

Quantum entanglement is a phenomenon in which two or more particles become correlated in such a way that the state of one particle cannot be described independently of the others, even when they are separated by large distances. Two particles connected by a wavy line, with arrows indicating their correlation. Quantum entanglement has fascinating implications for quantum computing, cryptography, and our understanding of reality itself!

In this unit student should be able to:

- Describe the concept of black body radiation.
- Describe how energy is distributed over the wavelength range for different temperatures.
- Describe Planck's hypothesis that radiation emitted and absorbed by the walls of a black body cavity is quantized.
- Elaborate the particle nature of electromagnetic radiation.
- Solve problems by using $E = hc/\lambda$
- Describe the phenomenon of photoelectric effect.
- Explain photoelectric Effect on the basis of quantum theory.
- Solve the problems by using $hf - \phi = KE$.
- Identify data sources, gather, process and present information to summarize the use of the photoelectric effect in solar cells and photocells.
- Explain the particle model of light in terms of photons with particular energy and frequency.
- Describe Compton's effect qualitatively.
- Solve problems by using $\Delta\lambda = h/moc (1 - \cos\theta)$
- Explain the process of pair production on the basis of conservation laws.
- Describe conservation laws in the annihilation of matter.
- Describe the impact of de Broglie proposal that any kind of particle has both wave and particle properties.
- Describe the confirmation of de Broglie proposal by Davisson and Germer experiment in which the diffraction of electrons by the surface layers of a crystal lattice was observed.
- Explain how the very short wavelength of electrons, and the ability to use electric and magnetic fields to focus them, allows electron microscope to achieve very high resolution.
- Search and describe the role of electron microscope to study the micro structures and properties of matter.
- Describe uncertainty principle.

Introduction:

Quantum physics is the study of matter and energy at atomic and sub-atomic level. It aims to discover the properties and behaviors of the very building blocks of nature. At these scales, the classical laws of physics no longer apply, and strange, seemingly random phenomena govern the behavior of particles. In early 20th century, the field of quantum physics was developed as a result of certain experimental discoveries which could not be described by classical physics. The fundamental discovery of quantum physics was that energy is radiated as discrete packets, or quanta. Quantum physics explores the principles of wave-particle duality, uncertainty, superposition and quantization, which form the basis of our understanding of the behavior of matter, energy, and the fabric of reality itself.

25.1 Quantum Theory of Radiation:

It was introduced by Max Planck and Albert Einstein. It explains how energy is emitted and absorbed in discrete packets (quanta), the key points of Quantum theory of radiation are:

- Energy is quantized (small packets, not continuous)
- Frequency (not amplitude) determines energy of radiation
- Radiation has both wave and particle properties (duality)

This theory revolutionized our understanding of light and radiation!

25.1.1 Black Body Radiation:

A black body is an idealized object which absorbs all incident radiation and also emits it in a continuous spectrum of colors, depending upon its temperature as illustrated in fig 25.1. This evidence encouraged physicists to explain the mechanisms through which entities like stars and heated metals produce both light and heat.

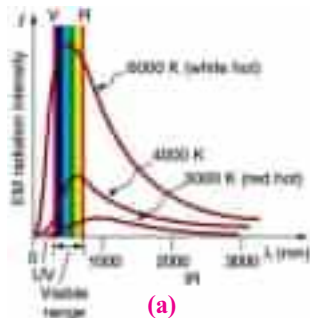


Figure 25.2 (a, b) Radiation intensity and red-hot horse

The spectrum of light emitted by heated objects was unexplained until 19th century. At normal temperature, we are not aware of this electromagnetic radiation due to low intensity. At higher temperatures, infrared radiation is emitted that we can feel heat if we are close to the object. At higher temperatures, objects actually glow, such as red-hot horse-shoe as shown in figure 25-2 (b). At temperatures more than 2000 K, objects glow with a yellow or whitish



Figure 25.1
Black body Radiation

color, such as white-hot iron. The light emitted contains a continuous range of wavelengths or frequencies, and the spectrum is a plot of intensity versus wavelength or frequency. As the temperature increases, the electromagnetic radiation emitted by objects not only increases in total energy but has its peak intensity and higher frequencies. Figure 25.2 Graphs of blackbody radiation (from an ideal radiator) at three different radiating temperatures.

The intensity or rate of radiation emission increases directly with temperature, and the peak of the spectrum shifts toward the visible and ultraviolet parts of the EM spectrum. The shape of the spectrum cannot be described with classical physics.

Some experimental facts about blackbody radiations:

- The blackbody spectrum depends only on the temperature of the object, and not on what it is made of i.e., material. An iron horseshoe, a ceramic vase, and a piece of charcoal --- all emit the same blackbody spectrum if their temperatures are the same. As the temperature of an object increases, it emits more intense blackbody radiations of all wavelengths.
- As the temperature of an object increases, the peak wavelength of the blackbody spectrum curve shifts towards shorter wavelength (Blue-shift). For example, blue stars are hotter than red stars.
- The blackbody spectrum peak shifted always becomes small at the left-hand side i.e., the short wavelength, high frequency.

25.1.2 Classical explanation of black body radiation:

Rayleigh-Jeans Law:

This Law states that the energy per unit volume per unit wavelength of blackbody radiation is inversely proportional to the fourth power of the wavelength (λ), not directly proportional to the square of the wavelength.

The law is given by the formula:

$$E(\lambda, T) = \frac{2ckT}{\lambda^4}$$

Mathematically expressed as:

$$E(\lambda, T) = \frac{\text{Constant}}{\lambda^4} \dots\dots 25.1$$

where:

- $E(\lambda, T)$ is the spectral radiance
- λ is the wavelength,

The failure of the Rayleigh-Jeans Law highlighted the need for a new theoretical framework, eventually leading to Max Planck's development of the quantum theory of radiation in 1900. Planck's work successfully resolved the ultraviolet catastrophe and laid the foundation for quantum physics.

- The graph in figure depicts that for large

DO YOU KNOW?

"Ultraviolet catastrophe," where the energy output would become infinite as the wavelength approached zero. This contradicted experimental observations and posed a significant challenge to classical physics.

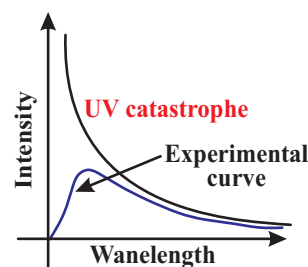


Figure 25.3 ultraviolet catastrophe

wavelengths it fitted the experimental data but it had major problems at shorter wavelengths.

