Unit 24 Relativity

Teaching Periods 07 **Weightage %** 05

The image depicts a black hole's gravitational pull causing time dilation, where time appears to slow down near the event horizon. A clock near the black hole is shown with slower-moving hands, while a clock farther away remains normal, illustrating the relative nature of time. The black hole's warped spacetime is represented by a swirling, curved background, and a nearby planet or object is shown in a distorted, time-dilated state, highlighting the extreme gravitational effects on time and space.

In this unit student should be able to:

- Describe Relative Motion with suitable examples (same and opposite direction)
- Distinguish between inertial and non-inertial frames of reference
- Predict the motion of an object relative to a different frame of reference e.g. dropping a ball in a moving vehicle observed from the vehicle and by a person standing on the side walk.
- Analyze and evaluate the evidence confirming or denying Einstein's two postulates
- Identify that if c is constant then space and time become relative.
- Explain qualitatively and quantitatively the consequence of special relativity in relation to the relativity of simultaneity length contraction time dilation mass increase the equivalence between mass and energy
- Discuss the limitation on the maximum velocity of a particle imposed by special relativity
- Explain the implications of mass increase, time dilation and length contraction for space travel.
- Identify the role of special theory of relativity in global positioning, NAVSTAR system.
- Solve problems using $\Delta t = \frac{\Delta t_o}{\sqrt{1-\frac{v^2}{c^2}}}$, $L = L_o\sqrt{1-\frac{v^2}{c^2}}$, $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$ and $E_o = mc^2$
- Describe the general relativity
- Understand gravity as space time continuum

Introduction:

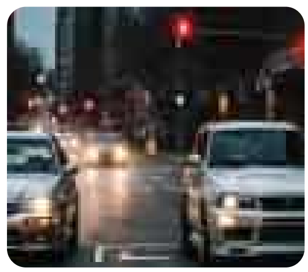
Relativity, a fundamental theory in physics developed by Albert Einstein, revolutionized our understanding of space, time, and energy. It consists of two main parts: Special Relativity and General Relativity. Special Relativity, and the idea that time and space are relative, varying according to the observer's motion. General Relativity, introduced extends these ideas to include gravity, describing it not as a force but as the curvature of spacetime caused by mass and energy. These theories have profound implications for our understanding of the universe, influencing modern physics, cosmology, and even technologies like GPS.

Have you ever been in a car at a stoplight and felt like your car was rolling backward because the car next to you started moving forward? This sensation can be disorienting and is another everyday example of relative motion. How can we describe the motion of objects accurately when the observer's perspective changes?

By exploring relative motion, we can better understand and describe the motion of objects from various perspectives, whether they are in the same frame of reference or moving relative to one another. This chapter will delve into the principles of relative motion, providing a foundational understanding that is essential for studying physics. Through examples, case studies, and thought-provoking questions, we will uncover how motion is not absolute but relative to the observer's point of view.

DO YOU KNOW?

"Albert Einstein's theory of relativity (1905, 1916) transformed our understanding of space, time, and physics. This chapter covers the key concepts of Special Relativity and General Relativity, two groundbreaking theories that revolutionized our understanding of the universe."



24.1 Frame of References

A reference frame is a fundamental concept that helps us to describe and understand the relative motion of objects and events. It consists of a set of coordinate axes (usually x, y, and z axes) and a clock, allowing observers to precisely define where and when events occur.

A rigid framework (usually x, y and z-axes) relative to which the position and motion of an object can be measured is called frame of references.

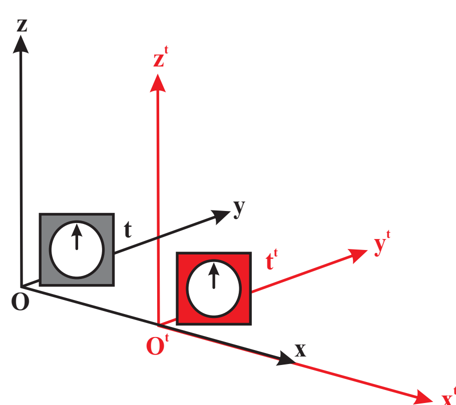


Figure 24. 1
Cartesian Co-ordinate System

The familiar Cartesian System of Co-ordinates, in which the position of the particle is specified by its three co-ordinates x, y, z, along the three perpendicular axes, is shown in Figure 24.1. We have indicated two observers

O and O′ and a particle P. These observers use frames of reference XYZ and X′Y′Z′, respectively. If O and O′ are at rest, they will observe the same motion of P. But if O and O′ are in relative motion, their observation of the motion of P will be different.

24.1.1 Relative Motion:

Relative motion is a concept that describes the motion of an object in relation to another observer or frame of reference. This viewpoint recognizes that motion is not absolute but depends on the observer's point of view.

Relative Motion: Same direction:

When two objects are moving in the same direction, their relative motion involves the difference in their velocities. The relative velocity is the velocity of one object as seen from the perspective of the other. The formula for relative velocity (v_{rel}) when the objects move in the same direction is given by:

$$v_{rel} = v_1 - v_2$$

where v_1 and v_2 are the velocities of the two objects, respectively.

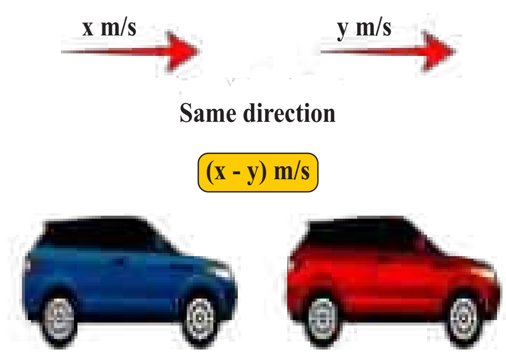


Figure 24.2 moving in same direction

Example:

Consider two cars, A and B, moving on a straight road. If car A has a velocity of 20 m/s, and car B has a velocity of 15 m/s in the same direction, the relative velocity of car B as observed from car A is

v_{rel}=20m/s – 15m/s=5m/s

Relative Motion: Opposite direction:

When two objects are moving in opposite directions, their relative motion involves the sum of their velocities. The formula for relative velocity (v_{rel}) when the objects move in opposite directions is given by:

$$v_{rel} = v_1 + v_2$$

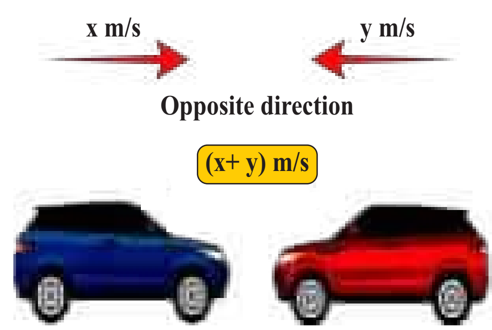


Figure 24.3 moving in different direction

Example:

Consider same two cars, A and B. But this time moving in opposite directions. Let's say car A is traveling at 80 m/s, and car B is now approaching it at 50 m/s.For a passenger on car A, car B appears to be moving towards them at relative velocity:

v_{rel}=80 m/s+50 m/s= 130 m/s

24.1.2 Types of Frames of References:

There are two types of frames of reference, Inertial frames of reference an dnon-inertial frames of reference.

Criteria	Inertial frame of Reference	Non-Inertial frame of Reference
Definition	A frame of reference with a constant	A frame of reference with an accelerating motion.
Motion of objects	Objects in uniform motion appear to follow straight lines or constant velocities.	Objects may appear to accelerate or experience fictitious forces.
Newton's first law	Newton's first law (Law of inertia) is valid in this frame.	Newton's first law is not valid due to accelerating motion.
Appearance of forces	Real forces are observed and can be directly measured.	Fictitious forces (e.g., centrifugal force, coriolis force) may appear due to the acceleration of the frame.
Equations of motion	Newton's laws of motion hold true in this frame.	Additional terms or transformations may be required to account of the frame.
Examples	A person inside a moving train, an object in free fall.	A person inside a spinning carousel, an object in circular motion.
Application	Often used in analyzing motion and dynamics in	Important in understanding phenomena in rotating systems, general relativity, and

Self-Assessment Questions:

1. Explain the difference between an inertial and a non-inertial frame of reference.
2. Provide an example of a commonly used frame of reference in everyday life.

24.1.3 Frames of references: Prediction of Motion:

Consider a vehicle moving at a constant speed v = 15 m/s with relative to the ground and a boy standing on the vehicle throws a ball with velocity v = 8 m/s in the same direction as the vehicle's motion. As shown in fig 24.4. If someone on the ground measures the ball's speed, it intuitively seems like it should be 23 m/s (15+8). Conversely, if you throw

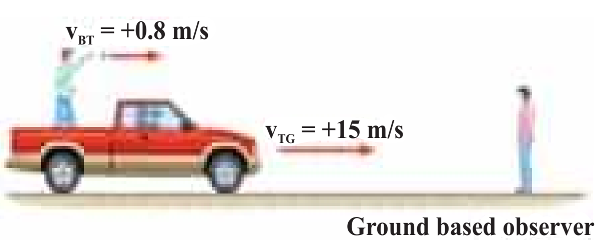


Figure 24.4: Moving and stationary frames of references

the ball in the opposite direction of the vehicle's speed, the ground observer would measure a speed of 7 m/s (15 − 8). Both observers are measuring the speed of the same object, but their relative motion leads to different results.

Now, let's derive a general equation applicable to any speeds. In our example, the vehicle's speed of 15 m/s becomes the relative speed of the two reference frames, denoted as u. The speed of the thrown ball is v′ (in the train's frame), and the measured speed on the ground is v. In a more formal way, we can express this relationship as:

$$v = v' + u$$

This equation is essentially the **Galilean transformation** of speeds. But, it is only applicable in mechanics.

Relative Motion: Galilean Transformation Equations:

The Galilean transformation equations describe the relationship between the coordinates of an event in two different inertial frames of references. These equations were formulated by Galileo and are applicable when the relative velocities involved are much smaller than the speed of light. With these equations, we can correctly measure the velocities of a ball in above example. Galileo concluded that:

"The laws of mechanics must be valid in all inertial frames of reference."

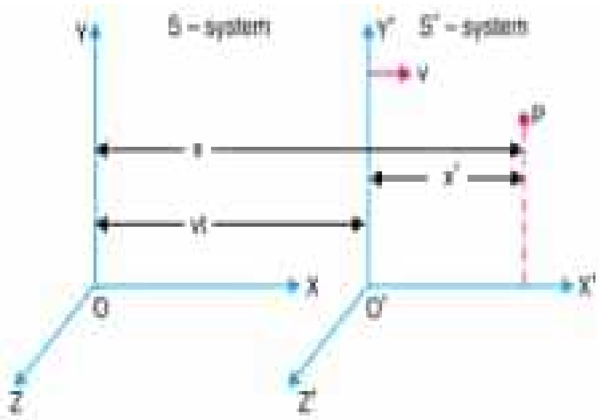


Figure 24. 5
Inertial frames of references: moving and stationary

This is also known as the "Principle of Galilean Relativity".

Let x and x′ be two inertial frames (Figure 24.5). Let x be at rest and x′ moves with uniform velocity v along the positive X direction. We assume that v << c. Let the origins of the two frames coincide at t = 0. Suppose some event occurs at the point P. The observer O in frame x determines the position of the event by the coordinates x, y, z. The observer O′ in frame x′ determines the position of the event by the coordinate's x′, y′, z′. Let the time elapsed at the same rate in both frames, i.e., t = t′. There is no relative motion between x and x′ along the axes of Y and Z. Measurements in the X direction made is x frame will be greater than those made in x′ frame by the amount v_t, which is the distance x′ has moved in the X direction. Therefore,

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

The set of above equations is known as the Galilean transformation equations.

24.2 Special Theory of Relativity:

Special theory of relativity explains how to interpret motion between different inertial frames of reference. In other words, it deals with the problems in which one frame of reference moves with a constant linear velocity relative to another frame of reference.

24.2.1 The Postulates of Special Relativity:

The special theory of relativity is based on two essential assumptions, commonly known as postulates.

Postulate I (Principle of Relativity):

The laws of Physics have the same form in all inertial frames of reference.

