UNIT

04

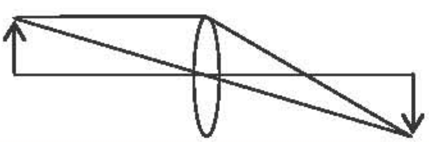
# FACTORIZATION AND ALGEBRAIC MANIPULATION

## In this unit the students will be able to:

- Recall factorization of expressions of the following types.
  $ka + kb + kc$, $ac + ad + bc + bd$, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $a^2 \pm 2ab + b^2 - c^2$
- Factorize the expressions of the following types:
  $a^4 + a^2b^2 + b^4$ or $a^4 + 4b^4$, $x^2 + px + q$, $ax^2 + bx + c$,
  $(ax^2 + bx + c)(ax^2 + bx + d) + k$, $(x + a)(x + b)(x + c)(x + d) + k$
  $(x + a)(x + b)(x + c)(x + d) + kx^2$
  $a^3 + 3a^2b + 3ab^2 + b^3$, $a^3 - 3a^2b + 3ab^2 - b^3$, $a^3 \pm b^3$
- Find highest common factor (HCF) and least common multiple (LCM) of algebraic expressions.
- Use factor or division method to determine highest common factor and least common multiple.
- Know the relationship between HCF and LCM.
- Solve real life problems related to HCF and LCM.
- Use highest common factor and least common multiple to reduce fractional expressions involving $+, -, \times, \div$.
- Find square root of algebraic expression by factorization and division.



Ansel Adams (1902 – 1984) was a famous American photographer known for his style of detailed and focused photos that showed its subjects simply and directly. To take sharp and clear pictures, Adams had to focus the camera precisely. The distance from the object to the lens '$p$' and the distance from the lens to the film '$q$' must be calculated accurately to ensure that sharp image. The focal length of the lens is '$f$'. The formula that relates these measurements is $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$. This formula involves addition of two algebraic fractions with the help of LCM.



## 4.1 Factorization of an Algebraic Expression

The process in which an algebraic expression can be expressed as the product of its factors is called its factorization. For example,

$$15x^2y = (5x)(3xy)$$

Here, $5x$ and $3xy$ are the factors of $15x^2y$.

Hence, the factorization process which converts expressions like $15x^2y$ into $(5x)(3xy)$ is essentially the opposite of the multiplication process.

Multiply

Polynomial $15x^2y$ = Factors $(5x)(3xy)$

Factorize

In the previous grades, we have learnt about the factorization of polynomials of the following types:

| | |
|---|---|
| (i) | $ka + kb + kc = k(a + b + c)$ |
| (ii) | $ac + ad + bc + bd = (c + d)(a + b)$ |
| (iii) | $a^2 \pm 2ab + b^2 = (a \quad b)^2$ |
| (iv) | $a^2 - b^2 = (a + b)(a - b)$ |
| (v) | $a^2 + 2ab + b^2 - c^2 = (a + b + c)(a + b - c)$ |
| (vi) | $a^2 - 2ab + b^2 - c^2 = (a - b + c)(a - b - c)$ |

Let us learn some more about the factorization of polynomials.

**Type-I** → **Factorizing Expressions of the Forms**

**(a)** $a^4 + a^2b^2 + b^4$ **(b)** $a^4 + 4b^4$

**(a)** To factorize $a^4 + a^2b^2 + b^4$ or $a^4 + 4b^4$, we shall modify it and try to make it in appropriate form to utilize the previous results for its factorization.

Consider: $a^4 + a^2b^2 + b^4 = (a^2)^2 + a^2b^2 + (b^2)^2$

Here first and last terms are perfect squares but middle term is not twice the product of the square root of first and last term. So, we shall add and subtract $a^2b^2$ to make it twice.

i.e. (including zero term)

$$\begin{aligned} a^4 + a^2b^2 + b^4 &= (a^2)^2 + a^2b^2 + a^2b^2 + (b^2)^2 - a^2b^2 \\ &= (a^2)^2 + 2a^2b^2 + (b^2)^2 - a^2b^2 \\ &= (a^2 + b^2)^2 - (ab)^2 \\ &= (a^2 + b^2 + ab)(a^2 + b^2 - ab) \end{aligned}$$

So, $a^4 + a^2b^2 + b^4 = (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

**(b)** Similarly, $a^4 + 64b^4 = (a^2)^2 + (8b^2)^2$

(zero term)

$$\begin{aligned} &= (a^2)^2 + 2(a^2)(8b^2) + (8b^2)^2 - 2(a^2)(8b^2) \\ &= (a^2 + 8b^2)^2 - 16a^2b^2 \\ &= (a^2 + 8b^2)^2 - (4ab)^2 \\ &= (a^2 + 8b^2 + 4ab)(a^2 + 8b^2 - 4ab) \end{aligned}$$

**Example 1:** Factorize the expression $8x^4 - 26x^2m^2 + 18m^4$.

**Solution:**

$$8x^4 - 26x^2m^2 + 18m^4$$
$$= 2(4x^4 - 13x^2m^2 + 9m^4)$$
$$= 2[(2x^2)^2 - 13x^2m^2 + (3m^2)^2]$$
$$= 2[(2x^2)^2 + 2(2x^2)(3m^2) + (3m^2)^2 - 2(2x^2)(3m^2) - 13x^2m^2]$$
$$= 2[(2x^2 + 3m^2)^2 - 12x^2m^2 - 13x^2m^2]$$
$$= 2[(2x^2 + 3m^2)^2 - 25x^2m^2]$$
$$= 2[(2x^2 + 3m^2)^2 - (5xm)^2]$$
$$= 2(2x^2 + 3m^2 + 5xm)(2x^2 + 3m^2 - 5xm)$$

> **Check Point**
> Can you factorize?
> $u^4 - u^2 + 1$

**Type-II** → **Factorization of a Trinomial of the Form**

**(i)** $x^2 + px + q$ **(ii)** $ax^2 + bx + c$

To factorize a trinomial of this form means to express the trinomial as the product of two binomials. To factorize such trinomials keep in mind the following steps:

(i) Make a list of all possible factors of 'product of extreme coefficients'.

(ii) Select a pair of factors among the list such that:

- their sum is equal to middle coefficient if extremes terms have same signs.
- their difference is equal to middle coefficient if extremes terms have opposite signs.

**Example 2:** Factorize the following polynomials.

**(a)** $y^2 - 7y + 12$ **(b)** $m^2 - 2m - 15$

**Solution: (a)**

$$y^2 - 7y + 12$$
$$= y^2 - 4y - 3y + 12$$
$$= y(y-4) - 3(y-4)$$
$$= (y-4)(y-3)$$

> Consider only negative factors when the middle term is negative and the coefficients of last term is positive.

So, $y^2 - 7y + 12 = (y-3)(y-4)$

product of –3 and –4 (→ 12)

sum of –3 and –4 (→ –7)

**(b)**

$$m^2 - 2m - 15$$
$$= m^2 - 5m + 3m - 15$$
$$= m(m-5) + 3(m-5)$$
$$= (m-5)(m+3)$$

> Consider one positive and one negative factor when the coefficient of middle term and of last term are negative.

**Example 3:** Factorize $3x^2 + 22x - 16$ ← when middle term is positive & last term is negative.

**Solution:**

$$3x^2 + 22x - 16$$
$$= 3x^2 + 24x - 2x - 16$$
$$= (3x^2 + 24x) - (2x + 16)$$
$$= 3x(x+8) - 2(x+8)$$
$$= (x+8)(3x-2)$$

## EXERCISE 4.1

Factorize the following polynomials.

| | | | |
|---|---|---|---|
| **1.** | $2x^2y^3 - 6x^2y^2 + 2xy^3$ | **2.** | $3nx - 3x - 3ny + 3y$ |
| **3.** | $18x^4 + 108x^2y^2 + 162y^4$ | **4.** | $(k+2)^2 - 8(k+2) + 16$ |
| **5.** | $9x^2 + 4 - 169y^2 - 12x$ | **6.** | $(x^2-1)(y+1) - (y+3)(x^2-1)$ |
| **7.** | $x^2 - 6ax + 9a^2 - 16b^2$ | **8.** | $1 - x^2 - 2xy - y^2$ |

**9.** Find a polynomial whose factorization is $(x + y - 2c)(x + 2c + y)$ by using an appropriate formula.

**10.** Show the expression $x^2 + 4y^2 - z^2 + 4xy$ as the difference of two squares.

**11.** Find the missing factor in the following.

**(a)** $(2y^2 - 3y - 27) = (y + 3)(\qquad)$ **(b)** $(5x^2 + 12x - 9) = (\qquad)(x + 3)$

Factorize the following expressions.

| | | | | | |
|---|---|---|---|---|---|
| **12.** | $x^4 + 4m^4$ | **13.** | $m^4 + m^2 + 1$ | **14.** | $3x^4 \quad 21x^3 + 24x^2$ |
| **15.** | $x^8 + x^4 + 1$ | **16.** | $4x^4 + 256y^4$ | **17.** | $12 - 7x + x^2$ |
| **18.** | $x^2 - 9x + 8$ | **19.** | $10z^2 - 29z + 10$ | **20.** | $-3y^2 + 13y - 4$ |
| **21.** | $x^2 - 21x + 90$ | **22.** | $x^2 + x - 2$ | **23.** | $3x^2 + 11x + 6$ |
| **24.** | $2x^2 - 5xy - 3y^2$ | **25.** | $8 + 6x - 5x^2$ | **26.** | $6 - 7x - 5x^2$ |
| **27.** | $2a^2 - 4a - 6$ | **28.** | $u^4 - 13u^2 + 36$ | **29.** | $y^4 - 12y^2 - 64$ |

**Type-III** → **Factorizing Expressions of the Forms**

**(a)** $(ax^2 + bx + c)(ax^2 + bx + d) + k$

**(b)** $(x + a)(x + b)(x + c)(x + d) + k$

**(c)** $(x + a)(x + b)(x + c)(x + d) + kx^2$

The process of factorizing expressions of above types will be explained in the following examples.