Wien's Displacement Law:

This law breaks down completely at low frequencies. Wien's displacement law states that the wavelength at which the intensity of blackbody radiation is maximum is inversely proportional to the temperature of the blackbody. As the temperature increases, the peak of the blackbody radiation curve shifts to shorter wavelengths. This relationship is mathematically expressed as:

$$\lambda_{\max} T = \text{Constant}$$

$$\lambda_{\max} = \frac{\text{Constant}}{T} \dots\dots 25.2$$

Where λ_{\max} is the wavelength at which a blackbody radiates at a given temperature T.

Stefan's Law:

The experimental relation is known as Stefan Boltzmann's law; it states that the total energy radiated per unit surface area of a black body is proportional to the fourth power of its temperature.

Mathematically, it is expressed as:

$$P = \sigma AT^4 \dots\dots\dots 25.3$$

Where A is the surface area of a blackbody, T is its temperature and σ is the Stefan's-Boltzmann constant,

$$\sigma = 5.67 \times 10^{-8} \text{ W / (m}^2 \cdot \text{K}^4\text{)}.$$



Stefan's law

It enables us to estimate how much energy a star is radiating by remotely measuring temperature.

25.1.3 Planck's Hypothesis:

According to Planck's Hypothesis, the energy levels of the oscillators in the walls of a blackbody are quantized, meaning they can only have specific, discrete values. This quantization of energy levels is what leads to the characteristic spectrum of blackbody radiation, which Planck's Law describes accurately.

Planck proposed that electromagnetic energy could only be emitted or absorbed in discrete packets, which he called "quanta." This assumption led to the concept of energy levels being quantized, meaning that the energy could only take on certain discrete values. The energy (E_n) of an oscillator in the cavity walls is given by the equation:

$$E_n = nhf \dots\dots\dots 25.4$$

where: E_n is the energy of the oscillator is an integer (1, 2, 3, ...), h is Planck's constant, f is the frequency of the oscillator.

This relationship, now known as Planck's Law, marked the birth of quantum theory, fundamentally changing our understanding of energy and leading to the development of quantum mechanics

An oscillator in the wall can receive energy from the radiation in the cavity (absorption), or it can emit energy to the radiation in the cavity (emission). The absorption process sends the oscillator to a higher quantum state, and the emission process sends the oscillator to a lower quantum state. Whichever way this exchange of energy goes, the smallest amount of energy allowed to exchange is $h\nu$. There is no upper limit to how much energy can

be exchanged, but whatever is exchanged must be an integer multiple of $h\nu$. If the energy packet does not have this exact amount, it is neither absorbed nor emitted at the wall of the blackbody.



Self-Assessment Questions:

1. How does the energy distribution of black body radiation change with temperature? Describe the relationship between temperature and the peak wavelength of radiation.
2. Describe the ultraviolet catastrophe problem in the context of black body radiation. How did Max Planck's quantum hypothesis resolve this issue?

Worked Example 25.1

At the Earth's surface, the energy flux in sunlight is $1.02 \times 10^3 \text{ watt/m}^2$. If a black sheet of paper faces the Sun, what is the equilibrium temperature of the paper? Assume that the bottom of the paper is insulated, so the only heat loss is by blackbody radiation from the top surface.

Solution:

Step 1: Energy = $P / A = 1.02 \times 10^3 \text{ watt/m}^2$

$T = ?$

$\sigma = 5.67 \times 10^{-8} \text{ watt/m}^2 \cdot \text{K}^4$

Step 2: Write down the formula and rearrange if necessary.

$E = \sigma T^4$

Step 2: Put the values in above formula

$$\sigma T^4 = 1.02 \times 10^3 \quad T = \left[\frac{1.02 \times 10^3}{5.67 \times 10^{-8}} \right]^{1/4}$$

$T = 362 \text{ K}$ or

$T = 90^\circ \text{C}$

Result: $T = 90^\circ \text{C}$

25.1.4 Particle Nature of Electromagnetic Radiation:

Electromagnetic radiation can be described in terms of a stream of **mass-less particles, called photons**. Each photon contains a certain amount of energy. The different types of radiation are defined by the amount of energy found in the photons.

The particle nature of light states that light consists of particles called 'Photons' (Particles do not interfere) That is, when the space is occupied by some particle, other particles cannot occupy the same space. Experiments from the last century, such as the **photoelectric effect** and **atomic line spectra**, can only be explained if EM radiation is assumed to behave as particles. The energy of a photon depends upon the frequency as described in Planck's hypothesis.

$$E = h\nu$$

$$\text{But } f = \frac{1}{t} \text{ and } V = \lambda \nu$$

$$c = \lambda \nu$$

$$E = \frac{h\nu}{\lambda} \dots\dots\dots 25.5$$

h = Planck's constant = $h = 6.626 \times 10^{-34}$

c = Speed of light = 3×10^8 m/s

λ = Wavelength of Photon particle

In EM spectrum as you move right wards the wavelength decreases and frequency increases; consequently, energy increases.

Worked Example 25.2

Calculate the energy of a photon with a frequency of 3×10^{18} Hz. Analyze the question Determine the energy of a photon given the frequency $f = 3 \times 10^{18}$ Hz. The frequency is in standard units and we know the relationship ($E = hf$) between frequency and energy.

Solution:

Step 1: given data

$$f = 3 \times 10^{18} \text{ Hz}$$

Energy = ?

Step 2: Write down the formula and rearrange if necessary.

$$E = hf$$

Step 2: Put the values in above formula

$$E = hf$$

$$= 6.63 \times 10^{-34} \text{ J} \cdot \text{s} \times 3 \times 10^{18} \text{ Hz} = 2 \times 10^{-15} \text{ J}$$

Result: $E = 2 \times 10^{-15} \text{ J}$



Self-Assessment Questions:

- In a microwave oven photon of energy $1.05 \times 10^{-5} \text{ eV}$ is used to heat food.
 - Calculate the frequency of the photons.
 - Calculate the wavelength of the photons.

25.2 Photoelectric Effect:

- In 1887, Heinrich Hertz observed the photoelectric effect in Germany, discovering that ultraviolet (UV) light shining onto metal surfaces causes the ejection of electrons, which are pushed with potential to reach an anode.
- In 1899, J.J. Thomson in England demonstrated that the incidence of UV light on a metal surface causes the ejection of electrons.
- In 1902, Philip Lenard measured the photoelectric effect quantitatively and concluded that light has electrical properties.

Though these experiments were able to describe phenomenon of photoelectric effect but some situations during experiments were not answered with scientific logic.

In 1905, Albert Einstein was able to resolve the phenomenon. Photoelectric effect is defined as:

When electromagnetic radiation like light is shined on certain metallic materials; electrons are emitted due to absorption of light by the electrons on the surface of material.

