss the representation of sets. We are already familiar with the three methods or forms of representing a set which we studied in previous classes. These are

1. Descriptive form.
2. Tabular form or Roster form.
3. Set Builder form.

Recall that, in descriptive form, a set is described by common characteristics of its elements in any common language, for instance

A = Set of natural numbers between 5 and 10.

In tabular form, a set is described by listing its elements, each is separated by comma and enclosed within braces. The above set is written in tabular form as $A = \{6,7,8,9\}$ while in set builder form a set is described by common characteristics of its elements using symbols. The set builder from has two parts, one determines the nature of the elements and other its range.

The given set A can be written in set builder form as $A = \{x \mid x \in \mathbb{N} \wedge 5 < x < 10\}$.

Example 1: Write the set $A = \{6, 8, 10, 12\}$ in descriptive and set builder form.

Solution:

Descriptive form: A = Set of even numbers between 5 and 13

Set builder form: $A = \{x \mid x \in \mathbb{E} \wedge 5 < x < 13\}$

Example 2: Write the set $B = \{y \mid y \in \mathbb{P} \wedge y < 10\}$ in tabular and descriptive form.

Solution:

Tabular form: $B = \{2, 3, 5, 7\}$

Descriptive form: B = Set of first four prime numbers

Now, we discuss some types of sets.

Empty set or Null set or void set:

A set having no element is called an empty or null set. It is denoted by $\varnothing$ or $\{\ \}$.

For example: (i) A = Set of even numbers between 5 and 6.

(ii) $B = \{x \mid x \in \mathbb{N} \wedge x < 1\}$

Singleton Set or Unit Set:

A set having single element is called singleton or unit set.

For example: $A = \{x \mid x \in W \wedge x < 1\}$ is a singleton.

Finite Set:

A set having limited number of elements is called a finite set.

For example: (i) $A = \{10, 12, 14, ..., 50\}$

(ii) B = Set of all countries of the world

Note: Empty set is considered as a finite set.

Infinite Set:

A set having unlimited number of elements is called an infinite set.

For example: (i) $P = \{10, 20, 30, ...\}$

(ii) $Q = \{x \mid x \in \mathbb{Q} \wedge 1 \leq x \leq 2\}$

Subset:

If every element of set A is also an element of set B then A is called a subset of B. Symbolically we write as $A \subseteq B$.

For example: If $A = \{2, 3, 5, 7\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ and $C = \{6, 7, 8, 9\}$

then $A \subseteq B$ but $A \not\subseteq C$ (A is not subset of C)

There are two types of subsets: (i) Proper subset (ii) Improper subset

(i) Proper Subset:

Let A and B be two non-empty sets such that $A \subseteq B$ but $B \not\subseteq A$ then A is called proper subset of B, denoted by $A \subset B$.

For Example: Let $A = \{2, 4, 6\}$ and $B = \{2, 4, 6, 8\}$

As $A \subseteq B$ but $B \not\subseteq A \Rightarrow A \subset B$.

Note: If $A \subset B \Rightarrow n(A) < n(B)$.

(ii) Improper Subset:

Let A and B be two non-empty sets such that $A \subseteq B$ but $B \subseteq A$ then A and B are called improper subsets of each other. Infact every set is a subset of itself i.e. an improper subset. Symbolically, $A \subseteq A$.

For Example: Let $A = \{0, 4, 8, 12\}$ and $B = \{0, 8, 12, 4\}$

Here $A \subseteq B$ and also $B \subseteq A$.

$\Rightarrow$ A and B are improper subsets of each other.

Notes: (i) Every set is subset of itself.

(ii) Empty set is a subset of every set.

(iii) Every non-empty set has at least two subsets.

(iv) Number of all subsets of a set containing in elements is 2^n.

Superset:

If set A is subset of set B then B is called superset of A.

Symbolically we write as $B \supseteq A$

For example: If $X = \{a, e, i, o, u\}$ and $Y = \{a, b, c, ..., z\}$ then $Y \supseteq X$

Equivalent Sets:

Two sets A and B are said to be equivalent sets if their orders are equal. Symbolically, we write as $A \sim B$, i.e., $A \sim B$ iff $O(A) = O(B)$

Thus, if $A \sim B$ then one-one correspondence between their elements can be established.

For example:

Let $A = \{x \mid x \in \mathbb{R} \wedge x^2 = 64\}$ and B = Set of prime numbers less than 5.

Here $A \sim B$ because $O(A) = O(B) = 2$

Equal Sets:

Two sets A and B of same order are called equal sets if all the elements of both the sets are same. Symbolically we write as $A = B$.

Thus $A = B$ iff $A \subseteq B$ and $B \subseteq A$ also $A = B$ iff $A \supseteq B$ and $B \supseteq A$

Example: Let $A = \{1, 2, 3, 6\}$, $B = \{x \mid x \in N \wedge x \leq 6\}$ and C = Set of divisions of 6

Here $A = C$ but $A \neq B$

Note: Order of set A means the number of elements of A. It is denoted by O(A) or n(A) or |A|.

Power Set:

The set of all the subsets of set A is called power set of A. It is denoted as P(A).

For example:

If $A = \{x, y, z\}$ then

$P(A) = \{\varnothing, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\}\}$

Note: Power set of an empty set is non-empty.

Universal Set:

The superset of all the sets under consideration is called universal set. It is denoted by U or X.

For example: If $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6\}$ then $U = \{1, 2, 3, 4, 5, 6\}$

Further types of sets will be discussed in section 17.1(iv).

Exercise 17.1

1. Write the following sets in tabular form.

(i) A = Set of all integers between -3 and 3

(ii) B = Set of composite numbers less than 11.

(iii) $C = \{x \mid x \in \mathbb{P} \wedge 5 < x \leq 13\}$ (iv) $D = \{y \mid y \in \mathbb{O} \wedge 7 < y < 17\}$

(v) $E = \{z \mid z \in \mathbb{R} \wedge z^2 = 121\}$ (vi) $F = \{p \mid p \in \mathbb{Q} \wedge p^2 = -1\}$

2. Write the following sets in set builder form.

(i) A = Set of all rational numbers between 5 and 6 (ii) $B = \{1, 2, 3, 4, 6, 12\}$

(iii) $C = \{0, \pm1, \pm2, ..., \pm40\}$ (iv) $D = \{-4, -2, 0, 2, 4\}$

(v) $E = \{1, 4, 9, 16, 25\}$ (vi) $F = \{-1, -3, -5, -7, ...\}$

3. Write any five examples of empty set.

4. Classify the following sets as finite and infinite sets.

(i) Set of Asian countries.

(ii) Set of all the medical universities of the world.

(iii) Set of all real numbers between 6 and 9.

(iv) Set of all the even prime numbers.

(v) Set of all odd integers less than 5.

5. Write an equivalent set, an improper subset and three proper subsets of each of the following sets.

(i) $P = \{a, e, i, o, u\}$

(ii) $Q = \{x \mid x \in \mathbb{Z} \wedge -2 \leq x \leq 2\}$

6. Write any two examples of singleton in set builder form.

7. Write power set of the following sets.

(i) $A = \{5, 10, 15\}$ (ii) $B = \{x \mid x \in Z \wedge -1 < x < 4\}$

8. Find a set which has

(i) only two proper subsets.

(ii) only one proper subset.

(iii) no proper subset.

17.1 (iii) Operations on sets

- Union
- Intersection
- Difference
- Complement

Union of two sets

The union of two sets X and Y, denoted as $X \cup Y$, is the set of all those elements which belong to X or to Y or to both X and Y.

i.e., $X \cup Y = \{x \mid x \in X \vee x \in Y\}$

For example: If $X = \{1,3,5\}$ and $Y = \{1,2,3,4\}$ then $X \cup Y = \{1,2,3,4,5\}$

Notes: (i) $X \subseteq (X \cup Y)$ (ii) $Y \subseteq (X \cup Y)$

Intersection of two sets

The intersection of two sets X and Y, denoted as $X \cap Y$, is the set of all those elements which belong to both X and Y.

i.e., $X \cap Y = \{x \mid x \in X \wedge x \in Y\}$

For example: If $X = \{2,4,6,8\}$ and $Y = \{1,2,3,6\}$ then $X \cap Y = \{2,6\}$

Notes: (i) $(X \cap Y) \subseteq X$ (ii) $(X \cap Y) \subseteq Y$

Difference of two sets

For any two sets X and Y, the difference X–Y is the set of all the elements which belong to X but do not belong to Y. It is also denoted as X\Y.

i.e., $X - Y = \{x \mid x \in X \wedge x \notin Y\}$

Similarly, the difference $Y - X$ is the set of all the elements which belong to Y but do not belong to X. It is also denoted as Y\X. i.e., $Y - X = \{y \mid y \in Y \wedge y \notin X\}$

For example:

If $X = \{4,6,8,9,10\}$ and $Y = \{2,4,6,8\}$

then $X - Y = \{9,10\}$

and $Y - X = \{2\}$

Notes: (i) $(X - Y) \subseteq X$ (ii) $(Y - X) \subseteq Y$

Complement of a set

If set A is a subset of universal set U then the complement of A is the set of all the elements of U which are not in A. It is denoted by A' or A^c.