**(a)** $(ax^2 + bx + c)(ax^2 + bx + d) + k$

**Example 4:** Factorize: $(x^2 + 3x - 4)(x^2 + 3x + 5) + 8$

We observe here that first two terms inside both the parentheses are same. i.e.

$$(\underbrace{x^2 + 3x}_{\text{same}} - 4)(\underbrace{x^2 + 3x}_{\text{same}} + 5) + 8$$

Let $x^2 + 3x = a$, then above expression will take the form

$$= (a-4)(a+5) + 8$$
$$= a(a+5) - 4(a+5) + 8$$
$$= a^2 + 5a - 4a - 20 + 8 = a^2 + a - 12$$
$$= a^2 + 4a - 3a - 12$$
$$= (a^2 + 4a) - (3a + 12)$$
$$= a(a+4) - 3(a+4) = (a+4)(a-3)$$
$$= (x^2 + 3x + 4)(x^2 + 3x - 3) \quad \text{.....by putting the value of a.}$$

**(b) $(x+a)(x+b)(x+c)(x+d)+k$**

To factorize such expressions, consider the the following examples.

**Example 5a:** Factorize: $(x+5)(x+3)(x+2)(x+6)-88$

**Solution:** $(x+5)(x+3)(x+2)(x+6)-88$ …… notice here, $5+3=2+6$

$$=[(x+5)(x+3)][(x+2)(x+6)]-88$$
$$=(x^2+8x+15)(x^2+8x+12)-88$$

Let $x^2+8x=a$, then above expression will take the form

$$=(a+15)(a+12)-88$$
$$=a^2+27a+180-88$$
$$=a^2+27a+92$$
$$=a^2+4a+23a+92$$
$$=(a^2+4a)+(23a+92)$$
$$=a(a+4)+23(a+4)=(a+4)(a+23)$$

$\therefore$ By back substitution

$$=(x^2+8x+4)(x^2+8x+23)$$

**Example 5b:** Factorize the expression

$$(z+1)(z-5)(z-9)(z-3)+44$$

**Solution:**

$$(z+1)(z-5)(z-9)(z-3)+44$$

Combine $(z+1)$ with $(z-9)$ and $(z-5)$ with $(z-3)$.

Re-arranging the given expression, we have

$$=(z+1)(z-9)(z-5)(z-3)+44$$
$$=(z^2-8z-9)(z^2-8z+15)+44$$

By putting $z^2-8z=x$ in the above expression,

it will take the form

$$=(x-9)(x+15)+44$$
$$=x^2-9x+15x-135+44$$
$$=x^2+6x-91$$
$$=x^2+13x-7x-91$$
$$=(x^2+13x)-(7x+91)$$
$$=x(x+13)-7(x+13)$$
$$=(x+13)(x-7)$$

Now replacing $x$ by $z^2-8z$, we have

$$=(z^2-8z+13)(z^2-8z-7)$$

**Check Point**

While assuming same binomials equal to another variable, you must have to consider that variable which is not already present in the given expression.
Do you know why?

**(c) $(x+a)(x+b)(x+c)(x+d)+kx^2$**

**Example 6:** **Factorize the expression.**

$$(x+1)(x+2)(x+3)(x+6)-3x^2$$

**Solution:** $(x+1)(x+6)(x+2)(x+3)-3x^2$

$$=(x^2+6+7x)(x^2+6+5x)-3x^2$$

let $x^2+6=y$

$$=(y+7x)(y+5x)-3x^2$$
$$=y^2+5xy+7xy+35x^2-3x^2$$
$$=y^2+12xy+32x^2$$
$$=y^2+8xy+4xy+32x^2$$
$$=y(y+8x)+4x(y+8x)$$
$$=(y+8)(y+4x)$$

putting the value of $y$

$$=(x^2+6+8x)(x^2+6+4x)$$
$$=(x^2+8x+6)(x^2+4x+6)$$

**Type-IV** → **Factorizing expressions of the forms:**

(a) $a^3 + 3a^2b + 3ab^2 + b^3$
(b) $a^3 - 3a^2b + 3ab^2 - b^3$

We have studied in the previous unit that:

(i) $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$
(ii) $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$

The technique of factorization is elaborated through following examples.

**Example 7:** Factorize the following expressions.

(i) $27a^3 + 1 + 27a^2 + 9a$

(ii) $x^3 + \dfrac{3}{x} - \dfrac{1}{x^3} - 3x$

(iii) $4x^2(2x - 15) - 25(5 - 6x)$

**Solution:**

(i) $27a^3 + 1 + 27a^2 + 9a$

$= (3a)^3 + (1)^3 + 3(3a)^2(1) + 3(3a)(1)^2$
$= (3a)^3 + 3(3a)^2(1) + 3(3a)(1)^2 + (1)^3$
$= (3a + 1)^3$

(ii) $x^3 + \dfrac{3}{x} - \dfrac{1}{x^3} - 3x$

$= x^3 - 3x + \dfrac{3}{x} - \dfrac{1}{x^3}$

$= (x)^3 - 3(x)^2\left(\dfrac{1}{x}\right) + 3(x)\left(\dfrac{1}{x}\right)^2 - \left(\dfrac{1}{x}\right)^3$

$= \left(x - \dfrac{1}{x}\right)^3$

(iii) $4x^2(2x - 15) - 25(5 - 6x)$

$= 8x^3 - 60x^2 - 125 + 150x$

$= 8x^3 - 60x^2 + 150x - 125$

$= (2x)^3 - 3(2x)^2(5) + 3(2x)(5)^2 - (5)^3$

$= (2x - 5)^3$

**Type-V** → **Factorizing the sum and difference of two cubes**

(a) $a^3 + b^3$  (b) $a^3 - b^3$

We have studied in the previous unit that:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad \text{and} \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

**Example 8:** Factorize: $x^3 + 27y^3$

**Solution:**

As, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

So, $x^3 + 27y^3 = (x)^3 + (3y)^3 = (x + 3y)[(x)^2 - (x)(3y) + (3y)^2]$
$= (x + 3y)(x^2 - 3xy + 9y^2)$

**Example 9:** Factorize $x^6 - y^6$.

**Solution:** $x^6 - y^6 = (x^3)^2 - (y^3)^2$
$= (x^3 + y^3)(x^3 - y^3)$

$$= (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$
$$= (x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$$

**Example 10:** Factorize $8a^3 - 125b^3 - 2a + 5b$

**Solution:** $8a^3 - 125b^3 - 2a + 5b$

$$= (2a)^3 - (5b)^3 - (2a - 5b)$$
$$= (2a - 5b)\ [(2a)^2 + (2a)(5b) + (5b)^2] - (2a - 5b)$$
$$= (2a - 5b)\ (4a^2 + 10ab + 25b^2) - (2a - 5b)$$
$$= (2a - 5b)\ (4a^2 + 10ab + 25b^2 - 1) \longleftarrow \text{factor out } (2a - 5b)$$

## EXERCISE 4.2

Factorize the following expressions completely.

1. $x^3 - 125$ 2. $8x^3 + 1$ 3. $3p^3q^3 - 81x^3$ 4. $27 + 512x^3$ 5. $t^6 - 64$
6. $x^6 + y^6$ 7. $(2-x)^3 + (y-2)^3$ 8. $64(x+y)^3 - z^3$
9. $27p^3 + 144pq^2 - 108p^2q - 64q^3$ 10. $8p^3 + q^3 + 12p^2q + 6pq^2$
11. $125x^3 - y^3 - 75x^2y + 15xy^2$ 12. $p^3 - 9p^2q + 27pq^2 - 27q^3$
13. $(2x^2 - 3x + 6)(2x^2 - 3x) - 55$ 14. $(y^2 + 2y - 3)(y^2 + 2y + 11) + 48$
15. $y(y-1)(y-3)(y-4) + 2$ 16. $(k+2)(k-3)(k+5)(k+10) + 375$
17. $(x-5)(x-6)(x+3)(x+2) + 12$ 18. $(x+1)(x+2)(x-3)(x-6) - 21x^2$
19. $(x-2)(x-6)(x-3)(x-4) - 2x^2$ 20. $(5-x)(2+x)(10-x)(1+x) - 7x^2$
21. The expression $a^6 + 729$ can be written in two ways as :
    (a) sum of two squares (b) sum of two cubes,
    which one will be used for factoring it and why? Also factorize the given expression.
22. Express $8 + 12t + 6t^2 + t^3$ as the product of three factors. Is each factor a binomial or a trinomial?

## 4.2 Highest Common Factor of Algebraic Expressions

As algebra is an extension of arithmetic, so we apply almost the same rules for finding HCF of two or more algebraic expressions (polynomials) as used in arithmetic.

### 4.2.1 Highest Common Factor by Factorization

A factor of a polynomial is an-other polynomial which divides it completely. The common factor of two or more polynomials is a polynomial which divides them exactly.

> The highest common factor of two or more than two polynomials is a highest degree polynomial which divides the given polynomials exactly.

## (a) HCF of Monomial Expressions

**To Find the HCF of Monomials:**

I. Determine the HCF of numerical coefficients by prime factorization.

II. Determine the common variables and select their lowest power that appears in all monomials.
This will be the HCF of variables.

