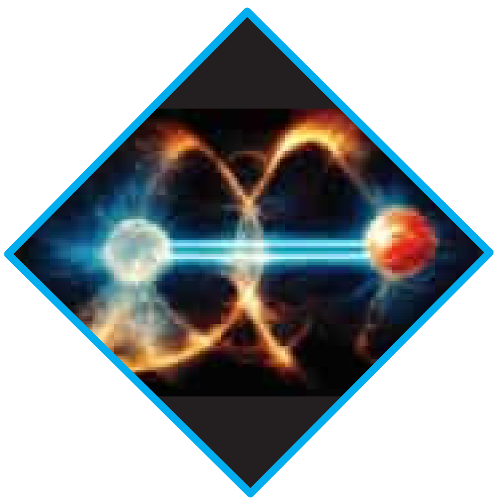
TEST EDITION

THE TEXTBOOK OF

PHYSICS

For Class XII



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PREFACE

The century we have stepped in, is the century of Science and technology. The modern disciplines of Physics are strongly influencing not only all the branches of science but each and every aspect of human life.

To keep the students abreast with the recent knowledge; it is must that the curricula at all the levels be updated. Moreover regularly by introducing the rapid and multidirectional development taking place in all the branches of Physics.

The recent book of Physics for Class - XII has been written in this preview and in accordance with the revised curriculum. Prepared by Ministry of Education, Govt of Sindh. Reviewed by independent team of Directorate of Curriculum Assessment and Research, Jamshoro Sindh. Keeping in view the importance of Physics, the topics have been revised and re-written according to the need of the time.

Among the new editions the introductory paragraphs, information boxes, summaries and a variety of extensive exercises have been included. Which I think will not only develop the interest but also add a lot to the utility of the book.

The Sind Textbook Board has taken great pains and incurred expenditure in publishing this book inspite to its limitations. A textbook is indeed not the last word and there is always room for improvement. While the authors have tried their level best to make the most suitable presentation, both in terms of concept and treatment. There may still have some deficiencies and omissions. Learned teachers and worthy students are therefore requested to be kind enough to point out the short comings of the text or diagrams and to communicate their suggestions and objections for the improvement of the next edition of this book.

In the end, I am thankful to our Authors, Editors and Subject specialist of Board for their relentless service rendered for the cause of education.

Chairman
Sindh Textbook Board

Unit 15

Molecular Theory of Gases

Teaching Periods 07 **Weightage %** 05

Popcorn is a fun way to learn about the kinetic molecular theory of gases, the phase change of water from a liquid to a gas. When heated, the water inside turns to steam, making the kernel pop and expand up to 50 times its size.

In this unit student should be able to:

- Recall concept of temperature
- Solve problems using scales of temperature and their conversion
- Explain triple point of water
- State general gas law
- Derive gas laws (Boyle's law, Charle's law and Avogadro's law)
- Solve problems using gas laws
- State the basic postulates of KTG
- Describe the molecular movement causes the pressure exerted by gas and derive pressure equation
- Describe the relation between kinetic energy of molecules and temperature
- Solve problems using relation between kinetic energy and temperature

Introduction:

We have based our study of Mechanics on the three fundamental quantities mass, length and time. Now we need to introduce the additional fundamental quantity temperature. We have used physical quantities such as total mass m, number of moles n, pressure P, Volume V and temperature T to describe the quality and condition of the material. We now want to pull several relationships together into more general formulation of the behavior of materials. A central concept in this synthesis is equation of state of a material. We begin with a general discussion of the molecular structure of matter then we develop the kinetic Molecular Model of an ideal gas, deriving the pressure of an ideal gas on the basis of kinetic Molecular Model.

15.1.1 Temperature:

Temperature is a measure of the average translational kinetic energy of the molecules of body.

How does Thermometer device help to measure the temperature of any other body? The answer to this question leads to the concept of thermal equilibrium. When two bodies at different temperatures are brought in thermal contact with each other, the heat start flowing from the hot body to the cold body till the temperature of both bodies becomes same then they are said to be in thermal equilibrium. The S.I Unit of temperature is Kelvin.

15.1.2 Scales of Temperature:

There are three scales of temperature

(i) Centigrade or Celsius scale
(ii) Fahrenheit scale
(iii) Kelvin or absolute scale are commonly used these days

DO YOU KNOW?