Consider a vehicle moving at a constant speed. Inside the vehicle, a passenger throws a ball straight up into the air. If we ignore any effects from the air, the passenger inside the moving vehicle sees the ball move up and then back down in a straight line, just as it would appear if someone standing still on the Earth threw a ball upwards. This means that the laws of physics that govern the motion of the ball, including gravity and equations for constant acceleration, work the same way whether the vehicle is moving or at rest.

Both observers, the one inside the vehicle and the one on the ground, see the ball go up and come back down. However, they see the path of the ball differently. The person on the ground sees the ball's path as a curved shape (a parabola), while the person in the vehicle sees it as a simple up-and-down motion. As shown in figure 24.6.

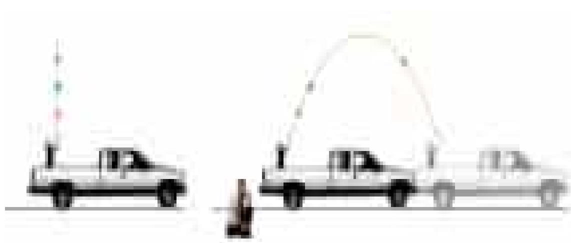


Figure 24.6: Principle of Relativity

Additionally, the person on the ground thinks that the ball has a horizontal component of velocity, which is the same as the vehicle's velocity. For the outside observer, the distance traveled along the parabolic path is longer than the path straight down but the time for the fall is the same. For the outside observer, the average velocity is greater.

Postulate II (Constancy of the speed of light):

The speed of light in vacuum has the same value, $c=3 \times 10^8 m/s$ in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

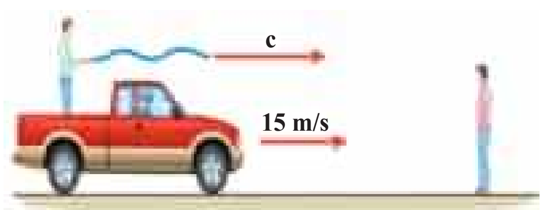


Figure 24.7 Two different frames of reference but speed of light is constant for both frames of references

Consider again the same example of throwing a ball in moving vehicle. As shown in figure 24.7, But this time, instead of throwing a ball, we shine a flashlight. If we were to do the same thing with light as we did with the ball, common sense might suggest that the speed of light would increase if the vehicle is moving in the same direction as the light according to Galilean Transformation Equation: v = c + u (if we replace ball's speed v′ with speed of light c). In fact, such an increase in the speed of light has never been found. In fact, in experiments carried out to test for the effect of the movement of the source on the speed of light (Michelson-Morley), the results indicate that the speed of light is completely unaffected by the motion of the source. It appears that the speed of light in a vacuum is constant regardless of relative motion.

In order to correct the Galilean equation of velocity addition, we have to consider velocity of light too. This correction was done by Hendrik Lorentz. The resultant equations are known as **Lorentz transformation equations**.

Relative Motion: Lorentz Transformation Equations:

We have to introduce new transformation equations which are consistent with the new concept of the invariance of light velocity in free space.

The Lorentz transformation equations for time and space coordinates (x,y,z,t) can be written as follows:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

where t and x,y,z are the time and space coordinates in the original frame, t′ and x′,y',z' are the time and space coordinates in the frame moving with velocity v relative to the original frame as shown in figure 24.8, c is the speed of light, and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

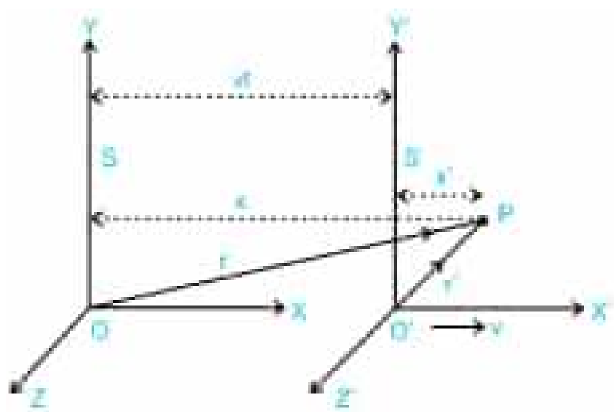


Figure 24.8 The Lorentz transformation

Self-Assessment Questions:

1. Calculate the value of Lorentz factor if the speed of car is 20m/s. Can we use Galilean Transformation equation for this value?
2. Why can't objects with mass exceed the speed of light?

24.2.2 Space and Time are Relative:

When the speed of light is constant, the time and space become relative concepts and are no longer absolute according to Newton.

The measurements of space and time intervals depend on the observer's relative velocity, leading to phenomena such as time dilation and length contraction. The constancy of the speed of light is a fundamental postulate in special relativity, challenging classical notions of absolute space and time.

24.2.3 Consequences of Special Theory of Relativity:

In Relativity, there is no such thing as an absolute length or absolute time interval. Here are some of the most important consequences of special relativity:

Relativity of Simultaneity:

The principle of relativity states that there is no preferred inertial frame of reference. It is defined as:

Two events that are simultaneous in one reference frame are in general not simultaneous in a second frame moving relative to the first. Simultaneity depends on the state of motion of the observer, and is therefore not an absolute concept.

Here's an example to illustrate this concept:

Imagine two observers, Boy A and B, standing on a train platform. They are equidistant from the center of the platform. At the exact moment when the train passes the center of the platform, lightning strikes both ends of the train. As shown in figure 24.9.

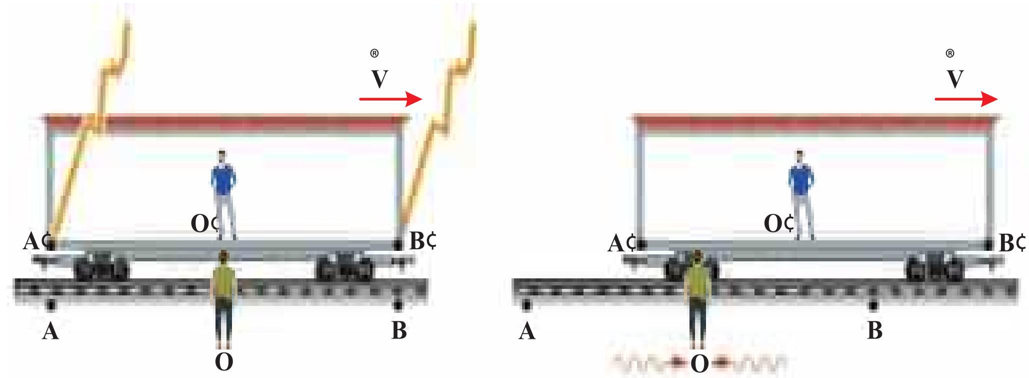


Figure 24. 9 Demonstration of Relativity of Simultaneity

From boy A's perspective:

Boy A sees the lightning strikes happen at the same time because she is stationary relative to the platform. Since light travels at a finite speed, the light from both lightning strikes reaches boy A simultaneously.