25.2.1 Photoelectric Effect Phenomenon:

The photoelectric phenomenon is shown in figure 25.4 in which an evacuated tube containing two electrodes cathode and anode; connected with a variable source voltage V . A monochromatic light source is shined onto cathode through a quartz window and anode (collecting electrode) is connected either positive or negative potential with respect to the cathode. An ammeter is connected with circuit to record the current due to photo-electrons.

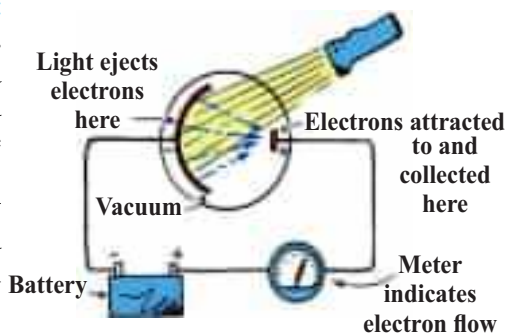


Figure 25.4 Photoelectric Effect

Following observations were made during this experiment:

1. For a constant potential difference between the cathode and anode, the number of electrons emitted from cathode increases with increasing intensity of radiation.
2. For a constant intensity and frequency of incident radiation the photoelectric current varies with the potential difference V between the cathode and anode and reaches a constant value beyond which further increase of potential difference does not affect the photoelectric current, instead, if the anode is made more and more negative with respect of the photocathode surface the current decreases. This negative potential (with respect to cathode) of the plate is called **retarding potential**. As shown in fig: 25.5. For a particular value of retarding potential, the photoelectric current becomes zero.

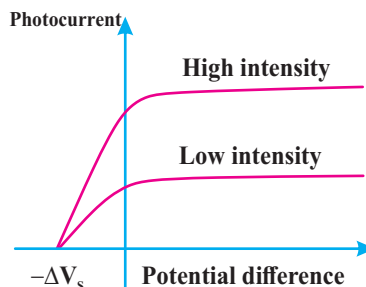


Figure 25.5
graph of photo current vs potential

This potential is called **cut-off or stopping potential** V_0 and is measure of maximum kinetic energy of

photo-electrons and we can write

$$K.E_{max} = eV_0$$

where $K.E_{max}$ (the maximum kinetic energy of the ejected electron.)

3. The stopping potential and hence the maximum kinetic energy $K.E_{max}$ of photo-electrons is independent of the intensity of incident radiation and depends only on the frequency ν of radiation.
4. For each substance there exists a characteristic frequency ν_0 such that for radiation with frequency below the photo-electrons are not ejected from the surface. This frequency is called the **threshold frequency** and the corresponding wavelength is called **Threshold wavelength**, $\lambda_0 \left(\frac{hc}{\phi} \right)$
5. The time lag between the incidence of radiation and the emission of a photoelectron is very small, less than 10^{-9} seconds.

**DO YOU
KNOW?**

Einstein was awarded the 1921 Nobel prize in physics for his work on the photoelectric effect.

25.2.2 Explanation of photo-electric effect on Quantum Theory:

Einstein proposed that radiation energy is not continuously distributed over the wave-front, but the light energy consists of discrete quanta of energy $h\nu$, which penetrates the surface of the cathode, all of its energy is transformed to an electron; it's depicted in figure 25.6. The photon's energy would be associated with its frequency ν , through a proportionality constant known as Planck's constant h , or alternately, using the wavelength λ and the speed of light c :

$$K.E = h\nu = \frac{hc}{\lambda}$$

or the momentum equation:

$$p = \frac{h}{\lambda}$$

When a photon is incident on the surface of a material, some of its energy is spent in making the electron free and the rest appears as kinetic energy of the electron. The electrons at the surface of the material are most loosely bound and require minimum energy for their liberation. This energy is called the **work function** ϕ of the material. The maximum kinetic energy of photo electrons ejected from the surface is given by

$$K.E_{\max} = \frac{1}{2}mv^2 = h\nu - \phi \quad \dots\dots\dots 25.6$$

So, **work function is the minimum energy that must be supplied to the electron for it to leave the Metal surface. As shown in fig: 25.7.**

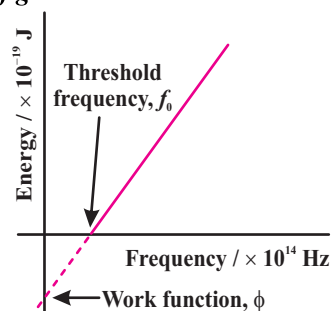


Figure 25.7 a minimum energy to escape electron from metal surface graph

Consequently there exists a minimum frequency that is independent of the intensity of light, below which electrons cannot be ejected from the metal. This proposal was proved by Millikan when in 1914 he published his results of the voltage required to stop photoelectrons ejecting from a metal surface.

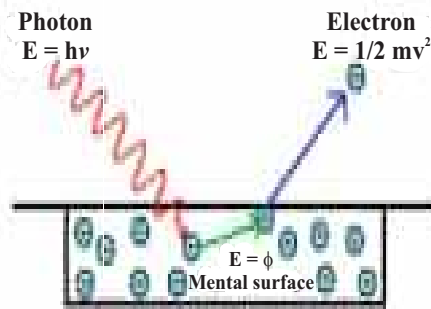


Figure 25.6 photo-electric effect

Table 25.1 Threshold frequencies value		
Metal	Work function ϕ	
Name	Symbol	eV
Sodium	Na	2.4
Calcium	Ca	2.8
Potassium	K	2.3
Copper	Cu	4.4
Silver	Ag	4.3
Lead	Pb	4.0
Platinum	Pt	6.4
Aluminum	Al	4.28
Gold	Au	5.10
Iron	Fe	4.7
Tungsten	W	4.55

**Self-Assessment Questions:**

1. How does the photoelectric effect demonstrate the particle nature of light?
2. Calculate the threshold frequency of incident radiation that will cause the emission of photo electrons from the surface of gold.

Worked Example 25.3

The work function for zinc is 4.24 eV. What is the threshold frequency for the ejection of photoelectrons from zinc?

Step 1: When an electron absorbs a photon at threshold frequency, it will just barely have enough energy to overcome the binding forces holding it in the metal and it will emerge with zero kinetic energy.

Step 2: Write down the formula and rearrange if necessary.

$$K.E_{max} = \frac{1}{2} mV^2 = h\nu - \phi$$

Step 3: First, we determine the frequency in terms of the wavelength.

$$\begin{aligned}
 K.E_{max} &= \frac{1}{2} mV^2 = h\nu - \phi \\
 0 &= h\nu - \phi_0 \\
 \nu &= \frac{\phi}{h} = \frac{4.24 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} \\
 \nu &= 1.02 \times 10^{15} \text{ Hz}
 \end{aligned}$$

Result: $\nu = 1.02 \times 10^{15} \text{ Hz}$

Photoelectric effect in solar cells and photocells:

Photovoltaic cells are made up of p-type and n-type silicon semiconductors. When light falls on the junction between p-type and n-type semiconductors, electrons are emitted according to the photoelectric effect. These electrons are collected in an external circuit to produce an electric current.