Thus $A' = U - A$

i.e., $A' = \{x \mid x \in U \wedge x \notin A\}$

For example: If $U = \{1,2,3,\ldots,20\}$ and $A = \{1,3,5,\ldots,19\}$ then $A' = \{2,4,6,\ldots,20\}$

17.1 (iv) Symmetric difference of two sets

The symmetric difference of two sets A and B, denoted $A \Delta B$, is the set of all the elements of A or B which are not common in both the sets.

i.e., $A \Delta B = \{x \mid x \in A \cup B \wedge x \notin A \cap B\}$

or $A \Delta B = (A \cup B) - (A \cap B)$

For example: If $A=\{1,2,3,4,5\}$ and $B=\{1,3,5,7\}$ then $A\Delta B=\{2,4,7\}$

In continuation of section 17.1(ii), further types of sets are given below:

Disjoint Sets:

Two non-empty sets A and B are called disjoint sets if they have no element in common

i.e., If $A\neq\phi, B\neq\phi$ and $A\cap B=\phi$ then A and B are called disjoint sets.

For example: The sets $A=\{1,3,5\}$ and $B=\{2,4,6\}$ are disjoint sets.

Overlapping Sets:

Two non-empty sets A and B are called overlapping sets if there exists at least one element common in both. Moreover neither of them is subset of other.

i.e., Two sets A and B are overlapping sets if $A\cap B\neq\phi$ and $A\not\subseteq B$ or $B\not\subseteq A$

For example: The sets $A=\{1,2,3,4\}$ and $B=\{2,4,6,8\}$ are overlapping sets.

Exhaustive Sets:

If two non-empty sets A and B are subsets of universal set U then A and B are called exhaustive sets if $A\cup B=U$.

For example: If $A=\{1,2,3,4\}$, $B=\{1,3,5\}$ and $U=\{1,2,3,4,5\}$ then A and B are exhaustive sets because $A\cup B=U$

Cells:

If A and B are two non-empty subsets of universal set U then A and B are called cells if they are disjoint as well as exhaustive sets.

i.e., A and B are called cells if A and B are non-empty subsets of U and

$A\cap B=\phi$ and $A\cup B=U$

For example: If $A=\{1,3,5,7,9\}$, $B=\{2,4,6,8,10\}$ and $U=\{1,2,3,...,10\}$ then A and B are cells.

Note: Sets A and A' are cells of universal set U.

Some important laws related to the operations on sets are as under.

Identity Laws:

For any set A

(i) $A\cup\phi=A$ (ii) $A\cup U=U$ (iii) $A\cap U=A$ (iv) $A\cap\phi=\phi$

Idempotent Laws:

For any set A

(i) $A\cup A=A$ (ii) $A\cap A=A$

Laws of the Complement:

For any set A

(i) $A\cup A'=U$ (ii) $A\cap A'=\phi$

(iii) $(A')'=A$ (iv) $U'=\phi$ and $\phi'=U$

De Morgan's Laws:

For any two sets A and B.

(i) $(A\cup B)'=A'\cap B'$ (ii) $(A\cap B)'=A'\cup B'$

Notes: (i) $A-B=A\cap B'$ (ii) $A-B'=A\cap B$

Exercise 17.2

1. **Which of the following sets are disjoint, overlapping, exhaustive and cells.**

(i) $\{1,2,3,5,7\}$ and $\{4,6,8,9,10\}$ (ii) $\{1,2,3,6\}$ and $\{1,2,4,8\}$

(iii) E and O when $U = Z$ (U denotes universal set)

(iv) $A = \{0,2,4, \ldots\}$, $B = N$ and $U = W$ (v) Q and Q' when $U = \mathbb{R}$

2. **If $A = \{1,2,3,4,5,6\}$ and $B = \{2,4,6,8,10\}$ then find**

(i) $A \cup B$ (ii) $B \cap A$ (iii) $A - B$
(iv) $B - A$ (v) $A \Delta B$

3. **If $U = \{1,2,3,\ldots,10\}$, $A = \{1,2,3,4,5\}$ and $B = \{1,3,5,7,9\}$ then find:**

(i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$

(v) $(A \cup B)'$ (vi) $(A \cap B)'$ (vii) $A' \Delta B'$ (viii) $(A \Delta B)'$

(ix) $A - B'$ (x) $A' - B$

4. **If $U = \{x \mid x \in Z \wedge -4 < x < 6\}$, $P = \{p \mid p \in E \wedge -4 < p < 6\}$**

$Q = \{q \mid q \in P \wedge q < 6\}$ then show that:

(i) $P - Q = P \cap Q'$ (ii) $Q - P = Q \cap P'$

(iii) $(P \cup Q)' = P' \cap Q'$ (iv) $(P \cap Q)' = P' \cup Q'$

5. **If $A = \{2n \mid n \in N\}$, $B = \{3n \mid n \in N\}$ and $C = \{4n \mid n \in N\}$ then find:**

(i) $A \cap B$ (ii) $A \cup C$ (iii) $B \cap C$ (iv) $A \cap C$

6. **Given that $A_n = \{n, n+1, n+2, \ldots\}$,** $\forall\, n \in N$ then find

(i) $A_3 \cup A_5$ (ii) $A_7 \cup A_{11}$ (iii) $A_{15} - A_{13}$ (iv) $A_9 \Delta A_8$

17.1.2(i) Properties of Union and Intersection.

Give formal proof of the following fundamental properties of union and intersection of two or three sets.

➢ **Commutative Property of Union**

We know that for any two sets A and B, $\boxed{A \cup B = B \cup A}$

This property is called commutative property of union.

Proof:

L.H.S $= A \cup B$

$= \{x \mid x \in A \text{ or } x \in B\}$ (By definition of union)

$= \{x \mid x \in B \text{ or } x \in A\}$ $\because$ Order of elements in a set is not preserved

$= B \cup A$ (By definition of union)

$=$ R.H.S

$\because$ L.H.S = R.H.S

$\therefore$ $A \cup B = B \cup A$

Hence proved.

➢ Commutative Property of Intersection

We know that for any two sets A and B, $\boxed{A \cap B = B \cap A}$

This property is called commutative property of intersection.

Proof:

$$
\begin{aligned}
\text{L.H.S} &= A \cap B \\
&= \{x \mid x \in A \text{ and } x \in B\} && (\text{By definition of intersection}) \\
&= \{x \mid x \in B \text{ and } x \in A\} && \because \text{ Order of elements in a set is not preserved} \\
&= B \cap A = \text{R.H.S} && (\text{By definition of intersection})
\end{aligned}
$$

$\because$ L.H.S = R.H.S

$\therefore$ $A \cap B = B \cap A$

Hence proved.

Note: The set operation difference (–) is not communicative in general, i.e $A - B \neq B - A$

➢ Associative property of union

We are already familiar with associative property of union which is as follows.

For any three sets A, B and C, $\boxed{A \cup (B \cup C) = (A \cup B) \cup C}$

Proof:

$$
\begin{aligned}
\text{L.H.S} &= A \cup (B \cup C) \\
&= \{x \mid x \in A \text{ or } x \in B \cup C\} && (\text{By definition of union}) \\
&= \{x \mid x \in A \text{ or } x \in B \text{ or } x \in C\} \\
&= \{x \mid x \in A \cup B \text{ or } x \in C\} && (\text{By definition of union}) \\
&= (A \cup B) \cup C = \text{R.H.S} && (\text{By definition of union})
\end{aligned}
$$

$\because$ L.H.S = R.H.S

$\therefore$ $A \cup (B \cup C) = (A \cup B) \cup C$

Hence proved.

➢ Associative property of intersection

We already know the associative property of intersection which states that, for any three sets A,B and C, $\boxed{A \cap (B \cap C) = (A \cap B) \cap C}$

Proof:

$$
\begin{aligned}
\text{L.H.S} &= A \cap (B \cap C) \\
&= \{x \mid x \in A \text{ and } x \in B \cap C\} && (\text{By definition of intersection}) \\
&= \{x \mid x \in A \text{ and } x \in B \text{ and } x \in C\} \\
&= \{x \mid x \in A \cap B \text{ and } x \in C\} && (\text{By definition of intersection}) \\
&= (A \cap B) \cap C = \text{R.H.S} && (\text{By definition of intersection})
\end{aligned}
$$

$\because$ L.H.S = R.H.S

$\therefore$ $A \cap (B \cap C) = (A \cap B) \cap C$

Hence proved.

Note: The set operation difference (–) is not associative in general,
i.e $A - (B - C) \neq (A - B) - C$

➢ Distributive property of union over intersection

We have already studied distributive property of union over intersection in previous classes which is as follows,

For any three sets A,B and C, $\boxed{A\cup(B\cap C)=(A\cup B)\cap(A\cup C)}$

Proof:

$$
\begin{aligned}
\text{L.H.S} &= A\cup(B\cap C) \\
&=\{x \mid x\in A \text{ or } x\in B\cap C\} && (\text{By definition of union}) \\
&=\{x \mid x\in A \text{ or } (x\in B \text{ and } x\in C)\} && (\text{By definition of intersection}) \\
&=\{x \mid (x\in A \text{ or } x\in B) \text{ and } (x\in A \text{ or } x\in C)\} \\
&=\{x \mid x\in A\cup B \text{ and } x\in A\cup C\} && (\text{By definition of union}) \\
&=(A\cup B)\cap(A\cup C) = \text{R.H.S} && (\text{By definition of intersection})
\end{aligned}
$$

$\because$ L.H.S = R.H.S

$\therefore$ $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence proved.

➢ Distributive property of intersection over union

We are already familiar with distributive property of intersection over union which is as under.

For any three sets A,B and C, $\boxed{A\cap(B\cup C)=(A\cap B)\cup(A\cap C)}$

Proof:

$$
\begin{aligned}
\text{L.H.S} &= A\cap(B\cup C) \\
&=\{x \mid x\in A \text{ and } x\in B\cup C\} && (\text{By definition of intersection}) \\
&=\{x \mid x\in A \text{ and } (x\in B \text{ or } x\in C)\} && (\text{By definition of union}) \\
&=\{x \mid (x\in A \text{ and } x\in B) \text{ or } (x\in A \text{ and } x\in C)\} \\
&=\{x \mid x\in A\cap B \text{ or } x\in A\cap C\} && (\text{By definition of intersection}) \\
&=(A\cap B)\cup(A\cap C) = \text{R.H.S} && (\text{By definition of union})
\end{aligned}
$$

$\because$ L.H.S = R.H.S

$\therefore$ $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Hence proved.