**Example 11:** Find the HCF of $18ab^3c$, $30a^2b^4c^3$, $24b^2c^5$.

**Solution:** First we find the HCF of numerical coefficients 18, 30 and 24 as

$$18 = 2 \times 3^2$$
$$30 = 2 \times 3 \times 5$$
$$24 = 2^3 \times 3$$

$\therefore$ HCF of 18, 30 and 24 = $2 \times 3 = 6$

HCF of $b^2$, $b^3$ and $b^4$ is $b^2$. ⟵ least common power of b

HCF of c, $c^3$ and $c^5$ is c. ⟵ least common power of c

Hence, required HCF = $6b^2c$

> **Food for Thought**
> What is HCF of $7x^2$ and $5y^2$?

## (b) HCF of Compound Polynomial Expressions

**To Find the HCF of Compound Expressions:**

I. Write each expression in complete factored form. Repeated factors should be expressed as powers.

II. Select the least power of each common factor.

III. The highest common factor (HCF) is the product of results of step-II.

**Example 12:** Find the HCF of $2m^2 - 2mn$, $4m^4 - 4m^2n^2$ and $2m^3 - 4m^2n + 2mn^2$.

**Solution:** First factor each polynomial expression completely as

$$2m^2 - 2mn = 2m(m-n)$$
$$4m^4 - 4m^2n^2 = 4m^2(m^2 - n^2)$$
$$= 2^2m^2(m+n)(m-n)$$
$$2m^3 - 4m^2n + 2mn^2 = 2m(m^2 - 2mn + n^2)$$
$$= 2m(m-n)^2$$

Common factors with least power are 2, m, m – n

$\therefore$ Required HCF = $2m(m-n)$

**Example 13:** Find the HCF of the following.

$$ax^2 + 7ax + 12a,\ ax^2 - 5ax - 24a,\ 2ax^2 + 5ax - 3a$$

**Solution:** $ax^2 + 7ax + 12a = a(x^2 + 7x + 12)$

$$= a(x^2 + 4x + 3x + 12)$$
$$= a[x(x+4) + 3(x+4)]$$
$$= a(x+3)(x+4)$$

$ax^2 - 5ax - 24a = a(x^2 - 5x - 24)$

$$= a(x^2 - 8x + 3x - 24)$$
$$= a[x(x-8) + 3(x-8)]$$

$= a(x+3)(x-8)$

$2ax^2 + 5ax - 3a = a(2x^2 + 5x - 3)$
$= a(2x^2 + 6x - x - 3)$
$= a[2x(x+3) - 1(x+3)]$
$= a(2x-1)(x+3)$

$\therefore$ Required HCF = product of all common factors with least power
$= a(x+3)$

**Food for Thought**

What would be the HCF of $a^3 - b^3$ and $a^3 + b^3$?

## EXERCISE 4.3

**1.(a)** Find the HCF of the following monomials by completing the table.

| Monomials | HCF of numerical coefficients | HCF of 'p' | HCF of 'q' | HCF of 'r' | Required HCF |
|---|---|---|---|---|---|
| $16p^3q$, $9pq^2r$ | | | | | |
| $10p^3q^2r$, $5p^2qr$, $15p^2qr^2$ | | | | | |
| $14p^4qr^4$, $28p^3qr^2$, $7p^2qr^2$, $21p^2q^2r^4$ | | | | | |

**(b)** If all common factors with least power of three unknown polynomials are $2^2$, 3, pq and $(p+q)^2$ then what would be their HCF?

**(c)** Write any two polynomials of your choice having HCF as 1.

**(d)** The only common factor of two polynomials is $m - n$ and the only uncommon factor is $m^2 + n^2$. Can you guess the unknown polynomials?

**(e)** Can you guess HCF of two polynomials $x^3 + 5x + 1$ and $1 + 5x + x^3$ without any procedure?

Find the HCF of the following by factorization.

2. $(x+y)^2$, $x^2 - y^2$
3. $(a-b)^3$, $a^2 - 2ab + b^2$
4. $a^3b - ab^3$, $a^5b^2 - a^2b^5$
5. $x^2 - 49$, $x^2 - 4x - 21$
6. $12x^2 + x - 1$, $15x^2 + 8x + 1$
7. $c^2x^2 - d^2$, $acx^2 - bcx + adx - bd$
8. $m^2 - n^2$, $m^4 - n^4$, $m^6 - n^6$
9. $ax^2 + 2a^2x + a^3$, $2ax^2 - 4a^2x - 6a^3$, $3(ax + a^2)^2$

### 4.2.2 Highest Common Factor by Division

Sometimes, it is difficult to factorize the given polynomials completely. In such cases, we adopt division method to find the HCF of these polynomials.
We will explain the procedure with the help of the following example.

**Example 14:** Find the HCF of two polynomials $x^3 + x^2 - 5x + 3$ and $x^2 + 3x$.

**Solution:** The following steps will be followed for finding HCF.

**Step-I:** Arrange the given polynomials in descending order w.r.t. the variables.

In this case, polynomials are already in descending order.

**Step-II:** Consider the higher degree polynomial as dividend and lower degree polynomial as divisor and start division process.

**Step-III:** Since degree of remainder has become smaller than the degree of divisor, so remainder will be taken as divisor and divisor will be considered as dividend.

$$
\begin{array}{l}
x^2+3x\,\big)\; x^3+x^2-5x+3\;\big(x-2 \\
\qquad\qquad \pm x^3 \pm 3x^2 \\
\hline
\qquad\qquad -2x^2-5x+3 \\
\qquad\qquad \mp 2x^2 \mp 6x \\
\hline
\qquad\qquad\qquad x+3\,\big)\; x^2+3x\;\big(x \\
\qquad\qquad\qquad\qquad \pm x^2 \pm 3x \\
\hline
\qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

Divisor

Continue the same process until we get 0 remainder. As $x + 3$ is the last divisor which gives remainder as '0'. Therefore, required HCF $= x + 3$

**Example 15:** Find the HCF of $12 + 16a + 7a^2 + a^3$ and $13a + 5a^2 + 14 + a^3$.

**Solution:** First arrange the given polynomials in descending order w.r.t. the variable. i.e. $a^3 + 7a^2 + 16a + 12$ and $a^3 + 5a^2 + 13a + 14$. Since degree of both the polynomials is same, so any one can be taken as a divisor or dividend.

$$
\begin{array}{l}
a^3+5a^2+13a+14\,\big)\; a^3+7a^2+16a+12\;\big(1 \\
\qquad\qquad\qquad \pm a^3 \pm 5a^2 \pm 13a \pm 14 \\
\hline
\qquad\qquad\qquad 2a^2+3a-2\,\big)\; a^3+5a^2+13a+14\;\big(a+7 \\
\qquad\qquad\qquad\qquad\qquad \times 2 \\
\hline
\qquad\qquad\qquad\qquad\qquad 2a^3+10a^2+26a+28 \\
\qquad\qquad\qquad\qquad\qquad \pm 2a^3 \pm 3a^2 \mp 2a \\
\hline
\qquad\qquad\qquad\qquad\qquad\quad 7a^2+28a+28 \\
\qquad\qquad\qquad\qquad\qquad\quad \times 2 \\
\hline
\qquad\qquad\qquad\qquad\qquad\quad 14a^2+56a+56 \\
\qquad\qquad\qquad\qquad\qquad\quad \pm 14a^2 \pm 21a \mp 14 \\
\hline
\qquad\qquad\qquad\qquad\qquad\qquad 35a+70 = 35(a+2)
\end{array}
$$

Since first term of the divisor is $2a^2$ and first term of the dividend is $a^3$. Therefore, for convenience, first we multiply the dividend by 2.

Again first we will multiply the remainder by 2.

Since 35 is not a factor of any polynomial so we can neglect it for convenience.

Now divide $2a^2 + 3a - 2$ by $a + 2$.

$$
\begin{array}{l}
a+2\,\big)\; 2a^2+3a-2\;\big(2a-1 \\
\qquad\quad \pm 2a^2 + 4a \\
\hline
\qquad\qquad\quad -a-2 \\
\qquad\qquad\quad \mp a \mp 2 \\
\hline
\qquad\qquad\qquad 0
\end{array}
$$

$\therefore$ Required HCF $= a + 2$

**Key Fact**

HCF of two polynomials P and Q will be the same as that of mP and nQ (where m and n are non zero constants). So in the process of finding the HCF, we can multiply or divide any divisor or dividend by any suitable number according to our requirement, but we cannot multiply or divide it by a variable.

## 4.3 Least Common Multiple of Algebraic Expressions

> The least common multiple of two or more polynomials is a polynomial of lowest degree that contains the factors of each polynomial.

### 4.3.1 Least Common Multiple by Factorization

The process of determining the LCM is almost identical to that for determining the HCF. Prime factorization is also useful to determine the LCM of two or more polynomials. The LCM is obtained by taking product of all factors (common and uncommon) with highest power.

**(a) LCM of Monomial Expressions**

To determine the LCM of two or more monomials, find LCM of the numerical coefficients and LCM of each variable. Then find their product for the required LCM.

To understand this concept, consider the two monomials $45x^2y$ and $60x^3y^2z$.

First find the LCM of numerical coefficients 45 and 60 which is 180.

Now consider variables $x$, $y$ and $z$.

LCM of $x^2$ and $x^3$ = highest power of $x$ appearing in any monomial = $x^3$

LCM of $y$ and $y^2$ = highest power of $y$ appearing in any monomial = $y^2$

LCM of $z$ = highest power of $z$ appearing in any monomial = $z$

Thus, required LCM = product of LCM of coefficients and LCM of each variable.