- Photocell, a device that uses the photoelectric effect to generate or control a current.
- Photovoltaic cells and photoconductive cells are examples of photoelectric cells.
- Solar cell, a device that converts electromagnetic energy into electrical energy.
- Photovoltaic cells are used to power a wide variety of devices, and are often used in conjunction with a battery that stores the power they produce.

25.3 Compton Effect:

A. H. Compton (1892-1962), aimed short-wavelength light (i.e., x-rays or γ -rays) at various materials, and detected light scattered at various angles. He observed that the scattered light had a slightly longer wavelength than did the incident light, and therefore a slightly lower frequency indicating a loss of energy. He explained this result based on the photon theory as incident photons colliding with electrons.

25.3.1 Particle Model of Photons with Energy and Frequency:

In Compton Effect experiment, there is increase in wavelength of photons, due to their scattering by an electron. The impact results in one of the fundamentals of quantum mechanics, which represents wave and particle properties of light.

Arthur Holly Compton successfully demonstrated that X-rays can be treated as discrete bundles, or quanta, of electromagnetic energy. This concept was later termed "photon" by American physicist Gilbert Lewis.

Photons exhibit both particle-like properties, such as energy and momentum, and wave-like properties, such as frequency and diffraction. The energy of a photon is dependent on its frequency, with lower energy photons having lower frequencies and longer wavelengths, as described by the equation $E = \frac{h\nu}{\lambda}$.

In Compton's experiment, photons collide with free or loosely bound electrons in matter. During these collisions, photons transfer part of their energy and momentum to the electrons, causing the electrons to recoil. This interaction results in the emission of new photons with lower energy and longer wavelengths, a phenomenon known as the Compton shift. The shift in frequency of the scattered photons depends on the energy transferred to the electrons and is independent of the initial wavelength of the incident photons.

25.3.2 Compton Effect Qualitatively:

Compton's Effect, as illustrated in Figure 25.8, can be qualitatively explained as follows:

Initial Interaction:

- A photon, which is a packet of electromagnetic energy, interacts with an electron in a material, typically a target like a metal or graphite.

Scattering Process:

- During the interaction, the photon transfers some of its energy and momentum to the electron. This transfer causes the photon to change its direction and wavelength (or equivalently, its frequency).

Change in Photon Energy:

- The scattered photon emerges with less energy (longer wavelength) than the initial incident photon. The amount of energy lost by the photon is directly related to the energy and momentum gained by the electron.

Quantum Nature:

- Compton's Effect cannot be explained using classical wave theory alone. Classical wave theory predicts that light should scatter uniformly without a change in wavelength. However, Compton's observations demonstrated that the scattered light has a shifted wavelength, indicating a particle-like interaction.

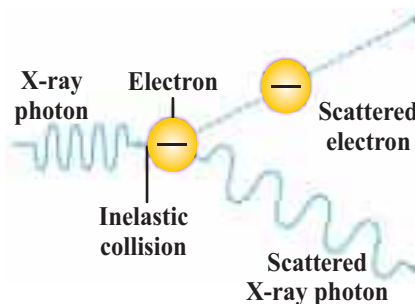


Figure 25.8 Compton Effect

Experimental Confirmation:

- Compton conducted experiments where X-rays were targeted at graphite and the scattered X-rays were observed. By measuring the scattering angle and change in wavelength, he confirmed that the results were consistent with the predictions of quantum theory.

Wavelength Shift:

- The wavelength shift observed in Compton scattering is directly proportional to the Compton wavelength, which depends on the mass of the electron and the Planck constant. This relationship provides crucial evidence for the particle nature of photons.

The Compton shift can be evolved as under:

$$\lambda_2 - \lambda_1 = \frac{hc}{E_o} (1 - \cos\theta) = \frac{h}{m_0c} (1 - \cos\theta)$$

$$\Delta\lambda = \frac{h}{m_0c} (1 - \cos\theta) \dots\dots\dots 25.7$$

This equation describes the phenomenon known as Compton Effect. $\Delta\lambda$ gives the change in photon wavelength due to scattering with a free electron and it is called **Compton shift**.

It's clear that the Compton Shift is independent of the wavelength of the incident photon and depends on scattering angle.

The term $\lambda_c = \frac{h}{m_0c} = 2.426 \times 10^{-12}m$, is called Compton Wavelength of the scattering particle i.e., electron.

**Self-Assessment Questions:**

1. Compare and contrast the photoelectric effect with the Compton effect and the Bohr model of the hydrogen atom in terms of their contributions to the development of quantum theory.
2. How does the Compton Effect demonstrate the particle nature of light?

Worked Example 25.4

A beam of x-rays of wavelength 0.01 \AA is incident on a block of graphite. What is the wavelength of the x-rays scattered at an angle of 30° ?

Step 1: given data and what to be found

$$\theta = 30^\circ$$

$$\lambda = 0.01 \text{ \AA}$$

Step 2: Write down the formula and rearrange if necessary.

$$\Delta\lambda = \frac{h}{m_0c} (1 - \cos\theta)$$

Step 3: $\Delta\lambda = 2.426 \times 10^{-12} (1 - \cos 30^\circ) = 0.003 \text{ \AA}$

Result: Hence, the final wavelength of these x-rays is $\lambda + \Delta\lambda = 0.01 + 0.003 = 0.013 \text{ \AA}$

25.4 Pair Production:

- Pair production is a phenomenon in quantum mechanics that occurs when a photon with sufficient energy interacts with matter and spontaneously transforms into a particle-antiparticle pair i.e., an electron and a positron, as shown in figure 25.9.

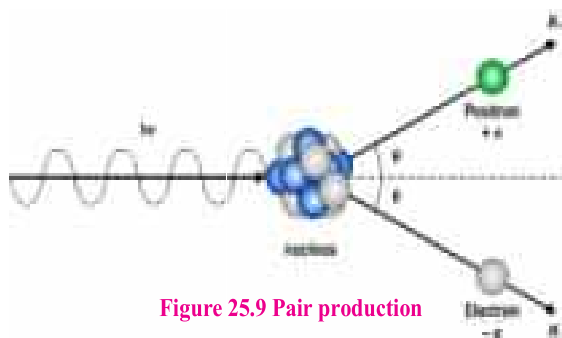
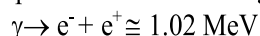


Figure 25.9 Pair production

25.4.1 Conservation laws of Pair-Production:

For pair production to occur, a photon must have a minimum energy of 1.022 MeV. This requirement arises because the rest mass of both an electron and a positron is 0.511 MeV each. Therefore, the combined energy needed to create an electron-positron pair is $0.511 \text{ MeV} \times 2$ (1.022 MeV); If the incident photon possesses energy exceeding 1.022 MeV, the surplus energy is distributed as kinetic energy between the electron and the positron.