A temperature scale that starts at absolute zero, the theoretical lowest possible temperature. It is measured in Kelvin (K),

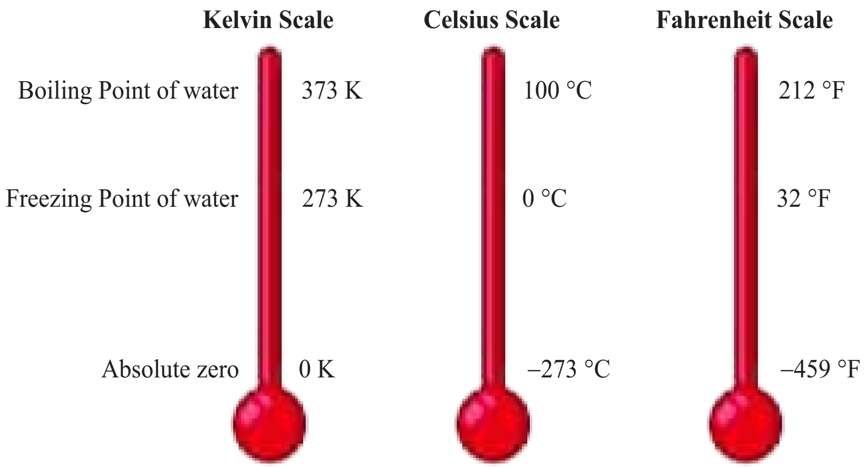


Figure 15.1
Scales of temperature mentioning the different temperatures.

Centigrade or Celsius Scale (Anders Celsius):

In the Celsius scale the freezing point of water or Melting point of ice is marked 0^0C and boiling point is 100^0C. The interval between these two points is divided into hundred equal parts. Each part thus represents one degree Celsius (1°C).

(i) Fahrenheit Scale (Daniel Fahrenheit):

In Fahrenheit scale the fixed points are marked 32°F and 212°F respectively and the interval between the fixed points is divided into 180 equal parts. Each part represents 1°F.

(ii) Kelvin Scale (James Lord Kelvin):

In this scale the melting point of ice is 273K and the boiling point of water is 373K. The temperature is given in units called Kelvin instead of degrees. The lowest temperature is 0K known as absolute zero.

In order to derive empirical formulae among Centigrade, Fahrenheit and Kelvin scales, let the three thermometers be placed in a bath tub and the mercury in each thermometer rises to the same level as shown in figure 15.1. We arrive at the relation.

$$\frac{\text{Temp: on one scale} - \text{F.P}}{\text{B.P} - \text{F.P}} = \frac{\text{Temp: on 2nd scale} - \text{F.P}}{\text{B.P} - \text{F.P}}$$

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273}{100} \ldots\ldots\ldots\ldots(15.1)$$

Centigrade temperature can be converted into Kelvin temperature by simply adding 273 to the centigrade temperature i.e.

$$T_K = 273 + T_C$$

DO YOU KNOW?

(i) The scale divisions on the Kelvin scale are equal in size to the divisions on Celsius scale. i.e 1k K= 1°C.
(ii) 1°C = 1.8°F

DO YOU KNOW?

If two systems are each in thermal equilibrium (equal temperature) with a third system, then the two initial systems are in thermal equilibrium with each other.

DO YOU KNOW?

(i) Fahrenheit and Celsius scales coincide at -400
(ii) Fahrenheit and Kelvin scales coincide at 574.25 or 574.25K

Worked Example 15.1

The normal human body temperature is 36.88°C. What is this temperature on Fahrenheit scale?

Solution:

Step 1: Write the known quantities and point out quantities to be found

T_C = 36.88°C

T_F?

Step 2: Write the empirical formula and rearrange if necessary

$$\frac{T_C}{5} = \frac{T_F - 32}{9}$$

$$T_F = \frac{9}{5}T_C + 32$$

Step 3: **Put the values and calculate.**

$T_F = \frac{9}{5} \times 36.88 + 32$

$T_F = 98.4°F$

Result: $T_F = 98.4°F$

Self-Assessment Questions:

1. Convert each of the following temperature from the centigrade scale to Kelvin scale and Fahrenheit 0°C, 20°C, 120°C, 500°C, –23°C, 200°C.
2. Convert each of the following temperature from the Kelvin scale to the Celsius scale and Fahrenheit 0K, 20K, 100K, 300K, 373K, and 500K.

15.1.3 The Triple Point of Water:

The vapor, liquid, and solid states of water can coexist in thermal equilibrium only at a specific pressure and temperature.

The triple point of water occurs at a specific temperature and pressure, where all three phases are in thermodynamic equilibrium. This means that at the triple point, ice, liquid water, and water vapor can exist together without any phase changing to another.

These conditions are used to define the Kelvin temperature scale, where the temperature at the triple point of water is precisely 273.16 K (0.01°C) and 4.58 mm of mercury (611.73 Pa) and is used to calibrate thermometer.

In a figure 15.3, the triple point is represented as the point where the lines separating the solid, liquid, and gas phases meet. This diagram visually demonstrates the relationship between temperature and pressure for the different phases of water.

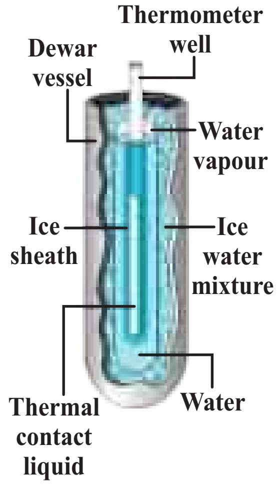


Figure 15.2 A triple point cell in which solid ice liquid water are water vapour co-exist in thermal equilibrium. The bulb of the constant volume gas thermometer is shown inserted into the well of the cell.