Note: The set operation difference ($-$) is neither distributive over union (U) nor over intersection ($\cap$) i.e. $A-(B\cup C)\neq(A-B)\cup(A-C)$ and $A-(B\cap C)\neq(A-B)\cap(A-C)$

De Morgan's Laws

We have already studied in section 17.1(iv) that there are two De Morgan's Laws which are as under.

For any two sets A and B

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Proof: (i) $(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)'$

$= \{x \mid x \in U \text{ and } x \notin A \cup B\}$ (By definition of complement)

$= \{x \mid x \in U \text{ and } (x \notin A \text{ and } x \notin B)\}$

$= \{x \mid (x \in U \text{ and } x \notin A) \text{ and } (x \in U \text{ and } x \notin B)\}$

$= \{x \mid x \in A' \text{ and } x \in B'\}$ (By definition of complement)

$= A' \cap B'$ (By definition of intersection)

$= R.H.S$

$\because$ $L.H.S = R.H.S$

$\therefore$ $(A \cup B)' = A' \cap B'$

Hence proved.

Proof: (ii) $(A \cap B)' = A' \cup B'$

$L.H.S = (A \cap B)'$

$= \{x \mid x \in U \text{ and } x \notin A \cap B\}$ (By definition of complement)

$= \{x \mid x \in U \text{ and } (x \notin A \text{ or } x \notin B)\}$

$= \{x \mid (x \in U \text{ and } x \notin A) \text{ or } (x \in U \text{ and } x \notin B)\}$

$= \{x \mid x \in A' \text{ or } x \in B'\}$ (By definition of complement)

$= A' \cup B'$ (By definition of union)

$= R.H.S$

$\because$ $L.H.S = R.H.S$

$\therefore$ $(A \cap B)' = A' \cup B'$

Hence proved.

17.1.2(ii) Verify the fundamental properties of given sets.

Let us verify the fundamental properties with the help of the following examples.

Example 1:

If $A = \{1, 2, 3, 4, 6, 12\}$ and $B = \{4, 6, 8, 9, 10, 12\}$ then verify commutative property of union and intersection.

Verification:

(a) Commutative property of union

i.e. $A \cup B = B \cup A$

$$\begin{aligned} L.H.S &= A \cup B \\ &= \{1,2,3,4,6,12\} \cup \{4,6,8,9,10,12\} \\ &= \{1,2,3,4,6,8,9,10,12\} \end{aligned}$$

$$\begin{aligned} R.H.S &= B \cup A \\ &= \{4,6,8,9,10,12\} \cup \{1,2,3,4,6,12\} \\ &= \{1,2,3,4,6,8,9,10,12\} \end{aligned}$$

$\because$ L.H.S = R.H.S

$\therefore$ $A \cup B = B \cup A$

Hence verified.

(b) Commutative property of intersection

i.e. $A \cap B = B \cap A$

$$\begin{aligned} L.H.S &= A \cap B \\ &= \{1,2,3,4,6,12\} \cap \{4,6,8,9,10,12\} \\ &= \{4,6,12\} \end{aligned}$$

$$\begin{aligned} R.H.S &= B \cap A \\ &= \{4,6,8,9,10,12\} \cap \{1,2,3,4,6,12\} \\ &= \{4,6,12\} \end{aligned}$$

$\because$ L.H.S = R.H.S

$\therefore$ $A \cap B = B \cap A$

Hence verified.

Example 2:

If $A = \{1,2,5,10\}$, $B = \{2,3,5,7\}$ and $C = \{1,3,5,7,9\}$ then verify associative property of union and intersection.

Verification:

(a) Associative property of union

i.e. $A \cup (B \cup C) = (A \cup B) \cup C$

$$\begin{aligned} L.H.S &= A \cup (B \cup C) \\ &= \{1,2,5,10\} \cup [\{2,3,5,7\} \cup \{1,3,5,7,9\}] \\ &= \{1,2,5,10\} \cup \{1,2,3,5,7,9\} \\ &= \{1,2,3,5,7,9,10\} \end{aligned}$$

$$\begin{aligned} R.H.S &= (A \cup B) \cup C \\ &= [\{1,2,5,10\} \cup \{2,3,5,7\}] \cup \{1,3,5,7,9\} \\ &= \{1,2,3,5,7,10\} \cup \{1,3,5,7,9\} \\ &= \{1,2,3,5,7,9,10\} \end{aligned}$$

$\because$ L.H.S = R.H.S

$\therefore$ $A \cup (B \cup C) = (A \cup B) \cup C$

Hence verified.

(b) Associative property of intersection

i.e. $A \cap (B \cap C) = (A \cap B) \cap C$

$$\begin{aligned} L.H.S &= A \cap (B \cap C) \\ &= \{1,2,5,10\} \cap [\{2,3,5,7\} \cap \{1,3,5,7,9\}] \\ &= \{1,2,5,10\} \cap \{3,5,7\} \\ &= \{5\} \end{aligned}$$

$$\begin{aligned} R.H.S &= (A \cap B) \cap C \\ &= [\{1,2,5,10\} \cap \{2,3,5,7\}] \cap \{1,3,5,7,9\} \\ &= \{2,5\} \cap \{1,3,5,7,9\} \\ &= \{5\} \end{aligned}$$

$\because$ L.H.S = R.H.S

$\therefore$ $A \cap (B \cap C) = (A \cap B) \cap C$

Hence verified.

Example 3: If $A=\{1,2,3,...,10\}$, $B=\{2,4,6,8,10\}$ and $C=\{3,6,9\}$ then verify

(a) Distributive property of union over intersection.

(b) Distributive property of intersection over union.

(a) Verification:

(a) Distributive property of union over intersection

i.e., $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$\text{L.H.S} = A\cup(B\cap C)$

$=\{1,2,3,...,10\}\cup[\{2,4,6,8,10\}\cap\{3,6,9\}]$

$=\{1,2,3,...,10\}\cup\{6\}$

$=\{1,2,3,...,10\}$

$\text{R.H.S} = (A\cup B)\cap(A\cup C)$

$=[\{1,2,3,...,10\}\cup\{2,4,6,8,10\}]\cap[\{1,2,3,...,10\}\cup\{3,6,9\}]$

$=\{1,2,3,...,10\}\cap\{1,2,3,...,10\}$

$=\{1,2,3,...,10\}$

$\because$ L.H.S = R.H.S

$\therefore$ $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence verified.

(b) Distributive property of Intersection over union

i.e., $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

$\text{L.H.S} = A\cap(B\cup C)$

$=\{1,2,3,...,10\}\cap[\{2,4,6,8,10\}\cup\{3,6,9\}]$

$=\{1,2,3,...,10\}\cap\{2,3,4,6,8,9,10\}$

$=\{2,3,4,6,8,9,10\}$

$\text{R.H.S} = (A\cap B)\cup(A\cap C)$

$=[\{1,2,3,...10\}\cap\{2,4,6,8,10\}]\cup[\{1,2,3,...,10\}\cap\{3,6,9\}]$

$=\{2,4,6,8,10\}\cup\{3,6,9\}$

$=\{2,3,4,6,8,9,10\}$

$\because$ L.H.S = R.H.S

$\therefore$ $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Hence verified.

Example 4: If $U=\{1,2,3,...,20\}$, $A=\{1,3,5,...,19\}$ and $B=\{2,4,6,...,20\}$ then verify De Morgan's laws.

i.e., (a) $(A\cup B)'=A'\cap B'$ (b) $(A\cap B)'=A'\cup B'$

Verification:

(a) $(A\cup B)'=A'\cap B'$

$$\begin{aligned}
L.H.S &= (A\cup B)' \\
&= U-(A\cup B) \\
&= \{1,2,3,...,20\}-[\{1,3,5,...,19\}\cup\{2,4,6...,20\}] \\
&= \{1,2,3,...,20\}-\{1,2,3,...,20\} \\
&= \phi
\end{aligned}$$

$$\begin{aligned}
R.H.S &= A'\cap B' \\
&= (U-A)\cap(U-B) \\
&= [\{1,2,3,...,20\}-\{1,3,5,...,19\}]\cap[\{1,2,3,...,20\}-\{2,4,6,...,20\}] \\
&= \{2,4,6,...,20\}\cap\{1,3,5,...,19\} \\
&= \phi
\end{aligned}$$

$\because$ L.H.S = R.H.S

$\therefore$ $(A\cup B)'=A'\cap B'$

Hence verified.

(b) $(A\cap B)'=A'\cup B'$

$$\begin{aligned}
L.H.S &= (A\cap B)' \\
&= U-(A\cap B) \\
&= \{1,2,3,...,20\}-[\{1,3,5,...,19\}\cap\{2,4,6...,20\}] \\
&= \{1,2,3,...,20\}-\{\ \} \\
&= \{1,2,3,...,20\}
\end{aligned}$$

$$\begin{aligned}
R.H.S &= A'\cup B' \\
&= (U-A)\cup(U-B) \\
&= [\{1,2,3,...,20\}-\{1,3,5,...,19\}]\cup[\{1,2,3,...,20\}-\{2,4,6,...,20\}] \\
&= \{2,4,6,...,20\}\cup\{1,3,5,...,19\} \\
&= \{1,2,3,...,20\}
\end{aligned}$$

$\because$ L.H.S = R.H.S

$\therefore$ $(A\cap B)'=A'\cup B'$

Hence verified.