Therefore, the pair production reaction is given as under:



Facts of pair production process are:

1. The pair production process obeys law of conservation of energy, momentum and electric charge respectively.
2. During collision, the antiparticle of an electron i.e., positron has the same physical properties as electron, except its charge, as both have opposite charge to each other. The sum of charges happens to be zero which is equal to photon before interaction. Therefore, **electric charge is conserved**.
3. The law of conservation of energy is
 - (a) $hf = (K.E)_{e^-} + (K.E)_{e^+}$
 - (b) $hf = 2m_0c^2 + (K.E)_{e^-} + (K.E)_{e^+}$

25.5 Annihilation of Matter:

Electron-positron annihilation occurs when an electron and a positron collide. The result of the collision is the conversion of the electron and positron to the creation of gamma-ray photons.

DO YOU KNOW?

“Pair production cannot take place in space!”

Pair Production Requires an External Object: Necessary for pair production to occur!

1. Absorbs recoil momentum
2. Conserves energy and momentum simultaneously

DO YOU KNOW?

Antimatter or Antiparticles:
Predicted by British Physicist Paul Dirac, discovered by Carl Anderson

- ✓ Same mass, opposite charge.
- ✓ Every particle has an antiparticle counterpart.
- ✓ Antiparticles have opposite:

Charge:

- Baryon Number
- Lepton Number
- Strangeness
- ✓ Made from anti-quarks (if particles are made from quarks)

25.5.1 Conservation laws of annihilation of matter:

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Each photon has an energy equal to the rest mass of electrons 0.51 MeV. The two photons are produced moving in opposite direction in order to conserve momentum and energy which forbids the creation of a single gamma-ray, as shown in figure 25.10. The charge is also conserved as net charge before and after is zero.

Pair production and Annihilation of matter supports that energy and mass are inter convertible i.e. $E=mc^2$.

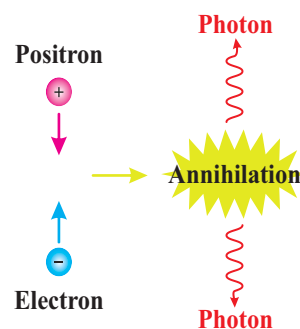


Figure 25.10 Annihilation of Matter

25.6 Wave Nature of Particles:

Classical physics traditionally treated particles and waves as separate and distinct entities.

However, the advent of quantum theory proposed a dual character for radiations, suggesting that they could exhibit characteristics of both waves and particles.

The wave nature of particles implies that particles can behave like waves, displaying properties such as reflection, refraction, interference, diffraction, and other wave-like characteristics. In essence, this duality suggests that matter and radiation can co-exist both particle and wave attributes.

Examples of particles that exhibit this dual nature include:

- (i) Matter particles, such as electrons, protons, and neutrons.
- (ii) X-rays.
- (iii) Photons.
- (iv) Electromagnetic radiation.

Particles, like above, can behave like waves. This means they can show wave-like properties, like diffraction and interference. This dual behavior (both wave-like and particle-like) is a fundamental principle of quantum mechanics, which helps us understand the behavior of matter and radiation at a tiny scale.

25.6.1 De Broglie wave:

Louis de Broglie's proposal of wave-particle duality for particles had a profound impact on the understanding of the nature of matter and laid the foundation for the development of quantum mechanics.

De Broglie's idea suggested that both matter and energy could exhibit both particle and wave properties. This unified description was a departure from the classical distinction between particles and waves, providing a more comprehensive understanding of the behavior of particles at the quantum level.

De Broglie's proposal was instrumental in the development of Niels Bohr's model of the atom. It helped explain why electrons in quantized orbits do not radiate energy continuously but exist in stable orbits, as their orbits are analogous to standing waves.

De Broglie Wavelength:

According to de Broglie hypothesis, **all matter particles like electrons, protons and neutrons in motion are associated with waves**. These waves are called de Broglie waves or matter waves. The momentum of photon of frequency f is given by

$$p = \frac{hf}{c} = \frac{h}{\lambda} \quad \text{since } c = v\lambda$$

The wavelength of a photon in terms of its momentum is

$$\lambda = \frac{h}{p} \quad \text{but } p = mv, \text{ then}$$

According to de Broglie, the above equation is completely applicable to matter particles as well. Therefore, for a particle of mass m travelling with speed v , the wavelength is given by

$$\lambda = \frac{h}{mv} \dots\dots\dots 25.8$$

This wavelength of the matter waves is known as de Broglie wavelength. This equation relates the wave character (the **wavelength**) and the particle character (the **momentum**) through 25-3, Planck's constant.

**Self-Assessment Questions:**

1. Calculate the de Broglie wavelength of an electron travelling through a space at a speed of 10^7 m/s. State whether or not these electrons be diffracted by solid materials (atomic spacing in solid materials is 10^{-10} m).
2. How does the De Broglie wavelength relate to the momentum of a particle?

25.6.2 Davisson – Germer Experiment:

De Broglie hypothesis of matter waves was experimentally confirmed by Clinton Davisson and Lester Germer in 1927. They demonstrated that electron beams are diffracted when they fall on crystalline solids. Since crystal can act as a three-dimensional diffraction grating for matter waves, the electron waves incident on crystals are diffracted off in certain specific directions. Figure 25.11 shows a schematic representation of the apparatus for the experiment.

The experimental parts are given as under:

1. An electron gun emits electrons via thermionic emission produced by the tungsten filament used in it, i.e., when heated to a specific temperature.
2. Two opposite charged plates known as Electrostatic Particle Accelerator, which accelerates the electrons at a certain potential.

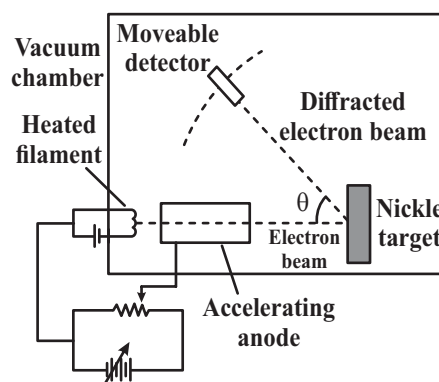


Figure 25.11 Davisson Germer Experiment

3. The accelerator is enclosed within a cylinder called Collimator which is a narrow passage for the electrons along its axis.
4. The target is a Nickel crystal on which the electron beam is fired normally.
5. When the electrons are scattered from Ni crystal, these are captured by the semicircular movable detector.