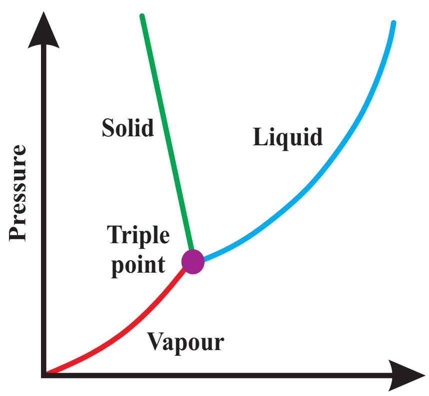


Figure 15.3 A triple point of water

The temperature at which Triple point of water has been set by international agreement to be T_3= 273.16K. In T_3 the subscript 3 means "Triple Point" this agreement also sets the size of the Kelvin as $\frac{1}{273.16}$ of the difference between the Triple point temperature of water and absolute zero.

Self-Assessment Questions:

1. What are the conditions (temperature and pressure) at which the triple point of water occurs?
2. Where is the triple point located in relation to the solid, liquid, and gas regions?

15.2 Gas Laws

Gases have no fixed volume or shape and their volume at same mass can be altered by changing the pressure as well as the temperature. Gases can be described with the help of four variables i.e. pressure, volume, mass and temperature. The relation between any two variables is found experimentally while keeping the other two constant.

15.2.1: Boyle's Law

Boyle's law states that "Volume V of given mass of a gas is inversely proportional to the pressure P, provided the temperature T of the gas remains constant."

Boyle's law can be written as:

$V \propto \frac{1}{P}$ (at constant temperature)

$PV = \text{constant}$ …………………..… (15.2)

We can also represent Boyle's law on a graph, as shown in Fig: 15.4(a) The graph plotted between P and V at constant temperature is a curve called hyperbola showing the inverse relation between them for two different states, while fig. 15.4(b) graph of P plotted against $\frac{1}{V}$ is a straight line passing through the origin, showing direct proportionality. Boyle's law can be written as:

$P_1V_1 = P_2 V_2 = \text{Constant}$………………………… (15.3)

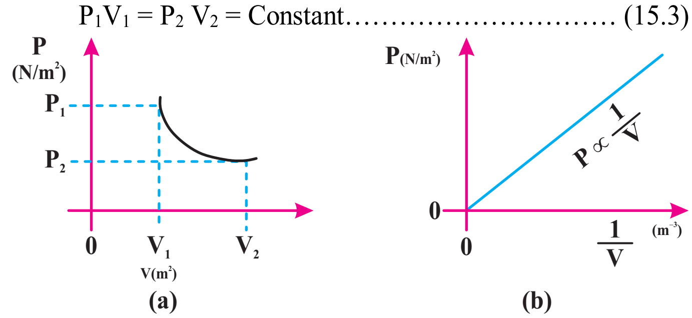


Figure 15.4 (a) and (b)
showing the relation between pressure and volume respectively

DO YOU KNOW?

"Liquid vapor equilibrium region" corresponds to the end point at the top of vaporization curve in fig: 15.4, it is called critical point and the corresponding values of P and T are called critical pressure Pc, and temperature Tc. A gas above the critical temperature does not separate into phases when it is compressed isothermally.

Charles's Law:

This law states that "The volume V of a given mass of a gas is directly proportional to the absolute temperature T at constant pressure P".

Charles law can be written as

$V \propto T$ (at constant pressure)

Or $\frac{V}{T}$ = constant ……………..…… (15.4)

If V_1 and V_2 are the volumes of the gas at temperature T_1 and T_2 respectively then for two different states, Charles's law is represented as

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ = constant ……………………..(15.5).

From the graph (fig: 15.5) at 0°C the gas still has a volume V_0. The graph between volume and temperature is a straight line. If the graph is extrapolated backward, it cuts the temperature axis at –237°C.This graph shows that volume of a gas is zero. Kevin selected this temperature (–273°C) as the zero called absolute zero (0K).

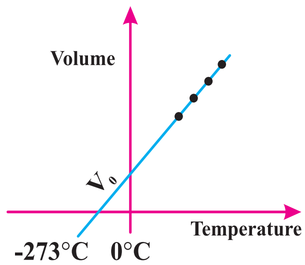


Figure 15.5
Shows relation between temperature and volume

DO YOU KNOW?

The temperature at which volume of a gas becomes zero and molecular motion ceases is termed as absolute zero.