$= 180x^3y^2z$

**Example 16:** Find LCM of $18a^3b^4c^5$, $60a^3b^4c^6$ and $42a^4b^3$.

**Solution:** First we find the LCM of 18, 60 and 42 by prime factorization as

$$18 = 2 \times 3^2, \quad 60 = 2^2 \times 3 \times 5, \quad 42 = 2 \times 3 \times 7$$

$\therefore$ LCM of 18, 60 and 42 = product of all factors with highest power

$= 2^2 \times 3^2 \times 5 \times 7 = 1260$

Now we find LCM of each variable as

LCM of $a^3$ and $a^4$ = $a^4$ (highest power of a)

LCM of $b^3$ and $b^4$ = $b^4$ (highest power of b)

LCM of $c^5$ and $c^6$ = $c^6$ (highest power of c)

$\therefore$ Required LCM = $1260a^4b^4c^6$

> **Thinking Zone**
>
> Haleema has $60(x+1)$ english story books, $45(x+1)$ math fun books and $75(x+1)$ science fun books. She wants to put all books in groups of same number. What do you think can be the biggest number of books to be put in each group.

**(b) LCM of Compound Polynomial Expressions**

To understand the procedure, let us consider the following examples.

**Example 17:** Find LCM of $x^4y^3 - x^3y^4$ and $x^3y - xy^3$

**Solution:** First we factorize each polynomial completely.

$$x^4y^3 - x^3y^4 = x^3y^3(x-y)$$

$$x^3y - xy^3 = xy(x^2 - y^2) = xy(x-y)(x+y)$$

We choose every factor with highest power as: $x^3y^3$, $x - y$ and $x + y$.

Then their product is $x^3y^3(x+y)(x-y)$

$\therefore$ Required LCM $= x^3y^3(x+y)(x-y)$

$= x^3y^3(x^2 - y^2)$

$= x^5y^3 - x^3y^5$

**To find the LCM of Compound Expressions:**

I. Write each polynomial in complete factored form. Repeated factors should be expressed as powers.

II. Select the highest power of every factor that appears.

III. The least common multiple is the product of the results of step II.

**Example 18:** Find the LCM of $2x^2 - 12xy + 16y^2$, $x^2 - 6xy + 8y^2$ and $3x^2 - 12y^2$.

**Solution:**

$$2x^2 - 12xy + 16y^2 = 2(x^2 - 6xy + 8y^2)$$
$$= 2(x - 4y)(x - 2y)$$
$$x^2 - 6xy + 8y^2 = (x - 4y)(x - 2y)$$
$$3x^2 - 12y^2 = 3(x^2 - 4y^2)$$
$$= 3(x + 2y)(x - 2y)$$

All factors with highest powers are 2, 3, $x - 4y$, $x - 2y$, $x + 2y$

$\therefore$ Required LCM $= 2 \times 3 \times (x - 4y)(x - 2y)(x + 2y)$

$= 6(x - 4y)(x^2 - 4y^2)$

**Note:** Same pattern will be followed for more than three polynomials.

**Key Fact**

- The key words in the two processes of HCF and LCM are:
  HCF $\longrightarrow$ lowest power and common factor.
  LCM $\longrightarrow$ highest power and every factor.
- LCM of two or more polynomials is a lowest degree polynomial that is exactly divisible by each of the given polynomials.

## EXERCISE 4.4

1. Give quick answers to these questions without doing any procedure.
   If HCF of two polynomials $x^3 + 5x^2 + 6x$ and $x^3 + 9x^2 + 14x$ is obtained as $x^2 + 2x$, then:

(i) What would be the HCF of $5(x^3 + 5x^2 + 6x)$ and $x^3 + 9x^2 + 14x$?

(ii) What would be the HCF of $x^3 + 5x^2 + 6x$ and $2(x^3 + 9x^2 + 14x)$?

(iii) What would be the HCF of $3(x^3 + 5x^2 + 6x)$ and $7(x^3 + 9x^2 + 14x)$?

(iv) What would be the HCF of $15(x^3 + 5x^2 + 6x)$ and $25(x^3 + 9x^2 + 14x)$?

(v) Does HCF of the given polynomials will remain unchanged if both are multiplied by $x$?

**2.** Find the LCM of the following monomials by completing the table.

| Monomials | LCM of numerical coefficients | LCM of '$x$' | LCM of '$y$' | LCM of '$z$' | Required LCM |
|---|---|---|---|---|---|
| (i) $8x^6y,\ 4x^2yz$ | | | | | |
| (ii) $12x^2y^4z, 24x^3z$ | | | | | |
| (iii) $18x^3z, 9xy^2z, 6x^6yz^3$ | | | | | |
| (iv) $xyz^3, z^5y^3x, 28x^3y^5z$ | | | | | |

Find the HCF of the following by division method.

**3.** $a^2 + a - 2, a^3 + 2a^2 + a + 2$

**4.** $x^3 + 2x^2 - 4x - 8, 2x^3 + 7x^2 + 4x - 4$

**5.** $2x^3 + x^2 - x - 2, 3x^3 - x^2 + x - 3$

**6.** $3 + 2p^4 + 5p^2, 5p + 5p^3 + 3 + 3p^2$

**7.** $24x^4 - 2x^3 - 60x^2 - 32x, 18x^4 - 6x^3 - 39x^2 - 18x$

**8.** $2x^3 + 6x^2 + x + 3, 3x^3 + 9x^2 - 2x - 6, x^3 + 3x^2 + 2x + 6$

Find the LCM of the following expressions.

**9.** $9a^2b - b, 6a^2 + 2a$

**10.** $p^3q - pq^3, p^5q^2 - p^2q^5$

**11.** $4x^2y - y, 2x^2 + x$

**12.** $x^2 - x - 6, x^2 + x - 2, x^2 - 4x + 3$

**13.** $m^6 - 1, m^4 - 1, m^3 - 1$

**14.** $x^3 + 2x^2 - x - 2, x^2 - x - 2, x^2 - 4$

**15.** $x^2 + x - 20, x^2 - 10x + 24, x^2 - x - 30$

## 4.3.2 Relation between HCF and LCM

Consider two polynomials $P = a^2 - b^2$ and $Q = a^2 - 2ab + b^2$. For their LCM and HCF, first factorize them as

$$P = a^2 - b^2 = (a + b)(a - b)$$

$$Q = a^2 - 2ab + b^2 = (a - b)^2$$

$\therefore$ HCF of P and Q = a – b

and LCM of P and Q = $(a + b)(a - b)^2$

Now $\quad$ HCF × LCM $= (a - b)(a + b)(a - b)^2$

$$= (a^2 - b^2)(a - b)^2$$

$$= (a^2 - b^2)(a^2 - 2ab + b^2) \quad \ldots\ldots\ldots\ \text{(i)}$$

Also, product of P and Q $= (a^2 - b^2)(a^2 - 2ab + b^2) \quad \ldots\ldots\ldots$ (ii)

Thus, $\quad$ LCM × HCF = P × Q ………. (1)

Hence, it can be generalized that:

product of their HCF and LCM = product of given polynomials

## 4.3.3 Finding of Least Common Multiple by Division

Sometimes, it is much difficult to find the LCM of given polynomials P and Q by factorization method. Then in that case, we can find the LCM by division as follows. From the relation (1) we have,

$$\text{LCM} = \frac{P\times Q}{HCF} = \frac{P}{HCF}\times Q = \frac{Q}{HCF}\times P$$

Procedure is illustrated through examples.

**Example 19:** Find the LCM of $P = 10x^4 + 3x^3 + 8$ and $Q = 8x^4 + 3x + 10$

**Solution:** First we will find their HCF as

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad\quad\quad 5 \\ 8x^4 + 3x + 10 \overline{)10x^4 + 3x^3 + 8} \\ \quad\quad\quad\quad\quad \times 4 \leftarrow \text{multiplying dividend by 4} \\ \quad\quad\quad\quad\quad 40x^4 + 12x^3 + 32 \\ \quad\quad\quad\quad\quad \underset{-}{+}40x^4 \quad \underset{-}{+}50 \underset{-}{+}15x \\ \quad\quad\quad\quad\quad\quad\quad 12x^3 - 18 - 15x \end{array}$$

where $(12x^3 - 18 - 15x) \div 3 = 4x^3 - 6 - 5x = 4x^3 - 5x - 6$

Now, we will divide $8x^4 + 3x + 10$ by $4x^3 - 5x - 6$

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad\quad 2x \\ 4x^3 - 5x - 6 \overline{)\ 8x^4 + 3x + 10} \\ \quad\quad\quad\quad \underset{-}{+}8x^4 \underset{+}{-}12x \quad \underset{+}{-}10x^2 \\ \quad\quad\quad\quad\quad\quad 15x + 10 + 10x^2 = 10x^2 + 15x + 10 \end{array}$$

Again, $(10x^2 + 15x + 10) \div 5 = 2x^2 + 3x + 2$

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad\quad 2x - 3 \\ 2x^2 + 3x + 2 \overline{)\ 4x^3 - 5x - 6} \\ \quad\quad\quad\quad \underset{-}{+}4x^3 \underset{-}{+}4x \quad \underset{-}{+}6x^2 \\ \quad\quad\quad\quad -6x^2 - 9x - 6 \\ \quad\quad\quad\quad \underset{+}{-}6x^2 \underset{+}{-}9x \underset{+}{-}6 \\ \quad\quad\quad\quad\quad\quad 0 \end{array}$$

$\therefore$ HCF is $2x^2 + 3x + 2$.