Davisson Germer Experiment and de Broglie Relation:

Let us consider an electron of mass (m_0), charge (e) accelerated from rest through a potential V . Then, the kinetic energy K . E of the electron equals to the work done (eV) on it by the electric field:

$$K. E = eV$$

$$\text{Now, } K. E = \frac{1}{2} m_0 v^2 = \frac{p^2}{2m_0}$$

$$p = \sqrt{2m_0 K. E} = \sqrt{2m_0 eV}$$

Then, the de Broglie wavelength of electron is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_0 eV}} \dots\dots\dots 25.9$$

By using $h = 6.626 \times 10^{-34}$ J-s, $m_0 = 9.11 \times 10^{-31}$ kg, and $e = 1.6 \times 10^{-19}$ C we get

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$$

Where V is the magnitude of accelerating potential in volts.

As, de Broglie wavelength is associated with electrons for $V = 54$ V from graph 25.12., then we have $\lambda = 0.167$ nm

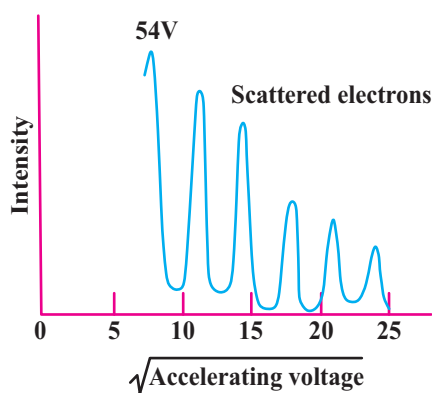


Figure 25 .12 de Broglie wavelength is associated with electrons graph

We conclude that this experiment confirms the wave nature of electrons and the de Broglie relation.

DO YOU KNOW?

Bragg's Equation

Electron Diffraction:

- Known: crystal spacing (d) from X-ray diffraction
- Measured: angles (θ) for diffraction peaks
- Calculated: wavelength (λ) of matter waves using: $n\lambda = 2d \sin(90^\circ - \theta/2)$
- Result: $\lambda = 0.165$ nm (wavelength of electron matter waves)

25.6.3 Electron Microscope:

De Broglie hypothesis are used in the field of electron optics. In the electron microscope design, the wave properties of electrons have been utilized with higher resolution, which is giving a great improvement in visualization.

A special type of microscope that uses an electron beam with its wavelike properties to illuminate a specimen able to magnify objects in high resolution (nanometers), which are formed by controlled use of electrons in a vacuum captured on a phosphorescent screen. **Ernst Ruska** (1906-1988), a German engineer and academic professor, built the first Electron Microscope (EM) in 1931, and the same principles behind his prototype still govern modern EMs.

De Broglie hypothesis of electron have wave paved the way for the development of the electron microscope (EM), which can produce images of much greater magnification than an optical microscope. There are two types of electron microscopes which are transmission electron microscope (TEM), which produces a two-dimensional image, and the scanning electron microscope (SEM), which produces images with a three-dimensional.

Working:

In both types i.e., TEM & SEM, the objective and eyepiece lenses are actually magnetic fields that exert force on the electrons to bring them to a focus. The fields are produced by carefully designed current-carrying coils of wire. EM's measure the intensity of electrons, producing monochromatic image. Color is often added artificially to highlight. The probe tip of scanning electron microscope, as it is moved horizontally, automatically moves up and down to maintain a constant tunneling current, and this motion is translated into an image of the surface.

Applications:

- Electron microscopes are used to investigate the ultra-structure of a wide range of biological and inorganic specimens including microorganisms, cells, large molecules, biopsy samples, metals, and crystals.
- Industrially, electron microscopes are often used for quality control and failure analysis.

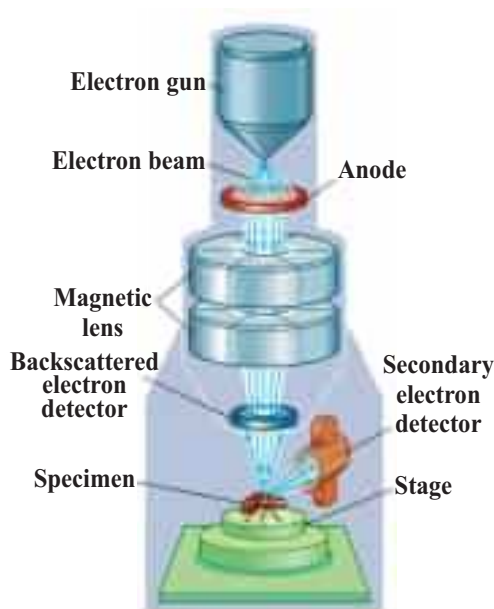


Figure 25.13 Electron Microscope

DO YOU KNOW?

Electron microscopes

As the wavelength of an electron can be up to 100,000 times shorter than that of visible light photons, electron microscopes have a higher resolving power than light microscopes and can reveal the structure of smaller objects.

- Modern electron microscopes produce electron micrographs using specialized digital cameras and frame grabbers to capture the images.
- The advancement of microbiology is significantly indebted to the electron microscope, which has revolutionized our understanding of microorganisms such as bacteria, viruses, and other pathogens, thereby greatly enhancing the effectiveness of disease treatments.

25.6.4 Role of Electron Microscope:

1. **Higher Resolution:** Electron microscope offers significantly higher resolution compared to optical microscopes, allowing observation at the nanometer scale due to the shorter wavelength of electrons.
2. **Transmission Electron Microscope (TEM):** TEMs study internal structures of thin specimens, producing detailed images of cells, organelles, and crystalline structures. Valuable in biology, materials science, and nanotechnology.
3. **Scanning Electron Microscope (SEM):** SEMs provide 3D surface images by scanning specimens and detecting emitted secondary electrons. Widely used in biology, geology, and materials science for surface analysis.
4. **Energy- Dispersive X-ray Spectroscopy (EDS):** With EDS detectors, electron microscopes analyze elemental composition by detecting X-rays emitted when high-energy electrons interact with the sample.
5. **Materials Science:** Crucial for studying the microstructure of materials, aiding understanding of relationships between microstructure and properties in metals, ceramics, and polymers.
6. **Nanotechnology:** Essential for imaging and characterizing nano-materials, contributing to the development of new materials and devices through observation and manipulation of nano-particles and nanostructures.
7. **Advancements in Medicine:** Contributes to medical research by providing insights into the structure of viruses, bacteria, and cellular organelles, aiding understanding of diseases and therapeutic development.
8. **Quality Control in Industry:** Used across industries for quality control and failure analysis, helping identify defects, analyze material composition, and ensure product integrity at the microscopic level.