Avogadro's Law:

In 1811, Italian scientist Amedeo Avogadro suggested his hypothesis regarding the relationship between volume and number of molecules of a gas. This hypothesis now called Avogadro's law which states that "equal volume of all gases contains the same number of molecules at the same temperature and pressure". Thus, the volume of a gas is directly proportional to number of moles of the gas at constant temperature and pressure.

In symbol, we can write as:

$V \propto n$ (at constant temperature and pressure)

$\frac{V}{n}$ = constant ………… (15.6)

Where V is volume and n is the number of moles of a gas. Thus $1dm^3$ (or cm^3, m^3) of oxygen contains the same number of molecules as $1dm^3$ or $1\ cm^3$) etc of hydrogen or of any other gas, provided the volumes are measured under the same conditions of temperature and pressure.

It can be written as

$\frac{V_1}{n_1} = \frac{V_2}{n_2}$ = constant ……………………..(15.7).

When V_1 and V_2 are volumes of gas and n_1 and n_2 are amount of gas.

DO YOU KNOW?

Avogadro's number:

A mole of any substance is that mass of substance that contains a specific No. of molecules called Avogadro's number.

$N_A = 6.022 \times 10^{23} \frac{\text{molecules}}{\text{mole}}$

Avogadro's number is defined to be the No. of carbon atoms in 12g of the isotope carbon-12. The number of moles of a substance is related to its mass m. $n = \frac{m}{M}$

Where M is the molecular mass of the substance usually expressed in g/mole.

Example:

Blowing up a balloon is an example of Avogadro's law, because it says as you blow more molecules of air into the balloon it expands.

15.2.2 General Gas Law:

In order to derive general gas law, we make use of Boyle's law, Charles's law and Avogadro's law. An interrelation among the physical quantities e.g. pressure, volume, temperature and amount of matter of a given sample of gas is termed as "equation of state" of gas or General gas law.

DO YOU KNOW?

Real or permanent gas is a gas that obeys gas Laws at high temperature and low Pressure only.

According to Boyle's law:

$V \propto \frac{1}{P}$ (when n number of mole and temperature T are kept constant)

According to Charles's law:

$V \propto T$ (when n and pressure P are kept constant)

According to Avogadro's law:

$V \propto n$ (when T and P are kept constant)

Consider for a moment that none of the variable are to be kept constant, then all the above three relationships can be joined together.

$$V \propto \frac{nT}{P}$$

$$V = constant \times \frac{nT}{P}$$

$$V = \frac{RnT}{P} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (15.8)$$

DO YOU KNOW?

$1dm = 0.1m$
Hence $1dm^3 = 10^{-3}m^3$
Ideal or perfect gas is a gas that obeys gas Laws at all temperatures and pressures.

Where **R** is constant of proportionally and is called General gas constant or universal gas constant and does not depend on the quantity of gas in the sample. If P is measured in Nm^{-2} V in m^3 and T in Kelvin then the volume of universal gas constant is $\mathbf{R = 8.314J\ mol^{-1},\ k^{-1}}$ Above equation is written as:

$$PV = nRT \quad \ldots\ldots\ldots\ldots\ldots\ldots \; (15.9)$$

Worked Example 15.2

A cylinder contains 0.80 dm^3 of nitrogen gas at a pressure of 1.2 atmosphere (1 atm = 1.01 10^5 N/m^2). A piston slowly starts compressing the gas to a pressure of 6.0 atm. The temperature of the gas remains constant. Calculate the final volume of the gas. (The temperature of the gas is constant and that its mass is fixed).

Solution:

Step 1: Write the known quantities and point out quantities to be found.

$P_1 = 1.2$ atm, $V_1 = 0.80dm^3$, $P_2 = 6.0$ atm, V_2=?

We don't worry about the particular units of pressure and volume being used here, so long as they are the same on both sides of the equation. The final value of V_2 will be in dm^3, because V_1 is in dm^3

Step 2: Write down the formula and rearrange if necessary.

$P_1V_1 = P_2V_2$

Substitute the values in equation and calculate.

$1.2 \text{ atm} \times 0.8dm^3 = 6.0 \text{ atm} \times V_2$

$V_2 = \frac{1.2 \text{ atm} \times 0.8dm^3}{6.0 \text{ atm}} = 0.16\ dm^3$

Result: The volume of the gas is reduced to 0.16dm3or 1/5th of original volume so the pressure increases by a factor of 5.

Worked Example 15.3

Find the mass of the air within the car tyre, given that it holds $0.020 m^3$ of air at 27°C and a pressure of $3.0 \times 10^5 N/m^2$. The molecular mass of air is 28.8g/mol.

Solution:

Step 1: Write the known quantities and quantities to be found.

$P = 3.0 \times 10^5 N/m^2$ R = 8.314J mol–1 k–1

T = 27°C = 300K i. No. of moles of air = n =?

$V = 0.020 m^3$ ii. Mass of the air = m =?

Molar mass of air = 28.8g/mol.