Now, we obtain the LCM using the following relation.

$$\text{LCM} = \frac{P\times Q}{HCF} = \frac{(10x^4 + 3x^3 + 8)(8x^4 + 3x + 10)}{2x^2 + 3x + 2}$$

(Since, the HCF of two polynomials divides both of them exactly. So divide any polynomial of numerator by denominator.)

$$\begin{array}{l} \quad\quad\quad\quad\quad\quad\quad 5x^2 - 6x + 4 \\ 2x^2 + 3x + 2 \overline{)\ 10x^4 + 3x^3 + 0x^2 + 0x + 8} \\ \quad\quad\quad\quad \underset{-}{+}10x^4 \underset{-}{+}15x^3 \underset{-}{+}10x^2 \\ \quad\quad\quad\quad\quad\quad -12x^3 - 10x^2 + 0x + 8 \\ \quad\quad\quad\quad\quad\quad \underset{+}{-}12x^3 \underset{+}{-}18x^2 \underset{+}{-}12x \\ \quad\quad\quad\quad\quad\quad\quad\quad 8x^2 + 12x + 8 \\ \quad\quad\quad\quad\quad\quad\quad \underset{-}{+}8x^2 \underset{-}{+}12x \underset{-}{+}8 \\ \quad\quad\quad\quad\quad\quad\quad\quad\quad 0 \end{array}$$

**Memory Plus**

HCF is not affected by multiplying or dividing any polynomial with any number during the process of finding HCF.

Hence, required LCM $= (5x^2 - 6x + 4)(8x^4 + 3x + 10)$

**Example 20:** Find the second polynomial Q when first polynomial $P = x^2 - 5x + 6$,
HCF $= x - 3$ and LCM $= x^3 - 9x^2 + 26x - 24$

**Solution:** $P = x^2 - 5x + 6$, $Q = ?$

LCM of P and Q $= x^3 - 9x^2 + 26x - 24$

HCF $= x - 3$

$$Q = \frac{HCF \times LCM}{P}$$

$$Q = \frac{(x-3)(x^3 - 9x^2 + 26x - 24)}{x^2 - 5x + 6}$$

$$Q = \frac{(x-3)(x^3 - 9x^2 + 26x - 24)}{(x-3)(x-2)}$$

$$Q = \frac{x^3 - 9x^2 + 26x - 24}{x-2} = x^2 - 7x + 12$$

$$\begin{array}{r|l} & x^2 - 7x + 12 \\ \hline x-2 & x^3 - 9x^2 + 26x - 24 \\ & \underline{\pm x^3 \mp 2x^2} \\ & -7x^2 + 26x - 24 \\ & \underline{\mp 7x^2 \pm 14x} \\ & 12x - 24 \\ & \underline{\pm 12x \mp 24} \\ & 0 \end{array}$$

**EXERCISE 4.5**

1. Find the HCF and LCM of the following.
   i. $16 - 4x^2,\ x^2 + x - 6$  ii. $a^4 - a^3 - a + 1,\ a^4 + a^2 + 1$
   iii. $x^3 + 2x^2 - 3x,\ 2x^3 + 5x^2 - 3x$
2. If HCF and LCM of two polynomials are $x - 7$ and $x^3 - 10x^2 + 11x + 70$ respectively. Then find product of two polynomials.
3. Product of two polynomials is $x^4 + 3x^3 - 12x^2 - 20x + 48$ and their HCF is $x - 2$. Find their LCM.
4. The product of two polynomials is $y^4 + 6y^3 - 3y^2 - 56y - 48$ and their LCM is $y^3 + 2y^2 - 11y - 12$. Find their HCF.
5. Find the second polynomial when,
   First polynomial $= x^4 + x^3 + x + 1$, HCF $= x + 1$ and LCM $= (x^3 + 1)(x^4 + x^3 - x - 1)$
6. Find the LCM of polynomials $4x^3 - 10x^2 + 4x + 2$ and $3x^4 - 2x^3 - 3x + 2$ if their HCF is $x - 1$.

## 4.4 Basic Operations on Algebraic Fractions

In this topic addition, subtraction, multiplication and division of algebraic fractions will be discussed.

### 4.4.1 Algebraic Fractions

Algebraic expressions of the type $\dfrac{x^2 + 2x + 1}{2x^2 - 5x - 3}, \dfrac{-1 - t}{\frac{1}{t} + 1}, \dfrac{p^2}{qp^2}$

are called algebraic fractions.

### 4.4.2 Multiplication of Algebraic Fractions

We multiply two or more algebraic fractions in the same way as common fractions in arithmetic.

If $\frac{R}{T}$ and $\frac{S}{U}$ are two algebraic fractions, where $T \neq 0, U \neq 0$.

Then, $$\frac{R}{T} \times \frac{S}{U} = \frac{RS}{TU}$$

In general, before finding the product of two or more algebraic fractions, we factorize the numerator and denominator of each fraction, if possible, and divide out all the common factors, to get the most reduced form of the resulting fraction.

**Key Fact**

In the process of multiplication, product of all the common factors being cancelled is in fact HCF of the expressions present in numerator and denominator.

**Example 21:** Find the indicated product.

$$\frac{x^2-1}{x^2+4x+4} \times \frac{x+2}{x^2+2x-3}$$

**Pointer to Ponder**

The key to success in simplifying an algebraic fraction lies in your ability to factor the polynomials.

**Solution:**

$$\frac{x^2-1}{x^2+4x+4} \times \frac{x+2}{x^2+2x-3}$$

$$= \frac{(x+1)(x-1)}{(x+2)(x+2)} \times \frac{x+2}{(x-1)(x+3)}$$

$$= \frac{x+1}{x+2} \times \frac{1}{x+3}$$

$$= \frac{x+1}{(x+2)(x+3)} = \frac{x+1}{x^2+5x+6}$$



### 4.4.3 Division of Algebraic Fractions

If $\frac{S}{U}$ and $\frac{T}{Q}$ are two algebraic fractions, then

$$\frac{S}{U} \div \frac{T}{Q} = \frac{S}{U} \times \frac{Q}{T} = \frac{SQ}{UT}, \text{ where } U \neq 0, Q \neq 0 \text{ and } T \neq 0$$

To divide one fraction by another, invert the divisor and proceed as in multiplication because 'to divide' by a fraction means 'to multiply' by its reciprocal.

**Example 22:** Find the indicated operation.

$$\frac{r^2 - s^2}{r} \div \frac{r - s}{s}$$

**Solution:** $= \frac{r^2 - s^2}{r} \div \frac{r - s}{s}$

$$= \frac{(r+s)(r-s)}{r} \times \frac{s}{r-s} \quad = \quad \frac{r+s}{r} \times \frac{s}{1}$$

$$= \frac{s(r+s)}{r} \quad = \quad \frac{sr+s^2}{r}$$

**Example 23:** Simplify: $\frac{3x^3 + 6x^2}{2x^2 + x - 6} \div (6x^2 - 15x)$

**Solution:** $\frac{3x^3 + 6x^2}{2x^2 + x - 6} \div (6x^2 - 15x)$

$$= \frac{3x^3 + 6x^2}{2x^2 + x - 6} \times \frac{1}{6x^2 - 15x}$$

$$= \frac{3x^2(x+2)}{(x+2)(2x-3)} \times \frac{1}{3x(2x-5)}$$

$$= \frac{x}{(2x-3)(2x-5)} \quad = \quad \frac{x}{4x^2 - 16x + 15}$$

### 4.4.4 Addition and Subtraction of Algebraic Fractions

Algebraic fractions are added or subtracted just like arithmetic fractions i.e., by manipulating LCM of their denominators.

**Example 24:** Simplify the following.

$$\frac{x^2 + 2x + 2}{2x} + \frac{(-2x-2)}{2x}$$

**Solution:** $\frac{x^2 + 2x + 2}{2x} + \frac{(-2x-2)}{2x}$

$$= \frac{x^2 + 2x + 2 + (-2x-2)}{2x}$$

$$= \frac{x^2 + 2x + 2 - 2x - 2}{2x}$$

$$= \frac{x^2}{2x} \quad = \quad \frac{x}{2}$$

**Enlighten Yourself**

Algebraic fractions with same denominators are called like fractions e.g., $\frac{a-2}{a+b}, \frac{b^2-2}{a+b}$

Fractions with different denominators are called unlike fractions e.g.,

$\frac{x^2-y}{ax+b}, \frac{y^2-x}{cx+d}$

**Example 25:** Perform the indicated operations.

$$\frac{-2x}{x+3}+\frac{3}{3-x}-\frac{8x-12}{x^2-9}$$

**Solution:** $\frac{-2x}{x+3}+\frac{3}{3-x}-\frac{8x-12}{x^2-9}$

$$=\frac{-2x}{x+3}+\frac{-3}{x-3}-\frac{8x-12}{(x+3)(x-3)}$$ ← $\because 3-x=-(x-3)$

$$=\frac{-2x(x-3)+(-3)(x+3)-(8x-12)}{(x+3)(x-3)}$$ ← $\because$ LCM is $(x+3)(x-3)$

$$=\frac{-2x^2+6x-3x-9-8x+12}{(x+3)(x-3)}$$ ← simplify the parentheses

$$=\frac{-2x^2-5x+3}{(x+3)(x-3)}$$ ← combine like terms

$$=\frac{-1(2x^2+5x-3)}{(x+3)(x-3)}$$ ← negative sign is taken as common

$$=\frac{-1(2x-1)(x+3)}{(x+3)(x-3)}$$

$$=\frac{-(2x-1)}{(x-3)}$$

$$=-\frac{2x-1}{x-3}$$ ← reduced form

### 4.4.5 Algebraic Fractions with Combined Operations

When two or more operations occur in any algebraic expression then the rule for order of operations (DMAS) must be followed.