Exercise 17.3

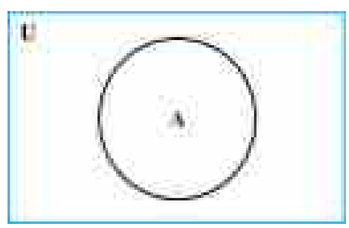
1. **Verify the commutative property of union and intersection for the following sets.**
   (i) $A=\{a,b,c,d,e\}$ and $B=\{a,e,i,o,u\}$
   (ii) $P=\{x \mid x\in Z \wedge -3<x<3\}$ and $Q=\{y \mid y\in E^{+} \wedge y\leq 4\}$
2. **Verify the associative property of union and intersection for the following sets.**
   (i) $A=\{1,2,4,5,10,20\}$, $B=\{5,10,15,20\}$ and $C=\{1,2,5,10\}$
   (ii) $A=\mathbb{N}$, $B=\mathbb{P}$ and $C=\mathbb{Z}$
3. **Verify**
   (a) Distributive property of union over intersection.
   (b) Distributive property of intersection over union for the following sets
   (i) $A=\{1,2,3,...,10\}$, $B=\{2,3,5,7\}$ and $C=\{1,3,5,7,9\}$
   (ii) $A=\mathbb{N}$, $B=\mathbb{P}$ and $C=\mathbb{W}$
4. **Verify De Morgan's laws if**
   $U=\{1,2,3...,12\}$, $A=\{1,2,3,4,6,12\}$ and $B=\{2,4,6,8\}$.
5. **If A and B are subsets of U then prove the following by using properties.**
   (i) $A\cup(A\cap B)=A\cap(A\cup B)$ (ii) $A\cup B=A\cup(A'\cap B)$
   (iii) $B=(A\cap B)\cup(A'\cap B)$ (iv) $B=A\cup(A'\cap B)$, if $A\subseteq B$

17.1.3 Venn Diagrams:

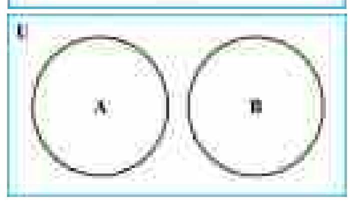
We have already studied in previous classes that the sets, their relations and operations can also be represented geometrically. This geometrical representation is called Venn Diagram named after the English mathematician John Venn who introduced it in 1881 AD. In Venn Diagram, a rectangle is usually used to represent the universal set whereas circles or ovals represent sets.

Recall some Venn diagrams

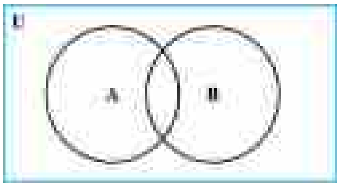
(i) Venn diagram showing a set A inside universal set U.



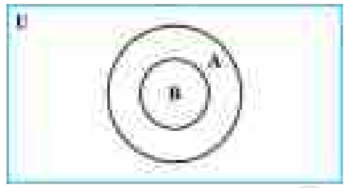
(ii) Venn diagram showing two disjoint sets A and B.



(iii) Venn diagram showing two overlapping sets A and B.



(iv) Venn diagram showing a subset B of A. i.e. $B \subseteq A$



17.1.3(i) Use Venn diagrams to represent

- Union and intersection of sets
- Complement of a set
- Symmetric difference of two sets

➢ Union and intersection of sets

In Venn diagram, the union of two sets A and B is represented by the entire region of both sets A and B. In Fig (i) the shaded or coloured region represents $A \cup B$.

Whereas the intersection of two sets A and B is represented by the common region of both A and B. In Fig (ii), the shaded or coloured region shows $A \cap B$.

$A \cup B$

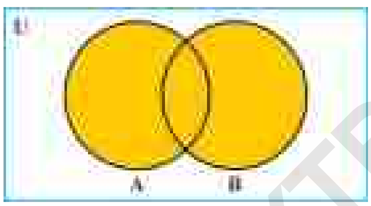


Fig (i)

$A \cap B$

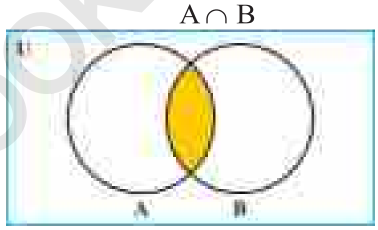


Fig (ii)

Example 1: If $U = \{1, 2, 3, ..., 10\}$, $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8\}$ then use Venn diagram to represent $A \cup B$ and $A \cap B$.

Solution:

In Fig (a), coloured or shaded region represents $A \cup B$.

From the Venn diagram, we have

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$

In Fig (b), the coloured or shaded region represents $A \cap B$

From the Venn diagram, we have

$A \cap B = \{2, 4\}$.

$A \cup B$

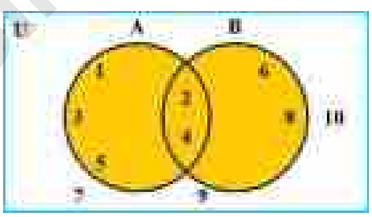


Fig (a)

$A \cap B$

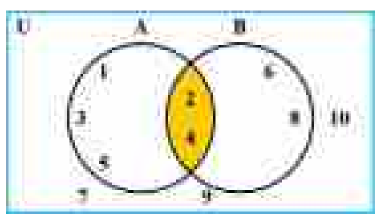
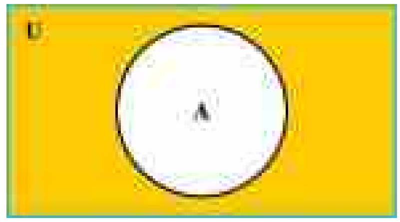


Fig (b)

- ## Complement of a set

In Venn diagram, the complement of set A is represented by the region of universal set excluding the region of A. In Fig (i) shaded or coloured region represents A'.

A'



For Example:

If $U = \{1, 2, 3, ..., 8\}$ and $A = \{1, 2, 3, 4\}$ then use Venn diagram to represent A'

Solution:

In the adjacent Venn diagram Fig (ii) the shaded or coloured region represents A'.

So we have $A' = \{5, 6, 7, 8\}$

A'

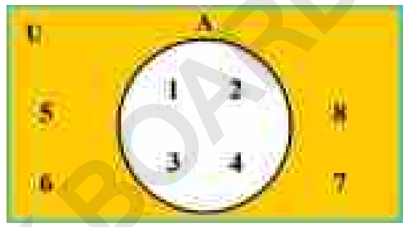


Fig (ii)

- ## Symmetric difference of two sets

In Venn diagram, the symmetric difference of two sets A and B is represented by the entire region of both A and B except the common region.
In Fig (i), the coloured or shaded region represents $A \Delta B$.

$A \Delta B$

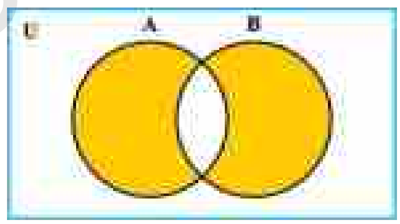


Fig (i)

Example:

If $U = \{a, b, c, d, e, f\}$, $A = \{a, c, e\}$ and $B = \{a, b, c\}$ then use Venn diagram to represent $A \Delta B$.

Solution:

In adjacent Venn diagram fig (ii), the shaded or coloured region represents $A \Delta B$.
i.e., $A \Delta B = \{b, e\}$

$A \Delta B$

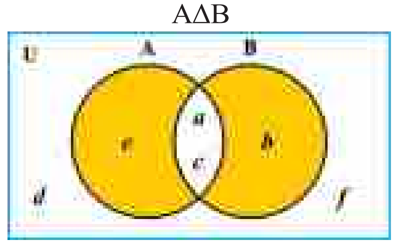


Fig (ii)

17.1.3(ii) Use Venn diagram to verify

- **Commutative property of union and intersection**
- **Associative laws**
- **Distributive laws**
- **De Morgan's laws**

➢ Commutative property of union and intersection

Let us verify the commutative property of union and intersection using Venn diagram with the help of the following example.

Example: If $A=\{1,2,3,4,5,6\}$ and $B=\{4,6,8\}$ then verify commutative property of union and intersection using Venn diagram.

Verification: (i) Commutative property of union, i.e., $A\cup B=B\cup A$

$A\cup B$

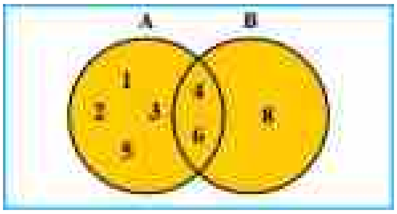


Fig (i)

From Venn diagram of Fig. i

$A\cup B=\{1,2,3,4,5,6,8\}$

$B\cup A$

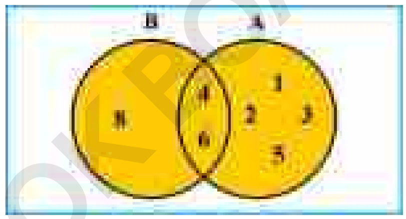


Fig (ii)

From Venn diagram of Fig. ii

$B\cup A=\{1,2,3,4,5,6,8\}$

$\because$ Shaded or coloured regions and their elements are equal as shown in the Fig (i) and Fig(ii).

$\therefore$ $A\cup B=B\cup A$

Hence verified.

Verification: (ii) Commutative property of intersection i.e. $A\cap B=B\cap A$.