Step 2: Write down the formula and rearrange if necessary.

$$n = \frac{PV}{RT}, \quad = \frac{3.0 \times 10^5 \times 0.020}{8.314 \times 300} = 2.41 mol$$

Step 3: Now we calculate the mass of air.

mass = number of moles × molar mass

m = 2.41 mol × 28.8g/mol = 69.4g ≅ 69g

Result: i. 2.41 mol ii. 69g

Self-Assessment Questions:

1. How does the pressure of a gas change if its volume is halved at constant temperature?
2. What happens to the volume of a gas when its temperature is increased while keeping the pressure constant?
3. What is the relationship between the volume and the number of moles of a gas at constant temperature and pressure?

15.3 Kinetic theory of gases:

We discussed the properties of an ideal gas by using such macroscopic variables such as pressure, volume and temperature. Such large-scale properties can be related to a description on a microscopic scale, where matter is treated as collection of molecules. Now we begin by developing a microscopic model of an ideal gas called Kinetic Theory of gases. The first step in the construction of a theory is to setup some sort of model which is simple enough to be treated analytically.

15.3.1 Basic Postulates of Kinetic Theory of Gases:

The basic postulates of Kinetic Theory of gases are as under

1. A gas contains a very large number of particles called molecules. Depending on the gases each molecule consists of an atom or a group of atoms.
2. A finite volume of a gas consists of very large number of molecules. This assumption is justified by experiments. At standard conditions there are 3×10^{25} molecules per cubic meter.
3. The size of the molecules is much smaller than separation between molecules; it is about 3×10^{-10}m.

4. The molecules move in all directions with various speeds collide elastically with one another and with the walls of the container.
5. The molecules exert no forces on one another except during collisions. In the absence of the external forces, they move freely in straight lines.
6. Laws of mechanics are assumed to be applicable to the motion of molecules.

15.3.2 Pressure of Gas:

The pressure exerted by a gas is merely the momentum transferred to the walls of the container per second per unit area due to the continuous collisions of molecules of the gas. In order to calculate the pressure of an ideal gas from Kinetic Theory. Let us consider a cube having side length L whose walls are perfectly elastic contains N number of molecules each of mass m as shown in figure 15.6.

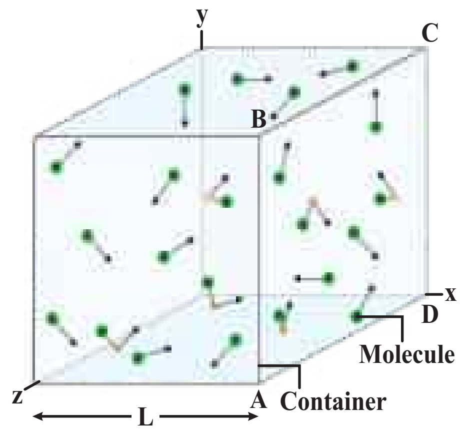


Figure 15.6
Shows gas molecules moving in random directions within container.

Consider a single molecule of mass m moving with velocity V_1 parallel to x-axis. It moves back and forth, colliding at regular intervals with the ends of the box and thereby contributing to the pressure of the gas. A molecule which has a velocity V_1 can be resolved into three rectangular components V_{1x}, V_{1y} and V_{1z} parallel to three co-ordinates axis x, y and z.

A molecule which collides with the face ABCDA of the cube, it will rebound elastically in opposite direction, such that x-component of the velocity V_{1x}, is reversed, the V_{1y} and V_{1z} remain unaffected. Therefore, the momentum before collision is mV_{1x} and after collision is $-mV_{1x}$ causing a change of momentum.

Change in momentum= $P_i - P_f$ = $= mV_{1x} - (-mV_{1x}) = mV_{1x} + mV_{1x}$

Change in momentum = $2mV_{1x}$ …………………. 15.10

After recoil the molecule travels to opposite face and collides with it, rebounds and travels back to the face ABCDA after covering a distance 2L. The time Δt between two successive collisions with face ABCDA is:

$$\Delta t = \frac{2L}{V_{1x}} \quad \ldots\ldots (15.11)$$

Now we can find the force that this one molecule exerts on face ABCDA, using Newton's 2nd law of motion. This says that the rate of change of momentum of the molecule is equal to force applied by the wall. According to Newton's 3rd law of motion, force F_1 exerted the molecule on face ABCDA is equal but opposite.

$$\text{Force} = F_1 = \frac{\text{change in momentum}}{\text{time taken}} = \frac{\Delta P}{\Delta t}$$

$$F_1 = \frac{2(mV_{1x})}{\frac{2L}{V_{1x}}} = \frac{mV_{1x}^2}{L}$$

Similarly, the forces due to all other molecules can be determined. Thus, the total x-directed F due to N number of molecules of the gas moving with velocities V_1, V_2, V_3,……………,V_n is:

$$F = F_1 + F_2 + F_3 + \ldots\ldots\ldots\ldots + F_n$$

$$F = \frac{mV_1^2\varkappa}{L} + \frac{mV_2^2\varkappa}{L} + \frac{mV_3^2\varkappa}{L} + \ldots\ldots\ldots\ldots + \frac{mV_n^2\varkappa}{L}$$