**Example 26:** Simplify.

$$\frac{3u^2+5uv+2v^2}{u^2+5uv+6v^2}+\frac{2u+6v}{u^2-4v^2}\div\frac{u^2+6uv+9v^2}{u^2v-2uv^2}$$

**Solution:** Division is performed before addition while simplifying an expression. Therefore, first we will simplify fractions having '÷' sign.

$$\frac{3u^2+5uv+2v^2}{u^2+5uv+6v^2}+\frac{2u+6v}{u^2-4v^2}\div\frac{u^2+6uv+9v^2}{u^2v-2uv^2}$$

$$=\frac{3u^2+5uv+2v^2}{u^2+5uv+6v^2}+\frac{2u+6v}{u^2-4v^2}\times\frac{u^2v-2uv^2}{u^2+6uv+9v^2}$$

$$=\frac{3u^2+5uv+2v^2}{u^2+5uv+6v^2}+\frac{2(u+3v)}{(u+2v)(u-2v)}\times\frac{uv(u-2v)}{(u+3v)^2}$$

$$=\frac{3u^2+5uv+2v^2}{u^2+5uv+6v^2}+\frac{2uv}{(u+2v)(u+3v)}$$

$$= \frac{3u^2+5uv+2v^2}{(u+2v)(u+3v)} + \frac{2uv}{(u+2v)(u+3v)}$$

$$= \frac{3u^2+5uv+2v^2+2uv}{(u+2v)(u+3v)}$$

$$= \frac{3u^2+7uv+2v^2}{(u+2v)(u+3v)}$$

$$= \frac{(3u+v)(u+2v)}{(u+2v)(u+3v)} = \frac{3u+v}{u+3v}$$

## EXERCISE 4.6

**1.** Answer these without calculations.

i. Product of what algebraic fraction and $x^3+7x-8$ is 1?

ii. Which algebraic fraction divided by $\frac{x^2}{x^2+y^2}$ gives 1?

iii. Sum of what algebraic fraction and $\frac{m}{m^2+n^2}$ is $\frac{m+n}{m^2+n^2}$ ?

iv. What is the product of an algebraic fraction and its reciprocal?

Simplify the following (where all the expressions in the denominator are non-zero).

**2.** $\frac{14x^2-7x}{12x^3+24x^2} \times \frac{x^2+2x}{2x-1}$

**3.** $\frac{a^2b^2+3ab}{4a^2-1} \times \frac{2a+1}{ab+3}$

**4.** $\frac{a-b}{a^2+ab} \times \frac{a^4-b^4}{a^2-2ab+b^2} \times \frac{a}{a^2+b^2}$

**5.** $\frac{6x^2y^2}{x^2-y^2} \div \frac{3xy}{x+y}$

**6.** $\frac{8a^3-1}{4a^3+2a^2} \div \frac{6a^2-13a+5}{15a-25} \times \frac{2a^4+a^3}{15a^2}$

**7.** $\frac{x^2-8x-9}{x^2-17x+72} \times \frac{x^2-25}{x^2-1} \div \frac{x^2+4x-5}{x^2-9x+8}$

**8.** $\frac{1}{2x-3y} - \frac{x+y}{4x^2-9y^2}$

**9.** $\frac{1}{x(x-y)} + \frac{1}{y(x+y)}$

**10.** $\frac{5x+5}{3(2x-1)} + \frac{6-2x}{2(1-2x)}$

**11.** $\frac{2a}{2a-3} - \frac{5}{6a+9} - \frac{4(3a+2)}{3(4a^2-9)}$

**12.** $\frac{5}{5+x-18x^2} - \frac{2}{2+5x+2x^2}$

**13.** $\frac{1-p^2}{1+q} \times \frac{1-q^2}{p+p^2} \times \left(1+\frac{p}{1-p}\right)$

## 4.5 Square Root of Algebraic Expressions

The square root of an algebraic expression is defined as one of its equal factors e.g., $x + y$ is the square root of $x^2 + 2xy + y^2$ because,

$$x^2 + 2xy + y^2 = (x + y)^2$$

> The square root of an algebraic expression P is another algebraic expression Q which, when squared, gives P. Thus, if $P = (-Q)^2$ then Q and – Q both are square roots of P.

Square root of an algebraic expression can be obtained in two ways.

i. Factorization Method  ii. Division Method

### 4.5.1 Square Root by Factorization Method

In this method, before applying the square root, given expression is written in the form of a complete square. For example, to find the square root of $4a^2 + 12ab + 9b^2$, first we will convert the given expression into a complete square as follows:

$$4a^2 + 12ab + 9b^2 = (2a)^2 + 2(2a)(3b) + (3b)^2$$
$$= (2a + 3b)^2$$
$$= [\pm\ (2a + 3b)]^2$$

Now, applying square root on both sides, we get.

$$\sqrt{4a^2 + 12ab + 9b^2} = \pm(2a + 3b)$$

**Key Fact**

The square root of an algebraic expression consists of two expressions, which are additive inverses of each other.

**Example 27:** Find the square root of $\left(a+\frac{1}{a}\right)^2 - 4\left(a-\frac{1}{a}\right)$.

**Solution:** To find square root of such type of expressions, we can adopt two methods.

**Method-I**

$$\left(a+\frac{1}{a}\right)^2 - 4\left(a-\frac{1}{a}\right) = a^2 + 2(a)\left(\frac{1}{a}\right) + \frac{1}{a^2} - 4\left(a-\frac{1}{a}\right)$$

$$= a^2 + 2 + \frac{1}{a^2} - 4\left(a-\frac{1}{a}\right)$$

$$= a^2 - 2 + \frac{1}{a^2} - 4\left(a-\frac{1}{a}\right) + 2 + 2$$

$$= \left(a-\frac{1}{a}\right)^2 - 4\left(a-\frac{1}{a}\right) + 4$$

$$= \left(a-\frac{1}{a}\right)^2 - 2\left(a-\frac{1}{a}\right)(2) + (2)^2$$

$$= \left(a-\frac{1}{a}-2\right)^2$$

$$\sqrt{\left(a+\frac{1}{a}\right)^2 - 4\left(a-\frac{1}{a}\right)} = \pm\left(a-\frac{1}{a}-2\right) \text{....(applying square root)}$$

**Method-II**

$$\left(a+\frac{1}{a}\right)^2-4\left(a-\frac{1}{a}\right) = a^2+2+\frac{1}{a^2}-4\left(a-\frac{1}{a}\right)$$

$$= \left(a^2+\frac{1}{a^2}\right)+2-4\left(a-\frac{1}{a}\right) \quad \text{........ (i)}$$

Let, $a-\frac{1}{a}=x$ ........ (a)

Then, $\left(a-\frac{1}{a}\right)^2=x^2$

or $a^2-2+\frac{1}{a^2}=x^2$

or $a^2+\frac{1}{a^2}=x^2+2$ ........ (b)

Substituting values from equations (a) and (b) in equation (i)

$$\left(a+\frac{1}{a}\right)^2-4\left(a-\frac{1}{a}\right) = (x^2+2)+2-4x$$

$$= x^2-4x+4$$

$$= (x-2)^2 \quad \text{........ (ii)}$$

Now, replace $x$ by $a-\frac{1}{a}$ in equation (ii)

$$\left(a+\frac{1}{a}\right)^2-4\left(a-\frac{1}{a}\right)=\left(a-\frac{1}{a}-2\right)^2$$

Applying square root on both sides, we get

$$\sqrt{\left(a+\frac{1}{a}\right)^2-4\left(a-\frac{1}{a}\right)} = \pm\left(a-\frac{1}{a}-2\right).$$

**Example 28:** Find the square root of $(2a+1)(2a+3)(2a+5)(2a+7)+16$.

**Solution:**

$$(2a+1)(2a+3)(2a+5)(2a+7)+16 = [(2a+1)(2a+7)][(2a+3)(2a+5)]+16$$

$$= (4a^2+16a+7)(4a^2+16a+15)+16$$

$$= (4a^2+16a+7)(4a^2+16a+7+8)+16$$

$$(2a+1)(2a+3)(2a+5)(2a+7)+16 = x(x+8)+16 \quad \text{.....(put } 4a^2+16a+7=x)$$

$$= x^2+8x+16$$

$$= x^2+2(x)(4)+(4)^2 = (x+4)^2$$

Now replace $x$ by $4a^2+16a+7$

$$(2a+1)(2a+3)(2a+5)(2a+7)+16 = (4a^2+16a+7+4)^2 = (4a^2+16a+11)^2$$

Applying square root on both sides, we get

$$\sqrt{(2a+1)(2a+3)(2a+5)(2a+7)+16} = \pm(4a^2+16a+11)$$

### 4.5.2 Square Root by Division Method

To understand this method, consider the following example.