$A\cap B$

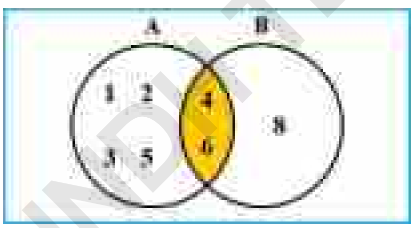


Fig (i)

From Venn diagram of Fig. i

$A\cap B=\{4,6\}$

$B\cap A$

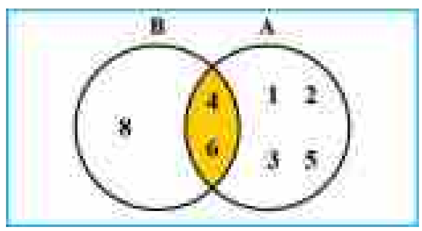


Fig (ii)

From Venn diagram of Fig. ii

$B\cap A=\{4,6\}$

$\because$ Shaded or coloured regions and their elements are equal as shown in Fig (i) and Fig (ii)

$\therefore$ $A\cap B=B\cap A$

Hence verified.

➢ Associative Laws

In order to verify associative laws using Venn diagram, we take the following example.

Example:

If $A=\{1,2,3,4,5,6\}$, $B=\{6,8,10\}$ and $C=\{5,6,7,8\}$ then verify associative laws of union and intersection.

Verification: Associative law of union i.e. $A\cup(B\cup C)=(A\cup B)\cup C$

$\underline{\text{L.H.S}=A\cup(B\cup C)}$

$B\cup C$

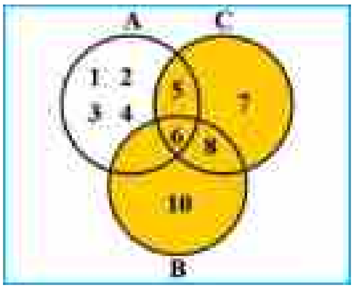


Fig (i)

$A\cup(B\cup C)$

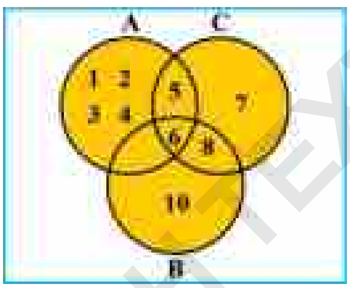


Fig (ii)

From Venn diagram of Fig: (ii)

$A\cup(B\cup C)=\{1,2,3,4,5,6,7,8,10\}$

$\underline{\text{R.H.S}=(A\cup B)\cup C}$

$A\cup B$

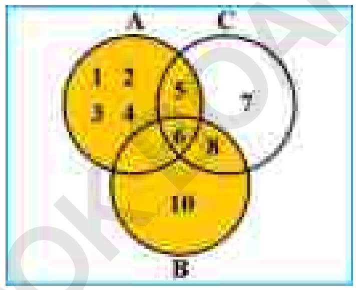


Fig (iii)

$(A\cup B)\cup C$

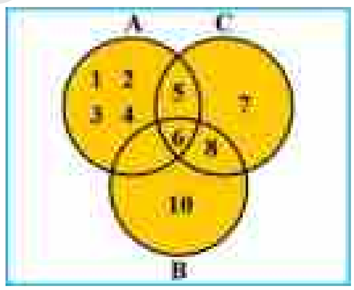


Fig (iv)

From Venn diagram of Fig. (iv)

$(A\cup B)\cup C=\{1,2,3,4,5,6,7,8,10\}$

$\because$ Shaded or coloured regions and their elements as shown in Fig: (ii) and Fig: (iv) are equal

$\therefore$ $A\cup(B\cup C)=(A\cup B)\cup C$

Hence verified.

Verification: Associative law of intersection i.e. $A\cap(B\cap C)=(A\cap B)\cap C$

$\underline{\text{L.H.S}=A\cap(B\cap C)}$ $\underline{\text{R.H.S}=(A\cap B)\cap C}$

$B\cap C$

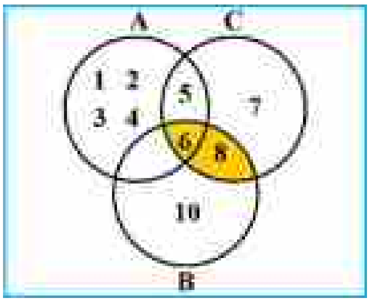


Fig (i)

$A\cap B$

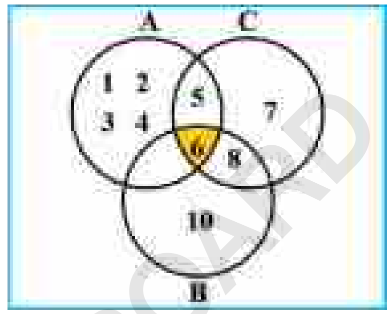


Fig (iii)

$A\cap(B\cap C)$

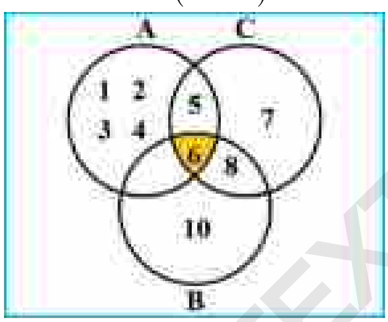


Fig (ii)

$(A\cap B)\cap C$

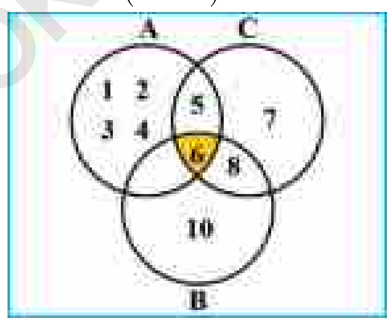


Fig (iv)

From Venn diagram of Fig. (ii)
$A\cap(B\cap C)=\{6\}$

From Venn diagram of Fig. (iv)
$(A\cap B)\cap C=\{6\}$

$\because$ Coloured or shaded regions and their elements of Fig:(ii) and Fig:(iv) are equal

$\therefore$ $A\cap(B\cap C)=(A\cap B)\cap C$

Hence verified.

➢ **Distributive Laws**

Now we verify distributive laws using Venn diagram with the help of the following example.

Example:

Verify distributive laws using Venn diagram

if $A=\{1,2,3,4,5,6\}$, $B=\{3,4,5,6,7\}$ and $C=\{2,4,6,8\}$.

Verification:

(i) Distributive law of union over intersection

i.e. $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$\underline{L.H.S = A\cup(B\cap C)}$

$B\cap C$

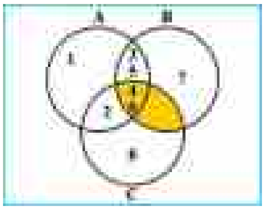


Fig (i)

$\underline{R.H.S=(A\cup B)\cap(A\cup C)}$

$A\cup B$

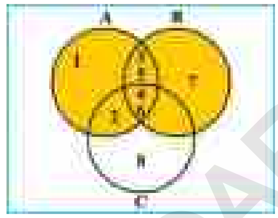


Fig (iii)

$A\cup C$

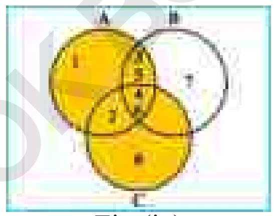


Fig (iv)

$A\cup(B\cap C)$

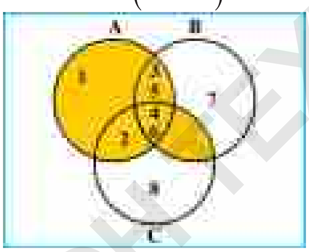


Fig (ii)

From Venn diagram of Fig.(ii)

$A\cup(B\cap C)=\{1,2,3,4,5,6\}$

$(A\cup B)\cap(A\cup C)$

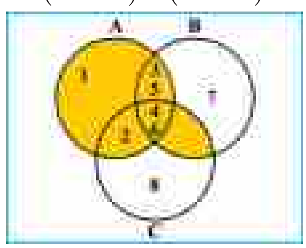


Fig (iv)

From Venn diagram of Fig.(v)

$(A\cup B)\cap(A\cup C)=\{1,2,3,4,5,6\}$

$\because$ Shaded or coloured region and their elements are equal as shown in Fig (ii) and Fig (v)

$\therefore$ $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Hence verified.

Verification:

(ii) Distributive law of intersection over union

i.e. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$\underline{L.H.S = A \cap (B \cup C)}$

$B \cup C$

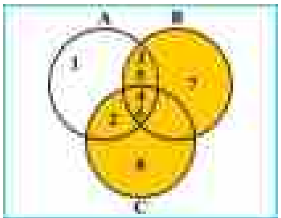


Fig (i)

$\underline{R.H.S = (A \cap B) \cup (A \cap C)}$

$A \cap B$

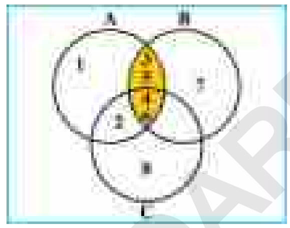


Fig (iii)

$A \cap C$

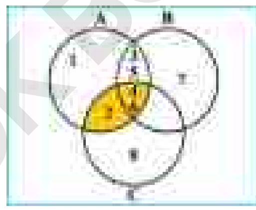


Fig (iv)

$A \cap (B \cup C)$

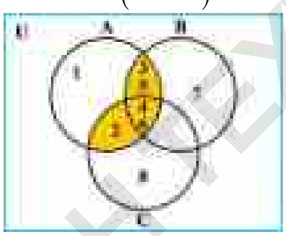


Fig (ii)

From Venn diagram of Fig. (ii)

$A \cap (B \cup C) = \{2, 3, 4, 5, 6\}$

$(A \cap B) \cup (A \cap C)$

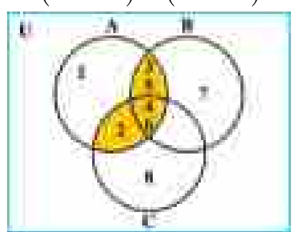


Fig (v)

From Venn diagram of Fig. (v)

$(A \cap B) \cup (A \cap C) = \{2, 3, 4, 5, 6\}$

$\because$ Shaded or coloured regions and their elements as shown in Fig. (ii) and Fig. (v) are equal

$\therefore$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,

Hence verified.