From boy B's perspective:

Boy B, however, is sitting on the moving train. Because the train is moving toward the lightning strike at the front of the train and away from the lightning strike at the back of the train, light takes longer to reach him from the back of the train than from the front.

As a result, Boy b perceives the lightning strike at the front of the train before the lightning strike at the back of the train. Thus, for boy B, the lightning strikes are not simultaneous.

This example demonstrates how the perception of simultaneity can differ between observers depending on their relative motion. In this case, what appears simultaneous to boy A (the lightning strikes) does not appear simultaneous to boy B due to his motion relative to the events.

Time Dilation:

The time interval, between two events occurring at a given point in the moving frame S′ appears to be longer to the observer in the stationary frame S. This effect is called time dilation.

Let Δt_o be the proper time measured by a clock that is at rest. The relative time t measured in another frame of reference is given by

$$\Delta t = \frac{\Delta t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where Δt is relativistic time and c is speed of light (3×10^8 m/s). Here, the value of $\sqrt{1-\frac{v^2}{c^2}}$ should not be equal to zero; this will only occur when the speed of an object is less than thespeed of light c. If the speed of an object v equals c in the above equation, then Δt becomes infinite. This implies that time will seemingly stop, which is impossible. Therefore, no material object can travel at the speed of light. As shown in figure 24.10.

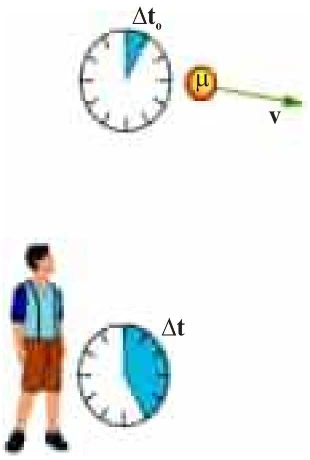


Figure 24. 10 Proper time elapsed by muon particle and relative time as measured by an observer

Length Contraction:

Let L_o be the length of a rod when the rod is stationary as shown in Figure 24.11 (a). If there is relative motion at speed v between an observer at rest and the rod along the length of the rod, then observer will calculate a relativistic length L given by

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

Where the length L_o is called proper length, L is relativistic length and c is speed of light (3×10^8 m/s). The relative motion causes a length contraction as shown in Figure 24.11 (b). A greater speed v results in a greater contraction. Space contracts in only one direction, the direction of motion as shown in Figure 24.12.

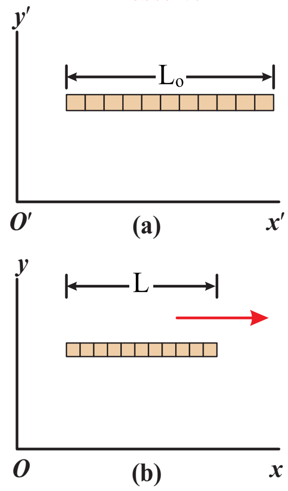


Figure 24. 11 Proper length and relative length of a rod moving with velocity v

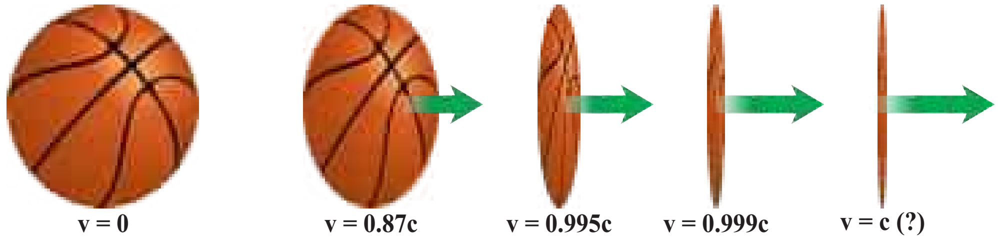


Figure 24. 12: Relative speed increases the contraction in only in one direction – the direction of motion. It doesn't affect other directions.

Mass Variation:

Let m_o be the rest mass of an object. If the object is moving at speed v then its relativistic mass m will be given by

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where m is relativistic mass and c is speed of light. The relative motion causes a mass variation.

Mass Energy Relationship:

According to Einstein's special theory of relativity, mass and energy are interchangeable. An object's mass m and the equivalent energy E_o are related by:

$$E_o = mc^2$$

where c is the speed of light. This equation gives the mass energy E_o that is associated with the object's mass m, regardless of whether the object is at rest or moving. If the object is moving, it has additional energy in the form of kinetic energy K. The total energy E is the sum of its mass energy and its kinetic energy:

$$E = E_o + K = mc^2 + K$$

$$E = \gamma mc^2$$

where γ is the Lorentz factor for the object's motion. And E is the total relativistic energy of the object. From above equations, relative kinetic energy can be calculated as:

$$K = E - E_o = E - mc^2 = \gamma mc^2 - mc^2$$

$$K = mc^2(\gamma - 1)$$

DO YOU KNOW?

Tachyons are hypothetical particles that travel faster than light. According to Einstein's theory of relativity, objects with real mass cannot reach the speed of light. Photons, which have no mass, can travel at this speed. For an object to travel at light speed, it would need imaginary mass. As a particle exceeds the speed of light, the denominator in the equation

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Becomes imaginary. Imaginary mass would counteract this, making the particle appear to have real mass in its rest frame, while always moving faster than light.