As pressure is normal force per unit area, hence pressure P on the face perpendicular to x-axis is:

$$P = \frac{F}{A} = \frac{F}{L^2}$$

$$P = \left(\frac{mV_1^2\varkappa}{L} + \frac{mV_2^2\varkappa}{L} + \frac{mV_3^2\varkappa}{L} + \ldots\ldots\ldots + \frac{mV_n^2\varkappa}{L}\right)/L^2$$

$$P = \frac{m}{L^3}(V_1^2\varkappa + V_2^2\varkappa + V_3^2\varkappa + \ldots\ldots\ldots + V_n^2\varkappa)\ldots\ldots\ldots\ldots(15.12)$$

The number of molecules in unit volume n_v is $\frac{N}{L^3}$. Where N is the total number of molecules. Therefore:

$L^3 = \frac{N}{n_v}$ and substituting this value in eq: (15.12)

$$P = mn_v\left(\frac{V_1^2\varkappa + V_2^2\varkappa + V_3^2\varkappa + \ldots\ldots\ldots + V_n^2\varkappa}{N}\right)\ldots\ldots\ldots (15.13)$$

Where mn_v is the mass per unit volume which we call density ρ and $\frac{V_1^2\varkappa + V_2^2\varkappa + V_3^2\varkappa + \ldots\ldots\ldots + V_n^2\varkappa}{N}$ is the average value of $\overline{V_x^2}$. for all the molecules in the container, we call this average square velocity $\overline{V_x^2}$. The square root of $\overline{V_x^2}$. is referred as V_{rms}. Eq: 15.13 can be written as:

$$P = \rho\overline{V_x^2} . \ \ldots\ldots\ldots\ldots\ldots\ldots (15.14)$$

The terms $\overline{V_x^2}$. is only one component of the total velocity. Since $\overline{V^2} = \overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}$ on the average. $\overline{V_x^2} = \overline{V_y^2} = \overline{V_z^2}$ due to randomness of the molecular motion.

$\overline{V^2} = 3\overline{V_x^2}$. and $\overline{V_x^2} = \frac{1}{3}\overline{V^2}$

Substituting this value into the above equation, we find that:

$$P = \frac{1}{3}\rho\overline{V^2} \ \ldots\ldots\ldots\ldots\ldots\ldots (15.15)$$

15.3.3 The relation between Kinetic Energy of Molecules and Temperature:

The relation between molecular Kinetic Energy and temperature can be derived by using eq: 15.15

$$P = \frac{1}{3}\rho\overline{V^2}$$

$$P = \frac{1}{3}mn_v\overline{V^2} \text{ since } \ell = (mn_v)\ldots\ldots\ldots\ldots (15.16)$$

Since n_v represents the number of molecules per unit volume $n_v = \frac{N}{V}$ Eq: (15.16) can be written as

$$PV = \frac{1}{3}m\frac{N}{V}\overline{V^2}\ldots\ldots\ldots\ldots (15.17)$$

Now we can compare the equation (15.17) with this ideal gas equation PV = nRT. The left-hand sides are the same. So, the two right-hand sides must also be equal.

$\frac{1}{3}\text{Nm}\,\overline{V^2} = \text{nRT}$ ………… (15.18)

Substituting $n = \frac{N}{N_A}$ and multiplying both sides by $\frac{3}{2}$, we obtain the relation.

$\frac{3}{2} \times \frac{1}{3}\text{Nm}\,\overline{V^2} = \frac{3}{2} \times \frac{N}{N_A}\text{RT}$

$\frac{1}{2}\text{m}\,\overline{V^2} = \left(\frac{3}{2}\right)\left(\frac{R}{N_A}\right)\text{T}$

Since $\frac{R}{N_A}$ = k (Boltzmann constant)

k= 1.38×10^{-23} J/Molecule k

Hence $\frac{1}{2}\text{m}\,\overline{V^2} = \frac{3}{2}\text{kT}$ …………… (15.19)

$\text{K.E}_{avg} = \frac{3}{2}$ kT

The mean translational Kinetic energy of a molecule of an ideal gas is proportional to the absolute temperature.

Hence, mean translational Kinetic energy of a molecule ∝ T.

DO YOU KNOW?

K (Boltzmann constant) is universal gas constant per molecule of a gas
R is a universal gas constant per mole of a gas.

Worked Example 15.4

The atoms in a gas have mean translational (K.E) equal to 5.0×10^{-21}J. Calculate the temperature of the gas in K and °C.

Solution:

Step 1: Write the known quantities and point out quantities to be found

Mean K.E = 5.0×10^{-21}J

K = 1.38×10^{-23}J/molecule. K

(i). T_k = ?