➢ De Morgan's Laws

Let us verify De Morgan's laws using Venn diagram with the help of the following example.

Example: Verify De Morgan's laws using Venn diagram.

If $U=\{1,2,3,...,10\}$, $A=\{1,2,3,4,5,6\}$ and $B=\{2,4,6,8,10\}$

Verification: (i) $(A\cup B)' = A'\cap B'$

$\underline{L.H.S=(A\cup B)'}$

$A\cup B$

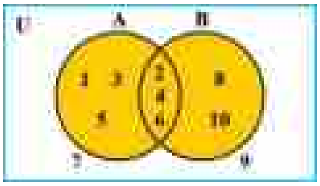


Fig (i)

$(A\cup B)'$

Fig (ii)

From Venn diagram of Fig. (ii)

$(A\cup B)' = \{7,9\}$

$\underline{R.H.S = A'\cap B'}$

A'

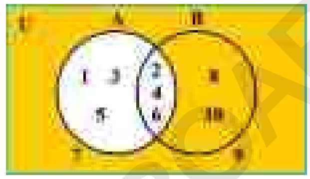


Fig (iii)

B'

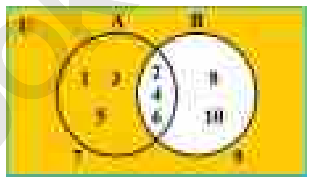


Fig (iv)

$A'\cap B'$

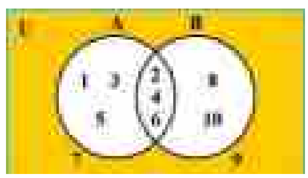


Fig (v)

From Venn diagram of Fig. (v)

$A'\cap B' = \{7,9\}$

$\because$ Shaded or coloured regions and their elements as shown in Fig. (ii) and Fig. (v) are equal

$\therefore$ $(A\cup B)' = A'\cap B'$

Hence verified.

Verification: (ii) $(A \cap B)' = A' \cup B'$

$\underline{\text{L.H.S} = (A \cap B)'}$

$A \cap B$

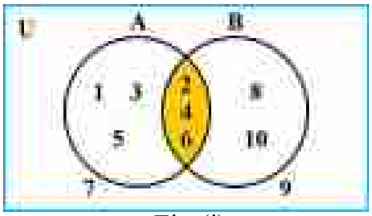


Fig (i)

$(A \cap B)'$

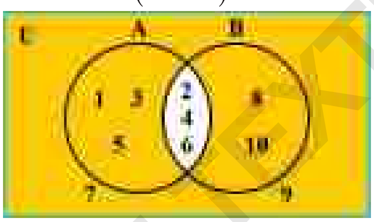


Fig (ii)

From Venn diagram of Fig. (ii)

$(A \cap B)' = \{1, 3, 5, 7, 8, 9, 10\}$

$\underline{\text{R.H.S} = A' \cup B'}$

A'

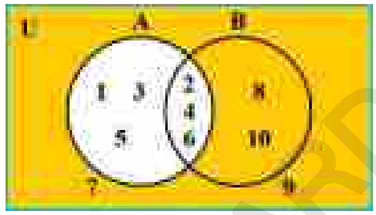


Fig (iii)

B'

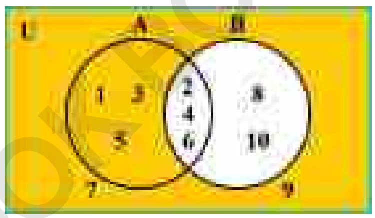


Fig (iv)

$A' \cup B'$

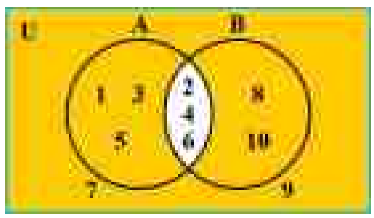


Fig (v)

From Venn diagram of Fig. (v)

$A' \cup B' = \{1, 3, 5, 7, 8, 9, 10\}$

$\because$ Shaded or coloured regions and their elements as shown in Fig: (ii) and Fig: (v) are equal

$\therefore$ $(A \cap B)' = A' \cup B'$

Hence verified.

Exercise 17.4

1. **If A and B are any two subsets of U then draw Venn diagrams in general of $A \cup B$, $A \cap B$, $A \Delta B$ and $A - B$, if**
   (i) A and B are disjoint sets.
   (ii) A and B are overlapping sets.
   (iii) A is subset of B.
2. **Verify commutative property of union and intersection using Venn diagram if**
   (i) $A = \{a, b, c, d, e\}$ and $B = \{a, e, i, o, u\}$
   (ii) $P = \{1, 2, 3, ..., 10\}$ and $Q = \{2, 4, 6, 8, 10\}$
3. **Verify De Morgan's laws using Venn diagram if**
   $\mathbf{A = \{1, 3, 5, 7, 9\}, B = \{5, 6, 7, 8\}}$ **and** $\mathbf{U = \{1, 2, 3, ..., 10\}}$
4. **Verify associative laws and distributive laws using Venn diagram**
   if $\mathbf{A = \{1, 2, 3, ..., 8\}}$, $\mathbf{B = \{4, 6, 7, 8, 9, 10\}}$ **and** $\mathbf{C = \{5, 7, 9, 11\}}$

17.1.4 Ordered pairs and Cartesian products

Cartesian product is named after French mathematician Rene Descartes who introduced Analytic Geometry. In Analytic Geometry ordered pairs are used to locate points in plane. Let us discuss ordered pair and Cartesian product in detail.

17.1.4 (i) Recognize ordered pair

A pair of numbers in which order is maintained is called an ordered pair. For any two real numbers *a* and *b*, if we regard ***a*** as first and ***b*** as second then the ordered pair of *a* and *b* is denoted as (a, b). In the ordered pair (a, b), *a* is called first component or element and b is called second component or element.

Two ordered pairs are equal if their corresponding components are equal

i.e., $(a, b) = (c, d)$ iff $a = c$ and $b = d$

Example: Find x and y if $(x+5, 8)$ and $(9, y-6)$ are equal

Solution:

We have $(9, y-6) = (x+5, 8)$

$\Rightarrow$	$9 = x + 5$	and		$y - 6 = 8$
$\Rightarrow$	$9 - 5 = x$		$\Rightarrow$	$y = 8 + 6$
or	$x = 4$		or	$y = 14$

So the value of x and y are 4 and 14 respectively.

17.1.4 (ii) To form Cartesian products

If A and B are any two non-empty sets then the Cartesian product of A with B, denoted as $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

i.e. $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

Similarly, Cartesian product of B with A is defined as

$B \times A = \{(b, a) \mid b \in B \text{ and } a \in A\}$

For example: If $P=\{1,2,4\}$ and $Q=\{5,10\}$

then $P\times Q=\{(1,5),(1,10),(2,5),(2,10),(4,5),(4,10)\}$

and $Q\times P=\{(5,1),(5,2),(5,4),(10,1),(10,2),(10,4)\}$

Note:

(i) $A\times B$ is read as "A cross B".

(ii) If $O(A)=m$ and $O(B)=n$ then $O(A\times B)=mn$

(iii) $A\times B\neq B\times A$ in general

17.2 Binary Relations

Define a binary relation and identify its domain and range.

- **Binary Relation**

If A and B are any two non-empty sets then any subset R of the Cartesian product $A\times B$ is called a binary relation from A to B.

Example: If $A=\{a,b,c\}$ and $B=\{2,4\}$ then find

(a) Two relations from A to B.
(b) Three relations from B to A.
(c) Four binary relations in B.

Solution (a) Two relations from A to B

Here, $A\times B=\{(a,2),(a,4),(b,2),(b,4),(c,2),(c,4)\}$

Now $R_1=\{(a,2),(b,4)\}$ and $R_2=\{(b,2),(c,2),(c,4)\}$

are any two relations from A to B.

(b) Three relations from B to A

Here, $B\times A=\{(2,a),(2,b),(2,c),(4,a),(4,b),(4,c)\}$

Now $R_1=\{(2,a)\}$, $R_2=\{(2,a),(2,b)\}$ and $R_3=\{(4,a),(4,b),(4,c)\}$

are any three relations from B to A.

(c) Four binary relations in B

Here, $B\times B=\{(2,2),(2,4),(4,2),(4,4)\}$

Now $R_1=\varnothing$, $R_2=\{(2,2)\}$, $R_3=\{(2,4),(4,2)\}$ and $R_4=\{(2,2),(2,4)\}$

are any four binary relations in B.

- **Domain and Range of a Binary Relation**

Let R be a binary relation from set A to set B. Then domain of R, denoted as Dom R, is the set of first components of all the ordered pairs of R.

The range of R, denoted as Range R, is the set of second components of all the ordered pairs of R. For example If $A=\{1,2,3,4\}$, $B=\{4,5,6,7\}$ and $R=\{(1,4),(2,4),(3,5),(3,6)\}$ is a binary relation from A to B.

then, $\text{Dom } R=\{1,2,3\}$ and $\text{Range } R=\{4,5,6\}$

Example:

If $x, y \in$ N and a binary relation R in N is given as $R = \{(x, y) \mid x + y = 5\}$ then write R in tabular form. Also find its domain and range.