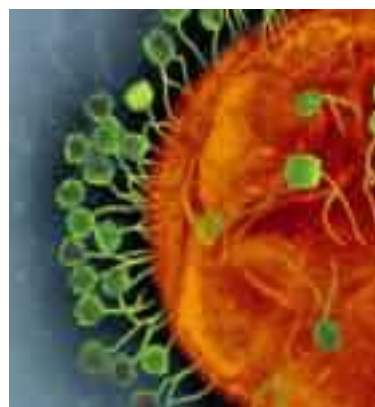


Figure 25.14 (a)
Viruses attacking a cell of bacterium
($\times 50000$) by TEM



Figure 25.14 (b)
Viruses attacking a cell of bacterium
($\times 35000$) by SEM

25.7 Uncertainty Principle:

In 1927, Werner Heisenberg suggested a principle that applies to measure the properties of quantum-sized objects (e.g., atomic and sub-atomic particles). The uncertainty principle mainly applied to experiments in physics labs, however some real-world effects are given. For example, in the sun's core, it is the uncertainty in the position of two hydrogen nuclei (protons) that allows for there to be a chance that they will overlap and fuse together. This fusing releases a tremendous amount of energy, which we receive as light. A pretty significant effect on our everyday lives.

25.7.1 Heisenberg Uncertainty Principle:

It states that it is impossible to measure simultaneously, certain pairs of properties of a subatomic particle, such as its position and momentum, with absolute certainty.

In other words the more precisely we know the position of a particle, the less precisely be able to know its momentum, and vice versa.

This principle develops from the wave-particle duality of particles in quantum physics, where particles can exhibit both wave-like and particle-like behaviors.

The formal inequality relating the standard deviation of position Δx and the standard deviation of momentum Δp are given here:

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \cdot \Delta p \geq \frac{h}{2} \dots\dots\dots 25.10$$

\hbar (pronounced "h-bar") is the reduced Planck constant
Heisenberg uncertainty principle can also be applied to other pairs of complementary quantum properties, such as energy and time

$$\Delta E \cdot \Delta t \geq \hbar/2 \dots\dots\dots 25.11$$

Here, ΔE and Δt represents the uncertainties in the energy and time respectively.

The Uncertainty Principle has profound consequences for understanding quantum world. It means that at the subatomic level, there are inherent limits to how precisely we can measure certain properties of particles. This is not due to restrictions in our measuring instruments but it is a fundamental aspect of the quantum nature of the universe.

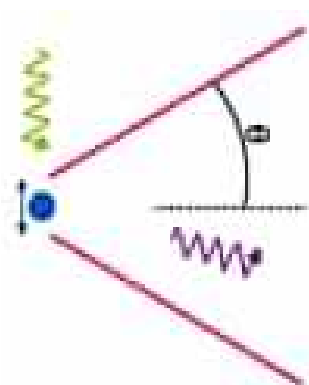


Figure 25.16 Uncertainty Principle

DO YOU KNOW?

Interesting Facts about Planck's Constant (h):

Universal constant: Appears in many areas of physics, like quantum mechanics, thermodynamics, and electromagnetism.

Tiny but mighty: Very small value (approximately 6.626×10^{-34} J s), but has a huge impact on our understanding of the universe.

Fundamental unit: Used to define the Planck units, a set of fundamental units for length, time, mass, and energy.

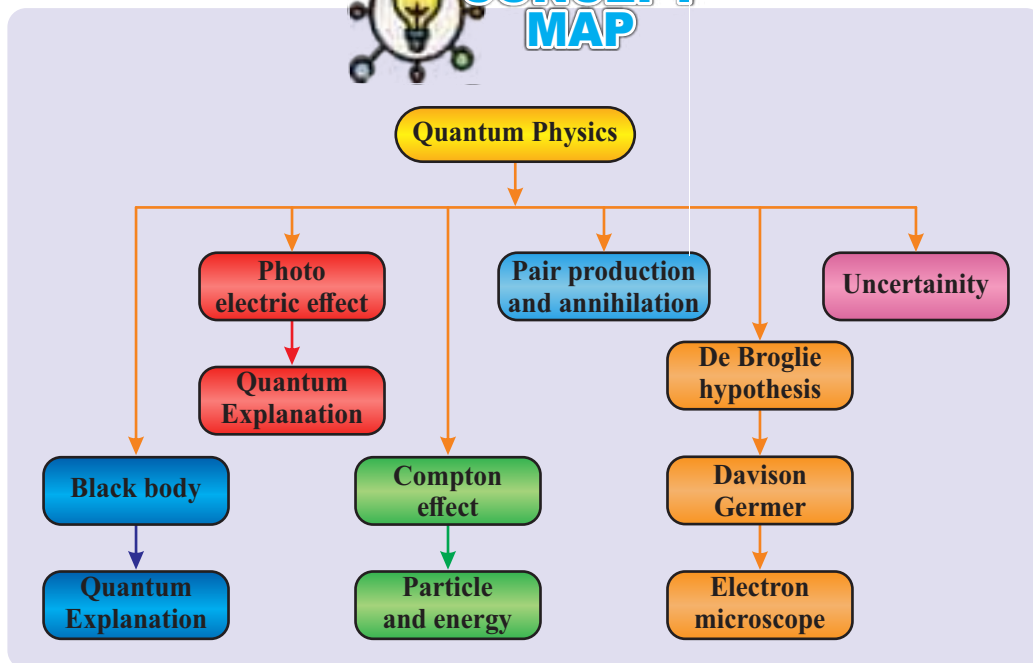


SUMMARY

- ✓ A black body absorbs all incident radiation and releases it in a continuous spectrum of colors, depending upon its temperature.
- ✓ The energy per unit volume per unit wavelength of black body radiation is directly proportional to the square of the wavelength and increases indefinitely as the wavelength becomes smaller.
- ✓ Different atoms and molecules can emit or absorb energy in discrete quantities only.
- ✓ When electromagnetic radiation like light is shinned on certain metallic materials; electrons are emitted due to absorption of light by the electrons on the surface of material.
- ✓ Work function, is the minimum energy that must be supplied to the electron for it to leave the metal.
- ✓ For each substance there exists a characteristic frequency f_0 such that for radiation with frequency below the photo-electrons are not ejected from the surface.
- ✓ Compton scattering, refers to X-ray or gamma-ray photons undergo a change in wavelength and momentum when they collide with charged particles, typically electrons.
- ✓ Change in wavelength gives the change in photon wavelength due to scattering with a free electron.
- ✓ When the photon interacts with the strong electric field around the nucleus, it undergoes a change of state and is transformed into two particles (creating matter from energy):
- ✓ Electron-positron annihilation occurs when an electron and a positron (the antiparticle of the electron) collide.
- ✓ All matter particles like electrons, protons and neutrons in motion are associated with waves.
- ✓ A microscope which uses a beam of accelerated electrons as a source of lighting.
- ✓ It is impossible to measure simultaneously all quantum properties of a particle with equal accuracy.