24.2.4 Maximum Velocity of A Particle:

A basic result of the special theory of relativity is that the speed of an object cannot equal or exceed the speed of light. That the speed of light is a natural speed limit in the universe can be seen from above equations of mass variation, length contraction, time dilation and mass-energy relationship. If an object is accelerated to greater speeds, its mass becomes larger and larger. Indeed, if v were equals to c, the denominator in the equation would be zero and the mass would become infinite. To accelerate an object up to v=c would thus require infinite energy, and so it is not possible.

Time Dilation:

Special relativity also predicts time dilation, meaning that as an object approaches the speed of light, time appears to slow down for the object relative to an observer at rest. This effect becomes more pronounced as the object's velocity approaches c. Consequently, from the perspective of an observer, it would take an infinite amount of time for a massive object to reach the speed of light.

Length Contraction:

Another consequence of special relativity is length contraction, where objects moving at relativistic speeds appear shorter in the direction of motion when observed from a stationary frame. This effect prevents objects from achieving relativistic velocities within a finite distance because, as the object's velocity increases, its length contracts in the direction of motion, making it increasingly difficult to accelerate further.

Self-Assessment Questions:

1. If a rectangular box is moving with the speed of 0.9c. Will it shrink to smaller one in all directions? Discuss.

24.2.5 Relativistic Effects in Space Travel:

The implications of mass increase, time dilation, and length contraction for space travel are profound and have significant consequences for our ability to explore the universe at relativistic speeds. Here's how each of these effects impacts space travel:

Mass Increase:

- The implication for space travel is that as spacecraft approach relativistic speeds, their mass increases exponentially. This makes it increasingly difficult to accelerate them further, requiring enormous amounts of energy.
- Overcoming the mass increase becomes a significant engineering challenge for spacecraft propulsion systems. Current propulsion technologies, such as chemical rockets, would become impractical at relativistic speeds due to the massive energy requirements.

Time Dilation:

- For space travelers moving at relativistic speeds, time dilation means that their perception of time differs from that of stationary observers. A journey that may take several years according to Earth-based observers could be experienced as much shorter by the travelers due to time dilation.
- Time dilation has implications for interstellar travel, where long-duration missions could be undertaken without experiencing the full effects of time passing, as perceived by Earth-based observers. However, it also presents challenges for communication and synchronization with mission control on Earth.

Length Contraction:

- For space travelers, length contraction means that distances along their direction of motion appear shorter. This has implications for navigation and spacecraft design, as distances may be perceived differently by travelers moving at relativistic speeds compared to stationary observers.
- Additionally, length contraction affects the perception of space travel distances. What may appear as a vast distance to stationary observers could be contracted from the perspective of travelers moving at relativistic speeds.

24.2.6 Navigation Satellite Timing and Ranging (NAVSTAR) SYSTEM):

It is the official name for the Global Positioning System (GPS). It's a network of 24 satellites orbiting Earth that allows us to determine our location on the planet with incredible accuracy. As shown in figure 24.13 (a,b).

Figure 24.13 (a) NAVSTAR SYSTEM

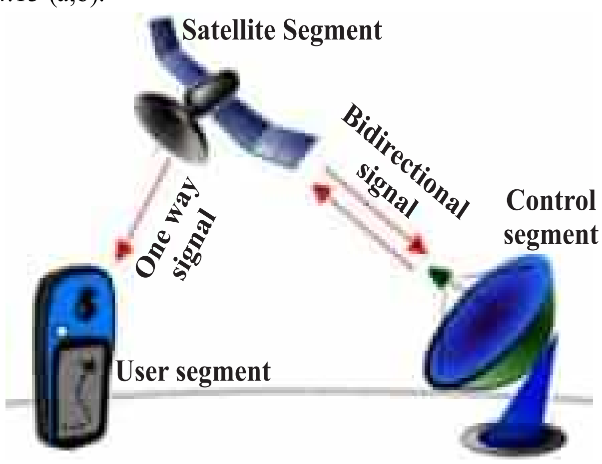


Figure 24. 13 (b): The NAVSTAR Global Positioning System (GPS)

One of the core principles of NAVSTAR system is synchronized clocks on the satellites. These clocks tick incredibly accurately, but due to the effects of relativity, this isn't as simple as it seems. Here's why:

Time Dilation:

As the satellites zoom around Earth at high speeds (roughly 11,000 km/h), their clocks actually run slightly slower compared to clocks on Earth.

Gravity:

The satellites being closer to Earth experience a stronger gravitational pull, which, according to General Relativity, also slows down their clocks compared to those on the ground.

If these effects weren't accounted for, our GPS calculations would be significantly off. But, because scientists understand space-time and its relativistic effects, they can account for this time discrepancy and maintain the accuracy of GPS positioning.

Using Equation of time dilation and the speed of the GPS satellites, we can calculate the difference between the dilated time interval and the proper time interval as a fraction of the proper time interval and compare the result to the stability of the GPS clocks:

$$\frac{\Delta t - \Delta t_o}{\Delta t_o} = \frac{1}{\sqrt{1 - v^2/c^2}} - 1 = \frac{1}{\sqrt{1 - (4000)^2/(3 \times 10^8)^2}} - 1$$

$$= \frac{1}{1.1 \times 10^{10}}$$

This indicates the level of stability. It means the clock deviates by one part in 10 trillion (10^{13}) over a certain period. In other words, the clock's timekeeping is extremely precise, with an error margin of just 0.0000000001%.

Worked Example 24.1

The period of a pendulum is measured to be 3.00 s in the inertial frame of the pendulum. What is the period as measured by an observer moving at a speed of 0.950c with respect to the pendulum?

Solution:

Step 1: Write down the formula and rearrange if necessary.

Using the equation of mass variation:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Step 2: Substitute the proper time and relative speed in the equation of time dilation:

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{3}{\sqrt{1 - \frac{0.95c^2}{c^2}}} = 9.61s$$

Result: The moving observer considers the pendulum to be moving, and moving clocks are observed to run more slowly: while the pendulum oscillates once in 3 s for an observer in the rest frame of the clock, it takes nearly 10 s to oscillate once according to the moving observer.

Worked Example 24.2

A starship is measured to be 125 m long while it is at rest with respect to an observer. If this starship now flies past the observer at a speed of 0.99c, what length will the observer measure for the starship?

Solution:

Step 1: Write down the formula and rearrange if necessary.