Solution:

In tabular form, $R = \{(1,4),(2,3),(3,2),(4,1)\}$

Here Dom $R = \{1,2,3,4\}$

and Range $R = \{1,2,3,4\}$

17.3 Functions:

Function is one of the basic concepts of calculus, the branch of Mathematics, which has revolutionized the field of science. Function, in fact, is a rule or relation which relates two sets or quantities, for instance, area "A" of circle is related with radius "r" by rule $A = \pi r^2$. At this level, we will only discuss function on the basis of sets.

17.3 (i) Define a function and identify its domain, co-domain and range

A function is a binary relation in which for each element of domain there is one and only one element of range. For example: $\{(1,3),(2,4),(3,5),(4,6)\}$ is a function.

Whereas $\{(1,5),(2,6),(2,7),(3,9)\}$ is not a function because the element "2" of domain is associated with more than one element of range.

- **Function from set A to set B**

Let A and B be any two non-empty sets and R be a binary relation from A to B. Then R is called a function from A to B if

(i) Domain of R = A

(ii) Every element of A is associated with unique element of B under R i.e. if $(a,b) \in R$ and $(a,c) \in R$ then $b = c$. Functions are usually denoted by letters of English and Greek alphabets like f, g, h etc and α, β, γ etc.

If f is a function from A to B then we write it as $f : A \to B$, and for every $(a,b) \in f$, b is called the image of a under f and we write it as $b = f(a)$.

Example: Let $A = \{1,2,3,4\}$ and $B = \{2,3,4,5\}$. Identify which one of the following is a function from A to B.

(i) $R_1 = \{(1,2),(2,3),(3,4)\}$

(ii) $R_2 = \{(1,2),(2,3),(2,4),(3,4),(4,5)\}$

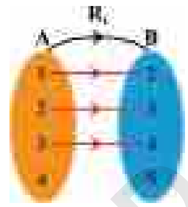
(iii) $R_3 = \{(1,2),(2,3),(3,4),(4,5)\}$

Solution:

Mapping Diagram

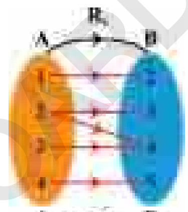
(i) $R_1 = \{(1,2),(2,3),(3,4)\}$

R_1 is not a function because $\text{Dom } R_1 \neq A$ as shown in the adjacent mapping diagram.



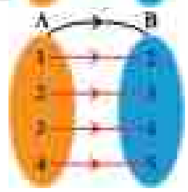
(ii) $R_2 = \{(1,2),(2,3),(2,4),(3,4),(4,5)\}$

R_2 is not a function because an element "2" of domain is not associated with unique element of B as shown in the adjacent mapping diagram.



(iii) $R_3 = \{(1,2),(2,3),(3,4),(4,5)\}$

R_3 is a function because $\text{Dom } R_3 = A$ and each element of domain is associated with unique element of B as shown in the adjacent mapping diagram.



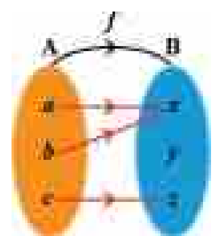
- **Domain, Co-domain and Range**

  If f is a function from A to B i.e. $f: A \to B$ then
  A is called its domain (First set)
  and B is called its co-domain (Second set)

Whereas,
Range is the set of all the images of f.

Example:

If f is a function from A to B as shown in the given mapping diagram then write down its domain, co-domain and range. Also write down the values of f(a) and f(c).



Solution:

From mapping diagram of f

Dom f $= A = \{a,b,c\}$, Co-domain of f $= B = \{x,y,z\}$ and Range f $= \{x,z\}$

Also $f(a) = x$ and $f(c) = z$

Note: (i) Range of f is subset of its co-domain.
(ii) Every function is a relation but converse is not true.

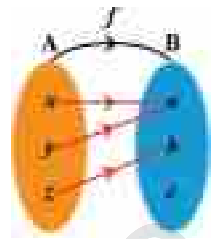
17.3 (ii) Demonstrate the following

- **Into and one-one function (Injective function)**
- **Onto function (Surjective function)**
- **One-one and onto function (Bijective function)**

First of all we define into function and one-one function.

➢ Into function:

For any two sets A and B, a function $f: A \rightarrow B$ is called into function if Range of f is proper subset of B $(\text{i.e. Range } f \subset B.)$

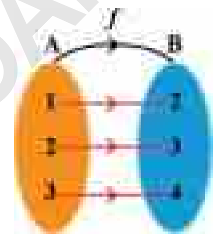


For example: For $A = \{x, y, z\}$ and $B = \{a, b, c\}$,
a function $f: A \rightarrow B$ defined as $f = \{(x,a),(y,a),(z,b)\}$ is an into function because Range $f \subset B$ as shown in the figure where Range of $f = \{a, b\}$.

➢ One-one function:

For any two sets A and B, a function $f: A \rightarrow B$ is called one-one function if distinct elements of A have distinct images in B.

For example: For $A = \{1,2,3\}$ and $B = \{2,3,4\}$, a function $f: A \rightarrow B$ defined as $f = \{(1,2),(2,3),(3,4)\}$ is one-one function because distinct elements of A have distinct images in B as shown in the mapping diagram.



➢ Into and one-one function or injective function:

A function which is into as well as one-one, is called an injective function.

For example: For $A = \{1,2,3\}$ and $B = \{2,3,4,5\}$ a function $f: A \rightarrow B$ defined as $f = \{(1,2),(2,3),(3,4)\}$ is an injective function because it is into as well as one-one as shown in the mapping diagram Fig (i).

A function $g: A \rightarrow B$ defined as $g = \{(1,3),(2,5),(3,4)\}$ is also an injective function as shown in mapping diagram Fig (ii).

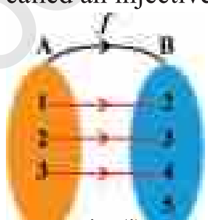


Fig (i)

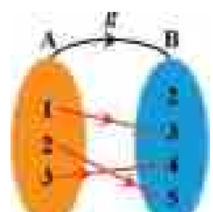


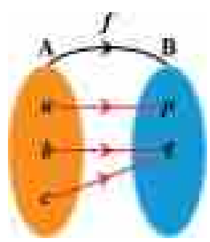
Fig (ii)

➢ Onto function or surjective function:

For any two sets A and B, a function $f: A \rightarrow B$ is called onto function or surjective function if Range $f = B$

For example:

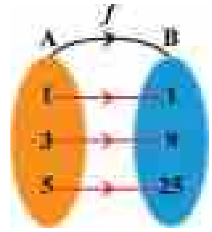
For $A = \{a,b,c\}$ and $B = \{p,q\}$ a function $f: A \rightarrow B$ defined as
$f = \{(a,p),(b,q),(c,q)\}$ is an onto function because Range $f = B$ as shown in the adjacent figure.



➢ One-one and onto function or bijective function:

A function which is one-one as well as onto, is called one-one and onto function or bijective function.

For example: For $A=\{1,3,5\}$ and $B=\{1,9,25\}$ a function $f:A\rightarrow B$ defined as $f=\{(1,1),(3,9),(5,25)\}$ is a bijective function because it is one-one as well as onto as shown in the adjacent figure.



17.3 (iii) Examine whether a given relation is a function or not

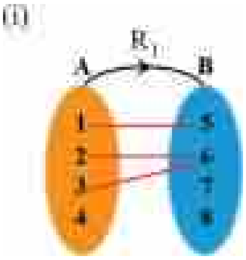
In order to examine whether a given relation is a function or not we have to concentrate on domain of the relation.

If there is any element of domain without image or any element of domain with more than one image in a relation then this relation is not a function.

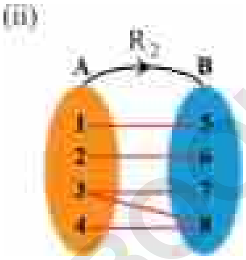
Example:

Examine the given relations from A to B whose mapping diagrams are given decide which one is function and which one is not.

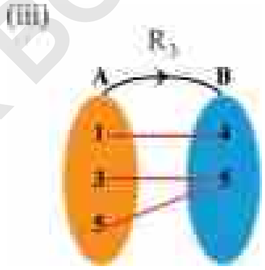
(i)



(ii)



(iii)



Solution:

R_1 is not a function because an element "4" of domain is without image.

R_2 is also not a function because an element "3" of domain has more than one images.

R_3 is a function because each element of domain has unique image.

17.3 (iv) Differentiate between one-one correspondence and one-one function.

Let A and B be two non-empty sets. In one-one correspondence between A and B, each element of either set is associated with exactly one element of the other set.

i.e. one-one correspondence always represents a bijective function.

Whereas one-one function is not always bijective.

For example, In figure (i) there is a one-one correspondence.

Whereas in Figure (ii), there is no one-one correspondence but it represents one-one function.