Quantum Physics



EXERCISE

Section (A): Multiple Choice Questions (MCQs)

Choose the correct answer:

- If h is the Planck constant, then \hbar is:
 - $2\pi h$
 - $2h$
 - $h/2$
 - $h/2\pi$
 - $2h/\pi$
- In a photoelectric effect experiment the stopping potential is:
 - the energy required to remove an electron from the sample
 - the kinetic energy of the most energetic electron ejected
 - the potential energy of the most energetic electron ejected
 - the electric potential that causes the electron current to vanish
- The work function for a certain sample is 2.3 eV. The stopping potential for electrons ejected from the sample by 7.0×10^{14} -Hz electromagnetic radiation is:
 - 0
 - 0.60 V
 - 2.3V
 - 2.9V
 - 5.2V
- In Compton scattering from stationary particles the maximum change in wavelength can be made smaller by using:
 - higher frequency radiation
 - lower frequency radiation
 - more massive particles
 - less massive particles

5. Of the following, Compton scattering from electrons is most easily observed for:
 - (a) infrared light
 - (b) visible light
 - (c) ultraviolet light
 - (d) X-rays
6. In the Compton scattering from stationary electrons the largest change in wavelength occurs when the photon is scattered through
 - (a) 0°
 - (b) 45°
 - (c) 90°
 - (d) 180°
7. A free electron has a momentum of $5.0 \times 10^{-24} \text{ kg} \cdot \text{m/s}$. The wavelength of its wave function is:
 - (a) $1.3 \times 10^{-8} \text{ m}$
 - (b) $1.3 \times 10^{-10} \text{ m}$
 - (c) $2.1 \times 10^{-11} \text{ m}$
 - (d) $2.1 \times 10^{-13} \text{ m}$
8. In Photoelectric Effect, when light fall on the surface of metal, the material should emit
 - (a) Electrons
 - (b) Protons
 - (c) Positrons
 - (d) Neutrons
9. Threshold frequency is defined as the frequency of incident light which can cause photo electric emission
 - (a) Maximum
 - (b) Minimum
 - (c) Average
 - (d) highest
10. The amount of energy which is necessary to start photo electric emission is called:
 - (a) Maximum
 - (b) Average
 - (c) Minimum
 - (d) Littlest

Section (B): CRQs (Short Answered Questions):

1. Differentiate between wave and particle.
2. Is it possible for the de Broglie wavelength of a particle?
3. Estimated Broglie wavelength of a cricket ball on the pitch?
4. Differentiate between the continuous and discrete emission of radiation?
5. What is threshold frequency?
6. How has the photoelectric effect been applied in real-world technologies or devices, and what are its practical implications?
7. What are the advantages of an electron microscope over an optical microscope?
8. Is it possible to create only an electron through matter and photon interaction?
9. Give construction of electron microscope?
10. Elaborate the particle nature of electromagnetic radiation.

Section (C): ERQs (Long Answered Questions):

1. Describe how energy is distributed over the wavelength range for different temperatures.
2. Explain the particle model of light in terms of photons with particular energy and frequency.
3. Describe conservation laws in pair production and annihilation of matter.
4. Describe Compton's effect qualitatively.
5. Explain how the very short wavelength of electrons, and the ability to use electrons and magnetic fields to focus them, allows electron microscope to achieve very high resolution.
6. Describe the impact of de Broglie proposal that any kind of particle has both wave and particle properties.
7. Describe the confirmation of de Broglie proposal by Davisson and Germer experiment in which the diffraction of electrons by the surface layers of a crystal lattice was observed.

Section (D): Numerical:

1. The Sun's surface temperature is 5700 K.
 - (i) How much power is radiated by the Sun?
 - (ii) Given that the distance to Earth is about 200 Sun radii, what is the maximum power possible from one square kilometer solar energy installation?
 - (iii) What is the wavelength of maximum intensity of solar radiation?
($5.98 \times 10^7 \text{ W/m}^2$, $3.6 \times 10^{26} \text{ Watts}$, $5.1 \times 10^{-7} \text{ m}$)
2. The temperature of your skin is approximately 32 °C. What is the wavelength at which the peak occurs in the radiation emitted from your skin? ($9.05 \times 10^{-5} \text{ m}$)
3. An FM radio transmitter has a power output of 100 kW and operates at a frequency of 94 MHz, How many photons per second does the transmitter emit?
($6.23 \times 10^{-26} \text{ J}$, $1.61 \times 10^{30} \text{ s}^{-1}$)
4. A light source of wavelength illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.0 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.0 eV. Determine the work function of the metal. (2 eV)
5. A 430 nm violet light is an incident on a calcium photo electrode with a work function of 2.71 eV. Find the energy of the incident photons and the maximum kinetic energy of ejected electrons. (2.88 eV , 0.17 eV)
6. Cut-off frequency for the photoelectric effect in some materials is $8 \times 10^{13} \text{ Hz}$. When the incident light has a frequency of $1.2 \times 10^{14} \text{ Hz}$, the stopping potential is measured as -0.16 V . Estimate a value of Planck's constant from these data and determine the percentage error of your estimation. (5.17 eV , 1.30 V)

7. The work function of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is:

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ in eV	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

(4 Metals)

8. X-rays with an energy of 300 keV undergo Compton scattering with a target. If the scattered X-rays are detected at 30° relative to the incident X-rays, determine the Compton shift at this angle, the energy of the scattered X-rays, and the energy of the recoiling electron.

(0.35 pm, 278keV, 22keV)

9. A photon with a wavelength of 6.0×10^{-12} m collides with an electron. After the collision the photon wavelength is found to have been changed by exactly one (Compton Wavelength is 2.43×10^{-12} m).

- What is the photon's wavelength after collision?
- Through what angle has been deflected in this collision?
- What is the angle for the electron after the collision?
- What is the electron's kinetic energy, in eV, after collision?

(8.4×10^{-12} m, 90° , $< 90^\circ$, 5.9×10^4 eV)

10. Find the de Broglie wavelength of an electron in the ground state of hydrogen.

(3.324 Å)

11. Determine the minimum uncertainties in the positions of the following objects if their speeds are known with a precision of 1.0×10^{-3} m/s: (a) an electron and (b) a bowling ball of mass 6.0 kg.

(5.8cm, 33 m)