Using the equation of mass variation:

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

Step 2: Substitute the proper length and relative speed in the equation of length contraction:

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}} = 125 \times \sqrt{1 - \frac{0.99c^2}{c^2}} = 17.6m$$

Result: $L = 17.6m$

Worked Example 24.3

At what speed will an object's relativistic mass be twice its rest mass?

Solution:

Step 1: Write down the formula and rearrange if necessary.

Using the equation of mass variation: $m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$

Step 2: Where its relativistic mass m = 2m$_o$. Then velocity of the object will be:

$$2m_o = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \qquad = 2 \times \sqrt{1-\frac{v^2}{c^2}} = 1$$

Step 3: Squaring both sides:

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$1 - \frac{1}{4} = \frac{v^2}{c^2}$$

$$v = \frac{\sqrt{3}c}{2}$$

Result: This speed is approximately 86.6% of the speed of light.

24.3 General Theory of Relativity:

Einstein worked on the fact that acceleration produced the same effect as gravitation. For example, if you were in an accelerating spaceship, or just an elevator, you could not tell if the force on you was from inertia or gravitation. While Newton's Law of Universal Gravitation works well for ordinary gravitational fields, it is inaccurate when the gravitational intensity is high.

27.3.1 General Theory of Relativity:

Albert Einstein's general theory of relativity is a new way to explain gravity. According to Newton, tossed balls curve because of a force of gravity. According to Einstein, tossed balls and light don't curve because of any force, but because the space time in which they travel is curved. General relativity describes how gravity is caused by the curvature of space-time.

The general theory of relativity is based on the following two postulates.

Spacetime is a four-dimensional continuum that includes three dimensions of space and one dimension of time. Imagine it as a two-dimensional rubber sheet. If you place a heavy object, like a bowling ball, on the sheet, it will cause the sheet to deform or bend around the object. Now, if you roll a smaller ball across the sheet, it will move along the curved paths created by the heavy object. This is analogous to how planets and other celestial bodies move along the curved space-time around massive objects like the Sun. As illustrated in Figure 24. 14.

Postulate I:

The laws of physics may be expressed in equations having the same form in all frames of reference, regardless of their states of motion.

Thus, the general theory covers uniform as well as accelerated motion. Hence it is able to describe gravitational phenomena.

Postulate II (Principle of equivalence):

There is no way to spot the difference between gravity and acceleration for an observer in a closed laboratory

OR

There is no way for an observer in a closed laboratory to distinguish between the effects produced by a gravitational field and those produced by an acceleration of the laboratory, This postulate follows from the experimental observation that the inertial mass of a body is always exactly equal to its gravitational mass.

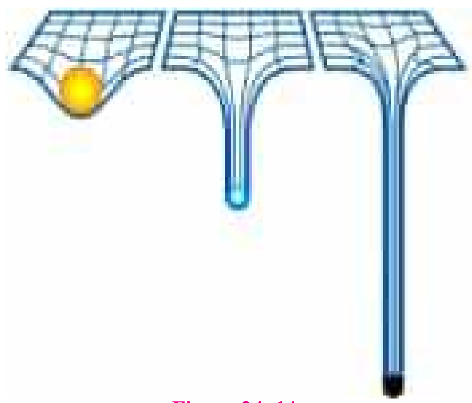


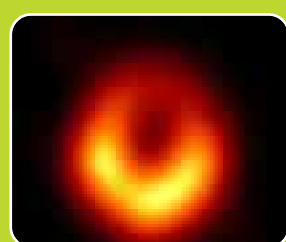
Figure 24. 14
Spacetime wrapping near heavy objects

General relativity has a number of consequences:

- Einstein's general theory of relativity states that time is a fourth dimension adding to thethree dimensions of space. Einstein called this four-dimensional geometry as space-time.
- The general theory of relativity also predicted light coming from a strong gravitational field would have its wavelength shifted toward longer wavelengths, called a red-shift.
- The theory also predicted that when gravity becomes great enough, it would produce objects called **black holes**. Black holes are objects whose gravity is so massive that light cannot escape from the surface at all. Since no light can escape, such objects would appear black
- Light is bent as it passes through curved space-time. This can cause distant objects to appear distorted or magnified. **Gravitational lensing** has been used to study distant galaxies and other objects.
- **The expansion of the universe:** General relativity predicts that the universe is expanding. This has been confirmed by astronomical observations.

DO YOU KNOW?

Dr. Katie Bouman captures the first image of a black hole. She and her team of NASA scientists developed a new technology to make the discovery possible. In 2019, astronomers observed the first direct evidence of existence of a black hole. (With first ever picture of black hole).



27.3.2 Gravity as Space Time Continuum:

According to General Theory of Relativity, when you put mass in space-time, it bends the geometry of space-time. According to this theory, gravity is not just a force between masses, as described by Newtonian physics, but rather a manifestation of c the curvature of space-time caused by mass and energy.
Here's a breakdown of what this concept entails:

Figure 24. 15
Heavier object bends the geometry of spacetime more than lighter object

Space-time Continuum:

- Space and time are unified into a single, four-dimensional entity known as space-time. In the absence of gravity or significant mass, space time is flat.
- However, the presence of mass and energy warps or curves the fabric of space-time. This curvature is what we perceive as the force of gravity.

DO YOU KNOW?

Masses placed in space cause the space to be curved. Curved space causes masses moving in a straight line to follow a curved path. Curved space also causes time to run more slowly.

Curvature of Space-time:

- Massive objects, such as stars, planets, and galaxies, curve the space-time around them. The greater the mass or energy density, the greater the curvature of space-time.
- Objects move along paths dictated by the curvature of space-time, which we perceive as being influenced by gravity. For example, Earth orbits the Sun because the Sun's mass curves the space-time around it, causing Earth to follow a curved path.

Effects of Gravity:

- In the presence of gravity, objects follow the shortest possible paths, known as geodesics, through the curved space-time. These paths are not necessarily straight.
- Lines in the traditional sense but are instead curved trajectories determined by the gravitational field.
- The strength of gravity is determined by the curvature of space-time. The steeper the curvature, the stronger the gravitational force experienced by nearby objects.