**Example 29:** Find the square root of $9x^2 - 42xy + 49y^2$ by division method.

**Solution:**

$$\begin{array}{l|l} & 3x - 7y \leftarrow \text{root} \\ \hline \left[\sqrt{9x^2} = 3x\right] \rightarrow \quad 3x & 9x^2 - 42xy + 49y^2 \\ \quad\quad\quad\quad\quad\quad \underline{+3x} & \pm 9x^2 \\ \hline \left[\dfrac{-42xy}{6x} = -7y\right] \quad 6x - 7y & -42xy + 49y^2 \leftarrow \text{remainder} \\ & \mp 42xy \pm 49y^2 \\ \hline & 0 \leftarrow \text{remainder} \end{array}$$

$\therefore$ Square root $= \pm(3x - 7y)$

**Example 30:** Find the square root of $4\left(x^2 + \frac{1}{x^2}\right) + 12\left(x + \frac{1}{x}\right) + 17, x \neq 0.$

**Solution:** $4\left(x^2 + \frac{1}{x^2}\right) + 12\left(x + \frac{1}{x}\right) + 17 = 4x^2 + \frac{4}{x^2} + 12x + \frac{12}{x} + 17$

$= 4x^2 + 12x + 17 + \frac{12}{x} + \frac{4}{x^2}$ ← arrange in descending order

$$\begin{array}{l|l} & 2x + 3 + \frac{2}{x} \\ \hline 2x & 4x^2 + 12x + 17 + \frac{12}{x} + \frac{4}{x^2} \\ & \pm 4x^2 \\ \hline 4x + 3 & +12x + 17 + \frac{12}{x} + \frac{4}{x^2} \\ & \pm 12x \pm 9 \\ \hline 4x + 6 + \frac{2}{x} & 8 + \frac{12}{x} + \frac{4}{x^2} \\ & \pm 8 \pm \frac{12}{x} \pm \frac{4}{x^2} \\ \hline & 0 \end{array}$$

$\therefore$ Required square root is $\pm\left(2x + 3 + \frac{2}{x}\right)$

Sometimes the given expression is not a perfect square and to make it a perfect square, we add (subtract) some value to (from) it. Consider the following example.

**Example 31:** To convert $x^4 + 4x^3 + 10x^2 + 10x + 5$ into a perfect square,

i. What should be added to it?
ii. What should be subtracted from it?
iii. What should be the value of $x$?

**Solution:** First we try to find the square root of the given expression as

$$\begin{array}{r|l}
 & x^2 + 2x + 3 \\ \hline
x^2 & x^4 + 4x^3 + 10x^2 + 10x + 5 \\
 & \pm x^4 \\ \hline
2x^2 + 2x & 4x^3 + 10x^2 + 10x + 5 \\
 & \pm 4x^3 \pm 4x^2 \\ \hline
2x^2 + 4x + 3 & 6x^2 + 10x + 5 \\
 & \pm 6x^2 \pm 12x \pm 9 \\ \hline
 & -2x - 4
\end{array}$$

For a perfect square, remainder must be zero, but here remainder is $-2x - 4$.
Hence,

i. Expression will be a perfect square if we add $-(-2x - 4) = 2x + 4$ in it.
ii. Expression will be a perfect square if we subtract $-2x - 4$ from it.
iii. For the value of $x$ put remainder equal to zero as

$$-2x - 4 = 0$$
$$-2x = 4 \quad \text{or} \quad x = -2$$

$\therefore$ For $x = -2$, expression will be a perfect square.

**Example 32:** For what values of $m$ and $n$, $9x^4 - 24x^3 - 14x^2 + mx + n$ is a complete square or perfect squre?

**Solution:** First we try to find the square root of $9x^4 - 24x^3 - 14x^2 + mx + n$.

$$
\begin{array}{r|l}
 & 3x^2 - 4x - 5 \\
\hline
3x^2 & 9x^4 - 24x^3 - 14x^2 + mx + n \\
 & \pm 9x^4 \\
\hline
6x^2 - 4x & -24x^3 - 14x^2 + mx + n \\
 & \mp 24x^3 \pm 16x^2 \\
\hline
6x^2 - 8x - 5 & -30x^2 + mx + n \\
 & \mp 30x^2 \pm 40x \pm 25 \\
\hline
 & mx - 40x + n - 25 \\
 & = x(m - 40) + (n - 25)
\end{array}
$$

The, given expression will be a complete square if the remainder is zero. This is only possible if,

$m - 40 = 0$ and $n - 25 = 0$

or m = 40 and $n = 25$

Hence, for $m = 40$ and $n = 25$ given expression will be a complete square

## EXERCISE 4.7

Find the square root of the following by factorization.

**1.** $16y^2 - 56y + 49$

**2.** $25a^4 - 30a^3 + 9a^2$

**3.** $\left(x^2 - \frac{1}{x^2}\right)^2 + 4\left(x^2 - \frac{1}{x^2}\right) + 4, \; x \neq 0$

**4.** $\left(a^2 + \frac{1}{a^2}\right) - 8\left(a - \frac{1}{a}\right) + 14, \; a \neq 0$

**5.** $(a + 2)(a + 4)(a + 6)(a + 8) + 16$

Find the square root of the following by division.

**6.** $x^4 + 8x^3 + 20x^2 + 16x + 4$

**7.** $x^4 + 10x^3 + 31x^2 + 30x + 9$

**8.** $49b^4 + 18a^2b^2 + 4a^3b + a^4 + 28ab^3$

**9.** $4x^4 - 12x^3 + 29x^2 - 30x + 25$

**10.** $1 - 10x + 27x^2 - 10x^3 + x^4$

**11.** $x^4 + \frac{1}{x^4} + 4x^2 - \frac{4}{x^2} + 2, \; x \neq 0$

**12.** $a^2 - 8a + 2 + \frac{56}{a} + \frac{49}{a^2}, \; a \neq 0$

**13.** $x^4 - 2x^3 + \frac{3x^2}{2} - \frac{x}{2} + \frac{1}{16}$

**14.** $4x^4 + 32x^2 + 96 + \frac{64}{x^4} + \frac{128}{x^2}$

15. To make $a^4 - 10a^3 + 27a^2 - 9a + 2$ a perfect square:
    i. What should be added in it? ii. What should be subtracted from it?
    iii. What will be the value of $a$?
16. Find the values of p and q if $x^4 - 12x^3 + px + q$ is a complete square.
17. For what value of $k$, the expression $y^4 + 4y^2 + k + \frac{8}{y^2} + \frac{4}{y^4}$ becomes a perfect square, where $y \neq 0$.

### 4.5.3 Application of Factorization in Daily Life

We use various basic principles of mathematics quite unknowingly in our daily life. Like, we are always using addition, subtraction, division and multiplication everywhere, from restaurants to public transport. When children learn about numbers and basic mathematics. This happens because, with time, we become familiar with the concepts. They can frequently apply these concepts in solving their real life problems. Factorization is a similar example of this, we have a bunch of real-life examples where we use factorization extensively, making our daily lives easier.

**Example 33:**

**Mustafa is working on a space project to make a cuboid with the volume as the difference of 2 cubes of sides $x$ and y respectively. Help him to find:**

a. Expression for volume of the required cuboid.
b. Expression for any one side of required cuboid.
c. Expression for Area of any one surface of required cuboid.

**Solution:**

Mustafa is working with cubes of volumes:

Volume 1: $x^3$ Volume 2 : $y^3$

Difference of 2 volumes = $x^3 - y^3$

a. Volume of required cuboid = $x^3 - y^3$

b. $x^3 - y^3 = (x-y)(x^2+xy+y^2)$ (by factorizing)

The volume of a cuboid is factorized in one linear and one quadratic factor. So the expression for length is "$x-y$"

c. The quadratic factor in above factorization represents Area of a surface. i-e Area of one surface is $x^2 + xy + y^2$.

**Example 34:**

Volume of a cubical container is given by expression $(x^3 - 6x^2y + 12xy^2 - 8y^3)\ m^3$. Find:

a. Expression for length of each side
b. Expression for area of the base
c. Expression for total surface area
d. Expression for cost of painting all outer surfaces at the rate of Rs.50/$m^2$.

**Solution:**

Volume of cube = $l^3 = x^3 - 6x^2y + 12xy^2 - 8y^3$
$= (x - 2y)^3$

a. Length of each side is $(x - 2y)$ m
b. Base area = $l^2 = (x - 2y)^2\ m^2$
c. Surface area = $6l^2 = 6\ (x - 2y)^2\ m^2$
d. Cost = rate × surface area = Rs. $50 \times 6 \times (x - 2y)^2$
= Rs. $300\ (x - 2y)^2$

## EXERCISE 4.8

1. In a map of an industry, expression of area of a rectangular veranda is given by $(x^2 - 2x - 3)m^2$. Find:
   a. expressions for both dimensions of veranda.
   b. expression for perimeter of veranda.
   c. expression for cost of fencing veranda @ Rs. 200/m.
   d. expression for the cost of carpeting veranda floor @ Rs. 250/m².
2. Area of a square shaped surface of a machine is given by the expression $(25x^2 - 30x + 9)m^2$. Find:
   a. expression for the length of the surface.
   b. expression for the boundary of the surface.
   c. expression for the cost of polishing the surface of machine @ Rs75/m².
   d. expression for the cost of edging around 2 sides of the machine surface @ Rs 28/m.
3. Volume of a cubical oil tank in an oil refinery is expressed as $(125x^3 - 150x^2 + 60x - 8)\ m^3$. Find:
   a. expression for height of oil tank.
   b. expression for surface area of oil tank.
   c. expression for painting it from outside @ Rs32/m².
4. A mechanical engineer working on wheels of a machine, finds their areas given by $A_1 = \pi x^2 - 6\pi x + 9\pi$ and $A_2 = \pi x^2 - 10\pi x + 25\pi$. Help him find radii of both wheels.
5. A machine has two squared shape pressers with areas expressed by $25\ m^2$ and $36n^2$ respectively. Difference of these areas describes a rectangular presser. Find dimension of the rectangular pressers.
6. Distance covered by a missile to hit the target is given by expression $(x^2 + 5x + 6)$m. Find:
   a. the possible expression for speed of missile
   b. the possible expression for time to reach the target

## KEY POINTS

- Factorization of expressions of the following types.