(ii). T_c = ?

Step 2: Write down the formula and rearrange if necessary.

Mean K.E = $\frac{3}{2}$kT

$T = \frac{\text{mean K.E}}{k} \times \frac{2}{3} = \frac{5.0\times10^{-21}}{1.38\times10^{-23}} \times \frac{2}{3}$

T = 242K

Thus, temperature in centigrade scale is:

T = K – 273

T = 242 – 273 = – 31°C.

Result: **(i).** 242K, **(ii).** –31°C.

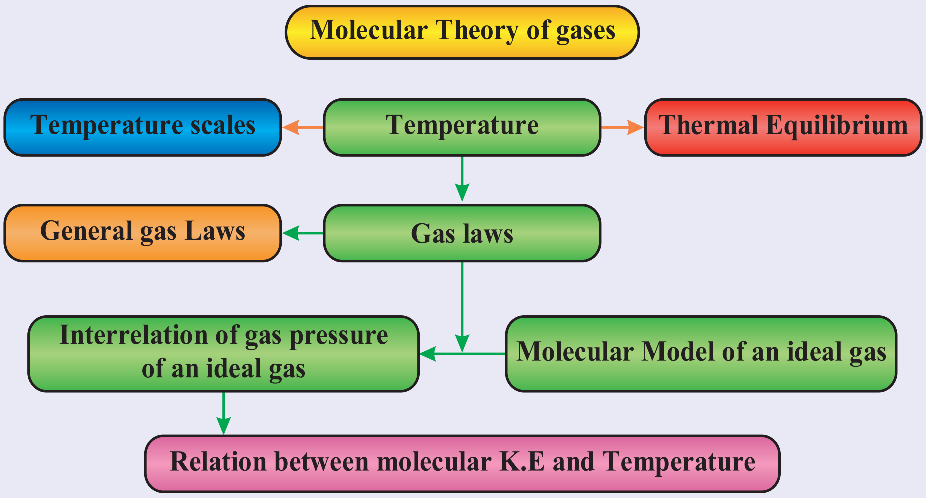
Self-Assessment Questions:

1. Describe the motion of molecules in a gas according to the kinetic molecular theory.
2. What is the relationship between temperature and molecular motion in a gas?

SUMMARY

- ✓ Temperature is a measure of the average kinetic energy of particles in a substance.
- ✓ The triple point of water is the temperature (0.01°C) and pressure (611.73 Pascal's) at which water coexists in all three phases (solid, liquid, and gas).
- ✓ The general gas law states that PV = nRT, relating pressure, volume, number of moles, and temperature of a gas.
- ✓ Boyle's Law: $P_1V_1 = P_2V_2$ (pressure and volume are inversely proportional at constant temperature)
- ✓ $V_1/T_1 = V_2/T_2$ (volume and temperature are directly proportional at constant pressure)
- ✓ V = RT/P (volume and number of moles are directly proportional at constant temperature and pressure)
- ✓ KTG (Kinetic Theory of Gases) Postulates:
    1. Gases consist of tiny particles (molecules) that are in constant motion.
    2. The molecules are very small compared to the distance between them.
    3. The molecules are in constant random motion.
    4. The molecules collide with each other and the container walls.
- ✓ Molecular movement and pressure: The pressure exerted by a gas is caused by the collisions of molecules with the container walls, and is related to the temperature and volume of the gas by the equation P = (2/3) nkT.
- ✓ The kinetic energy of molecules is directly proportional to the temperature of the gas, and is described by the equation KE = (3/2) kT.

CONCEPT MAP



EXERCISE

Section (A): **Multiple Choice Questions (MCQs)**
Choose the correct answer:

1. The relationship between temperature and average kinetic energy of particles in a gas is:
   (a) temperature is inversely proportional to average kinetic energy
   (b) temperature is directly proportional to average kinetic energy
   (c) temperature is independent of average kinetic energy
   (d) temperature is proportional to the square of average kinetic energy

2. Standard conditions of temperature and pressure (STP) refer to a gas at:
   (a) 0°C and 1 atm (b) 20°C and 1 atm
   (c) 25°C and 1 atm (d) 30°C and 101.3 kPa (1 atm)

3. If the temperature is kept constant and the volume of a gas is doubled, then pressure of a gas is:
   (a) Reduced to $\frac{1}{2}$ of the original value. (b) Doubled
   (c) Reduced to $\frac{1}{4}$ of the original value (d) Quadrupled

4. The Avogadro's number is the number of molecules in:
   (a) One mole of a substance
   (b) One kg of a substance
   (c) One m^3 of a gas
   (d) One kilogram of hydrogen gas

5. Mean translational K.E. per molecule of an ideal at temperature T is:
   (a) $\frac{3}{2}kT$
   (b) $\frac{1}{2}kT$
   (c) $\frac{2}{3}kT$
   (d) kT^4