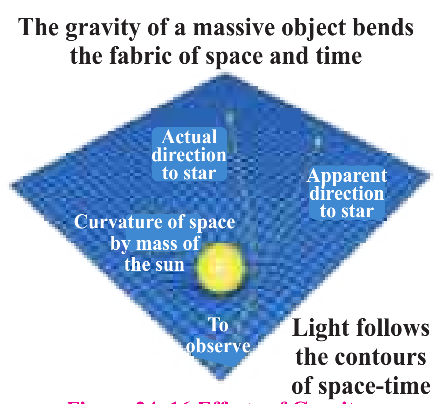


Figure 24. 16 Effects of Gravity

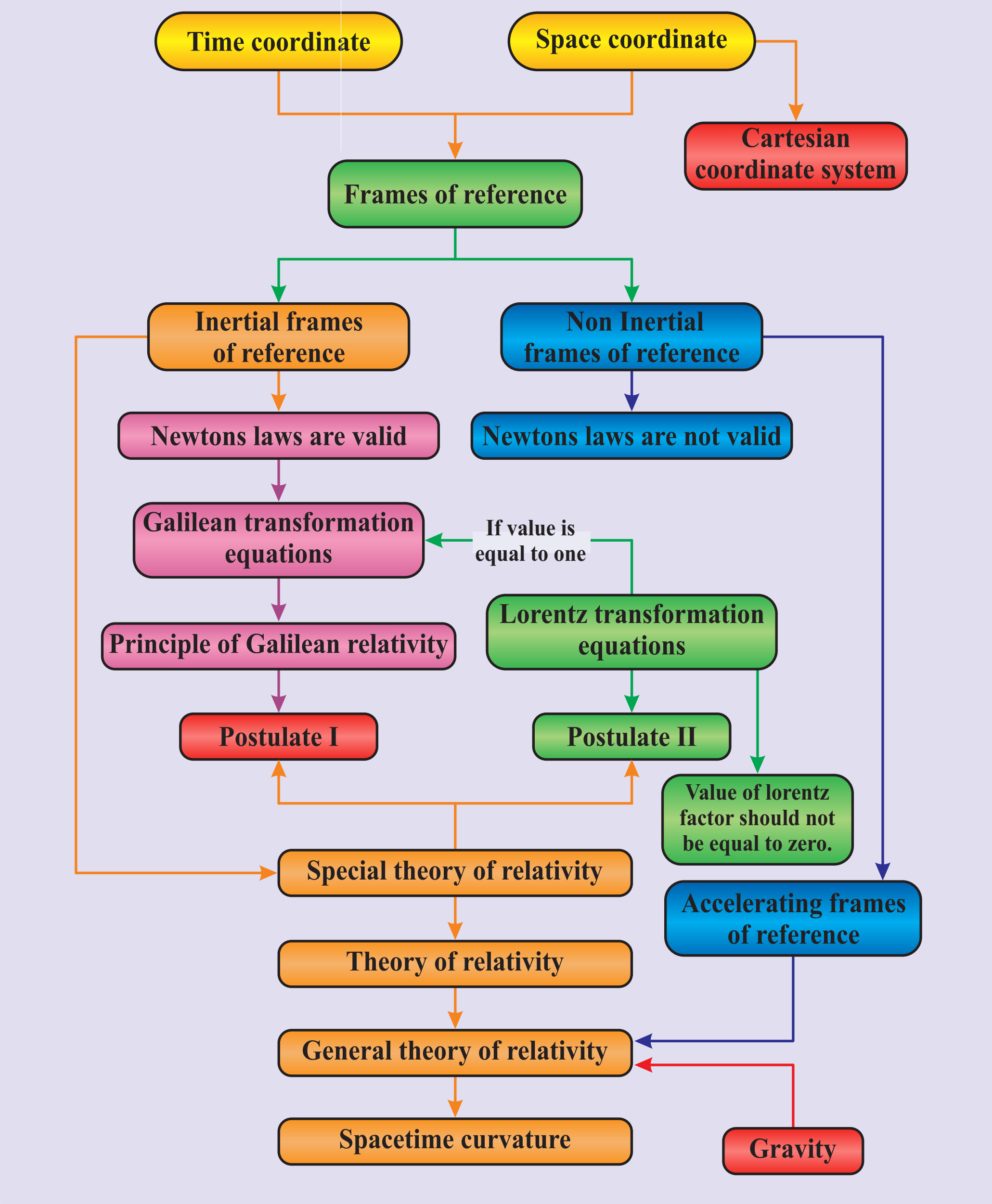
Physical Interpretation:

- Gravity is thus interpreted as the result of objects moving through the curved space-time created by mass and energy. Massive objects "warp" the space-time around them, causing other objects to move in response to this curvature.
- This concept provides a unified explanation for both the gravitational force and the motion of objects in space, incorporating both space and time into a single framework.

SUMMARY

- ✓ Special Reactivity applies to objects moving at constant velocity relative to each other. It introduces the concept of reference frames and explains how the laws of physics are the same in all inertial reference frames.
- ✓ Two important postulates of Special Relativity are:
  - The laws of physics are the same in all inertial frames of reference.
  - The speed of light in a vacuum is the same for all observers regardless of their relative motion or the motion of the source of the light.
- ✓ Consequences of Special Relativity include:
  - Relativity of simultaneity: Two events that are simultaneous in one frame of reference may not be simultaneous in another frame moving relative to the first.
  - Time dilation: Time intervals between events appear longer for objects in motion relative to an observer at rest.
  - Mass variation refers to the change in the mass of an object as its speed approaches the speed of light. According to relativity, the mass of an object increases with its velocity, becoming infinite as it reaches the speed of light.
  - Length contraction: Objects in motion appear shorter in the direction of their motion relative to an observer at rest.
  - Mass-energy equivalence: Mass and energy are interchangeable $E=mc^2$.
- ✓ General Relativity applies to objects under any motion, including acceleration. It explains gravity as a curvature of space-time caused by mass and energy.
- ✓ Two postulates of General Relativity are:
  - The laws of physics may be expressed in equations having the same form in all frames of reference, regardless of their states of motion.
  - There is no way for an observer in a closed laboratory to distinguish between the effects produced by a gravitational field and those produced by an acceleration of the laboratory itself.
- ✓ Consequences of General Relativity include:
  - Space-time: Space and time are unified into a single four-dimensional fabric.
  - Gravitational red-shift: Light coming from a strong gravitational field has a longer wavelength (red-shifted).
  - Black holes: Extremely massive objects whose gravity is so strong that not even light can escape.
  - Gravitational lensing: Light bends as it travels through curved spacetime, which can distort, diminish or magnify distant objects.
  - Expansion of the universe: General relativity predicts the universe is expanding, which has been confirmed by astronomical observations.