  Type-I: $ka + kb + kc = k(a + b + c)$

  Type-II: $ac + ad + bc + bd = (a + b)(c + d)$

  Type-III: $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$

  Type-IV: $a^2 - b^2 = (a + b)(a - b)$

  Type-V: $a^2 + 2ab + b^2 - c^2 = (a + b + c)(a + b - c)$ and

  $a^2 - 2ab + b^2 - c^2 = (a - b + c)(a - b - c)$

  Type-VI: $a^4 + a^2b^2 + b^4 = (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

  $a^4 + 4b^4 = (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$

  Type-VII: $x^2 + px + q$ (factorize it write p as the sum of the factors of q)

  Type-VIII: $ax^2 + bx + c$ (factorize it write b as the sum of the factors of ac)

  Type-IX:
  - $(ax^2 + bx + c)(ax^2 + bx + d) + k$
  - $(x + a)(x + b)(x + c)(x + d) + k$ (where $a + b = c + d$)
  - $(x + a)(x + b)(x + c)(x + d) + kx^2$

  Type-X: $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$ and $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$

  Type-XI: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

- HCF of two or more polynomials is a highest degree polynomial which divides the given polynomials exactly.
- LCM of two or more polynomials is a least degree polynomial which is exactly divisible by the given polynomials.
- Product of two polynomials P and Q = Product of their HCF and LCM i.e.

  $P \times Q = \text{HCF} \times \text{LCM}$
- An algebraic expression of the form $\frac{P}{Q}$ where P and Q are two expressions and $Q \neq 0$ is called an algebraic fraction.
- A fraction having rational expression in its denominator or numerator or both, is called a complex fraction.
- In case of division, after converting the fractions into multiplication form, follow the same rules for

  simplification as applied in the product of algebraic fractions.
- Use 'DMAS' rule while simplifying the algebraic fractions having more than one operation.
- Square root of an algebraic expression is defined as one of its equal factors.
- Square root of an algebraic expression 'P' is another expression 'Q' which, when squared, gives 'P'

  i.e., if $P = (\pm Q)^2$ then, $+Q$ and $-Q$ both are square roots of P.

## MISCELLANEOUS EXERCISE 4

**1. Encircle the correct option in the following.**

(i) Factors of $-2 - a + a^2$ are

(a) $(a-2)(a-1)$ (b) $(a+1)(a+2)$ (c) $(a+2)(a-1)$ (d) $(a+1)(a-2)$

(ii) Factorization of $x^2 - x + \frac{1}{4}$ is

(a) $\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)$ (b) $\left(x+\frac{1}{2}\right)(x-1)$

(c) $\left(x-\frac{1}{2}\right)(x+1)$ (d) $\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2}\right)$

(iii). $(x^2+m^2)(x+m)(x^4+m^4)(x-m)$ is the factored form of

(a) $x^4 - m^4$ (b) $x^8 + m^8$ (c) $x^8 - m^8$ (d) $x^4 + m^4$

(iv). $a^4 + 64b^4$ is the product of

(a) $a^2 - 4ab + 8b^2$ and $a^2 + 4ab + 8b^2$ (b) $a^2 + 8b^2 + 4ab$ and $a^2 - 8b^2 + 4ab$

(c) $(a^2 + 8b^2)^2$ (d) none of these

(v). $x(2a+3b+7) + x(2a+3b+5)$ equals

(a) $2x(2a+3b+6)$ (b) $4x(2a+3b+6)$

(c) $x(2a+3b+7)(2a+3b+5)$ (d) $(x+x)(2a+3b+12)$

(vi) Which is the highest common factor of $-12x^2y^2$, $6xy^3$, $24x^2y^2$?

(a) $-6xy$ (b) $6x^2y^3$ (c) $6xy^2$ (d) $-24x^2y^3$

(vii) What is the least common multiple of $12x^2y^2$, $6xy^3$, $24x^2y^2$?

(a) $6xy$ (b) $6x^2y^3$ (c) $6xy^2$ (d) $24y^3x^2$

(viii) What is the highest common factor of $7x - 6xy$ and $5xy^3 - 3x^2$?

(a) $(7-6y)(5y^3-3x)$ (b) $(7x-6xy)(5y^3x-3x^2)$

(c) $x$ (d) $x(7-6y)(5y^3-3x)$

(ix) Least common multiple of $7x - 6xy$ and $5y^3x - 3x^2$ is:

(a) $(7-6y)(5y^3-3x^2)$ (b) $(7x-6xy)(5y^3-3x^2)$

(c) $x$ (d) $x(7-6y)(5y^3-3x)$

(x) HCF of $7x^3 - 8y^3$ and $3x^3 - 5y^3$ is:

(a) 1 (b) $(7x^3-8y^3)(3x^3-5y^3)$

(c) $7x^3 - 8y^3$ (d) $3x^3 - 5y^3$

(xi) LCM of $7x^3 - 8y^3$ and $3x^3 - 5y^3$ is:

(a) 1 (b) $(7x^3 - 8y^3)(3x^3 - 5y^3)$

(c) $7x^3 - 8y^3$ (d) $3x^3 - 5y^3$

(xii) What is the product of $\frac{uv^2}{3w^3}$ and $\frac{6w^4}{u^2v^3}$?

(a) $\frac{uv^2}{3w^2}\times\frac{u^2v^3}{6w^4}$ (b) $\frac{2w}{uv}$ (c) $\frac{uv}{2w}$ (d) both a & c

(xiii) What is the quotient of $\frac{3y^2}{10}\div\frac{y^3}{2}$?

(a) $\frac{3y^5}{20}$ (b) $\frac{3}{5y}$ (c) $\frac{3y^2}{10}\div\frac{2}{y^3}$ (d) $\frac{5y}{3}$

(xiv) What is the sum of $\frac{2a}{a^2-1}$ and $\frac{-a}{a^2-1}$?

(a) $\frac{3a}{a^2-1}$ (b) $\frac{a}{a^2-1}$ (c) $\frac{2a-a}{(a^2-1)+(a^2-1)}$ (d) $\frac{-2a}{a^2-1}$

(xv) What is the difference of $\frac{-3x}{x+y}$ and $\frac{x}{x+y}$?

(a) $\frac{-2x}{x+y}$ (b) $\frac{-3x-x}{2x+2y}$ (c) $\frac{3x^2}{x+y}$ (d) $\frac{-4x}{x+y}$

(xvi) If product of two polynomials is $(a-b)^2(a^2+ab+b^2)$ and their HCF is $a-b$, what is their LCM?

(a) $a^3 - b^3$ (b) $(a-b)^3(a^2+ab+b^2)$

(c) $(a-b)^2(a^2+ab+b^2)$ (d) $a^2+ab+b^2$

(xvii) If product of HCF and LCM of two polynomials is $(x^3 - y^3)(x + y)$ then what will be the product of these polynomials?

(a) $x^4 - y^4$ (b) $(x-y)(x^2+y^2)$

(c) $(x^2-y^2)(x^2+xy+y^2)$ (d) $(x^2-y^2)(x^2-xy+y^2)$

(xviii) What is the square root of $36x^6y^{16}$?

(a) $6x^6y^{16}$ (b) $6xy$ (c) $6x^3y^8$ (d) $16x^3y^8$

(xix) What is the square root of $(15x^2 - 7y^2)^4$ ?

(a) $\pm(15x^2 - 7y^2)^2$ (b) $\pm(15x^2 - 7y^2)$

(c) $\pm(15x - 7y)^2$ (d) $\pm(15x - 7y)$

(xx) What is the square root of $\left[-\left(2x+\frac{1}{x}+1\right)\right]^2$ ?

(a) $\pm\left(2x+\frac{1}{x}+1\right)^2$ (b) $\left(2x+\frac{1}{x}+1\right)^2$ (c) $2x+\frac{1}{x}+1$ (d) $\sqrt{2x+\frac{1}{x}+1}$

Factorize the following.

**2.** $(a^2-5)^2-13(a^2-5)+36$

**3.** $6x^2+19x+15$

**4.** $(x+1)(x-3)(x-5)(x-9)+44$

**5.** $x^4-12x^2+16$

**6.** $(3x^2+4x-5)(3x^2-2+4x)-4$

**7.** $2m^4+m^2n^2-3n^4$

**8.** $x^4+10x^2y^2-56y^4$

**9.** $x^2+2ax-bx-2ab$

**10.** $a^{12}-b^{12}$

**11.** $28x^4y+64x^3y-60x^2y$

**12.** $4(x-y)^3-(x-y)$

**13.** $x^3p^2-8y^3p^2-4x^3q^2+32y^3q^2$

**14.** $x^4+y^4-7x^2y^2$

**15.** Find the highest common factor of the following.

(i) $x^3+3x^2-8x-24$ , $x^3+3x^2-3x-9$

(ii) $3x^4-3x^3-2x^2-x-1$ , $9x^4-3x^3-x-1$

**16.** Find the LCM of the following.

(i) $2x^2+3x+1$ , $2x^2+5x+2$ , $x^2+3x+2$

(ii) $3x^2+11x+6$ , $3x^2+8x+4$ , $x^2+5x+6$

**17.** Find the HCF and LCM of the following expressions.

$a(a+c)-b(b+c)$ , $b(b+a)-c(c+a)$ , $c(c+b)-a(a+b)$

**18.** Find the square root of $\frac{9a^2}{x^2}-\frac{6a}{5x}+\frac{101}{25}-\frac{4x}{15a}+\frac{4x^2}{9a^2}$

**19.** Simplify the following (all the expressions in the denominator are non-zero).

(i) $\frac{x^2+x-2}{x^2-x-20}\times\frac{x^2+5x+4}{x^2-x}\div\left(\frac{x^2+3x+2}{x^2-2x-15}\times\frac{x+3}{x^2}\right)$

(ii) $\frac{1}{x+1}-\frac{1}{(x+1)(x+2)}+\frac{1}{(x+1)(x+2)(x+3)}$

**20.** Find the values of *a* and *b* if

$x^4+ax^3+bx^2-4x+4$ is a perfect square.