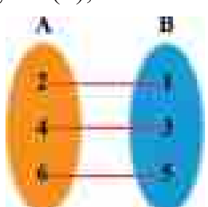


Fig (i)

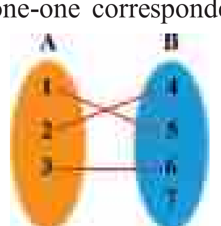


Fig (ii)

Exercise 17.5

1. **Find the values of x and y if:**
   (i) $(x-5,10)=(11,y-7)$
   (ii) $(5x+8,5y-4)=(3x+10,2y+2)$
   (iii) $(2x-3y,5x+y)=(3,16)$
2. **If set P has 10 elements and Q has 15 elements. Find the number of elements of $P\times Q, Q\times P, P\times P$ and $Q\times Q$.**
3. **If $A=\{1,2\}, B=\{2,3\}$ and $C=\{1,3,5\}$ then find:**
   (i) $A\times B$ (ii) $B\times C$ (iii) $A\times(B\cup C)$
   (iv) $B\times(A\cup C)$ (v) $(A\cap B)\times(B\cap C)$
4. **If $A=\{5,6\}$ and $B=\{1,2,3\}$ then find:**
   (i) Three relations in $A\times B$ (ii) Four relations in $B\times A$
   (iii) Five relations in B (iv) All relations in A
5. **For two sets A and B, if $O(A)=3$ and $O(B)=4$, then find number of all binary relations in $A\times B$.**
6. **Write the following binary relations of $A\times B$ in tabular form where $A=\{0,1,2,3\}$ and $B=\{2,4,6,8\}$ such that $a\in A$ and $b\in B$.**
   (i) $R_1=\{(a,b)\mid b<5\}$ (ii) $R_2=\{(a,b)\mid a+b=9\}$
   (iii) $R_3=\{(a,b)\mid a-b=1\}$ (iv) $R_4=\{(a,b)\mid ab=6\}$
7. **If relation $R=\{(x,y)\mid y=2x+5\}$ is in Z then:**
   (i) find range if domain is $\{-2,-1,0,1,2\}$
   (ii) find domain if range is $\{11,13,15,17\}$
8. **If x,y represent elements of W, find domain and range of following relations**
   (i) $\{(x,y)\mid 3x+y=11\}$ (ii) $\{(x,y)\mid x-y=6\}$
9. **If $f: A\to B$ is a function given by $f=\{(1,5),(2,6),(3,7),(4,8)\}$** where **$A=\{1,2,3,4\}$ and $B=N$** then write its domain, co-domain and range. Also write the values of **$f(2)$ and $f(4)$.**
10. **If $A=\{0,1,2,3,4\}$ and $B=\{2,4,6,8,10\}$** then find whether the following relation from A to B are functions or not. If they are functions find their types:
    (i) $R_1=\{(0,2),(1,4),(2,6),(3,8)\}$

(ii) $R_2 = \{(0,10),(1,8),(2,6),(2,4),(3,4),(4,2)\}$

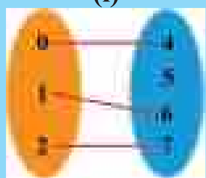
(iii) $R_3 = \{(0,2),(1,4),(2,6),(3,8),(4,8)\}$

(iv) $R_4 = \{(0,2),(1,2),(2,2),(3,2),(4,8)\}$

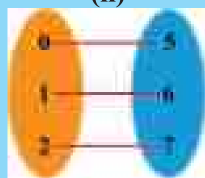
(v) $R_5 = \{(0,2),(1,4),(2,6),(3,8),(4,10)\}$

11. Which of the following is one-one function or one-one correspondence or neither of them

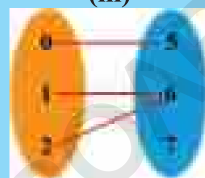
(i)



(ii)



(iii)



12. If $P = \{a, b, c\}$, $Q = \{x, y, z\}$ and $R = \{p, q, r, s\}$ then find:

(i) a function f from P into Q.
(ii) a function g from R onto P.
(iii) a function h from P to R which is injective.
(iv) a function k from Q to P which is bijective.

Review Exercise 17

1. Choose the correct option:

(i) Which one is subset of N
(a) Z (b) Q (c) E (d) P

(ii) Tabular form of $A = \{x \mid x \in Z \wedge x^2 = 25\}$ is ________.
(a) $\{1,5\}$ (b) $\{-5,5\}$
(c) $\{1,5,25\}$ (d) $\{-5\}$

(iii) For two sets A and B, if $O(A) = O(B)$ then
(a) $A \subseteq B$ (b) $B = A$
(c) $A \sim B$ (d) $A \subset B$

(iv) $\{x \mid x \in N \wedge x < 1\}$ is ______
(a) Singleton (b) Infinite set
(c) Empty set (d) Super set

(v) If $A = \{1,2,3,4\}$ and $B = \{2,4,6\}$ then $A \Delta B =$ ________.
(a) $\{1,2\}$ (b) $\{6\}$
(c) $\{1,3\}$ (d) $\{1,3,6\}$

(vi) Which operation is not commutative

(a) Symmetric difference (b) Union

(c) Difference (d) Intersection

(vii) $\{x \mid x \in P \vee x \in Q\} =$ ________.

(a) $P \cap Q$ (b) $P \cup Q$

(c) $P - Q$ (d) $P \Delta Q$

(viii) $\{x \mid x \in U \wedge x \notin A \cap B\} =$ ________.

(a) $A \cap B$ (b) $(A \cup B)'$

(c) $(A \cap B)'$ (d) $A' \cap B'$

(ix) For any two non-empty sets A and B, if $A \cup B = U$ and $A \cap B = \varnothing$ then A and B are ________.

(a) Cells (b) Overlapping sets

(c) Equal sets (d) None

(x) $\left((A')'\right)' =$ ________.

(a) $(A')'$ (b) A'

(c) A (d) B

(xi) If $A \supset B$ then $A \cup B =$ ________.

(a) A (b) B

(c) $\varnothing$ (d) U

(xii) In Venn diagram, ________ is used to represent universal set.

(a) rectangle (b) circle

(c) oval (d) all of these

(xiii) If $(x, 6) = (2, y-6)$ then $x + y =$ ________.

(a) 8 (b) 10 (c) 12 (d) 14

(xiv) If $O(A \times B) = 100$ and $O(B) = 5$ then $O(A) =$ ________.

(a) 10 (b) 15 (c) 20 (d) 25

(xv) A function is called surjective if range ________ co-domain

(a) $\supset$ (b) $\subset$

(c) $=$ (d) $\neq$

(xvi) A function is called into if range________ co-domain

(a) $\supset$ (b) $\subset$

(c) $=$ (d) none of these

(xvii) Given that A = $\{0,1,2\}$ and B = $\{3,4\}$ then decide which of the followings is a function f:A→B ?

(a) $\{(0,3),(1,4)\}$ (b) $\{(0,3),(1,4),(2,3),(2,4)\}$

(c) $\{(0,3),(1,4),(2,4)\}$ (d) none of these

(xviii) Which one is always bijective

(a) Onto function (b) One-one function

(c) Into function (d) One-one correspondence

(xix) If $f:\mathbb{N}\rightarrow\mathbb{R}$ then which one is not possible

(a) $f(5)=6$ (b) $f(7)=8$

(c) $f(-2)=6$ (d) $f(9)=0$

(xx) Which one of the followings is a commutative set operation?

(a) Difference (b) Union

(c) Cartesian product (d) None

2. If $A=\{1,2,5\}$, $B=\{2,4,6\}$, $U=\{1,2,3,4,5,6\}$, then find the following

(a) $A\cup B$ (b) $A\cap B$ (c) $A\Delta B$

(d) $A-B$ (e) A' (f) B'

3. Verify De Morgan's laws for the sets of question 2.

4. Verify commutative properties of union and intersection for the sets.

$A=\{a,b,d\}$ and $B=\{a,d,e\}$

5. For sets $P=\{1,3,5\}$ and $Q=\{1,3,7\}$ represent the following set by using Venn diagram.

(a) $P\cup Q$ (b) $P\cap Q$ (c) $P\Delta Q$ (d) $P-Q$

6. If $A=\{a,c\}$ and $B=\{b,d\}$ then find

(a) Two relations in A (b) Three relations in B

(c) Three relations in $A\times B$ (d) Two relations in $A\cup B$

7. If $A=\{1,2,5\}$ and $B=\{6,8\}$ then find function from A to B, which is

(a) Onto function (b) Into function

Summary

- Tabular, descriptive and set builder forms are the methods of representation of a set.
- Empty set has no element.
- If A is subset of B then B is superset of A.
- If $A \subseteq B$ and $B \subseteq A$ then $A = B$.
- Equivalent sets have one-one correspondence.
- If $A \subseteq B$ and $A \neq B$ then $A \subset B$.
- Union, intersection, difference, symmetric difference, complementation and Cartesian product are operation on sets.
- If $A \cap B = \varnothing$ then A and B are disjoint sets.
- If $A \cup B = U$ then A and B are exhaustive sets.
- Cells are always disjoint and exhaustive.
- Difference and Cartesian product are not commutative.
- Union, intersection and symmetric difference are commutative and associative.
- Union and intersection are distributive over each other.
- De Morgan's Laws: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$
- Geometrical representation of sets, their relations and operations is called Venn diagram.
- Venn diagrams were introduced by John Venn in 1881 AD.
- Sets and universal set in Venn diagram are represented by circles or ovals and rectangle respectively.
- If $(a,b) = (c,d) \Leftrightarrow a = c$ and $b = d$
- $O(A \times B) = O(A) \cdot O(B)$
- $A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$
- Every subset of Cartesian product is called binary relation.
- Function is a binary relation in which each element of domain has unique image.
- If $f: X \to Y$ is a function then X and Y are domain and co-domain respectively whereas range is the set of all images of f.
- A function is called (i) Into if range $\subset$ co-domain (ii) Onto if range $=$ co-domain
  (iii) One-one if distinct elements of domain have distinct images in co-domain.
- A function is called
  (i) Injective if it is one-one and into. (ii) Surjective if it is onto.
  (iii) Bijective if it is one-one and onto
- One-one correspondence is always bijective.
- One-one function is not always bijective.