CONCEPT MAP



Time coordinate
Space coordinate
Cartesian coordinate system
Frames of reference
Inertial frames of reference
Non Inertial frames of reference
Newtons laws are valid
Newtons laws are not valid
Galilean transformation equations
If value is equal to one
Lorentz transformation equations
Principle of Galilean relativity
Postulate I
Postulate II
Value of lorentz factor should not be equal to zero.
Special theory of relativity
Accelerating frames of reference
Theory of relativity
General theory of relativity
Spacetime curvature
Gravity

EXERCISE

Section (A): Multiple Choice Questions (MCQs)
Choose the correct answer:

1. The General Theory of Relativity was a new way of understanding of
   (a) The speed of light (b) gravity
   (c) mass (d) force
2. An object at rest has a mass of 1 kg. What is its mass when it is moving at a speed of 0.9c?
   (a) Infinite (b) 1.2 kg (c) 2.3 kg (d) 1 kg
3. The equivalence of principle in general relativity is:
   (a) The equivalence of inertial and gravitational mass
   (b) The equivalence of electric and magnetic fields
   (c) The equivalence of space and time
   (d) The equivalence of matter and energy
4. Which of the following phenomena is NOT predicted by special relativity?
   (a) Time dilation (b) Length contraction
   (c) Gravitational waves (d) Relativistic mass increase
5. What does the equivalence principle state about acceleration and gravity?
   (a) They are completely different forces (b) They are indistinguishable for an observer
   (c) Gravity is stronger than acceleration (d) Acceleration cancels out gravity
6. Light passing near a massive object like a star will bend due to:
   (a) Gravitational lensing (b) refraction
   (c) reflection (d) diffraction
7. What is the main reason astronomers cannot directly observe black holes?
   (a) They are too small (b) They deflect light
   (c) They are too far away (d) their immense gravity traps light
8. Objects cannot exceed the speed of light because:
   (a) Their mass becomes infinite (b) Their length becomes zero
   (c) Time slows down to zero (d) they lose all energy
9. Why is the Galilean transformation not valid at high speeds?
   (a) It cannot handle accelerating frames
   (b) It violates the constancy of the speed of light
   (c) It only works in flat spacetime
   (d) It neglects time dilation effects
10. The example of inertial frame in our everyday life is:
    (a) A car accelerating on a highway
    (b) A train moving smoothly at constant speed
    (c) A person standing on a spinning platform
    (d) an airplane encountering turbulence

Section (B): CRQs (Short Answered Questions):

1. Show that for values of v << c, Lorentz transformation reduces to the Galilean transformation.
2. If a particle could move with the velocity of light, how much K.E. would it possess?
3. Explain the difference between Special and General Relativity in simple terms.
4. Differentiate between Inertial Frames of Reference and Non- Inertial Frames of Reference.
5. Why can't any object move at the speed of light?
6. What is the limitation in the Galilean Transformation Equation, and how did Lorentz solve it?
7. Calculate the value of γ (Lorentz factor) if the object is moving at the speed of light.

Section (C): ERQs (Long Answered Questions):

1. State and explain the basic postulates of Einstein's special theory of relativity. Discuss length-contraction, mass variation and time-dilation.
2. Explain the concept of mass-energy equivalence. Derive Einstein's mass-energy relation and demonstrate that 1 atomic mass unit (u) is equivalent to 931 MeV.
3. Discuss the important conclusions derived from General Theory of relativity. What are the experimental observations in favour of these conclusions?
4. How does it principle of relativity differ from the classical Galilean view?
5. Explain the concept of spacetime curvature in general relativity and how it is used to visualize the effects of gravity.
6. Derive the basic equations of the Galilean transformation and explain how they relate the positions and velocities of objects in different inertial frames.
7. Discuss the Lorentz transformation equations in special relativity and how they describe the relationship between space and time coordinates in different inertial frames. Give examples to illustrate their application.

Section (D): Numerical:

1. A rod 1 meter long is moving along its length with a velocity 0.6c. Calculate its length as it appears to an observer (a) on the earth (b) moving with the rod itself.
   [Ans: (a) 0.8m, (b) 1m]
2. How fast would a rocket have to go relative to an observer for its length to be contracted to 99% of its length at rest? **[Ans: 42.45×10^6 m/s]**
3. A particle with a proper lifetime of 1μs moves through the laboratory at 2.7×10^8 m/s. (a) What is its lifetime, as measured by observers in the laboratory? (b) What will be the distance traversed by it before disintegrating? **[Ans: (a) $2.3 \times 10^{-6} s$, (b) 620m]**
4. At what speed is a particle moving if the mass is equal to three times its rest mass?
   [Ans: $\frac{2\sqrt{2}}{3}$c]

5. If 4 kg of a substance is fully converted into energy, how much energy is produced?
[Ans: 3.6×10^{17}J]
6. Calculate the rest energy of an electron in joules and in electron volts.
[Ans: 8.2×10^{-14}J, 0.511MeV]
7. Calculate the K.E. of an electron moving with a velocity of 0.98 times the velocity of light in the laboratory system. **[Ans: 3.3×10^{-13}J]**
8. At what velocity does the K.E. of a particle equal its rest energy? **[Ans: $\frac{\sqrt{3}}{2}c$]**
9. A particle of rest mass m_0 moves with speed $c/\sqrt{2}$. Calculate its mass, momentum, total energy and kinetic energy. **[Ans: $\sqrt{2}m_0$, m_0c, $\sqrt{2}\ m_0c^2$, $0.41\ m_0c^2$]**
10. The nearest star to Earth is Proximal Centauri, 4.3light- years away. A spaceship with a constant speed of 0.800c relative to the Earth travels toward the star.
    (a) How much time would elapse on a clock as measured by travelers on the spacecraft?
    [Ans: (a) 3.22 years]
    (b) How long does the trip take according to Earth observers?
    [Ans: (b) 5.38 years]