6. The normal human body temperature is:
   (a) 98.6°F (37°C)
   (b) 99.6°F (37.4°C)
   (c) 100.4°F (38°C)
   (d) 101°F (38.3°C)

7. The pressure P, the density ρ and the average speed of molecules of an ideal gas are related by the equation.
   (a) $P = \frac{2}{3} m\ell^2$
   (b) $P = \frac{1}{3} m\ell^2$
   (c) $P = \frac{1}{3}\rho\overline{V^2}$
   (d) $P = \frac{2}{3}\ell^2$

8. In air at S.T.P, the average speed of the:
   (a) Oxygen molecules is greater than Nitrogen molecules
   (b) Nitrogen molecules is greater than Oxygen molecules
   (c) Oxygen molecules is approximately equal to Nitrogen molecules
   (d) Helium atoms is greater than both Oxygen and Nitrogen molecules

9. If the absolute temperature of a gas is increased 3 times, the rms velocity of the molecules will be:
   (a) 3 times
   (b) 9 times
   (c) $\sqrt{3}$ times
   (d) $\frac{1}{3}$ times

10. A gas is enclosed in an isolated container which is placed on a fast-moving train uniformly. The temperature of the gas:
    (a) Increases due to the motion of the train
    (b) Decreases due to the motion of the train
    (c) Remains constant
    (d) Fluctuates, depending on the train's speed and direction

Section (B): CRQs (Short Answered Questions):

15.1. Why the earth is not in thermal equilibrium with the sun?

15.2. Describe the relationship between temperature and kinetic energy of molecules.

15.3. It is observed that when mercury in glass thermometer is put in a flame, the column of mercury first descends and then rises. Explain.

15.4. What is standard temperature, pressure?

15.5. A thermometer is placed in direct sun light. What will it read the temperature?

15.6. The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen. Why?

15.7. When a sealed thermos bottle full of coffee is shaken, what are the changes occur?

15.9. How does the Kinetic theory account for the following observed facts:
(a) A gas exerts pressure
(b) The pressure of a gas depends upon its temperature.

15.10 Calculate the average speed of an air molecule at room temperature (20°C) and compare it to the speed of sound in air (330 m/s).

Section (C): **ERQs (Long Answered Questions):**

15.1. What is temperature? Explain the scales of temperature in detail.

15.2. Define and explain Boyle's law, Charles's and Avogadro's law.

15.3. Derive general gas law by making use of gas laws.

15.4. Describe the molecular movement causes the pressure exerted by gas, derive pressure equation.

15.5. Interpret mathematically that temperature is a measure of average translational K.E of the molecules of a gas.

Section (D): **Numerical:**

15.1. The freezing point of mercury is –39°C.Convert it into °F and the comfort level temperature of 20° into Kelvin. **(Ans: –38.2°F, 293K)**

15.2. The boiling point of liquid nitrogen is -321°F. Change it into equivalent Kelvin temperature. **(Ans: 77K)**

15.3. Calculate the volume occupied by a gram-mole of a gas at 0°C and a pressure of 1.0 atmosphere. **(Ans: 22.4 liters/mole)**

15.4. An air storage tank whose volume is 112 liters contain 3kg of air at a pressure of 18 atmospheres. How much air would have to be forced into the tank to increase the pressure to 21 atmospheres, assuming no change in temperature? **(Ans: 0.5kg)**

15.5. A balloon contains 0.04m^3 of air at a pressure of 120KPa. Calculate the pressure required to reduce its volume to 0.025 m^3 at constant temperature. **(Ans: 1.9×10^5Pa)**

15.6. The molar mass of nitrogen gas N_2 is 28gmol^{-1}. For 100g of nitrogen, calculate.
(a) the number of moles. **(Ans: (a) 3.57 mole)**
(b) the volume occupied at room temperature (20°C) and pressure of 1.01×10^5 Pa.
(Ans: (b) 0.086m^3 or 86dm^3)

15.7. A sample of a gas contains 3.0×10^{24} atoms. Calculate the volume of the gas at a temperature of 300K and a pressure of 120K Pa. **(Ans: 0.104m^3)**

15.8. Calculate the root mean square speed of hydrogen molecules at 0°C and 1.0 atm pressure. Assuming hydrogen to be an ideal gas. The density of hydrogen is 8.99×10^{-2} kg/m^3. **(Ans: 1835.86ms^{-1})**

15.9. Calculate the root mean square speed of hydrogen molecule at 500K (mass of proton = 1.67×10^{-27}kg and K = 1.38×10^{-23}J/molecule.-K) **(Ans: 2489.49ms^{-1})**

15.10. (a) Determine the average value of the Kinetic energy of the particles of an ideal gas at 10°C and at 40°C. **(Ans: 5.86x10^{-21}J, 6.48x10^{-21}J)**
(b) What is the Kinetic energy per mole of an ideal gas at thesetemperatures?
(Ans: 3526.57J, 3901J)